Lecture #2:
Split Hopkinson Bar Systems

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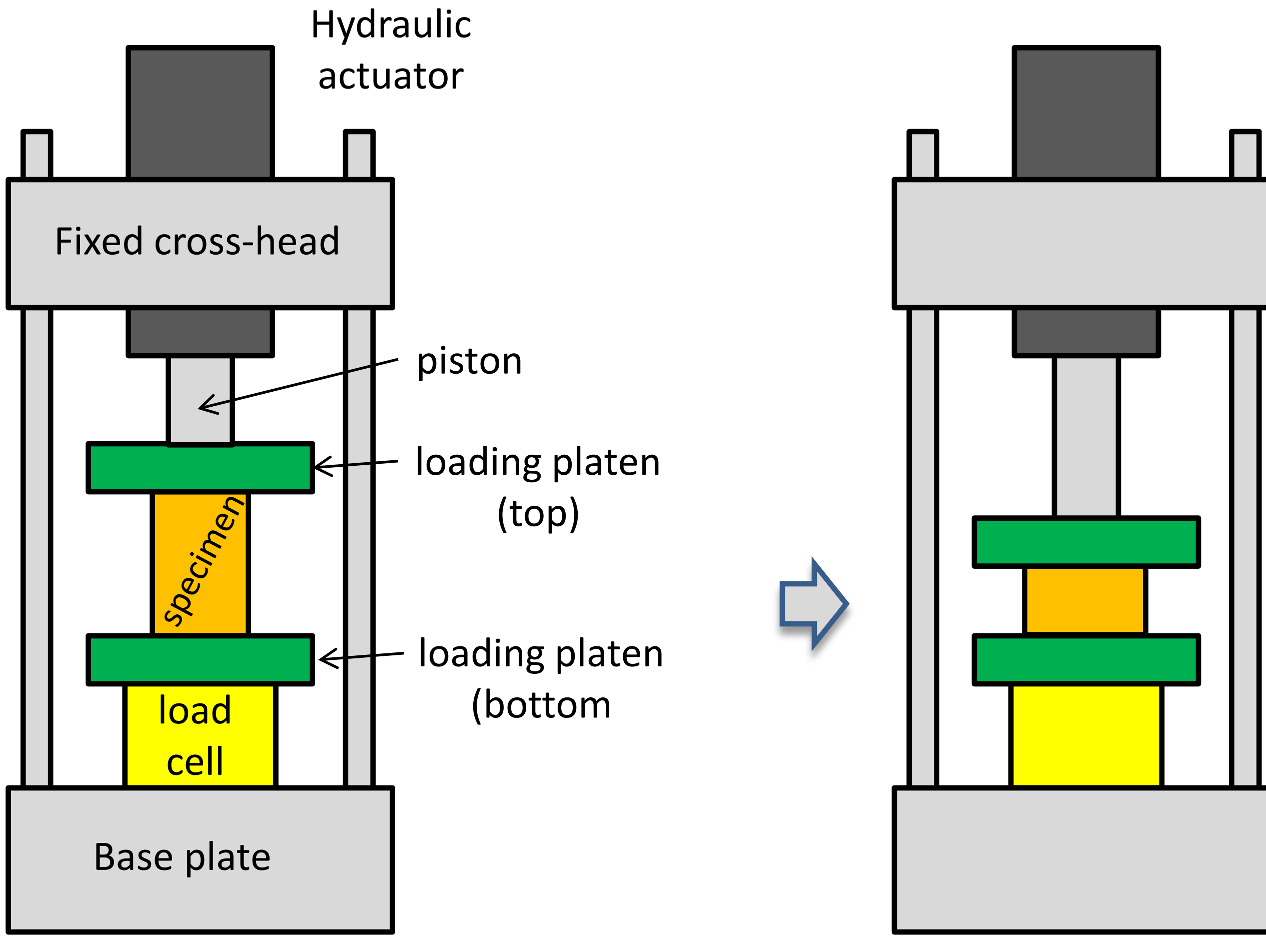
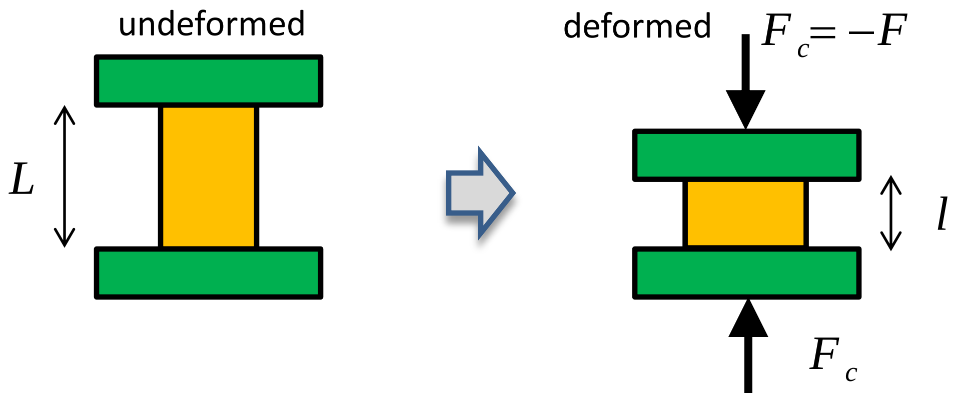
Uniaxial Compression Testing

- Useful experiment to characterize the flow behavior of materials at large strains



Uniaxial Compression Testing



F_c = applied compression force

l = current length

L_0 = initial length

l = current length

L_0 = initial length

- Axial **strain** definitions

$$\varepsilon_{eng} = \frac{l}{L} - 1 \text{ (engineering or nominal strain)}$$

$$\varepsilon = \ln[1+\varepsilon_{eng}] \quad \text{(true or logarithmic strain)}$$

$\varepsilon > 0$ (extension)
$\varepsilon < 0$ (shortening)

- Axial **stress** definitions

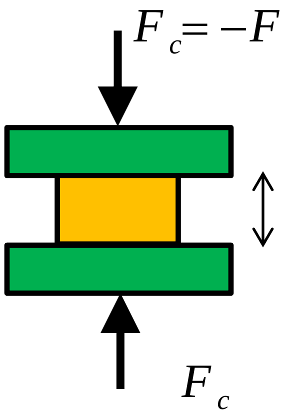
$$\sigma_{eng} = \frac{F}{A_0} \text{ (engineering or nominal stress)}$$

$$\sigma = \frac{F}{A} = \sigma_{eng}[1+\varepsilon_{eng}] \quad \text{(true stress)}$$

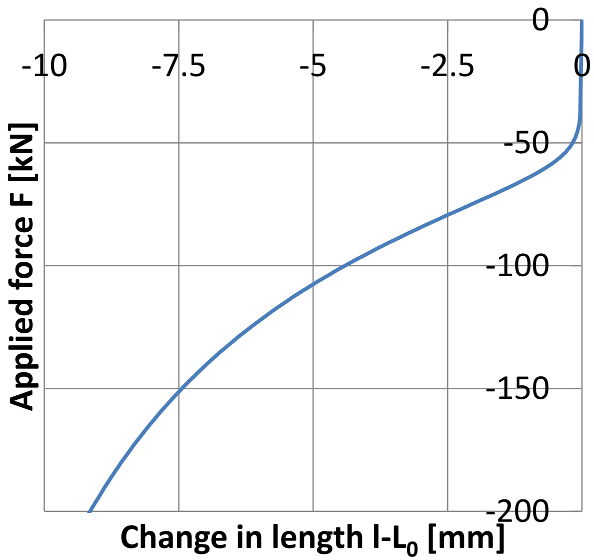
Only valid for incompressible materials

$\sigma > 0$ (tension)
$\sigma < 0$ (compression)

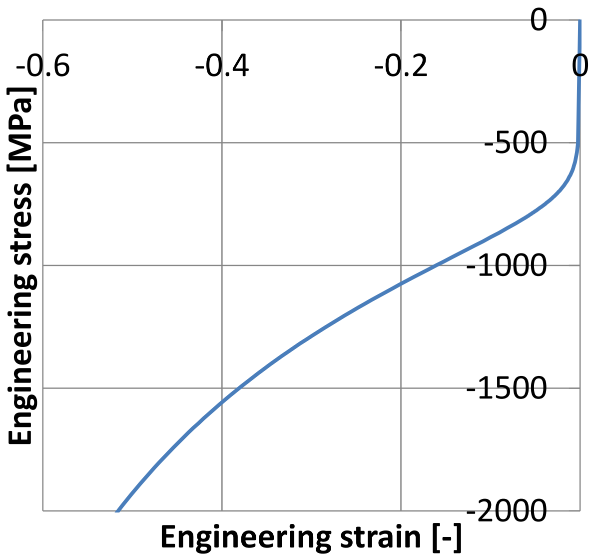
Uniaxial Compression Testing



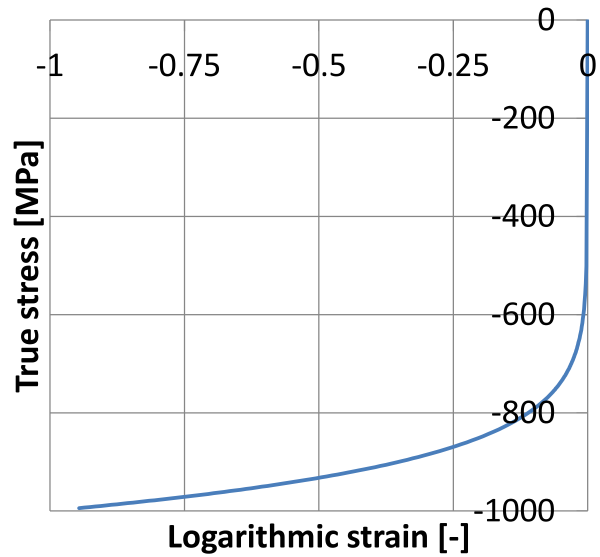
Force-displacement curve



Engineering stress-strain curve

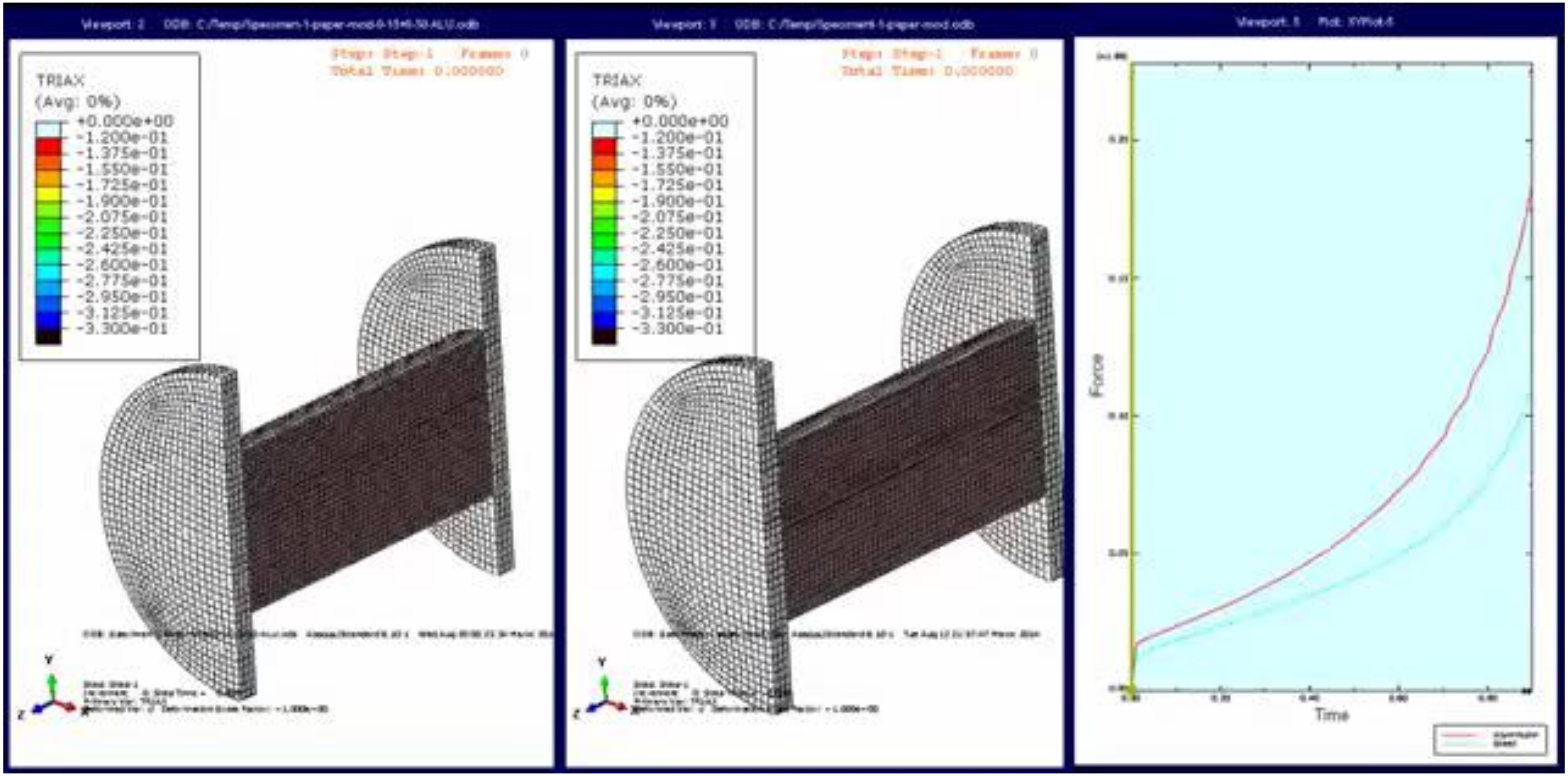


True stress-strain curve



Uniaxial Compression Testing

- Numerical Simulation (→ also topic of computer lab #2)

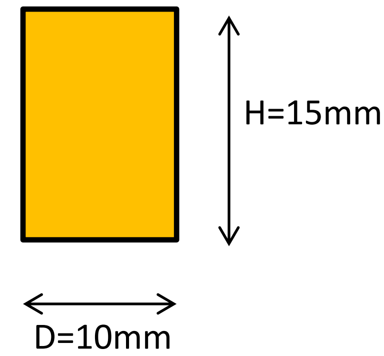


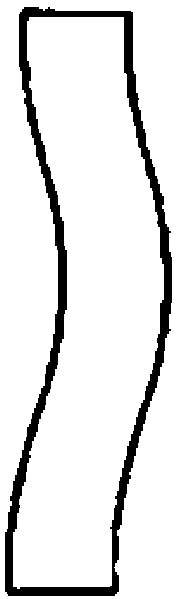
Source: https://www.youtube.com/watch?feature=player_detailpage&v=OzzWc-SfF-w

Uniaxial Compression Testing

- Experimental challenges

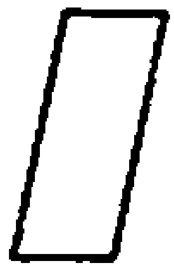
- Recommended size for metals testing:





Plastic buckling

- H/D too large

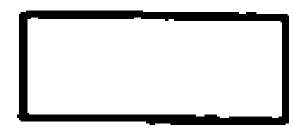


Shear buckling

- Poor alignment
- anisotropy

barreling

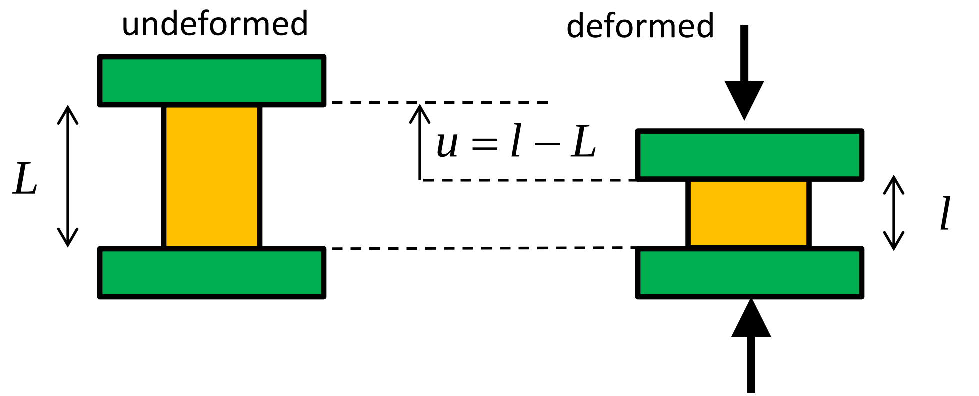
- Friction too high
- H/D too small



optimal

- Excellent lubrication
- H/D ~ 1.5

Strain rate in a compression experiment



$u > 0$ (extension)
$u < 0$ (shortening)

- Nominal strain rate

$$\varepsilon_{eng} = \frac{l}{L} - 1 \quad \Rightarrow \quad \dot{\varepsilon}_{eng} = \frac{\dot{l}}{L} = \frac{\dot{u}}{L}$$

- True strain rate

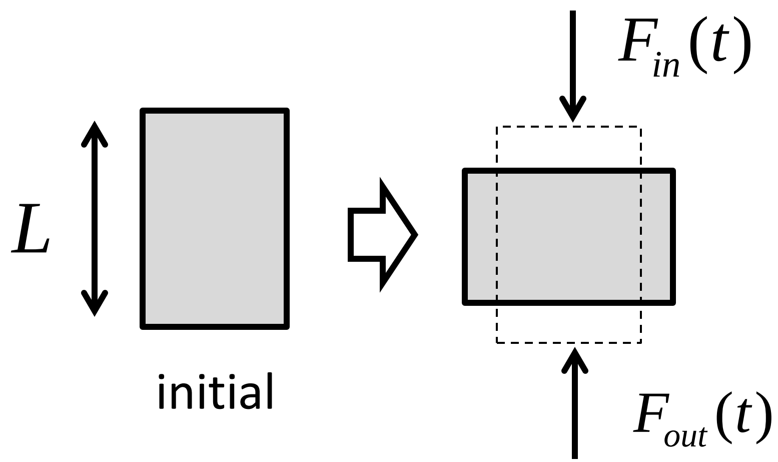
$$\varepsilon = \ln[1 + \varepsilon_{eng}] \quad \Rightarrow \quad \dot{\varepsilon} = \frac{\dot{\varepsilon}_{eng}}{1 + \varepsilon_{eng}}$$

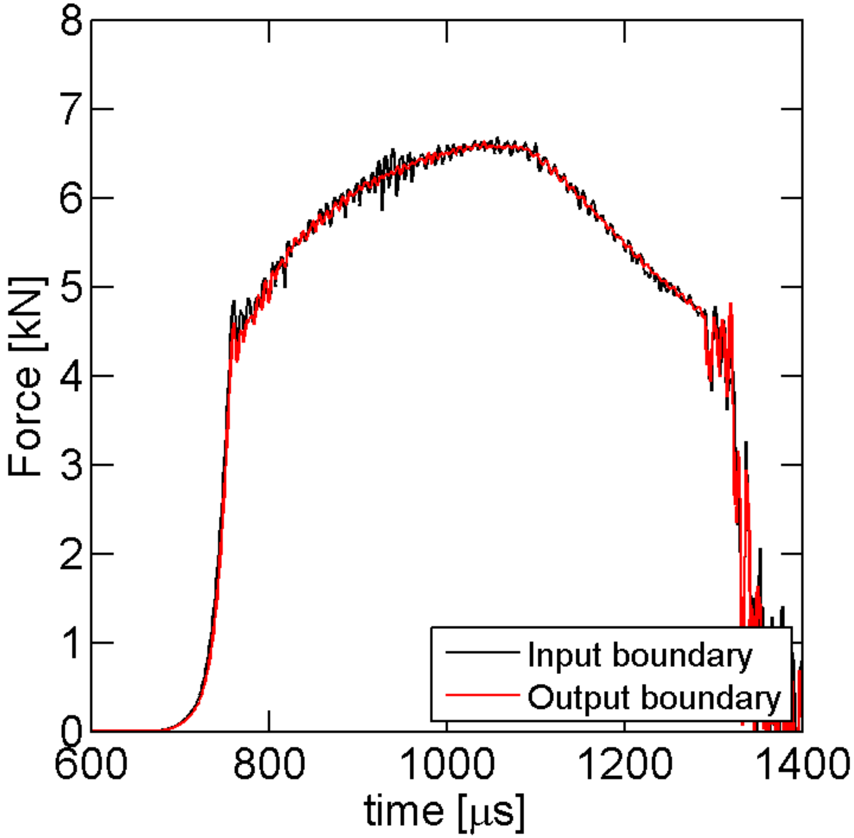
- Equiv. strain rate

$$\dot{\varepsilon} = \frac{|\dot{\varepsilon}_{eng}|}{1 + \varepsilon_{eng}}$$

Principle of Quasi-static Equilibrium

Compression of a cylindrical specimen

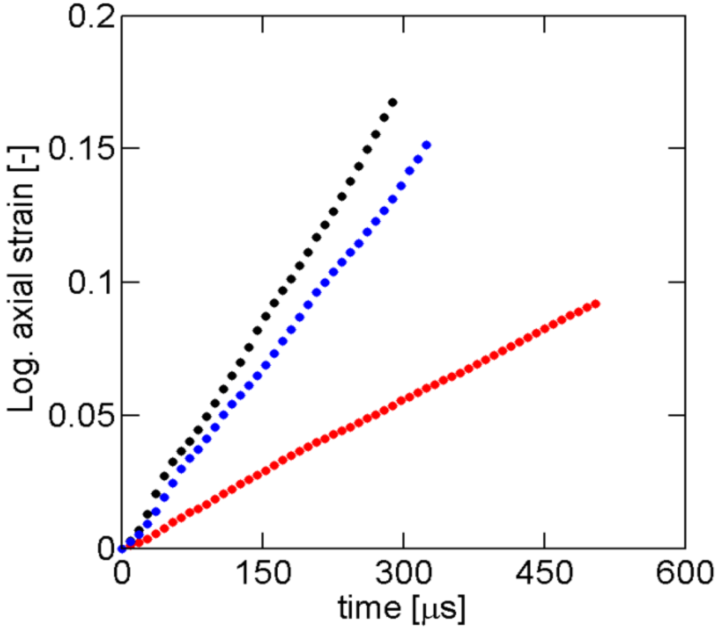




Quasi-static equilibrium

$$F_{in}(t) \cong F_{out}(t)$$

Time scale #1



Strain at the end of the experiment: $\varepsilon_{\max}$

Average strain rate (over time): $\dot{\varepsilon}_{av}$

⇒ Duration of experiment $T = \dfrac{\varepsilon_{\max}}{\dot{\varepsilon}_{av}}$

Note: T is independent of specimen dimensions!

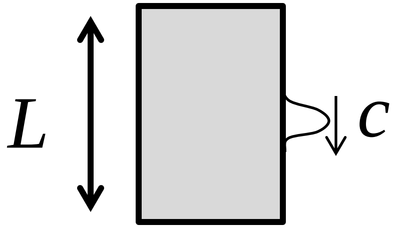
Example:

$$\left.\begin{aligned} \varepsilon_{\max} &= 0.15 \\ \dot{\varepsilon}_{av} &= 500/s \end{aligned}\right\} \quad T = \frac{0.15}{500} = 0.0003s = 0.3ms = 300\mu s$$

Time scale #2

Wave propagation speed: $c = \sqrt{\frac{E}{\rho}}$

Specimen length: L



Wave travel time $\Delta t = \frac{L}{c}$

Note: Δt does depend on specimen dimensions!

Example:

$$\left.\begin{aligned} c &\cong 5000 m/s \\ L &= 10 mm \end{aligned}\right\} \quad \Delta t = \frac{10}{5 \times 10^6} = 2 \times 10^{-6} s = 2 \mu s$$

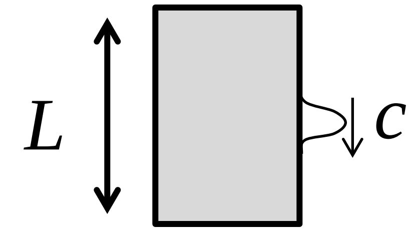
Principle of Quasi-static Equilibrium

Long time scale:

⇨ Duration of experiment $T = \frac{\varepsilon_{\max}}{\dot{\varepsilon}_{av}}$

Short time scale:

⇨ Wave travel time $\Delta t = \frac{L}{c}$



Condition for quasi-static equilibrium:
(when testing an elasto-plastic material)

$$F_{in}(t) \cong F_{out}(t) \iff \Delta t << T$$

Principle of Quasi-static Equilibrium

Condition of quasi-static equilibrium

$$\Delta t << T \quad \Leftrightarrow \quad \frac{L}{c} << \frac{\varepsilon_{\max}}{\dot{\varepsilon}_{av}}$$

Example for challenging experiments (w/ regards to quasi-static equilibrium)

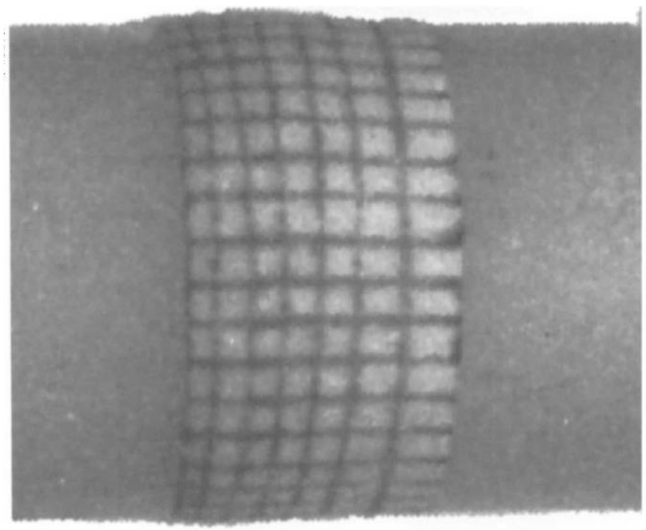
– Brittle materials, e.g. ceramics $\varepsilon_{\max} \sim 0.01$

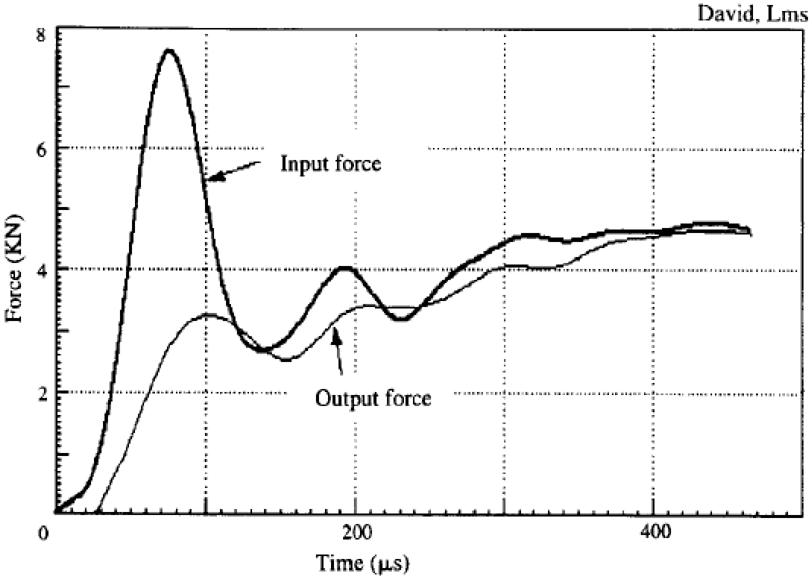
– Soft materials, e.g. polymers $c \sim 1000 m/s$

– Materials of coarse microstructure, e.g. metallic foams $L >> 1mm$

Dynamic testing of foams

Example for challenging experiments (w/ regards to quasi-static equilibrium)

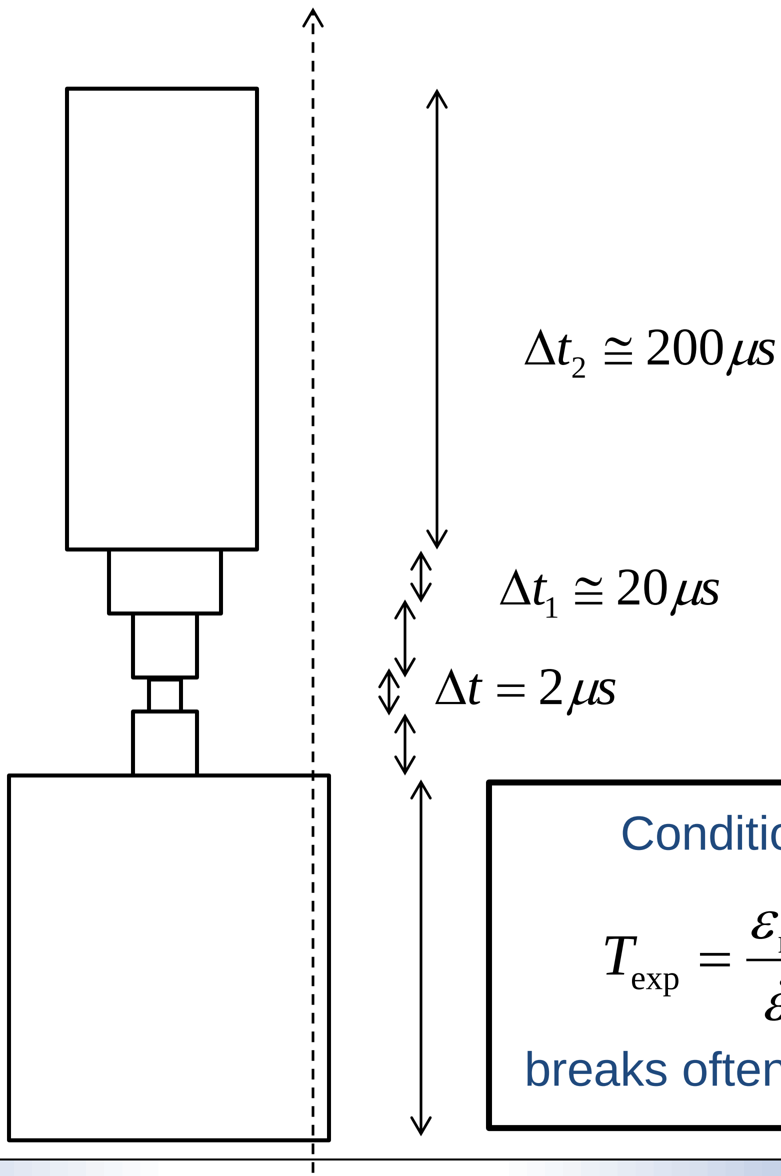




Zhao et al. (1997)

LIMITATION OF UNIVERSAL TESTING MACHINES

- Vibration issues



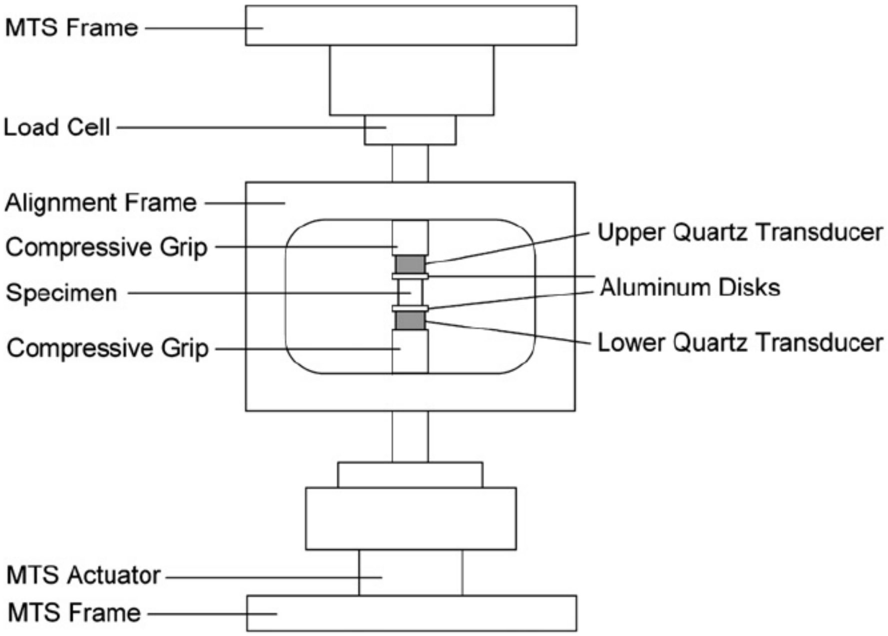
Condition of validity

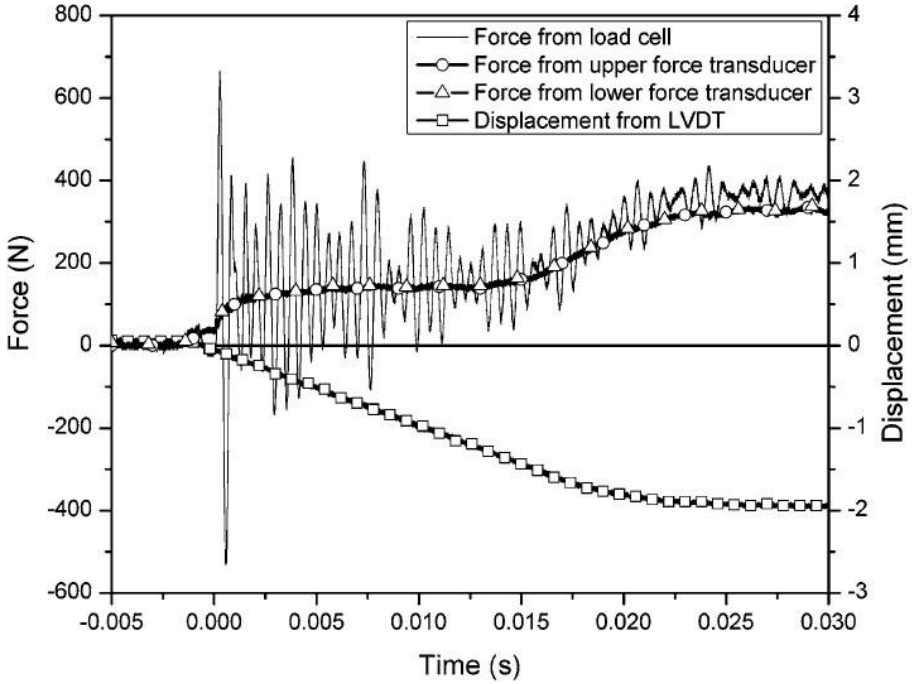
$$T_{\exp} = \frac{\varepsilon_{\max}}{\dot{\varepsilon}_{av}} >> \max[\Delta t_i]$$

breaks often down at around 10/s

LIMITATION OF UNIVERSAL TESTING MACHINES

Ringing of conventional load cell



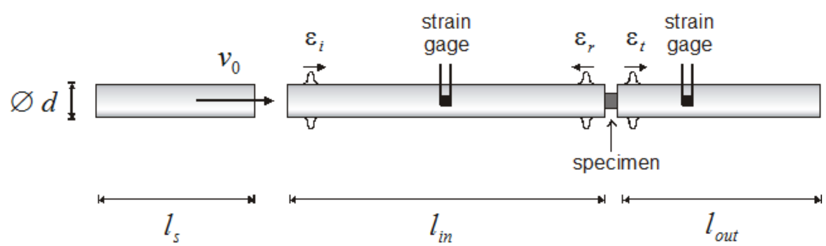


SEPARATION OF TIME SCALES

Characteristic time scale of testing system Δt_{sys}

versus ⇔

Duration of experiment $T_{\exp} = \dfrac{\varepsilon_{\max}}{\dot{\varepsilon}_{av}}$



Universal testing machines

SHPB

$\Delta t_{sys} << T_{\exp}$

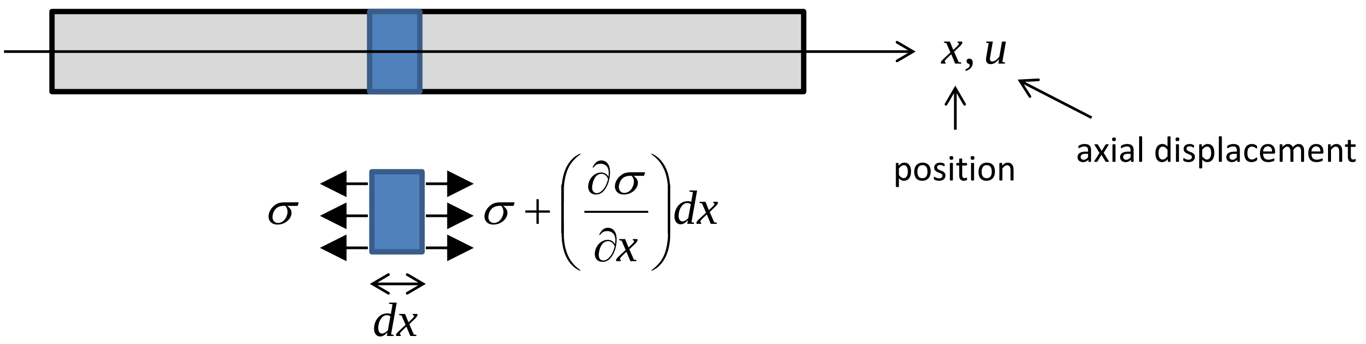
$\Delta t_{sys} \geq T_{\exp}$

$\dot{\varepsilon}\,[s^{-1}]$

10^{-3} 10^{-2} 10^{-1} 1 10 10^{2} 10^{3} 10^{4}

Wave Equation

- Derived wave equation for bars under the assumption of uniaxial stress



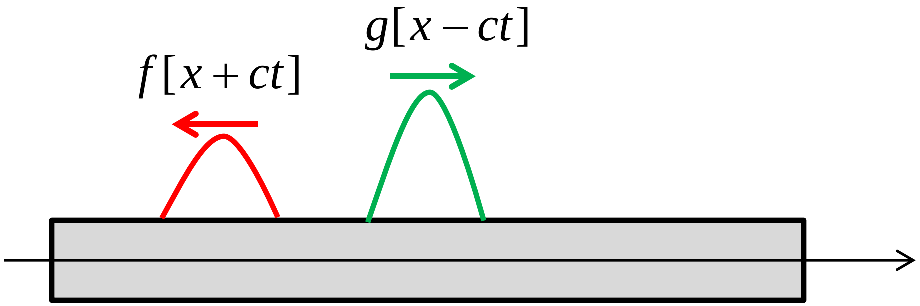
Differential equation for axial displacement u[x,t]:

$$c^2\left(\frac{\partial^2 u}{\partial x^2}\right) - \frac{\partial^2 u}{\partial t^2} = 0$$

- longitudinal elastic **wave velocity**: $c = \sqrt{\frac{E}{\rho}}$
- the **particle velocity**: $v[x,t] = \dot{u}[x,t] = \frac{\partial u}{\partial t}$

General Solution

- General solution of wave equation



$$c^2\left(\frac{\partial^2 u}{\partial x^2}\right)-\frac{\partial^2 u}{\partial t^2}=0$$

⇨

- displacement:

$$u[x,t]=\underbrace{f[x+ct]}_{\text{Leftward traveling}}+\underbrace{g[x-ct]}_{\text{rightward traveling}}$$

- strain

$$\varepsilon[x,t]=u'[x,t]=\frac{\partial f}{\partial x}+\frac{\partial g}{\partial x}=\varepsilon^l[x,t]+\varepsilon^r[x,t]$$

- particle velocity

$$v[x,t]=\dot{u}[x,t]=c\frac{\partial f}{\partial x}-c\frac{\partial g}{\partial x}=c\left(\varepsilon^l[x,t]-\varepsilon^r[x,t]\right)$$

Elastic Wave Velocities

- Particle velocity depends on applied loading, while the **wave velocity** is an intrinsic material property

Material	Density [g/cm³]	Young's modulus [GPa]	Wave velocity [m/s]
Air			340
Steel	7.8	210	5188
Aluminum	2.7	70	5091
Magnesium	1.7	44	5087
Tungsten	19.3	400	4552
Lead	10.2	14	1171
PMMA	1.2	3	1581
Concrete	3	30	3162

All values are rough estimates and may vary depending on the exact material composition and environmental conditions

Hopkinson Bar Experiment

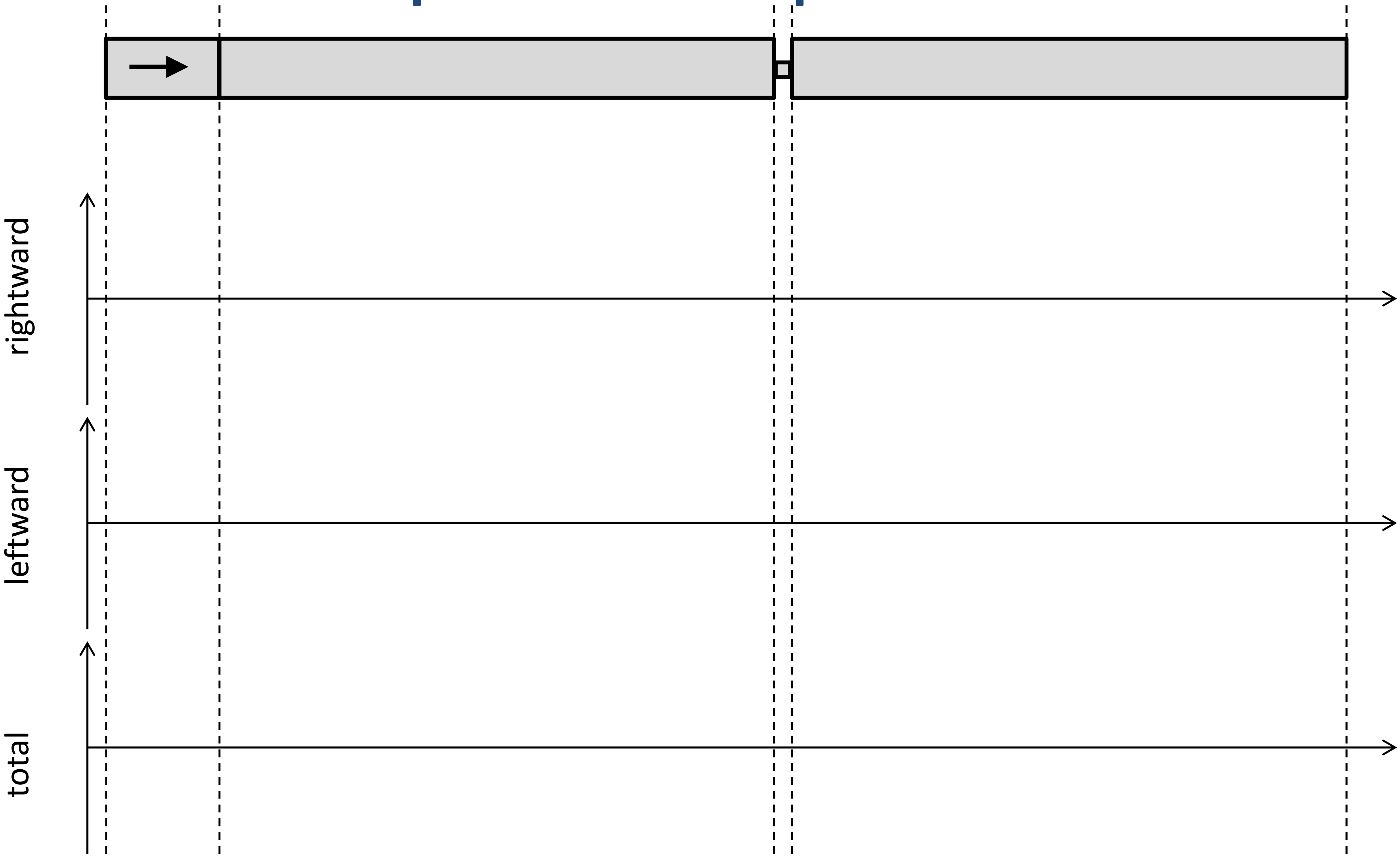
Typical system characteristics:

- Striker bar length: L_s =1000 *mm*
- Input bar length: L_i= 3000 *mm*
- Output bar length: L_o= 3000 *mm*
- Bar diameter: D= 20 *mm*

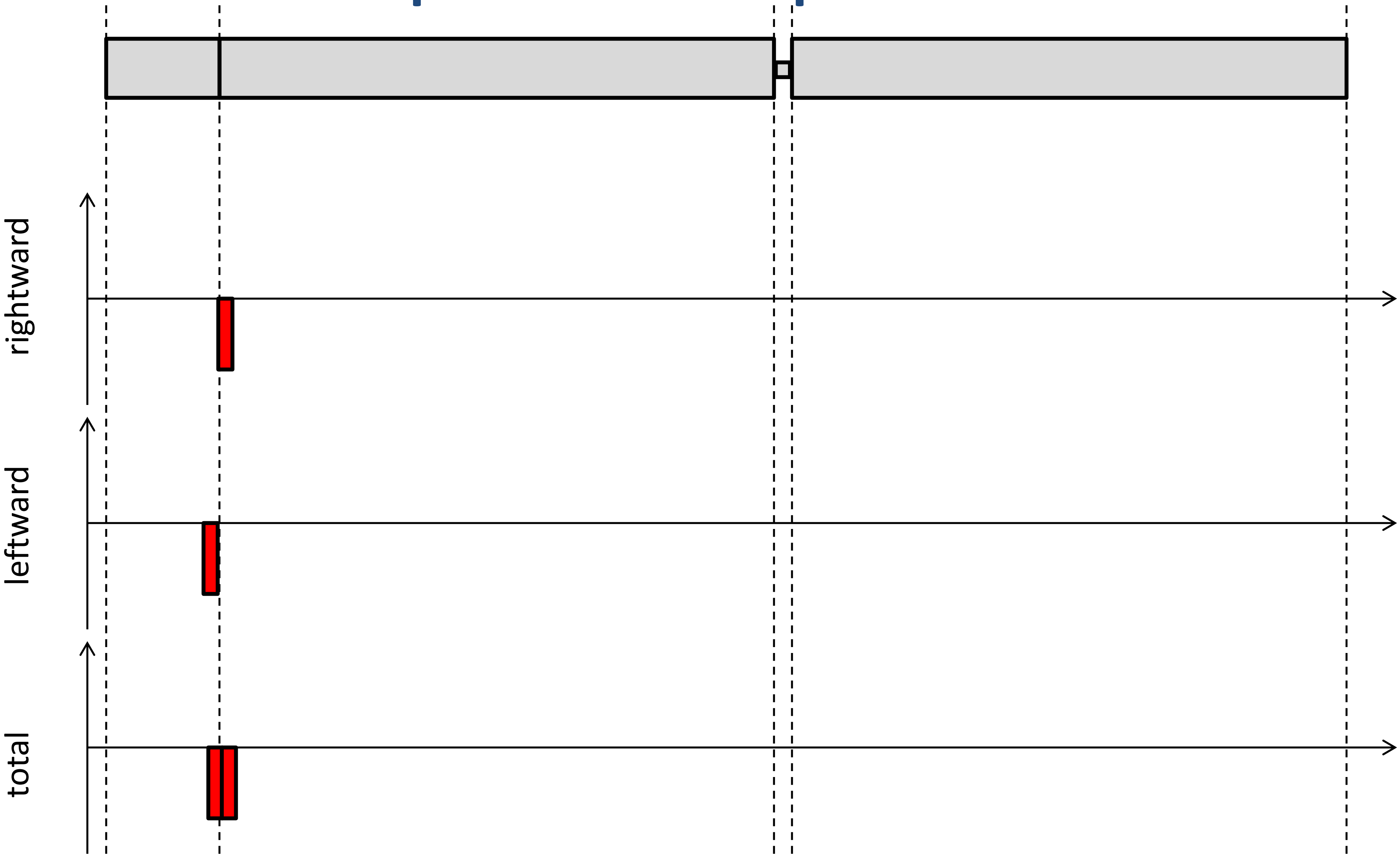
Specimen characteristics (for simplified theoretical analysis):

- Ideal plastic, constant force

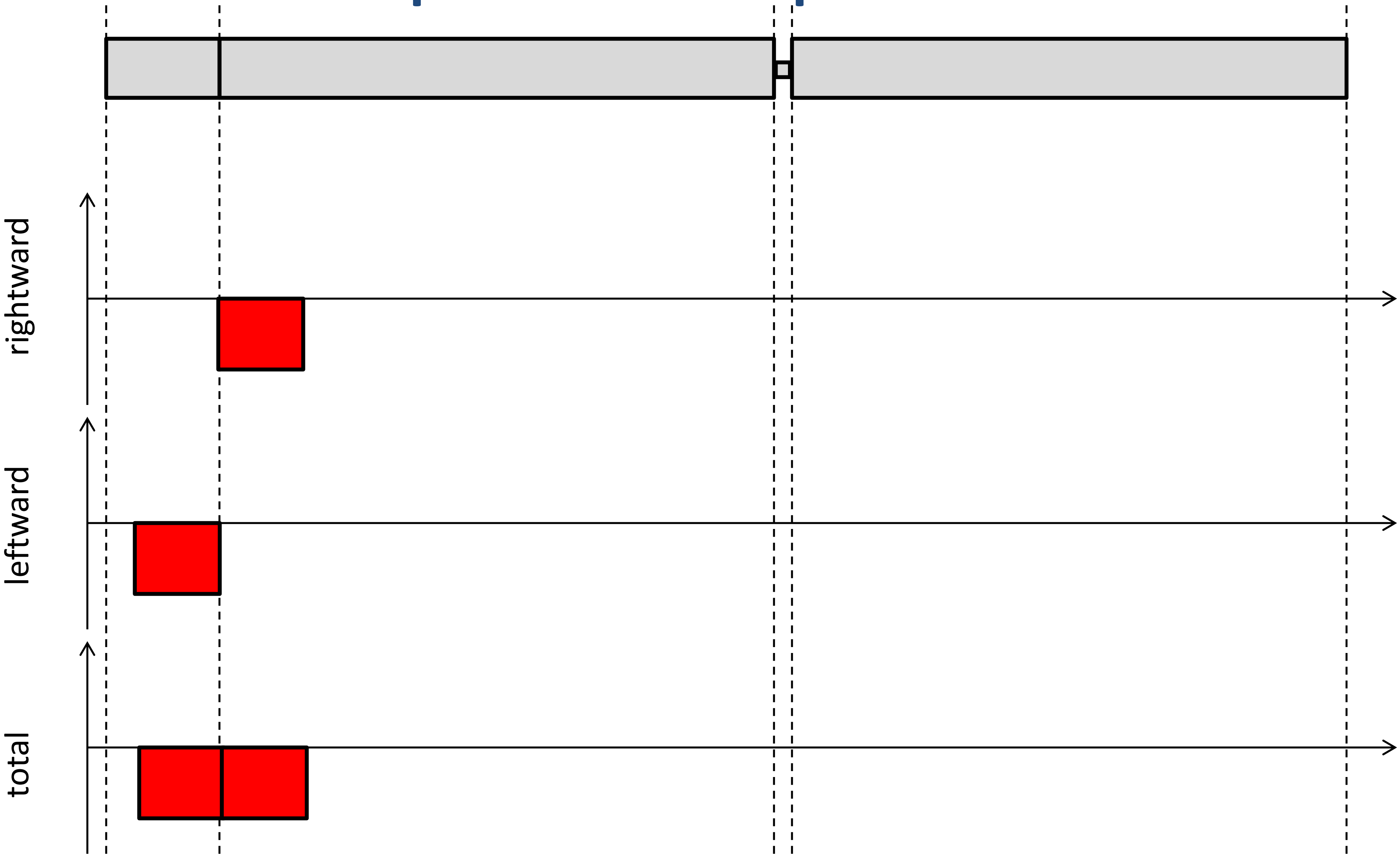
Hopkinson Bar Experiment



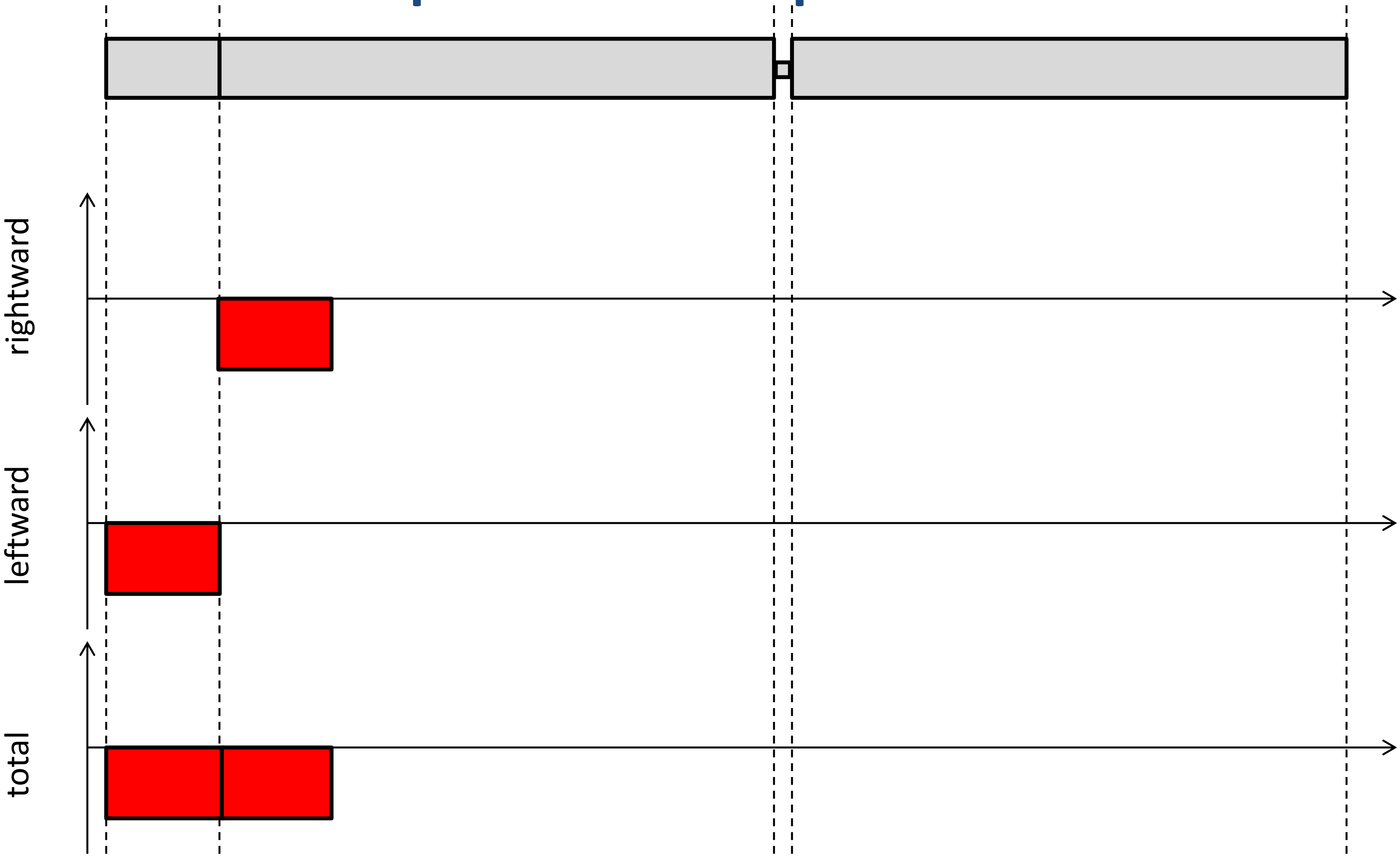
Hopkinson Bar Experiment



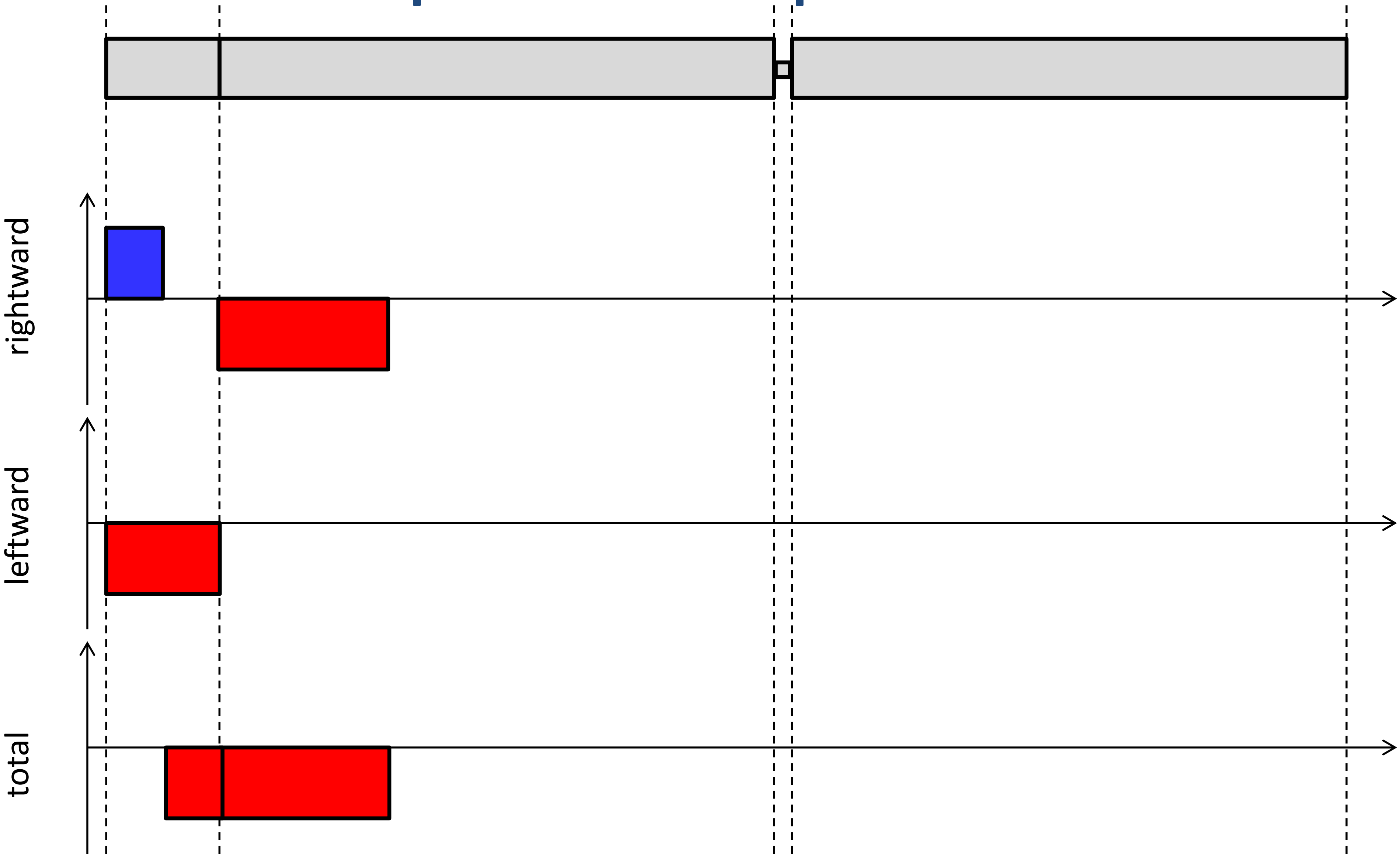
Hopkinson Bar Experiment



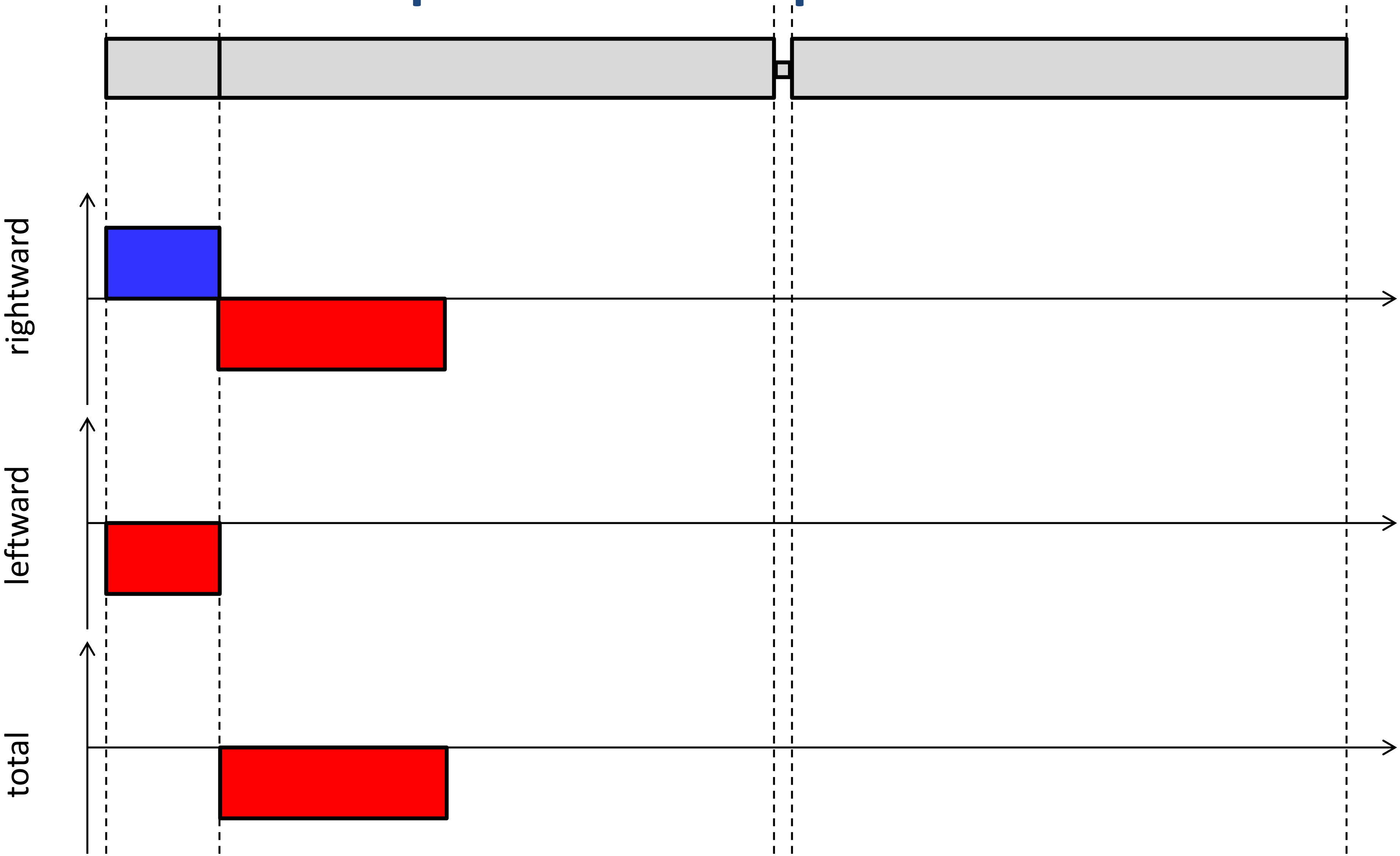
Hopkinson Bar Experiment



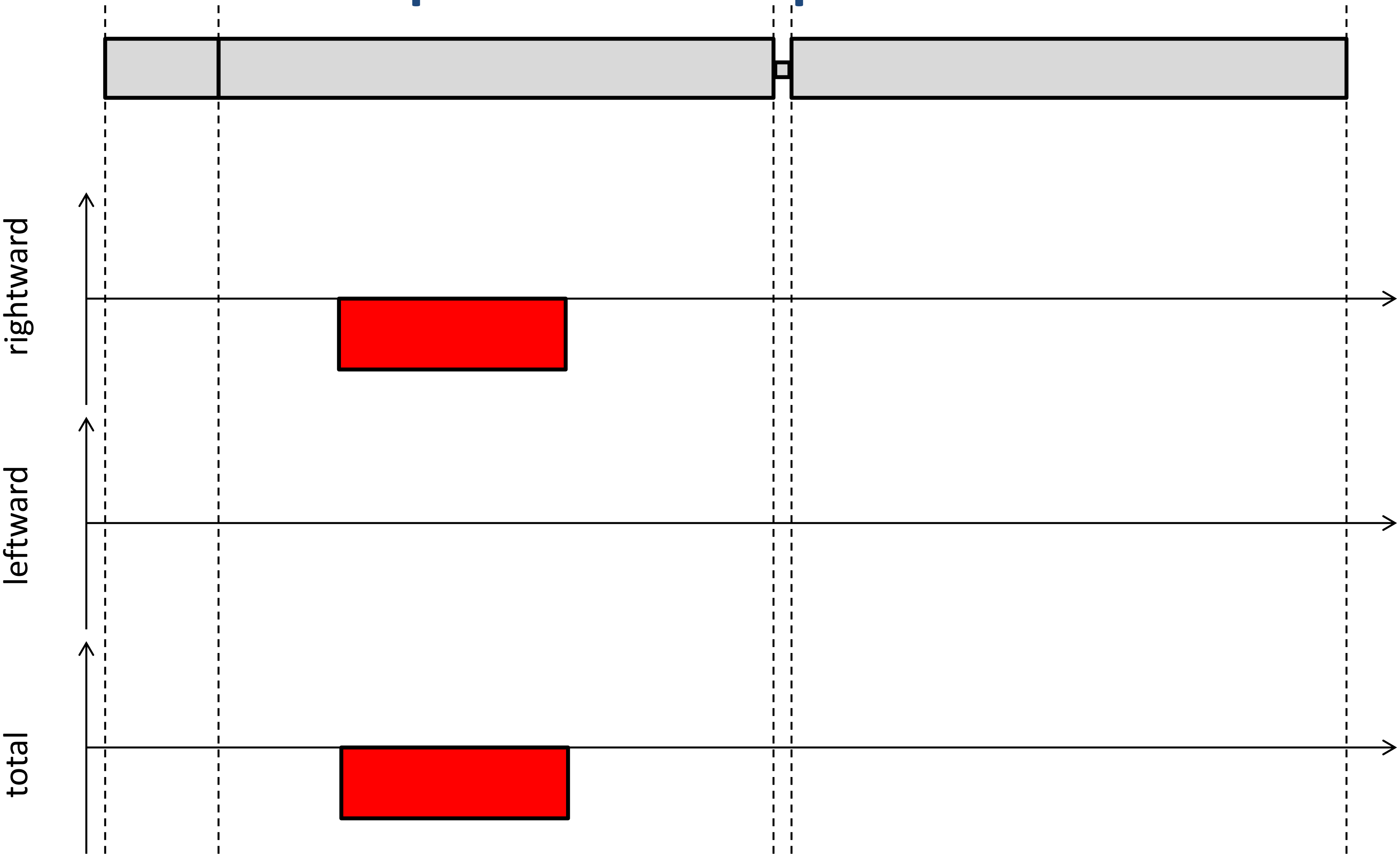
Hopkinson Bar Experiment



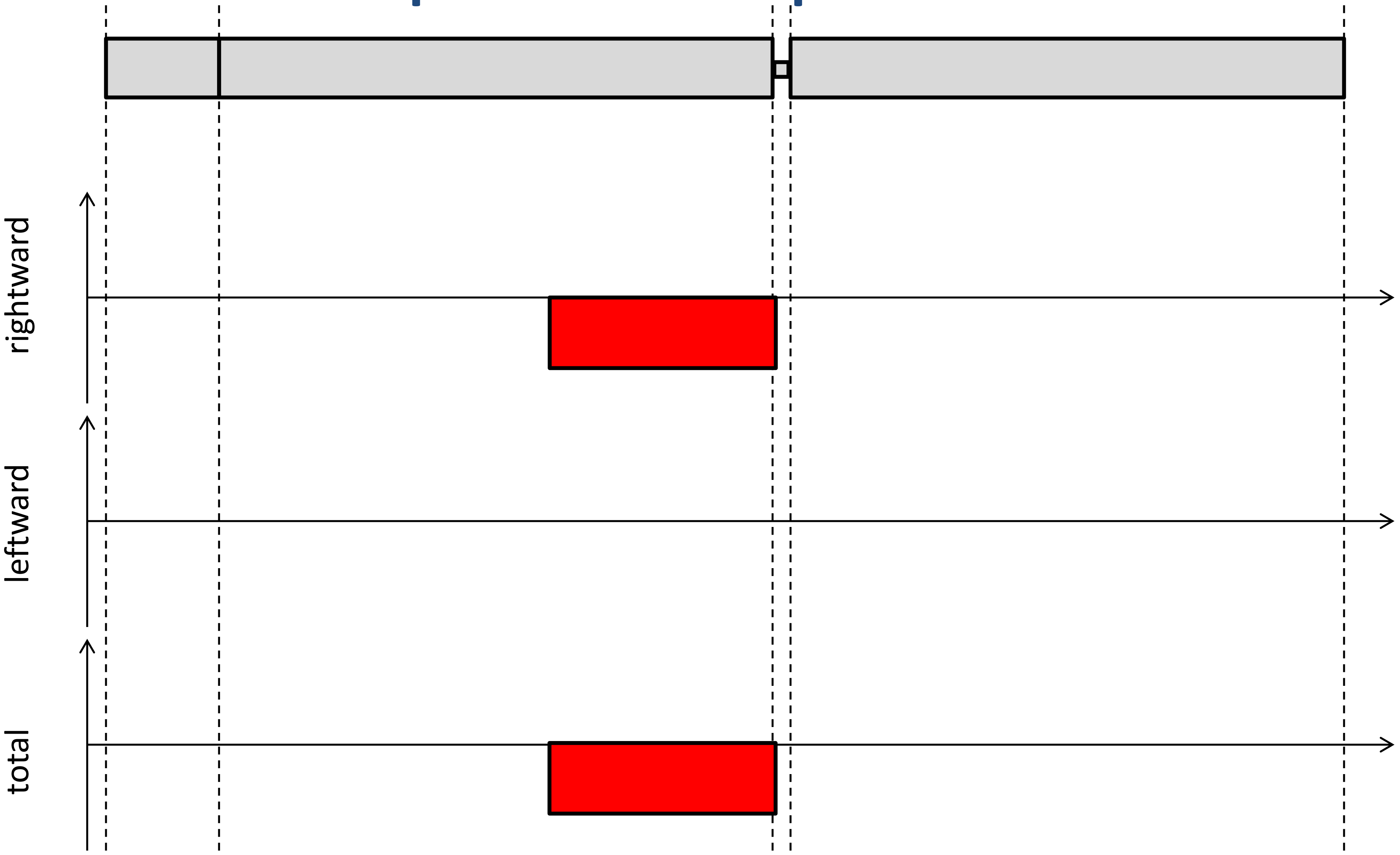
Hopkinson Bar Experiment



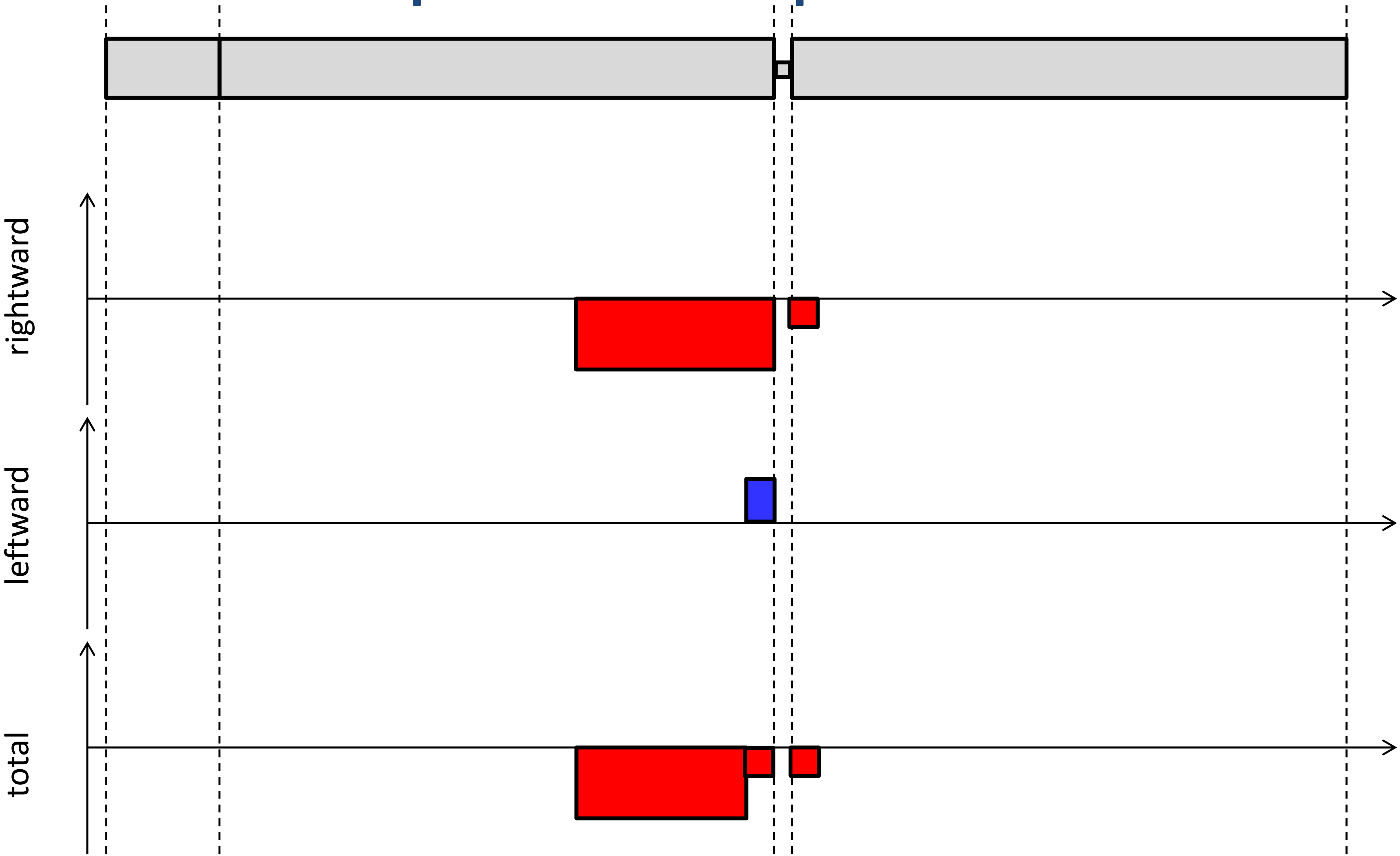
Hopkinson Bar Experiment



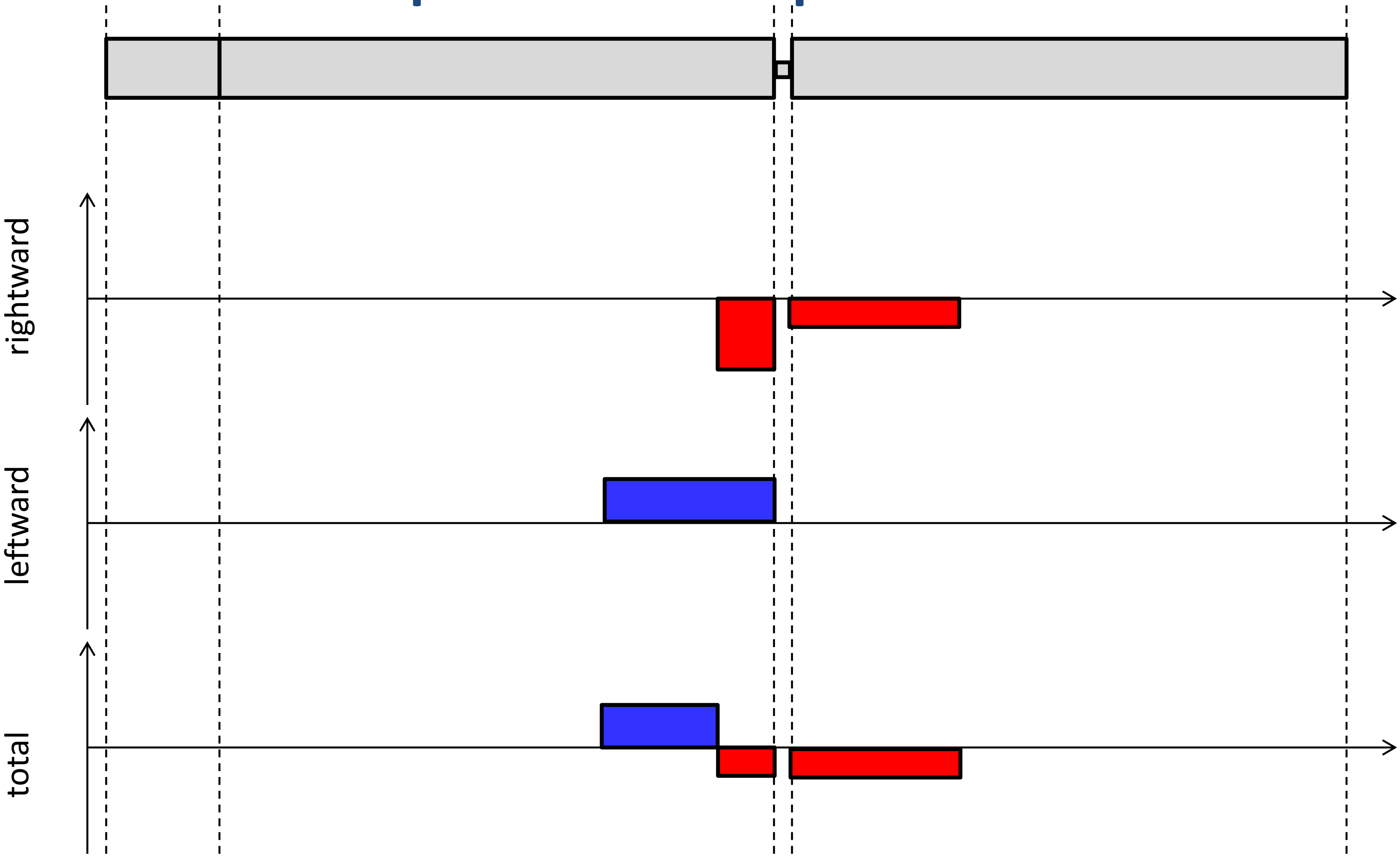
Hopkinson Bar Experiment



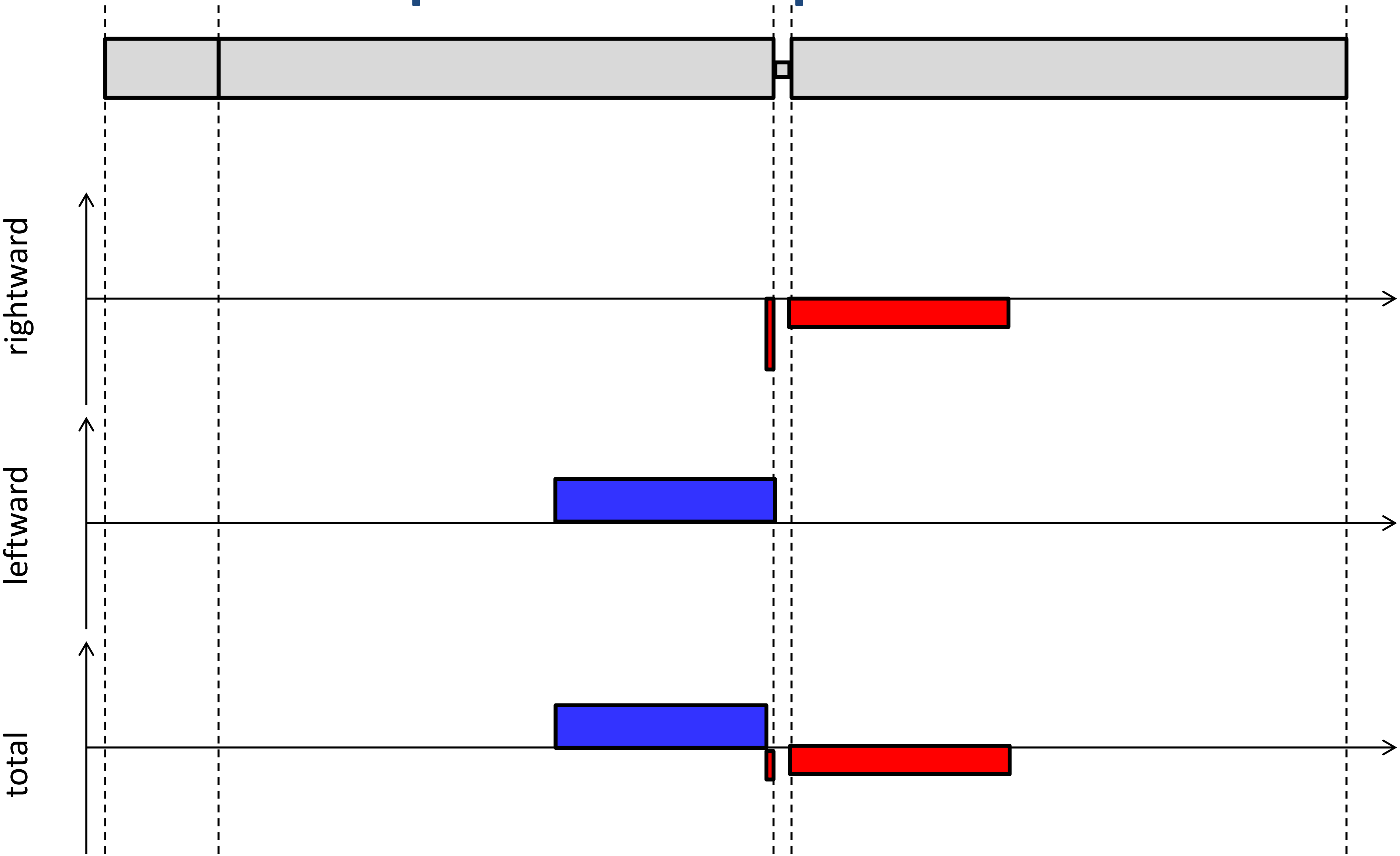
Hopkinson Bar Experiment



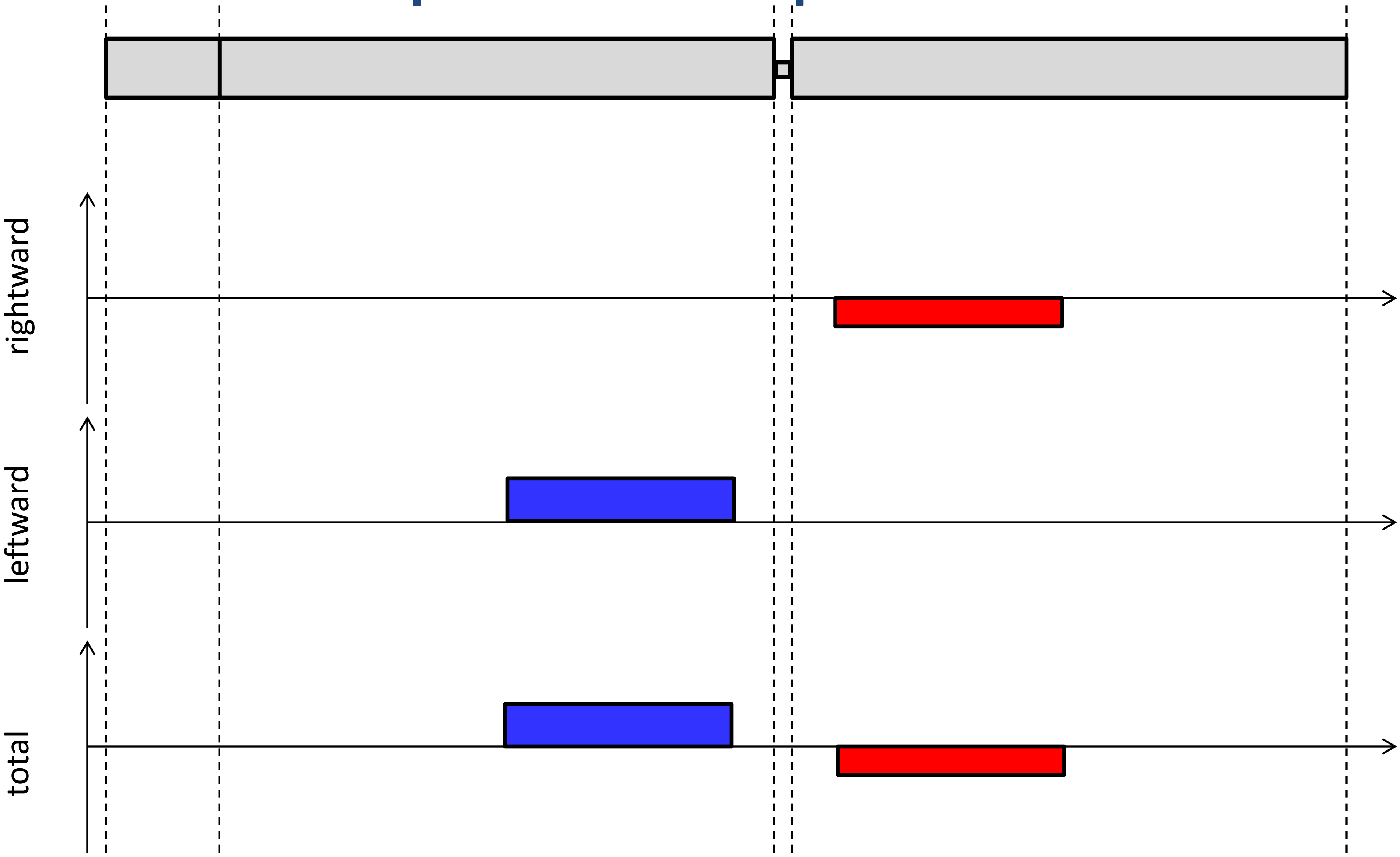
Hopkinson Bar Experiment



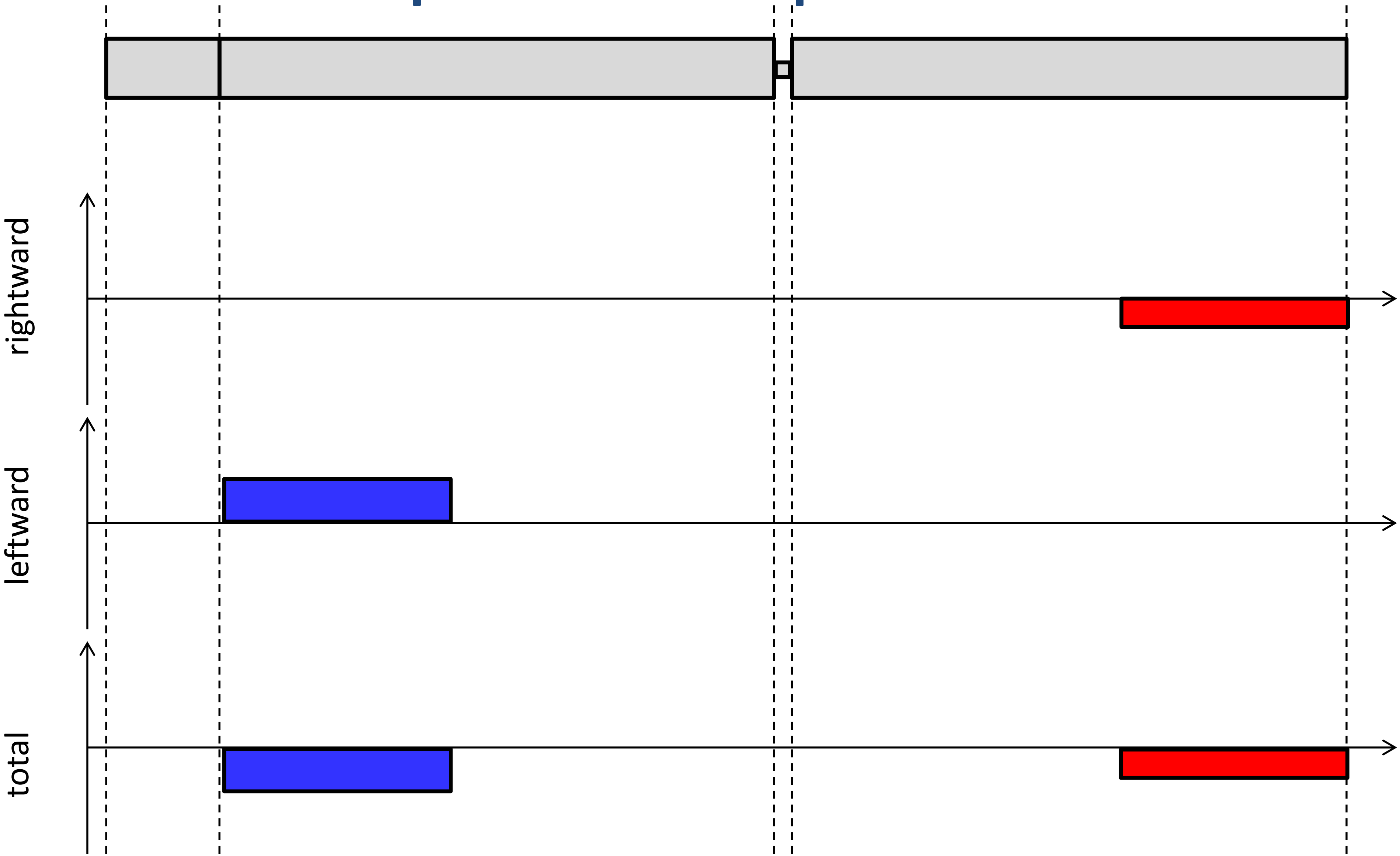
Hopkinson Bar Experiment



Hopkinson Bar Experiment

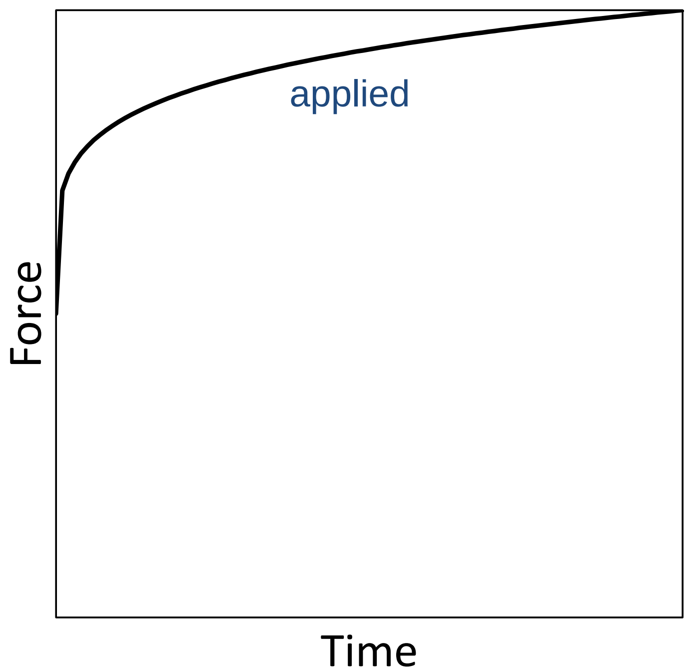


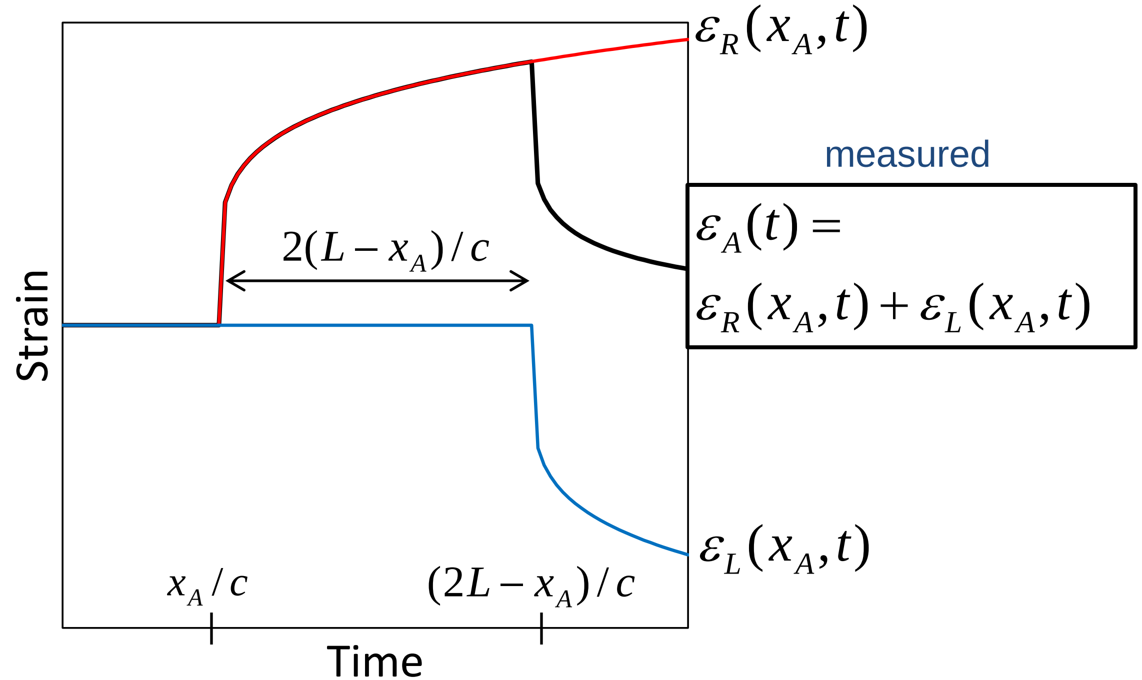
Hopkinson Bar Experiment

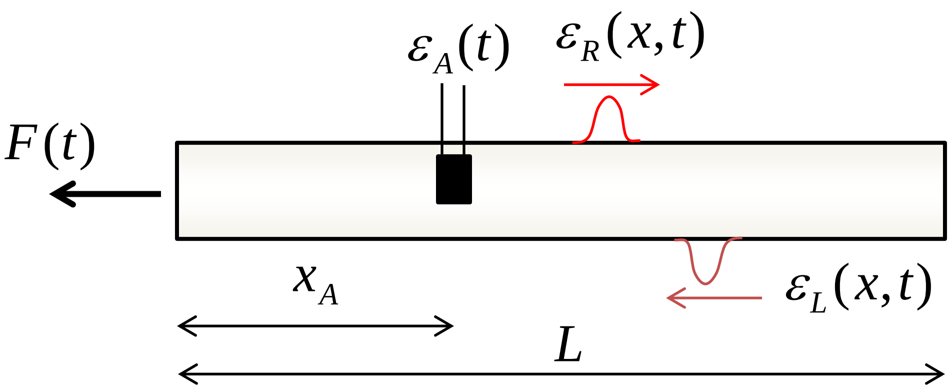


HOPKINSON BAR TECHNIQUE

Measuring force with a slender bar







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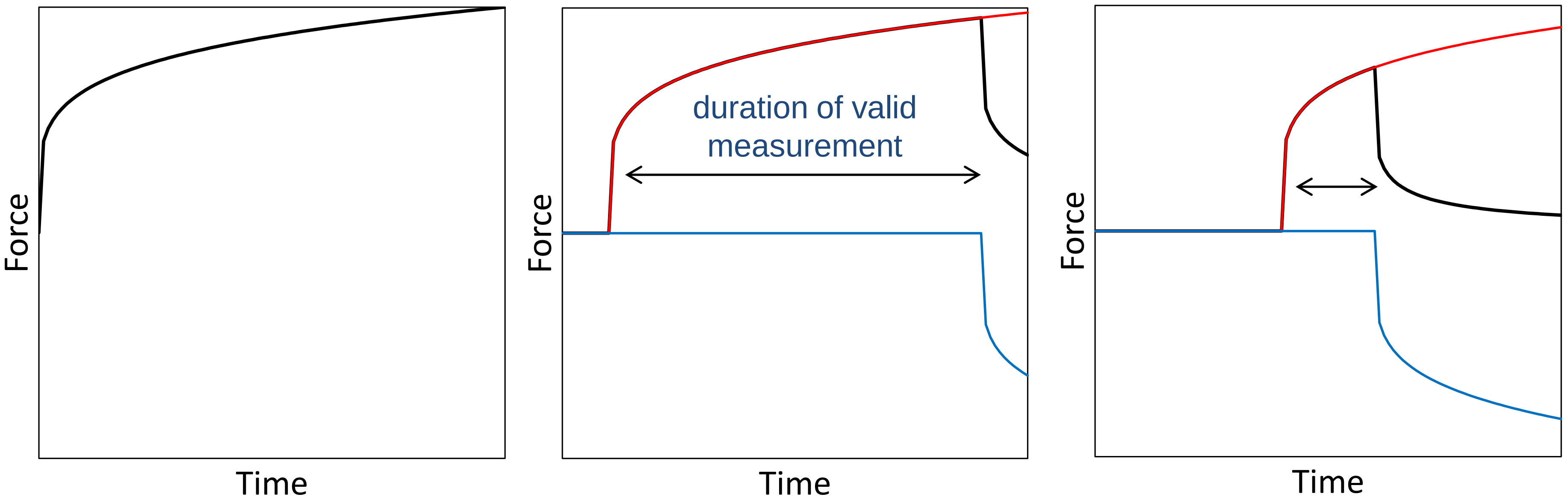
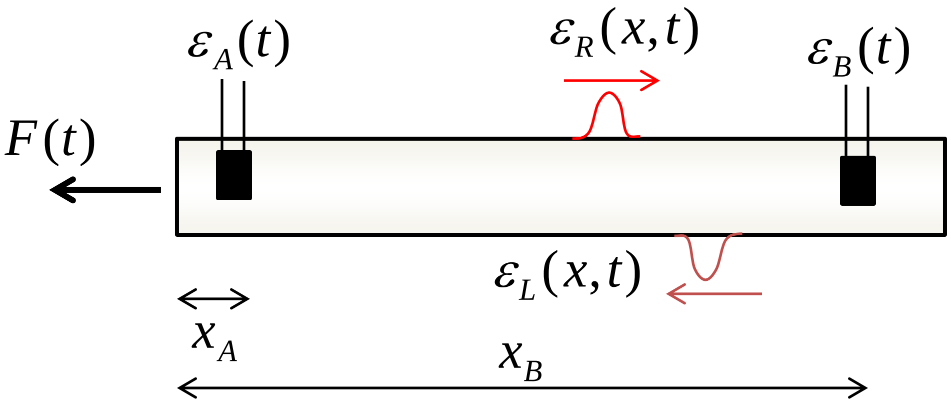
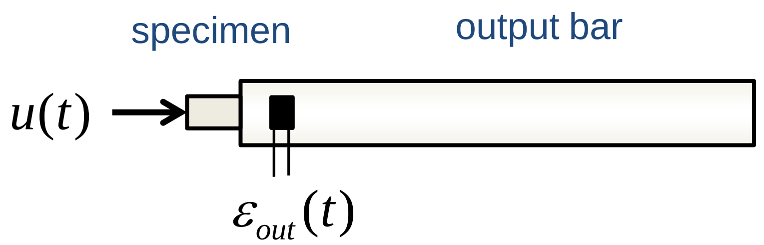


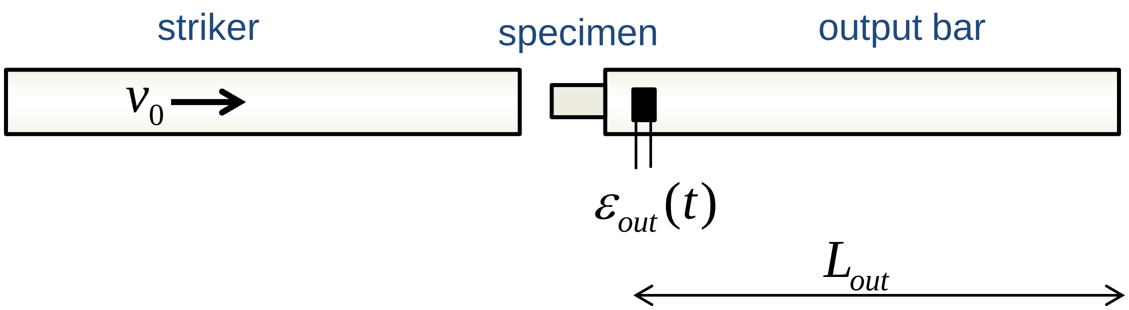


HOPKINSON BAR TECHNIQUE

Direct impact experiment:

One force measurement (output bar only)





Strategy: Experiment ends before leftward traveling wave in output bar reaches strain gage

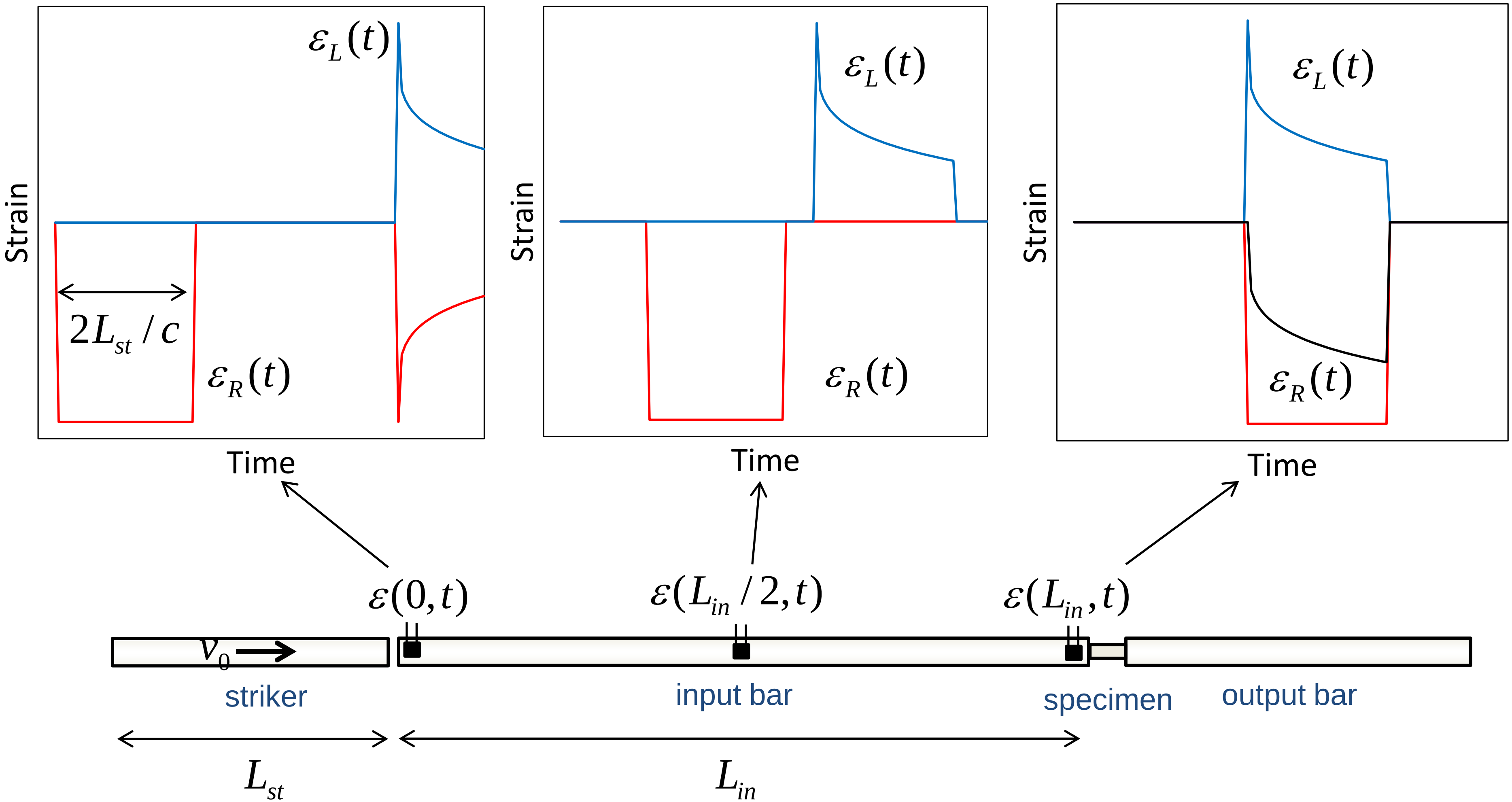
Length of output bar ↘ ↙ duration of the experiment

$$L_{out} > \frac{T}{2c_{out}}$$

HOPKINSON BAR TECHNIQUE

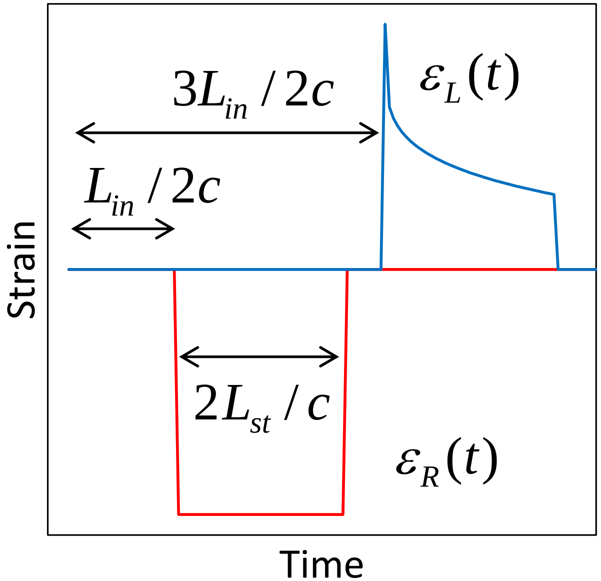
Split Hopkinson Pressure Bar (SHPB) experiment

Two force measurements



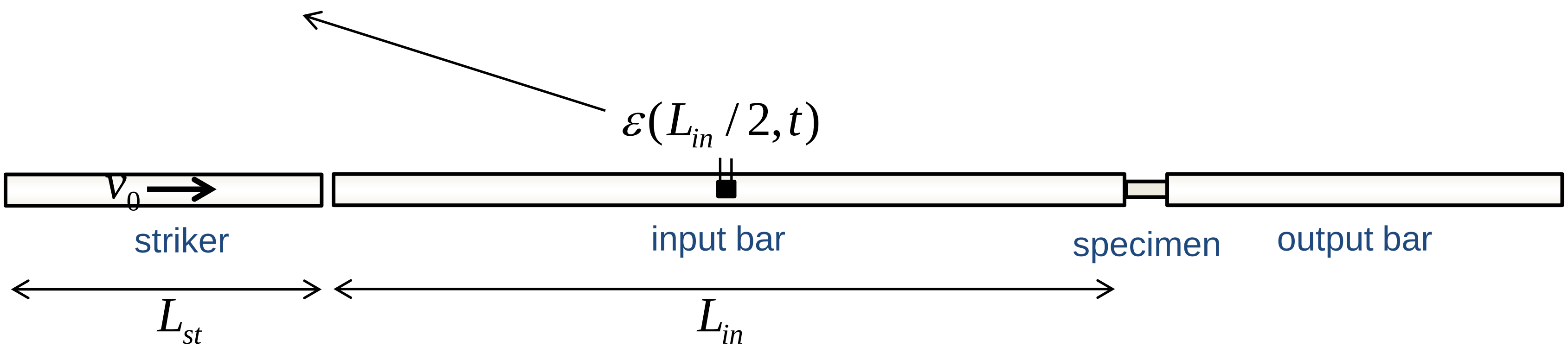
HOPKINSON BAR TECHNIQUE

Criterion to avoid wave superposition at strain gage position



$$3L_{in}/2c > L_{in}/2c + 2L_{st}/c$$

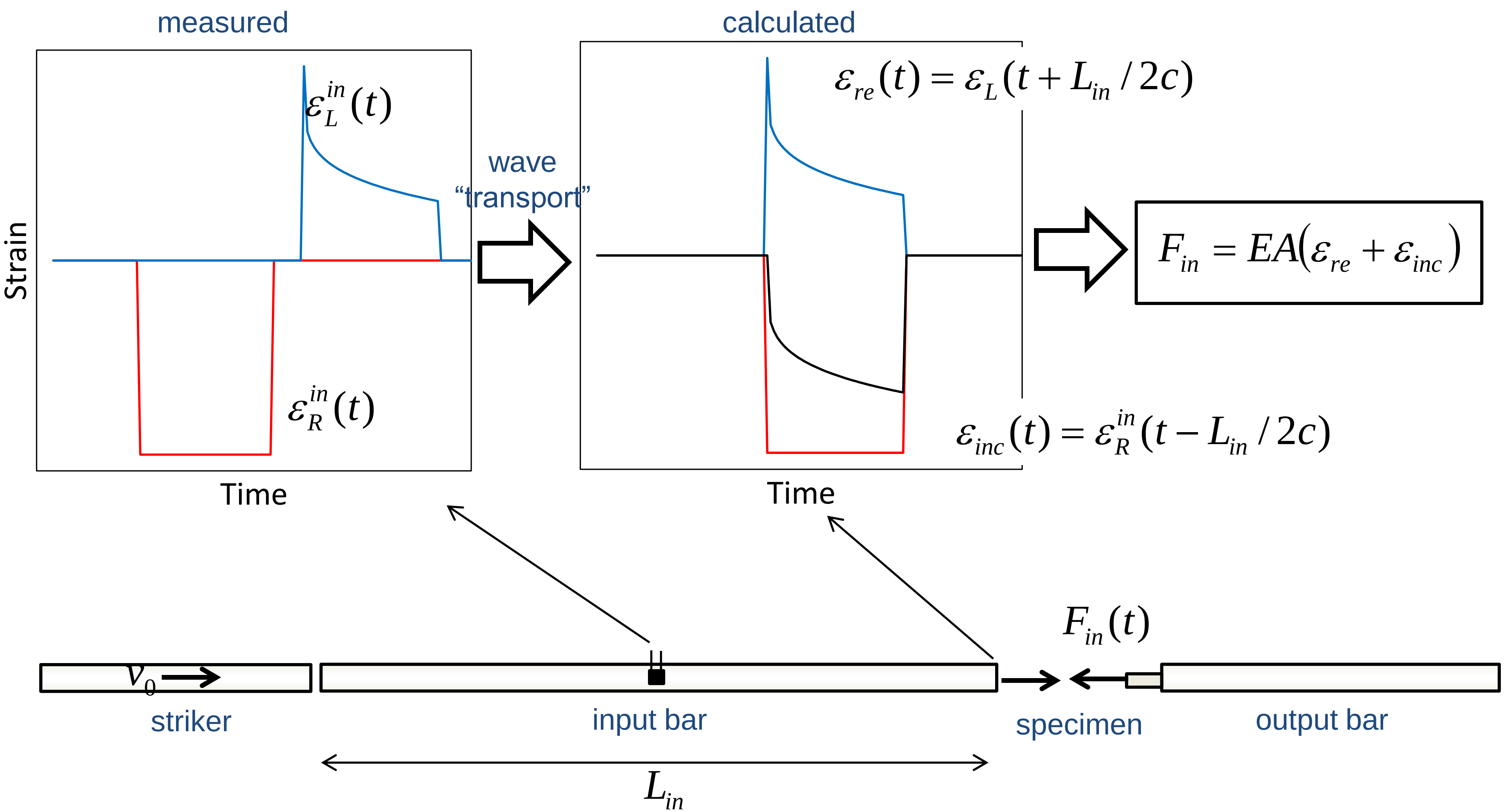
$$\Rightarrow \boxed{L_{in} > 2L_{st}}$$



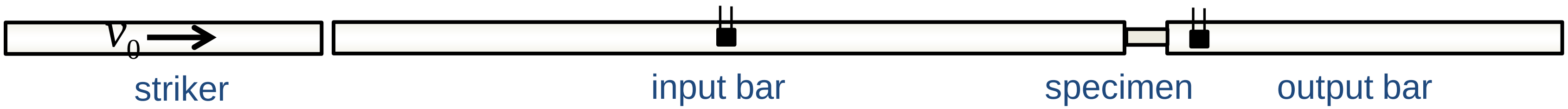
SPLIT HOPKINSON PRESSURE BAR (SHPB) SYSTEM

Kolsky (1949)

Input force measurement:



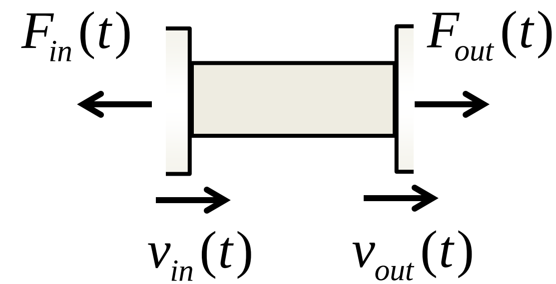
SPLIT HOPKINSON PRESSURE BAR (SHPB) SYSTEM



Calculate forces and velocities at specimen boundaries

Input bar/specimen interface:

$$\begin{matrix} \varepsilon_R^{in}(t) \\ \varepsilon_L^{in}(t) \end{matrix} \Rightarrow \begin{matrix} \varepsilon_{inc}(t) \\ \varepsilon_{re}(t) \end{matrix} \Rightarrow \begin{matrix} F_{in} = EA(\varepsilon_{re} + \varepsilon_{inc}) \\ v_{in} = c(\varepsilon_{re} - \varepsilon_{inc}) \end{matrix}$$



Output bar/specimen interface:

$$\varepsilon_R^{out}(t) \Rightarrow \varepsilon_{tra}(t) \Rightarrow \begin{matrix} F_{out} = EA\varepsilon_{tra} \\ v_{out} = -c\varepsilon_{tra} \end{matrix}$$

Specimen specific post-processing

- Verify quasi-static equilibrium
- Coupling with other measurements (e.g. high speed photography)
- Calculate stress, strain and strain rate

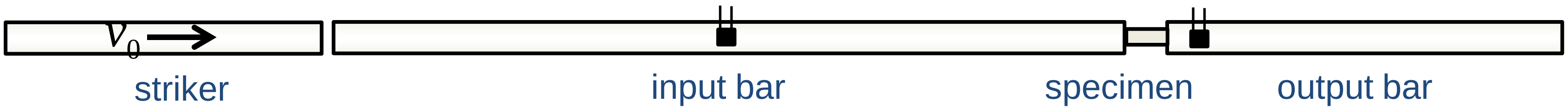
DESIGN OF A STEEL SHPB SYSTEM

Duration of the experiment: $T = \frac{\varepsilon_{\max}}{\dot{\varepsilon}_{av}} = \frac{0.5}{1000} = 500\mu s$

Minimum output bar length: $L_{out} = Tc/2 = 0.0005s \times 5000m/s/2 = 1.25m$

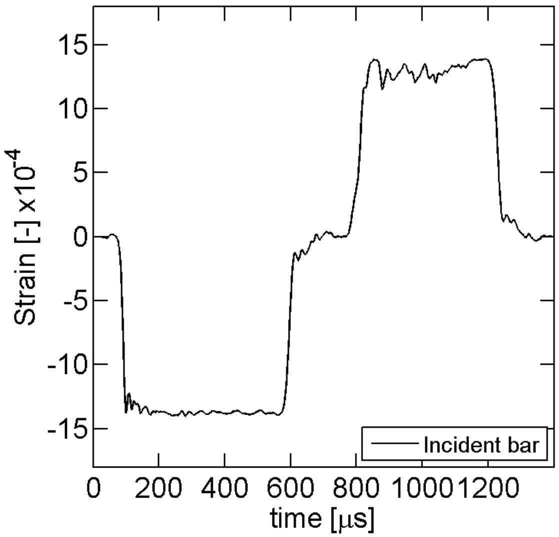
Minimum striker bar length: $L_{st} = Tc/2 = 1.25m$

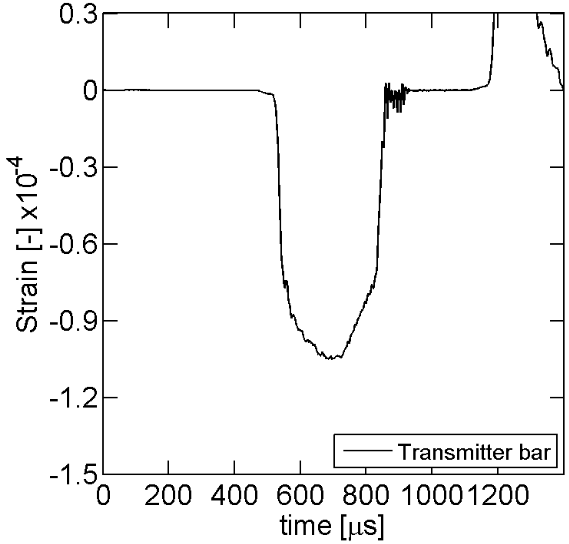
Minimum input bar length: $L_{in} = 2L_{st} = 2.5m$

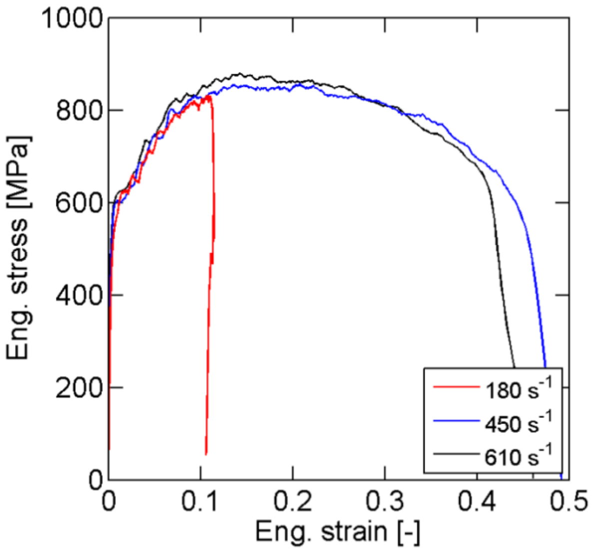


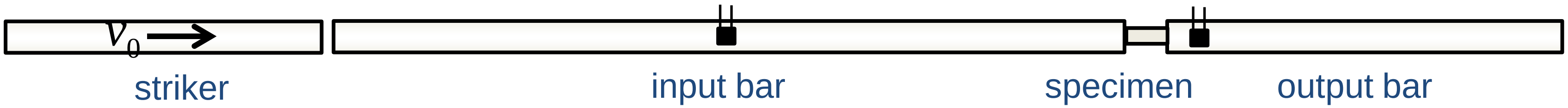
The bar diameters need to be chosen in accordance with the forces required to deform the specimen (force associated with incident wave should be much higher than the specimen resistance)

EXAMPLE EXPERIMENT



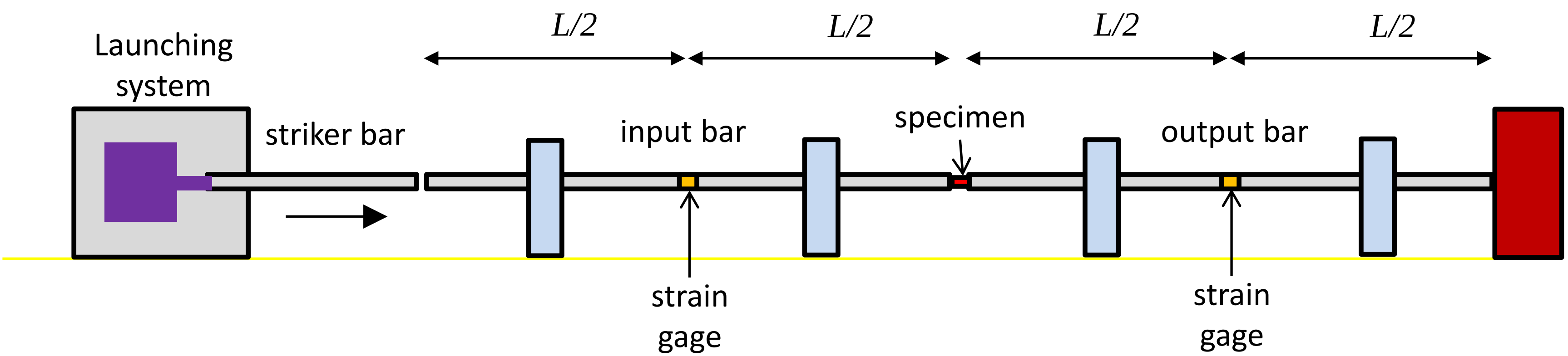




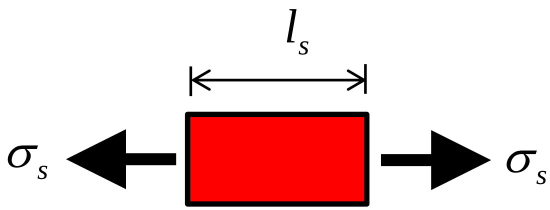


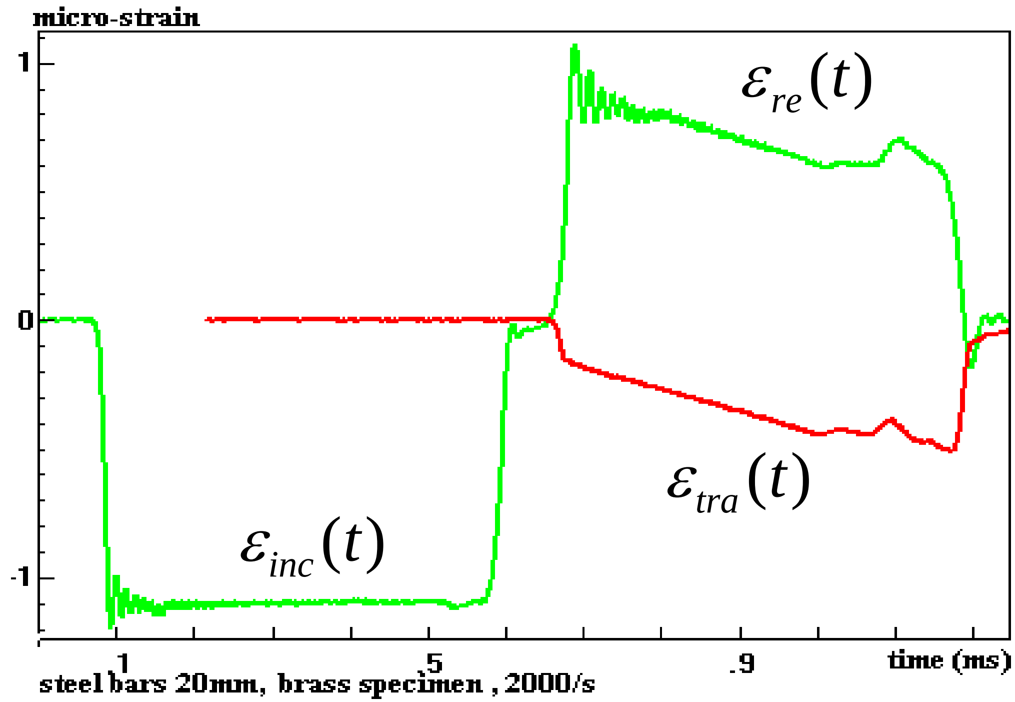
Kolsky bar system

Requirements:
- Striker, input and output bar made from the same bar stock (i.e. same material, same diameter)
- Length of input and output bars identical
- Striker bar length less then half the input bar length
- Strain gages positioned at the center of the input and output bars



Kolsky bar formulas



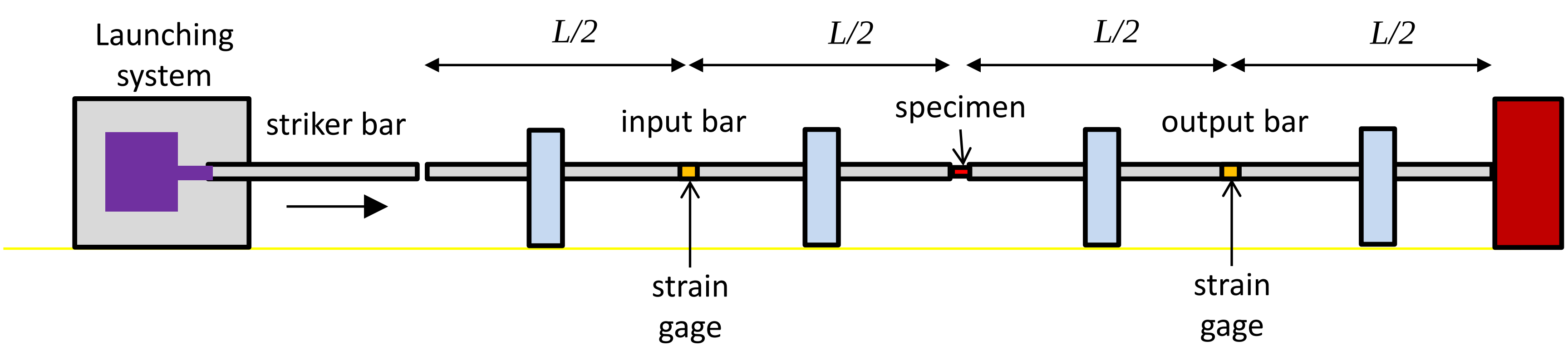


- Stress in specimen:

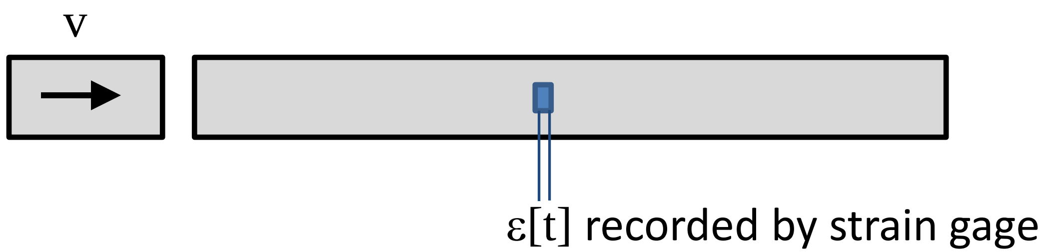
$$\sigma_s(t) = \frac{EA}{A_s}\varepsilon_{tra}(t)$$

- Strain rate in specimen:

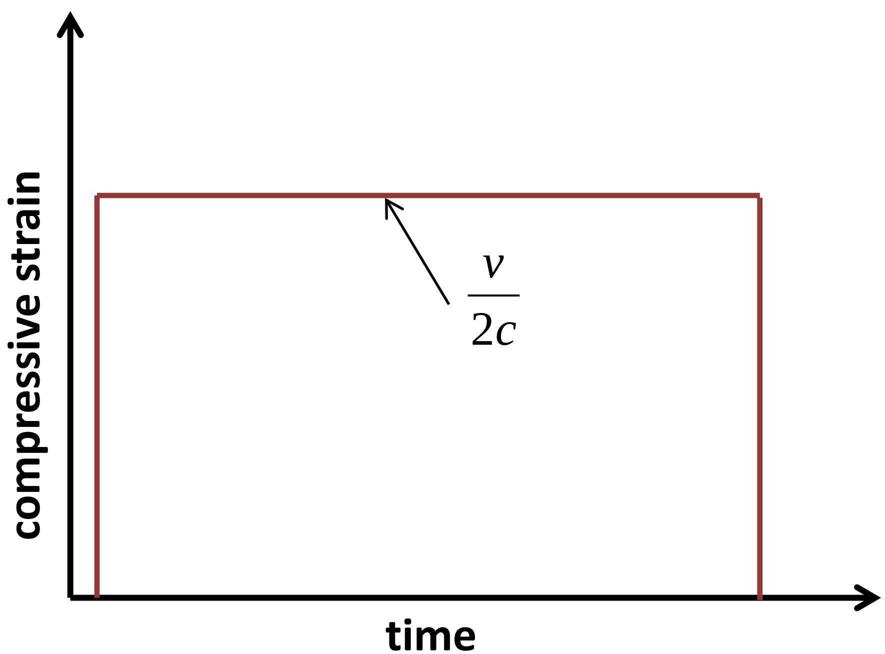
$$\dot{\varepsilon}_s(t) = -\frac{2c}{l_s}\varepsilon_{re}(t)$$



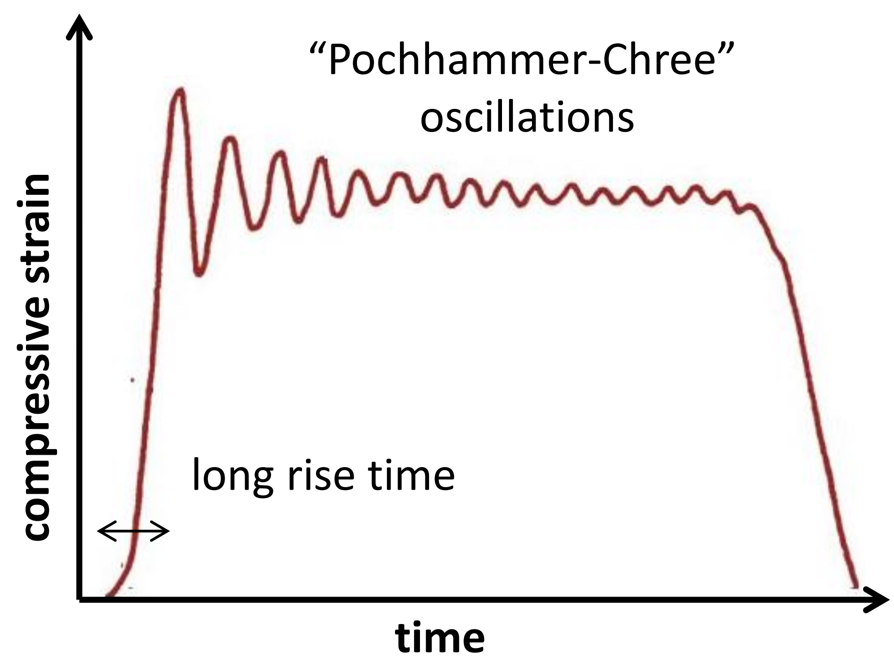
Wave Dispersion Effects



1D THEORY



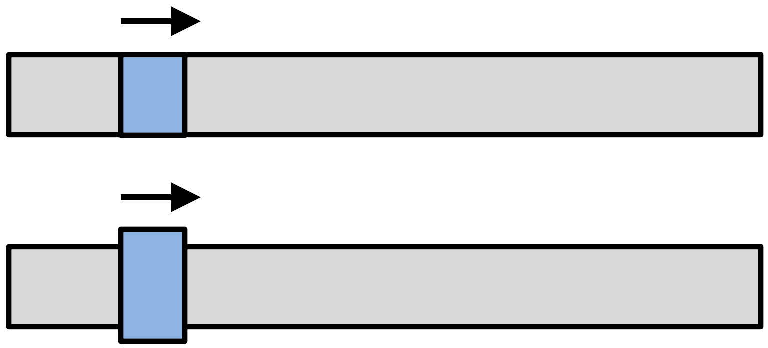
EXPERIMENT



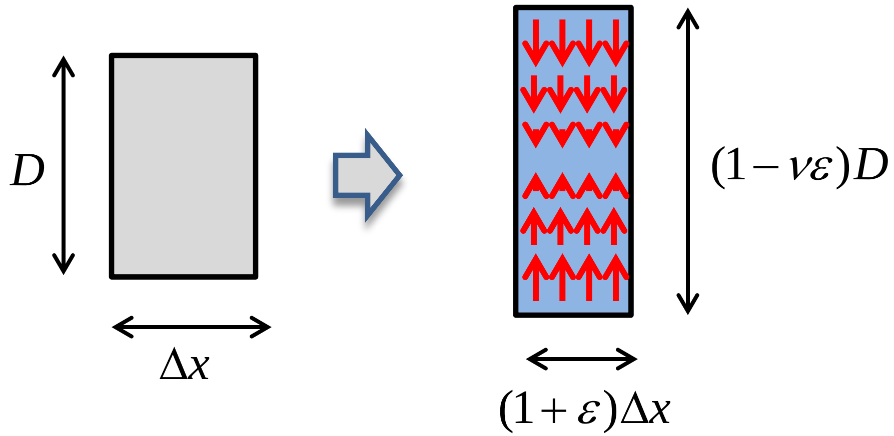
Wave Dispersion Effects

Simplified model:
axial compression only:

Reality:
axial compression &
radial expansion:



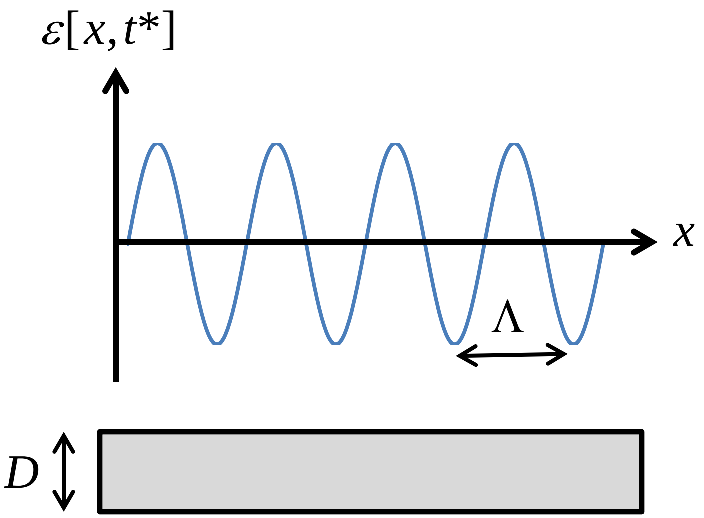
In reality, the wave propagation in a bar is a 3D problem and lateral inertia effects come into play due to the Poisson's effect!



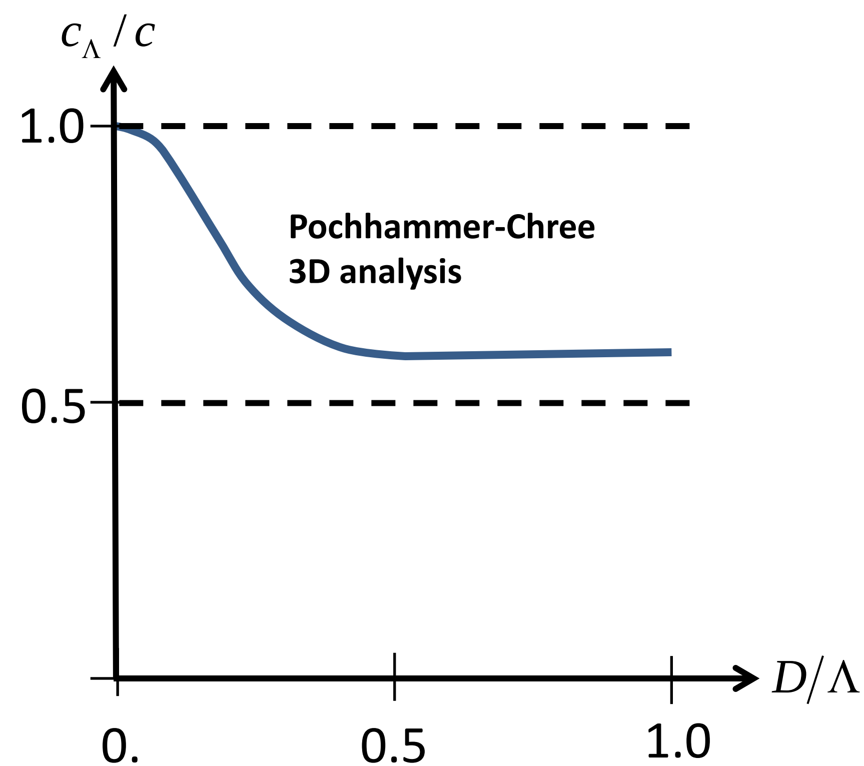
Inertia forces along the radial direction delay the radial expansion upon axial compression

Geometric Wave Dispersion

- Consider a rightward traveling sinusoidal wave train of wave length Λ in an infinite bar of radius a raveling at wave speed c_Λ



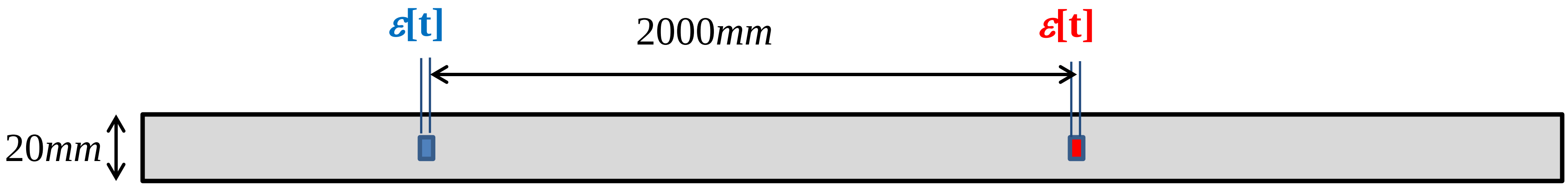
- Theoretical wave speed (1D analysis): $c=\sqrt{E/\rho}$
- Theoretical wave speed (3D analysis):



- The wave propagation speed depends on wave length!
- The 1D theory only true for very long wave lengths (or very thin bars)
- High frequency waves propagate more slowly than low frequency waves

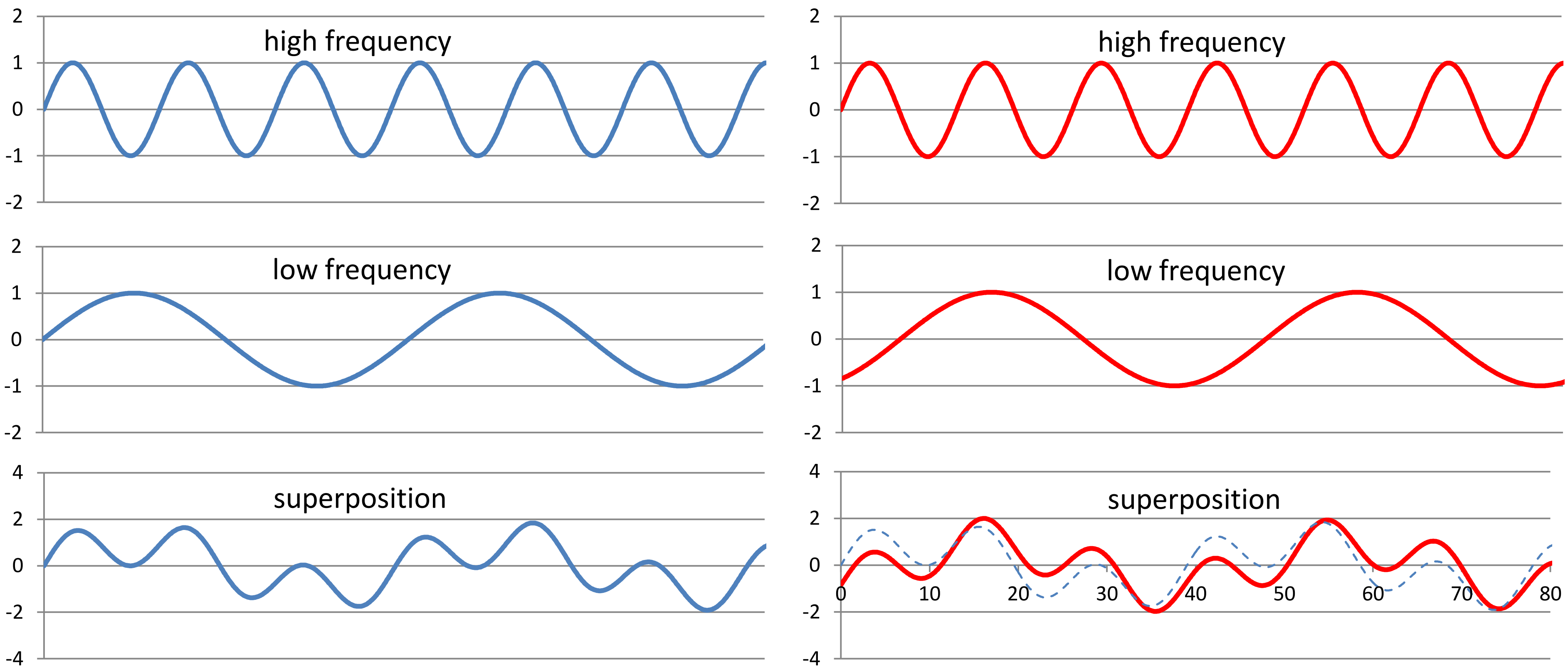
Geometric Wave Dispersion

- Example: Rightward propagating wave in a steel bar



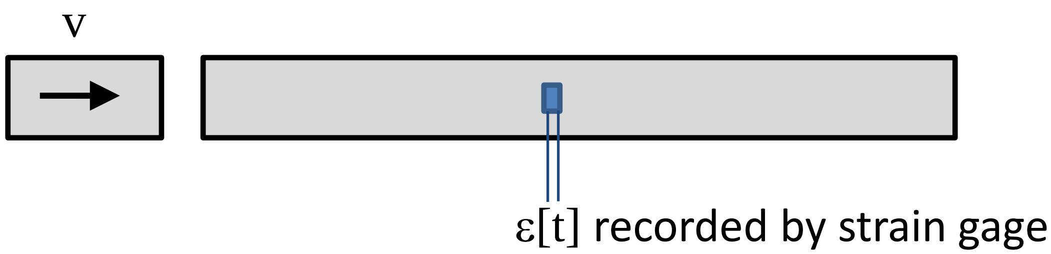
$\Lambda_1 = 40mm$ ⇒ $D/\Lambda_1 = 0.5$ ⇒ $c_1 \cong 3.1km/s$ ⇒ $f_1 \cong 78kHz$ ⇒ $\Delta T_1 \cong 642\mu s$

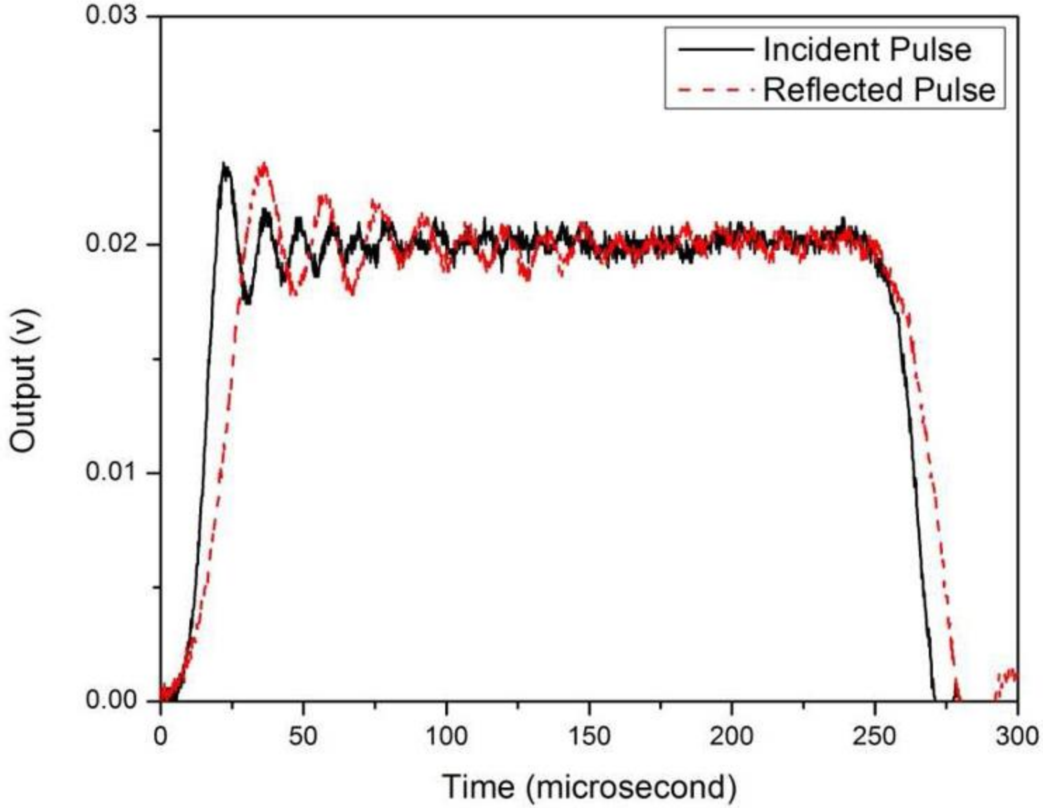
$\Lambda_2 = 200mm$ ⇒ $D/\Lambda_2 = 0.1$ ⇒ $c_2 \cong 4.9km/s$ ⇒ $f_2 \cong 25kHz$ ⇒ $\Delta T_2 \cong 406\mu s$



Geometric Wave Dispersion

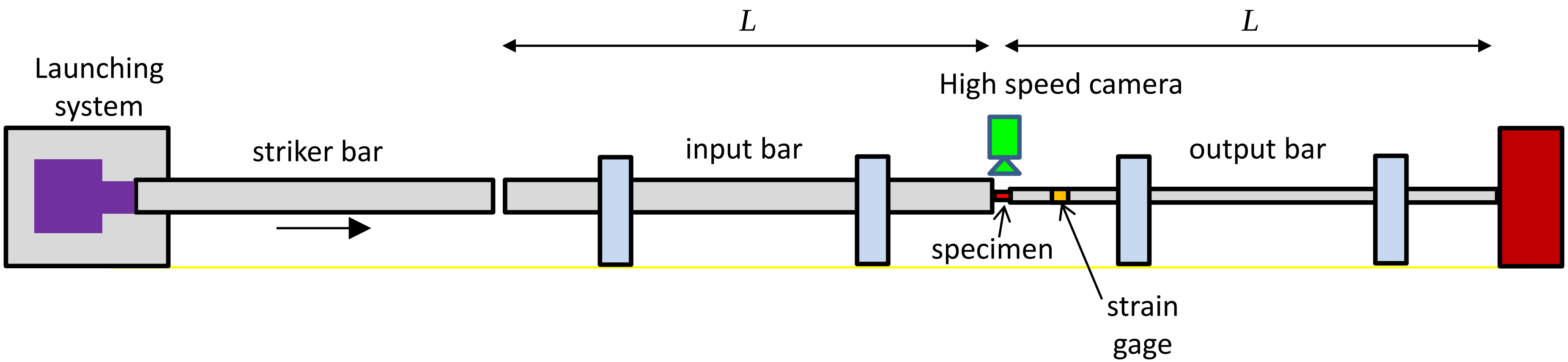
- Example





Chen & Song (2010)

Modern Hopkinson Bar Systems



Features:
- Striker and input typically made from the same bar stock (i.e. same material, same diameter)
- Small diameter output bar for accurate force measurement
- Similar length of all bars
- Output bar strain gages positioned near specimen end
- Wave propagation modeled with dispersion
- Strains are measured directly on specimen surface using Digital Image Correlation (DIC)

ADVANCED TOPICS related to SHPB technique

- Accurate wave transport taking geometric wave dispersion into account
- Use of visco-elastic bars (slower wave propagation than in metallic bars, more sensitive for soft materials)
- Torsion and tension Hopkinson bar systems
- Lateral inertia at the specimen level
- Friction at the bar/specimen interfaces
- Dynamic testing of materials (where quasi-static equilibrium cannot be achieved)
- Pulse shaping
- Intermediate strain rate testing
- Infrared temperature measurements
- Experiments to characterize brittle fracture
- Multi-axial ductile fracture experiments
- Experiments under lateral confinement

… and many others.

Reading Materials for Lecture #2

- George T. Gray, “High-Strain-Rate Testing of Materials: The Split-Hopkinson Pressure Bar”: http://onlinelibrary.wiley.com/doi/10.1002/0471266965.com023.pub2/abstract
- M.A. Meyers, “Dynamic behavior of Materials” (chapter 2): http://onlinelibrary.wiley.com/book/10.1002/9780470172278
- W. Chen, B. Song, “Split Hopkinson (Kolsky) Bar”: http://link.springer.com/book/10.1007%2F978-1-4419-7982-7