ETH zürich

Autonomous Mobile Robots

Exercise 2: Mobile Robot Kinematics and Control

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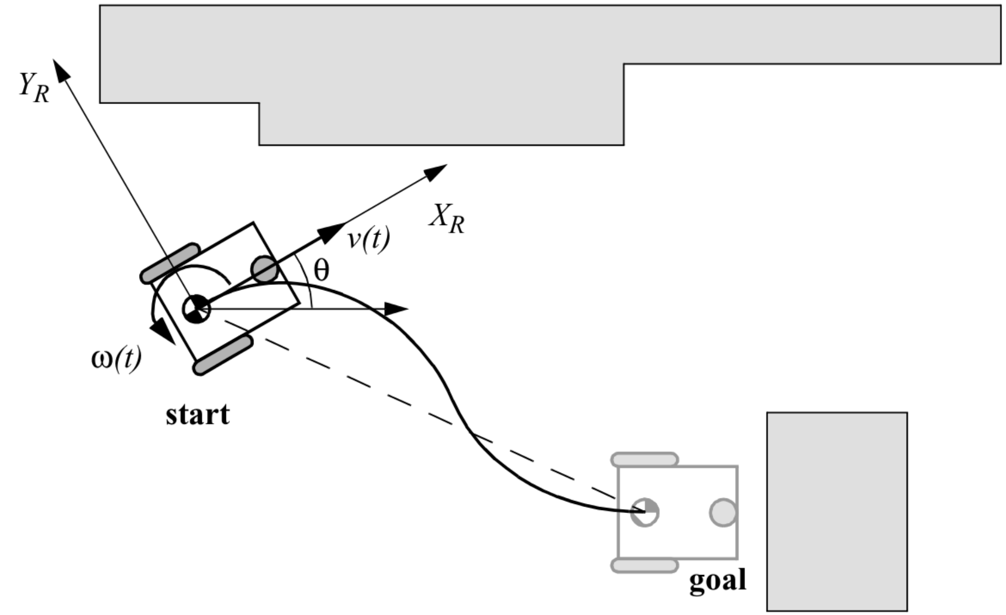
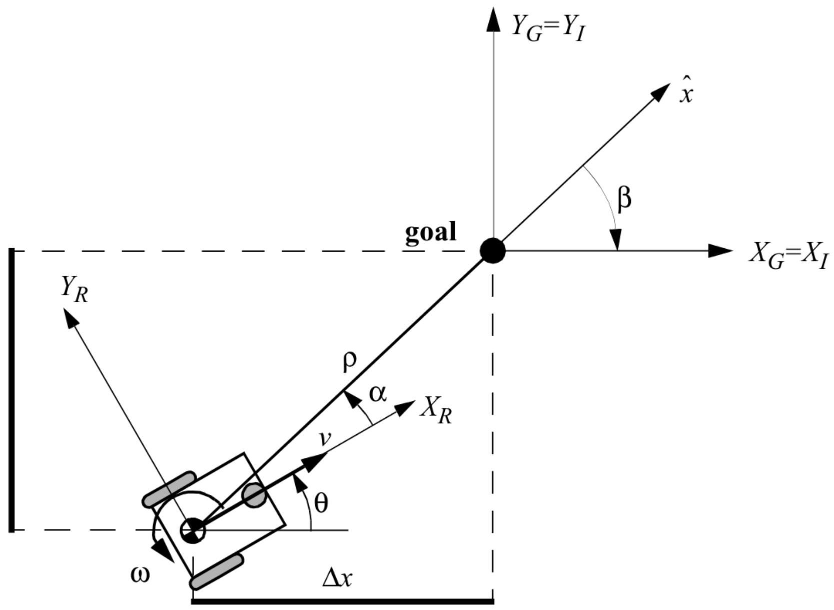
Based on slides by Anurag Vempati

Mobile Robot Control

- How can the robot reach a goal position from its current state?



Mobile Robot Control



- **Kinematic model**: Wheel speeds needed for achieving particular robot motion
- **Control**: What robot motion (speeds) will be needed to get to the goal state

Kinematics

- Differential drive kinematic model:

$$v = \frac{r\dot{\phi}_r}{2} + \frac{r\dot{\phi}_l}{2}$$
$$\omega = \frac{r\dot{\phi}_r}{2l} - \frac{r\dot{\phi}_l}{2l}$$

- Robot description:
  - *wheel speeds* + *intrinsics* ↔ *robot speed*

- More complex descriptions:
  - Include forces, dynamic constraints

Control

- Error : $e = {}^{R}[x, y, \theta]^{T}$

$$\begin{bmatrix} v(t) \\ \omega(t) \end{bmatrix} = K \cdot e = K \, {}^{R}\begin{bmatrix} x \\ y \\ \theta \end{bmatrix} \Longrightarrow K = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \end{bmatrix} \quad \text{with } k_{ij} = k(t, e)$$

- Find K such that $\lim_{t \to \infty} e(t) = 0$
- Proportional control: Robot not actuated when at the goal position

Control

- Motion of the robot in inertial frame
- Kinematic of differential-drive robot in Inertial frame:

$$^{I}\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$$

- Cartesian → Polar coordinates
  - Easier description for the given problem

- What does the controller need?
  - Position → decide speed
  - Heading → for alignment to goal location
  - Orientation → for aligning with goal orientation

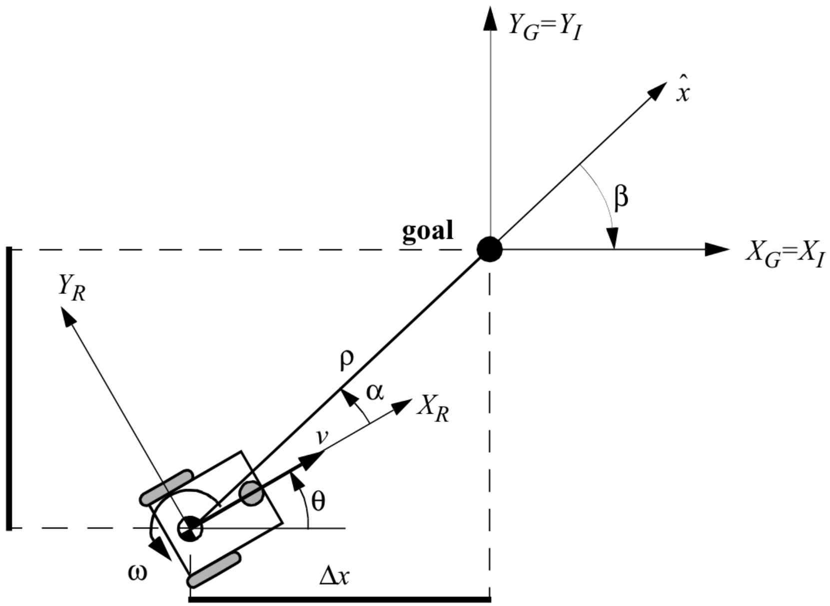
Control

- Inertial → Polar transformation

$$\rho = \sqrt{\Delta x^2 + \Delta y^2}$$

$$\alpha = -\theta + \operatorname{atan2}(\Delta y, \Delta x)$$

$$\beta = -\theta - \alpha$$



- Kinematics model:

$$\begin{bmatrix} \dot{\rho} \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} -\cos\alpha & 0 \\ \frac{\sin\alpha}{\rho} & -1 \\ -\frac{\sin\alpha}{\rho} & 0 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$$

NOTE:
heading ↔ speed relation

Control law

- Simple linear control law:

$$v = k_\rho \rho$$

$$\omega = k_\alpha \alpha + k_\beta \beta$$

- Closed-loop system description:

$$\begin{bmatrix} \dot{\rho} \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} -k_\rho \rho \cos \alpha \\ k_\rho \sin \alpha - k_\alpha \alpha - k_\beta \beta \\ -k_\rho \sin \alpha \end{bmatrix}$$

Local stability

- Linearizing the system around equilibrium
- (for *x ~0: cos(x) ~ 1; sin(x) ~ x):*

$$\begin{bmatrix}\dot{\rho}\\ \dot{\alpha}\\ \dot{\beta}\end{bmatrix} = \underbrace{\begin{bmatrix}-k_\rho & 0 & 0\\ 0 & -(k_\alpha - k_\rho) & -k_\beta\\ 0 & -k_\rho & 0\end{bmatrix}}_{\boldsymbol{A}} \begin{bmatrix}\rho\\ \alpha\\ \beta\end{bmatrix}$$

- Locally exponentially stable if the eigenvalues of the matrix *A* all have negative real part.
- Characteristic polynomial of A (*det(A-λ*I)*):

$$(\lambda + k_\rho)(\lambda^2 + \lambda(k_\alpha - k_\rho) - k_\rho k_\beta)$$

Local stability

- Negative real parts for all the roots of characteristic polynomial:

$$k_\rho > 0 \;;\quad -k_\beta > 0 \;;\quad k_\alpha - k_\rho > 0$$