Lecture #13:

• Damage of fiber-reinforced composites

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Modeling Damage of Fiber-reinforced Composites
Recall from last lecture

In the last lecture, we established criteria to estimate the mechanical loads under which a single lamina fails. For this we considered four failure modes:

1. Fiber tension failure
2. Matrix failure under combined compression and shear
3. Matrix failure under combined tension and shear
4. Fiber compression failure (kinking)

In the context of laminates, such criteria are useful to predict the so-called first ply failure load, i.e. the load at which the first lamina fails.
The first ply failure criteria also serve as **damage initiation criteria**. It is often assumed that a lamina does not loose its entire load carrying capacity instantaneously. Instead it is assumed that it undergoes a **damage process** throughout which it looses its load carrying capacity gradually.

![Diagram showing loading phase (fully elastic) and damage phase (dissipation) with damage initiation at $\epsilon_0$ and $\epsilon_u$.]
Continuum Damage Mechanics (CDM)

In the elastic loading phase, the entire internal energy (work performed by stress) is recovered upon unloading. The particular feature of the damage phase is that it is dissipative. At the instant of damage initiation, 100% of the internal energy can still be recovered upon unloading. This percentage decreases gradually to 0% as the material is strained from $\varepsilon_0$ to $\varepsilon_f$. 
A **scalar damage variable** $d$ is introduced to represent material damage. For example, consider the reduction of the effective load carrying cross-section due to voids.

The presence of voids is usually neglected when defining the axial stress in tension experiments,

$$ \sigma = \frac{F}{A} $$

However, if $d$ denotes the volume fraction of isotropically distributed voids, the effective cross-section is only $(1-d)A$. Consequently, the **local stresses are higher**,

$$ \sigma_{loc} = \frac{F}{(1-d)A} = \frac{\sigma}{(1-d)} $$
Reduced modulus due to damage

The elastic stress-strain relationship then applies for the local stresses:

\[ \sigma_{loc} = \frac{F}{(1 - d)A} = \frac{\sigma}{(1 - d)} = E \epsilon \]

The macroscopic stress-strain relationship then reads

\[ \sigma = (1 - d)E \epsilon \]

which corresponds to a “damaged modulus” of

\[ E_d = (1 - d)E \]

In other words, damage is associated with a modulus reduction.
Elastic strain energy

The repartition of the internal energy into elastic strain energy and dissipation can be calculated after introducing the internal damage variable. Note that at any instant of loading the **recoverable elastic strain energy** is defined as

$$
\psi_e = \frac{1}{2} E_d \varepsilon^2 = \frac{1}{2} (1 - d) E \varepsilon^2
$$
If we assume a linearly decreasing relationship in the damage phase, we have

$$\sigma = \sigma_0 \left( 1 - \frac{\varepsilon - \varepsilon_0}{\varepsilon_u - \varepsilon_0} \right)$$

and thus

$$E_d = \frac{\sigma}{\varepsilon} = E \frac{\varepsilon_0}{\varepsilon} \frac{\varepsilon_u - \varepsilon}{\varepsilon_u - \varepsilon_0}$$

According to the definition of the damage variable, we then have the damage evolution under monotonic loading:

$$d = 1 - \frac{E_d}{E} = \frac{\varepsilon_u}{\varepsilon} \frac{\varepsilon - \varepsilon_0}{\varepsilon_u - \varepsilon_0}$$
Incremental damage evolution law

To account for the irreversibility of the damage process, the damage evolution law is written as

\[
\dot{d} = \begin{cases} 
\left( \frac{\varepsilon_u}{\varepsilon_u - \varepsilon_0} \frac{\varepsilon_0}{\varepsilon^2} \right) \dot{\varepsilon} & \text{if } \varepsilon \geq \varepsilon_0 \text{ and } \dot{\varepsilon} > 0 \\
0 & \text{if } \varepsilon < \varepsilon_0 \text{ or } \dot{\varepsilon} \leq 0
\end{cases}
\]
To account for the **irreversibility of the damage process**, the damage evolution law is written as

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0 & \text{if } \varepsilon < \varepsilon_0 \text{ or } \dot{\varepsilon} \leq 0
\end{cases}
\]
Multiple sources of damage

For each failure mode, we define:

- a stress-based damage initiation criterion of the form \( f_{(i)} = 1 \)
- an effective strain measure \( \bar{\varepsilon}_{(i)} \)

<table>
<thead>
<tr>
<th>Failure Mode</th>
<th>Initiation criterion</th>
<th>Equivalent strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiber tension</td>
<td>( f_{ft} = \frac{&lt;\sigma_{11}&gt;}{X_t} )</td>
<td>( \bar{\varepsilon}<em>{ft} = &lt;\varepsilon</em>{11}&gt; )</td>
</tr>
<tr>
<td>Fiber compression</td>
<td>( f_{fc} = \frac{-\sigma_{11}}{X_c} )</td>
<td>( \bar{\varepsilon}<em>{fc} = &lt;-\varepsilon</em>{11}&gt; )</td>
</tr>
<tr>
<td>Matrix tension &amp; shear</td>
<td>( f_{mt} = \left( \frac{\sigma_{22}}{Y_t} \right)^2 + \left( \frac{\sigma_{12}}{S_L} \right)^2 )</td>
<td>( \bar{\varepsilon}<em>{mt} = \sqrt{&lt;\varepsilon</em>{22}&gt;^2 + (2\varepsilon_{12})^2} )</td>
</tr>
<tr>
<td>Matrix comp. &amp; shear</td>
<td>( f_{mc} = \left( \frac{\sigma_{22}}{2S_T} \right)^2 + \left( \frac{Y_c}{2S_T} \right)^2 - 1 ) ( \frac{\sigma_{22}}{Y_c} + \left( \frac{\sigma_{12}}{S_L} \right)^2 )</td>
<td>( \bar{\varepsilon}<em>{mc} = \sqrt{&lt;-\varepsilon</em>{22}&gt;^2 + (2\varepsilon_{12})^2} )</td>
</tr>
</tbody>
</table>

Note: the Macauley brackets \(<..>\) are defined as: \( <x> = \begin{cases} x & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases} \)
Multiple sources of damage

Subsequently, a damage variable is defined for each failure mode using the same generic damage evolution law. For example, for monotonic fiber tension, we have

\[
\text{For } \bar{\varepsilon}_f^t \geq \bar{\varepsilon}_0^f: \quad d_{ft} = \frac{\bar{\varepsilon}_u^f}{\bar{\varepsilon}_f^t} \left( \frac{\bar{\varepsilon}_ft - \bar{\varepsilon}_0^f}{\bar{\varepsilon}_u^f - \bar{\varepsilon}_0^f} \right) \quad \text{with the damage parameters } \{\bar{\varepsilon}_u^f, \bar{\varepsilon}_0^f\}
\]

Note that \( \bar{\varepsilon}_0^f \) is a dependent parameter which can be calculated from the elastic constitutive equation and the tensile strength \( X_t \).
Interaction of failure modes

In close analogy, we can then define the damage variables for all other failure modes:

\[
\begin{align*}
    d_{fc} &= \frac{\bar{E}_u^{fc}}{\bar{E}_{fc}} \left( \frac{\bar{E}_u - \bar{E}_0^{fc}}{\bar{E}_u^{fc} - \bar{E}_0^{fc}} \right) \\
d_{mt} &= \frac{\bar{E}_u^{mt}}{\bar{E}_{mt}} \left( \frac{\bar{E}_u - \bar{E}_0^{mt}}{\bar{E}_u^{mt} - \bar{E}_0^{mt}} \right) \\
d_{mc} &= \frac{\bar{E}_u^{mc}}{\bar{E}_{mc}} \left( \frac{\bar{E}_u - \bar{E}_0^{mc}}{\bar{E}_u^{mc} - \bar{E}_0^{mc}} \right)
\end{align*}
\]

The stiffness of the lamina is affected by all failure modes. For example, the stiffness along the fiber direction is reduced through both fiber tensile failure and fiber compression failure. This accumulation of damage is then taken into account through three “global damage variables”:

- **Fiber damage**: \( d_f = 1 - (1-d_{ft})(1-d_{fc}) \)
- **Matrix damage**: \( d_m = 1 - (1-d_{mt})(1-d_{mc}) \)
- **Shear damage**: \( d_s = 1 - (1-d_{ft})(1-d_{fc})(1-d_{mt})(1-d_{mc}) \)
Damaged compliance matrix

Recall the **undamaged compliance matrix** for an orthotropic lamina

\[
\begin{align*}
\begin{bmatrix}
\varepsilon_{11} \\
\varepsilon_{22} \\
\varepsilon_{12}
\end{bmatrix}
&= 
\begin{bmatrix}
\frac{1}{E_1} & -\nu_{21} & 0 \\
-\nu_{12} & \frac{1}{E_2} & 0 \\
0 & 0 & \frac{1}{2G_{12}}
\end{bmatrix}
\begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{12}
\end{bmatrix}
\end{align*}
\]

With the help of the global damage variables, the corresponding **damaged compliance matrix** is then defined as

\[
\begin{align*}
\begin{bmatrix}
\varepsilon_{11} \\
\varepsilon_{22} \\
\varepsilon_{12}
\end{bmatrix}
&= 
\begin{bmatrix}
\frac{1}{(1-d_f)E_1} & -\nu_{21} & 0 \\
-\nu_{12} & \frac{1}{(1-d_m)E_2} & 0 \\
0 & 0 & \frac{1}{2(1-d_s)G_{12}}
\end{bmatrix}
\begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{12}
\end{bmatrix}
\end{align*}
\]
Damaged stiffness matrix

The **damaged stiffness matrix** is then given by the inverse of the damaged compliance matrix,

\[
\begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{12}
\end{bmatrix} = \frac{1}{D} \begin{bmatrix}
(1 - d_f)E_1 & (1 - d_f)(1 - d_m)\nu_{21}E_1 & 0 \\
(1 - d_f)(1 - d_m)\nu_{12}E_2 & (1 - d_m)E_2 & 0 \\
0 & 0 & 2D(1 - d_s)G_{12}
\end{bmatrix} \begin{bmatrix}
\varepsilon_{11} \\
\varepsilon_{22} \\
\varepsilon_{12}
\end{bmatrix}
\]

with

\[
D = 1 - (1 - d_f)(1 - d_m)\nu_{12}\nu_{21}
\]
Localization of deformation

Up to the point of onset of damage, there is a uniform solution to the boundary value problem for a bar subject to an axial load. However, beyond the point of onset of damage, two types of domains emerge:

- **Damage band** in which the axial strain increases rapidly and where dissipation takes place
- **Elastically unloaded domains** in which the axial strain decreases and where the material remains undamaged
Localization of deformation

In finite element simulations, the deformation will localize in the row of elements where the largest imperfections (of physical or numerical origin) prevail. As a result, the **width of the damage band is set by the element size**. Consequently, the numerical solutions are mesh-size dependent:

Source: I. Lapczyk, J. Hurtardo (Simulia, ppt, 2006)
The energy dissipated in a uniformly-strained single cubic element of edge length $L$ for a given failure mode is

$$\psi_d = L^3 \int_0^{\bar{\varepsilon}_u} \bar{\sigma} d\bar{\varepsilon} = \frac{L^3}{2} \bar{\sigma}_0 \bar{\varepsilon}_u$$
Fracture energy

Assuming that the damage localizes is a single row of elements, the fracture energy per unit area of the created crack would be

\[ G = \frac{\psi_d}{L^2} = \frac{L}{2} \overline{\sigma_0} \overline{\epsilon_u} \]

From a physical point of view, this fracture energy per unit area may be associated with micro cracks that form within a narrow band around the crack (fracture process zone). It is a material property which is independent of the element size.
To ensure the independence of the predicted fracture energy per unit area from the element size, we define the damage model parameter $\bar{\varepsilon}_u$ as a function of the element size

$$\bar{\varepsilon}_u = \bar{\varepsilon}_u[L] = \frac{2G_f}{L\bar{\sigma}_0}$$

while the fracture energy per unit area, $G_f$, is introduced as additional material model parameter.
Regularization

As a result of this regularization, the width of the band of localization still corresponds to a single row of elements (and is hence element size dependent). However, the displacement to fracture $\Delta u_f$ associated with the deformation in this band is now element size independent:

$$\Delta u_f = \overline{\varepsilon}_u L = \frac{2G_f}{\sigma_0}$$

Large strains are shown for visualization purposes, small strains are expected to prevail in reality.
Regularization

**Fine mesh**

- \( L = 0.3 \)
- \( \bar{\varepsilon}_u = \frac{1.0}{0.3} - 1 = 2.33 \)
- \( \Delta u_f = \bar{\varepsilon}_u L = 0.7 \)

**Coarse mesh**

- \( L = 0.6 \)
- \( \bar{\varepsilon}_u = \frac{1.3}{0.6} - 1 = 1.17 \)
- \( \Delta u_f = \bar{\varepsilon}_u L = 0.7 \)

Large strains are shown for visualization purposes, small strains are expected to prevail in reality.

- \( L = 0.6 \)
- \( \bar{\varepsilon}_u = \frac{2.0}{0.6} - 1 = 2.33 \)
- \( \Delta u_f = \bar{\varepsilon}_u L = 1.4 \)
Solution after regularization

Source: I. Lapczyk, J. Hurtardo (Simulia, ppt, 2006)
Summary: Lamina Material Parameters

Aside from basic characteristics such as the lamina orientation, density and thickness, the following material properties must be specified:

**Undamaged Elasticity**
- Modulus along fiber direction
- Modulus along transverse direction
- In-plane Poisson’s ratio
- In-plane shear modulus

\[ E_1, E_2, v_{12}, G_{12} \]

**Damage Initiation**
- Tensile strength along fiber direction
- Compressive strength along fiber direction
- Tensile strength along transverse direction
- Compression strength along transverse direction
- In-plane shear strength
- Out-of-plane shear strength

\[ X_t, X_c, Y_t, Y_c, S_L, S_T \]

**Ultimate Failure**
- Fracture energy per unit area for fiber tensile failure
- Fracture energy per unit area for fiber compression failure
- Fracture energy per unit area for matrix tensile failure
- Fracture energy per unit area for matrix compression failure

\[ G_{ft}, G_{fc}, G_{mt}, G_{mc} \]
Modeling Choices & Limitations

- **intralaminar**
  - element: layered (thin/thick) shells
  - one solid element per ply
  - material: plasticity / damage models

**layered thin shell elements**
- numerical „cheap“ (thickness does not influence the critical time step size)
- combination of single layers to sub-laminates
- no stresses in thickness dir. (no delamination)

**layered thick shell elements**
- 3D stress state
- combination of single layers to sub-laminates
- thickness influences the critical time step size

**solid elements**
- 3D stress state
- one element for every single layer (no layering)
  \[\rightarrow\] numerical „expensive“

**interlaminar (delamination)**
- cohesive elements
- tiebreak contacts

Source: Stefan Hartmann, DYNAmore GmbH, Composite Berechnung in LS-DYNA, Stuttgart (2013)
Composite Modeling Summary

- Laminate properties are computed automatically by the FE code (after user specifies the lay-up)
- Material properties need to be provided by the user at the lamina level


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Lecture #13 – Fall 2015
Reading Materials for Lecture #13


• A. Matzenmiller, J. Lubliner, R.L. Taylor (1995), A constitutive model for anisotropic damage in fiber-composites, Mechanics of Materials 20, 125-152


• S.J. Hiermaier (2008), Structures under crash and Impact, Springer.