Short-term Deviations from Simple Majority Voting

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I discuss instances where a committee wants to deviate from the simple majority rule by adopting an alternative voting scheme for two consecutive binary ballots. The alternative voting rule, called Minority Voting as an Exception (MVE), works as follows: In the first ballot a β-majority rule is used, where $\beta < \frac{1}{2}$ is equal to the minority fraction that favors some project, say project 1. This allows the minority to induce the adoption of project 1. After the first ballot all voting winners, i.e. the minority of project winners, lose their voting rights for the upcoming second ballot, where the simple majority rule is used. Hence, MVE may benefit both project losers and winners and may thus be unanimously accepted. The analysis of this short-term deviation is presented with a potential application in the sphere of communal politics.

Keywords: voting, minority, communal politics

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1 Introduction

Voting rules applied in committees, city councils, or elections usually require a majority to change the status quo. The simple majority rule is the most common voting scheme. The question addressed in this paper is whether a committee may want to deviate from the simple majority rule and use an alternative voting scheme. Suppose that such a deviation from simple majority voting is possible as long as it is accepted unanimously. Then an alternative voting scheme for one single yes-no ballot would never be chosen, as some committee members would benefit while others would be disadvantaged. A deviation for at least two ballots, however, may be possible, as losses in one ballot might be compensated by benefits in another ballot. Alternative voting rules can be of two types. Either different vote thresholds are used so that the voting rule favors one alternative over the other, or voting rights in future ballots are changed.

In this paper I propose a new voting scheme that covers two consecutive ballots and serves as a potential deviation from the simple majority rule. It combines both types of alternative voting rules and works as follows: In the first (open) ballot a β-majority rule is used, where $\beta < \frac{1}{2}$ is equal to the minority fraction that favors some project, say project 1. This allows the minority to induce the adoption of project 1. After the first ballot all voting winners, i.e. the minority of project winners, lose their voting rights for the upcoming second ballot, where the simple majority rule is used. I call this new voting scheme MVE, which stands for 'Minority Voting as an Exception'.[1] The use of MVE, i.e. the deviation from simple majority voting, has to be accepted unanimously.

I will show that individuals who favor a project will prefer MVE over simple majority voting if their own project benefits are large enough and if it would fail under simple majority voting. Vice versa, individuals who suffer from the project also have incentives to deviate by using MVE if their losses are compensated by higher expected utility in the second ballot. As the deviation has to be accepted unanimously, all individuals need to expect some utility gain for the deviation to be approved, i.e. expected aggregate utility will increase if MVE is applied. In addition, it is possible to show that under complete uncertainty regarding the second project, a project leading to an application of MVE is socially desirable in connection with committee sizes typical of community councils in Germany. The community councils example serves to illustrate how MVE can be applied.

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[1]MVE gives the minority a new opportunity to realize its preferences. The idea of MVE was inspired by Minority Voting as introduced by Fahrenberger and Gersbach (2008). They develop a voting scheme that serves to protect a minority against repeated exploitation.

The subject of this paper has two main links to the literature. First, the new voting scheme serves minority protection as it gives small groups an additional possibility to realize their own preferences. Minority protection is a widely discussed topic in the literature. New voting schemes have been proposed, many of them increasing the influence of the minority by allocating more than one vote per person and ballot. There are several variations of this basic idea. One famous example is Cumulative Voting (see e.g. Sawyer and MacRae (1962), Brams (1975), Cox (1990), Guinier (1994) or Gerber, Morton, and Rietz (1998)), where all individuals obtain as many votes as there are candidates or issues to vote for. Only one ballot is carried out. Individuals can cast more than one vote for one of the alternatives, which is a way of expressing preferences intensity and also serves to protect minorities. This idea is extended to a row of n decisions by Hortala-Vallve (2007), who introduced Qualitative Voting. Storable Votes, as proposed by Casella (2005), is a voting mechanism where individuals can store votes if a proposal has only a minor effect on their utility. They can then use this extra vote on a proposal that they find more important. Fahrenberger and Gersbach (2008) introduce Minority Voting where the influence of a minority is increased by reducing the size of the group with voting rights.

Second, MVE is related to log-rolling, as can be seen in the following alternative formulation for the new voting scheme: Individuals with a strong interest in a project propose that the project be accepted (i.e. other individuals vote in favor of the project winners). Accordingly, these project winners do not use their voting rights in the upcoming ballot. Under log-rolling, individuals with different payoffs and hence different preferences agree upon strategic voting ensuring that decisions are taken in favor of the individual who has the stronger interest in it. Whether log-rolling is welfare-improving or not is discussed e.g. by Brams and Riker (1973), Tullock (1974), Bernholz (1978), and Coleman (1983). One crucial problem here is that the individual who supports the other agent by voting strategically in the first ballot has to rely on the other individual to abide by the agreement during the second ballot. Mueller (1967) suggests that literally changing ballots paper can solve the reliability problem. MVE differs from log-rolling in one important way. Individuals with high payoffs from the first project have no voting rights in the second ballot. This leads to the expected utility gain for individuals who keep their voting rights, and cannot be changed. Hence reliability is given.

The paper is structured as follows: In section 2, the current legal background in communal politics is introduced, as this example will be used in the discussion of MVE later on. Section 3 covers the model, the voting scheme, and a discussion of optimal voting behavior of individuals under MVE. The conditions necessary for an application of MVE, i.e. deviation from simple

majority voting, are examined in section 4. A discussion of these results with respect to communal politics follows in section 5. In section 6 I discuss whether and how strategic voting can occur if some limitations of the basic model are weakened. Section 7 concludes.

2 Communal Politics in Germany

In the literature, communal politics is widely discussed. However, the focus is not on the voting scheme used in decision-making but on the distribution of power within a community and its relation to certain rules that vary between different states in Germany ('Bundesländer'), e.g. whether a mayor is directly elected by the inhabitants of the community or by the delegates in the council. The structure of administration and council is also analyzed. There are several books that give a good overview, e.g. Naßmacher and Naßmacher (2007) or Kost and Wehling (2003). So far, the application of the voting rules (either simple majority voting or an α-majority rule with $\alpha = \frac{2}{3}$) has not been questioned.

Community responsibilities, voting and election rules, composition of the community council and the administration, etc. are set down in the municipal codes.[2] Each federal state has its own municipal code for the communities and cities within the range of its jurisdiction. Despite differences in details, there are common characteristics such as:

(i) In regular decisions simple majority voting is applied and the ballot is open.

(ii) A two-thirds majority is required if the decision has to do with a change in the geographical area of the community, the community's name, a change of the agenda, non-open ballots, deselection of delegates or of the governing mayor.

(iii) A community council consists of anything between 8 and 60 delegates, depending on the number of inhabitants.

(iv) Committee members know the agenda for each meeting.

(v) A committee is quorate if at least half of the committee members are present.

In the next section I present the model containing the new voting scheme, utility functions, and the timing of events. The quality of the model with respect to an application in communal politics will be discussed in section 3.5.

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[2]The municipal codes ('Gemeindeordnungen') can be viewed e.g. on http://www.jura.uni-osnabrueck.de/institut/jkr/kronline.htm, as at December 11, 2008.

3 The Model

3.1 The Alternative Voting Scheme

Minority Voting as an Exception is defined as follows:

MVE:

The voting scheme MVE applies in two successive ballots. In the first ballot a β-fraction with $\beta < \frac{1}{2}$ has to support a proposal to change the status quo for it to be accepted. All voting winners from this ballot lose their voting rights for the upcoming second ballot. In the second ballot the simple majority rule applies. All ballots are open.

In addition, a constitution is needed that allows deviation from simple majority voting upon request. In particular, I assume that the committee acts under the following rules, defining **constitution $\mathcal{C}$**:

- The simple majority rule is the default voting scheme. Ballots are open.

- Before a ballot upon some project x takes place, committee members may propose an application of MVE for the two upcoming ballots, where the choice of β has to be included in the proposal.[3]

- The committee has to be unanimous on the application of MVE.

- Within two MVE ballots, no further application of MVE can be proposed.

Constitution $\mathcal{C}$ allows for a deviation from simple majority voting that always yields an increase in expected aggregate utility. However, it may also lead to time delays if MVE is proposed but not accepted.

3.2 Utility

Assume a committee of N individuals (with N odd) deciding upon several projects one after the other.

- Utility of a project x is given by $u_{ix} = a_x \cdot z_{ix}$ with $z_{ix} \in [-1, 1]$. a_x denotes the decision of the committee: $a_x = 1$ if project x is accepted, $a_x = 0$ if it is rejected.

- When a ballot on a project x takes place, the payoffs z_{ix} are common knowledge.

- Overall utility is given by the sum of all realized project payoffs. Discounting is ne-

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[3]For the analysis I restrict the choice of β such that no different proposals can be submitted. I comment on the case of a free choice of β in section 6.

glected as the decisions take place in one meeting. Therefore the realization of all projects that have been accepted may also take place at the same time.

The deviation voting scheme MVE extends across two ballots. Accordingly, I restrict the analysis to two projects, which means that only one decision about an application of MVE will be analyzed. The corresponding projects are denoted by project 1 and project 2. The first project is crucial for the application of MVE. In the following I introduce some assumptions on these projects.

3.2.1 Assumption on Project 1

The first-project payoffs are common knowledge and have already been distributed when the decision about MVE takes place. This project divides the committee into two subgroups, the group of project winners W and the group of project losers L. For simplicity I assume

A1: *First-project payoffs are either $z_{i1} = z_W \in [0,1]$ for all individuals $i \in W$ and $z_{i1} = z_L \in [-1,0)$ for all $i \in L$, where W denotes the group of project winners and L denotes the group of project losers.*

An immediate consequence of A1 is that all L-members and all W-members have identical strategic considerations when it comes to a decision on MVE.[4]

3.2.2 Assumptions on Project 2

Here I use a universal approach allowing the later choice of different parameter constellations reflecting different levels of information within the committee. Assumptions:

- W-members have probability ϕ_W of benefiting from the second project as well, i.e. $\phi_W = \mathbb{P}[z_{i2} \geq 0 | i \in W]$. Probability ϕ_W is common knowledge.

- L-members have probability ϕ_L of benefiting from the second project, i.e. $\phi_L = \mathbb{P}[z_{i2} \geq 0 | i \in L]$. Probability ϕ_L is common knowledge.

- All individuals know the density functions $u_-(z_{i2})$ on $[-1,0)$ and $u_+(z_{i2})$ on $[0,1]$. Note that mass 1 is put on each subinterval of $[-1,1]$ because of the conditional probabilities ϕ_W and ϕ_L.

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[4]A more general approach with different z_{i1} within each group W and L implies that the individuals with the lowest utility in each group determine the thresholds that yield an application of MVE. See section 4.

An explicit density function of the second-project payoffs is not needed for the analysis. Instead, the expected values of z_{i2} restricted to being (non-)negative are sufficient. I denote these values by $z_E^+ = \int_0^1 u_+(z)dz$ and $z_E^- = \int_{-1}^0 u_-(z)dz$ respectively.

3.3 Timing

The time structure is given as follows:

1. Project 1 is proposed. Individuals learn about payoffs z_{i1} from project 1, i.e. the payoffs are common knowledge, determining the W-group and the L-group.

2. All members can propose the application of MVE for the upcoming two ballots, i.e. the first ballot under MVE is the decision on project 1, while the second ballot includes the decision on some unknown project 2.

3. The committee decides unanimously on the application of MVE.

4. If MVE is rejected, the simple majority rule is applied in both ballots on project 1 and project 2.[5] If MVE is accepted, the β-rule is applied in the first ballot. Voting rights are distributed according to the definition of MVE.

Note that time is neglected in the sense that there is no other ballot that has to be canceled for the additional decision about an application of MVE. Here we can take city councils as an example. They generally meet to discuss certain topics that are constituted before. This may take more or less time.

W-members have no incentive to deviate from simple majority voting if project 1 is realized in a regular ballot under the simple majority rule. Therefore the only interesting case occurs if the W-group forms a minority. This imposes the following assumption:

A2: *W-members form a minority, i.e. $|W| \in \{1, \ldots, \frac{N-1}{2}\}$.*[6]

For completeness a tie-breaking rule is needed in the case of indifference between MVE and the simple majority rule:

A3: *Committee members will propose MVE if and only if the expected utility under MVE is strictly higher than under the simple majority rule.*

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[5]As the analysis is restricted to two projects, no further application of MVE after the first ballot is reasonable. Without the restriction to two projects, the committee would return to the default voting scheme after the two MVE ballots. Furthermore, no further application of MVE can be proposed within an MVE cycle.

[6]$|S|$ denotes the number of pairwise different elements in a set S.

I assume strict improvement under MVE, as proposing MVE may make for a time delay if rejected. This may be costly and hence should be avoided.

Note that the proposed β-fraction has to be smaller than or equal to the minority fraction in the society, i.e. $\beta \in (0, \frac{|W|}{N}]$. To avoid the possibility of strategic choices of β, I simplify the analysis by assuming

A4: *The only β that can be proposed for an application of MVE is given by $\beta = \frac{|W|}{N}$.*

This requirement is plausible, as all committee members know the minority fraction. Free choice of β will be discussed in section 6.

3.4 Voting Equilibria

I apply the concept of Bayesian Nash equilibria. Weakly dominated strategies are excluded. Under simple majority voting all committee members vote sincerely in both ballots. Regarding optimal voting behavior under MVE the following lemma holds:

Lemma 1 *Suppose that MVE with $\beta = \frac{|W|}{N}$ has been proposed and unanimously accepted. Then,*

- *in the first ballot under MVE, all minority (W) members will vote for the project and all majority (L) members will vote against the project;*

- *in the ballot on project 2 all individuals with voting rights will vote sincerely.*

The first point holds as the W-group has to achieve the required β-threshold. No W-member has an incentive to deviate from this strategy, as project 1 will otherwise not be realized, while the L-group together with the deviating W-members turns into the group of voting winners, thus losing their voting rights. Vice versa, voting against project 1 ensures all L-members voting rights for the upcoming second ballot.

In the vote on project 2 voting sincerely weakly dominates voting strategically.

3.5 Relation to the Application Example

After introducing the model, I now briefly compare the theoretical assumptions with the predefined setting of communal politics as presented in section 2.

- The simple majority rule as a default voting scheme is reasonable.

- The application of MVE implies a change of the agenda that again requires a two-thirds majority, as stated in the municipal codes. This requirement is satisfied by constitution $\mathcal{C}$, as MVE has to be accepted unanimously.

- Standard assumptions include complete uncertainty about upcoming projects. However, this does not adequately reflect the procedure in community councils, as council members have knowledge about the agenda and may even work on proposals themselves. The universal setting presented in section 3.2.2 covers this issue. Under this approach, the choice of parameters can be used to illustrate uncertainty as well as ex-ante knowledge, strong party affiliation, or linked projects.

- All W-members lose their voting rights, which implies that more than half of the committee keep the voting rights for the second ballot. This property goes hand in hand with the municipal codes by requirement (v) as stated in section 2: A community council is quorate only if at least half of the committee members are present at the meeting.

The next step is to compare MVE with simple majority voting (henceforth SM) to derive conditions on first-project payoffs that yield an application of MVE.

4 Conditions for an Application of MVE

This section derives the conditions necessary for an application of MVE and hence for incentives to deviate from simple majority voting. I determine the individual expected utility under MVE for W- and L-members and compare it with the corresponding terms without the application of MVE, i.e. under SM. The calculations require the individual probability of winning in the second ballot, as described in the following.

4.1 Probability of Winning in the Second Ballot

The winning probabilities of W- and L-members in the second ballot using the voting rule $x \in \{\text{MVE}, \text{SM}\}$ are denoted by

- $\mathbb{P}^x_{W+}$ = winning probability of a W-member i given $z_{i2} \geq 0$.

- $\mathbb{P}^x_{W-}$ = winning probability of a W-member i given $z_{i2} < 0$.

- $\mathbb{P}^x_{L+}$ = winning probability of an L-member i given $z_{i2} \geq 0$.

- $\mathbb{P}^x_{L-}$ = winning probability of an L-member i given $z_{i2} < 0$.

As the assumptions on the second project are very general the probabilities are complex expressions. The calculation of all winning probabilities is given in the Appendix. The basic concept is explained within reference to the example of $\mathbb{P}_{L+}^{MVE}$ in the following.

Under MVE only L-members are allowed to vote in the second ballot. Thus the probability of winning for an L-member i under MVE is determined by the question whether at least half of the other L-members have payoffs tending in the same direction as the individual under consideration. If $N - |W|$, i.e. the number of individuals with a voting right, is odd, then i will win if at least $\frac{N-|W|-1}{2}$ other individuals vote the same way. If $N - |W|$ is an even number, then i will win if either more than $\frac{N-|W|}{2}$ individuals favor the same alternative or if individual i wins the tie-break (with probability $\frac{1}{2}$) in the case of $\frac{N-|W|}{2} - 1$ other individuals voting in the same way. Let TB be the case where a tie-break can occur. It is given by[7]

$$TB = 1 - \left\lceil \frac{N-|W|}{2} - \left\lfloor \frac{N-|W|}{2} \right\rfloor \right\rceil = \begin{cases} 1, & \text{if } N - |W| \text{ is even (i.e. } \frac{N-|W|}{2} \in \mathbb{N}) \\ 0, & \text{if } N - |W| \text{ is odd.} \end{cases}$$

Suppose that i observes $z_{i2} \geq 0$, i.e. she is a project winner. Then her probability of winning in the second ballot under MVE is given by

$$\begin{aligned} \mathbb{P}_{L+}^{MVE} &= \sum_{k=\lfloor \frac{N-|W|-1}{2} \rfloor}^{N-|W|-1} \phi_L^k (1-\phi_L)^{N-|W|-1-k} \binom{N-|W|-1}{k} \\ &\quad - TB \cdot \frac{1}{2} \phi_L^{\frac{N-|W|}{2}-1} (1-\phi_L)^{\frac{N-|W|}{2}} \binom{N-|W|-1}{\frac{N-|W|}{2}-1}. \end{aligned}$$

In the formula, winning a tie-break is included in the first term, i.e. in the sum over k. Therefore the probability of losing a tie-break is substracted in the second term.

The next step is to compare the expected utility for W- and L-members under MVE and SM.

4.2 Comparison

A comparison between MVE and SM requires the calculation of expected utility. I give an example for an L-member if MVE is applied. All remaining calculations are given in the proof of the upcoming Proposition 1.

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[7] $\lfloor x \rfloor = \max\{y \in \mathbb{N}: \ y \leq x\}$ and $\lceil x \rceil = \min\{y \in \mathbb{N}: \ y \geq x\}$.

$$U_L^{MVE} = z_L + \phi_L \mathbb{P}_{L+}^{MVE} \int_0^1 u(z)dz + \phi_L(1 - \mathbb{P}_{L+}^{MVE}) \cdot 0 + (1 - \phi_L)\mathbb{P}_{L-}^{MVE} \cdot 0$$

$$+ (1 - \phi_L)(1 - \mathbb{P}_{L-}^{MVE}) \int_{-1}^0 u(z)dz$$

$$= z_L + \phi_L \mathbb{P}_{L+}^{MVE} z_E^+ + (1 - \phi_L)(1 - \mathbb{P}_{L-}^{MVE})z_E^-.$$

Expected utility consists of the following parts: the utility from the first project realized under MVE, i.e. z_L, and the expected utility from the second project. The latter can be split up into four subparts: (1) having a positive z_{i2} and winning, i.e. project 2 will be realized, (2) positive z_{i1} and losing, (3) negative z_{i2} and winning, i.e. project 2 will be rejected, and (4) negative z_{i1} and losing.

Comparing expected utility under MVE and SM for L- and W-members yields the following proposition:

Proposition 1 *Given a distribution of z_{i2} on $[-1, 1]$, represented by z_E^+ and z_E^-, the committee's size N, the minority's size $|W|$, and the conditional probabilities of being a second-project winner ϕ_W and ϕ_L, the following statements hold:*

- *L-members will prefer an application of MVE if and only if*

$$z_L > \phi_L(\mathbb{P}_{L+}^{SM} - \mathbb{P}_{L+}^{MVE})z_E^+ + (1 - \phi_L)(\mathbb{P}_{L-}^{MVE} - \mathbb{P}_{L-}^{SM})z_E^- = z_L^*.$$

- *W-members will prefer an application of MVE if and only if*

$$z_W > \phi_W(\mathbb{P}_{W+}^{SM} - \mathbb{P}_{W+}^{MVE})z_E^+ + (1 - \phi_W)(\mathbb{P}_{W-}^{MVE} - \mathbb{P}_{W-}^{SM})z_E^- = z_W^*.$$

The proof of Proposition 1 is given in the Appendix. Note that both threshold values have the same structure: expected utility difference between MVE and SM when being a project winner regarding the second project, plus expected utility difference when being a project loser.

5 Discussion

As indicated, the general setting has the advantage of being interpretable for several situations in community councils:

- Having a (relatively) stable partition into majority and minority ($|\phi_W - \phi_L|$ large).

This occurs notably in political committees where party membership determines the overall partition of the group. Other examples include international committees where nationality determines preferences.

- Linked projects, i.e. individuals are either project winners of both projects or project losers (ϕ_W large while ϕ_L small, or vice versa). Examples include related projects such as building a new airport and thus having to build a new highway to ensure access. Both projects might induce noise pollution for the same group of people, while guaranteeing improved transport connection for another group.

- Uncertainty about the second project ($\phi_W = \phi_L = \frac{1}{2}$ and $z_E^+ = -z_E^- = \frac{1}{2}$). This assumption is standard.

- While ϕ_W and ϕ_L describe individual beliefs about own preferences on projects, the parameters z_E^+ and z_E^- illustrate whether the project imposes large or small utility impacts in general. They represent different distributions of z_{i2} over $[-1, 1]$. Examples are projects that benefit only one group but cause other individuals no harm, such as building a new gymnasium for a school, i.e. z_E^- is close to zero and z_E^+ large.

Two scenarios will now be analyzed in more detail. The first, called S1, reflects uncertainty about the second project. In the second, denoted by S2, project winners (losers) have a high probability of being project winners (losers) again. Plots are used to derive some observations summarized in the following lemma. The plots are given in the Appendix.

Lemma 2 *Given committee sizes typical of German community councils, i.e. $N \in \{7, \ldots, 61\}$, the following observations can be made:*

(S1) If $z_E^+ = \frac{1}{2}$, $z_E^- = -\frac{1}{2}$ and $\phi_L = \phi_W = \frac{1}{2}$, then

- *$\tilde{z}_L$ is negative, tends to increase in N, is decreasing in $|W|$, and is mainly zero for very small $|W|$.*

- *$\tilde{z}_W$ is positive and decreasing in N.*

- *Projects leading to an application of MVE are socially desirable.*

(S2) If $z_E^+ = \frac{1}{2}$, $z_E^- = -\frac{1}{2}$, $\phi_L = 0.2$ and $\phi_W = 0.8$, then

- *$\tilde{z}_L$ is decreasing in $|W|$ and becomes positive for small minorities, i.e. small $|W|$.*

- *$\tilde{z}_W$ is positive and tends to decrease in N.*

Lemma 2 illustrates how much influence the level of information regarding future payoffs has on the effects of voting schemes, as the results differ widely between S1 and S2. All observations are discussed in the following.

The threshold $\tilde{z}_L$

The observation that for small W-groups $\tilde{z}_L > 0$ in S2 and $\tilde{z}_L \approx 0$ in S1 implies that L-members will not want to deviate from simple majority voting, i.e. they would not accept MVE. Proposing MVE only makes for costly delay. The reason is that a small W-group leads to a smaller increase in the probability of winning in the second ballot, i.e. MVE is less attractive for L-members. This property is tightened in S2 as this setting leads to a relatively stable partition into minority and majority. Therefore L-members, i.e. majority members, are most likely to win in the second ballot, whether first-project minority members have voting rights or not. Their incentive to accept MVE decreases ($\tilde{z}_L$ increases).

The threshold $\tilde{z}_W$

Similar reasoning explains the observation that $\tilde{z}_W$ is decreasing in $|W|$ under S2. All W-members again have a very high probability of belonging to the minority when it comes to the ballot on the next project. This implies that they will most likely lose in the next ballot, especially if $|W|$ is small. Deviating from simple majority voting by using MVE becomes more attractive, as it guarantees at least the realization of their own preferences on project 1. For similar reasons the threshold for W-members tends to be more relaxed (i.e. smaller) under S2 than under S1, where projects 1 and 2 are technologically independent.

Social desirability of project 1

Note that an application of MVE improves every individual's expected utility, as the change in the voting scheme has to be accepted unanimously. However, it also leads to the realization of a project, namely project 1, that benefits only a minority. It might be interesting to ask whether this project is socially desirable, i.e. whether project 1 implies an aggregate welfare increase: $|W|z_W + (N - |W|)z_L > 0$. Using the conditions for an application of MVE, i.e. $z_W \geq \tilde{z}_W$ and $z_L \geq \tilde{z}_L$, and rearranging this inequality yields a sufficient condition for social desirability:

$$|W| > N \cdot \frac{-\tilde{z}_L}{\tilde{z}_W - \tilde{z}_L}. \tag{1}$$

Under S1, condition (1) is always fulfilled, i.e. MVE yields the realization of a socially desirable project and is, in addition, Pareto-improving in terms of expected utility. Under S2, social desirability of project 1 depends crucially on the size of the W-group. A larger W-group increases aggregate welfare due to $z_W \geq 0$. The fact that, under S1, all projects leading to

an application of MVE are socially desirable, stems from the observation that the threshold $\tilde{z}_W$ is higher than under S2, notably for small W-groups. In the Appendix I provide plots underlining these results.

To sum up, the incentives to deviate from simple majority voting are very small for majority members if the partition of the committee into majority and minority is rather stable, or if the majority group is very large. Vice versa, deviation incentives are high for minority members in the two cases. These cases may in fact make for timely and costly delays if minority members propose MVE while majority members reject its application.

6 Implications of a Free Choice of β

By assumption A4 the choice of β is restricted to $\frac{|W|}{N}$. This might lead to strategic considerations on the part of W-members. Pretending to belong to the L-group may ensure the voting right due to a smaller β, while it still yields an application of MVE and thus a realization of project 1. This strategic behavior can be legalized by allowing the required minority fraction β to be any number below $\frac{1}{2}$ with the following consequences:

- $\beta > \frac{|W|}{N}$ will never be unanimously accepted, as some L-members would have to vote for project 1, thus losing their voting rights while a decision conflicting with their preferences is made.

- If $\beta < \frac{|W|}{N}$ is used, some W-members may keep their voting rights. A coordination device among W-members is required to ensure that the β-threshold is reached, while simultaneously guaranteeing that the influence of W-members in the second ballot is maximized. This is of particular interest if W and L are stable groups, as in political committees.

- In equilibrium, W-members will propose the minimal $\beta \leq \frac{|W|}{N}$ so that the requirements $z_L > \tilde{z}_L$ and $z_W > \tilde{z}_W$ are still fulfilled, while L-members will propose the largest possible β, i.e. $\beta = \frac{|W|}{N}$. Therefore a free choice of β requires additional constitutional rules that apply in the case where two different MVE proposals are present, or else exclude different proposals. For example, one could restrict the right to propose MVE to project winners only, or one might use a recognition rule ensuring that only one committee member is allowed to propose MVE including β.

- The calculation of the threshold values $\tilde{z}_W$ and $\tilde{z}_L$ is based on the assumption that $\beta = \frac{|W|}{N}$. If a smaller β is chosen, the calculation changes in the following way:

(i) The number of individuals with voting rights in the second ballot under MVE increases, i.e. winning probability decreases. However, it remains higher than the value of winning probability under SM.

(ii) The expected utility from the second project decreases for all L-members while it increases for all W-members – directly, for those who keep their voting rights, indirectly for those who lose it. The indirect utility increase stems from the fact that a larger number of individuals with voting rights in the second ballot better reflects the society.

(iii) Overall, threshold $\tilde{z}_W$ will be weakened, i.e. it will decrease, whereas threshold $\tilde{z}_L$ will be stronger, i.e. it will increase.

7 Conclusion

I have argued that individuals may have an incentive to deviate from simple majority voting if an alternative voting scheme is provided for. The alternative I have proposed is a voting procedure called MVE that favors the acceptance of a project in the sense that only a minority has to approve its application. It covers two ballots, i.e. deviation from simple majority voting is temporally restricted. The alternative voting procedure can only be applied upon request. The incentives to use MVE instead of SM are higher for minority members who are in favor of a project than for majority members, in particular in cases where either the partition of the committee into majority and minority is fairly stable or where the minority group is very small.

The main advantage of MVE is that it is only used when it is needed and desired. This ensures aggregate utility gain. In addition, I demonstrate for middle-sized committees that a project leading to an application of MVE is always socially desirable if the minority is large enough, as it involves a high utility gain for project winners and a small loss for project losers. Under uncertainty, all projects leading to MVE are socially desirable, regardless of whether the W-group is minimum or maximum in size.

MVE is a short-term voting rule and is easily applied. This makes it particularly suitable for middle-sized committees like community councils. The model introduced in this paper is an adequate setting for community councils. Together with the results, i.e. increase in expected aggregate utility and realization of socially desirable projects that otherwise get rejected, MVE offers an attractive alternative to the standard voting rule for community and city councils. Hence deviation from simple majority voting can be welfare-improving and deserves more discussion.

8 Appendix

Calculation of the probability of winning under MVE and SM

We use $TB := (1 - \lceil \frac{N-|W|}{2} - \lfloor \frac{N-|W|}{2} \rfloor \rceil)$ to denote the function that is 1 if $N - |W|$ is an even number and 0 if not, i.e. it identifies the parameter constellation where a tie-break (TB) occurs.

MVE:

- If a W-member i observes $z_{i2} \geq 0$, his probability of winning is given by

$$
\mathbb{P}_{W+}^{MVE} \quad := \quad \sum_{k=\lfloor \frac{N-|W|}{2} \rfloor}^{N-|W|} \phi_L^k (1-\phi_L)^{N-|W|-k} \binom{N-|W|}{k}
$$
$$
+ \, TB \cdot \frac{1}{2} \phi_L^{\frac{N-|W|}{2}} (1-\phi_L)^{\frac{N-|W|}{2}} \binom{N-|W|}{\frac{N-|W|}{2}}.
$$

- If a W-member i observes $z_{i2} < 0$, his probability of winning is given by

$$
\mathbb{P}_{W-}^{MVE} \quad := \quad \sum_{k=\lfloor \frac{N-|W|}{2} \rfloor}^{N-|W|} (1-\phi_L)^k \phi_L^{N-|W|-k} \binom{N-|W|}{k}
$$
$$
+ \, TB \cdot \frac{1}{2} (1-\phi_L)^{\frac{N-|W|}{2}} \phi_L^{\frac{N-|W|}{2}} \binom{N-|W|}{\frac{N-|W|}{2}}.
$$

- If an L-member i observes $z_{i2} \geq 0$, his probability of winning is given by

$$
\mathbb{P}_{L+}^{MVE} \quad := \quad \sum_{k=\lfloor \frac{N-|W|-1}{2} \rfloor}^{N-|W|-1} \phi_L^k (1-\phi_L)^{N-|W|-1-k} \binom{N-|W|-1}{k}
$$
$$
- \, TB \cdot \frac{1}{2} \phi_L^{\frac{N-|W|}{2}-1} (1-\phi_L)^{\frac{N-|W|}{2}} \binom{N-|W|-1}{\frac{N-|W|}{2}-1}.
$$

- If an L-member i observes $z_{i2} < 0$, his probability of winning is given by

$$
\mathbb{P}_{L-}^{MVE} \quad := \quad \sum_{k=\frac{N-|W|-1}{2}}^{N-|W|-1} (1-\phi_L)^k \phi_L^{N-|W|-1-k} \binom{N-|W|-1}{k}
$$
$$
- \, TB \cdot \frac{1}{2} (1-\phi_L)^{\frac{N-|W|}{2}-1} \phi_L^{\frac{N-|W|}{2}} \binom{N-|W|-1}{\frac{N-|W|}{2}-1}.
$$

SM:

- If a W-member i observes $z_{i2} \geq 0$, his probability of winning is given by

$$\mathbb{P}^{SM}_{W+} := \sum_{k=\frac{N-1}{2}}^{N-1} \sum_{m=0}^{|W|-1} \phi_W^m (1-\phi_W)^{|W|-1-m} \phi_L^{k-m} (1-\phi_L)^{N-|W|-(k-m)} \binom{|W|-1}{m} \binom{N-|W|}{k-m}.$$

- If a W-member i observes $z_{i2} < 0$, his probability of winning is given by

$$\mathbb{P}^{SM}_{W-} := \sum_{k=\frac{N-1}{2}}^{N-1} \sum_{m=0}^{|W|-1} (1-\phi_W)^m \phi_W^{|W|-1-m} (1-\phi_L)^{k-m} \phi_L^{N-|W|-(k-m)} \binom{|W|-1}{m} \binom{N-|W|}{k-m}.$$

- If an L-member i observes $z_{i2} \geq 0$, his probability of winning is given by

$$\mathbb{P}^{SM}_{L+} := \sum_{k=\frac{N-1}{2}}^{N-1} \sum_{m=0}^{|W|} \phi_W^m (1-\phi_W)^{|W|-m} \phi_L^{k-m} (1-\phi_L)^{N-|W|-1-(k-m)} \binom{|W|}{m} \binom{N-|W|-1}{k-m}.$$

- If an L-member i observes $z_{i2} \geq 0$, his probability of winning is given by

$$\mathbb{P}^{SM}_{L-} := \sum_{k=\frac{N-1}{2}}^{N-1} \sum_{m=0}^{|W|} (1-\phi_W)^m \phi_W^{|W|-m} (1-\phi_L)^{k-m} \phi_L^{N-|W|-1-(k-m)} \binom{|W|}{m} \binom{N-|W|-1}{k-m}.$$

Proof of Proposition 1

The expected utility from MVE for W-members has the following structure: utility of the first project (z_W) plus the expected utility of the second project that is a sum over all possible situations, i.e. being a project winner ($z_{i2} \geq 0$) with probability ϕ_W and winning with probability $\mathbb{P}^{MVE}_{W+}$, being a project loser with probability $1 - \phi_W$ and winning with probability $\mathbb{P}^{MVE}_{W-}$, and so forth.

This yields

$$
\begin{aligned}
U^{MVE}_W &= z_W + \phi_W \mathbb{P}^{MVE}_{W+} \int_0^1 u(z) dz + \phi_W (1 - \mathbb{P}^{MVE}_{W+}) \cdot 0 \\
&\quad + (1-\phi_W) \mathbb{P}^{MVE}_{W-} \cdot 0 + (1-\phi_W)(1 - \mathbb{P}^{MVE}_{W-}) \int_{-1}^0 u(z) dz \\
&= z_W + \phi_W \mathbb{P}^{MVE}_{W+} z_E^+ + (1-\phi_W)(1 - \mathbb{P}^{MVE}_{W-}) z_E^-.
\end{aligned}
$$

and

$$
\begin{aligned}
U_W^{SM} &= \phi_W \mathbb{P}_{W+}^{SM} \int_0^1 u(z)dz + \phi_W(1 - \mathbb{P}_{W+}^{SM}) \cdot 0 \\
&\quad + (1 - \phi_W)\mathbb{P}_{W-}^{SM} \cdot 0 + (1 - \phi_W)(1 - \mathbb{P}_{W-}^{SM}) \int_{-1}^0 u(z)dz \\
&= \phi_W \mathbb{P}_{W+}^{SM} z_E^+ + (1 - \phi_W)(1 - \mathbb{P}_{W-}^{SM})z_E^-.
\end{aligned}
$$

A comparison between U_W^{MVE} and U_W^{SM} gives us the threshold $\tilde{z}_W$ at which MVE is more attractive for W-members than abstaining from the utility of project 1 while retaining the voting right for the second ballot, i.e.

$$
\begin{aligned}
U_W^{MVE} &> U_W^{SM} \\
\Leftrightarrow \quad z_W &> \phi_W(\mathbb{P}_{W+}^{SM} - \mathbb{P}_{W+}^{MVE})z_E^+ + (1 - \phi_W)(\mathbb{P}_{W-}^{MVE} - \mathbb{P}_{W-}^{SM})z_E^- =: \tilde{z}_W
\end{aligned}
$$

The same calculations hold for L-members by replacing W by L.

$\blacksquare$

Plots illustrating the threshold values $\tilde{z}_L$ and $\tilde{z}_W$ as described in Lemma 2

The first plots (figures 1–3) show the threshold values $\tilde{z}_W$ and $\tilde{z}_L$ for both settings, S1 and S2, with different minority sizes.[8] N runs from 7 to 61, hence reflecting committee sizes in communal politics. Part (a) always corresponds to the uncertainty case (S1), i.e. $\phi_W = \phi_L = \frac{1}{2}$ and $z_E^+ = \frac{1}{2}$, $z_E^- = -\frac{1}{2}$. Part (b) corresponds to a biased project 2 (S2) where $\phi_W = 0.8$ and $\phi_L = \frac{1}{2}$.

---

[8] The oscillating behavior stems from the fact that the probabilities of winning include binomial coefficients. This implies that they take the same values for an even number and the next-higher odd number, i.e. they are a weakly decreasing step function.

(a) Uncertainty about project 2   (b) Biased project 2
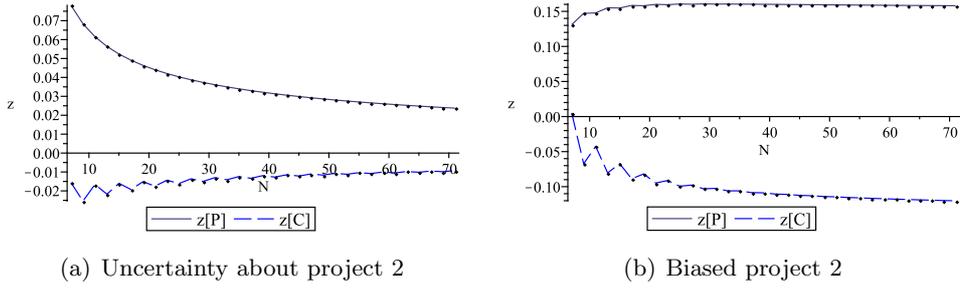
Figure 1: Thresholds when the group of project winners W is of maximum size, i.e. $|W| = \frac{N-1}{2}$. If the payoff for project winners is larger than $\tilde{z}_W$ and the payoff for project losers is larger than $\tilde{z}_L$, then MVE is proposed and accepted and yields higher expected aggregate welfare than the simple majority rule.


(a) Uncertainty about project 2   (b) Biased project 2
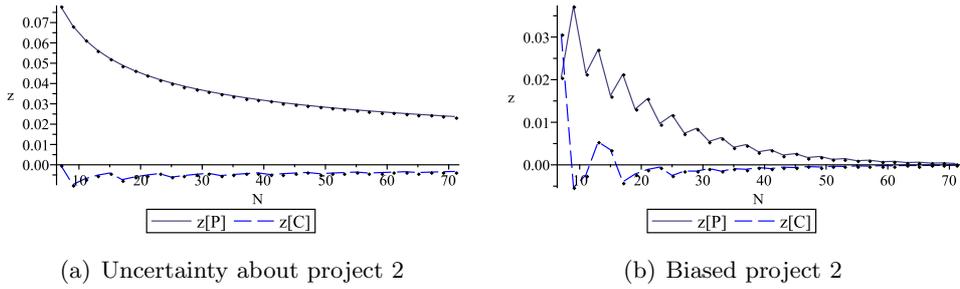
Figure 2: Thresholds in the case where W is of medium size, i.e. $|P| = \frac{N-1}{4}$.

(a) Uncertainty about project 2   (b) Biased project 2
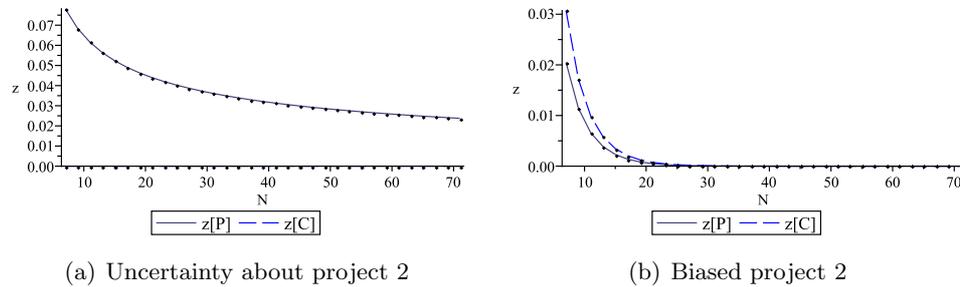
Figure 3: Thresholds in the case where W is of minimum size, i.e. $|P| = 1$.

Plots for the condition on social desirability of project 1, i.e. inequality (1)

Figure 4 shows the LHS and the RHS of inequality (1): $|W| \geq N \cdot \frac{-\tilde{z}_L}{\tilde{z}_W - \tilde{z}_L}$.[9] The RHS is denoted by S1 or S2 in the plots, according to the settings of Lemma 2. Again $N \in \{7, \dots, 61\}$ and $|W| \in \{1, \frac{N-1}{4}, \frac{N-1}{2}\}$ to indicate extreme cases.

(a) Large minority: $|W| = \frac{N-1}{2}$.

(b) Middle-sized minority: $|W| = \frac{N-1}{4}$.

(c) Small minority: $|W| = 1$.
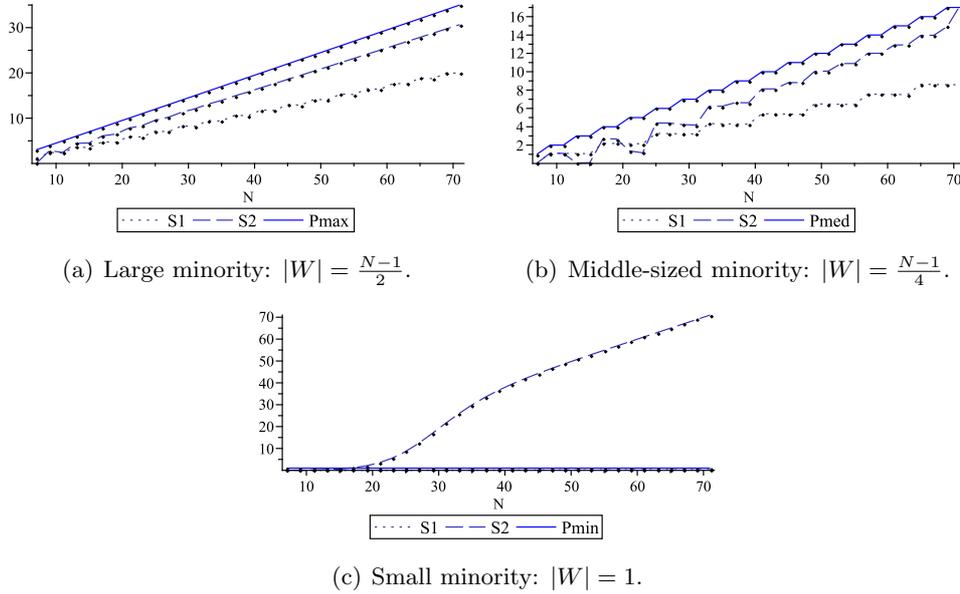
Figure 4: LHS ('$W_{min/med/max}$') and RHS ('S1','S2') of the condition on social desirability of project 1, where LHS > RHS has to hold. Note that the RHS also depends on the number of project winners W.

For large W-groups, the LHS (W_{med}, W_{max}, see figure 4(a) and 4(b)) is always larger than the RHS, i.e. project 1 is socially desirable. Thus the plots prove numerically that under S1 all MVE projects are socially desirable and that a project 1 leading to MVE under S2 is socially desirable if $N \leq 23$ and $|W| > 1$. For larger N the cutting point between $|W|$ and $N \cdot \frac{-\tilde{z}_L}{\tilde{z}_W - \tilde{z}_L}$ moves from $|W| = 1$ to approximately $|W| = \frac{N-3}{4}$ for the largest assumed values of N.

---

[9]We use $\min\{-0.0001, \tilde{z}_L\}$, as $\tilde{z}_L$ can be positive under S2.

References

BERNHOLZ, P. (1978): "On the Stability of Logrolling Outcomes in Stochastic Games," *Public Choice*, 33, 65–82.

BRAMS, S. (1975): *Game Theory and Politics*. Free Press, New York.

BRAMS, S., AND W. RIKER (1973): "The paradox of vote trading," *The American Political Science Review*, 67, 1235–1247.

CASELLA, A. (2005): "Storable Votes," *Games and Economic Behavior*, 51, 391–419.

COLEMAN, J. (1983): "Recontracting, Trustworthiness, and the Stability of Vote Exchanges," *Public Choice*, 40(1), 89–94.

COX, G. (1990): "Centripetal and Centrifugal Incentives in Electoral Systems," *American Journal of Political Science*, 34, 903–935.

FAHRENBERGER, T., AND H. GERSBACH (2008): "Minority Voting and Long-term Decisions," CESifo Working Paper No. 2198.

GERBER, E., R. MORTON, AND T. RIETZ (1998): "Minority Representation in Multimember Districts," *The American Political Science Review*, 92(1), 127–144.

GUINIER, L. (1994): *The Tyranny of the Majority*. Free Press, New York.

HORTALA-VALLVE, R. (2007): "Qualitative Voting," Discussion paper, Economics Series Working Papers No. 320, University of Oxford.

KOST, A., AND H.-G. WEHLING (2003): *Kommunalpolitik in den deutschen Ländern*. Westdeutscher Verlag.

MUELLER, D. (1967): "The Possibility of a Social Welfare Function: Comment," *The American Economic Review*, 57(5), 1304–1311.

NASSMACHER, H., AND K.-H. NASSMACHER (2007): *Kommunalpolitik in Deutschland*. VS Verlag für Sozialwissenschaften.

SAWYER, J., AND D. MACRAE (1962): "Game Theory and Cummulative Voting in Illinois: 1902-1954," *The American Political Science Review*, 56(4), 936–946.

TULLOCK, G. (1974): "Paradox Lost (comment on Riker-Brams article)," *The American Political Science Review*, 68(4), 1687–1688.

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