Localization | the Kalman Filter and Markov Approach

Autonomous Mobile Robots

https://edge.edx.org/courses/course-v1%3AETHZ%2BAMRx_Internal1_2015%2B2015_T1/

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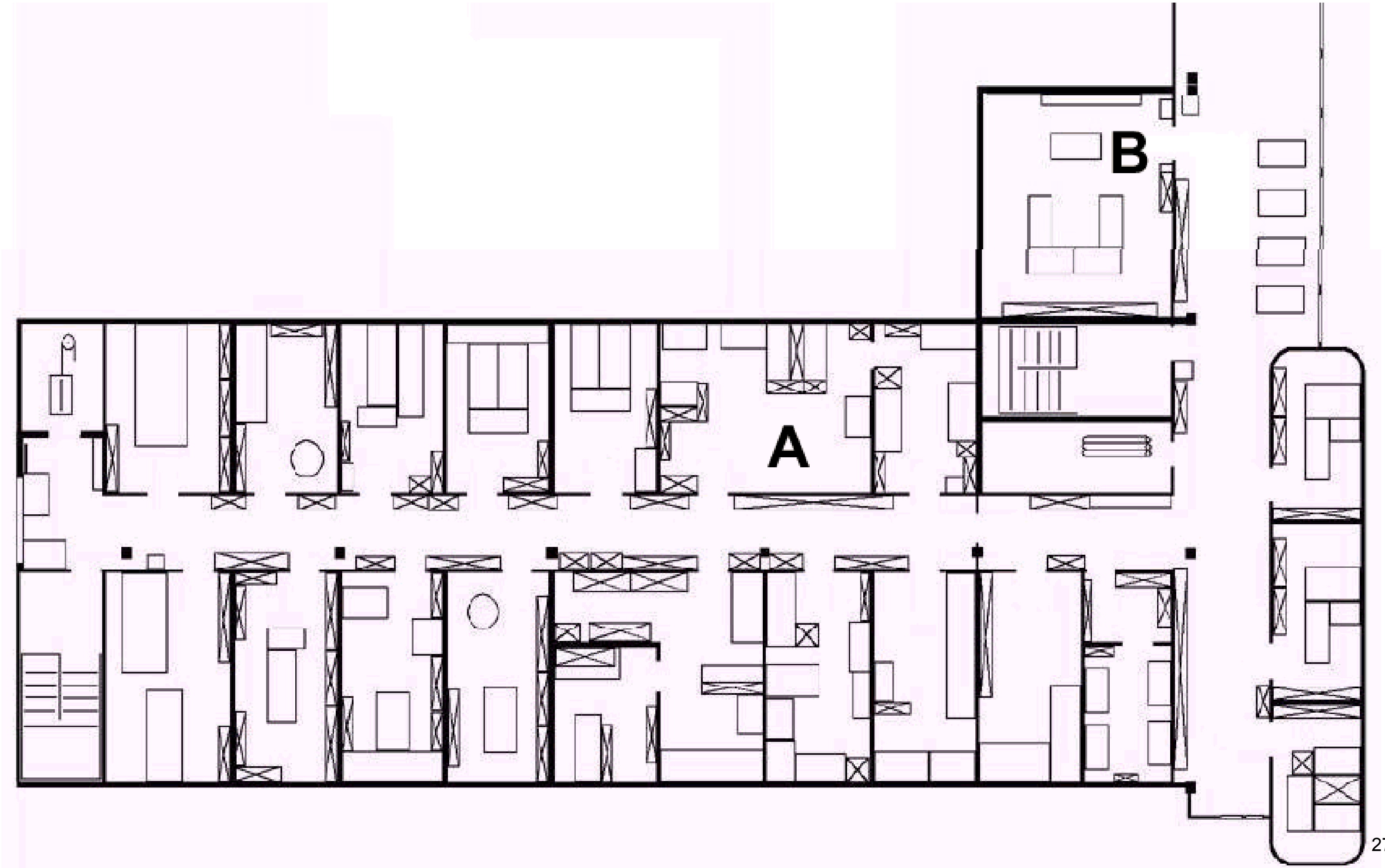
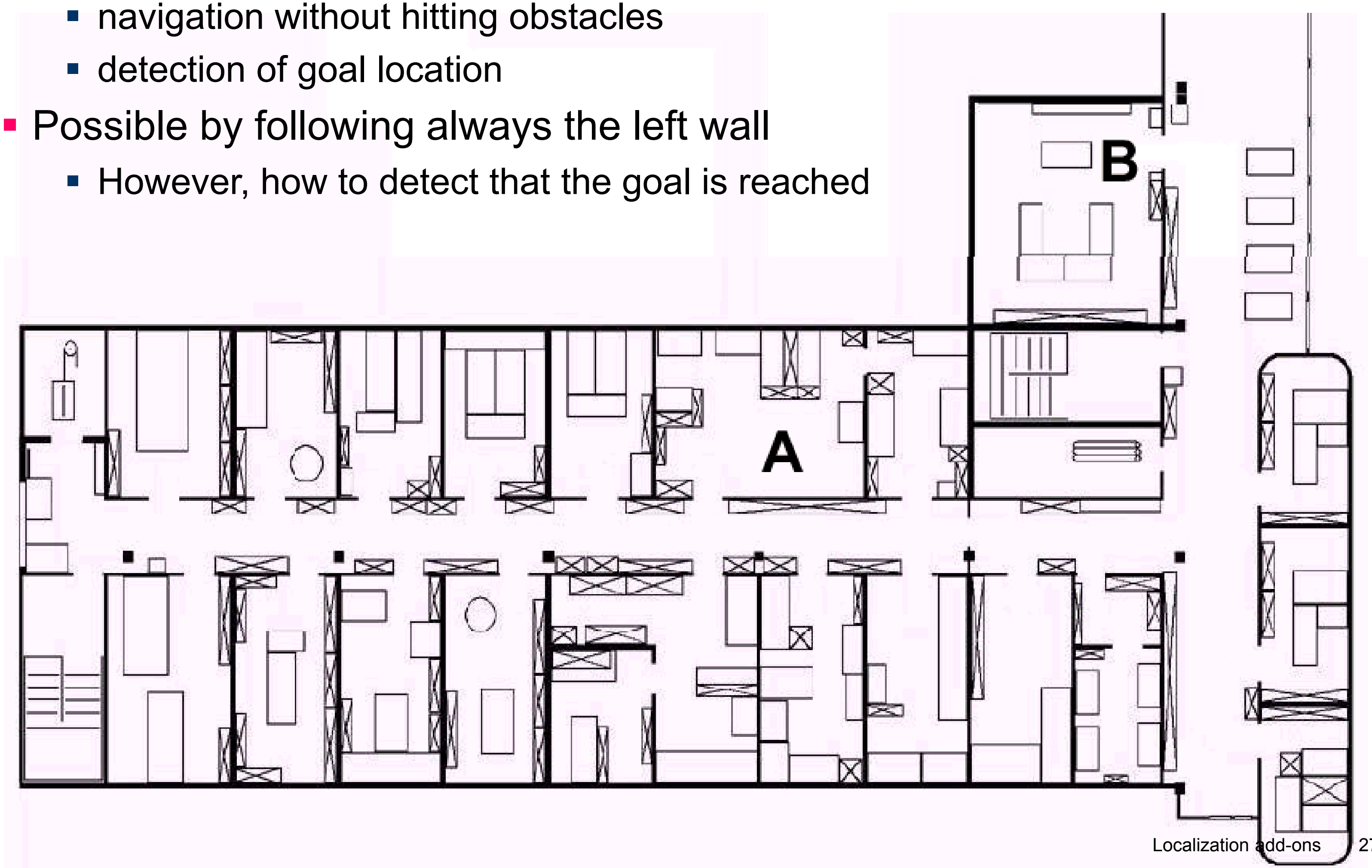
Introduction
Do we need to localize or not?

- To go from A to B, does the robot need to know where it



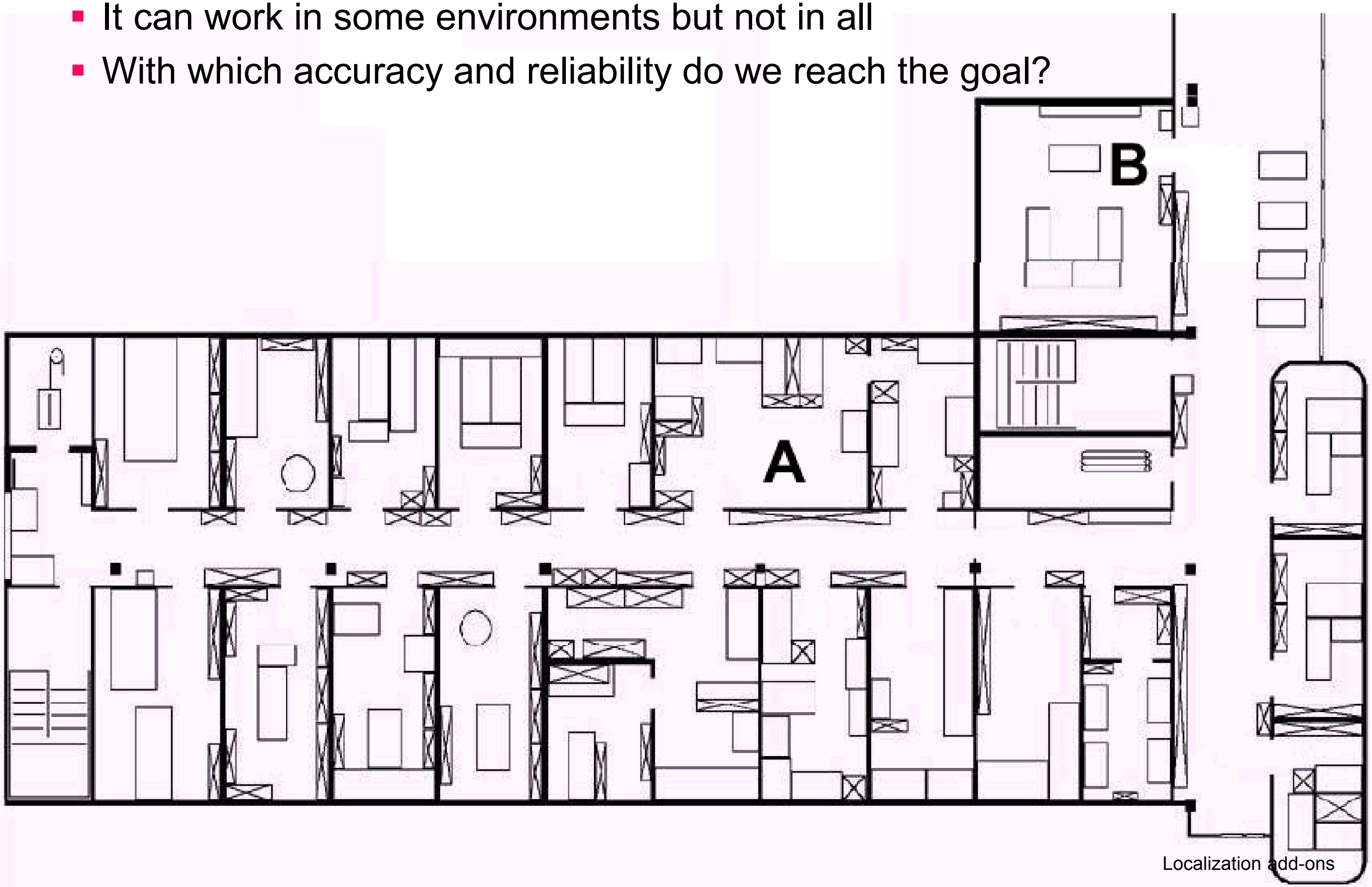
Introduction
Do we need to localize or not?

- How to navigate between A and B
  - navigation without hitting obstacles
  - detection of goal location
- Possible by following always the left wall
  - However, how to detect that the goal is reached



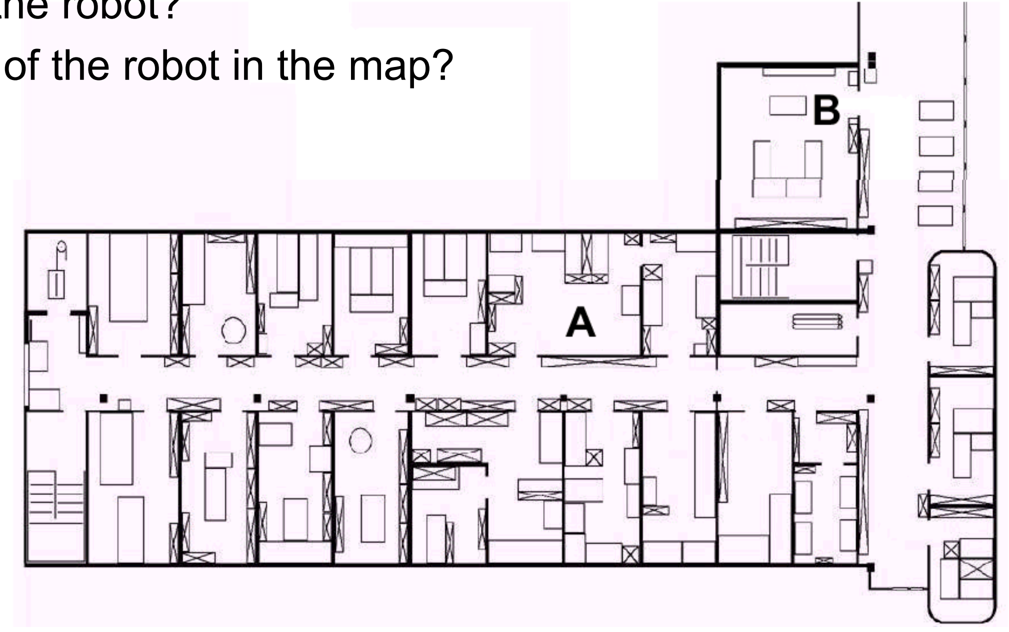
Introduction
Do we need to localize or not?

- Following the left wall is an example of “behavior based navigation”
  - It can work in some environments but not in all
  - With which accuracy and reliability do we reach the goal?



Introduction
Do we need to localize or not?

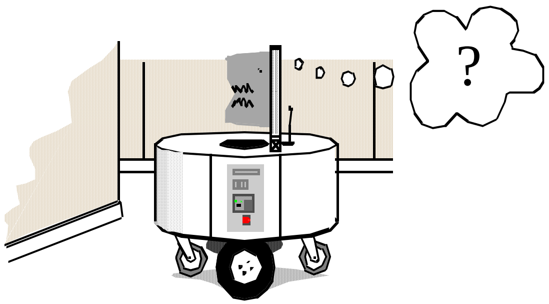
- As opposed to behavior based navigation is "map based navigation"
    - Assuming that the map is known, at every time step the robot has to know where it is. How?
        - If we know the start position, we can use wheel odometry or dead reckoning. Is this enough? What else can we use?
- But how do we represent the map for the robot?
- And how do we represent the position of the robot in the map?



Introduction
Definitions

- Global localization
  - The robot is not told its initial position
  - Its position must be estimated from scratch

- Position Tracking
  - A robot knows its initial position and “only” has to accommodate small errors in its odometry as it moves

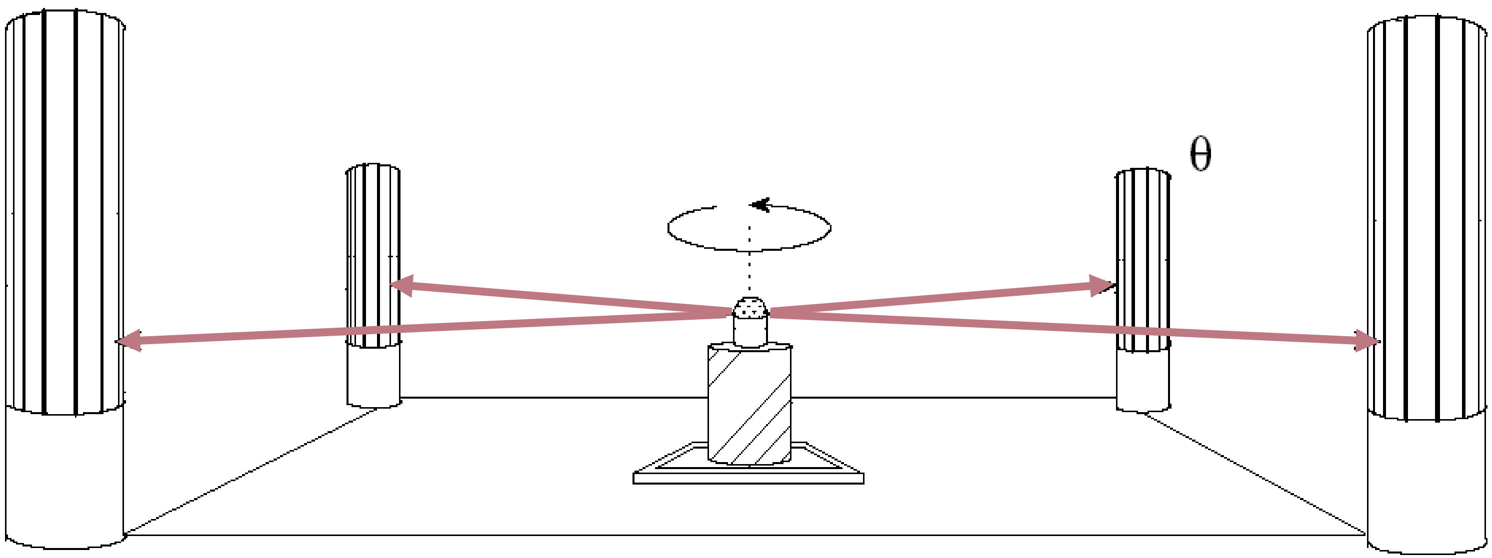


Introduction
How to localize?

- Localization based on external sensors, beacons or landmarks
- Odometry
- Map Based Localization
  - without external sensors or artificial landmarks
  - just use robot onboard sensors
  - Example: Probabilistic Map Based Localization

Introduction
Beacon Based Localization

- Triangulation
  - Ex 1: Poles with highly reflective surface and a laser for detecting them
  - Ex 2: Coloured beacons and an omnidirectional camera for detecting them (example: RoboCup or autonomous robots in tennis fields)



Introduction
Beacon Based Localization

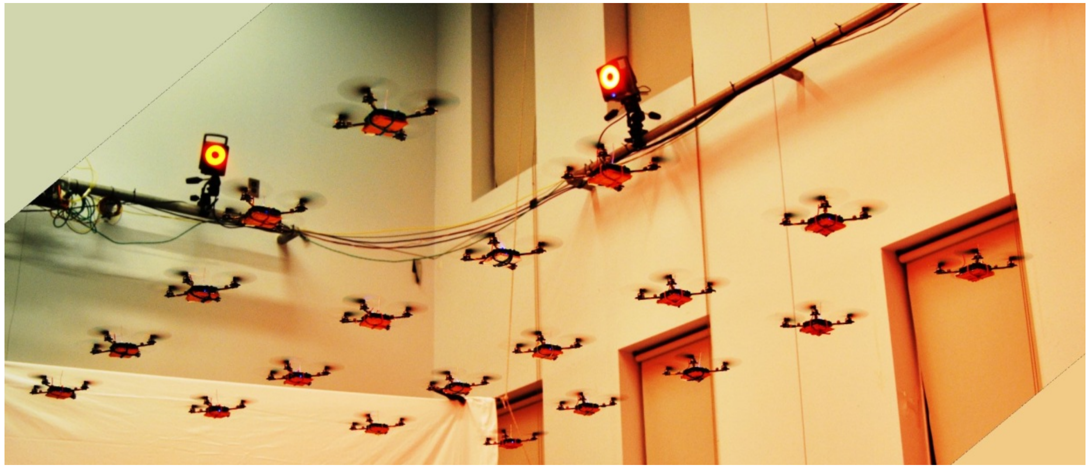
- KIVA Systems, Boston (MA) (acquired by Amazon in 2011)

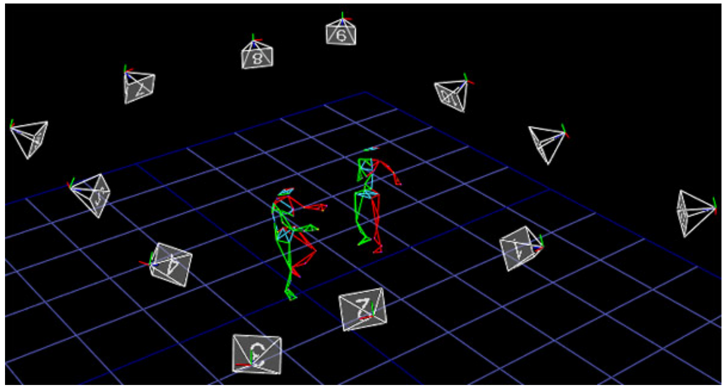
Unique marker with known absolute 2D position in the map

Prof. Raff D'Andrea, ETH

Introduction
Motion Capture Systems

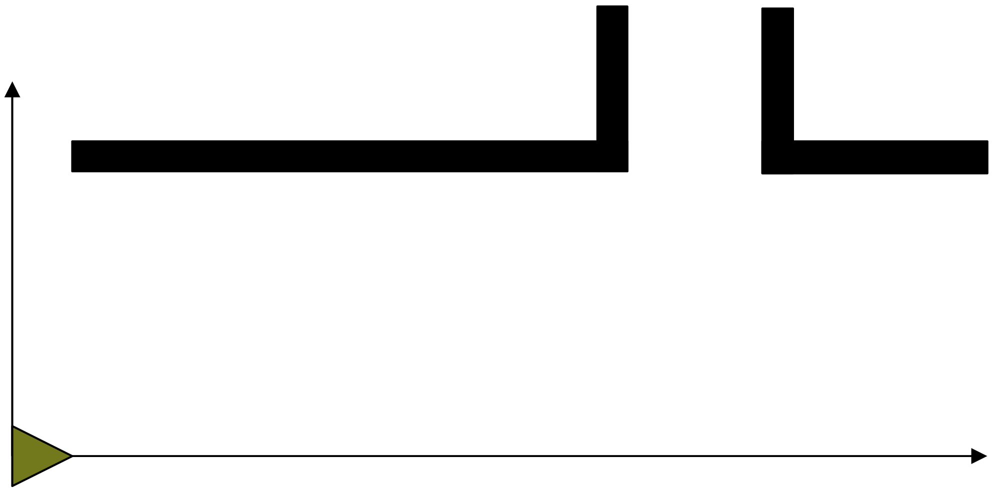
- High resolution (from VGA up to 16 Mpixels)
- Very high frame rate (several hundreds of Hz)
- Good for ground truth reference and multi-robot control strategies
- Popular brands:
  - VICON (10kCHF per camera),
  - OptiTrack (2kCHF per camera)





Introduction
Map-based localization

- Consider a mobile robot moving in a known environment.



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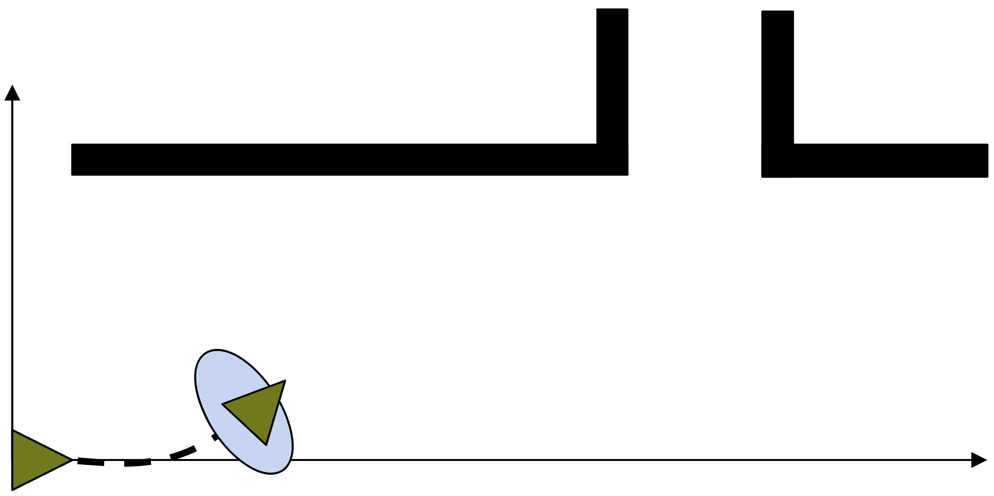
Map-based localization

- Consider a mobile robot moving in a known environment.
- As it starts to move, say from a precisely known location, it can keep track of its motion using odometry.



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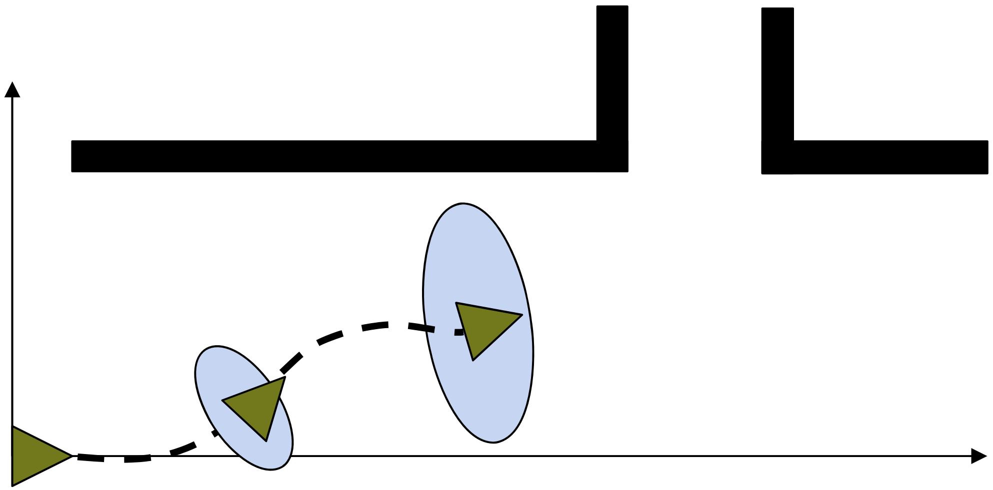

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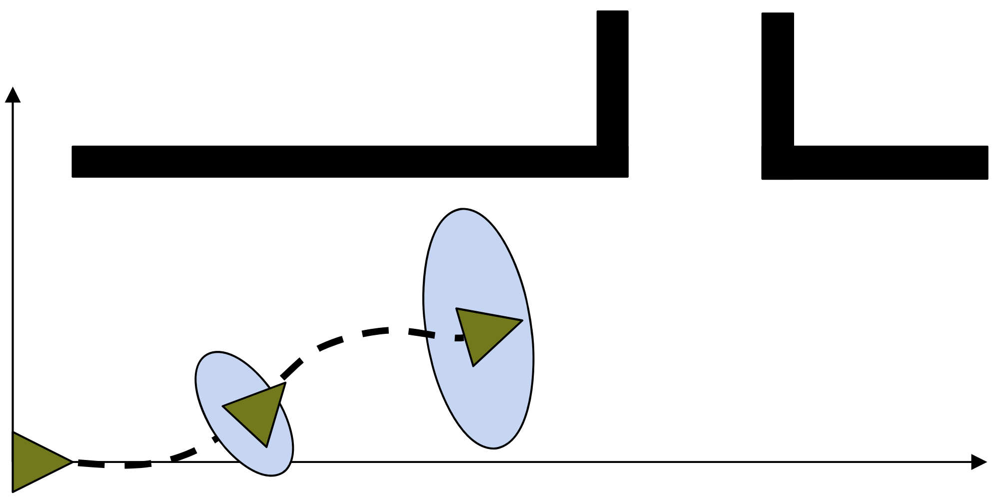
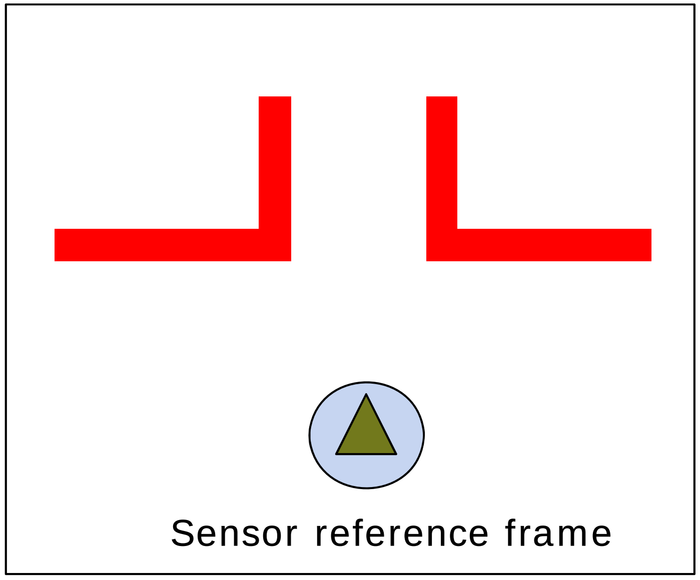
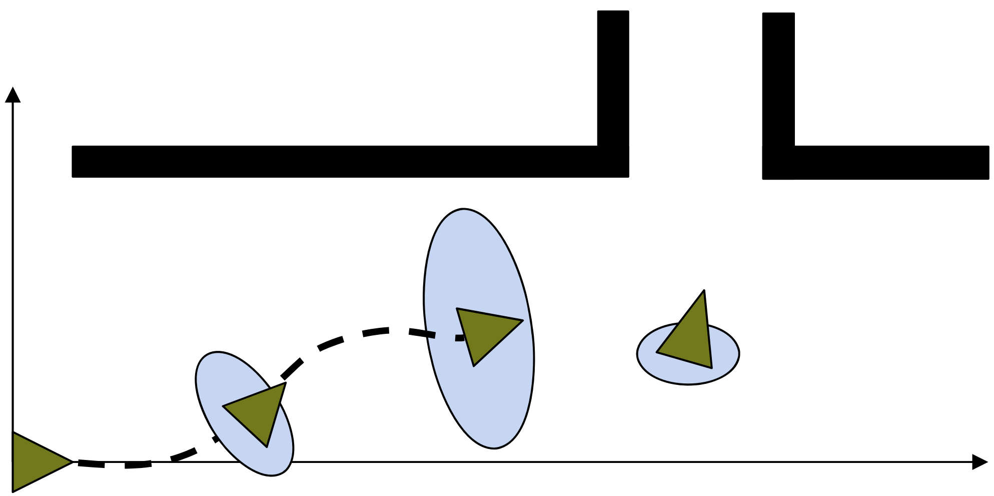
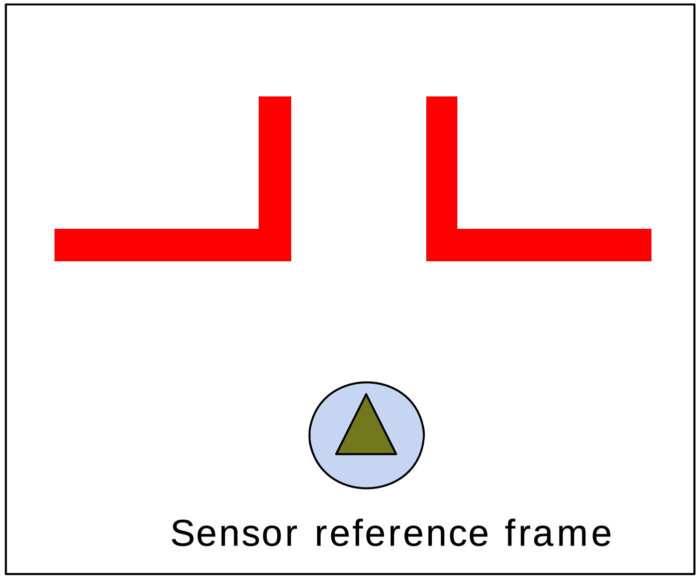




Introduction
Map-based localization

- Consider a mobile robot moving in a known environment.
- As it starts to move, say from a precisely known location, it can keep track of its motion using odometry.
- The robot makes an observation and updates its position and uncertainty





Ingredients
Probabilistic Map-based localization

- Probability theory → error propagation, sensor fusion
- Belief representation (map/position) → discrete / continuous
- Motion model → odometry model
- Sensing → measurement model

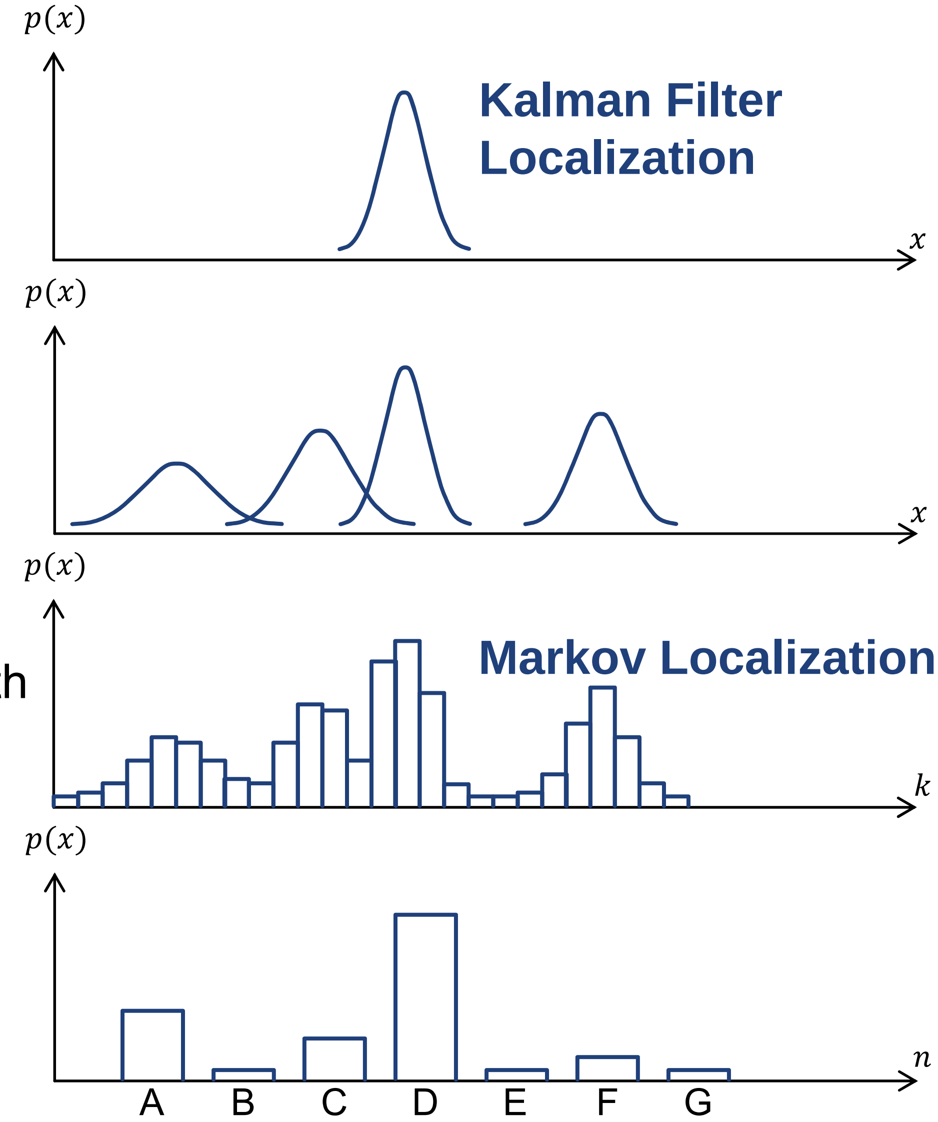
Probabilistic localization belief representation

a) Continuous map with single hypothesis probability distribution $p(x)$

b) Continuous map with multiple hypotheses probability distribution $p(x)$

c) Discretized metric map (grid k) with probability distribution $p(k)$

d) Discretized topological map (nodes n) with probability distribution $p(n)$



Belief Representation
Characteristics

- Continuous
  - Precision bound by sensor data
  - Typically single hypothesis pose estimate
  - Lost when diverging (for single hypothesis)
  - Compact representation and typically reasonable in processing power.

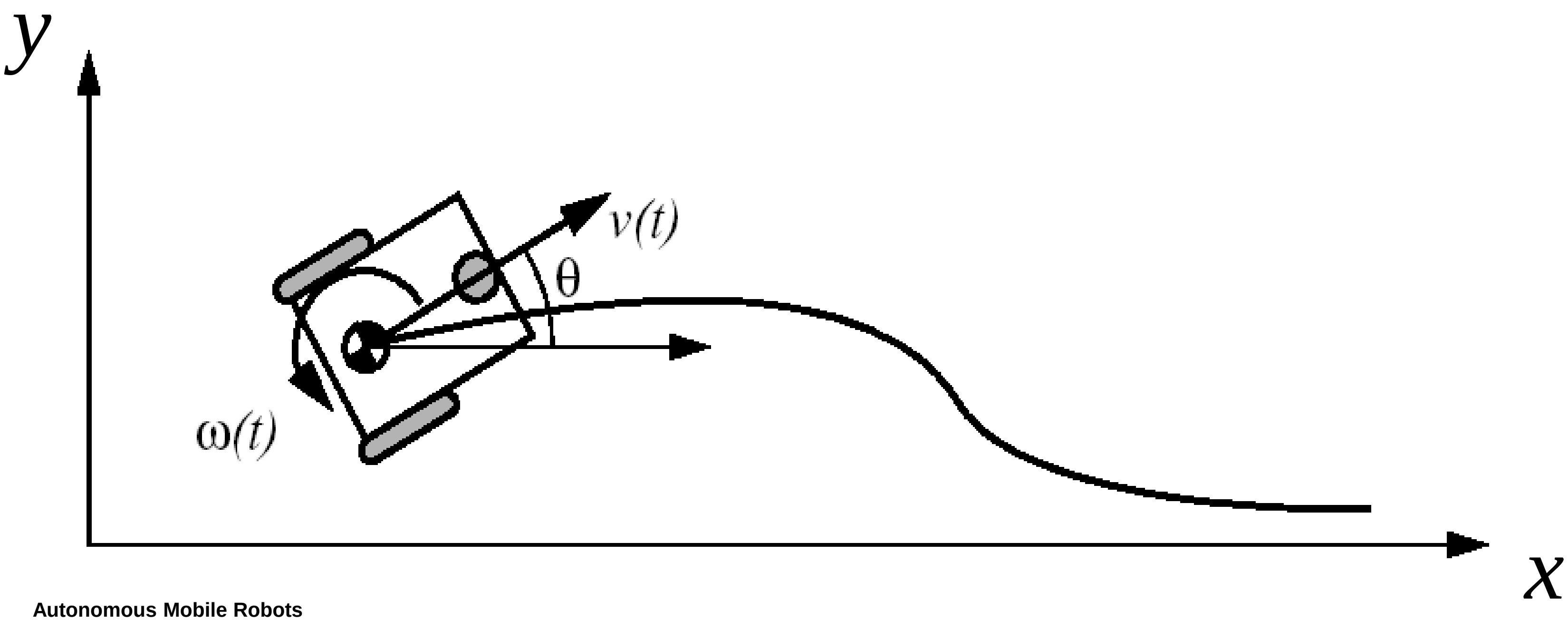
- Discrete
  - Precision bound by resolution of discretisation
  - Typically multiple hypothesis pose estimate
  - Never lost (when diverges converges to another cell)
  - Important memory and processing power needed. (not the case for topological maps)

Odometry

- Definition
  - **Dead reckoning** (also **deduced reckoning** or **odometry**) is the process of calculating vehicle's current position by using a previously determined position and estimated speeds over the elapsed time
- Robot motion is recovered by integrating proprioceptive sensor velocities readings
  - Pros: Straightforward
  - Cons: Errors are integrated -> unbound
- Heading sensors (e.g., gyroscope) help to reduce the accumulated errors but drift remains

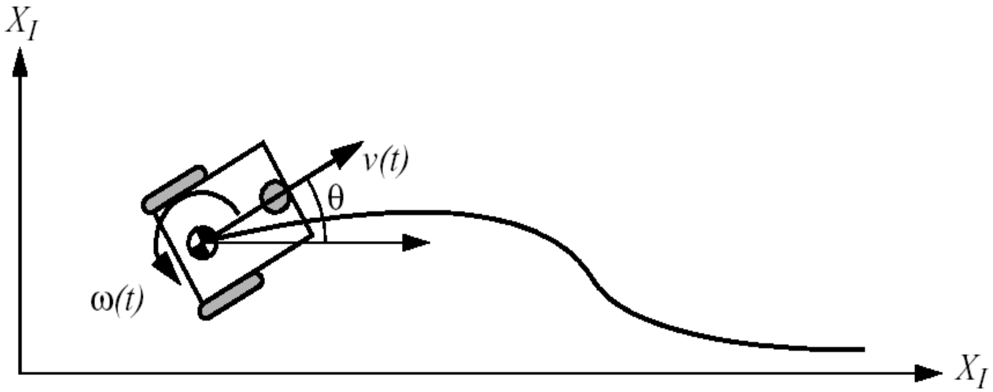
Odometry
The Differential Drive Robot (1)

$$x = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} \qquad \hat{x}_t = x_{t-1} + \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta\theta \end{bmatrix} = f(x_{t-1}, u_t)$$



Odometry
Wheel Odometry

- Kinematics



$$\hat{x}_t = f(x_{t-1}, u_t) = \begin{bmatrix} x_{t-1} \\ y_{t-1} \\ \theta_{t-1} \end{bmatrix} + \begin{bmatrix} \Delta s \cos(\theta + \frac{\Delta\theta}{2}) \\ \Delta s \sin(\theta + \frac{\Delta\theta}{2}) \\ \Delta\theta \end{bmatrix}$$

→ This term comes from the application of the Instantaneous Center of Rotation

Can you demonstrate these equations?

$$\Delta s = \frac{\Delta s_r + \Delta s_l}{2}$$

$$\Delta\theta = \frac{\Delta s_r - \Delta s_l}{b}$$

Odometry
Odometric Error Propagation

- Error model

$$P_t = F_{x_{t-1}} \cdot \Sigma_{x_{t-1}} \cdot F_{x_{t-1}}{}^T + F_{\Delta S} \cdot \Sigma_{\Delta S} \cdot F_{\Delta S}{}^T$$

$$\Sigma_{\Delta S} = \begin{bmatrix} k_r|\Delta s_r| & 0 \\ 0 & k_l|\Delta s_l| \end{bmatrix}$$

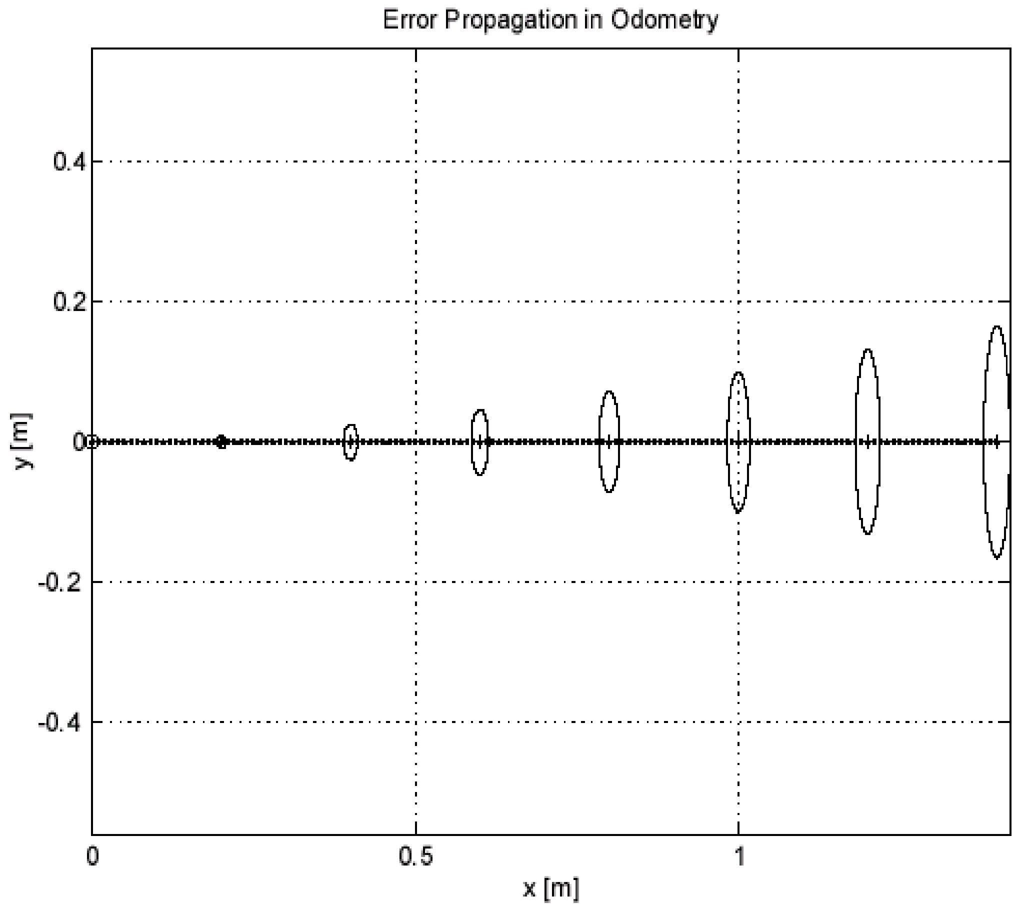
$$F_{x_{t-1}} = \nabla f_{x_{t-1}} = \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial f}{\partial \theta} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\Delta s \sin(\theta + \Delta\theta/2) \\ 0 & 1 & \Delta s \cos(\theta + \Delta\theta/2) \\ 0 & 0 & 1 \end{bmatrix}$$

$$F_{\Delta S} = \begin{bmatrix} \frac{1}{2}\cos\left(\theta + \frac{\Delta\theta}{2}\right) - \frac{\Delta s}{2b}\sin\left(\theta + \frac{\Delta\theta}{2}\right) & \frac{1}{2}\cos\left(\theta + \frac{\Delta\theta}{2}\right) + \frac{\Delta s}{2b}\sin\left(\theta + \frac{\Delta\theta}{2}\right) \\ \frac{1}{2}\sin\left(\theta + \frac{\Delta\theta}{2}\right) + \frac{\Delta s}{2b}\cos\left(\theta + \frac{\Delta\theta}{2}\right) & \frac{1}{2}\sin\left(\theta + \frac{\Delta\theta}{2}\right) - \frac{\Delta s}{2b}\cos\left(\theta + \frac{\Delta\theta}{2}\right) \\ \frac{1}{b} & -\frac{1}{b} \end{bmatrix}$$

Odometry

Growth of Pose uncertainty for Straight Line Movement

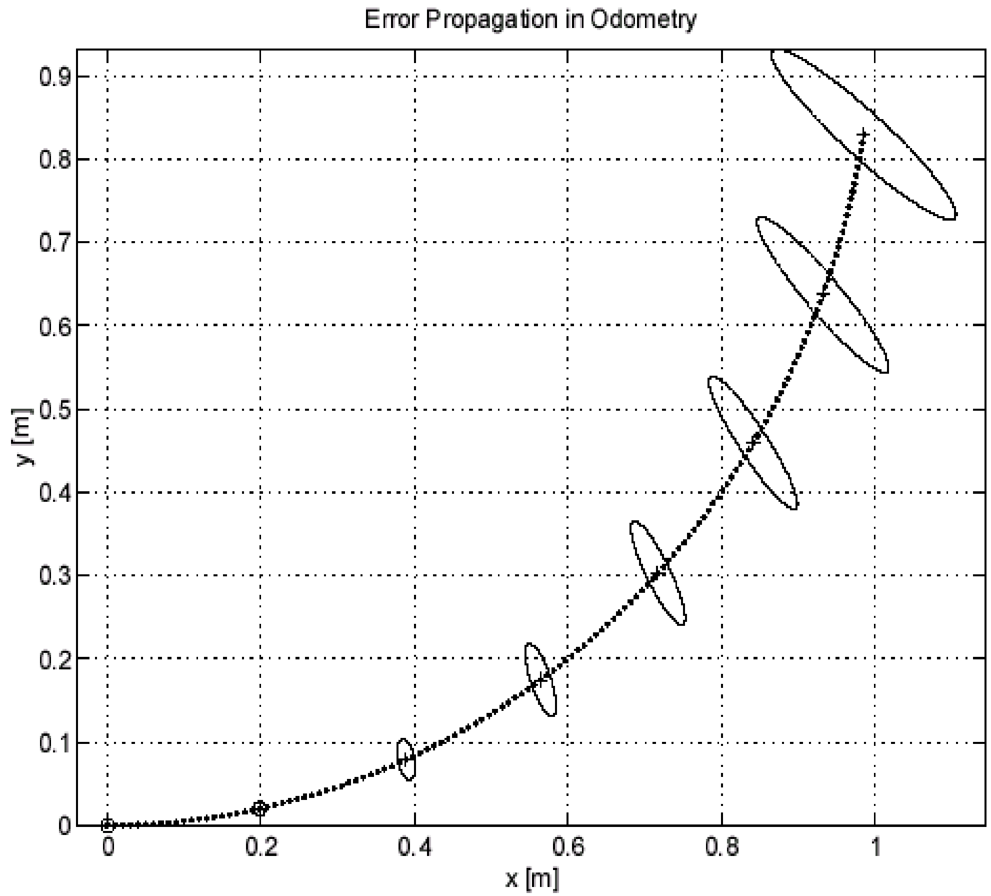
- Note: Errors perpendicular to the direction of movement are growing much faster!



Odometry

Growth of Pose uncertainty for Movement on a Circle

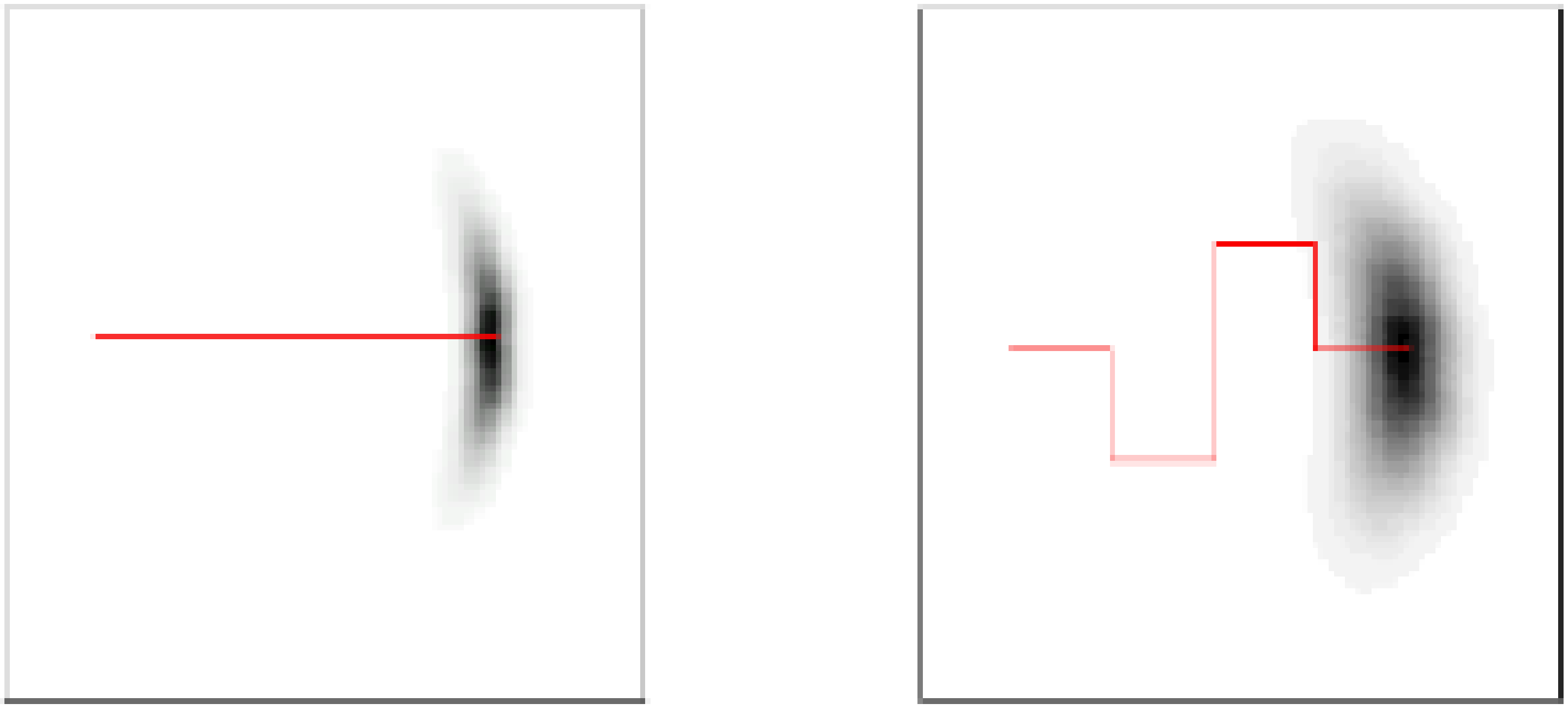
- Note: Errors ellipse does not remain perpendicular to the direction of movement!



Odometry
Example of non-Gaussian error model

- Note: Errors are not shaped like ellipses!

Courtesy AI Lab, Stanford



[Fox, Thrun, Burgard, Dellaert, 2000]

Odometry
Error sources

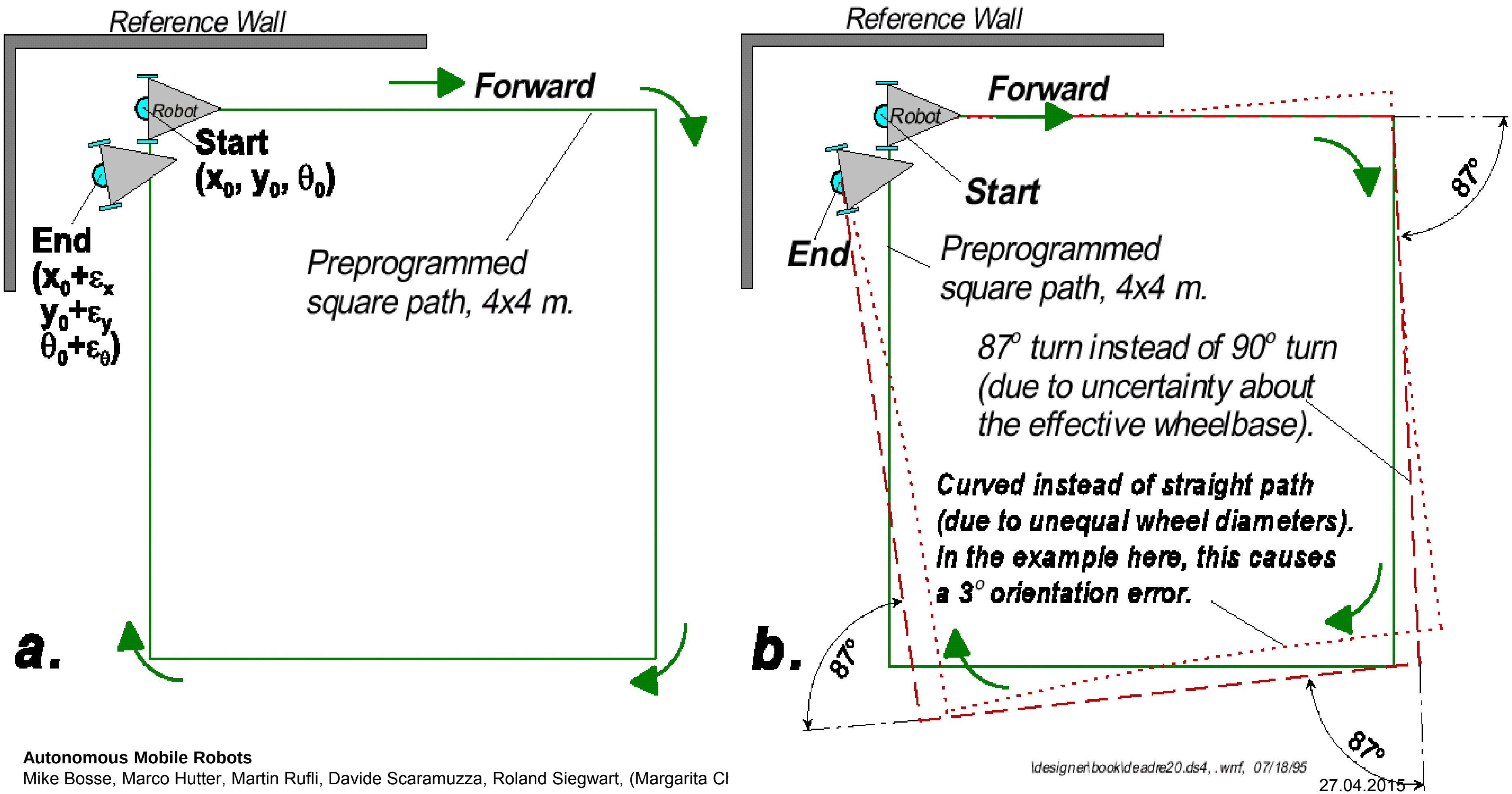
Deterministic (Systematic) ⟷ Non-Deterministic (Non-Systematic)

- **Deterministic** errors can be eliminated by proper **calibration** of the system.
- **Non-Deterministic** errors are **random errors**. They have to be described by **error models** and will always lead to uncertain position estimate.

- Major Error Sources in Odometry:
  - Limited resolution during integration (time increments, measurement resolution)
  - Misalignment of the wheels (deterministic)
  - Unequal wheel diameter (deterministic)
  - Variation in the contact point of the wheel (non deterministic)
  - Unequal floor contact (slippage, non planar …) (non deterministic)

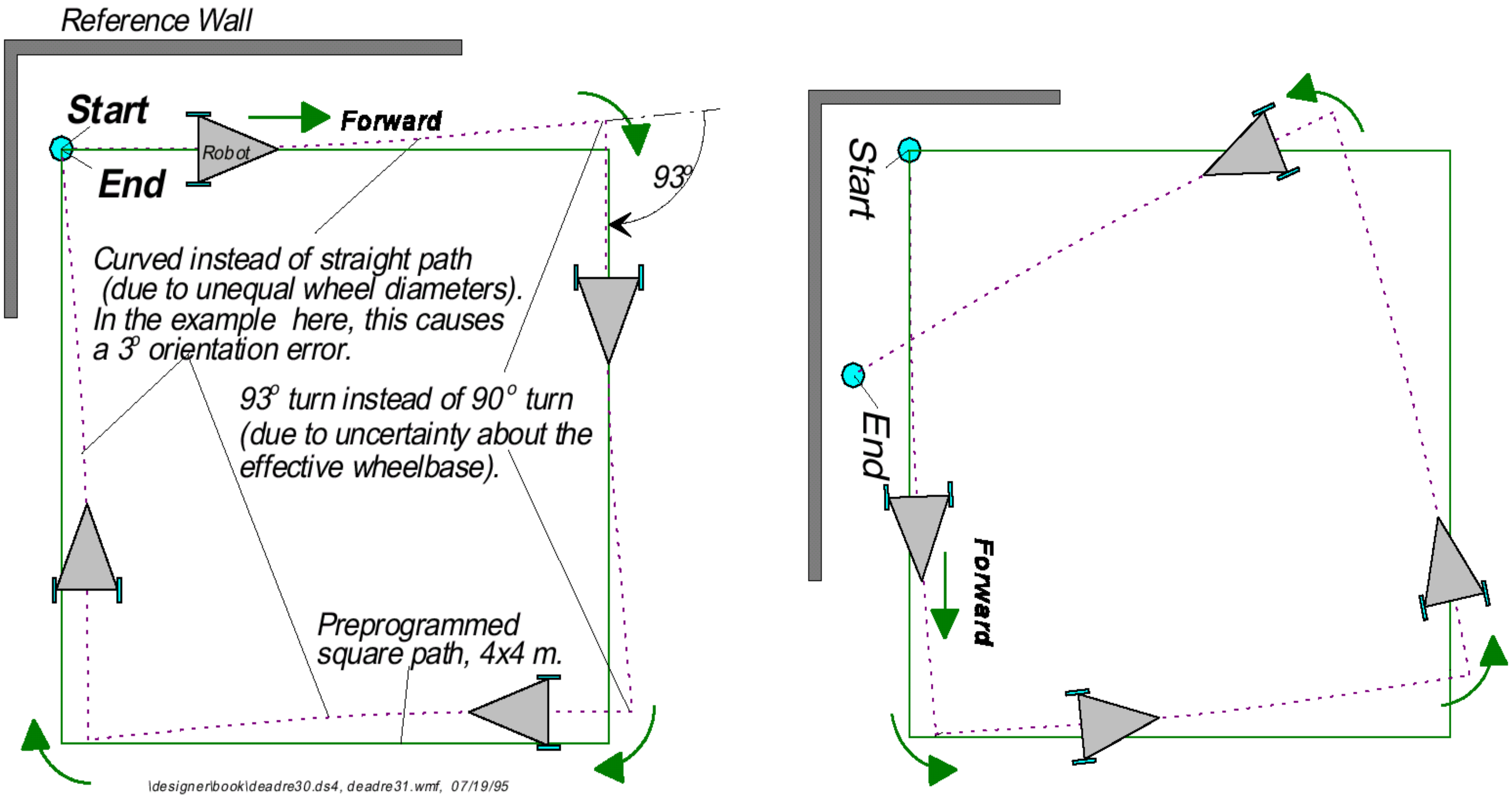
Odometry
Calibration of systematic errors [Borenstein 1996]

- The unidirectional square path experiment



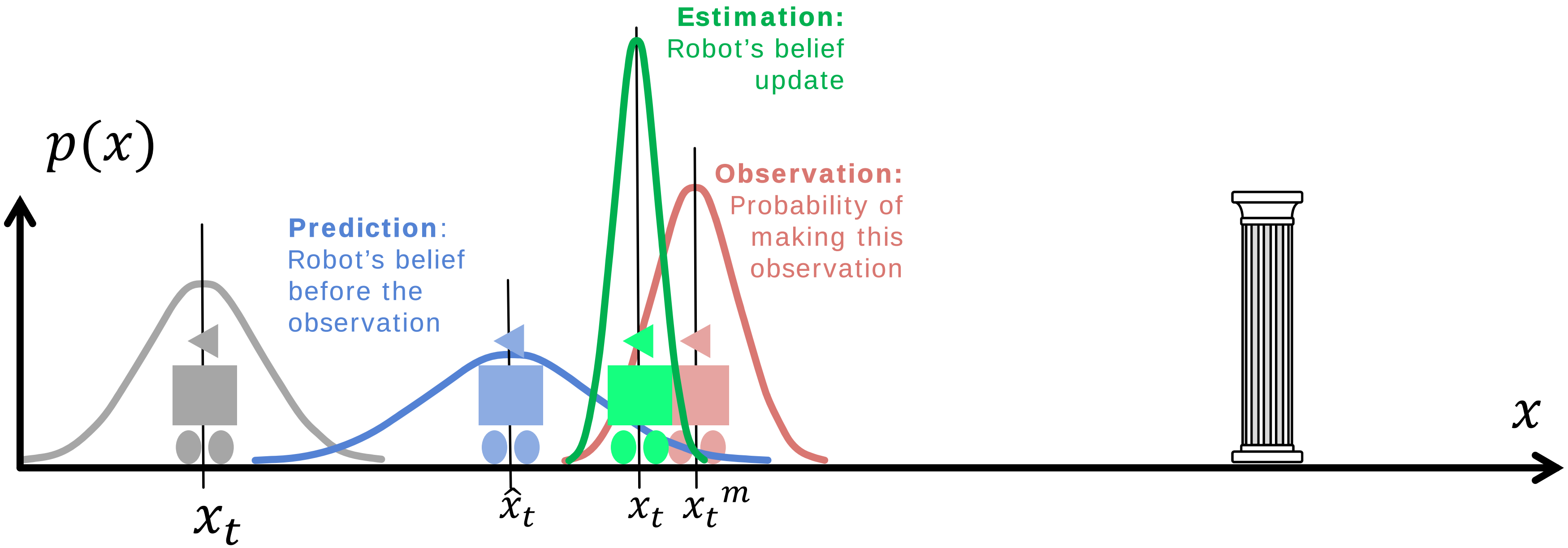
Odometry
Calibration of Errors II (Borenstein [5])

- The bi-directional square path experiment

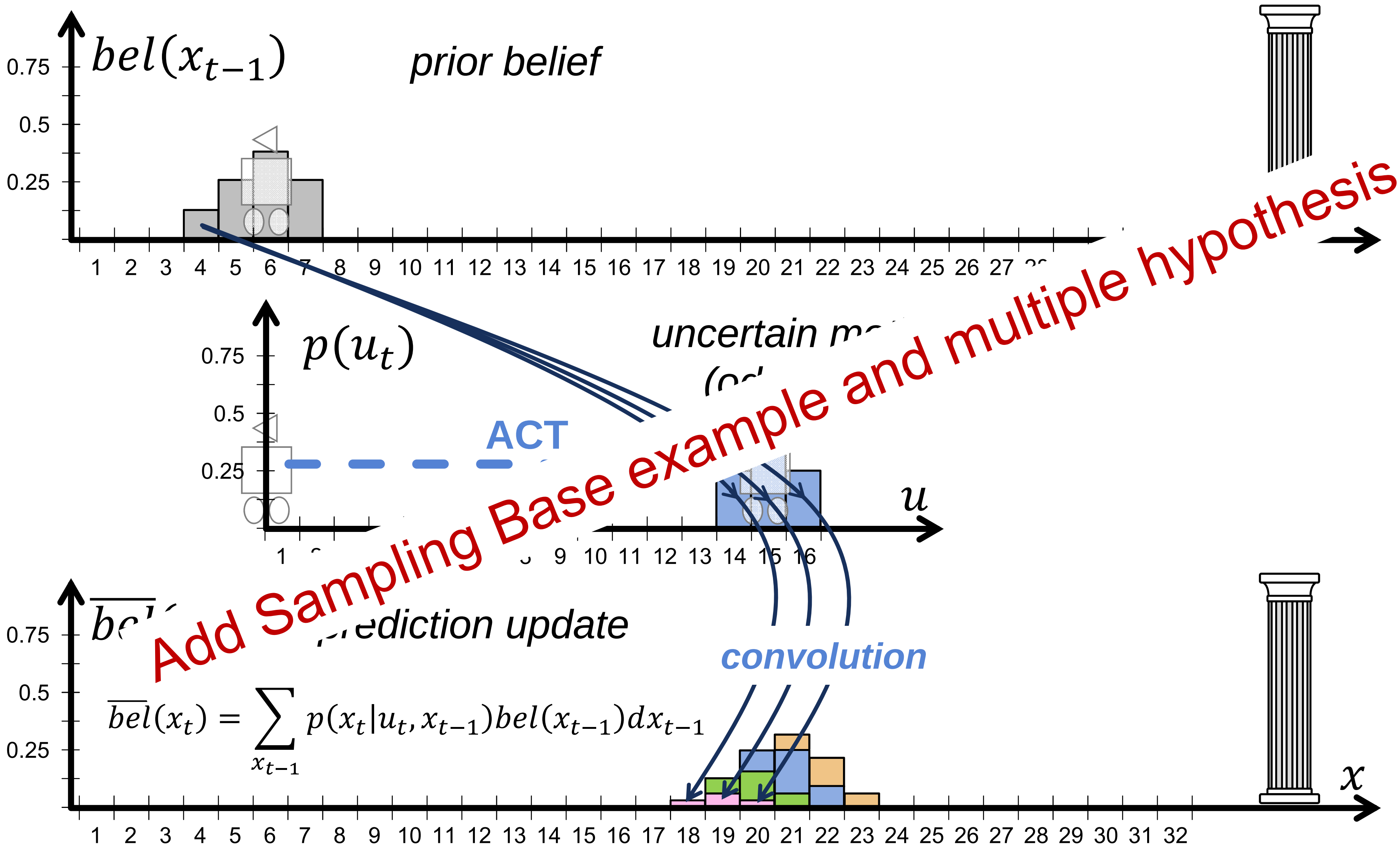


Kalman Filter Localization | in summery

1. **Prediction (ACT)** based on previous estimate and odometry
2. **Observation (SEE)** with on-board sensors
3. **Measurement prediction** based on prediction and map
4. **Matching** of observation and map
5. **Estimation** → position update (posteriori position)

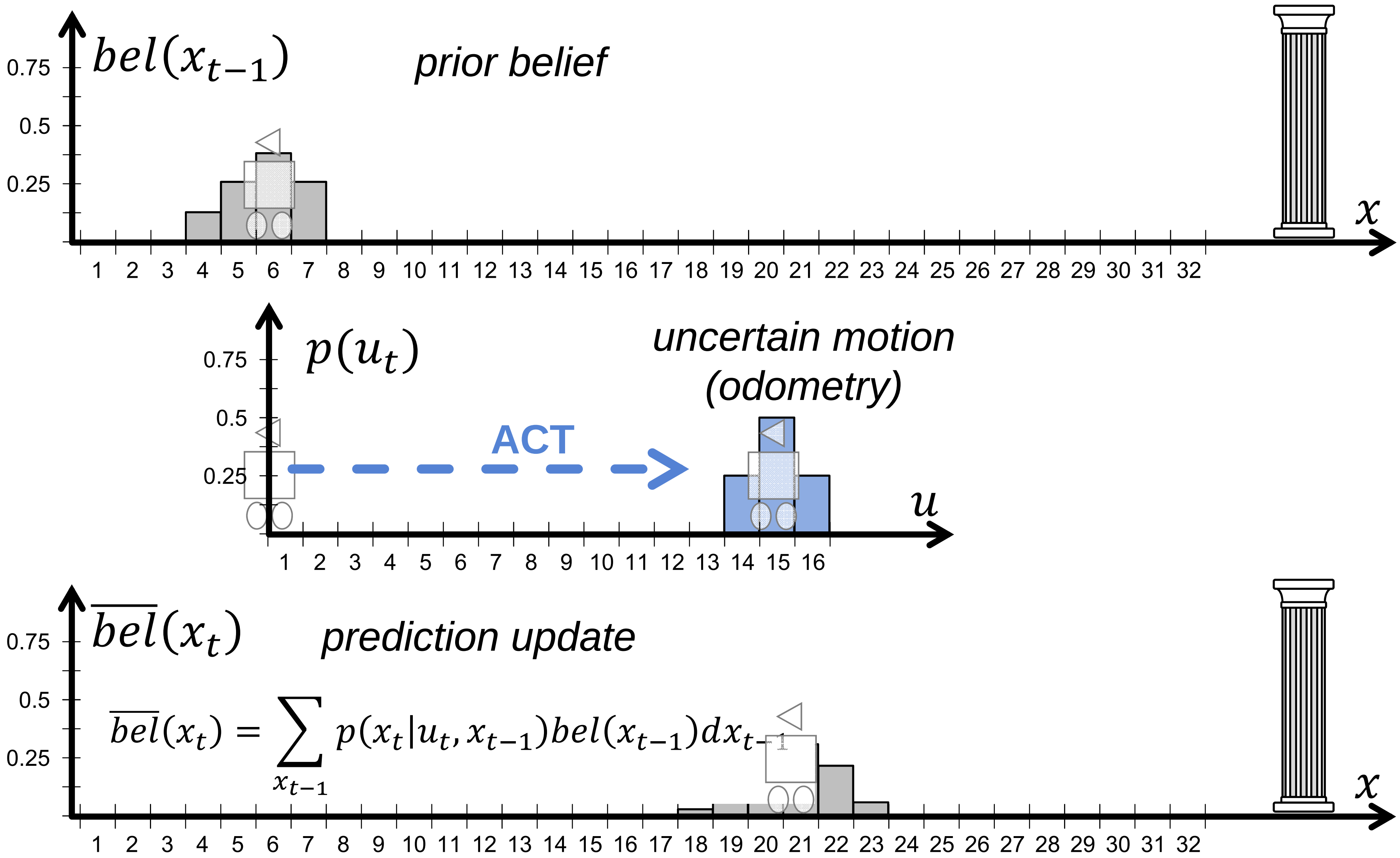


ACT | using motion model and its uncertainties



$$\overline{bel}(x_t) = \sum_{x_{t-1}} p(x_t|u_t, x_{t-1}) bel(x_{t-1}) dx_{t-1}$$

ACT | using motion model and its uncertainties



SEE | estimation of position based on perception and map