Robot Dynamics - Fixed Wing UAS
Exercise 1: Aircraft Aerodynamics & Flight Mechanics

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Abstract

This exercise analyzes the performance of the Techpod UAV during steady-level and gliding flight conditions. A decoupled longitudinal model is further derived from its full six-degrees-of-freedom representation, and the necessary assumptions leading to said model are elaborated.

1 Aircraft Flight Performance

Use the parameters given in Table 3 and the following environmental and platform specific information to answer the following questions. Do *not* interpolate, simply take the closest value. Assume the aircraft is in steady conditions (i.e. ${}_{\mathcal{B}}\dot{\mathbf{v}} = {}_{\mathcal{B}}\omega = \mathbf{0}$) at all times, and level (i.e. $\gamma = 0$), if powered.
Enivronment: density $\rho = 1.225 kg/m^3$ (assume sea-level values)
Aircraft specs: wing area $S = 0.39 m^2$, mass $m = 2.65 kg$

a) Calculate the minimum level flight speed of the Techpod UAV. Is this a good choice of operating airspeed?

As we are assuming steady-level flight conditions, we may equate the lifting force with the force of gravity, i.e. $L = 0.5\rho V^2 S c_L = mg$. Solving this equation for airspeed, we see that maximizing c_L, i.e. $c_L = c_{L_{max}} = 1.125$ (from Table 3), minimizes speed.

$$V_{min} = \sqrt{\frac{2mg}{\rho S c_{L_{max}}}} = 9.83 m/s$$

As this operating speed would be directly at the stall point of the aircraft, this is a **highly dangerous** condition. For sure not recommended as an operating speed.

b) Suppose the motor fails. What maximum glide ratio can be reached? And at what speed is this maximum achieved?

Conveniently, an aircraft's lift-to-drag ratio is numerically equivalent to its glide ratio, whether assuming steady-level (powered) or steady (un-powered / gliding) flight. Using this fact, we can directly pick out the maximum glide ratio from Table 3, $(c_L/c_D)_{max} = 9.691$. Note this is not a particularly high glide ratio, modern sailplanes range between 40-60 (depending on span). But viewing the Techpod's design in Figure 1, you can see how the plane is not particularly built for excellent gliding flight!

Now that we have the glide ratio, we can calculate the speed at which we can achieve it by balancing forces without thrusting effects, and taking the corresponding lift and drag coefficients from the table, i.e. $c_L = 0.735, c_D = 0.0759$.

$$V_{glide} = \sqrt{\frac{2mg}{\rho S \sqrt{c_L^2 + c_D^2}}} = 12.13 m/s$$

c) Show (derive) how aircraft endurance is maximized when c_L^3/c_D^2 is maximized.

As we showed in the lecture for the maximum range example, we start by defining what we want to optimize. Maximum endurance entails staying aloft as long as possible. So we want to maximize ΔT (time). Using the relationship of power (with some efficiency) to change in energy over time, we can obtain the following:

$$\frac{\Delta E}{\Delta T} = P/\eta \Rightarrow \Delta T = \frac{\Delta E}{P/\eta}$$

$$max\,(1/P) = max\left(\tfrac{1}{VD}\right) = max\left(\tfrac{1}{VD}\tfrac{L}{mg}\right) = max\left(\tfrac{1}{Vmg}\left(\tfrac{c_L}{c_D}\right)\right) =$$
$$= max\left(\sqrt{\tfrac{\rho S c_L}{2mg}}\tfrac{1}{mg}\left(\tfrac{c_L}{c_D}\right)\right) = max\left(\tfrac{c_L^3}{c_D^2}\right)$$

d) If Techpod is attempting a new flight endurance record, what airspeed should it choose? How many degrees away from stall is this operating point? What magnitude of a **local** short term downdraft (i.e. additional Δw body velocity component) would it take to push the effective angle of attack to stall?

As we determined in the previous problem, endurance is maximized when $c_L^3/c_D^2 = \left(c_L^3/c_D^2\right)_{max}$. From Table 3, we can find $\left(c_L^3/c_D^2\right)_{max} = 78.126$, and the corresponding lift coefficient from the table, i.e. $c_L = 0.904$. Plug, and chug!

$$V_{min.sink} = \sqrt{\frac{2mg}{\rho S c_L}} = 10.97m/s$$

The effective angle of attack acting on the wing or aircraft may be different than the global angle of attack with respect to the free stream in the event of short gusts. If we consider the example of a downdraft, e.g. Δw body-z velocity component added breifly to the current flight operating point, we can determine how large this should be to reach the stall point.

Let us first determine the angle of attack at the current condition from the table, $\alpha = 0.0698rad(4deg)$. We can use this value to determine the current u and w body velocity components. (let us assume sideslip is zero)

$$u = V_{min.sink}\cos\alpha = 10.94m/s$$
$$w = V_{min.sink}\sin\alpha = 0.765m/s$$

Now we can reapply these values with the instantaneous gust addition and solve.

$$\alpha_{stall} = 0.174(10deg) = tan^{-1}\frac{w + \Delta w}{u} \Rightarrow \Delta w = u\tan\alpha_{stall} - w = 1.164m/s$$

Note this is not so large! This means the Techpod (as many small UAVs) is very susceptible to up/down drafts during flight. A fast reactive controller should be in place to ensure safe flight through turbulent conditions.

2 Aircraft Model Derivation

The following equations describe the longitudinal dynamics of a glider in still air (i.e. no thrust force and no thrust moment). The x and z position states have been omitted for simplicity.

$$
\begin{aligned}
\dot{u} &= -qw + \frac{1}{m}\left(-D\cos\alpha + L\sin\alpha\right) - g\sin\theta \\
\dot{w} &= qu + \frac{1}{m}\left(-D\sin\alpha - L\cos\alpha\right) + g\cos\theta \\
\dot{q} &= M_m/I_{yy} \\
\dot{\theta} &= q
\end{aligned}
$$

Discuss the simplifications and assumptions made that resulted in the above equations compared to the full 6DoF rigid body dynamics.

When decoupling a longitudinal model from the full rigid-body dynamics, we are essentially setting lateral-directional states to zero (or constant) values. This is accomplished, in this case, by assuming our lateral-directional states $\phi = p = r = \beta = 0$. Note: $\beta = 0 \Rightarrow v = 0$. This assumption says that these lateral-directional states now do not effect our longitudinal dynamics.

Note that for the lateral-directional case, there are actually some longtitudinal states which must remain as constants when decoupling. Can you untuit which ones? It would be a good exercise to try this on your own! :)

Figure 1: Techpod UAV.

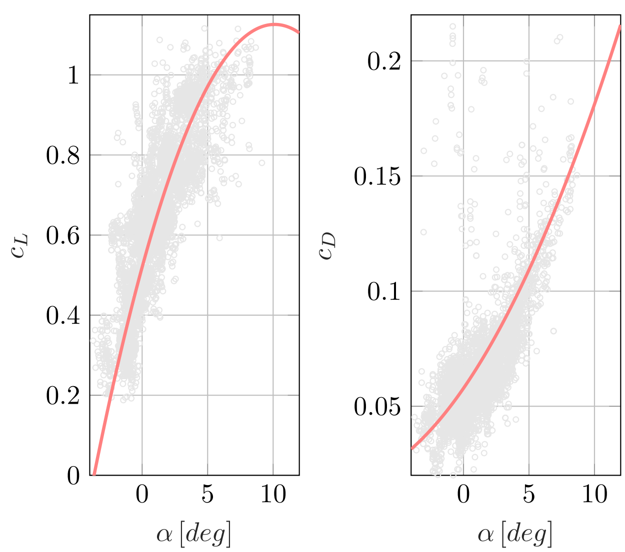

Figure 2: Static lift and drag identified for the Techpod UAV from flight data.

α [deg]	c_L	c_D	c_L/c_D	c_L^3/c_D^2
-5	-0.227	0.0268	-8.486	-16.393
-4	-0.0544	0.0313	-1.733	-0.163
-3	0.106	0.0367	2.908	0.904
-2	0.256	0.0429	5.969	9.136
-1	0.394	0.0499	7.885	24.503
0	0.519	0.0578	8.992	42.041
1	0.633	0.0664	9.536	57.641
2	0.735	0.0759	9.691	69.120
3	0.826	0.0862	9.581	75.847
4	0.904	0.0973	9.293	78.126
5	0.971	0.109	8.888	76.716
6	1.025	0.121	8.408	72.517
7	1.068	0.135	7.882	66.404
8	1.099	0.149	7.333	59.137
9	1.118	0.165	6.774	51.343
10	1.125	0.181	6.215	43.504
11	1.121	0.197	5.664	35.979
12	1.105	0.215	5.124	29.016
13	1.0767	0.234	4.599	22.777
14	1.036	0.253	4.090	17.348
15	0.984	0.273	3.600	12.761

Figure 3: Aerodynamic Data