ETH zürich

Robot Dynamics

Fixed Wing UAS: Review/Refresher

151-0851-00 V

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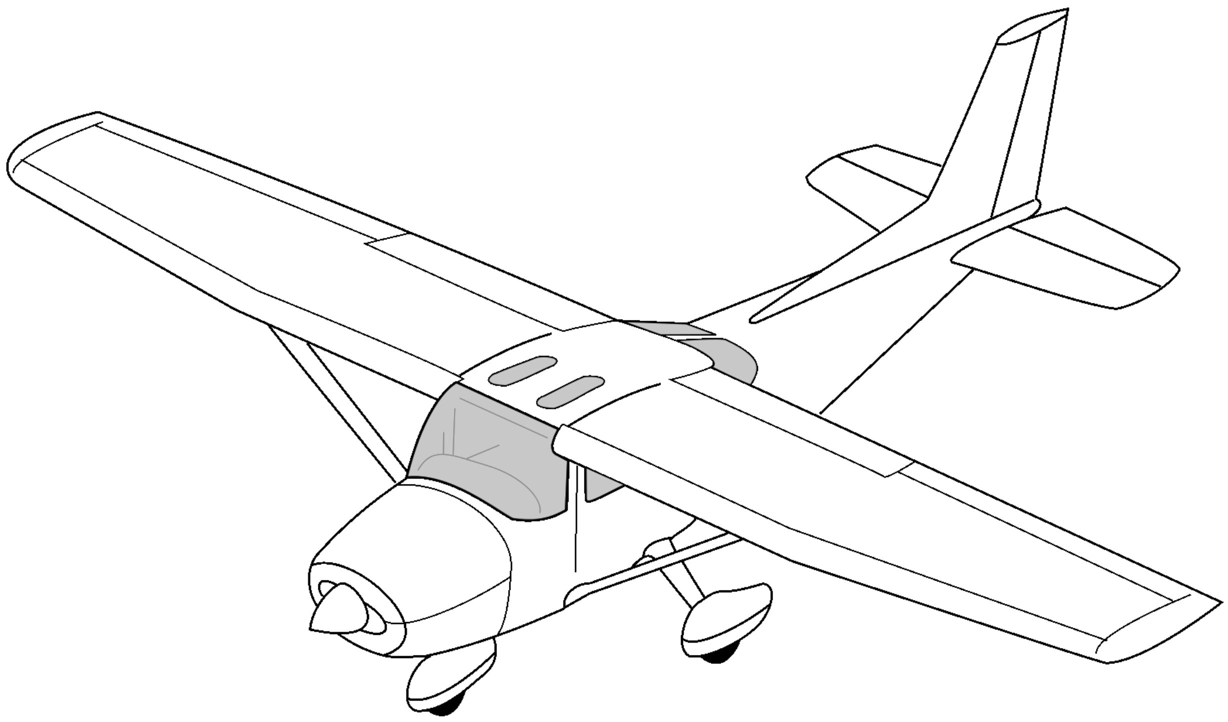
Autonomous Systems Lab

Frames

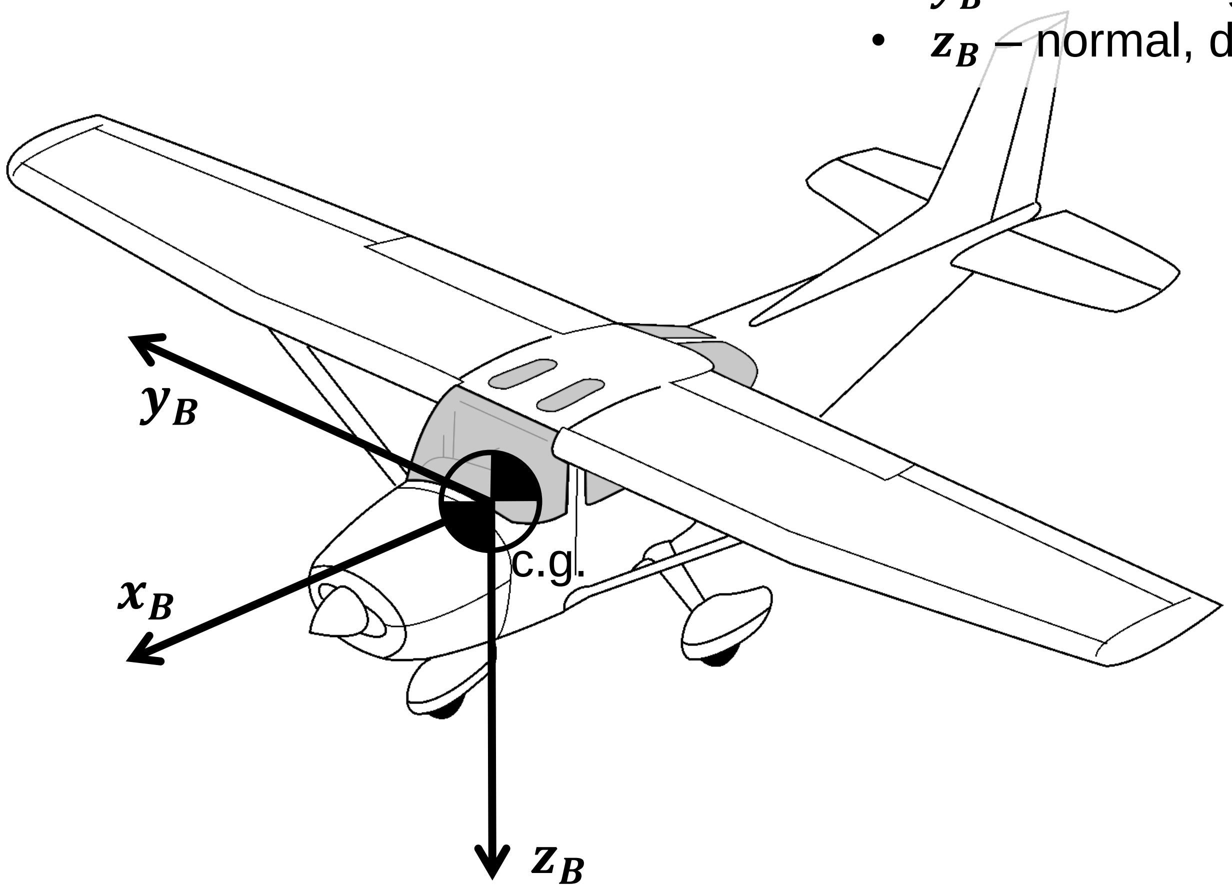


Background image:
http://upload.wikimedia.org/wikipedia/commons/ 5/5c/C_172_line_drawing_oblique.svg

Frames

Body-fixed Frame:

- $\boldsymbol{x_B}$ – out the nose
- $\boldsymbol{y_B}$ – out the right wing
- $\boldsymbol{z_B}$ – normal, down



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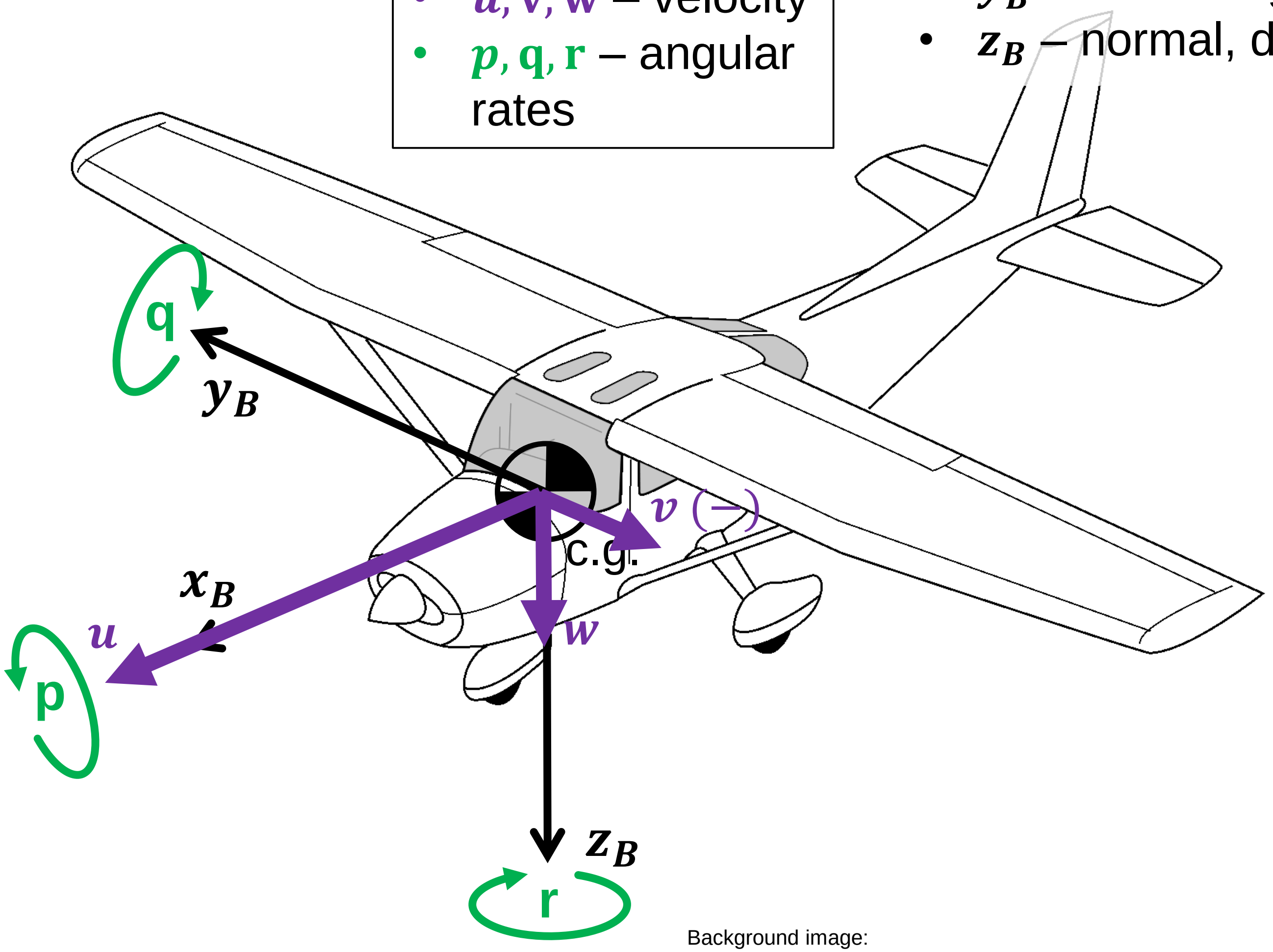

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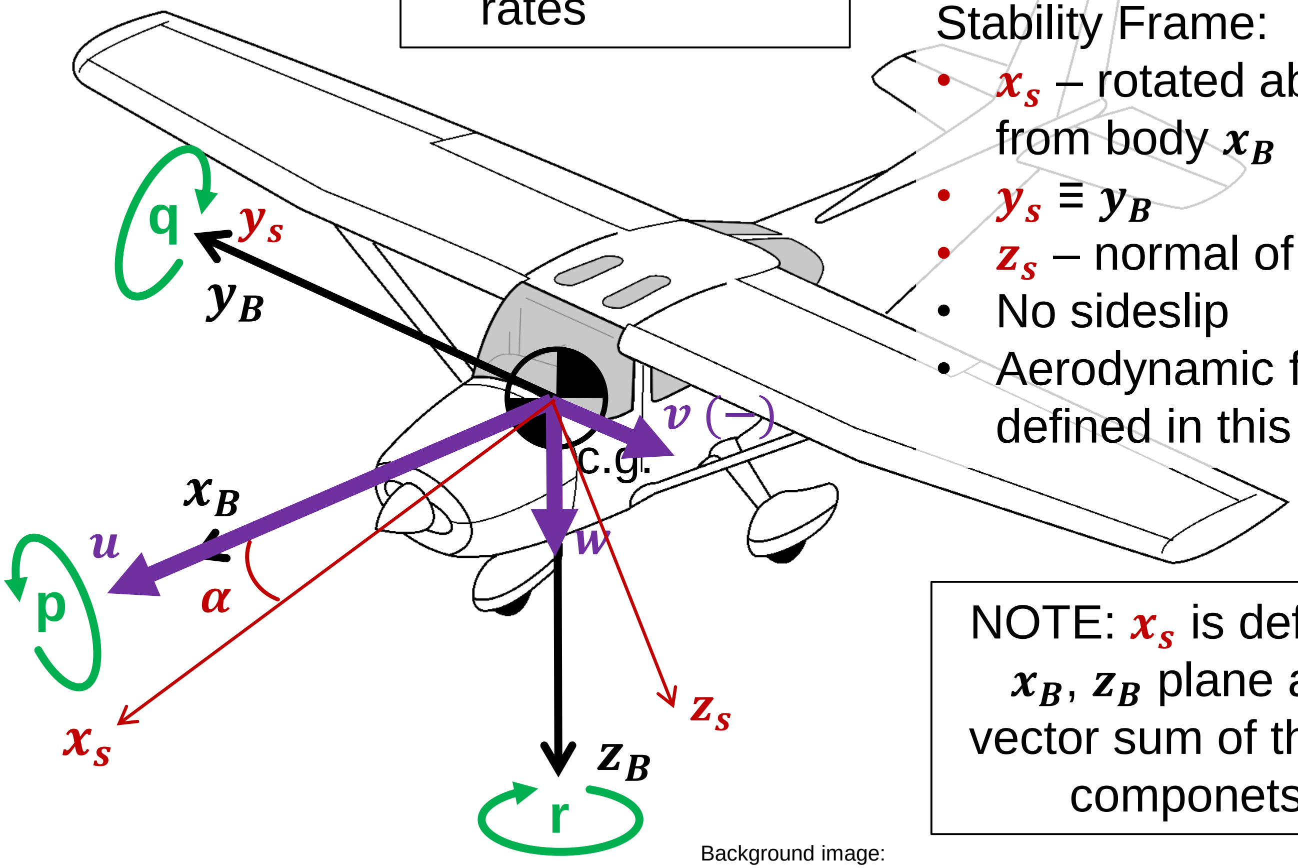
Stability Frame:
- x_s – rotated about α (angle of attack) from body x_B
- $y_s \equiv y_B$
- z_s – normal of x_s, y_s
- No sideslip
- Aerodynamic forces and moments defined in this frame.



NOTE: x_s is defined in the x_B, z_B plane along the vector sum of the u and w componets of V_T

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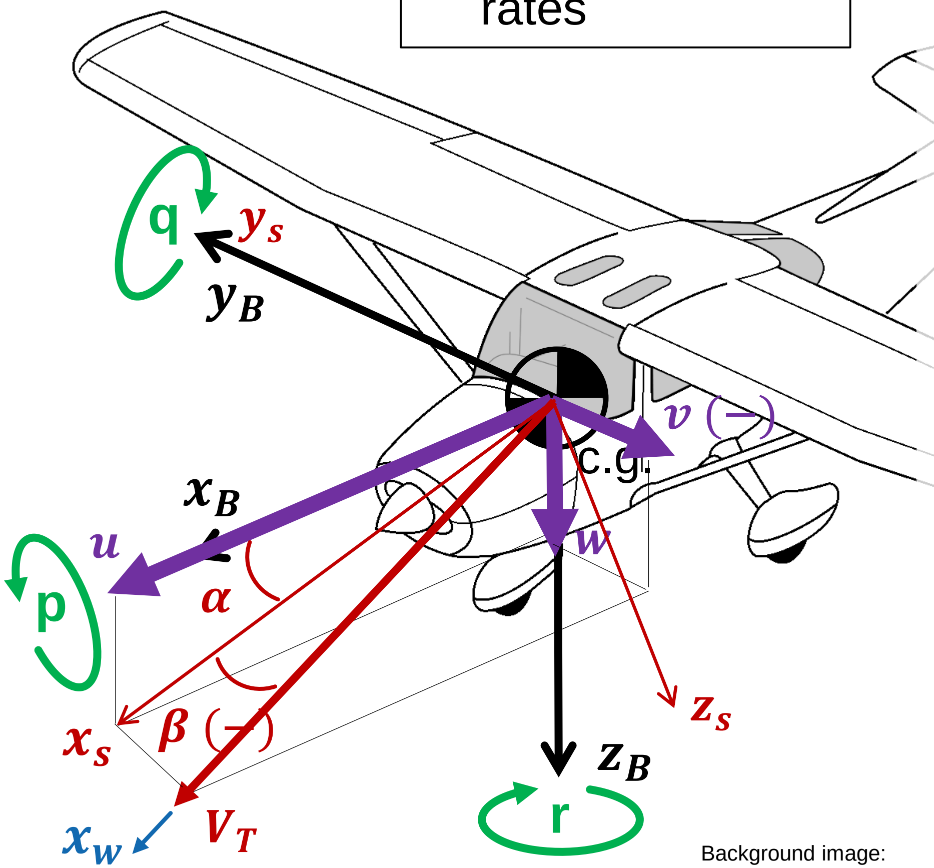

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"Wind" Frame:
- x_w defined on the total velocity vector (airspeed vector)
- Same as stability, but with extra sideslip rotation
- Opposite to "free-stream" V_∞

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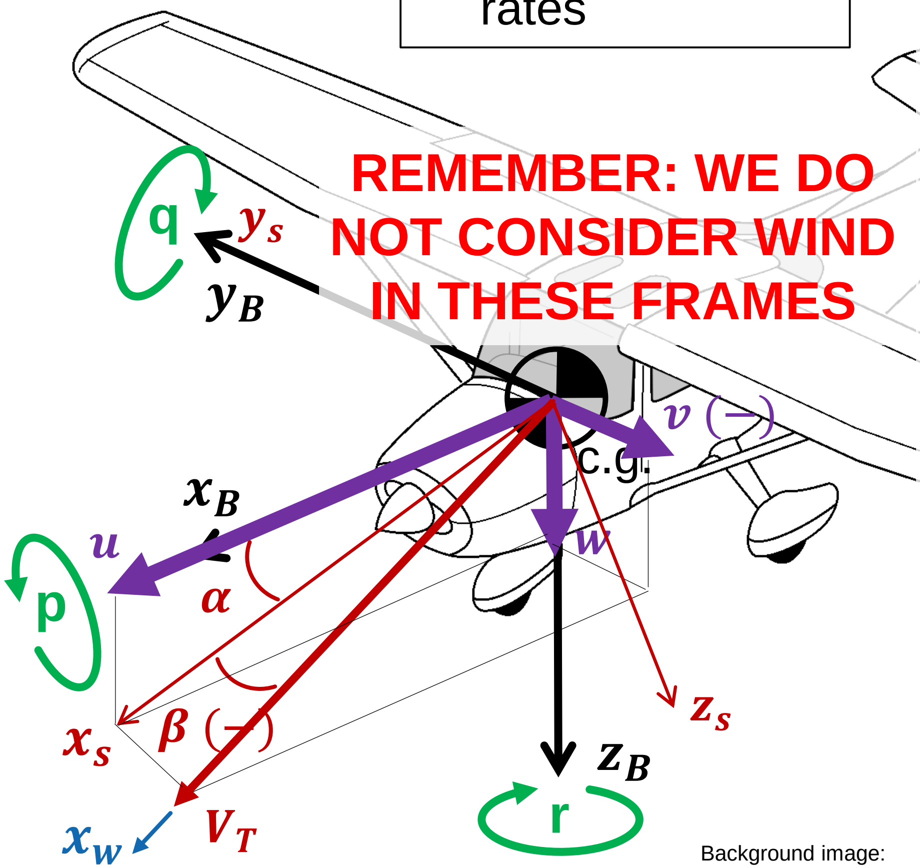

REMEMBER: WE DO NOT CONSIDER WIND IN THESE FRAMES

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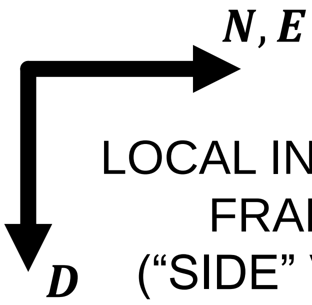
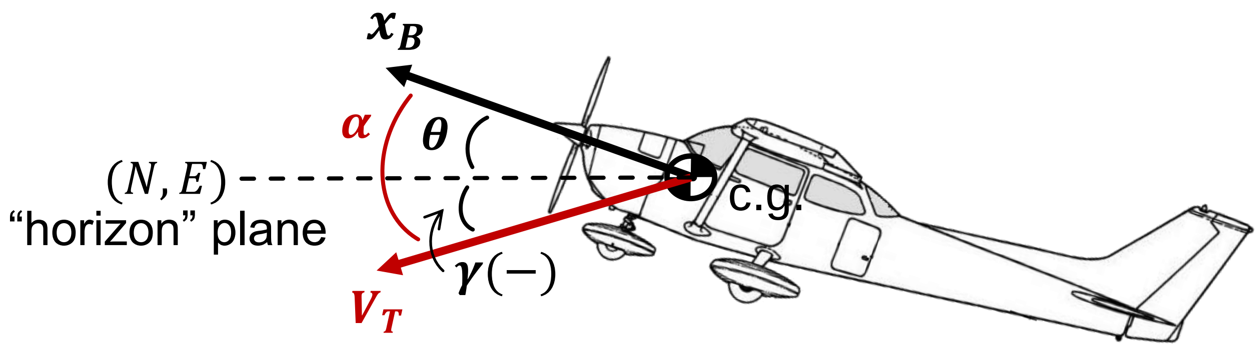
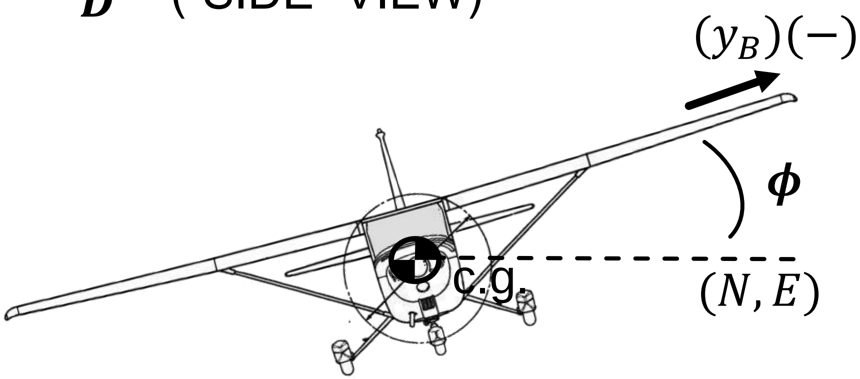


LOCAL INERTIAL FRAME
("SIDE" VIEW)



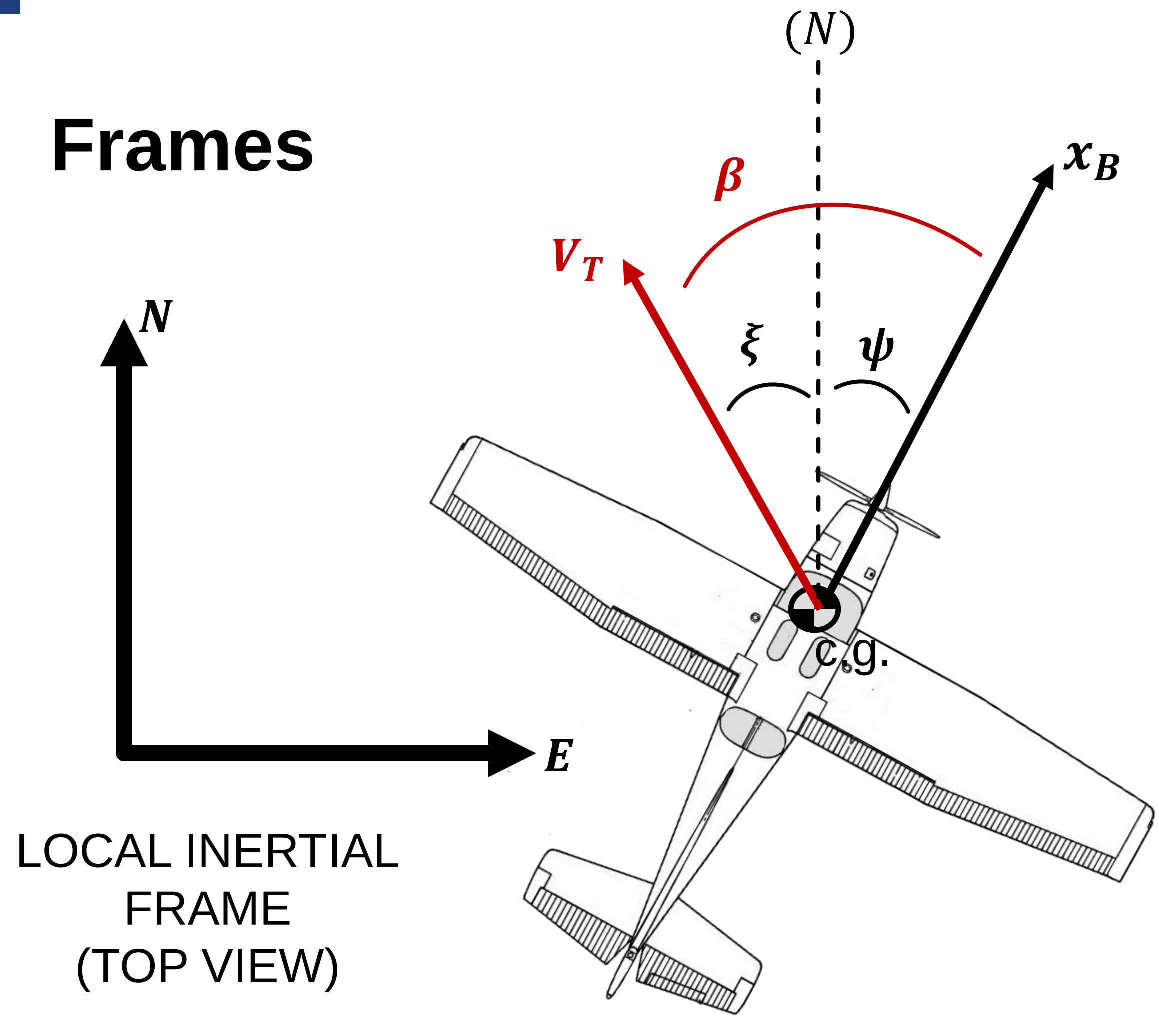


Longitudinal view in the inertial frame:

- $\boldsymbol{\theta}$ – pitch angle, defined from the horizon to $\boldsymbol{x_B}$
- $\boldsymbol{\gamma}$ – flight path angle, defined from the horizon to $\boldsymbol{V_T}$
- $\boldsymbol{\alpha}$ – angle of attack, defined from $\boldsymbol{V_T}$ to $\boldsymbol{x_B}$
- $\boldsymbol{\phi}$ – roll angle, defined from the horizon to $\boldsymbol{y_B}$.
- Note the $\boldsymbol{x_B}$ axis is pointing out of the page in the rolling figure.

http://atallguy.com/Simple-Multi-View-Plans/C-images/cessna_172_3v.jpg

Frames



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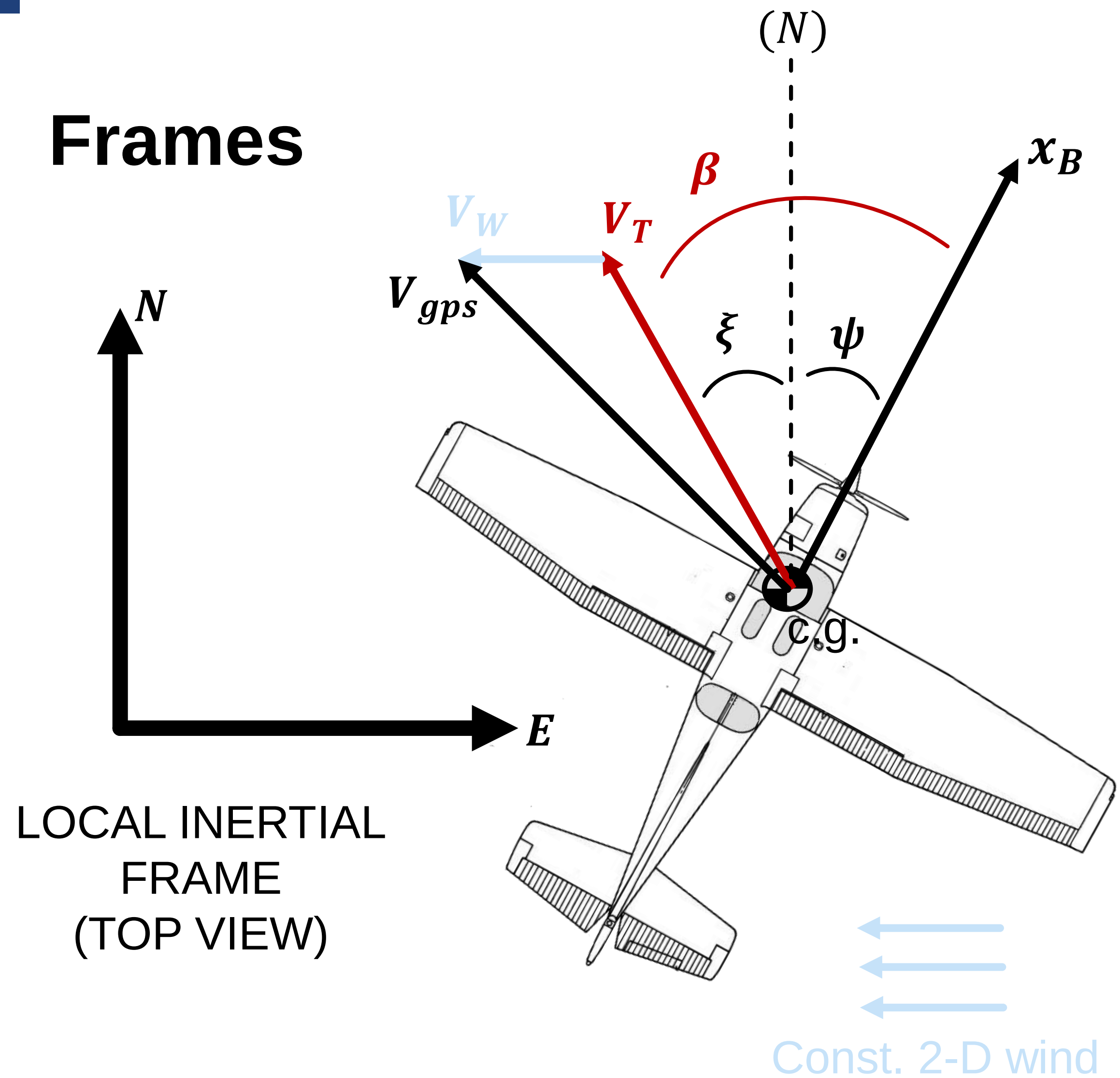
Lateral-directional view in the inertial frame:

- ψ – yaw angle, defined from North to x_B
- ξ – Heading angle, defined from North to airspeed vector V_T
- β – sideslip angle
- Note the heading angle is defined to the lateral-directional projection of the airspeed vector on the N,E plane (or the "horizon plane"). I.e. if the aircraft is flying at a non-zero flight path angle γ, the V_T and β shown here will be smaller than their true magnitudes.

Frames



Lateral-directional view in the inertial frame:

- If a constant wind is added in the inertial frame (i.e. the air-mass in which we are flying is now moving with some velocity with respect to the ground), note the velocity a GPS reads will differ in magnitude from the airspeed vector and point in a direction that is no longer equal to the heading of the aircraft.

http://atallguy.com/Simple-Multi-View-Plans/C-images/cessna_172_3v.jpg

Development of the Model

- **Summarized equations of motion:**
  - Translation

$$\dot{u} = rv - qw + \frac{1}{m}\left(F_T \cos\varepsilon - D\cos\alpha + L\sin\alpha\right) - g\sin\theta$$

$$\dot{v} = pw - ru + \frac{1}{m}Y + g\sin\varphi\cos\theta$$

$$\dot{w} = qu - pv + \frac{1}{m}\left(F_T \sin\varepsilon - D\sin\alpha - L\cos\alpha\right) + g\cos\varphi\cos\theta$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \mathbf{C}_{EB} \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

Development of the Model

- Rotation (simplified with $I_{xz} \approx 0$):

$$\dot{p} = \frac{1}{I_{xx}}\left[L + L_T - qr\left(I_{zz} - I_{yy}\right)\right]$$

$$\dot{q} = \frac{1}{I_{yy}}\left[M + M_T - pr\left(I_{xx} - I_{zz}\right)\right]$$

$$\dot{r} = \frac{1}{I_{zz}}\left[N + N_T - pq\left(I_{yy} - I_{xx}\right)\right]$$

$$\begin{bmatrix} \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \mathbf{J}_r^{-1} \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} p + q \tan\theta \sin\varphi + r \tan\theta \cos\varphi \\ q \cos\varphi - r \sin\varphi \\ q \dfrac{\sin\varphi}{\cos\theta} + r \dfrac{\cos\varphi}{\cos\theta} \end{bmatrix}$$

Decoupling Dynamics

- **Longitudinal states:**
  - $\boldsymbol{u, w, q, \theta}$ – note: these are defined in the body-fixed axes
  - Auxillary state: $\boldsymbol{\alpha = tan^{-1}(w/u)}$
  - Set remaining states to zero.
- **Lateral-directional states:**
  - $\boldsymbol{v, p, r, \phi, \psi}$ – note: these are defined in the body-fixed axes
  - Auxillary state: $\boldsymbol{\beta = sin^{-1}\left(v/V_T\right)}$
  - Note yaw angle, $\boldsymbol{\psi}$, has no effect on the dynamic.
  - Another note: $\boldsymbol{\theta = \alpha = const}$. NOT necessarily zero.

Decoupling Dynamics

- **Longitudinal dynamics:**
  - Translation

$$\dot{u} = rv - qw + \frac{1}{m}\left(F_T \cos\varepsilon - D\cos\alpha + L\sin\alpha\right) - g\sin\theta$$

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Decoupling Dynamics

- **Longitudinal dynamics:**
  - Translation

$$\dot{u} = \cancel{rv} - qw + \frac{1}{m}\left(F_T \cos\varepsilon - D\cos\alpha + L\sin\alpha\right) - g\sin\theta$$

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$$\dot{w} = qu - \cancel{pv} + \frac{1}{m}\left(F_T \sin\varepsilon - D\sin\alpha - L\cos\alpha\right) + g\cancel{\cos\varphi}\cos\theta$$

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\end{bmatrix} = \mathbf{C}_{EB}\begin{bmatrix}u\\ v\\ w\end{bmatrix}$$

Decoupling Dynamics

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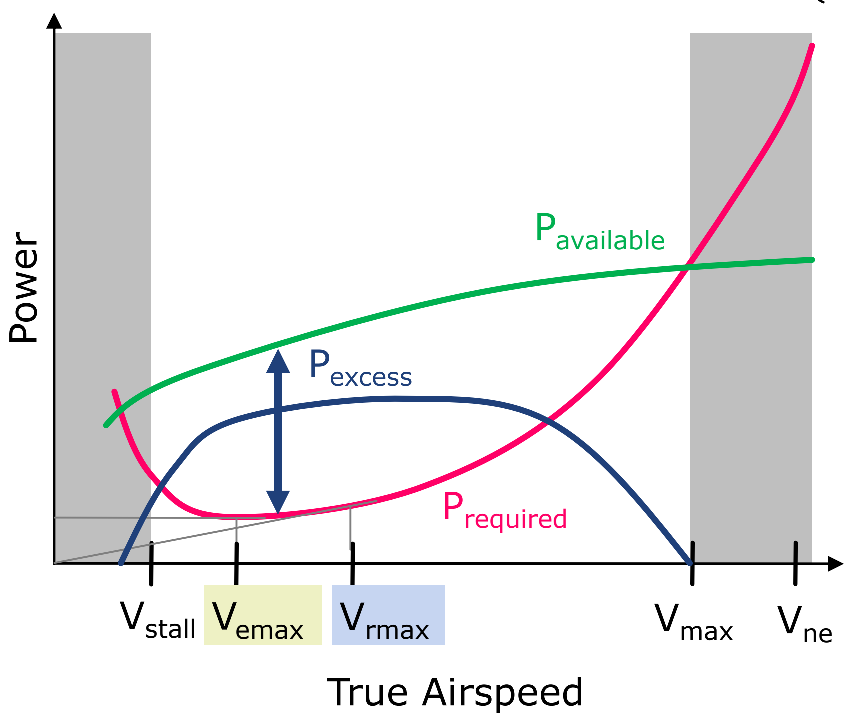
Power Required and Available for Level Flight

- Given: drag coeff. as a funcion of lift coeff.: $C_L(V) = \frac{2mg}{A\rho V^2}$, $C_D(C_L)$
- Required power: $P_{required} = D \cdot V = \frac{1}{2}\rho V^3 A C_D$
- Specific Excess Power: $SEP = \left(P_{available} - P_{required}\right)/(mg) \approx V_{climb,achievable}$



* Assuming constant propulsive efficiency η

Max. Range*(v_{rmax}): $\Delta s = V\Delta T = V\frac{\Delta E}{P/\eta} \rightarrow \max$

$$\Leftrightarrow \frac{P}{V} = D = mg\left(\frac{D}{L}\right) = mg\left(\frac{C_D}{C_L}\right) \rightarrow \min \Leftrightarrow \boxed{\frac{C_L}{C_D} \rightarrow \max}$$

Best glide ratio

Max. Endurance*(v_{emax}): $P/\eta \rightarrow \min$

$$\Leftrightarrow P = VD = V \cdot mg\left(\frac{C_D}{C_L}\right) = \sqrt{\frac{2mg}{\rho A \cdot C_L}} \cdot mg\left(\frac{C_D}{C_L}\right) \Leftrightarrow$$

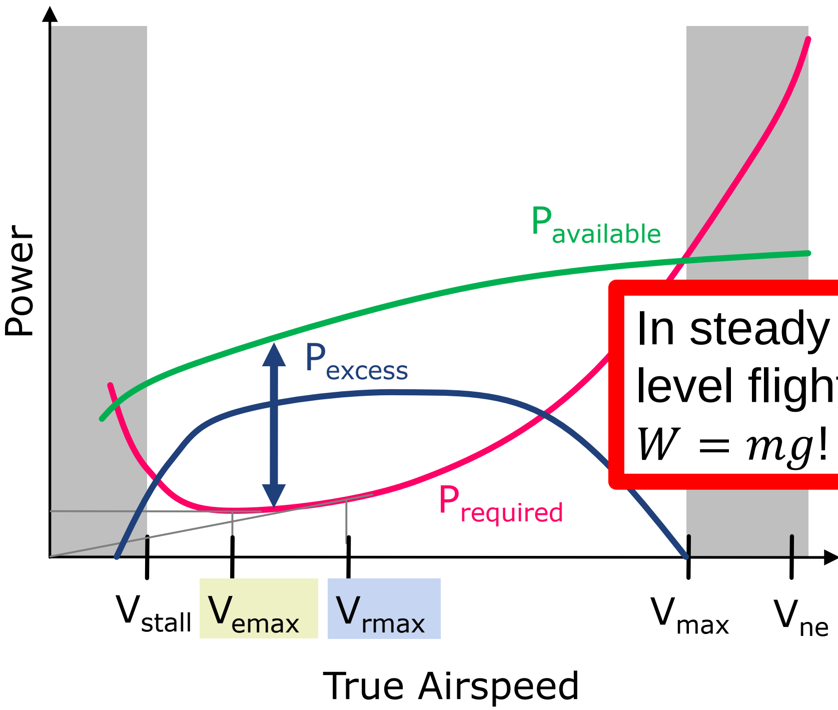
$$\boxed{\frac{C_L{}^3}{C_D{}^2} \rightarrow \max}$$

Minimum sink in gliding mode

Power Required and Available for Level Flight

- Given: drag coeff. as a funcio… $2mg$ …
- Required power: $P_{required} = D \cdot V$ …
- Specific Excess Power: SEP … $(P_{available} \quad P_{required})/(mg)$ … $v_{climb,achievable}$

Generally, power is a change in energy over a change in time: $P = \frac{\Delta E}{\Delta T}\eta$ Note: propulsion systems are not 100% efficient!



In steady state, level flight, $L = W = mg$!

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Best glide ratio

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$$\frac{{C_L}^3}{{C_D}^2} \to \max$$

Minimum sink in gliding mode

* Assuming constant propulsive efficiency η

Ralph Paul (Solar Impulse) next week!

- Exercise today. HG E 27