Optimal Technology Choice and Investment Timing: A Stochastic Model of Industrial Cogeneration vs. Heat-Only Production

Reinhard Madlener and Marcel Wickart

CEPE Working Paper No. 37
First version Dec 2004
revised version Dec 2006,
former title “Risk and Uncertainty in Industrial Large-Scale Cogeneration Investment”

CEPE
Zurichbergstrasse 18 (ZUE E)
CH-8032 Zurich
www.cepe.ethz.ch
Optimal technology choice and investment timing:
A stochastic model of industrial cogeneration vs.
heat-only production

Marcel Wickart and Reinhard Madlener

Centre for Energy Policy and Economics (CEPE),
Department of Management, Technology, and Economics, ETH Zurich,
Zurichbergstrasse 18 (ZUE E), 8032 Zurich, Switzerland

CEPE Working Paper No. 37
(first version December 2004, revised version December 2006, former title
“Risk and Uncertainty in Industrial Large-Scale Co-generation Investment”)

Corresponding author. Tel.: +41 44 632 0652, fax: +41 44 632 1050; E-mail address: rmadlener@ethz.ch (R. Madlener).
Abstract

In this paper we develop an economic model that explains the decision-making problem under uncertainty of an industrial firm that wants to invest in a process technology. More specifically, the decision is between making an irreversible investment in a combined heat-and-power production (cogeneration) system, or to invest in a conventional heat-only generation system (steam boiler) and to purchase all electricity from the grid. In our model we include the main economic and technical variables of the investment decision process. We also account for the risk and uncertainty inherent in volatile energy prices that can greatly affect the valuation of the investment project. The dynamic stochastic model presented allows us to simultaneously determine the optimal technology choice and investment timing. We apply the theoretical model and illustrate our main findings with a numerical example that is based on realistic cost values for industrial oil- or gas-fired cogeneration and heat-only generation in Switzerland. We also briefly discuss expected effects of a CO$_2$ tax on the investment decision.

Keywords: Cogeneration, Irreversible investment, Risk, Uncertainty, Real options

JEL Classification: C61; D21; D81; D92; O33; Q41; Q48
1 Introduction

Cogeneration, i.e. the combined production of useful heat and power (CHP), enables a more rational use of primary energy by the utilization of waste heat. It allows for fuel savings between 10-40% and is therefore considered a powerful carbon abatement technology that can be employed in many different applications (Madlener and Schmid, 2003a).

Early model-guided work on the economics of cogeneration in the 1980s has focused on peak-load pricing and capacity planning for CHP installations in different market structures (Dobbs, 1983), cogeneration adoption (investment) decisions of cost-minimizing representative agents (Joskow and Jones, 1983; Joskow, 1984), investment policy (tax credits) impacts and social welfare considerations (Anandalingam, 1985), and the pricing behavior of utilities for cogenerated electricity fed into the grid (Zweifel and Beck, 1987; Woo, 1988). Much of this research was influenced by the significance of the 1978 Public Utility Regulatory Policies Act (PURPA). In the 1990s, research dealt, for instance, with the implications of PURPA, state-level regulation and fuel and electricity prices on aggregate (state-level) investment in cogeneration (Fox-Penner, 1990). Other work has focused on the joint planning of optimal capacity expansion and operation of industrial CHP plants and electricity production by utilities, and the influence of buy-back rates on the optimal level of self-generation (Kwun and Baughman, 1991). Rose and McDonald (1991), Dismukes and Kleit (1999), Bonilla et al. (2002, 2003) and Madlener and Schmid (2003b) have studied the impact of various technical and economic variables on cogeneration adoption by applying econometric model specifications. Kwon and Yun (1991) have specifically explored economies of scope of cogeneration systems. Throughout the 1990s and in recent years, the influence of market liberalization and deregulation has gained importance and created new types of uncertainty, thus fundamentally changing the boundary conditions for industrial cogenerators. Unsurprisingly, this development has also started to increasingly attract researchers.

In this paper, which is an improved and extended version of Wickart and Madlener (2004), we take the main risks and uncertainties of industrial heat and electricity generation explicitly into account.\footnote{We use the two terms ‘risk’ and ‘uncertainty’ rather loosely in this paper. Knight (1921), a frequently} In particular, we develop an economic model that explains the
decision-making problem of an industrial firm contemplating investment in cogeneration technology in an uncertain environment. Moreover, the decision is focused on the problem of either making an irreversible investment in a cogeneration system for self-producing heat and electricity, or to invest in some heat-only generation system (typically a steam boiler) and to purchase all electricity from the grid. Finally, we also study the joint decision, i.e. of choosing both the optimal technology and the optimal investment timing, in a single model.

Technically, the work presented here builds upon our earlier modeling study on cogeneration investment within a deterministic framework (Madlener and Wickart, 2006), a policy-focused study on current and expected future drivers, barriers and regulatory issues surrounding cogeneration technology adoption and diffusion in the Swiss industrial sector (Madlener and Wickart, 2004), and a Finnish study on the investment in alternative energy conversion technologies where the prices of the input fuels considered (biomass and natural gas) are subject to uncertainty (Murto and Nese, 2003). We introduce a dynamic stochastic model that captures the major variables of interest, and analyze the impact of different sources of input price uncertainty on the value of the investment and the relative attractiveness of the technological alternatives studied. Our modeling approach is based on the theory of irreversible investment under uncertainty, also referred to as real options theory (Dixit and Pindyck, 1994; Trigeorgis, 1996; Schwartz and Trigeorgis, 2001), which presupposes that the investment can indeed be postponed. It explicitly accounts for the value of deferring an investment project that derives from the opportunity to take a better informed decision in the future rather than to invest immediately. Put differently, when uncertainty about the costs and benefits of an investment project is ignored, the value of the project is incorrectly assessed, leading to misguided investment decisions. In our case we restrict the analysis to the price uncertainty related to the input fuel used and the electricity produced or purchased, which is measured by the annual volatility $\sigma$ of the price process. The larger the volatility, the larger is the option value of the project (and hence cited reference in this context from the economics literature, distinguishes between ‘risk’, referring to situations where the decision-maker can assign mathematical probabilities to the randomness she is confronted with, and ‘uncertainty’, which refers to situations where this randomness cannot be expressed in terms of specific mathematical probabilities.
the value of deferring the project), which decreases the incentive of the potential adopter to invest immediately.

The original contribution of this paper is the development and numerical application of a dynamic stochastic technology adoption model for industrial cogeneration plants, assessed against the alternative investment into a steam boiler. Moreover, in fuel (electricity) price over fuel (electricity) price volatility plots, we study in detail the no-waiting and waiting regions for investments in cogeneration and steam boiler plants, and how these may be affected by the introduction of a CO₂ tax. Overall, we are able to show that the effect of policies which do not affect uncertainty directly may still have a huge impact on technology adoption due to the presence of uncertainty.

The strength and appeal of our approach lies in the modeling of the decision between joint or separate production of heat and electricity by means of two distinct process technologies (one for combined-heat-and-power production and one for heat-only generation, respectively) with very different techno-economic characteristics. The focus is on the optimal technology choice and optimal time of investment, not on the optimal operation of the equipment. Separate production implies that electricity has to be bought via the grid. Due to the lack of an external heat supplier, both options require that the company’s heat demand must be met at all times. The weakness of the approach can be ascribed to several strong assumptions that had to be made regarding the prevailing operating conditions, in order to keep the model reasonably simple and analytically tractable (e.g. we rule out excess electricity production and feed-in to the grid; investment is considered totally irreversible; and, once installed, the generation plant is operated for an infinite period of time without tackling the possibility of shut-down and restart²). Note that in reality industrial cogeneration units may be run up to 8200 hours (out of a total of 8760 h per year), the difference being due to some scheduled down-times for maintenance and repair, including capital improvements (plus maybe some unscheduled outages, which seem to be rather rare). Since our main focus here is on optimal technology adoption decisions, rather than

²For many industries with heat-demand-driven operation of the cogeneration unit this seems to be a reasonable assumption, since the CHP units are run on a continuous basis, irrespective of the development of the input fuel price or the electricity price. In other words, if there is no alternative heat supply unit, the cogeneration unit has to be operated even in situations where the electricity price is very low.
optimal dispatching, we have refrained from including this detail. We are convinced that the nature of the results would not be changed in any substantial way.

The remainder of this paper is structured as follows. In section 2 we first provide a review of the literature related to ours. In section 3 we introduce the model as well as the sensitivity of both the option value and the investment threshold with respect to uncertainty. In section 4 we model the optimal time of investment and technology choice as a joint decision problem. In order to apply the theoretical model and to illustrate our main findings regarding optimal technology choice and investment timing under uncertainty, a numerical example is presented in section 5 that is based on realistic cost figures for oil- or natural-gas-fired CHP and steam boilers in Switzerland (see Appendix). In Section 6 we discuss the implications of our model-based theoretical reasoning on energy policy, especially CO$_2$ taxation. Section 7 concludes.

2 Literature review (real options theory and energy conversion technologies)

Research interest in the application of real options theory to energy conversion technologies has grown considerably in recent years. This section covers a brief literature review of some related research work, in chronological order, and restricted to the combination of real options modeling and energy conversion technologies. Note that most of the studies considered have focused on power generation, and a great many of them on the optimal operation of the plants, not the investment decision. For a somewhat more comprehensive survey on the literature dealing with the economics of cogeneration in particular, and a useful summary table, see Madlener and Wickart (2006). Useful surveys of the literature on optimal timing of investment under uncertainty not specifically related to energy issues can be found, for instance, in Dixit and Pindyck (1994) and references contained therein.

Gardner and Zhuang (2000) describe a stochastic dynamic programming model for power plant valuation that accounts for minimum on- and off-times, ramp time, non-constant heat rates, response rate, and minimum electricity dispatch level. The application is for electricity supply data from the power pool in New England. The results indicate
that operating constraints can have a significant impact on power plant valuation and optimal operating regimes.

Deng et al. (2001) develop a methodology for valuing electricity derivatives by constructing replicating portfolios from electricity futures and the risk-free asset. A replication with futures is used because of the non-storable nature of electricity, which rules out any storage-based method of valuing commodity derivatives based on spot market data. Valuation is done for both spark and locational spark options, and for geometric Brownian motion as well as mean reverting price processes. The results from the valuation are then used to construct a real options model for valuing power generation and transmission assets. The model is applied to a sample set of generation assets, and the theoretical values calculated are compared with the achieved selling price.

Frayer and Uludere (2001) perform a real options analysis for two generation assets (one for peak-load and one for mid-load demand) in a regional electricity market (Northeast of the U.S.A.) and for volatile electricity prices. The authors find that a peak-load gas-fired plant may be more valuable than a mid-load coal-fired plant, even though conventional investment valuation methodologies would favor the coal-fired asset because of its lower marginal cost.

Tseng and Barz (2002) introduce a real options approach for the short-term valuation of power plants with unit commitment constraints. The problem is formulated as a multi-stage stochastic problem. The solution procedure integrates backward-moving dynamic programming with forward-moving Monte Carlo simulation. Power plant operators are assumed to maximize expected profit based on hourly dispatching optimization, where lead times for the commitment and de-commitment decisions are taken into account (for start-up and shut-down of the unit, for which the costs are also incorporated). Commitment decisions are subject to physical constraints, such as minimum uptime and downtime. For both the fuel and the electricity markets it is assumed that hourly markets exist, and that prices follow Itô processes. In a numerical simulation exercise, the authors show that failure to consider physical constraints may lead to significant overvaluation of power plants.

Deng and Oren (2003) study ‘real options’-based valuation of electricity generation capacity that incorporates certain operational constraints and start-up costs. Stochastic
prices of electricity and fuel are represented by re-combinations of multi-nomial trees. Power generation units are modeled as a strip of cross-commodity call options with a delay and a cost imposed on each option exercise. The authors discuss and illustrate the implications of operational characteristics on the valuation of generation assets under different assumptions about the energy commodity prices. They find that the impacts of operational constraints on valuation depend on both the model specification and the nature of the operating characteristics.

Keppo and Lu (2003), also using a real options modeling framework, consider the situation of a large producer of electricity and what price effect it has on the electricity market. This effect influences the generation assets owned by the producers and also the investment opportunities, if the companies are unable to hedge the price effect in the financial markets, and if there is no significant competition about the investment opportunity. Firms optimize three variables: time of adoption of the investment, electricity production, and the choice of the portfolio of financial instruments. The former two are optimized by maximizing the company value and choosing the optimal portfolio hedge.

Murto and Nese (2003) consider the investment choice between fossil fuel- and biomass-fired power production technologies for the case of input price uncertainty and an opportunity to shut down the plant whenever production is unprofitable. By means of a dynamic model they show the effect of fossil fuel input price uncertainty on the choice of the technology and the (optimal) timing of investment. A numerical example with realistic cost estimates for the two power plant types considered illustrates the way the model works.

Nässäkkälä and Fleten (2005) focus on two types of gas-fired power plant investments: base-load and peak-load plants. Particularly, building upon the work of Deng et al. (2001) mentioned above, they model the spark spread (i.e. the difference between the price of electricity and the input fuel cost for generating electricity) by means of a two-factor model. The model allows for mean reversion in short-term fluctuations and uncertainty in the equilibrium price to which prices revert. Base-load plants generate electricity at all levels of the spark spread, while peak-load plants only generate electricity when the spark spread exceeds emission costs. Base-load plants can be upgraded to become peak load plants. The authors focus on the optimal choice of production strategy by determining the
threshold value for upgrading, and by modeling the optimal type of gas-fired plant as a function of the spark spread. They also model the investment threshold, i.e. the threshold value below which it is optimal to build the plant with the previously determined optimal technology. Again, a numerical example rounds off the analysis. The authors find that an increase in the variability of the spark spread has an ambiguous effect on the investment decisions: it increases the value of a peak-load plant, making investments in such plants more attractive, while uncertainty at the same time delays investment. The model can be used to measure the relative strength of these two opposing effects.

Siddiqui and Marnay (2005) model the operating decisions of a commercial enterprise that aims to satisfy its electricity demand either by on-site distributed generation (DG) or by purchases from the wholesale market. The former option involves electricity generation at relatively high and possibly stochastic costs from a set of (capacity-constrained) DG technologies, while the latter implies unlimited open market transactions at stochastic prices. The resulting optimization problem is solved by stochastic dynamic programming. The option values of the DG units are found by solving the problem with and without the availability of DG units. The authors find that using a capacity-weighted portfolio of options overstates the implied option value of the portfolio, since it ignores the crowding out of the more expensive by the less expensive DG unit. This effect is amplified when minimum run-time constraints are imposed, and mitigated when the input fuel price is stochastic.

Siddiqui and Marnay (2006) study the problem of investing in a DG unit that runs on natural gas and that is part of a micro-grid.\(^3\) The long-term generation costs are stochastic. Initially, it is assumed that the micro-grid may purchase electricity at a fixed retail rate from its utility. Using a real options approach the authors find generation cost thresholds that trigger investment in DG. Operational flexibility by the micro-grid is found to accelerate DG investment, while the option to disconnect from the grid is found

\(^3\)A micro-grid is defined as a semi-autonomous group of electricity and heat loads interconnected with the existing (macro-) grid but able to operate independently as well. The micro-grid minimizes the cost of meeting its heat and electricity requirements by optimizing the installation and operation of DG capacity and purchasing residual energy (electricity, natural gas) from the local multi-utility (Siddiqui and Marnay, 2006).
to be unattractive (and only exercised in case of sustained natural gas price decreases). By allowing the electricity price to be stochastic, the authors determine a boundary for the investment threshold. From this they find that high (low) electricity price volatility relative to that of natural gas generating costs delays (accelerates) investment, while at the same time increasing (decreasing) the value of the investment.

Fleten et al. (in press) focus on optimal investments in decentralized renewable power generation technologies under price uncertainty. The investor has the deferrable opportunity (or option) to invest in a local power generating unit, with the aim to maximize profits from the investment. The aim of the study is to find the price intervals and the optimal capacity for investment. From a case study for wind power generation for an office building the authors conclude that, given price uncertainty, it is optimal to wait for higher prices than the break-even price that accrues from the net present value calculation, and that optimal capacity choice is indeed influenced by the current market price and price volatility. Moreover, they find that with low price volatility there can be more than one investment price interval for different units, with intermediate waiting regions between them. In contrast, high price volatility increases the value of the investment option, which makes it more attractive to postpone the investment until larger units become profitable.

3 The model

In this section we first introduce the basic model set-up used (section 3.1), followed by a determination of the option value of investment (section 3.2), and some sensitivity analysis for different levels of uncertainty of both the investment threshold and the option value (section 3.3).

3.1 Basic model set-up

We consider two investment alternatives for industrial large-scale heat generation. The investor can either adopt a new cogeneration plant or a heat-only generation system, both of which are assumed to be fossil-fuelled.\textsuperscript{4} We assume that the production facility of the

\textsuperscript{4}Most cogeneration plants today use fossil fuels, although biomass-fired cogeneration units have been gaining importance in recent years, often in response to dedicated energy policies aimed at fostering the
industrial firm is connected to the electrical grid but not to any district heating network. Therefore, we assume further that the facility is operated in such a way that heat supply always meets heat demand. Moreover, we restrict our analysis to the case where electricity must be purchased from the grid, i.e. there is no delivery of (excess) power to the grid. This assumption allows us to exclude considerations about buy-back rates and related uncertainty. At the same time it restricts the analysis to the case where the electrical capacity of cogeneration is below the firm’s demand for electricity.\(^5\) Additionally, we consider the investment to be totally irreversible, and assume further that if the system is installed and put into operation, it cannot be shut down and restarted anymore (for costless shut-down and restart see McDonald and Siegel, 1985). Based on our assumption that heat demand must always be met by the on-site heat generation facility, this simplification can be justified by the fact that, within a reasonable range of energy prices, it does not pay off to shut down the whole production of the firm. While this assumption does not change the qualitative results of our analysis, it greatly simplifies the analysis since we do not have to consider optimal operation of an installed facility in terms of real options theory. Finally, the investor faces uncertainty in the price for the fossil fuel input and for electricity, respectively.

In the following, we expound our model in its general form with two stochastic prices – for the input fuel and for electricity. In order to confine the complexity of the analysis (and to keep the model analytically tractable), we solve the model by fixing one price at a time, such that uncertainty arises only from one source.\(^6\) We start with a comparison between the two options (cogeneration system, steam boiler) for different parameter values describing the uncertainty of the investment. We then analyze the joint determination of optimal technology choice and optimal timing of investment.

Let us now take a closer look at the fossil fuel price, \(P_F\), and the electricity price, \(P_E\).
We assume that both prices follow a geometric Brownian motion:

\[
\frac{dP_F(t)}{P_F(t)} = \mu_F dt + \sigma_F dz_1(t),
\]

\[
\frac{dP_E(t)}{P_E(t)} = \mu_E dt + \sigma_E \rho_{FE} dz_1(t) + \sigma_E \sqrt{1 - \rho_{FE}^2} dz_2(t),
\]

(1)

(2)

where \( \mu_i \) and \( \sigma_i, i = \{F, E\} \), are constants describing the drift and the volatility, respectively, of the price process \( P_i \) concerned, \( dt \) is an infinitesimal time increment, and \( dz_1 \) and \( dz_2 \) are two uncorrelated standard Brownian motion increments. The prices in \( t = 0 \) are known, and the correlation between \( P_F \) and \( P_E \) is given by \( \rho_{FE} \).

Since we focus on the effects of energy price volatility on the optimal choice of technology in a real options framework, we only consider the stochastic components of the operating costs. That is, we focus on the fuel costs to operate the heat generation system and the electricity costs (i.e. the costs of purchasing electricity from the grid), which represent the main fraction of the variable costs. The unit costs for heat of the heat-only generation system (typically a steam boiler), \( C_{SB} \), and of the co-generation system, \( C_{CG} \), are defined as:

\[
C_j(P_F(t), P_E(t)) = \frac{P_F(t)}{\eta_j} + \frac{1 - \lambda \varsigma_j}{\lambda} P_E(t),
\]

(3)

where \( \eta_j \) denotes the thermal efficiency of the heat generation system, \( \lambda \) represents the heat intensity of the firm’s demand for heat and electricity (i.e. the number of heat units needed per electricity unit used), which is assumed to be constant, and \( \varsigma_j \) is the electricity rate of the heat generation system (i.e. the ratio between electric efficiency and thermal efficiency; hence \( \varsigma_{SB} \equiv 0 \)). The unit cost function can be written in a more compact form as:

\[
C_j(P_F(t), P_E(t)) = \gamma_{F,j} P_F(t) + \gamma_{E,j} P_E(t),
\]

(4)

where \( \gamma_{F,j} \equiv \frac{1}{\eta_j} \) and \( \gamma_{E,j} \equiv \frac{1 - \lambda \varsigma_j}{\lambda} \) denote the cost parameters. According to our assumption, no electricity is delivered to the grid, which requires that the cost parameter for electricity, \( \gamma_{E,j} \), is greater than zero.

Since we want to analyze the implications of energy price uncertainties, we assume for simplicity that the firm earns a constant net revenue stream, \( \Pi \), from company-wide operations. This revenue stream can be interpreted as the net income stream of the firm...
without electricity and fuel costs. Hence, the cash flow of the firm investing in technology
$j$, $\pi_j(P_F(t), P_E(t))$, is defined as:

$$\pi_j(P_F(t), P_E(t)) = \Pi - C_j(P_F(t), P_E(t)).$$

(5)

Investment in such a generation system is equivalent to swapping a fixed amount of money – the sunk investment costs $I_j$ – with a perpetual cash flow stream, $\pi_j(P_F(t), P_E(t))$. In a full-scale model we would also have to consider replacement and retrofit decisions, and to extend our analysis to sequential decisions. Since we want to keep our model as simple as possible, we exclude those aspects and assume that the heat-only system and the cogeneration system can be operated infinitely. In this case, the value of the firm installing the generation system at time $t$ is equal to the discounted cash flow stream over the whole lifetime of the plant,

$$V_j(t) = E_t \left[ \int_t^\infty \pi_j(P_F(\tau), P_E(\tau)) e^{-\rho(\tau-t)} d\tau \right] = \frac{\Pi}{\rho} - \frac{\gamma_{F,j} P_F(t)}{\rho - \mu_F} - \frac{\gamma_{E,j} P_E(t)}{\rho - \mu_E},$$

(6)

where $p_F(t)$ and $p_E(t)$ are known realizations of the prices and $\rho$ is the investor’s discount rate.

In the following, we analyze and compare the following two investment options:

1. The firm invests in a cogeneration system, and thus is subject to a lower degree of electricity price uncertainty for the electricity purchased from the grid, since it self-generates part of its electricity needs. At the same time, more fossil fuel is needed to supply the heat demanded and, consequently, the firm is more exposed to fossil fuel price fluctuations.

2. The firm invests in a heat-only generation system, and is to a lower degree subject to input fuel price uncertainty, but more exposed to volatile electricity prices (due to the lower amount of self-generation). Note that for the two options considered not only the cost structure is different but also the risk profile.

### 3.2 Option value of the investment

The decision to invest in a generation system can be specified as an optimal stopping problem and solved by using the technique of dynamic programming (cf. Dixit and Pindyck,
1994). The value of the option to invest in the generation system at time $t$ as a function of $P_F(t)$ and $P_E(t)$ is given by:

$$
\rho F_j(P_F(t), P_E(t)) = \frac{E[dF_j]}{dt}.
$$

(7)

This equation can be interpreted as a no arbitrage condition. Holding an option with value $F(x)$ over the period $(t, t + dt)$ yields an expected gain, $E[dF]$. This expected gain has to equal the return $\rho F(x)dt$. We can apply Itô’s Lemma and derive the following partial differential equation:

$$
\frac{1}{2} \sigma_F^2 P_F^2 \frac{\partial^2 F_j}{\partial P_F^2} + \rho_F \sigma_F \sigma_E P_F P_E \frac{\partial^2 F_j}{\partial P_F \partial P_E} + \frac{1}{2} \sigma_E^2 P_E^2 \frac{\partial^2 F_j}{\partial P_E^2} + \alpha_F P_F \frac{\partial F_j}{\partial P_F} + \alpha_E P_E \frac{\partial F_j}{\partial P_E} - \rho F_j = 0.
$$

Now the investor has to decide whether she should exercise the investment option or keep it alive. Hence, she has to decide whether she should swap a fixed amount of money, $I_j$, for an asset with value $V_j(t)$, which pays a perpetual stochastic cash flow stream, $\pi_j(P_F(t), P_E(t))$. If there are no time restrictions, this option is similar to an American call option, which never expires. The payoff function of such an option at any time $t$ is given by:

$$
H_j(P_F(t), P_E(t)) = \max \left[ \frac{\Pi}{\rho} - \frac{\gamma_{F,j} P_F(t)}{\rho - \mu_F} - \frac{\gamma_{E,j} P_E(t)}{\rho - \mu_E} - I_j, 0 \right],
$$

where $I_j$ denotes the sunk investment costs. Since the option can be exercised at any time, the option value $F_j(P_F(t), P_E(t))$ must always dominate its intrinsic value, i.e.:

$$
F_j(P_F(t), P_E(t)) \geq H_j(P_F(t), P_E(t)).
$$

There exists a boundary $(\hat{p}_{F,j}, \hat{p}_{E,j})$ at which it is optimal to exercise the option. If the input prices are below this boundary, then it is optimal to exercise the option immediately, and the option value equals the net present value of the investment. In this case the return per unit of time of the investment option exceeds the capital gain of holding the option, and the equilibrium condition becomes:

$$
\rho F_j(P_F(t), P_E(t)) > \frac{E[dF_j]}{dt}.
$$

In contrast, if the input prices are above their threshold values, then it is optimal to keep the option, in which case the value of the option is determined by the equilibrium condition.
Since we are mainly interested in the implications of stochastic input prices, we neglect
the deterministic drift parameter and set $\mu_F = \mu_E = 0$. Furthermore, for simplicity we set
$\rho_{FE} = 0$. If we only consider the cases where one variable is deterministic, i.e. $\sigma_F > 0$ and
$\sigma_E = 0$, or $\sigma_F = 0$ and $\sigma_E > 0$, we have to solve the ordinary differential equation:

$$\frac{1}{2} \sigma_i^2 P_i^2 \frac{\partial^2 F_{ij}}{\partial P_i^2} - \rho F_{ij} = 0,$$
with the boundary conditions:

$$F_{ij}(P_i(t)) \geq \max \left[ \frac{\Pi_{ij} - \gamma_{ij} P_i(t)}{\rho} - I_j, 0 \right]$$

$$F_{ij}(\tilde{p}_{ij}) = V_{ij}(\tilde{p}_{ij}, \tilde{p}_{-i}) - I_j,$$

$$\frac{\partial F_{ij}(\tilde{p}_{ij})}{\partial P_i} = \frac{\partial V_{ij}(\tilde{p}_{ij}, \tilde{p}_{-i})}{\partial P_i},$$

where $F_{ij}(P_i(t)) \equiv F_j(P_i(t), \tilde{p}_{-i})$ denotes the option value of the heat generation system
$j$ if price $i$ is stochastic, $\Pi_{ij} \equiv \Pi - \gamma_{ij} \tilde{p}_{-i}$ and $V_{ij}(P_i(t), \tilde{p}_{-i}) \equiv \frac{\Pi_{ij} - \gamma_{ij} P_i(t)}{\rho}$. The second
condition is known as the value matching condition, while the third condition is called the
smooth pasting condition. Both together guarantee the optimality of the solution (e.g.
Sødal, 1998).

The option value of such a non-dividend paying underlying has been solved by Merton
(1973). The solution to this problem is:

$$F_{ij}(P_i) = A_{ij} P_i^{\alpha_i}\quad (10)$$

$$A_{ij} = \left(-\frac{\gamma_{ij}}{\rho \alpha_i}\right)^{\alpha_i} \left(\frac{\Pi_{ij} - \rho I_j}{\rho(1 - \alpha_i)}\right)^{1-\alpha_i}$$

$$\alpha_i = \frac{1}{2} - \sqrt{\frac{1}{4} + \frac{2\rho}{\sigma_i^2}} < 0.$$

The threshold price $\tilde{p}_{ij}$ at which the value of the investment equals the option value is
defined as:

$$\tilde{p}_{ij} = \frac{\alpha_i}{\alpha_i - 1} \frac{\Pi_{ij} - \rho I_j}{\gamma_{ij}}.\quad (11)$$

If the current price is lower than this threshold price, then it is optimal for the firm to invest
today. On the other hand, if the current price is above the threshold, then it is optimal to
wait and get more information about the possible future energy price level. Inspecting the
expression for the threshold price (11), we see that the threshold decreases as $\sigma$ increases.
Thus, the more uncertain future energy prices are, the lower must the energy prices for realizing the investment be. Furthermore, if $\sigma$ goes to zero, then the investment threshold is equal to the break even price of the net present value calculation, i.e. $\tilde{p}_{ij} = \frac{\Pi_{ij} - \rho I_j}{\gamma_{ij}}$.

### 3.3 Sensitivity of investment threshold and option value

Figures 1 and 2 show the option value of the two generation systems (expressed in million Euros per Megawatt of installed heat capacity) considered for alternative values of price volatility per annum, $\sigma_i$. It is a standard result that the option price increases with higher volatility. Put differently, the firm requires a higher net present value of the investment at higher levels of volatility.

Next we show – for economically interesting parameter settings – that there exists a critical level of price volatility, above which the value of one investment option always dominates that of the other one. We assume that the graphs for the net present value of the cogeneration and steam boiler investment options cross in the first quadrant, i.e.

\[ V_{i,SB}(p_i, p_{-i}) - I_{SB} = V_{i,CG}(p_i, p_{-i}) - I_{CG} > 0. \]

If this crossing property is satisfied, then there exists a price range for which the steam boiler yields a higher net present value than the cogeneration system and vice versa. Since $\gamma_{F,CG} > \gamma_{F,SB}$, this implies for the case of stochastic fuel prices that

(i) at $p_F = 0$ the net present value of cogeneration is higher than for the steam boiler,
Figure 2: Option value as a function of electricity price for different levels of electricity price volatility

Figure 3: Threshold prices

\[ \Pi_{F,CG} - \rho I_{CG} > \Pi_{F,SB} - \rho I_{SB} > 0, \text{ and,} \]

(ii) the break even price under the net present value evaluation is higher for the steam boiler than for the cogeneration unit, i.e. \( \frac{\Pi_{F,SB} - \rho I_{SB}}{\gamma_{F,SB}} > \frac{\Pi_{E,CG} - \rho I_{CG}}{\gamma_{F,CG}} > 0 \).

An immediate consequence of the second implication is that the threshold price, \( \tilde{p}_{F,SB} \), is greater than \( \tilde{p}_{F,CG} \) (see Figure 3(a)). The same holds, with opposite signs since \( \gamma_{E,SB} > \gamma_{E,CG} \), in the case of stochastic electricity prices: \( \Pi_{E,SB} - \rho I_{SB} > \Pi_{E,CG} - \rho I_{CG} > 0 \) and \( \frac{\Pi_{E,CG} - \rho I_{CG}}{\gamma_{E,CG}} > \frac{\Pi_{E,SB} - \rho I_{SB}}{\gamma_{E,SB}} > 0 \), which implies that \( \tilde{p}_{E,CG} > \tilde{p}_{E,SB} \) (see Figure 3(b)).

If the crossing property is satisfied, then there exist values for \( \sigma \) such that the option
value of one investment option dominates the other one. If fuel prices are stochastic, then the cogeneration option always dominates the steam boiler if $F_{F,CG} \geq F_{F,SB}$, which implies that

$$\alpha_F \leq \frac{\ln \left( \frac{\Pi_{F,CG} - \rho_{CG}}{\Pi_{F,SB} - \rho_{SB}} \right)}{\ln \left( \frac{\gamma_{F,SB}(\Pi_{F,CG} - \rho_{CG})}{\gamma_{F,CG}(\Pi_{F,SB} - \rho_{SB})} \right)} < 0.$$  

On the other hand, if electricity prices are stochastic, then the steam boiler option dominates the cogeneration option if $F_{E,SB} \geq F_{E,CG}$, i.e.

$$\alpha_E \leq \frac{\ln \left( \frac{\Pi_{E,SB} - \rho_{SB}}{\Pi_{E,CG} - \rho_{CG}} \right)}{\ln \left( \frac{\gamma_{E,CG}(\Pi_{E,SB} - \rho_{SB})}{\gamma_{E,SB}(\Pi_{E,CG} - \rho_{CG})} \right)} < 0.$$  

Thus, the critical value of $\sigma_i$ is given by $F_{i,SB} = F_{i,CG}$:

$$\hat{\sigma}_i^2 = \frac{2\rho}{\hat{\alpha}_i (\hat{\alpha}_i - 1)},$$

$$\hat{\alpha}_i = \frac{\ln \left( \frac{\Pi_{i,CG} - \rho_{CG}}{\Pi_{i,SB} - \rho_{SB}} \right)}{\ln \left( \frac{\gamma_{i,SB}(\Pi_{i,CG} - \rho_{CG})}{\gamma_{i,CG}(\Pi_{i,SB} - \rho_{SB})} \right)} < 0.$$  

If the price volatility is equal to or higher than this critical value, either the value of the dominating technology or the value of postponing the investment project is always higher than the option value of the alternative investment. Figure 4 graphically illustrates the option values at the critical volatility levels for the case of a stochastic fuel price and the case of a stochastic electricity price.
4 Optimal time of investment and technology choice

In this section we analyze the joint determination of the optimal timing of the investment and the technology chosen. For both energy conversion systems the arbitrage argument implies that the ordinary differential equation has to be satisfied:

$$\frac{1}{2} \sigma_i^2 P_i^2 \frac{\partial^2 F_i}{\partial P_i^2} - \rho F_i = 0.$$  \hfill (13)

The general solution to this equation is:

$$F_i(P_i) = B_{1i} P_i^{\alpha_i} + B_{2i} P_i^{\beta_i},$$  \hfill (14)

where

$$\alpha_i = \frac{1}{2} - \sqrt{\frac{1}{4} + 2 \frac{\rho}{\sigma_i^2}} < 0, \quad \beta_i = \frac{1}{2} + \sqrt{\frac{1}{4} + 2 \frac{\rho}{\sigma_i^2}} > 1.$$  \hfill (15)

The higher the electricity price, the more favorable is cogeneration. Hence the optimal solution of the investor’s decision problem can be expressed by two threshold levels for the electricity price, \( \hat{p}_{E,SB} \leq \hat{p}_{E,CG} \). In between the two threshold prices postponement of the investment decision is optimal. If the electricity price is above \( \hat{p}_{E,CG} \), then it is optimal to invest in cogeneration technology. Conversely, if it is below \( \hat{p}_{E,SB} \), then it is optimal to adopt the conventional steam boiler technology. The same argument, vice versa, applies for the case of stochastic fuel prices. The boundary conditions, i.e. the value matching and smooth pasting conditions, of the problem are:

$$F_i(\hat{p}_{ij}) = V_{ij}(\hat{p}_{ij}) - I_j,$$

$$\frac{\partial F_i(\hat{p}_{ij})}{\partial P_i} = \frac{\partial V_{ij}(\hat{p}_{ij})}{\partial P_i},$$  \hfill (16)

or put explicitly,

$$B_{1i}^{\alpha_i} \hat{p}_{i,SB} + B_{2i}^{\beta_i} \hat{p}_{i,SB} = \frac{\Pi_{i,SB} - \gamma_{i,SB} \hat{p}_{i,SB}}{\rho} - I_{SB},$$  \hfill (17)

$$B_{1i}^{\alpha_i} \hat{p}_{i,CG} + B_{2i}^{\beta_i} \hat{p}_{i,CG} = \frac{\Pi_{i,CG} - \gamma_{i,CG} \hat{p}_{i,CG}}{\rho} - I_{CG},$$  \hfill (18)

$$\alpha_i B_{1i}^{\alpha_i - 1} \hat{p}_{i,SB} + \beta_i B_{2i}^{\beta_i - 1} \hat{p}_{i,SB} = - \frac{\gamma_{i,SB}}{\rho},$$  \hfill (19)

$$\alpha_i B_{1i}^{\alpha_i - 1} \hat{p}_{i,CG} + \beta_i B_{2i}^{\beta_i - 1} \hat{p}_{i,CG} = - \frac{\gamma_{i,CG}}{\rho}.$$  \hfill (20)
which solves for the unknowns $B_1^i$, $B_2^i$, $\hat{p}_{i,CG}$, and $\hat{p}_{i,SB}$. These equations are linear in the coefficients $B_1^i$ and $B_2^i$, but highly nonlinear in the threshold values. Hence this system of equations cannot be solved explicitly. In order to solve it, we have parameterized the model and calculated the investment threshold prices. The parameters used and parameter values chosen are listed in the Appendix. These values have been taken from an unpublished study of a cogeneration plant in the Swiss chemical industry sector.

5 The effect of uncertainty

In the previous discussion we have seen that under certain circumstances there exists a critical value for the price volatility where one technology always dominates the alternative. At this boundary we have a non-monotone change in the optimal investment regions. If the volatility is below this critical boundary, both investment alternatives are reasonable options. But if the volatility is above the critical boundary, one option is always dominated and falls out of the set of possible investment alternatives, since it is always more favorable to either invest into the dominating alternative or to postpone the investment project.

In Figure 5 the alternative investment regions and the waiting region are depicted for different volatility levels of the fuel and the electricity prices, respectively. Figure 5(a) shows that with low fuel prices, cogeneration is more attractive since the difference between the electricity price and the cost of self-generation is higher than the additional fuel costs.
of a cogeneration plant. As the volatility of the fuel price increases, the investment region for the steam boiler technology decreases. Again, the upper bound of the investment region for the steam boiler technology decreases, since postponing the investment is more valuable than investing today. On the other hand, the lower bound of the investment region for the steam boiler increases, since postponing the investment is more attractive than immediate investment. In this case, the investor can gain more information about future fuel prices, which allows her to determine which option is more economical. In our example, if volatility becomes larger than the critical value of 21.5%, then the cogeneration technology dominates the steam boiler technology for all possible fuel prices.

Figure 5(b) depicts that if electricity prices are low, then a steam boiler is more economical, since self-generation of electricity by using a fossil-fuelled cogeneration plant is too costly. As the volatility of the electricity price increases, the waiting region becomes larger. Furthermore, the investment region for cogeneration becomes narrower as the price volatility increases. First, the upper bound of the investment region decreases, since with increasing volatility the investment project becomes more and more risky and the value of postponing the project at high prices rises. Second, the lower bound of the investment region increases, since for high volatilities the investor is uncertain about which of the alternatives is more favorable, and thus postpones the investment, in order to obtain more information about the future electricity price level. In our example calculation, if volatility becomes larger than a critical value of about 24%, then the cogeneration option is dominated by the steam boiler technology for all possible electricity prices.

6 The effect of a CO$_2$ tax

Increased concentration of CO$_2$ in the atmosphere and the expected global warming is one of the main environmental concerns today. The taxation of CO$_2$ from fossil fuels, the most important greenhouse gas, has frequently been recommended by economists as an important means to mitigate anthropogenic global climate change in a cost-effective manner. A carbon tax, imposed on fossil fuels proportionally to their carbon contents (or alternatively the amount of carbon dioxide released during combustion), would stimulate firms to reduce the fossil fuel intensity of production and to shift the fuel mix towards
less carbon-intensive fuels, such as natural gas or biomass (cf. Jorgenson et al., 1992, for a useful discussion on CO$_2$ taxation).

Studies on the impact of a CO$_2$ tax on cogeneration technology adoption are still rare. Siddiqui et al. (2005), for instance, examine the effect of carbon taxation on the adoption and carbon emissions of distributed generation (DG) assets by a hypothetical micro-grid consisting of several commercial buildings. The DG technologies available are reciprocating engines, micro-turbines, and fuel cells, with or without cogeneration equipment, such as water and space heating and/or absorption cooling. Introduction of a carbon tax shows that carbon emissions are lower if cogeneration technologies are being used in the micro-grid, thus providing some evidence for the potential benefits of small-scale CHP technology for climate change mitigation.

In this section we want to demonstrate that a policy, which alters the relative energy prices, changes the investment thresholds in a non-linear way. To this end, we introduce a CO$_2$ tax of EUR 40 per ton of CO$_2$. Let us assume that the heat generation system is operated with natural gas. Furthermore, we assume that the electricity price rises due to the CO$_2$ tax. In order to calculate the expected price increases, we have used a CO$_2$ intensity of natural gas of 55 tons of CO$_2$ per Terajoule (TJ), and for the UCTE mix 143 tons of CO$_2$ per TJ (cf. UCTE, 2004). The induced price increase, therefore, was around 0.5 cents per kWh for natural gas and about 1.5 cents per kWh for electricity. We have used the prices including the CO$_2$ tax for recalculating the investment regions, which are shown in Figure 6.

From a comparison with Figure 5 we can see that the waiting region becomes markedly larger. Under uncertainty, changes in energy prices have a non-linear effect on investment regions. Where one option is dominated by the alternative, a CO$_2$ tax induces a decrease in the value of the critical volatility. This implies that the investment region of the dominated technology becomes smaller.

---

7 The “Union for the Co-ordination of Transmission of Electricity” (UCTE) is the association of transmission system operators in continental Europe. The so-called ‘UCTE power mix’ is the country-specific average value of power generation by primary energy carrier, reported by the UCTE on an annual basis. Only three categories of power generation are distinguished: thermal, hydro, and nuclear.
7 Conclusions

Many industrial firms face the decision problem of whether they should invest in a heat-only generating technology for meeting their heat demand, or whether they should invest in cogeneration technology, thus allowing them to produce at least part of their electricity needs on site (self-generation).

In this paper we have introduced the decision-making problem of an industrial firm that, in a situation of uncertainty regarding either the input fuel price or the electricity price, aims at investing in a cogeneration system or a heat-only generating system. In a numerical example we have applied the dynamic stochastic model developed to realistic values for the economic and technical parameters considered.

We show that for critical price volatility levels (in our numerical example 24% for electricity price and 21% for fuel price), one technology always dominates the other. In this case the choice of the firm is either to adopt the dominating technology or to postpone the investment. For the case of a CO$_2$ tax of 40 EUR/ton we show that investment thresholds are altered in a non-linear fashion, and that the investment region of the dominated technology becomes significantly smaller.

Methodologically, it is well known that traditional net present value models cannot capture the effects of risks on the valuation of investment projects if there exists a waiting option. For the case of industrial large-scale cogeneration we have analyzed the impacts of
volatile energy prices on the firm’s decision problem contemplating cogeneration adoption, i.e. the joint determination of technology choice and investment timing. Thus, for policy making, our model suggests that the interrelation between policy measures and energy price volatilities should be taken into account. On the one hand, since investment and waiting regions depend on energy price volatility, policies that change energy price volatilities, e.g. market deregulation, affect both the optimal technology choice and the optimal investment timing. But also policy measures that do not affect energy price volatilities directly, such as the introduction of a CO₂ tax, can change the optimal choice in a non-linear way. Therefore, using net present value based calculations to assess current or future policy measures related to cogeneration, and neglecting the effects of volatile energy prices, might result in misleading findings (in that net present value calculations would ignore the non-linear shape of the region where it is optimal to postpone the investment, and its changing size due to changes in the level of risk).

In order to highlight the role of energy price volatilities in industrial cogeneration investment decisions we made several simplifying assumptions, which can be relaxed in future research. In particular, if the model is to be used in applied work, the following aspects have to be taken into account:

First, we have excluded any operational risks from our analysis (e.g. unsatisfactory plant performance). However, if operational risks are high, investors are likely to prefer less capital-intensive investments, which would make the steam boiler option relatively more attractive.

Second, in order to exclude the problem of optimal operation from our analysis, we have assumed that there is no shut-down and restart option, i.e. the firm is inflexible in the operation of the heat generation system due to heat demand rigidity. However, in practice many firms run cogeneration and steam boilers in combination – possibly also having the opportunity of fuel switching – and can therefore adjust the optimal operation plan of their heat and power generation system according to prevailing energy prices. Clearly, the optimal dispatching decision can also be analyzed within a real options framework, as research by others has shown. In this case, we would have to analyze the problem sequentially, i.e. before studying the joint determination of technology choice and investment
timing, we have to consider the optimal dispatching policy which determines the value of the investment.

Third, allowing both the fuel price and the electricity price to enter simultaneously as stochastic variables into the model would complicate model solving markedly. As Murto and Nese (2003) have already pointed out, the nature of the solution would remain the same, but the state space would be two-dimensional instead of one-dimensional. Of course, adding more than two sources of uncertainty enlarges the state space further. Having more than one stochastic variable also raises questions how the correlation between these stochastic variables affects the investment decision. In case of stochastic fuel and electricity prices, positive correlation implies higher risk exposure for both cogeneration and steam boiler technology. Thus, the direction and size of the effect of correlated fuel and electricity prices on the joint determination of technology choice and investment timing is not obvious \textit{a priori}. Stochastic models that are based on the difference between the electricity price and fuel costs (so-called 'spark spread models') seem to be a useful alternative. Note, however, that these models are again only analytically tractable if the assumptions are highly restrictive regarding the stochastic process that mimics the spark spread (e.g. geometric Brownian motion).

8 Acknowledgements

Comments received from Lorna Greening and three anonymous referees are gratefully acknowledged.

References


### Appendix: Economic and technical parameters used in the numerical example

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount rate</td>
<td>$\rho$</td>
<td>5</td>
<td>%/a</td>
</tr>
<tr>
<td>Demand load</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heat</td>
<td>$L_H$</td>
<td>13</td>
<td>MW</td>
</tr>
<tr>
<td>Electricity</td>
<td>$L_E$</td>
<td>9</td>
<td>MW</td>
</tr>
<tr>
<td>Heat intensity</td>
<td>$\lambda$</td>
<td>$\approx 1.44$</td>
<td></td>
</tr>
<tr>
<td>Thermal efficiency</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heat-only generation</td>
<td>$\eta_{SB}$</td>
<td>0.9</td>
<td>-</td>
</tr>
<tr>
<td>Cogeneration</td>
<td>$\eta_{CG}$</td>
<td>0.55</td>
<td>-</td>
</tr>
<tr>
<td>Electricity rate</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heat-only generation</td>
<td>$\varsigma_{SB}$</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>Cogeneration</td>
<td>$\varsigma_{CG}$</td>
<td>$\approx 0.36$</td>
<td>-</td>
</tr>
<tr>
<td>Specific investment costs</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heat-only generation</td>
<td>$I_{SB}$</td>
<td>130</td>
<td>EUR/kW$_{th}$</td>
</tr>
<tr>
<td>Cogeneration</td>
<td>$I_{CG}$</td>
<td>1100</td>
<td>EUR/kW$_{el}$</td>
</tr>
<tr>
<td>Revenue</td>
<td>$\Pi$</td>
<td>14</td>
<td>million EUR/a</td>
</tr>
</tbody>
</table>
CEPE Working Papers

1999


2000


2001


2002


2003


2005


2006


CEPE Reports and CEPE Working Papers can mostly be downloaded free of charge in pdf-format from the CEPE Website (www.cepe.ethz.ch). Alternatively, they may be ordered from: CEPE, Secretariat, Zurichbergstrasse 18 (ZUE E), CH-8032 Zurich, Switzerland.