Fenske's equation: derivation

Multicomponent distillation column design

The method is applied to the light key (LK) and to the heavy key (HK) components, under column's condition of total reflux. The scheme for such a column is here below sketched.

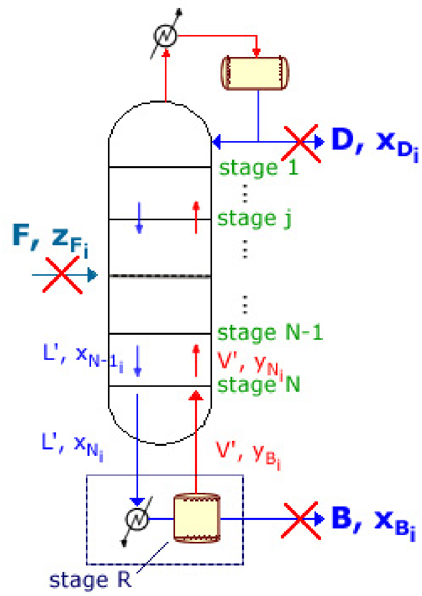

Figure 1: Fenske's distillation column with total reflux

Step 1

- Equilibrium equations at the reboiler (stage R):

$$(y_{LK})_R = k_{LK} \cdot (x_{LK})_R \tag{1}$$

$$(y_{HK})_R = k_{HK} \cdot (x_{HK})_R \tag{2}$$

Dividing (1) and (2) term by term, yields:

$$\left(\frac{y_{LK}}{y_{HK}}\right)_R = \alpha_R \cdot \left(\frac{x_{LK}}{x_{HK}}\right)_R \tag{3}$$

where

$$\alpha_R = \frac{k_{LK}}{k_{HK}} \tag{4}$$

Step 2

- Mass balance around the reboiler.

  Overall mass balance:

$$V' = L' \tag{5}$$

mass balances on the key components:

$$V' \cdot (y_{LK})_R = L' \cdot (x_{LK})_N \tag{6}$$

$$V' \cdot (y_{HK})_R = L' \cdot (x_{HK})_N \tag{7}$$

substituting eq. (5) in eqs. (6) and (7), yields:

$$(y_{LK})_R = (x_{LK})_N \tag{8}$$

$$(y_{HK})_R = (x_{HK})_N \tag{9}$$

Step 3

- The liquid composition of the reboiler stage, which is a given specification, is correlated with the liquid composition of the previous stage N, which can be consequently calculated.

  Dividing eq. (8) and (9) term by term and substituting eq. (3), yields:

$$\left(\frac{x_{LK}}{x_{HK}}\right)_N = \alpha_R \cdot \left(\frac{x_{LK}}{x_{HK}}\right)_R \tag{10}$$

Step 4

- This iteration proceeds backwards from stage N, to N-1, to N-2, and so on so forth until it is reached the liquid composition of the distillate stage (stage D), which is known as well.

  For the stage N-1, it is:

$$\left(\frac{x_{LK}}{x_{HK}}\right)_{N-1} = \alpha_N \cdot \alpha_R \cdot \left(\frac{x_{LK}}{x_{HK}}\right)_R \tag{11}$$

...until stage D for which it can be written:

$$\left(\frac{x_{LK}}{x_{HK}}\right)_D = \alpha_1 \cdot \alpha_2 \cdot ...\alpha_{N-1} \cdot \alpha_N \cdot \alpha_R \cdot \left(\frac{x_{LK}}{x_{HK}}\right)_R \quad (12)$$

assuming an average value of α valid for all the stages, eq. (12) can be written as:

$$\left(\frac{x_{LK}}{x_{HK}}\right)_D = \alpha_{aver.}^{N_{min}} \cdot \left(\frac{x_{LK}}{x_{HK}}\right)_R \quad (13)$$

in eq. (13) the only unknown is the minimum number of stage (included the reboiler), which can be easily derived.