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Computer Lab #5:

Stress update algorithm on von Mises plasticity

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Part I: Material subroutine

The material model for rate-independent isotropic plasticity has been implemented into the FE code Abaqus. Its main features are the von Mises yield function, an associated flow rule and mixed Swift-Voce strain hardening. The implementation was done through a user material subroutine programmed in FORTRAN language.

During a time increment $\Delta t = t_{n+1} - t_n$ the stress tensor along with the internal variables of the model need to be updated. At the beginning of the time increment, the strain increment tensor $\Delta\boldsymbol{\varepsilon}$, the stress tensor in the previous increment $\boldsymbol{\sigma}_n$ and the internal variable $\bar{\varepsilon}_{p\ n}$ in the previous increment are given.

- In a first step, we assume that the material deforms elastically only (trial state assumption). In other words, we have:

$$\boldsymbol{\varepsilon}_{n+1}^{e\ \text{trial}} = \boldsymbol{\varepsilon}_n^e + \Delta\boldsymbol{\varepsilon}$$

- Next, the strain tensors are decomposed into their deviatoric and hydrostatic parts::

Hydrostatic part

$$\varepsilon_v^{e\ \text{trial}} = tr\left(\boldsymbol{\varepsilon}_{n+1}^{e\ \text{trial}}\right)$$

$$\Delta\varepsilon_v = tr\left(\Delta\boldsymbol{\varepsilon}\right)$$

Deviatoric part

$$\boldsymbol{\varepsilon}_{d\ n+1}^{e\ \text{trial}} = \boldsymbol{\varepsilon}_{n+1}^{e\ \text{trial}} - \frac{1}{3}\varepsilon_{v\ n+1}^{e\ \text{trial}}\boldsymbol{I}$$

$$\Delta\boldsymbol{\varepsilon}_d = \Delta\boldsymbol{\varepsilon} - \frac{1}{3}\Delta\varepsilon_v\boldsymbol{I}$$

1. ELASTIC PREDICTOR $\left(\bullet\right)_{n+1}^{\text{trial}}$

- The trial elastic predictor is:

$$\boldsymbol{\sigma}_{n+1}^{\text{trial}} = \boldsymbol{\sigma}_n + \mathbf{C} : \Delta\boldsymbol{\varepsilon}$$

Using the above decomposition, we have:

Hydrostatic part

$$\sigma_{h\ n+1}^{\text{trial}} = K\varepsilon_{v\ n+1}^{e\ \text{trial}}$$

$$\Delta\sigma_h = K\Delta\varepsilon_v$$

Deviatoric part

$$\boldsymbol{s}_{n+1}^{\text{trial}} = 2G\boldsymbol{\varepsilon}_{d\ n+1}^{e\ \text{trial}}$$

$$\Delta\boldsymbol{s} = 2G\Delta\boldsymbol{\varepsilon}_d$$

$$\boldsymbol{\sigma}_{n+1}^{\text{trial}} = \boldsymbol{\sigma}_n + 2G\Delta\boldsymbol{\varepsilon}_d + K\Delta\varepsilon_v\boldsymbol{I}$$

2. CHECK YIELDING

- We compute the equivalent trial stress:

$$\bar{\sigma}_{n+1}^{\text{trial}} = \sqrt{\frac{3}{2}\boldsymbol{s}_{n+1}^{\text{trial}} : \boldsymbol{s}_{n+1}^{\text{trial}}} = \sqrt{\frac{3}{2}\text{tr}\left[\left(\boldsymbol{s}_{n+1}^{\text{trial}}\right)^T \boldsymbol{s}_{n+1}^{\text{trial}}\right]}$$

- and the flow stress according to the Swift-Voce hardening law

$$k_{n+1}^{\text{trial}} = k_n = \alpha\left\{A\left(\varepsilon_0 + \bar{\varepsilon}_{p\,n}\right)^n\right\} + \left(1-\alpha\right)\left\{\sigma_0 + Q_1\left[1-\exp\left(-C_1\bar{\varepsilon}_{p\,n}\right)\right]\right\}$$

- to evaluate the von Mises yield function

$$f_{n+1}^{\text{trial}} = \bar{\sigma}_{n+1}^{\text{trial}} - k_{n+1}^{\text{trial}}$$

3. PLASTIC CORRECTOR $\left(\bullet\right)_{n+1}$ if $f_{n+1}^{\text{trial}} > 0$

- Solution of the discrete consistency condition to determine the plastic multiplier (using a Newton-Raphson scheme)

$$f_{j+1} \equiv \bar{\sigma}_{j+1}^{\text{trial}} - 3G\Delta\gamma - k_{j+1} = 0$$

I. Initial guess

$$\Delta\gamma = 0$$
$$k_{j+1} = k_{n+1}^{\text{trial}}$$

II. Equivalent plastic strain

$$\delta\gamma = \frac{f_{j+1}}{3G + \dfrac{\partial k_{j+1}}{\partial\gamma}} = \frac{f_{j+1}}{3G + H_{j+1}}$$
$$\Delta\gamma = \Delta\gamma + \delta\gamma$$
$$\bar{\varepsilon}_{p\,j+1} = \bar{\varepsilon}_{p\,j} + \Delta\gamma$$

III. Check convergence

If $\left|f_{j+1}\right| / k_n \le TOL$ then

$$\left(\bullet\right)_{n+1} = \left(\bullet\right)_{j+1}$$

exit the loop

IV. Go to the beginning

$$j = j + 1$$

- Update the stress tensor

$$m = \frac{\mathbf{s}_{n+1}}{\mathbf{s}_{n+1}^{\text{trial}}} = 1 - \frac{3G\Delta\gamma}{\bar{\sigma}_{n+1}^{\text{trial}}}$$

$$\boldsymbol{\sigma}_{n+1} = \mathbf{s}_{n+1}^{\text{trial}} m + \sigma_{h\ n+1}\boldsymbol{I}$$

4. ELASTIC PREDICTOR INSIDE THE ELASTIC DOMAIN $(\bullet)_{n+1}$ if $f_{n+1}^{\text{trial}} \leq 0$

- Update the stress tensor

$$\boldsymbol{\sigma}_{n+1} = \boldsymbol{\sigma}_{n+1}^{\text{trial}}$$

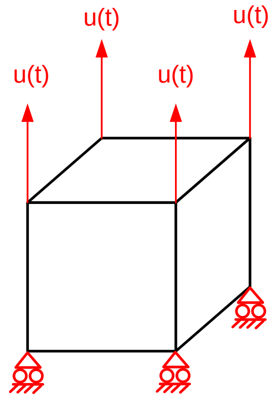
Part II: Simulation of a single element with the von Mises plasticity material model

A 5 mm single cubic element made out of steel is tested in a tensile configuration as shown in the figure. The following material constants of the Swift-Voce hardening law were found to represent adequately the experimental behavior as seen in ComputerLab#4.

A (MPa)	ε_0	n	σ_0 (MPa)	Q_1 (MPa)	C_1	α
1310.00	0.00128	0.199	350.00	324.25	25.92	0.64

The input file “ONE.inp” that contains all the details of the simulation is provided. The material parameters, termination time of the calculation, boundary conditions and output data acquisition times are empty fields that can be filled out depending on the case.

The “VON_MISES_AFR.for” material subroutine file that contains the von Mises plasticity and Swift-Voce hardening law is provided. The stress update is done according to a Backward-Euler integration scheme as outlined in Part I.



Part II: Convergence study

1) Run the single element model knowing that the final displacement is 1.5 mm after 10 ms employing a TOL=1.0e-5. Plot the residual f_{n+1}/k_n against time. Why is it sometimes positive and sometimes negative?
2) Employing a tolerance value of TOL=1.0e-7 change the convergence criterion from $|f_{n+1}|/k_n$ to $|f_{n+1}|$. Plot the number of iterations until convergence of both cases against time. Comment on the difference between them. What is the reason of such difference?
3) Using the same boundary conditions as in the previous section, run the single element test with TOL1=1.0e-3, TOL2=1.0e-5, TOL3=1.0e-7 and TOL4=1.0e-9. Plot the following graphs:
   - Equivalent stress-plastic strain
   - Number of iterations against equivalent plastic strain

   Taking the case of TOL=1.0e-5 as the reference case. Compare the four cases and comment the differences. Which tolerance value would you choose in this particular case?

Part III: Forward-Euler

1) Change the stress update algorithm from Backward-Euler (BE) to Forward-Euler (FE). After implementing such a scheme, what do you think is the main advantage of it?
2) Simulate the single element as detailed in section 1 of Part II using the BE (TOL=1.0e-7) and the FE scheme. Plot the following graphs:

   - Equivalent stress-plastic strain
   - Residual ($|f_{n+1}|/k_n$) against equivalent plastic strain

   Comment the differences between both cases. Which one of the two schemes would you implement to simulate a large model that of a car crash impact? Give arguments that justify your choice.