Combining Rights and Welfarism: a new approach to intertemporal evaluation of social alternatives

Ngo Van Long[1] and Vincent Martinet[2]

November 3, 2014

[1]Department of Economics, McGill University, Montreal H3A 2T7, Canada.
E-mail: ngo.long@mcgill.ca, phone: 1-514-398-4844, fax: 1-514-398-4938.
[2]INRA, UMR210 Economie Publique, F-78850 Thiverval-Grignon, France.
E-mail: vincent.martinet@grignon.inra.fr, phone: 33-1-3081-5357, fax: 33-1-3081-5368.

Combining Rights and Welfarism: a new approach to intertemporal evaluation of social alternatives

Abstract

We propose a new criterion which reflects both the concern for rights and the concern for welfare in the evaluation of economic development paths. The concern for rights is captured by a pre-ordering over combinations of thresholds corresponding to floors or ceilings on various quantitative indicators. The resulting constraints on actions and on levels of state variables are interpreted as minimal rights to be guaranteed to all generations, for intergenerational equity or legacy purposes. They are endogenously chosen within the set of all feasible thresholds, accounting for the "cost in terms of welfare" of granting these rights. Such a criterion could embody the idea of sustainable development. We apply the criterion to the standard Dasgupta-Heal-Solow model of resource extraction and capital accumulation. We show that if the weight given to rights in the criterion is sufficiently high, the optimal solution is on the threshold possibility frontier. The development path is then "driven" by the rights. In particular, if a minimal consumption is considered as a right, constant consumption can be optimal even with a positive utility discount rate. The shadow values of rights constraints play an important role in the determination of the rate of discount to be applied to social investment projects.

JEL-Classifications: D63, H43, Q01.

Keywords: Rights, Welfare, Intergenerational Equity, Sustainability.

1 Introduction

Much of normative economic theory is built on the premises that individuals seek to maximize their "utility" and that social welfare is the (weighted) sum of individual utilities. Under utilitarianism, or more generally welfarism, it is legitimate to prescribe policies that lead to increase the utility of some individuals at the expense of that of other individuals, as long as "social welfare" rises. At some extreme, the life of a person could be sacrificed for "the greater good" of the society. In an intergenerational context, the welfare of a generation could be sacrificed without limits to increase the intertemporal welfare by raising the welfare of other generations. Many philosophers have expressed the concern that welfarism does not take "rights" seriously. They argue that all individuals should be entitled to some basic rights, such as life, health, and a "decent standard of living." Rawls (1971) pointed out that "optimal growth" (under some utilitarian objectives) may unreasonably require too much savings from poor generations for the benefits of their wealthier descendants. More recently, the same rationale has led environmentalists to argue that the present generations, in their pursuit of wealth and wellbeing, are depriving future generations of their rights to natural assets.

Sustainable development has been described in the Brundtland report (WCED, 1987) as development "that meets the needs of the present without compromising the ability of future generations to meet their own needs." Current growth patterns induce concerns for sustainability, in particular with respect to environmental degradations. Intergenerational equity and environmental concerns are cornerstones of sustainability. Reflecting the concerns for rights, environmental issues are often addressed with quantitative approaches on physical measures and thresholds. Along these lines, it is argued that society should impose constraints, in the form of floors or ceilings, on various variables. For example biodiversity should not fall below a certain level, while emissions of pollutants should not exceed a certain level.[1] These thresholds can be interpreted as *minimal rights* to be guaranteed to all generations (Martinet, 2011). Of course, if floors are too high and ceilings are too low, the set of possible actions will be empty. To rule out such a case, one has to address the trade-offs among minimal rights. Moreover, these constraints, when they are effective, induce some costs in terms of welfare growth. In the climate

[1] Socio-economic thresholds could also be mentioned, for instance on health and education.

change debate, a ceiling on green house gases concentration would impose restrictions on the current growth pattern as emissions would have to be curtailed. This is the cost of providing future generations the right to live in a more or less tolerable climate. When defining such an environmental constraint, current generations trade off this cost and the level of the environmental objective they agree to sustain for future generations. There is a tension between rights and welfare. The definition of rights to be guaranteed to all generations should account both for the trade-offs among rights levels and the trade-offs between rights and welfare.

While the question of the trade-offs among several sustainability objectives (i.e., quantities which should be sustained) has been formalized in Martinet (2011), the welfare cost of such minimal rights have not been considered. Alvarez-Cuadrado and Long (2009), however, emphasized that imposing a sustainability constraint on the growth path induces a cost in terms of welfare. In a different context, not referring to rights but to a concern for the utility of the worse-off generation, they show that there is a trade-off between welfare and the sustained level of utility optimally chosen by the society.[2]

In this paper, we put forward the view that society may not only seek to maximize "welfare" (in a standard sense offered by welfare economics), but may also be concerned with rights. Society thus makes trade-offs between rights and welfare. To represent these trade-offs, we propose a criterion for ranking social alternatives, based on an indicator called "Rights and Welfare Indicator" (RWI for short). This indicator combines an index of rights (such as the right to satisfy basic needs or the right to have access to some natural assets) with a welfare index (based on the conventional utilitarian objective of maximizing the integral of the discounted stream of utility derived from the consumption of goods and services). The index of rights is an aggregate measure of various thresholds representing "sustainability" in a broad sense. The optimization problem *endogenously* determines the rights guaranteed to all generations, accounting for their cost in terms of welfare. The optimal development path that satisfies these rights is also characterized, allowing us to describe the implications of this objective on the path of resource use.

We show that, depending on the preferences and the relative weight accorded to min-

[2] Alvarez-Cuadrado and Long (2009) proposed a "Mixed Bentham Rawls Criterion" to address this issue. We shall see that if the utility of the worse-off generation is interpreted as a *minimal right*, their criterion is a special case of ours, which considers several minimal rights.

imal rights, the optimal development path may either be a constrained utilitarian path, or switch to a development path fully characterized by the minimal rights guaranteed to all generations ("*right-based sustainable development*"). We illustrate the general results by applying the criterion to the standard Dasgupta-Heal-Solow model of resource extraction and capital accumulation. In this model, we find that, when the minimal rights constraints are effective, the implied social discount rate is different from that obtained under the classical utilitarian formulation.

The remaining of the paper is organized as follows. The motivation of our approach is detailed in Section 2. We present therein the tension between rights and welfare, as well as a brief history of sustainability criteria that puts our criterion in perspective. Section 3 presents the studied criterion and characterizes its solution in a general economic framework. The results are illustrated in the Dasgupta-Heal-Solow model of nonrenewable resource depletion and capital accumulation in Section 4. We provide concluding remarks in Section 5.

2 Motivation

2.1 Rights versus Welfare

The tension between rights and welfarist considerations has long been a subject of debate among philosophers, thinkers and economists. The Rawlsian theory of justice places rights above welfare.[3] In fact, Rawls's first principle of justice is that everyone should have equal rights: "each person is to have an equal right in the most extensive scheme of equal basic liberties compatible with a similar scheme of liberties for others." His second principle of justice, the difference principle, insists that social and economic inequalities are acceptable only if they are arranged so that they are "both (a) to the greatest expected benefit of the least advantaged and (b) attached to offices and positions open to all under conditions of fair equality of opportunity." In particular, difference in income is acceptable only if it

[3]Rawls's conception of justice has its foundation in the theory of social contract advanced by Locke, Rousseau, and Kant. The initial position conceived by Rawls is a hypothetical situation in which the contracting parties are individuals hidden behind the veil of ignorance: none of them knows his place in society, his natural talents, intelligence, strength, and the like. In other words, the principles of justice are agreed to in an initial situation that is fair.

improves the life prospects of the least advantaged.[4]

This concern extends to the intergenerational equity, for example when optimal growth requires too much savings from poor generations for the benefits of their wealthier descendants.[5] In that intertemporal context, Rawls' difference principle would result in a constant utility path, with no growth (Solow, 1974; Burmeister and Hammond, 1977). Rawls (1971) acknowledged that economic growth is necessary, because without adequate material resources a society cannot develop institutions that guarantee equal liberties to all.[6] Wealth creation is necessary for the effective defense of rights and liberties. Welfarist considerations cannot be ignored.

The welfarist approach to development is different from the right-based approach. It is based on intertemporal welfare functions (i.e., criteria) ranking the intertemporal performance of the economy. In this context, the emphasis is on the weight of the various generations in the objective function, with strong implications on discounting and intergenerational equity. These questions have been extensively addressed in the sustainability literature.

[4]Another influential philosopher who stressed the preponderance of rights is Nozick (1974). He emphasized the importance of property rights, from a somewhat different perspective. Nozick's work has inspired alternative articulations of libertarian rights with a game-theoretic flavor. See Gärdenfors (1981); Sugden (1985); Gaertner et al. (1992); Deb (1994); Hammond (1995); Hammond (1996); Peleg (1998); Suzumura and Yoshihara (2008), among others, and the overview proposed by Suzumura (2005); these papers acknowledge the fundamental contribution of Sen (1970a,b, 1976). In our paper, we abstract from game-theoretic considerations.

[5]Immanuel Kant (1724-1804) found it disconcerting that earlier generations should carry the burdens for the benefits of later generations. In his essay, "Idea for a Universal History with a Cosmopolitan Purpose," Kant put forward the view that nature is concerned with seeing that man should work his way onwards to make himself worthy of life and well-being. He added: "What remains disconcerting about all this is firstly, that the earlier generations seem to perform their laborious tasks only for the sake of the later ones, so as to prepare for them a further stage from which they can raise still higher the structure intended by nature; and secondly, that only the later generations will in fact have the good fortune to inhabit the building on which a whole series of their forefathers ... had worked without being able to share in the happiness they were preparing." See Reiss (1970, p. 44).

[6]The need for adequate savings is a major concern for Rawls, because, "to establish effective, just institutions within which the basic liberties can be realized"society must have a sufficient material base. As a unmodified difference principle would lead to "no savings at all," he pointed out that the difference principle must be modified to allow for economic growth. For this purpose, he sketched a theory of "just saving" in which generations must "carry their fair share of the burden of realizing and preserving a just society." See Long (2007).

2.2 Sustainability and minimal rights for future generations

The tension between rights and welfare is particularly important when trying to define "sustainable development." In this respect, there is an important difference between weak and strong sustainability (Neumayer, 2013). Proponents of weak sustainability generally seek to sustain some notion of welfare, and define sustainability criteria to address the intergenerational equity issue. On the other hand, proponents of strong sustainability argue for the sustainment of environmental assets, and are concerned with the rights of future generations to inherit a good environmental quality. In practice, environmental issues are often addressed in setting quantitative targets (e.g., the cap in the Kyoto Protocol for climate change, or the habitat conservation objectives in the Nagoya Protocol for the Convention on Biological Diversity), which can be interpreted as a way to set environmental rights for future generations.

The traditional criterion for evaluating intertemporal development paths is the discounted utility criterion. According to this criterion, a decrease in the utility level of a generation (no matter how disadvantaged this generation already is and how large is the considered sacrifice) can be justified by a sufficient increase in the utility level of some other generations. This criterion is strongly inequitable, and has been shown to display "dictatorship of the present," a term coined by Chichilnisky (1996). For example, in the case of the Dasgupta-Heal-Solow model, the optimal consumption path under discounted utilitarianism decreases toward zero in the long run while the resource is exhausted (Dasgupta and Heal, 1974). Defining a criterion that accounts for intergenerational equity, and in particular for the long run, has been a challenge for economists addressing the sustainability issue (Heal, 1998; Martinet, 2012).

Another, extreme alternative is the Green Golden Rule (Beltratti et al., 1995), which defines the development path reaching and sustaining the highest possible development level. Giving weight only to the very long-run, this criterion has been criticized by Chichilnisky (1996) as permitting "dictatorship of the future". To avoid dictatorships of the present and the future, Chichilnisky (1996) suggested to use as a welfare function the weighted sum of the usual discounted stream of utilities and a measurement of the limiting behavior of the utility sequence. Alvarez-Cuadrado and Long (2009) proposed to modify Chichilnisky's criterion by replacing the second term with the minimal level of utility of the trajectory over time, in line with the maximin criterion (Solow, 1974; Cairns and Long, 2006). The maximin criterion, considered alone, is insensitive to the utilities of generations that are not the poorest.[7] Moreover, if it is possible to smooth utility over time, the maximin principle leads to no growth, no matter how small is the initial maximal sustainable utility. There is no concern for growth, which may be an issue if capital accumulation is needed to develop and sustain just institutions. By considering a weighted average of the standard sum of discounted utilities and a Rawlsian part which places special emphasis on the utility of the least advantaged generation, the "Mixed Bentham-Rawls"(MBR) criterion provided by Alvarez-Cuadrado and Long (2009) satisfies both non-dictatorship of the present and of the future, just as Chichilnisky's welfare function.[8] The maximization of the MBR criterion determines endogenously a minimal utility level to be sustained forever. Without referring to rights, the criterion introduces the idea that the welfare of some generations (in particular future generations) cannot be sacrificed too much for other generations (in particular present generations).

All the described intertemporal welfare functions weigh the utility of the various generations differently. This has strong implications in terms of discounting. More specifically, the discount rate to be used to evaluate project investment with long run impacts is strongly influenced by the criterion chosen. None, however, encompasses a concern for rights.

A challenge for strong sustainability and the practice of sustainable development is to define the quantities of environmental assets to be conserved or the levels of environmental indicators that have to be sustained. Martinet (2011) proposed a criterion which defines several *minimal rights* to be guaranteed to all generations. These rights are represented by a set of constraints on indicators $\mathcal{I}_i$, and the associated sustainability thresholds μ_i, which must be chosen optimally.[9] This criterion does not permit intergenerational trade-offs. All generations have the same minimal rights. By not accounting for the utility levels

[7]The maximin criterion has been strengthened to eliminate some maximin paths that are Pareto dominated by other paths that have the same minimum level of utility. See Asheim and Zuber (2013); Long (2011).

[8]It also implies that social welfare is increasing in U_t, ensuring that the strong Pareto property is satisfied. The utility of the least advantaged is thus not the only thing that matters. One may say that this rules out "dictatorship of the least advantaged."

[9]Formally, the indicators are functions of a set of state variables x and control variables c, resulting in constraints of the form $\mathcal{I}_i(x(t), c(t)) \geq \mu_i$ at all times t, where the μ_i's are to be chosen optimally.

of different generations in an intertemporal welfare function, the criterion solely defines minimal rights representing sustainability, without considering welfare and the "cost" of satisfying these rights. This is an important limitation for the scope of application.

In the present paper, we aim at developing an approach which encompasses both welfarist considerations and the concern for rights. In particular, we emphasize that, if minimal rights are imposed to the development path, one should account for the consequences of these rights on welfare when setting their levels.

The criterion studied in the present paper combines a welfare index with an index based on minimal rights. The levels of the minimal rights are defined endogenously, and come at a "cost" in term of present-value welfare. This criterion could represent the choice of a society defining (economic and environmental) minimal rights to be guaranteed over time to embody the idea of sustainable development.[10]

3 The Rights and Welfare Indicator

3.1 Economic modeling framework

We assume that the economy is composed of infinitely many generations. We make the simplifying assumption that each generation can be assimilated to a representative agent, and do not address intragenerational equity, in order to focus on the intertemporal dimension of the problem.

We consider a stylized, general economic model in a continuous time framework. Let x be a vector of n state variables, and c a vector of m control variables. Denote the instantaneous utility function by $U(x(t), c(t))$. The transition equations are $\dot{x}_k(t) = g_k(x(t), c(t))$, for $k = 1, \ldots, n$. Given the values of the state variables, the control variables at time t must belong to a technologically feasibility set $A(x(t))$ which is characterized by a set of

[10]It is important to acknowledge that the criteria proposed by Chichilnisky (1996), Alvarez-Cuadrado and Long (2009) and Martinet (2011) all have the property that in general the optimal solution is time-inconsistent. As time goes by, the utilities in the distant future, which were negligible at the time the plan was conceived, become important, and their weights in the trade-off (against minimum consumption right, as in Avarez-Cuadrado and Long, or against the golden rule utility, as in Chichilnisky) are no longer negligible. Even in a framework without discounting, as in Martinet, the time-inconsistency problem can also arise, because the maximin level of consumption is in general sensitive to the current stock level. Our RWI proposed in this paper shares with the above criteria this important issue. The readers are referred to Alvarez-Cuadrado and Long (2009); Martinet (2011) for further discussion on this issue.

s inequality constraints:

$$h_j(x(t), c(t)) \geq 0,\ j = 1, 2, \ldots, s. \tag{1}$$

For state and decisions, a time path is denoted by $x(\cdot)$ or $c(\cdot)$. Following the standard control theoretic treatment, we require $x(\cdot)$ to be piece-wise differentiable and $c(\cdot)$ to be piece-wise continuous.

3.2 Welfare and rights measurement

Welfare Assuming a constant utility discount rate $\delta > 0$, a feasible time path $(x(\cdot), c(\cdot))$ starting from state x_0 yields a welfare level

$$\mathcal{W}(x(\cdot), c(\cdot)) \equiv \int_0^\infty e^{-\delta t} U\left(x(t), c(t)\right) dt\ . \tag{2}$$

Rights We suppose that society places values on some minimal rights guaranteed at all times. We assume that, for each right, it is possible to construct an indicator function showing at each point of time how society is faring in terms of meeting that right. An indicator is a function of a set of state variables and control variables. A threshold for an indicator is the numerical level below which the indicator is not allowed to fall.[11] The rights are represented by constraints on the indicators' level. Consider a finite number (I) of issues, each represented by an indicator $\mathcal{I}_i$ and a threshold μ_i, with $i = 1, \ldots, I$.[12] We distinguish two types of rights, corresponding to *legacy* considerations and to the *satisfaction of current needs.*

Legacy constraints One may require that for some capital stocks, at least certain minimum levels ought to be transmitted to future generations. It could be the case for natural resources, in a strong sustainability perspective. Among the I issues related to rights, the first p are considered to be legacy constraints. Without loss of generality, we

[11] By formulating thresholds as floors rather than ceilings, we are normalizing so that indicators are "goods" rather than "bads."

[12] The number of issues considered can be as large as one wants. If some issues are not important enough, the associated thresholds are endogenously set at levels that are not constraining for welfare maximization. The number of effective rights can thus be considered as endogenous and optimal.

suppose that a legacy constraint applies to each of the first p state variables, where $p \leq n$.[13] The indicator associated to each legacy minimal rights is simply the corresponding state variable: $I_i \equiv x_i$ where $i = 1, 2, ..., p$.

Current needs satisfaction rights The other type of rights corresponds to the satisfaction of some needs at all times. The associated indicators depend on decision variables (or on decision and state variables), and are of the form $\mathcal{I}_i(x(t), c(t))$, with $i = p + 1, \ldots, I$.

The constraints associated to both types of rights read

$$x_i(t) \geq \mu_i \ , \qquad i = 1, \ldots, p \ , \qquad \forall t \ . \tag{3}$$

$$\mathcal{I}_i(x(t), c(t)) \geq \mu_i \ , \qquad i = p + 1, \ldots, I \ , \quad \forall t \ . \tag{4}$$

For any set of thresholds $(\mu_1, \ldots, \mu_I)$, define $\mathcal{F}(x_0; \mu_1, \ldots, \mu_I)$ as the set of all the economic paths $(x(\cdot), c(\cdot))$ starting from initial state x_0 and satisfying all constraints (3 - 4), i.e.,

$$\mathcal{F}(x_0; \mu_1, \ldots, \mu_I) = \left\{ (x(\cdot), c(\cdot)) \left| \begin{array}{ll} x_k(0) = x_{k0} \ , & k = 1, \ldots, n \ \text{ and } \forall t : \\ \dot{x}_k(t) = g_k(x(t), c(t)) \ , & k = 1, \ldots, n \ ; \\ h_j(x(t), c(t)) \geq 0 \ , & j = 1, \ldots, s \ ; \\ x_i(t) \geq \mu_i, & i = 1, \ldots, p \ ; \\ \mathcal{I}_i(x(t), c(t)) \geq \mu_i \ , & i = p + 1, \ldots, I \end{array} \right. \right\} \tag{5}$$

Clearly, given the initial stock x_0, the set $\mathcal{F}(x_0; \mu_1, \ldots, \mu_I)$ may be empty if the thresholds μ_i are too high. It is sensible to consider only thresholds that are consistent with the economic endowment x_0. Since the maintenance of an indicator above a threshold level typically requires the use of resources, it is plausible to argue that for any given level of resource endowment, there is a well-defined *set of feasible thresholds* within which a vector of optimal thresholds would be chosen.

[13] This convention simplifies notation. It is only a harmless re-arrangement of subscripts.

Definition 1 (Set of feasible thresholds) *Given an initial state x_0, the set of feasible thresholds is defined as the set of thresholds for which there are feasible economic paths starting from state x_0 and satisfying constraints (3 - 4) at all times, i.e.,*

$$\mathcal{M}(x_0) = \{(\mu_1, \ldots, \mu_I) |\ \mathcal{F}(x_0; \mu_1, \ldots, \mu_I) \neq \emptyset\} \ .$$

We assume that the set of feasible thresholds $\mathcal{M}(x_0)$ is delimited by a *threshold possibility frontier*. This upper boundary is represented by the equality $\phi(\mu_1, \ldots, \mu_I; x_0) = 0$, with the convention that points below this frontier yield $\phi(\mu_1, \ldots, \mu_I; x_0) > 0$, where ϕ is a real-valued differentiable function, as illustrated in Fig. 1.[14] Given x_0, an infeasible threshold vector $\mu_1, \ldots, \mu_I$ would yield $\phi(\mu_1, \ldots, \mu_I; x_0) < 0$. Then, for any feasible threshold vector, the real number $\phi(\mu_1, \ldots, \mu_I; x_0)$ can be thought of as a measure of how much leeway there is left for satisfying objectives other than the chosen minimal rights. Since higher thresholds reduce the leeway, we suppose that $\partial\phi/\partial\mu_i \leq 0$.[15]

The preferences of society over the minimal rights formalized by the thresholds are represented by a function $\mathcal{R}(\mu_1, \ldots, \mu_I)$, which is non-decreasing in all its arguments. This function can be interpreted as a *right index*. This index of rights is an aggregate measure of the threshold levels, not of the extent to which society exceeds the various thresholds.

It is likely that increasing any threshold will reduce the welfare index given by eq. (2). In this sense, there is a tension between rights and welfare. To emphasize this, we define a constrained welfare value function giving the maximal welfare level which can be achieved given some rights constraints.

Definition 2 (Constrained welfare value function) *For any vector of feasible thresholds $(\mu_1, \ldots, \mu_I) \in \mathcal{M}(x_0)$, we define the associated* constrained welfare value func-

[14] We assume here the existence of a function ϕ that is differentiable, to represent the boundary of the set of feasible minimal rights.

[15] Thus, in the case of two thresholds, the slope of the threshold possibility frontier is $\frac{d\mu_2}{d\mu_1} = -\frac{\phi_{\mu_1}}{\phi_{\mu_2}} < 0$.

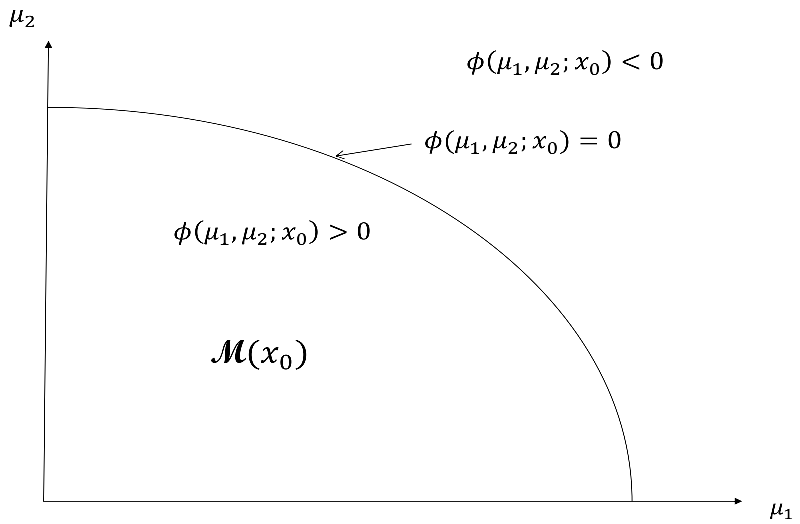

Figure 1: Set of feasible thresholds for two minimal rights

tion $V(x_0;\mu_1,\ldots,\mu_I)$ *as*

$$V(x_0;\mu_1,\ldots,\mu_I) \equiv \max_{c(\cdot)} \mathcal{W}(x(\cdot),c(\cdot)) \ , \tag{6}$$

$$s.t. \quad \begin{array}{ll} x_k(0)=x_{k0} \ , & k=1,\ldots,n \quad \textit{and}\ \forall t: \\ \dot{x}_k(t)=g_k(x(t),c(t)) \ , & k=1,\ldots,n \ ; \\ h_j(x(t),c(t))\geq 0 \ , & j=1,\ldots,s \ ; \\ x_i(t)\geq \ \mu_i, & i=1,\ldots,p \ ; \\ \mathcal{I}_i(x(t),c(t))\geq \ \mu_i \ , & i=p+1,\ldots,I \ . \end{array}$$

The following proposition states that increasing one or several minimal rights thresholds reduces the potential welfare.[16]

Proposition 1 (Tension between rights and welfare) *For any set of thresholds*

[16]Proofs are in the Appendix.

$(\mu_1, \ldots, \mu_I)$ *and* $(\mu'_1, \ldots, \mu'_I)$ *such that* $\mu'_i \geq \mu_i$ *for* $i = 1, \ldots, I$, *one has* $V(x_0; \mu'_1, \ldots, \mu'_I) \leq V(x_0; \mu_1, \ldots, \mu_I)$.

Society may choose thresholds inside the feasibility set, because the cost of being on the frontier, measured in terms of forgone consumption of some goods and services, may outweigh the value of guaranteeing a high level of the rights represented by the thresholds. An optimal threshold vector should precisely balance the "costs" of thresholds in terms of welfarist consequences and the "moral worth" of thresholds in terms of rights. A social ranking criterion is required for this.

3.3 The criterion

We assume that the social problem of defining the optimal growth path along with the minimal rights level can be represented as the maximization of a criterion that we call a *Rights and Welfare Indicator (RWI)*. It is defined as the weighted sum of the right index $\mathcal{R}(\mu)$ and the welfare index $\mathcal{W}(x(\cdot), c(\cdot))$ defined by eq.(2), the relative weight given to "rights" being defined by a parameter $0 < \theta < 1$. The parameter θ is taken as given and can be interpreted as the political weight of the "non-welfarist" proponents. While the trade-offs are captured by a scalar measure, this measure should not be interpreted as a measure of "generalized welfare."

The resulting optimal control problem with endogenous constraints is defined as follows.

$$
\begin{aligned}
\max_{c(\cdot),\mu} \quad & \theta \mathcal{R}(\mu_1, \ldots, \mu_I) + (1-\theta)\mathcal{W}(x(\cdot), c(\cdot)) \qquad (7) \\
s.t. \quad & x(t=0) = x_0, \quad \text{and} \quad \forall t \\
& \dot{x}_k(t) = g_k(x(t), c(t)) \;, k = 1, \ldots, n \\
& h_j(x(t), c(t)) \geq 0 \;, j = 1, \ldots, s \\
& x_i(t) \geq \mu_i \;, i = 1, \ldots, p \\
& \mathcal{I}_i(x(t), c(t)) \geq \mu_i \;, i = p+1, \ldots, I \\
& \phi(\mu_1, \ldots, \mu_I) \geq 0.
\end{aligned}
$$

This problem defines both the optimal growth path and the optimally chosen levels of the constraints. Increasing the threshold for any right indicator involves a cost in terms of welfare. This cost is accounted for in the optimization.

3.4 The necessary conditions

Maximizing criterion (7) is equivalent to maximizing the following expression by choosing optimal μ and $c(\cdot)$:

$$\int_0^\infty \left\{\delta\theta\mathcal{R}(\mu_1,\ldots,\mu_I) + (1-\theta)U(x(t),c(t))\right\} e^{-\delta t} dt \; . \tag{8}$$

Since $(\mu_1,\ldots,\mu_I)$ are constants to be chosen optimally, this optimization problem is an optimal control problem with $(\mu_1,\ldots,\mu_I)$ treated as control parameters. The necessary conditions for such problems can be derived from Hestenes' Theorem.[17]

Let $\pi(t)$ denote the vector of co-state variables, $\lambda(t)$ the vector of multipliers associated with the inequality constraints $h_j(x(t),c(t)) \geq 0, \quad j = 1,\ldots,s$, and $\omega(t)$ the vector of multipliers associated with the right-based constraints

$$\mathcal{I}_i(x(t),c(t)) - \mu_i \geq 0, \quad i = 1,\ldots,I \; . \tag{9}$$

where the first p of these right-based constraints are pure-state-variable constraints, with $\mathcal{I}_i(x(t),c(t)) \equiv x_i$. Let $\psi(t) = e^{\delta t}\pi(t)$, $\Delta(t) = e^{\delta t}\lambda(t)$ and $w(t) = e^{\delta t}\omega(t)$ represent the current values of these variables. The current-value Hamiltonian of this infinite horizon problem is

$$H^c = \delta\theta\mathcal{R}(\mu_1,\ldots,\mu_I) + (1-\theta)U(x,c) + \psi g(x,c) \; , \tag{10}$$

and the current-value Lagrangian is

$$L^c = H^c + \Delta h(x,c) + w\left[\mathcal{I}(x,c) - \mu\right] . \tag{11}$$

The first-order conditions of the optimization problem are as follows.[18]

[17] See Leonard and Long (1991, Theorem 7.11.1) or Takayama (1985) for an exposition of Hestenes' Theorem which deals with optimal control problems involving control parameters and various constraints.

[18] We here consider the first order, necessary conditions only, for the sake of simplicity. The sufficiency

The control variables maximize the Hamiltonian subject to the inequality constraints (1) and (9)., i.e.,

$$\frac{\partial L^c}{\partial c} = (1-\theta)U_c' + \psi g_c' + \Delta h_c' + w\mathcal{I}_c' = 0 \ .$$

The shadow-values satisfy

$$\dot{\psi} = \delta\psi - \frac{\partial L^c}{\partial x} \ .$$

The stock dynamics imply

$$\dot{x} = \frac{\partial L^c}{\partial \psi} \ .$$

The satisfaction of the admissibility constraints implies

$$\Delta \geq 0, \quad h(x,c) \geq 0, \quad \Delta h(x,c) = 0 \ . \tag{12}$$

The satisfaction of the right-based constraints implies

$$w \geq 0, \quad \mathcal{I}(x,c) - \mu \geq 0, \quad w\left[\mathcal{I}(x,c) - \mu\right] = 0 \ . \tag{13}$$

Moreover, when pure state constraints like (3) are imposed for all t, technically this implies specifying a 'terminal manifold' for the associated state variables:

$$\lim_{t\to\infty} x_i(t) \geq \mu_i, \quad i = 1,2,...,p. \tag{14}$$

The transversality conditions with respect to these stocks are $\lim_{t\to\infty} \pi_i(t) \geq 0$ and $\lim_{t\to\infty} \pi_i(t)(x_i(t) - \mu_i) = 0$

The optimality conditions with respect to the choice of the control parameters μ_i are

$$\int_0^\infty e^{-\delta t}\frac{\partial L^c}{\partial \mu_i}dt + \gamma\frac{\partial \phi}{\partial \mu_i} = \overline{\pi}_i \ , i = 1,2,...,p \tag{15}$$

$$\int_0^\infty e^{-\delta t}\frac{\partial L^c}{\partial \mu_i}dt + \gamma\frac{\partial \phi}{\partial \mu_i} = 0 \ , i = p+1, p+2, ..., I \tag{16}$$

with $\gamma \geq 0$, $\phi(\mu_1, \ldots, \mu_I; x_0) \geq 0$, and $\gamma\phi(.) = 0$, and $\overline{\pi}_i \equiv \lim_{t\to\infty} \pi_i(t)$.

Conditions (15) and (16), which are specific to our problem with endogenous param-

conditions (concavity conditions) can be derived as in Leonard and Long (1991). We also assume that constraint qualifications are satisfied (see Takayama, 1985).

eters choice, deserve an interpretation. First, consider conditions (16), which are the conditions that thresholds μ_i associated with current needs satisfaction ($i = p+1, ..., I$) must satisfy to maximize the objective function (8). The first term (the integral) expresses the marginal contribution of the parameter level to the overall objective, i.e., to rights and welfare. It balances the trade-off between rights and welfare. Standard optimization intuition is to set the level of μ_i such that its marginal contribution to the overall objective is nil, i.e, the marginal gain in terms of rights of increasing the threshold equals its marginal cost in terms of welfare. As the parameters must belong to the set of feasible thresholds $\mathcal{M}(x_0)$, it may not, however, be possible to increase μ_i up to that point. This feasibility constraint is encompassed in the second term. This second term is nil when the considered vector of parameters μ is in the interior of the feasible set (one then has $\gamma = 0$), allowing to "increase" the thresholds up to the point to which their marginal contributions to the objective are nil. When this is not possible, the vector μ is on the boundary of the feasibility set $\mathcal{M}(x_0)$, meaning that $\phi(.) = 0$. One then has $\gamma > 0$. The second term then represents the trade-offs among minimal rights. Optimality is reached while the marginal contribution of the parameter to the objective is still positive (given that $\frac{\partial \phi}{\partial \mu_i} < 0$).

Conditions (15) apply only when the rights are in the form of pure state constraints, i.e., for legacy constraints (3). The two terms on the left-hand side of the equality have the same interpretation as for conditions (16). The right-hand side element represents the welfare cost of preserving a part of the capital stock forever. Requiring that a stock x_i must not fall below some strictly positive threshold level μ_i involves a welfare cost, measured by $\overline{\pi}_i$, such that $\overline{\pi}_i \geq 0$, with strict inequality holding only if $\lim_{t\to\infty} x_i(t) = \mu_i$. Even when $x_i(t) > \mu_i$ for all finite t (i.e., $w_i = 0$ at all times), the constraint can be binding in the limiting sense.

From these optimality conditions, we can derive the following general results to our optimization problem. These are all correlated propositions.

Proposition 2 (Optimal minimal rights) *Consider the vector of optimal minimal rights* $(\mu_1^\star, \ldots, \mu_I^\star)$ *and the optimal development path* $(x^\star(\cdot), c^\star(\cdot))$ *defined by the maximization of criterion (7). Minimal rights satisfy the following properties.*

Prop2a *A minimal right μ_i has a zero marginal value at the optimum, i.e., $\mathcal{R}'_{\mu_i}(\mu^\star) = 0$, if and only if the associated constraint is never binding*[19] and *the vector of optimal minimal rights is not on the boundary of the set of feasible rights $\mathcal{M}(x_0)$.*

Legacy-driven rights

Prop2b *If a legacy constraint is binding in the limiting sense and the optimal solution $\mu^\star$ is in the interior of the threshold possibility set, the marginal contribution of this right $\theta R'_{\mu_i}(\mu^\star)$ is equal to its marginal cost in terms of forgone welfare $\overline{\pi}_i$.*

Current needs satisfaction rights

Prop2c *A minimal right μ_i associated with the satisfaction of current needs has a positive marginal value at the optimum, i.e., $\mathcal{R}'_{\mu_i}(\mu^\star) > 0$ with $i = p+1, \ldots, I$, if and only if the associated constraint is binding* or *the vector of optimal minimal rights is on the boundary of the set of feasible rights $\mathcal{M}(x_0)$.*

Prop2d *If the vector of optimal minimal rights is not on the boundary of the set of feasible rights $\mathcal{M}(x_0)$, any constraint associated with the satisfaction of current needs for which the associated minimal right has a strictly positive marginal value must be binding for at least some time interval along the optimal development path, i.e., for any $i = p+1, \ldots, I$ such that $\mathcal{R}'_{\mu_i}(\mu^\star) > 0$, there is a time interval $[t_i, t_i + \varepsilon_i]$ for some $\varepsilon_i > 0$ such that $\mathcal{I}_i(x^\star(t), c^\star(t)) = \mu_i^\star$ for all $t \in [t_i, t_i + \varepsilon_i]$.*

Prop2e *Assume there are no pure-state-variable rights constraints. Then, if one (or more) of the right-based constraints is never binding along the optimal development path $(x^\star(\cdot), c^\star(\cdot))$ while the associated minimal right has a strictly positive marginal value (i.e., $\mathcal{R}'_{\mu_i}(\mu^\star) > 0$), the vector of optimal minimal rights is necessarily on the boundary of the set of feasible rights $\mathcal{M}(x_0)$.*

We can interpret these results as follows: One would increase the level of a minimal right up to the point at which its marginal value for society is nil if and only if it has no opportunity costs, neither in terms of welfare maximization (the constraint is not

[19] In the case of legacy-driven rights associated to pure-state-variable constraint, 'never binding' means that both $w_i(t) = 0$ for all t and $\overline{\pi}_i = 0$.

binding) nor in terms of rights (other rights could be increased without reducing this one) [Prop2a]. Such a right is not constraining welfare or conflicting with other rights. If either of the two conditions does not hold, the level of this minimal right is restricted. The corresponding threshold is increased to a point at which the minimal right still has a positive marginal value [Prop2b and Prop2c]. When the vector of optimal minimal rights is not on the boundary of the set of feasible rights and a minimal right associated with the satisfaction of current needs ($i = p+1, \ldots, I$) has a strictly positive marginal value, the associated constraint is binding [Prop2d]. It is not possible to increase this right without decreasing welfare. This illustrates the tension between rights and welfare. Last, in a problem without legacy constraints, if at least one of the constraint associated with the satisfaction of current needs is never binding while the associated minimal right has a positive marginal value, it is because it is not possible to increase the threshold level without reducing another minimal right, the vector of optimal minimal rights being on the boundary of the set of feasible rights [Prop2e]. This illustrate the trade-offs among minimal rights.

To better understand the implications of the criterion, we shall apply it to a canonical model often used in the sustainability literature, the Dasgupta-Heal-Solow model (Dasgupta and Heal, 1974; Solow, 1974). Before that, let us discuss some axioms the criterion satisfies.

3.5 Discussion on axiomatic foundation

We have postulated that the objective function is a weighted sum of welfare and rights. While we do not intend to derive this objective function from a set of axioms, it is useful to discuss the types of axioms that would be consistent with our criterion. For this purpose, it is convenient to use the familar discrete time framework, which has been used in, e.g., Chichilnisky (1996) and Alvarez-Cuadrado and Long (2009).

For this section, let time be denoted by $t = 0, 1, 2, 3, \ldots$. There are infinitely many generations, each living for one period, without overlapping. Each generation consists of a single (representative) individual. The consumption of the generation t yields a utility level u_t. For simplicity, assume that u_t is a real number in the unit interval $[0, 1]$.

We define a 'utility allocation' to be a sequence $\mathbf{u} \equiv \{u_t\}_{t=0,1,2,\ldots} \equiv \{u_0, u_1, u_2, \ldots\}$.

Let S be the set of all utility sequences with $0 \leq u_t \leq 1$ for all t. Let $\boldsymbol{\mu} = (\mu_1, \ldots, \mu_I)$ denote a threshold vector in $\mathbb{R}^I$.

A "social alternative" is a couple $(\boldsymbol{u}, \boldsymbol{\mu}) \in S \times \mathbb{R}^I$ corresponding to a utility sequence $\mathbf{u}$ and a vector of minimal rights $\boldsymbol{\mu}$.

A social ranking criterion (SRC), denoted by G, is a preference ordering $\succsim$ over the set $S \times \mathbb{R}^I$. We require G to satisfy the following properties.

Property 1 (Completeness) *G must rank all possible social alternatives. For any pair $(\boldsymbol{u}, \boldsymbol{\mu})$ and $(\boldsymbol{u'}, \boldsymbol{\mu'})$, either $(\boldsymbol{u}, \boldsymbol{\mu}) \succsim (\boldsymbol{u'}, \boldsymbol{\mu'})$, or $(\boldsymbol{u'}, \boldsymbol{\mu'}) \succsim (\boldsymbol{u}, \boldsymbol{\mu})$.*

Property 2 (Transitivity) *If $(\boldsymbol{u}, \boldsymbol{\mu}) \succsim (\boldsymbol{u'}, \boldsymbol{\mu'})$ and $(\boldsymbol{u'}, \boldsymbol{\mu'}) \succsim (\boldsymbol{u}^\sharp, \boldsymbol{\mu}^\sharp)$, then $(\boldsymbol{u}, \boldsymbol{\mu}) \succsim (\boldsymbol{u}^\sharp, \boldsymbol{\mu}^\sharp)$*

Property 3 (Monotonicity in right thresholds) *$(\boldsymbol{u}, \boldsymbol{\mu'}) \succsim (\boldsymbol{u}, \boldsymbol{\mu})$ if $\mu'_i \geq \mu_i$ for all $i \in \{1, \ldots, I\}$.*

Property 4 (Strong Pareto in welfare) *$(\boldsymbol{u'}, \boldsymbol{\mu}) \succ (\boldsymbol{u}, \boldsymbol{\mu})$ if $u'_t \geq u_t$ for all t, with strict inequality for some t.*

Since our social preferences include right thresholds as well as utilities, we define the concepts of *dictatorship of rights* and *dictatorship of welfare.*

Definition 3 (Dictatorship of rights) *A social ranking criterion is said to display 'dictatorship of rights' if and only if*

$$(\mathbf{u}, \boldsymbol{\mu}) \succ (\mathbf{u}', \boldsymbol{\mu}') \text{ implies } (\mathbf{u}^{\#}, \boldsymbol{\mu}) \succ (\tilde{\mathbf{u}}, \boldsymbol{\mu}')$$

for all utility sequences $\mathbf{u}^{\#}$ and $\tilde{\mathbf{u}}$.

Definition 4 (Dictatorship of welfare) *A social ranking criterion is said to display 'dictatorship of welfare' if and only if*

$$(\mathbf{u}, \boldsymbol{\mu}) \succ (\mathbf{u}', \boldsymbol{\mu}') \text{ implies } (\mathbf{u}, \boldsymbol{\mu}^{\#}) \succ (\mathbf{u}', \tilde{\boldsymbol{\mu}})$$

for all vectors of minimal rights $\boldsymbol{\mu}^{\#}$ and $\tilde{\boldsymbol{\mu}}$.

In the literature on sustainability, the concepts of dictatorship of the present and dictatorship of the future are crucial (Chichilnisky, 1996). In the familiar welfarist framework of Chichilnisky, a social welfare function is said to display 'dictatorship of the present' if modifications in the utility levels of generations that live in the distant future cannot reverse the ranking of any pair of utility streams. Similarly, a social welfare function is said to display 'dictatorship of the future' if modifications in the utility levels of present generations cannot reverse the ranking of any pair of utility streams. Chichilnisky defined a social welfare function as 'sustainable' if and only if it displays both non-dictatorship of the present and non-dictatorship of the future. Similarly to Chichilnisky, we consider the requirement that social ranking is sufficiently sensitive to the utility levels of present and future generations. For this purpose, let us introduce two further definitions.

Definition 5 (Insensitivity to the welfare of future generations) *A social ranking criterion is said to display* insensitivity to the welfare of future generations *if and only if, given a common* $\boldsymbol{\mu}$*,* $(\mathbf{u^1}, \boldsymbol{\mu}) \succ (\mathbf{u^2}, \boldsymbol{\mu})$ *implies that, for a sufficiently large integer* T'*, if* $T > T'$ *then* $({}_T\mathbf{u^1}, \mathbf{a^1}_T, \boldsymbol{\mu}) \succ ({}_T\mathbf{u^2}, \mathbf{a^2}_T, \boldsymbol{\mu})$*, for all pairs of utility sequences* $(\mathbf{a^1}, \mathbf{a^2})$*, where* $({}_T\mathbf{u^i}, \mathbf{a^i}_T)$ *means that all elements of* $\mathbf{u^i}$ *except the first* $T+1$ *elements are replaced by the corresponding elements of the sequence* $\mathbf{a^i}$*.*

Definition 6 (Insensitivity to the welfare of present generations) *A social ranking criterion is said to display* insensitivity to the welfare of present generations *if and only if, given a common* $\boldsymbol{\mu}$*,* $(\mathbf{u^1}, \boldsymbol{\mu}) \succ (\mathbf{u^2}, \boldsymbol{\mu})$ *implies that there exists some* $T' > 0$ *such that for all* $T < T'$*,* $({}_T\mathbf{a^1}, \mathbf{u^1}_T, \boldsymbol{\mu}) \succ ({}_T\mathbf{a^2}, \mathbf{u^2}_T, \boldsymbol{\mu})$*, for all pairs of utility sequences* $(\mathbf{a^1}, \mathbf{a^2})$*, where* $({}_T\mathbf{a^i}, \mathbf{u^i}_T)$ *means that the first* $T+1$ *elements of* $\mathbf{u^i}$ *are replaced by the vector* ${}_T\mathbf{a^i} \equiv (a^1_0, a^1_1, \ldots, a^1_T)$*.*

Now consider the following Axioms.

Axiom 1 (Non-dictatorship of rights) *The social ranking criterion must not display dictatorship of rights.*

Axiom 2 (Non-dictatorship of welfare) *The social ranking criterion must not display dictatorship of welfare.*

Axiom 3 (Sensitivity to the welfare of present generations) *The social ranking criterion must not display insensitivity to the welfare of present generations.*

Axiom 4 (Sensitivity to the welfare of future generations) *The social ranking criterion must not display insensitivity to the welfare of future generations.*

Theorem 1 *Let $\mathcal{R}(\mu_1, \ldots, \mu_I)$ be an increasing function. Consider the following objective function*

$$G = \max_{(\mathbf{u},\boldsymbol{\mu})} \left(\theta \mathcal{R}(\mu_1, \ldots, \mu_I) + (1-\theta) \sum_{t=0}^{\infty} \beta^t u_t \right), \qquad \textit{with } \beta, \theta \in (0,1)$$

This criterion satisfies Properties 1, 2, 3, 4 and Axioms 1, 2, 3. It also satisfies Axiom 4 if one of the rights corresponding to the satisfaction of current needs, say μ_{p+1}, is associated to a finite measure strictly increasing with the utility of the current generation, such that for all t, $u_t \geq \mu_{p+1}$, i.e., $\inf\{u_t\}_{t=0,1,2,\ldots} \geq \mu_{p+1}$.

The proof of Theorem 1 is quite straightforward, except for the last statement, the proof of which is similar to that of Alvarez-Cuadrado and Long (2009) and is therefore omitted.

4 An example: The production-consumption economy with a nonrenewable resource

4.1 The model

Consider the Dasgupta-Heal-Solow model of nonrenewable resource extraction and capital accumulation (Dasgupta and Heal, 1974, 1979; Solow, 1974). Capital stock is denoted by $K(t)$, resource stock by $S(t)$, resource extraction by $r(t)$ and consumption by $c(t)$. We assume a Cobb-Douglas production function, i.e., $F(K, r) = K^{\alpha} r^{\beta}$, with $0 < \beta < \alpha < 1$. The dynamics of this economy are as follows:

$$\dot{K}(t) = K(t)^{\alpha} r(t)^{\beta} - c(t), \tag{17}$$

$$\dot{S}(t) = -r(t). \tag{18}$$

Instantaneous utility is derived only from consumption, i.e., $U(c(t))$, with $U' > 0$ and $U'' < 0$.

We consider the following sustainability indicators of consumption and resource stock,[20]

$$\begin{aligned} \mathcal{I}_1(c, r, S, K) &\equiv c \,, \\ \mathcal{I}_2(c, r, S, K) &\equiv S \,, \end{aligned}$$

as well as the following rights/sustainability constraints:

$$c(t) \geq \mu_c \,, \tag{19}$$
$$S(t) \geq \mu_S \,. \tag{20}$$

These constraints state that every generation has the right to a minimal consumption at level μ_c, i.e., to a minimal utility level, and the right to a minimal preserved resource stock μ_S. Note that condition (19) rules out insensitivity to the welfare of future generations, in accordance with our axiomatic discussion.

The set of achievable minimal consumption and preserved resource stock $(\mu_c, \mu_S) \in \mathcal{M}(K_0, S_0)$ is characterized by the following relationship (see Martinet and Doyen, 2007; Martinet, 2011):

$$\phi(\mu_c, \mu_S, K_0, S_0) \equiv (1-\beta)\big((S_0 - \mu_S)(\alpha - \beta)\big)^{\frac{\beta}{1-\beta}} K_0^{\frac{\alpha-\beta}{1-\beta}} - \mu_c \geq 0 \,. \tag{21}$$

The upper boundary of this set satisfies $\phi(\mu_c, \mu_S, K_0, S_0) = 0$. It can be represented

[20] Several authors have used this simple production-consumption economy to address the climate change issue (e.g., Stollery, 1998; D'Autume et al., 2010). The nonrenewable resource is related to fossil energy. Stabilizing green house gas (GHG) concentrations requires limiting the cumulative emissions over time. The in-ground resource stock is used as a proxy for non-emitted GHG. A limit on cumulative emissions can be represented by a constraint on resource extraction: a part of the stock has to be preserved.

by the following "threshold possibility frontier":[21]

$$\mu_c = (1-\beta)\big((S_0-\mu_S)(\alpha-\beta)\big)^{\frac{\beta}{1-\beta}} K_0^{\frac{\alpha-\beta}{1-\beta}} \ . \tag{22}$$

4.2 The RWI criterion

Assume that $\mathcal{R}(\mu_c, \mu_S) \equiv \eta_c\mu_c + \eta_S\mu_S$, where η_c and η_S are positive parameters, and consider the objective function:

$$J \equiv \theta\left[\eta_c\mu_c + \eta_s\mu_S\right] + (1-\theta)\int_0^\infty e^{-\delta t}U(c(t))dt \ , \tag{23}$$

subject to

$$\dot{K}(t) = K(t)^\alpha r(t)^\beta - c(t) \ , \ K(0) = K_0, K(t) \geq 0 \ , \tag{24}$$

$$\dot{S}(t) = -r(t), \ S(0) = S_0, \tag{25}$$

$$c(t) - \mu_c \geq 0 \ , \tag{26}$$

$$S(t) - \mu_S \geq 0 \ , \tag{27}$$

and

$$\phi(\mu_c, \mu_S, S_0, K_0) \geq 0 \ .$$

Using the same notations as in the general case for co-state variables and constraint multipliers, the current value Hamiltonian is

$$H^c = \delta\theta(\eta_c\mu_c + \eta_S\mu_S) + (1-\theta)U(c(t)) + \psi_K\left[K(t)^\alpha r(t)^\beta - c(t)\right] - \psi_S r(t) \ .$$

The associated Lagragian is

$$L^c = H^c + w_c(c-\mu_c) + w_S(S-\mu_S) \ .$$

[21]This curve has a negative slope and is concave: for all $\mu_S < S_0$, as $\frac{\partial\mu_c}{\partial\mu_S} = -\beta\left(S_0-\mu_S\right)^{\frac{-1}{1-\beta}}\big(\alpha-\beta\big)^{\frac{\beta}{1-\beta}} K_0^{\frac{\alpha-\beta}{1-\beta}} < 0$, and $\frac{\partial^2\mu_c}{(\partial\mu_S)^2} = -\frac{\beta}{1-\beta}\left(S_0-\mu_S\right)^{\frac{-2+\beta}{1-\beta}}\big(\alpha-\beta\big)^{\frac{\beta}{1-\beta}} K_0^{\frac{\alpha-\beta}{1-\beta}} < 0$.

The necessary conditions of this problem are[22]

$$
\begin{aligned}
\frac{\partial L^c}{\partial c} = 0 \quad &\Leftrightarrow \quad (1-\theta)U'_c - \psi_K + w_c = 0 \ , && (28)\\
\frac{\partial L^c}{\partial r} = 0 \quad &\Leftrightarrow \quad \psi_S = \psi_K F'_r \ , && (29)\\
\dot{\psi}_K = \delta\psi_K - \frac{\partial L^c}{\partial K} \quad &\Leftrightarrow \quad \frac{\dot{\psi}_K}{\psi_K} = \delta - F'_K \ , && (30)\\
\dot{\psi}_S = \delta\psi_S - \frac{\partial L^c}{\partial S} \quad &\Leftrightarrow \quad \dot{\psi}_S = \delta\psi_S - w_S \ , && (31)\\
\int_0^\infty e^{-\delta t}\frac{\partial L^c}{\partial \mu_c}dt + \gamma\frac{\partial \phi}{\partial \mu_c} = 0 \quad &\Leftrightarrow \quad \theta\eta_c - \int_0^\infty e^{-\delta t} w_c dt - \gamma = 0 \ , && (32)\\
\int_0^\infty e^{-\delta t}\frac{\partial L^c}{\partial \mu_S}dt + \gamma\frac{\partial \phi}{\partial \mu_S} = \lim_{t\to\infty} e^{-\delta t}\psi_S(t) \quad &\Leftrightarrow \quad \theta\eta_S - \int_0^\infty e^{-\delta t} w_S dt \ldots \\
&- \quad \gamma\beta(\alpha-\beta)^{\frac{\beta}{1-\beta}}(S_0-\mu_S)^{\frac{\beta}{1-\beta}-1}K_0^{\frac{\alpha-\beta}{1-\beta}} = \overline{\pi}_S \\
& && (33)
\end{aligned}
$$

with $\gamma \geq 0$, $\phi(\mu_c, \mu_S; S_0, K_0) \geq 0$ and $\gamma\phi(.) = 0$, as well as conditions (24), (25), (26), (27), $\overline{\pi}_S = \lim_{t\to\infty} e^{-\delta t}\psi_S(t)$ and

$$w_c \geq 0, \ w_c(c-\mu_c) = 0 \ ,$$

$$w_S \geq 0, \ w_S(S-\mu_S) = 0 \ .$$

Eq. (28) characterizes the capital shadow value as being the marginal utility weighted by the share of welfare in the RWI, plus the cost of the minimal consumption constraint. Eq. (29) characterizes the natural resource shadow value as being the marginal productivity of the resource times the capital shadow value. Eqs. (30) and (31) are related to Keynes-Ramsey's rule and Hotelling's rule (see our discussion at the end of the section). Eq. (32) and (33) correspond to the optimality conditions on minimal rights.

[22] Note that if we had specified a *Logarithmic valuation of rights*, with $\mathcal{R}(\mu_c, \mu_S) \equiv \eta_c \ln \mu_c + \eta_S \ln \mu_S$, then the terms $\theta\eta_c$ and $\theta\eta_S$ in Eqs. (32) and (33) would have to be replaced by $\theta\eta_c/\mu_c$ and $\theta\eta_S/\mu_S$. All other equations would remain unchanged.

4.3 Characterization of the optimal solution

Using the optimality conditions, and focusing on the case where $U'(0) = \infty$, we can prove the following results.[23]

Result 1 *Given that $\eta_c > 0$, the optimal consumption threshold $\mu_c^\star$ is strictly positive.*

To prove this result, suppose μ_c were zero, and $\mu_S \geq 0$. Then the optimal path would be the same as the utilitarian optimum in an economy with a stock $S(0) = S_0 - \mu_S > 0$. Along this path, consumption is always positive, approaching zero only asymptotically. Therefore, raising μ_c infinitesimally above zero will have effects only on the consumption in the far future, and thus, since $\delta > 0$, will have negligible marginal effect on welfare.[24] Yet, the marginal effect on minimum consumption rights is $\eta_c > 0$. It follows that it is optimal to raise μ_c above zero, i.e. $\mu_c^\star > 0$. (This is reflected in the necessary condition (32): $w_c(t)$ must be positive for t sufficiently large.)

Result 2 *Along the optimal path, $S(t) > \mu_S^\star$ for all finite t.*

This follows from the fact that at all points of time, a positive amount of resource must be extracted to meet a positive consumption that is at least as high as $\mu_c^\star > 0$.

Result 3 *As long as the resource is extracted, the following efficiency condition is satisfied: The rate of increase in the marginal product of the resource input is equal to the marginal product of capital*

$$\frac{1}{F_r'}\frac{d}{dt}(F_r') = F_K'. \tag{34}$$

To prove this result, use eqs (29), (30), and (31), bearing in mind that $w_S(t) = 0$ for all t. Thus, the introduction of the optimal thresholds on rights does not interfere with the dynamic efficiency condition (eq. 34), i.e., Hotelling's rule holds.

Result 4 *If the point $(\mu_c^\star, \mu_S^\star)$ is not on the threshold possibility frontier, the optimal shadow price of the resource, $\psi_S(0)$, is exactly equal to $\theta\eta_S$.*

[23]When $U'(0)$ is finite, the results are slightly different. In particular, Hotelling's rule (Result 3 below) may be modified if a stationary state is reached.

[24]Notice that μ_S can be kept constant when μ_c is raised marginally from zero: only the consumption path has to be marginally perturbed.

To prove this, note that Result 2 implies $w_S(t) = 0$ for all finite t, and thus we have (a) the integral in condition (33) is equal to zero along the optimal path, and (b) $\dot{\psi}_S = \delta\psi_S$ always, and therefore $\lim_{t\to\infty} e^{-\delta t}\psi_S(t) = \psi_S(0)$. Then, condition (33) reduces to

$$\theta\eta_S = \gamma\beta(\alpha-\beta)^{\frac{\beta}{1-\beta}}(S_0 - \mu_S^\star)^{\frac{\beta}{1-\beta}-1}K_0^{\frac{\alpha-\beta}{1-\beta}} + \psi_S(0) \; .$$

From this equation, we deduce that, if the point $(\mu_c^\star, \mu_S^\star)$ is not on the threshold possibility frontier (i.e., $\gamma = 0$), then the optimal shadow price of the resource, $\psi_S(0)$, is exactly equal to $\theta\eta_S$.

Comparative Statics Results

The optimal threshold point $(\mu_c^\star, \mu_S^\star)$ may lie on the threshold possibility frontier or below it. If the relative weight $\theta/(1-\theta)$ is sufficiently large, i.e., if the weight accorded to minimal rights is relatively high with respect to that of welfare in the RWI indicator, $(\mu_c^\star, \mu_S^\star)$ will lie on the threshold possibility frontier. To see this, consider the constrained welfare value function $V(S_0 - \mu_S, K_0, \mu_c)$ associated with the thresholds μ_S and μ_c

$$\begin{aligned} V(S_0 - \mu_S, K_0, \mu_c) \;\; &= \;\; \max_{c(\cdot),r(\cdot)} \int_0^\infty U(c(t))e^{-\delta t}dt \; , && (35) \\ s.t. \qquad & \dot{K}(t) = K(t)^\alpha r(t)^\beta - c(t) \; , \\ & \dot{S}(t) = -r(t) \; , \\ & K(0) = K_0 \; , \\ & S(0) = S_0 - \mu_S \\ & c(t) \geq \mu_c && (36) \end{aligned}$$

The constrained value function $V(.)$ is decreasing in μ_S and in μ_c. On the other hand, $R(.,.)$ is strictly increasing and linear in μ_S and μ_c. Since the RWI assigns a weight of θ to the right indicator, it follows that if θ is sufficiently closed to unity, the maximization of RWI with respect to μ_c and μ_S will give an optimal minimal rights vector on the threshold possibility frontier (Martinet, 2011). We conclude that if $\theta/(1-\theta)$ is large enough, the optimal thresholds point $(\mu_c^\star, \mu_S^\star)$ is on the threshold possibilities frontier. Conversely, if $\theta/(1-\theta)$ is not too high, the optimal thresholds point $(\mu_c^\star, \mu_S^\star)$ is in the interior of the threshold possibility frontier. In the limiting case where θ tends toward zero, one gets the usual unconstrained discounted utility solution and $(\mu_c^\star, \mu_S^\star)$ is at the origin $(0, 0)$.

Fig. 2 illustrates the trade-off between rights and welfare. Welfare (maximized constrained welfare value) is represented as a function of the thresholds levels. Note that for $\mathcal{R}(\mu_c, \mu_S) \equiv \eta_c\mu_c + \eta_S\mu_S$, the iso-value RWI curves correspond to planes in the space of welfare index and rights (with relative slopes depending on $(1-\theta)$, $\theta\eta_c$, and $\theta\eta_S$). Depending on the relative importance of Rights and Welfare in the RWI indicator, the

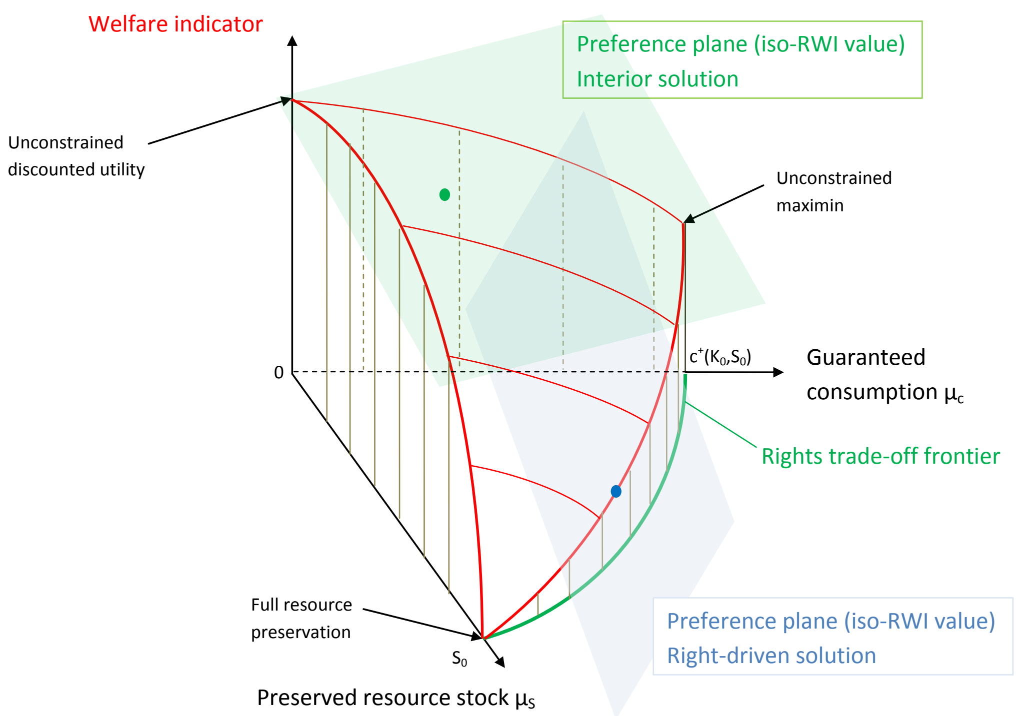

Figure 2: Trade-offs between welfare and rights in the DHS model.

optimal solution is either on the threshold possibility frontier or within the set of feasible thresholds, allowing us to distinguish two types of development paths: Right-based sustainable development paths, and Constrained utilitarian paths.

Right-based sustainable development path

Let us now turn to a full characterization of the case where the optimal thresholds point $(\mu_c^\star, \mu_S^\star)$ is on the threshold possibility frontier.[25] In this case, it is clear that the optimal consumption is constant.[26] The solution corresponds to the maximin consumption under a resource preservation constraint (Solow, 1974; Cairns and Long, 2006; Martinet and Doyen, 2007; Martinet, 2011). The consumption is constant, at a level

$$\begin{aligned} c^+(K_0, S_0, \mu_S^\star) &= (1-\beta)\big((S(t)-\mu_S^\star)(\alpha-\beta)\big)^{\frac{\beta}{1-\beta}} K(t)^{\frac{\alpha-\beta}{1-\beta}} \\ &= (1-\beta)\big((S_0-\mu_S^\star)(\alpha-\beta)\big)^{\frac{\beta}{1-\beta}} K_0^{\frac{\alpha-\beta}{1-\beta}} \\ &= \mu_c^\star\,. \end{aligned} \tag{37}$$

It yields a welfare $\mathcal{W} = \frac{1}{\delta}U(\mu_c^\star)$ and the constraints yield a right index $\mathcal{R}(\mu_c^\star, \mu_S^\star)$, so that the maximized RWI level is $J = \theta\mathcal{R}(\mu_c^\star, \mu_S^\star) + (1-\theta)\frac{1}{\delta}U(\mu_c^\star)$.

We know $\mu_S^\star$ as a function of $\mu_c^\star$ when these parameters are on the boundary of the feasibility set from the expression $\phi = 0$. We can define the function $\mu_S^\star = \bar{\mu}_S(\mu_c^\star)$ from eq. (37). From the expression of J, and the condition on the optimal choice of the parameters on the boundary, we can derive the solution. It satisfies the following condition:[27]

$$\frac{dJ}{d\mu_c} = 0 \quad \Leftrightarrow$$

$$\underbrace{(1-\theta)\frac{1}{\delta}U'(\mu_c^\star)}_{\substack{\text{Net present value gain} \\ \text{from increasing the} \\ \text{constant consumption level}}} \quad + \quad \underbrace{\theta\mathcal{R}'_{\mu_c}(\mu_c^\star, \bar{\mu}_S(\mu_c^\star))}_{\substack{\text{Gain in terms of} \\ \text{guaranteed consumption}}} \quad = \underbrace{-\theta\bar{\mu}'_S(\mu_c^\star)\mathcal{R}'_{\mu_S}(\mu_c^\star, \bar{\mu}_S(\mu_c^\star))}_{\text{Loss in terms of preserved stock}} \tag{38}$$

[25] It is shown in the appendix that this feasible solution may satisfy the optimal conditions of the original optimization problem.

[26] Suppose, on the contrary, that $c(t) > \mu_c^\star + \varepsilon$ over some time interval, where $\varepsilon > 0$. Then by re-arranging investment and consumption, it is feasible to ensure that $c(t) > \mu_c^\star + \kappa\varepsilon$ for some small number $\kappa > 0$, for all t. This means that the point $(\mu_c^\star + \kappa\varepsilon, \mu_S^\star)$ belongs to the feasible set $\mathcal{M}(x_0)$. But this contradicts the hypothesis that the solution $(\mu_c^\star, \mu_S^\star)$ is on the threshold possibility frontier.

[27] Providing an explicit expression of the optimal thresholds is possible from this condition given a specific utility function.

This equation can be re-arranged as follows

$$\frac{(1-\theta)\delta^{-1}U'(\mu_c^\star)+\theta\eta_c}{\theta\eta_S} = -\overline{\mu}'_S(\mu_c^\star) \tag{39}$$

It has a familiar interpretation: The left-hand side is the marginal rate of substitution of μ_c for μ_S along a RWI indifference curve, and the right-hand side is the marginal rate of transformation of the consumption right threshold into the resource-legacy right threshold along the threshold possibility frontier. From this equation, we can obtain comparative statics results: how do small changes in the preferences parameters δ, θ, η_c and η_S affect the optimal threshold μ_c, assuming that the changes are small enough so that the solution pair $(\mu_c^\star, \mu_S^\star)$ remains on the threshold possibility frontier. We obtain the following comparative statics results, the proofs of which are in the appendix.

Result CS 1 *A small increase in the discount rate δ leads to a lower guaranteed consumption threshold and a higher resource-legacy threshold.*

The intuition behind this result is that, as the consumption threshold contributes to the RWI in two ways, as a contribution to rights and to welfare, an increase in the discount rate diminishes the contribution of the guaranteed consumption to welfare. This favors the resource conservation legacy constraint in the trade-off between the two rights. The importance of consumption diminishes when society discounts the future consumption stream more heavily.

Result CS 2 *A small increase in θ (the weight of the Rights Indicator) leads to a lower consumption threshold and a higher resource-legacy threshold.*

The intuition behind this result is somewhat similar to that of Result 1. If the weight of rights increases at the expense of the weight on welfare, the relative contribution of the guaranteed consumption relative to that of the resource preservation diminishes: On the one hand the weight of the minimal consumption increases as the weight of rights increases, but the effect on its overall contribution to the RWI is ambiguous as the positive effect is mitigated by the decrease of the weight of welfare. This favors the conservation threshold.

Result CS 3 *An increase in the weight of the legacy constraint, η_S, will reduce the optimal consumption threshold $\mu_c^\star$.*

Result CS 4 *An increase in the weight of the minimum consumption, η_c, will increase the optimal consumption threshold $\mu_c^\star$.*

These last results are intuitive.

Interior solution: Constrained utilitarian path

We now turn to the case where the optimal choice is not on the *threshold possibility frontier*, $\phi(\mu_c, \mu_S, K_0, S_0) = 0$.[28] As marginal utility is infinite, consumption is positive at all times, and a part of the stock $(S_0 - \mu_S^\star)$ is depleted asymptotically.

Conditional on a given $\mu_c^\star$, the optimal conservation threshold $\mu_S^\star$ must solve

$$\max_{\mu_S} J(\mu_S) \equiv (1-\theta)V(S_0 - \mu_S, K_0, \mu_c) + \theta\eta_S\mu_S + \theta\eta_c\mu_c \tag{40}$$

The value function $V(S_0 - \mu_S, K_0, \mu_c)$ can, in principle, be computed, and for an interior solution the optimal conservation level $\mu_S^\star$ satisfies

$$\frac{\partial J}{\partial \mu_S} = 0 \quad \Leftrightarrow$$
$$\underbrace{-(1-\theta)V_S'(S_0 - \mu_S^\star, K_0, \mu_c^\star)}_{\substack{\text{Net present value loss}\\ \text{from increasing the}\\ \text{preservation constraint}}} \;+\; \underbrace{\theta\mathcal{R}'_{\mu_S}(\mu_c^\star, \mu_S^\star)}_{\substack{\text{Gain}\\ \text{in terms of}\\ \text{preserved stock}}} = 0 \tag{41}$$

which is equivalent to

$$\frac{\partial}{\partial \mu_S}(V(S_0 - \mu_S^\star, K_0, \mu_c^\star)) = -\frac{\theta}{(1-\theta)}\eta_S\ . \tag{42}$$

Similarly,

$$\frac{\partial}{\partial \mu_c}V(S_0 - \mu_S^\star, K_0, \mu_c^\star) = -\frac{\theta}{(1-\theta)}\eta_c$$

We cannot characterize further the expression of $\mu_S^\star$ and $\mu_c^\star$ without knowing more

[28]The optimal trajectory of this case is very difficult to determine, as discussed in the appendix.

details about the second order cross derivatives of the value function.[29] We can say, however, that there is a unique solution, as the value function is monotonic increasing and concave in the state variable, given that the utility function is strictly increasing and concave in the consumption.[30]

In general, we cannot exclude corner solutions. On the one hand, if $V'_S(S_0, K_0, \mu_c) \geq \frac{\theta}{(1-\theta)}\eta_S$, it is optimal to preserve none of the resource stock, i.e., $\mu_S^\star = 0$. On the other hand, if $V'_S(0, K_0, \mu_c) \leq \frac{\theta}{(1-\theta)}\eta_S$, it is optimal to preserve all the initial resource stock, i.e., $\mu_S^\star = S_0$.

4.4 Implications for discounting

This section provides some economic interpretations of the necessary conditions of our maximization problem, in particular on discounting. In the absence of minimal-rights constraints, the Keynes-Ramsey rule states that the rate of growth of consumption is equal to the product of the elasticity of intertemporal substitution $\sigma \equiv \frac{-U'_c}{cU''_c}$ and the difference between the interest rate facing consumers, $\rho(t)$, and the utility discount rate δ. In a competitive economy without externalities and policy intervention, the consumption rate of interest $\rho(t)$ is equal to the marginal productivity of capital. The Keynes-Ramsey rule reads $\frac{\dot{c}}{c} = \sigma(\rho(t) - \delta) = \sigma(F'_K - \delta)$. This rule can also be expressed as follows,

$$\frac{\dot{U}'_c}{U'_c} \equiv -\frac{1}{\sigma}\left(\frac{\dot{c}}{c}\right) = \delta - \rho(t) \ . \tag{43}$$

If the consumption discount rate (the interest rate) is larger than the impatience represented by the utility discount rate, consumption increases over time (i.e., the rate of change of marginal utility is negative and marginal utility decreases). Alternatively, expressing the consumption discount rate as a function of the utility discount rate, the growth rate and the elasticity of intertemporal substitution, i.e.,

$$\rho(t) = \delta + \frac{1}{\sigma}\frac{\dot{c}}{c} \ ,$$

[29]It is usually not possible to have a close-form solution to problem (35), except under some restrictive conditions (Benchekroun and Withagen, 2011).

[30]For a proof, see Long (1979).

one gets the usual expression of the discount rate to apply to investment project. It is equal to the sum of pure preference for the present plus the wealth effect.

When the minimum consumption constraint is binding such that $c = \mu_c^\star > 0$ and $w_c > 0$, the wealth effect is modified and one has a "modified Keynes-Ramsey Rule"(from Eqs. 28 and 30):

$$\begin{aligned}
-\frac{1}{\sigma}\left(\frac{\dot{c}}{c}\right) &= \frac{1}{\psi_K - w_c}\frac{d}{dt}\left(\psi_K - w_c\right) \\
&= \left(\frac{\psi_K}{\psi_K - w_c}\right)\left(\frac{\dot{\psi}_K}{\psi_K}\right) - \left(\frac{w_c}{\psi_K - w_c}\right)\left(\frac{\dot{w}_c}{w_c}\right) \\
&= \left(\frac{\psi_K}{\psi_K - w_c}\right)\left[\delta - F'_K\right] - \left(\frac{w_c}{\psi_K - w_c}\right)\left(\frac{\dot{w}_c}{w_c}\right)
\end{aligned}$$

If individuals are price-takers in a perfectly competitive capital market, intertemporal consumption smoothing (eq.43) implies that the left-hand side of the above expression should be equal to $\delta - \rho^c(t)$, where $\rho^c(t)$ is the rate of interest facing the consumers in terms of the consumption good c (the consumption discount rate). It follows that if the planner's allocation is to be achieved by a decentralized mechanism, the implied rate of interest $\rho^c(t)$ facing the consumers must satisfy the following condition:

$$\rho^c(t) = \delta - \left(\frac{\psi_K}{\psi_K - w_c}\right)\left[\delta - F'_K\right] + \left(\frac{w_c}{\psi_K - w_c}\right)\left(\frac{\dot{w}_c}{w_c}\right)$$

On the other hand, let $\rho^I(t)$ be the interest rate used to discount the future returns on investment. It is equal to the marginal product of capital. Then

$$\rho^I(t) = F'_K = \delta + \frac{1}{\sigma}\frac{\dot{c}}{c}\left(\frac{\psi_K - w_c}{\psi_K}\right) - \frac{\dot{w}_c}{\psi_K} \quad \neq \rho^c(t) \equiv \delta + \frac{1}{\sigma}\frac{\dot{c}}{c}$$

This wedge between producer's interest rate and consumer's interest rate implies tax or subsidy on savings, to ensure minimal consumption rights. For example, if it is socially optimal under the RWI criterion to have constant consumption for ever, then in the decentralised implementation of the social optimal program the interest facing private households should be $\rho^c(t) = \delta$, while the interest rate facing producers (the rental rate) should be F'_K.We know that F'_K is greater than δ earlier in the program, when the capital stock is low, and F'_K converges to zero toward the end of the program, when the capital stock tends to infinity, along the constant consumption path of the DHS model. Then, in the decentralised implemenation, there must be a wedge between the consumer's interest rate and the producer's rental rate. This wedge is negative early in the program, and positive toward the end of the program.

5 Concluding Remarks

The present paper introduces a criterion that accounts for rights and welfare in ranking social alternatives of development paths. The criterion is a weighted sum of an index of minimal rights guaranteed to all generations and a welfare index. Such a criterion could represent the development of a society that collectively defines minimal rights to be guaranteed over time (e.g., rights related to the environmental quality) while individuals make their own private decisions (e.g., on consumption and investment) without considering these rights explicitly. These latter are implemented by the social planner as collective constraints. Such collective constraints, when applied to environmental issues, could represent the objectives of a sustainable development.

We illustrate the general results in the canonical model of production-consumption with nonrenewable resource developed after Dasgupta and Heal (1974, 1979) and Solow (1974). Our example highlights the possibility that, at some point, minimal rights may be so important that the willingness to satisfy these minimal rights intertemporally drives the development path (right-based sustainable development). The development trajectory may strongly differ from the competitive unconstrained path. In particular, if sustaining a positive consumption level is desired, one has to modify the Keynes-Ramsey rule to smooth consumption over time and adjust investment. This can be done by influencing the discount rate the consumers face, and make it different from the producers' discount rate as defined by the marginal productivity of capital. This may imply some wedge between consumers and producers interest rates, possibly implemented by tax or subsidy on savings. These results have important implications in the definition of the discount rate to be applied on investment projects, as they are discussed in the climatic change mitigation debate.

Acknowledgments: We thank the participants of the CIREQ Natural Resource Economics workshop (Montreal, January 2012), IFO lunch time seminar (Munich, January, 2012), SURED 2012 (Ascona), EAERE 2012 (Prague), and Ecole Polytechnique seminar (Palaiseau, December 2012). We thank Bob Cairns, Hassan Benchekroun, Hans-Werner Sinn, and Karen Pittel, as well as two referees and the editor for comments. All errors are our owns.

A Appendix

A.1 Proof of Proposition 1

The optimization problem (6) is equivalent to

$$V(x_0;\mu_1,\ldots,\mu_I) = \max_{(x(\cdot),c(\cdot))\in\mathcal{F}(x_0;\boldsymbol{\mu_1,\ldots,\mu_I})} \mathcal{W}(x(\cdot),c(\cdot)) \; .$$

For any set of thresholds $(\mu_1,\ldots,\mu_I)$ and $(\mu_1',\ldots,\mu_I')$ such that $\mu_i' \geq \mu_i$ for $i=1,\ldots,I$, we have $\mathcal{F}(x_0;\mu_1',\ldots,\mu_I') \subseteq \mathcal{F}(x_0;\mu_1,\ldots,\mu_I)$. This implies that $V(x_0;\mu_1',\ldots,\mu_I') \leq V(x_0;\mu_1,\ldots,\mu_I)$.

A.2 Proof of Proposition 2

Consider the optimality conditions (16) and (15), that we recall here:

$$\int_0^\infty e^{-\delta t}\frac{\partial L^c}{\partial \mu_i}dt + \gamma\frac{\partial \phi}{\partial \mu_i} = \overline{\pi}_i \; , i=1,2,...,p$$

$$\int_0^\infty e^{-\delta t}\frac{\partial L^c}{\partial \mu_i}dt + \gamma\frac{\partial \phi}{\partial \mu_i} = 0 \; , i=p+1,...,I$$

with $\gamma \geq 0$, $\phi(\mu_1,\ldots,\mu_I;x_0) \geq 0$, and $\gamma\phi(.) = 0$, and $\overline{\pi}_i = \lim_{t\to\infty} e^{-\delta t}\psi_i(t)$, for $i = 1,2,..,p$.

From Eqs. (10) and (11), for all i we have $\frac{\partial L^c}{\partial \mu_i} = \delta\theta\mathcal{R}'_{\mu_i}(\mu^\star) - w_i$. As $\mathcal{R}$ does not depend on time, neither do its derivatives. The optimality conditions read

$$\theta \mathcal{R}'_{\mu_i} - \int_0^\infty e^{-\delta t} w_i(t)dt + \gamma \frac{\partial \phi}{\partial \mu_i} = \overline{\pi}_i \ , i = 1, ..., p \tag{44}$$

$$\theta \mathcal{R}'_{\mu_i} - \int_0^\infty e^{-\delta t} w_i(t)dt + \gamma \frac{\partial \phi}{\partial \mu_i} = 0 \ , i = p+1, ..., I \tag{45}$$

If a legacy constraint is binding in the limiting case, $w_i(t) = 0$ at all times and Eq. (44) becomes

$$\theta \mathcal{R}'_{\mu_i}(\mu^\star) + \gamma \frac{\partial \phi}{\partial \mu_i} = \overline{\pi}_i \ .$$

i) If $\mu^\star$ is in the interior of the threshold possibility set, i.e., $\phi(\mu_1^\star, \dots, \mu_I^\star; x_0) > 0$, one has $\gamma = 0$ and the previous conditions reduce to

$$\theta R'_{\mu_i}(\mu^\star) = \overline{\pi}_i \quad i = 1, 2, ..., p$$

$$\theta \mathcal{R}'_{\mu_i} = \int_0^\infty e^{-\delta t} w_i(t)dt \ , \quad i = p+1, ..., I$$

For any i such that $\mathcal{R}'_{\mu_i} > 0$, this implies that either $w_i(t) > 0$ on some time interval, or $\lim_{t\to\infty} e^{-\delta t}\psi_i(t) > 0$. The associated constraint $\mathcal{I}_i(x, c) \geq \mu_i$ is binding, either on an interval for $i = p+1, \dots, I$, or in the limiting sense for $i = 1, \dots, p$.

ii) Assume that the right-based constraint i, for $i = p+1, \dots, I$, is never binding. This implies from condition (13) that $w_i(t) = 0$ for all t. Condition (45) reduces to

$$\theta \mathcal{R}'_{\mu_i} + \gamma \frac{\partial \phi}{\partial \mu_i} = 0 \ .$$

If $\mathcal{R}'_{\mu_i}(\mu^\star) > 0$, this implies that $\gamma > 0$ (given that $\frac{\partial \phi}{\partial \mu_i} < 0$), and thus that the optimal rights are on the boundary of the feasibility set.

iii) If there is no legacy constraints and for some i, one has $\mathcal{R}'_{\mu_i}(\mu^\star) = 0$, the associated condition (45) reduces to

$$-\int_0^\infty e^{-\delta t} w_i(t)dt + \gamma \frac{\partial \phi}{\partial \mu_i} = 0, i = p+1, ..., I$$

As the two terms of the sum are non-positive, the equality holds only if both are nil. This means that $w_i(t) = 0$ at all times, i.e., the associated constraint is never binding, and that $\gamma = 0$, i.e., the optimal minimal rights are not on the boundary of the set of feasible rights.

A.3 Proofs of the Comparative Static results

Proof of CS1: Since the optimality condition (39) holds as an identity, we can differentiate both sides with respect to δ, treating $\mu_c^\star$ as an implicit function of δ. We obtain

$$\frac{\partial \mu_c^\star}{\partial \delta} = \frac{1}{G} \frac{(1-\theta)\delta^{-2} U'}{\theta \eta_S} < 0$$

where

$$G \equiv \frac{(1-\theta)\delta^{-1} U''}{\theta \eta_S} + \overline{\mu}_S''(\mu_c^\star) < 0$$

Proof of CS2: Re-write the optimality condition (39) as

$$\frac{(1-\theta)\delta^{-1} U'(\mu_c^\star)}{\theta \eta_S} + \frac{\eta_c}{\eta_S} = -\overline{\mu}_S'(\mu_c^\star)$$

Then

$$\frac{\partial \mu_c^\star}{\partial \theta} = -\frac{1}{G} \delta^{-1} U'(\mu_c^\star) \frac{d}{d\theta} \left(\frac{1-\theta}{\theta} \right) < 0$$

The proofs of **CS3** and **CS4** are similar, and are therefore omitted.

A.4 Characterization of the optimal development path in the DHS model

A.4.1 Optimality of the right-based development path

Let us show that the constant consumption path associated with a solution located on the threshold possibility frontier is consistent with the optimality conditions. Consider a given $\mu_c^\star > 0$. As we have argued, when $\mu_S^\star = \bar{\mu}_S(\mu_c^\star)$, it is impossible to have a phase $[0, T]$ where $c(t) > \mu_c^\star$ for all $t \in [0, T]$.

Then, given $\mu_c^\star > 0$ and $\mu_S^\star = \bar{\mu}_S(\mu_c^\star)$, consider the following problem:

$$\max(1-\theta)\int_0^\infty e^{-\delta t}U(c)dt \tag{46}$$

s.t.

$$\dot{K} = K(t)^\alpha r(t)^\beta - c(t)\,,\ K(0) = K_0,\ K(t) \geq 0$$

$$\dot{S} = -r(t),\ \ S(0) = S_0,\ \lim_{t\to\infty} S(t) \geq \bar{\mu}_S(\mu_c^\star)$$

$$c(t) - \mu_c^\star \geq 0$$

By construction, we know this problem has a feasible solution (as described above) where $c(t) = \mu_c^\star$ for all t, and the value of this feasible program is $\frac{1}{\delta}U(\mu_c^\star)$. It is the Solow-Hartwick constant consumption program.We check here that this feasible solution satisfies the necessary conditions for problem (46). We use the superscript M to distinguish this problem from problem (23). The necessary conditions for problem (46) are derived below. The current value Hamiltonian is

$$H^M = (1-\theta)U(c(t)) + \psi_k^M\left[K(t)^\alpha r(t)^\beta - c(t)\right] - \psi_S^M r(t)$$

The Lagragian is

$$L^M = H^M + w_c^M(c - \mu_c^\star)\ .$$

The necessary conditions of this problem are

$$\frac{\partial L^M}{\partial c} = 0 \Leftrightarrow (1-\theta)U_c' - \psi_K^M + w_c^M = 0$$

$$\frac{\partial L^M}{\partial r} = 0 \Leftrightarrow \psi_S^M = \psi_K^M F_r'$$

$$\dot{\psi}_K^M = \delta\psi_K^M - \frac{\partial L^M}{\partial K} \Leftrightarrow \frac{\dot{\psi}_K^M}{\psi_K^M} = \delta - F_K'$$

$$\dot{\psi}_S = \delta\psi_S - \frac{\partial L^c}{\partial S} \Leftrightarrow \dot{\psi}_S = \delta\psi_S$$

and also (24), $\dot{S} = -r(t),\ \ S(0) = S_0$, $\lim_{t\to\infty} S(t) \geq \bar{\mu}_S(\mu_c^\star), c(t) - \mu_c^\star \geq 0$, and

$$\omega_c^M(t) \geq 0, w_c^M(t)[c(t) - \mu_c^\star] = 0$$

Setting $c(t) = \mu_c^\star$ for all t, we have

$$(1-\theta)U_c'(\mu_c^\star) - \psi_K^M(t) + w_c^M(t) = 0\ ,$$

where $w_c^M(t) \geq 0$ is satisfied if weight given to welfare, $(1-\theta)$, is sufficiently small relative to the weight given to minimum consumption rights, $\theta\eta_c$.

A.4.2 Characterization of the constrained utilitarian paths

Infinite marginal utility case In the case where $U'(0) = \infty$, consumption is never nil and there is some $\mu_S > 0$ that is set aside from the beginning. To determine μ_S and μ_c we can proceed using a two-step procedure.

Step 1: Consider the discounted utility maximization a la Dasgupta and Heal, coupled with a minimum consumption constraint, $c(t) \geq \mu_c$. This gives rise to an associated value function for an initial stock of resource $S_0 - \mu_S$:

$$V(S_0 - \mu_S, K_0, \mu_c) \equiv \max_{c,r} \int_0^\infty e^{-\delta t} U(c(t))dt\ , \tag{47}$$

s.t.

$$\dot{K} = K(t)^\alpha r(t)^\beta - c(t)\ ,\ K(0) = K_0,\ K(t) \geq 0\ ,$$

$$\dot{S} = -r(t),\ \ S(0) = S_0,\ \ \lim_{t\to\infty} S(t) = \mu_S\ .$$

$$c(t) \geq \mu_c.$$

This function can in principle be calculated (though not in closed form).[31]

Step 2: Choose μ_c and μ_S to maximize

$$(1-\theta)V(S_0 - \mu_S, K_0, \mu_c) + \theta(\eta_c\mu_c + \eta_S\mu_s).$$

[31] For some special cases of problem (47), it is possible to obtain a closed form solution for the value function. In this case, using the expression of the value function, it is possible to solve explicitely problem (40).

Since the function $V(S_0 - \mu_S, K_0, \mu_c)$ is not analytically tractable, one will have to rely on numerical solutions.

References

Alvarez-Cuadrado F. and Long N.V. (2009) A mixed Bentham–Rawls criterion for intergenerational equity: Theory and implications. *Journal of Environmental Economics and Management* **58**:154-168.

Asheim G.B. and Zuber S. (2013) A complete and strongly anonymous leximin relation on infinite streams. *Social Choice and Welfare* **41**:819-834.

Beltratti A., Chichilnisky G. and Heal G. (1995) The Green Golden Rule. *Economics Letters* **49(2)**:175-179.

Benchekroun H. and Withagen C. (2011) The optimal depletion of exhaustible resources: A complete characterization. *Resource and Energy Economics*, **33(3)**:612-636.

Burmeister E. and Hammond P.J. (1977) Maximin paths of heterogeneous capital accumulation and the instability of paradoxical steady states. *Econometrica* **45**: 853-870.

Cairns R. and Long N.V. (2006) Maximin, a direct approach to sustainability. *Environmental and Development Economics* **11**:275-300.

Chakravorty U., Moreaux M. and Tidball M. (2008) Ordering the extraction of polluting nonrenewable resources. *American Economic Review* **98(3)**:1128-1144.

Chichilnisky G. (1996) An Axiomatic Approach to Sustainable Development. *Social Choice and Welfare* **13**:231-257.

Dasgupta P. and Heal G. (1974) The Optimal Depletion of Exhaustible Resources. *Review of Economic Studies* **41**:1-28.

Dasgupta P. and Heal G. (1979) *Economic Theory and Exhaustible Resources.* Cambridge University Press.

D'Autume A., Hartwick J. and Schubert K. (2010) The zero discounting and maximin optimal paths in a simple model of global warming. *Mathematical Social Sciences*, **59**:193–207.

Deb R. (1994) Waiver, Effectivity, and Rights as Game Forms. *Economica* **61**:167-178.

Gaertner W., Pattanaik P.K. and Suzumura K. (1992) Individual Rights Revisited. *Economica* **59**:161-177.

Gärdenfors P. (1981) Rights, Games and Social Choice. *Noûs* **15**:341-356.

Hammond P.J. (1995) Social Choice of Individual and Group Rights. In Barnett W., Moulin H., Salles M. and Schofield N. (eds.), *Social Choice, Welfare, and Ethics*, Cambridge: Cambridge University Press, pp.55-77.

Hammond P.J. (1996) Game Forms versus Social Choice Rules as Models of Rights. In Arrow K.J., Sen A.K. and Suzumura K. (eds.), *Social Choice Re-examined*, Vol.2, London: Macmillan, pp.82-95.

Heal G. (1998) *Valuing the Future. Economic Theory and Sustainability.* Columbia University Press, New York.

Harstad B. (2012) Buy Coal! A Case for Supply-Side Environmental Policy. *Journal of Political Economy*, **120(1)**:77-115.

Long N.V. (1979) Two Theorems on Generalized Diminishing Returns and their Applications to Economic Analysis. *Economic Record*, **55**:58-63.

Long N.V. (2007) Toward a Theory of a Just Savings Principle. In J. Roemer and K. Suzumura (Eds.), *Intergenerational Equity and Sustainability*, Palgrave, London.

Long N.V. (2011) A Dynamic Game of Environmental Exploitation between Two Countries with Sequential Maximin Objectives. *International Journal of Development and Conflict* **1(3)**:1-15.

Leonard D. and Long N.V. (1991) *Optimal Control Theory and Static Optimization in Economics*, Cambridge University Press, Cambridge.

Martinet V. (2011) A Characterization of Sustainability with Indicators. *Journal of Environmental Economics and Management* **61**:183-197.

Martinet V. (2012) *Economic Theory and Sustainable Development. What Can we preserve for future generations?*, Routledge, London and New York.

Martinet V. and Doyen L. (2007) Sustainability of an economy with an exhaustible resource: A viable control approach. *Resource and Energy Economics* **29**:17-39.

Neumayer E. (2013) *Weak Versus Strong Sustainability: Exploring the Limits of Two Opposing Paradigms*. Edward Elgar Publishing.

Nozick R. (1974) *Anarchy, State and Utopia*, New York: Basic Books.

Peleg B. (1998) Effectivity Functions, Game Forms, Games, and Rights. *Social Choice and Welfare* **15**:67-80.

Rawls J. (1971) *A theory of Justice*. Oxford, England: Clarendon.

Reiss H. (1970) *Kant's Political Writings*. Cambridge University Press, Cambridge.

Sen A.K. (1970a) *Collective Choice and Social Welfare*, San Francisco: Holden-Day. Republished, Amsterdam: North-Holland, 1979.

Sen A.K. (1970b) The Impossibility of a Paretian Liberal. *Journal of Political Economy* **78**:152-157.

Sen A.K. (1976) Liberty, Unanimity and Rights. *Economica* **43**:217-241.

Solow R.M. (1974) Intergenerational Equity and Exhaustible Resources. *Review of Economic Studies* **41**:29-45.

Stollery K. (1998) Constant utility paths and irreversible global warming. *Canadian Journal of Economics*, **31(3)**:730–742.

Sugden R. (1985) Liberty, Preference and Choice. *Economics and Philosophy* **1**:213-229.

Suzumura K. (2005) An interview with Paul Samuelson: welfare economics, "old" and "new" and social choice theory. *Social Choice and Welfare* **25(2)**:327-356.

Suzumura K. and Yoshihara N. (2008) On Initial Conferment of Individual Rights. *Working Paper*, Institute of Economic Research, Hitotsubashi University.

Takayama A. (1985) *Mathematical Economics* (2nd edition). Cambridge University Press.

World Commission on Environment and Development – WCED (1987) *Our common Future*, Oxford University Press.