Endogenous Growth, Asymmetric Trade and Resource Taxation

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Abstract

Since 1980, the aggregate income of oil-exporting countries relative to that of oil-poor countries has been remarkably constant despite structural gaps in productivity growth rates. This stylized fact is analyzed in a two-country model where resource-poor (Home) and resource-rich (Foreign) economies display productivity differences but stable income shares due to terms-of-trade dynamics. We show that Home's income share is positively related to the national tax on domestic resource use, a prediction confirmed by dynamic panel estimations for sixteen oil-poor economies. National governments have incentives to deviate from both efficient and laissez-faire allocations. In Home, increasing the oil tax improves welfare through a rent-transfer mechanism. In Foreign, subsidies (taxes) on domestic oil use improve welfare if R&D productivity is lower (higher) than in Home.

1 Introduction

The functioning of modern economies crucially relies on the primary inputs obtained from exhaustible natural resources and, due to the uneven distribution of endowments of fossil fuels and minerals, many industrialized economies heavily depend on imports from resource-rich countries. The relevance of this form of trade dependence is emphasized by the statistics – fossil fuels and minerals now account for 22.5% of world merchandise trade (World Trade Organization, 2009: p.43) – and is increasingly regarded as a crucial determinant of the growth performance of resource-rich economies (Lederman and Maloney, 2007). Few studies, however, analyze in detail the implications of asymmetric trade between resource-rich and resource-poor countries from the perspective of modern growth theory. In this paper, we exploit the endogenous growth framework to analyze the determinants of national income shares and the effects of resource taxation on welfare distribution.

Our empirical reference is the economic performance of the world's top net exporters of oil – henceforth labelled as OEX group – relative to that of big oil-poor economies, i.e., the world's top importers whose domestic oil production is zero or negligible – henceforth labelled as OIM group. Since 1980, the ratio between the aggregate incomes of the two groups has been constant: the OIM share over the total income of the two groups is 73%, and its time profile over the last three decades is remarkably flat (cf. section 2). This result is surprising in view of the productivity differentials observed during the same period: labor productivity has stagnated or declined in most OEX countries while it has substantially increased in OIM economies.

In line with these empirical facts, we build a two-country model in which productivity growth rates differ across countries but the world income distribution is stable over time. A resource-poor economy (henceforth called Home) imports a final good and an exhaustible primary input from a resource-rich economy (henceforth called Foreign) and only exports its final consumption good. Both countries exhibit endogenous growth driven by R&D: in equilibrium, productivity growth differentials are compensated by the dynamics of the prices of traded goods, and income shares are constant.

Our first aim is to study the determinants of national income shares and check whether the theoretical predictions are supported by empirical evidence. The model predicts that the income share of OIM economies is (i) positively related to the domestic rate of R&D investment, (ii) negatively related to the outher country's investment rate, (iii) positively influenced by the domestic tax on resource use due to a peculiar rent-extraction mechanism. Our panel estimations for sixteen OIM economies in relation to an aggregate group of ten OEX economies confirm the positive (negative) effect of domestic (foreign) saving rates as well as the positive impact of domestic oil taxes on income shares.

Our second aim is to link the model predictions to the policy debate. The recent up-surge in oil prices revived the interest for the analysis of strategic tax policies between resource-rich and resource-poor economies. The crucial question is: do economies involved in asymmetric trade have peculiar incentives to implement inefficient taxes on domestic resource use? We show that these incentives exist and are particularly strong

for oil importers. First, if the initial state of affairs is an efficient equilibrium in which all domestic market failures are internalized, the Home government can increase domestic welfare by raising the national resource tax above the efficient level: due to the rent-extraction mechanism, a higher resource tax increases Home's relative income and enhances consumption possibilities. Second, if the initial state of affairs is a laissez-faire equilibrium, productivity differences come into play: in the empirically plausible case where productivity growth is higher in the resource-poor economy, Home's incentive to raise the resource tax is reinforced whereas Foreign has an incentive to subsidize domestic resource use. Both these results appear consistent with the behavior that is typically observed in reality (Gupta et al. 2002; Metschies, 2005).

As mentioned above, few studies analyze the implications of asymmetric trade between resource-rich and resource-poor countries from the perspective of modern growth theory. The trade-and-growth literature typically neglects asymmetric trade structures induced by uneven endowments of primary inputs. In the early resource economics literature, two-country models assumed that the accumulation of man-made capital inputs was either absent (Kemp and Suzuki, 1975; Brander and Djajic, 1983) or subject to diminishing returns (Chiarella, 1980; Van Geldrop and Withagen, 1993). The parallel literature on endogenous growth with natural resources as inputs, pioneered by Barbier (1999) and Scholz and Ziemes (1999), generally refers to closed or small open economies: to our knowledge, the two-country setting is only considered in two recent papers by Daubanes and Grimaud (2006) and Peretto and Valente (2010) that differ from the present analysis in both aims and means.[1]

2 Empirical Facts

Our theoretical analysis focuses on tradeable exhaustible resources and can be applied to several types of minerals and fossil fuels. The main empirical reference, however, is the relative economic performance of oil-rich and oil-poor economies. The first column of Table 1 lists a group of countries, labeled as OEX, which comprises the seventeen top oil exporters at the world level over the period 1980-2008.[2] These economies satisfy two requirements: during the relevant period, each country (i) has never been a net oil importer and (ii) steadily appeared in the top exporters list in each single year. In

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[1]Daubanes and Grimaud (2006) use a North-South model where oil generates pollution and economic growth is exclusively driven by the technology of the oil-poor economy: there are no productivity gaps and terms of trade are excluded by the homogeneity of the final consumption good. Peretto and Valente (2009) assume identical R&D technologies between countries in a non-scale model of endogenous growth featuring both vertical and horizontal innovations. They analyze the effects of resource booms, i.e. unexpected discoveries of new resource stocks on innovation rates and relative welfare.

[2]We consider seventeen countries and draw the line below Kazakhstan because the subsequent positions are occupied by countries exporting much less oil in absolute terms. Over the 1980-1992 period, the Russian Federation would be replaced by USSR, and the 17th top exporter would be Egypt, whose average yearly net exports have been nearly one half of the preceding country, Canada. Over the 1992-2008 period, the 18th top exporter is Colombia, with similar figures in proportion to Kazakhstan. Indonesia and United Kingdom are excluded from the computations since they both turned from net exporters in the 1980s to net importers nowadays.

Table 1, the ratio between oil consumption and production in physical terms highlights different degrees of dependence and/or specialization. Due to data availability, real output growth rates for OEX countries are calculated for two subsets: OEX-15, which excludes Iraq and Lybia, and OEX-13, which also excludes Russia and Kazakhstan.

OEX Countries	Oil Net Exports*		Oil Cons./Prod.**	Real GDP Growth***	
	1980-2008	1992-2008	1992-2008	1980-2006	1990-2006
Saudi Arabia	7042	8022	16.9%	1.57%	2.89%
Russian Federation	-	4751	37.9%	-	-0.16%
Norway	1957	2752	7.2%	2.80%	2.98%
Iran	2089	2483	34.9%	3.39%	4.17%
Venezuela	2103	2432	17.8%	1.92%	2.57%
United Arab Emirates	1926	2232	13.6%	3.07%	5.04%
Kuwait	1632	1964	11.2%	2.89%	5.95%
Nigeria	1666	1901	12.8%	2.77%	3.78%
Mexico	1420	1482	57.0%	2.47%	2.86%
Algeria	1244	1422	13.6%	2.62%	2.53%
Libya	1233	1319	14.0%	-	-
Iraq	1236	1124	30.0%	-	-
Angola	639	899	4.1%	3.72%	4.44%
Oman	653	778	6.8%	6.14%	4.50%
Canada	536	752	73.1%	2.69%	2.63%
Qatar	592	751	7.5%	3.89%	7.42%
Kazakhstan	-	594	29.1%	-	1.36%

Table 1. Selected top oil exporters. * Average yearly net exports of oil, thousands barrels per day; ** Average yearly oil cosumption-to-production ratio; *** Average yearly growth rate of real GDP; Sources - * and ** EIA (2009); *** World Bank (2009) and IMF (2009) for Angola and Qatar.

We compare the OEX group with seventeen economies that, during the same period, (i) steadily appeared in the list of top oil-importers *and* (ii) relied on imported oil for domestic use. Specifically, we have excluded all countries producing more than 10% of the oil they consume domestically.[3] The resulting list of oil-poor, oil-importing countries is labelled as OIM and is reported in Table 2: the third column shows the ratio between net imports and domestic oil consumption, which does not fall short of 90% (the limit case is Netherlands). Data on real GDP growth for the whole set of countries, labeled as OIM-17, cover the 1990-2006 period. The subset OIM-15, which excludes Germany and Poland, covers the 1980-2006 period.

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[3]Excluded oil-importing countries (and their yearly average Net Import/Consumption ratio over 1992-2008) are: United States (53%), China (31%), India (62%), Thailand (74%), Brazil (26%), Ukraine (77%), South Africa (57%), Pakistan (81%), Australia (25%).

OIM Countries	Oil Net Imports*		Oil Net Imp./Cons.**	Real GDP Growth***	
	1980-2008	1992-2008	1992-2008	1980-2006	1990-2006
Japan	5076	3113	98.0%	2.24%	1.26%
Germany	-	1542	94.9%	-	1.57%
South Korea	1495	1202	99.8%	6.49%	5.19%
France	1838	1105	95.4%	2.01%	1.78%
Italy	1714	999	92.5%	1.71%	1.29%
Spain	1178	797	97.9%	2.87%	2.83%
Taiwan	660	486	99.8%	6.12%	5.04%
Netherlands	702	452	89.0%	2.37%	2.39%
Singapore	499	387	99.1%	6.65%	6.20%
Belgium	534	350	98.2%	1.94%	1.90%
Turkey	474	335	91.0%	4.37%	3.87%
Poland	359	227	94.9%	-	3.50%
Greece	326	225	97.7%	2.08%	2.90%
Sweden	376	218	99.4%	2.18%	2.17%
Philippines	269	189	96.9%	2.77%	3.42%
Portugal	261	179	98.9%	2.48%	2.03%
Switzerland	267	158	99.2%	1.56%	1.19%

Table 2. Selected oil-poor net importers. * Average yearly net imports of oil, thousands barrels per day; ** Average yearly oil net imports-to-consumption ratio; *** Average yearly growth rate of real GDP. Sources - * and ** EIA (2009); *** World Bank (2009) and IMF (2009) for Taiwan.

The first empirical fact concerns the behavior of gross domestic product (GDP) and gross national income (GNI) calculated in purchasing power parity. Computing the income shares of each group over the total income of both groups in each year, all the resulting time paths are remarkably flat. This result is robust to alternative PPP-adjusted measures of GNI and GDP, both in constant and in current prices. An example is reported in Figure 1: the GDP share of OEX-13 versus OIM-15 countries in 2006 is 73%, almost identical to the value observed in 1980, and there is little variation during the whole period. The same result is obtained for GDP shares of OEX-15 versus OIM-17 countries between 1990 and 2006.

The second empirical fact is related to productivity growth. Using the series of labor productivity levels calculated by the International Labor Organization (2009),[4] we normalize the 1980 level to unity for each country and obtain the time paths depicted in Figure 2. There are substantial productivity growth differentials in favor of OIM economies: apart from Canada and Norway, OEX countries have been falling behind oil-importing economies over the last three decades.

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[4] The ILO calculations are based on the estimates of the Total Economy Database of the Conference Board. The data used in Figure 2 refer to the "LP person GK" time series reported in the Total Economy Database.
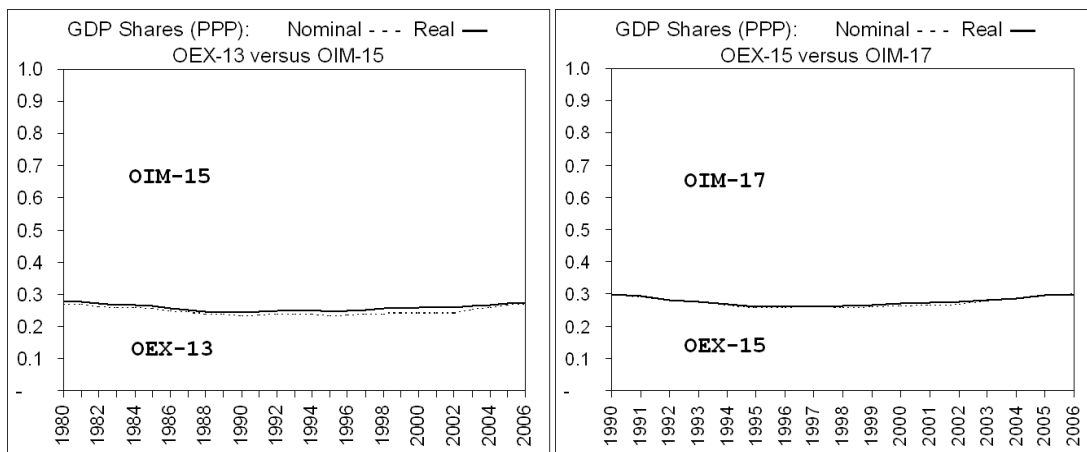
Figure 1. Current price (dotted line) and constant price (bold line) GDP shares (PPP) of oil-exporters and oil-importers. Source: authors calculations on World Bank (2009) data (IMF data for Angola 1980-1984 and Taiwan 1980-2006).
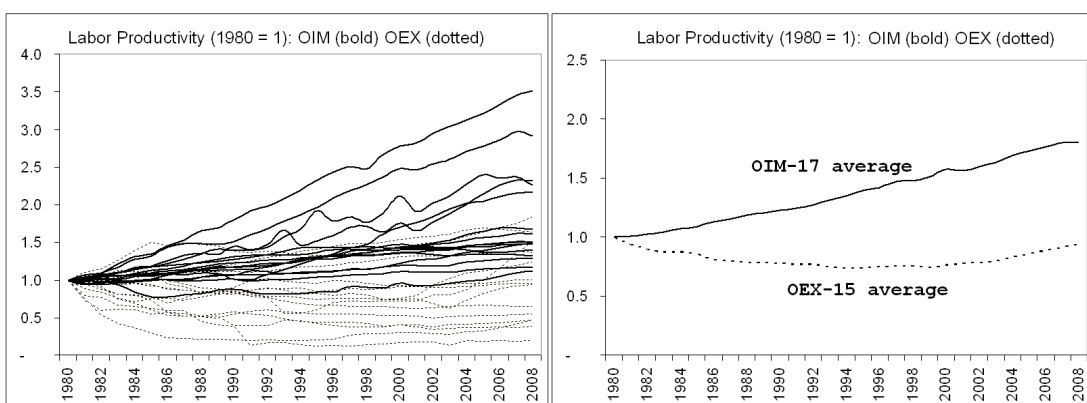

Figure 2. Normalized (1980 = 1) labor productivity levels in all countries (left graph) and group averages OEX versus OIM (right graph). Source: authors calculations on data from the International Labor Organization and The Conference Board.

The existence of substantial gaps in productivity growth rates in conjunction with balanced GDP growth at the international level suggests that stationary income shares may originate, at least in part, in the compensating effect of the prices of traded goods. In the next section, we present a general equilibrium model of endogenous growth in which income shares are constant because structural gaps in productivity growth are compensated by the dynamics of terms of trade.

3 The Model

The world comprises two countries, Home and Foreign, indexed by $i = h, f$. Each economy produces a tradable final good, consumed by the residents of both countries, using man-made intermediate inputs and an exhaustible natural resource. As the resource stock is entirely owned by Foreign residents, the structure of trade is asymmetric: Home only exports its final good whereas Foreign exports its final good and natural resource units. The engine of growth is represented by R&D activity that expands the number of varieties of intermediate inputs – e.g., as in Rivera-Batiz and Romer (1991). Intermediates' producers earn monopoly rents, and the productivity of R&D firms is enhanced by knowledge spillovers that eliminate scale effects. National governments have access to three fiscal instruments – a subsidy to R&D investment, a tax on final producers, and a tax on domestic resource use – that can be used to correct for domestic market failures. Our primary interest, however, is the role of national resource taxes as potential strategic instruments: we will analyze in detail the welfare effects of resource taxes when governments deviate from both laissez-faire and efficient allocations.

Using conventional notation, the time-derivative and the growth rate of variable $g(t)$ are respectively denoted by $\dot{g}(t) \equiv dg(t)/g(t)$ and $\hat{g}(t) \equiv \dot{g}(t)/g(t)$. All Propositions are proved in Appendix.

3.1 Final Producers, Intermediate Sectors and R&D

Final Sector. In each economy, the final sector produces Y_i units of a country-specific final good by means of M_i varieties of differentiated man-made intermediate products, L_i units of labor, and R_i units of an exhaustible resource, according to the production function

$$Y_i = \int_0^{M_i} (X_i(m_i))^\alpha \, dm_i \cdot (v_i L_i)^\beta R_i^\gamma, \qquad i = h, f, \tag{1}$$

where $X_i(m_i)$ is the quantity of the m_i-th variety of intermediate input employed in production, and v_i denotes the productive efficiency of each worker. Parameters satisfy $\alpha + \beta + \gamma = 1$, with $0 < \alpha, \beta, \gamma < 1$. As the engine of growth is represented by increases in the number of intermediate products, we assume that workers' efficiency v_i grows at the exogenous constant rate $\hat{v}_i = \eta_i$, and that labor is supplied inelastically: L_h and L_f are fixed amounts and coincide with population size in Home and Foreign, respectively. The law of one price holds for all traded goods: the quantities (Y_h, Y_f) are sold at the respective world prices (P_Y^h, P_Y^f) and the exhaustible resource is sold to all final producers at the same world price P_R. Labor and intermediates are not traded so that

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the wage rate and the price of each intermediate, respectively denoted P_L^i and $P_{X(m_i)}^i$, are country-specific. Production costs are affected by fiscal policy: we denote by b_i the tax on the purchases of intermediates, and by τ_i the tax on domestic resource use.[5] The profit-maximizing conditions on resource use and intermediates respectively imply

$$P_R R_i \left(1 + \tau_i\right) = \gamma P_Y^i Y_i, \tag{2}$$

$$P_{X(m_i)}^i \left(1 + b_i\right) = \alpha P_Y^i \left(X_i \left(m_i\right)\right)^{\alpha-1} \left(v_i L_i\right)^\beta R_i^\gamma, \tag{3}$$

where (3) is valid for each $m_i \in [0, M_i]$.

Intermediate Sector. Each variety of intermediates is produced by a monopolist who holds the relevant patent and maximizes instantaneous profits $\Pi_i\left(m_i\right)$ taking the demand schedule (3) as given. Production requires $\bar{\varsigma}\left(X_i\left(m_i\right)\right)$ units of final good, where $\bar{\varsigma}\left(.\right)$ is the cost function. Assuming the linear form $\bar{\varsigma}\left(X_i\right) = \varsigma X_i$ for each variety, profit maximization requires

$$P_{X(m_i)}^i = \left(\varsigma/\alpha\right) P_Y^i \text{ for each } m_i \in [0, M_i], \tag{4}$$

and therefore symmetric quantities and profits across monopolists (see Appendix).

R&D Sector. The number of intermediates' varieties M_i grows over time by virtue of R&D activity pursued by competitive firms that develop new blueprints and sell the relevant patent to an incumbent intermediate producer. R&D firms can be represented as a consolidated sector earning zero profits due to free-entry.[6] Developing blueprints requires investing units of the domestic final good: each unit has a constant marginal productivity $\phi_i > 0$, taken as given by R&D firms, and for each unit invested, R&D firms receive from the national government a subsidy at constant rate $a_i > 0$. Denoting by Z_i the total amount invested by R&D firms, total investment in country i is $Z_i\left(1 + a_i\right)$, and the increase in the number of varieties equals

$$\dot{M}_i\left(t\right) = \phi_i \cdot \left(1 + a_i\right) Z_i\left(t\right). \tag{5}$$

The productivity of the R&D sector is affected by externalities whereby the current marginal productivity of investment, ϕ_i, is positively influenced by past research effort. These externalities take the form of knowledge spillovers, exactly as in models *à la* Lucas (1988) where the diffusion of public knowledge implies an un-compensated transmission of human capital across generations. In the present context, the productivity of each R&D firm is higher the better the 'current state of technology attained by virtue of previous research'. This concept of state-of-the-art in research is conveniently measured

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[5]Both b_i and τ_i are assumed to be constant in order to preserve the balanced-growth properties of the world equilibrium. This assumption does not affect the generality of our results: as shown in section 5, both efficient allocations and laissez-faire equilibria exhibit balanced growth in each instant. Decentralizing efficient allocations thus requires implementing constant taxes.

[6]This is due to the symmetry in intermediate producers' profits. See the Appendix for the derivation of the zero-profit condition in the R&D sector.

by the ratio between the number of existing varieties and current output levels, M_i/Y_i. We thus posit a linear function

$$\phi_i(t) \equiv \varphi_i \cdot (M_i(t)/Y_i(t)) \tag{6}$$

where $\varphi_i > 0$ is a constant proportionality factor determining the social productivity of R&D. From from (5) and (6), the growth rate of intermediates' varieties increases linearly with the economy-wide rate of R&D investment:

$$\hat{M}_i(t) = \varphi_i(1 + a_i) \cdot (Z_i(t)/Y_i(t)). \tag{7}$$

As Barro and Sala-i-Martin (2004: p.300-302) point out, the linear law (7) generally exhibits two desirable properties: it eliminates scale effects by making the equilibrium growth rate of output independent of the size of endowments, and is empirically plausible since, in most industrialized economies, productivity growth appears to be positively related to the rate of R&D investment.

3.2   Resource Extraction in Foreign

In each instant, total resource extraction $R(t)$ equals the sum of the resource units employed in the two countries, $R(t) \equiv R_h(t) + R_f(t)$. The resource stock $Q(t)$ is non-renewable and is given at $t = 0$. Extracting firms are competitive and take the world price of the resource P_R as given. For simplicity, extraction costs are zero. The owners of extracting firms are households in Foreign, each of whom earns the same fraction $1/L_f$ of rents. Normalizing the mass of firms to unity[7], the representative firm maximizes the present-value stream of profits

$$\int_0^\infty P_R(t) R(t) e^{-\int_t^\infty r_f(v)dv} dt, \tag{8}$$

subject to the dynamic resource constraint $\dot{Q}(t) = -R(t)$. The solution to this dynamic problem is characterized by the conditions

$$\hat{P}_R(t) = r_f(t), \tag{9}$$

$$Q_0 = \int_0^\infty R(t) dt. \tag{10}$$

Equation (9) is the standard Hotelling rule: the resource price must grow over time at a rate equal to the rate of return to investment in the economy. Equation (10) is the intertemporal resource constraint requiring asymptotic exhaustion of the resource stock.

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[7]We would obtain identical results if we assumed that the resource stock is incorporated in private wealth: in this case, each household is endowed with a fraction $1/L_f$ of the initial stock and directly extracts the resource in accordance with Hotelling' s rule (9).

3.3   Governments, Households and Trade Balance

Governments. The public sector in country $i = h, f$ finances public R&D subsidies by means of the ad valorem taxes on intermediates' purchases and resource use. Ruling out public debt, balanced budget is achieved in each instant by compensating possible imbalances with a lump-sum transfer F_i imposed on each household.

Households. Economy i is populated by L_i homogeneous households that solve a standard consumer problem in two steps. In the first step, agents decide how to allocate expenditures between imported and domestically-produced final goods. Denoting by c_i^j the quantity of the good produced in country j and individually consumed in country i, the instantaneous utility of each resident in country i reads

$$u_i(c_i^h, c_i^f) = \ln\left[(c_i^h)^\epsilon (c_i^f)^{1-\epsilon}\right], \qquad 0 < \epsilon < 1, \tag{11}$$

where the weighting parameters, ϵ and $1 - \epsilon$, indicate the preference taste for Home and Foreign goods, respectively. Maximizing (11) subject to the expenditure constraint

$$E_i^c / L_i = P_Y^h c_i^h + P_Y^f c_i^f, \tag{12}$$

where E_i^c is aggregate consumption expenditure in country i, we obtain the indirect utility function $\bar{u}_i = \ln[\omega \cdot (E_i^c / L_i)]$, where $\omega \equiv \omega(P_Y^h, P_Y^f)$ is a weighted average of final goods' prices (see Appendix). In the second step, agents choose the time profile of expenditures by maximizing present-value utility

$$U_i \equiv \int_0^\infty e^{-\rho t} \cdot \ln[(\omega(t) \cdot (E_i^c(t) / L_i)] dt, \tag{13}$$

where $\rho > 0$ is the pure time-preference rate, and the path of $\omega(t)$ is taken as given by the household. Objective (13) is maximized subject to the dynamic wealth constraint of the household (see Appendix). The resulting optimality conditions yield

$$\hat{E}_i^c(t) = r_i(t) - \rho, \tag{14}$$

which is the standard Keynes-Ramsey rule.

Trade. Ruling out asset mobility, trade is balanced in each instant: the value of Foreign total exports – resources plus exported consumption goods – equals the value of final goods imported from Home,

$$P_R R_h + P_Y^f L_h c_h^f = P_Y^h L_f c_f^h. \tag{15}$$

The resource-rich economy exhibits a structural deficit in final-goods trade: this asymmetric structure of international trade will be crucial for the results.

Aggregate Constraints. To simplify the notation, denote aggregate R&D expenditures of country i as $E_i^d \equiv P_Y^i Z_i (1 + a_i)$ and aggregate expenditures in intermediates'

production as $E_i^x \equiv P_Y^i \varsigma M_i X_i$. The total expenditure index of country i is defined as $E_i \equiv E_i^c + E_i^d + E_i^x$, and the aggregate income constraints of the two economies read

$$E_h \equiv E_h^c + E_h^d + E_h^x = P_Y^h Y_h - P_R R_h, \qquad (16)$$

$$E_f \equiv E_f^c + E_f^d + E_f^x = P_Y^f Y_f + P_R R_h. \qquad (17)$$

From (16), total incomes in Home equal the value of final output less the value of resource rents paid to Foreign resource owners. In (17), resource rents are added to the value of final production in Foreign to obtain total Foreign incomes.

3.4 Competitive World Equilibrium

In each country, the equilibrium rates of return read (see Appendix)

$$r_i = \hat{P}_Y^i + \Omega_i = \hat{P}_Y^i + \left[\frac{\alpha (1-\alpha)(1+a_i)}{1+b_i} \varphi_i + \frac{\beta}{1-\alpha} \eta_i + \frac{\gamma}{1-\alpha} \hat{R}_i \right], \qquad (18)$$

where Ω_i denotes the last term in square brackets and is a measure of *physical productivity growth* in the final sector of country i. Productivity growth incorporates the effect of three country-specific characteristics: the social productivity of R&D (crucially determined by φ_i), the growth rate of labor efficiency (η_i), and the growth rate of domestic resource use ($\hat{R}_i$). From (18), we can decompose the interest rate differential between Home and Foreign into a price component and a productivity term:

$$r_h - r_f = (\hat{P}_Y^h - \hat{P}_Y^f) + (\Omega_h - \Omega_f). \qquad (19)$$

The price component is a standard terms-of-trade effect, determined by differences in the growth rates of final goods' prices. The term $(\Omega_h - \Omega_f)$ reflects gaps in physical productivity growth – henceforth, *structural gaps*. The following Proposition establishes that terms-of-trade effects exactly compensate for structural gaps in each point in time, implying the equalization of equilibrium interest rates:

Proposition 1 *In the world competitive equilibrium, interest rates are equalized, $r_h(t) = r_f(t)$ in each $t \in [0, \infty)$, and terms of trade compensate for structural gaps:*

$$\hat{P}_Y^h - \hat{P}_Y^f = \Omega_f - \Omega_h. \qquad (20)$$

Equal interest rates imply that consumption grows at the same rate in the two countries. We now show that this, in turn, implies balanced growth at the world level in each instant and therefore constant income shares. The key variable determining the equilibrium distribution is the ratio between resource-use flows, which we denote as $\theta(t) \equiv R_h(t) / R_f(t)$, and henceforth call *relative resource use*. Given this definition, the world competitive equilibrium is characterized as follows.

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Proposition 2 *The world competitive equilibrium exhibits a constant relative resource use $\theta(t) \equiv R_h(t)/R_f(t) = \bar{\theta}$ in each $t \in [0, \infty)$. Output, resource use, expenditures and the mass of varieties grow at rates*

$$\hat{Y}_h = \Omega_h - \rho \quad and \quad \hat{Y}_f = \Omega_f - \rho, \tag{21}$$

$$\hat{R}_h = \hat{R}_f = -\rho, \tag{22}$$

$$\hat{E}_h = \hat{E}_f = r_i - \rho \ with \ r_h = r_f, \tag{23}$$

$$\hat{M}_i = \varphi_i \alpha (1 - \alpha)(1 + a_i)(1 + b_i)^{-1} - \rho. \tag{24}$$

The final output ratio $(P_Y^h Y_h)/(P_Y^f Y_f)$ and the income ratio E_h/E_f are constant.

Result (21) shows that differences between the growth rates of physical final output in the two countries are due to structural gaps between the respective productivity indices ($\Omega_h \neq \Omega_f$), which reflect possible differences in R&D productivity ($\varphi_h \neq \varphi_f$), in taxes on intermediates' purchases ($b_h \neq b_f$), or in labor efficiency growth ($\eta_h \neq \eta_f$). Income shares are constant by virtue of the terms-of-trade mechanism already emphasized in Proposition 1: price dynamics compensate for structural gaps, yielding balanced growth at the world level. Conceptually, this mechanism is similar to that emphasized by Acemoglu and Ventura (2002) in a different model where countries produce a homogeneous good but exhibit heterogeneous technologies.[8]

In the world equilibrium, the determinants of relative resource use and market shares in final output are crucially determined by the willingness to invest of the two economies. One possible definition of willingness to invest is (see Appendix)

$$I_i \equiv \frac{\varphi_i \alpha (1 - \alpha)(1 + a_i) - \rho(1 + b_i)}{\varphi_i (1 + b_i)} + \frac{\alpha^2}{1 + b_i}, \tag{25}$$

where the right hand side equals the fraction of final output invested in R&D activity (first term) plus the fraction of output spent in producing intermediates (second term) in country i. Given (25), the equilibrium level of relative resource use equals (see Appendix)

$$\bar{\theta} = \frac{1 + \tau_f}{1 + \tau_h} \cdot \frac{\epsilon}{1 - \epsilon} \cdot \frac{1 - I_f}{1 - I_h}. \tag{26}$$

Equation (26) shows that, in each country, relative resource use increases with the domestic investment rate and declines with the domestic resource tax. However, resource taxes do not affect the respective market shares in final goods: the output ratio equals (see Appendix)

$$\frac{P_Y^h Y_h}{P_Y^f Y_f} = \frac{\epsilon}{1 - \epsilon} \cdot \frac{1 - I_f}{1 - I_h}, \tag{27}$$

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[8] Considering a model in which countries exploit linear technologies with different productivity paramaters, Acemoglu and Ventura (2002) show that the world income distribution is stable over time because terms of trade effects in the international market for intermediates compensate for structural gaps. In our model, national income shares are constant because structural gaps are offset by the dynamics of final goods' prices.

which does not depend on τ_h nor on τ_f. The reason for this result is that if a country raises the domestic resource tax, resource use and physical output fall but the relative price of the produced final good increases. The net effect on market shares is zero because terms of trade exactly compensate for physical quantities:

Proposition 3 *An increase in the Home (Foreign) resource tax reduces Home (Foreign) resource use and physical output, increases the relative price of the Home (Foreign) good,*

$$\frac{d\bar{\theta}}{d\tau_h} < 0, \quad \frac{d(Y_h/Y_f)}{d\tau_h} < 0, \quad \frac{d(P_Y^h/P_Y^f)}{d\tau_h} > 0,$$

$$\frac{d\bar{\theta}}{d\tau_f} > 0, \quad \frac{d(Y_h/Y_f)}{d\tau_f} > 0, \quad \frac{d(P_Y^h/P_Y^f)}{d\tau_f} < 0,$$

but leaves final output shares unchanged:

$$\frac{d(P_Y^h Y_h)/(P_Y^f Y_f)}{\partial \tau_h} = \frac{d(P_Y^h Y_h)/(P_Y^f Y_f)}{\partial \tau_f} = 0.$$

Proposition 3 establishes the neutrality of resource taxes with respect to final output shares. However, resource taxation is not neutral with respect to relative income and welfare levels. Indeed, the fundamental difference between our framework and standard two-country models – e.g., Grossman and Helpman (1991) – is the asymmetric structure of trade. This implies that income shares are affected by the degree of dependence of the Home economy on the exhaustible resources supplied by Foreign, and that Home and Foreign taxes on resource use have asymmetric effects on relative income and welfare levels. The following sections address each point in turn.

4   Income Shares: Theory and Evidence

The asymmetric structure of international trade implies that income shares differ the market shares in world final output and depend on the degree of Home dependence on imported resources. Specifically, the income share of the Home economy equals (see Appendix)

$$s_h \equiv \frac{E_h}{E_h + E_f} = \underbrace{\frac{(P_Y^h Y_h)/(P_Y^f Y_f)}{1 + (P_Y^h Y_h)/(P_Y^f Y_f)}}_{\text{Final output share}} \cdot \underbrace{(1 - \tilde{\gamma}_h)}_{\text{Net of rents to Foreign}}, \tag{28}$$

where $\tilde{\gamma}_h \equiv \gamma (1 + \tau_h)^{-1}$ is the tax-adjusted resource elasticity in final production in Home. The income share of Foreign residents is obviously symmetric and equals $s_f \equiv 1 - s_h$. Expression (28) shows that Home's income share is the product of two factors. The first is Home's final output share. The second represents the effect of dependence on resource imports: Home producers must use a fraction $\tilde{\gamma}_h$ of revenues from final-good sales to finance resource imports.[9] Hence, the income share equals the final output

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[9]From (2), resource rents paid by Home producers to Foreign resource owners equal $qR_h = \tilde{\gamma}_h P_Y^h Y_h$.

share net of the rents paid to Foreign resource owners. Starting from (28), we show that Home resource taxes increase Home's relative income, and that the model predictions regarding the determinants of income shares find empirical support in OIM countries.

4.1 Income Shares and Resource Taxes

A peculiar feature of our model is that Home and Foreign resource taxes have asymmetric effects on relative income levels:

Proposition 4 *An increase in the Home resource tax increases Home's income share relative to Foreign. An increase in the Foreign resource tax leaves income shares unchanged:*

$$\frac{ds_h}{d\tau_h} > 0, \qquad \frac{ds_h}{d\tau_f} = 0.$$

The intuition behind Proposition 4 is as follows. If Home increases the domestic resource tax, τ_h, domestic final output declines in physical terms but its relative price increases and the net effect on final output shares is zero (cf. Proposition 3). However, the increase in the resource tax also implies that a lower fraction of domestic output is spent on resource rents. Hence, an increase in τ_h increases Home's relative income through the reduction of $\tilde{\gamma}_h$ – i.e., the term respresenting Home's dependence on resource imports in (28). Due to the asymmetric structure of trade, changes in the Foreign resorce tax have different consequences. On the one hand, an increase in τ_f leaves final output shares unchanged (cf. Proposition 3). On the other hand, τ_f does not influence Home's resource dependence, $\tilde{\gamma}_h$, so that variations in the Foreign resource tax do not affect the income shares of the two countries. For future reference, we will label the positive income effect of the Home resource tax as a *rent-extraction* effect: higher resource taxes in Home restrict domestic resource use and thereby the value of the rents paid to Foreign owners.

4.2 Determinants of Income Shares: An Empirical Test

Our results on the determinants of income shares can be summarized as follows. Substituting (27) in (28), we can rewrite Home's income share as a function of the domestic resource tax and of investment rates:

$$s_h \equiv \Psi\left(I_h, I_f, \tau_h\right) \text{ with } \Psi_{I_h} > 0, \ \Psi_{I_f} < 0, \ \Psi_{\tau_h} > 0. \tag{29}$$

That is, the income share of the resource-poor economy is (i) positively related to the domestic investment rate, (ii) negatively related to the investment rate of the resource-rich economy, and (iii) positively related to the national tax on domestic resource use. We now test this prediction empirically, using a dynamic panel-estimation technique.

The time period is 1980-2008, and the countries for which we have data are sixteen OIM countries – namely Belgium, France, Germany, Greece, Italy, Japan, South Korea, Netherlands, Philippines, Poland, Portugal, Singapore, Spain, Sweden, Switzerland,

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Turkey – and the ten OEX countries – i.e., Algeria, Canada, Iran, Kuwait, Mexico, Norway, Oman, Saudi Arabia, United Arab Emirates, and Venezuela. This is the country sample which best represents the framework of our theoretical model and for which the relevant data are nearly completely available, except for taxes in the Philippines and Singapore.

In order to focus on long-run effects and to avoid the impact of business cycles, we build five-year averages; the considered periods are: 1980-84, 1985-89, 1990-94, 1995-99, 2000-04, and 2005-08. To capture the dynamic development we include lags of the dependent variable. By construction, the emerging unobserved panel-level effects are then correlated with the lagged dependent variables, making standard estimators inconsistent. That is why the Arellano-Bond dynamic panel-data estimation is used; it provides a consistent generalized method-of-moments (GMM) estimator for the parameters of this model.

We use online data from the World Bank (2009) for the macroeconomic variables and from the International Energy Agency (IEA, 2009) for resource taxes. Incomes shares are calculated for each oil-importing country as the ratio between its GNI level and the sum of the GNIs of all oil-exporting countries, which we label by *shareoim*. For the investment rates, we take gross capital formation as a percentage of GDP for both oil-importing and -exporting countries. In the case of oil importers, the variable is denoted by *investoim*; for oil exporters we calculate the average investment rate – with population size used as the weighting factor – to get the parameter *investoex*. Resource taxes are measured by taxes on light fuel oil and labelled with *oiltax*. Further control variables are education expenditures as a percentage of GDP (*eduexp*, the investment rate for human capital), research expenditures as a percentage of GDP (*rdexp*, the investment rate for knowledge capital), population size *pop*, and central government debt as a percentage of GDP *cgovdebt*.

The results are presented in Table 3, which includes six representative equations [1]-[6]. In all equations we include the (first) lag of the endogenous variable which is significant at the 1%-level in all specifications; this confirms that the estimation method is appropriate. In [1] we start by testing the impact of the investment shares in both types of countries. As can be seen from the results, the theoretical model is confirmed by the estimations as domestic investment affects the oil-importers' income share positively while the opposite holds true for the impact of foreign investment rates. The next equation [2] exhibits that also the domestic investment rate in human capital *eduexp* is positive for the income share, which also holds for all the other specifications.

In [3], oil taxes are included. It appears as very favorable for the theoretical model that taxation has the predicted positive sign; the significance is 5% or 10% according to the specification. Thus according to the empirical results, oil-importing countries can indeed increase their share of total income by raising domestic oil taxes, which is a remarkable finding.

Population size *pop*, i.e. the scale of the economy, has no significant effect in any specification, mainly because the endgenous lagged variable already captures this effect. Similarly, research expenditures *rdexp* as well as central government debt *cgovdebt* have no significant impact and do not change our major findings. In view of these results, the

model predictions regarding the determination of income shares appear to be consistent with the available empirical evidence on oil-importing and oil-exporting countries.

Table 3: Estimation results for income shares of oil-importing countries
Arellano-Bond dynamic panel-data estimation
Endogenous variable: shareoim

	(1)	(2)	(3)	(4)	(5)	(6)
shareoim						
shareoim (-1)	0.559***	0.551***	0.529***	0.595***	0.604***	0.601***
	(0.0940)	(0.0953)	(0.0990)	(0.104)	(0.106)	(0.103)
investoim	0.482***	0.500***	0.554***	0.524***	0.516***	0.543***
	(0.108)	(0.109)	(0.141)	(0.155)	(0.158)	(0.158)
investoex	-0.212***	-0.235***	-0.316***	-0.347***	-0.277***	-0.333***
	(0.0447)	(0.0460)	(0.0730)	(0.0807)	(0.0820)	(0.0819)
eduexp		1.427***	1.576***	1.747***	1.747***	1.738***
		(0.465)	(0.492)	(0.508)	(0.506)	(0.511)
oiltax			0.00568*	0.00784**	0.00734*	0.00766**
			(0.00335)	(0.00386)	(0.00384)	(0.00387)
pop				5.02e-08	1.74e-08	5.15e-08
				(1.98e-07)	(1.96e-07)	(2.00e-07)
rdexp					-0.953	
					(1.205)	
cgovdebt						0.0171
						(0.0350)
Constant	6.004	0.519	0.790	-2.733	-1.935	-4.580
	(3.729)	(4.160)	(5.737)	(9.310)	(9.328)	(9.932)
Observations	64	64	56	56	56	56
Nr. of countries	16	16	14	14	14	14
Wald χ^2	85.22	92.28	96.23	110.09	112.70	109.18

*** p<0.01, ** p<0.05, * p<0.1
Standard errors in parentheses

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5 Efficiency and Policy

Given the general characteristics of the world competitive equilibrium, the effects of public policy can be studied with reference to two benchmark regimes. In this section, we briefly describe the characteristics of laissez-faire equilibria – in which all taxes and subsidies are set to zero – and efficient allocations – i.e., allocations in which all the domestic market failures, induced by R&D externalities and monopolistic competition, are neutralized by fiscal authorities through fiscal instruments. This analysis provides the basis for studying the welfare consequences of discretionary variations in national resource taxes.

5.1 Laissez-Faire Equilibria

Suppose that taxes and subsidies are set to zero in each country: $\tau_i = b_i = a_i = 0$. From (25) and (26), the level of relative resource use under laissez-faire equals

$$\bar{\theta}_{(\text{Laissez-faire})} = \frac{\epsilon}{1-\epsilon} \cdot \frac{1-\alpha+(\rho/\varphi_f)}{1-\alpha+(\rho/\varphi_h)}. \tag{30}$$

From (30), relative resource use is positively (negatively) related to R&D productivity in Home (Foreign) through the terms φ_h and φ_f. This result is peculiar to the laissez-faire equilibrium, which is however inefficient by construction: in each country, monopolistic competition in the intermediate sector and knowledge spillovers in the R&D sector imply that the economy misallocates domestic final output between R&D investment, consumption and production of intermediates. The following subsection describes the set of taxes and subsidies through which fiscal authorities can restore efficiency at the country level.

5.2 Conditional Efficiency

Suppose that a government internalizes all the domestic market failures generated by monopoly pricing and R&D spillovers. The resulting allocation is called conditionally efficient, according to the following

Definition 5 *An allocation is conditionally efficient for country i if domestic output is allocated so as to maximize present-value utility U_i subject to the technology, income, and resource constraints faced by country i at given international prices.*

At the formal level, the conditionally efficient allocation (CE-allocation, hereafter) is similar to the welfare-maximizing allocation that characterizes social optimality in closed-economy models. However, in the present context, conditional efficiency and optimality are different concepts. In closed economies, the welfare-maximizing allocation is chosen by a social planner endowed with full control over all the elements of the allocation. The CE-allocation in country i, instead, implies the maximization of domestic utility at given international prices. The crucial point is that international prices are influenced by the fiscal policies of both countries. There is no general presumption that

a government actually wishes to implement the CE-allocation: this depends on the assumed information set of the policymaker. If a government actually takes international prices as given, achieving the CE-allocation is an overriding political target. If, instead, the government could infer all the general-equilibrium effects generated by domestic fiscal instruments, it may be desirable to deviate from conditional efficiency because non-efficient policies may increase domestic welfare to the detriment of the other country's welfare. Indeed, we will later show that well-informed governments have concrete incentives to deviate from conditional efficiency.

We characterize CE-allocations by denoting the relevant variables by tildas. In Home, the CE-allocation is represented by the paths of imported resource flows and expenditures (in consumption, intermediates' production and R&D activity), that maximize Home's indirect utility subject to the final-good technology, the intermediate-good technology, the R&D technology, and Home's expenditure constraint:

$$\left\{ \tilde{R}_h, \tilde{E}_h^c, \tilde{E}_h^x, \tilde{E}_h^d \right\}_{t=0}^{\infty} = \arg\max U_h \ \text{s.t.} \ (1), (7), (16)$$

where U_h in (13) is maximized taking prices as given and the R&D externality is fully taken into account through constraint (7). In Foreign, the CE-allocation is represented by the paths of domestic resource use, exported resources, and expenditures that maximize Foreign utility subject to the technology constraints, the aggregate expenditure constraint, and the exhaustible resource constraint:

$$\left\{ \tilde{R}_h, \tilde{R}_f, \tilde{E}_f^c, \tilde{E}_f^x, \tilde{E}_f^d \right\}_{t=0}^{\infty} = \arg\max U_f \ \text{s.t.} \ (1), (7), (16) \text{ and } \dot{Q} = -R_h - R_f.$$

Solving these maximization problems, we obtain two results. First, if a government wishes to decentralize the CE-allocation, it must implement an efficient policy that consists of the following subsidies and taxes

$$\tilde{a}_i = (\varphi_i/\rho) - (1-\alpha)^{-1} > 0, \tag{31}$$

$$\tilde{b}_i = \alpha (1-\alpha) (\varphi_i/\rho) - 1 > 0, \tag{32}$$

$$\tilde{\tau}_i = (1-\alpha) (\varphi_i/\rho) - 1 > 0. \tag{33}$$

The role of subsidies to R&D investment is clear: research activity generates positive externalities and must therefore be encouraged by public authorities through $\tilde{a}_i > 0$. This policy must be accompanied by positive taxes on resource use and intermediates' purchases because private agents exhibit inefficiently low saving rates and, hence, excessive demand for the inputs employed in final production. The second result is: if both economies display conditional efficiency, relative resource use equals[10]

$$\bar{\theta}_{\text{(Conditional Efficiency)}} = \frac{\epsilon}{1-\epsilon}. \tag{34}$$

Expression (34) shows that the efficient level of resource use is exclusively determined by preference parameters, with no role played by technology. As a consequence, it

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[10]It is easily verified from (25) and (26) that if the governments of both countries implement the efficient policy $(\tilde{a}_i, \tilde{b}_i, \tilde{\tau}_i)$, the equilibrium relative resource use $\bar{\theta}$ coincides with the efficient level in (34).

differs from (coincides with) the laissez-faire level (30) when the technological parameters governing R&D productivity, φ_h and φ_h, are different (equal).[11] We now exploit these and the previous results in order to address the issue of strategic taxation.

6   Welfare

Do the governments of Home and Foreign have particular incentives to implement inefficient taxes? This section shows that if fiscal authorities recognize all the general-equilibrium effects of national taxes – and, hence, calculate the final effect on domestic welfare levels – they have concrete incentives to deviate from efficient allocations. Further incentives arise in laissez-faire equilibria, especially when the two countries exhibit different rates of R&D productivity.

6.1   National Welfare and Resource Taxes

The reaction of utility levels to variations in domestic resource taxes is represented by two welfare-tax relationships that can derived in explicit form. Present-value utilities in the two countries equal (see Appendix)

$$U_h = \varkappa_h + \frac{1}{\rho} \ln \left[p_0^{1-\epsilon} \cdot Y_h \left(0 \right) \cdot \bar{\sigma}_h^c \right], \tag{35}$$

$$U_f = \varkappa_f + \frac{1}{\rho} \ln \left[p_0^{-\epsilon} \cdot Y_f \left(0 \right) \cdot \bar{\sigma}_f^c \right], \tag{36}$$

where $\varkappa_i$ is a constant factor independent of resource taxes, $p_0 \equiv P_Y^h \left(0 \right) / P_Y^f \left(0 \right)$ is the initial relative price of the Home final good, and the constants $\bar{\sigma}_i^c \equiv E_i^c / (P_Y^h Y_i)$ equal the ratios between consumption expenditures and final output in the two countries.

The three variables appearing in the square brackets in (35)-(36) generally depend on resource taxes. Hence, the marginal effect of an increase in the domestic resource tax on domestic welfare, $dU_i/d\tau_i$, can be split in three components: (i) the terms-of-trade effect, (ii) the physical output effect, and (iii) the consumption-share effect. The direction of the first two effects is known from Proposition 3: an increase in the Home (Foreign) resource tax increases the relative price of the Home (Foreign) good and reduces Home (Foreign) physical output. The direction of the consumption-share effect – that is, the sign of $d \ln \bar{\sigma}_i^c / d\tau_i$ – is asymmetric instead. In Home, an increase in the domestic resource tax increases the ratio between consumption and final output:

$$\rho \cdot \frac{dU_h}{d\tau_h} = \underbrace{(1 - \epsilon) \frac{d \ln p_0}{d\tau_h}}_{\text{Terms of Trade } (+)} + \underbrace{\frac{d \ln Y_h \left(0 \right)}{d\tau_h}}_{\text{Physical Output } (-)} + \underbrace{\frac{d \ln \bar{\sigma}_h^c}{d\tau_h}}_{\text{Consumption Share } (+)} . \tag{37}$$

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[11] Obviously, this does not mean that laissez-faire equilibria are efficient under homogeneous technologies: when $\varphi_h = \varphi_h$ and all taxes and subsidies are zero, the level of $\bar{\theta}$ coincides with the efficient one but the competitive equilibrium is still inefficient since domestic output is misallocated among its competing uses – i.e., consumption, intermediates' production and R&D investment – by virtue of the existing market failures.

In Foreign, an increase in the domestic resource tax leaves the consumption-output ratio unchanged. Given $d\ln\bar{\sigma}_f^c/d\tau_f = 0$, the marginal welfare effect of the Foreign tax only depends on the relative strength of the variations in terms of trade and physical output,

$$\rho \cdot \frac{dU_f}{d\tau_f} = \epsilon \underbrace{\frac{d\ln p_0^{-1}}{d\tau_f}}_{(+)} + \underbrace{\frac{d\ln Y_f(0)}{d\tau_f}}_{(-)}. \tag{38}$$

The asymmetric effects of Home and Foreign taxes on the respective consumption-output ratios are directly linked to the asymmetric effects on relative incomes emphasized in Proposition 4. In Home, the rent-extraction mechanism increases relative income and allows consumers to increase the value of their consumption relative to the value of domestic final output, all other things being equal. In Foreign, the resource tax does not influence world income shares and thus leaves the consumption-output ratio unchanged.

The contrasting effects of resource taxes on terms of trade and physical output imply that, in each country, the welfare-tax relationship $U_i(\tau_i)$ is hump-shaped: there exists a unique level of the domestic resource tax, $\tau_i^{\max}$, that maximizes domestic welfare for a given state of affairs in the other country. In particular, for each country i, the welfare-maximizing tax rate is always associated to a specific level of relative resource use, which we denote by $\theta_i^{\max}$. The following Proposition establishes that the welfare-maximizing taxes of the two countries are necessarily associated with different allocations: the two governments cannot simultaneously implement the respective $\tau_i^{\max}$ because Home would prefer a lower level of resource use.

Proposition 6 *In Foreign, implementing the welfare-maximizing resource tax $\tau_f^{\max}$ always implies*

$$\bar{\theta} = \theta_f^{\max} = \frac{\epsilon}{1-\epsilon}. \tag{39}$$

In Home, implementing the welfare-maximizing resource tax $\tau_h^{\max}$ always implies

$$\bar{\theta} = \theta_h^{\max} < \frac{\epsilon}{1-\epsilon}. \tag{40}$$

Proposition 6 implies that if both national governments fully recognize all the general-equilibrium effects of the respective taxes on resource use, the independent pursuit of maximal domestic welfare results into conflicting objectives: each government seeks a different equilibrium level of relative resource use. This is a very general conclusion since neither (40) nor (39) assume that the two economies are starting from a specific equilibrium. The result originates in the asymmetric effects of national resource taxes on utility levels – see (37)-(38) above. If the consumption-share effect in Home were zero, the welfare-maximizing resource tax in Home would be associated to $\bar{\theta} = \epsilon/(1-\epsilon)$, exactly as in the Foreign economy. Hence, the economic mechanism behind Proposition 6 is the rent-transfer effect, which allows Home residents to increase the ratio between consumption expenditures and domestic final output.

6.2 Deviations from Efficiency and Laissez-Faire

Proposition 6 can be applied to specific contexts in which the initial state of affairs is a given equilibrium. We now analyze the incentives to deviate from two benchmark regimes: symmetric CE-allocations and laissez-faire equilibria. First, suppose that both governments consider as their reference equilibrium a symmetric CE-allocation: the efficient taxes (31)-(32)-(33) are implemented in both countries and relative resource use is given by (34). Under this state of affairs, the following result holds.

Proposition 7 *In an efficient world equilibrium, $dU_h/d\tau_h > 0$ and $dU_f/d\tau_f = 0$.*

Proposition 7 establishes that, given an efficient world equilibrium, Home would gain from increasing the resource tax. Foreign, instead, would not gain from deviating from the initial state of affairs: from (39), the welfare-maximizing Foreign tax is always associated to an equilibrium in which relative resource use coincides with the efficient level.[12] The incentive to increase the tax in Home clearly hinges on the possibility of earning additional income through the rent-extracting mechanism and thereby improve domestic consumption possibilities.

Now consider the laissez-faire case: the reference state of affairs is an inefficient equilibrium where all taxes and subsidies are zero. In this scenario, productivity differences matter for the incentive scheme, which falls in three possible cases:

Proposition 8 *Given a laissez-faire equilibrium, higher (lower) R&D productivity in Home implies an incentive for Foreign to subsidize (tax) domestic resource use:*

$$
\begin{aligned}
&i. &&\text{If } \varphi_h > \varphi_f \text{ then } dU_h/d\tau_h > 0 \text{ and } dU_f/d\tau_f < 0; \\
&ii. &&\text{If } \varphi_h = \varphi_f \text{ then } dU_h/d\tau_h > 0 \text{ and } dU_f/d\tau_f = 0; \\
&iii. &&\text{If } \varphi_h < \varphi_f \text{ then } dU_h/d\tau_h \gtreqless 0 \text{ and } dU_f/d\tau_f > 0;
\end{aligned}
$$

When R&D productivity is higher in Home, relative resource use is strictly greater than the efficient level: from (30), having $\varphi_h > \varphi_f$ implies $\bar{\theta} > \epsilon/(1-\epsilon)$. In this situation, both countries have incentives to deviate – in particular, Foreign would gain from subsidizing domestic resource use. With respect to this result, we emphasize two points. First, hypothesis and conclusions are empirically plausible. On the one hand, oil-poor countries exhibit on average faster productivity growth than oil-rich countries. On the other hand, there is evidence of positive taxes on imported primary resources in OIM economies as well as of direct or indirect subsidies to domestic oil consumption in OEX economies (Gupta et al. 2002; Metschies, 2005).

The two other cases reported in Proposition 8 are easily interpreted. If R&D technologies are identical in the two countries, relative resource use coincides with the efficient level and this implies, similarly to Proposition 7, an incentive to raise a tax in

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[12]Note that this does not mean that $\tau_f^{\max}$ is always associated to an efficient equilibrium: relative resource use may be equal to $\epsilon/(1-\epsilon)$ also in inefficient equilibria. For example, when $\varphi_h = \varphi_f$, laissez-faire conditions would imply $\bar{\theta} = \epsilon/(1-\epsilon)$ – see equation (30) above – but this equilibrium is inefficient due to the market failures induced by R&D externalities and monopolistic competition in the two economies.

Home and no incentive to deviate from laissez-faire in Foreign. Finally, if R&D productivity is higher in Foreign, relative resource use falls short of the efficient level: Foreign would gain from raising a resource tax whereas Home would gain by implementing either a resource tax or a subsidy, depending on the width of the productivity gap.

6.3 Remarks

Our results concerning the welfare effects of taxation suggest that, if domestic welfare represents the payoff of each government in a political game, inefficient equilibria may well be the outcome. This is an issue that we leave for further research. More generally, our interest in welfare-tax relationships stems from the fact that oil taxes are extensively used in oil-importing economies and are often interpreted as stategic policies. The theoretical literature made a first recognition of the topic after the oil shocks of 1970s, and Bergstrom (1982) suggested that the rent-extraction effect of oil taxes is virtually unbounded. This is not the case in our model, where the welfare-maximizing tax is finite. The reason is that Bergstrom (1982) only considered taxes on imported oil in a partial equilibrium model with no trade in final goods – and hence without the terms-of-trade effects that play a crucial role in our analysis. A related paper by Brander and Djajic (1983) shows that oil importers tend to impose strategic taxes in response to monopolistic behavior of oil producers. In this respect, our results differ in several respects. In particular, we have shown that oil-poor economies have an incentive to raise the domestic tax starting from both laissez-faire and efficient allocations – not just in response to the monopolistic behavior of oil exporters, which we have ruled out – because the rent-extraction mechanism guarantees enhanced consumption possibilities. Another original result of our paper is that Foreign has an independent incentive to subsidize domestic oil consumption when there is a structural productivity gap in favor of Home. This result is specific to our model and, to our knowledge, is novel to the literature: the rationale for subsidizing domestic resource use in oil-exporting economies may be exclusively due to low productivity growth in final sectors relative to that observed in other countries.

7 Conclusion

Since 1980, the aggregate income of oil-exporting countries relative to that of oil-poor countries has been remarkably constant despite structural gaps in labor productivity growth rates. We rationalized this behavior in a two-country model of asymmetric trade where growth rates are endogenously determined by R&D activity, and productivity differences between countries are compensated by terms-of-trade dynamics. The model predictions regarding the basic determinants of income shares are supported by empirical evidence: the share of each oil-poor economy is positively (negatively) related to the domestic (average foreign) investment rate and to the national tax on domestic resource use. Our results regarding the marginal effects of taxation are also relevant for the current policy debate. Oil taxes are extensively used in oil-importing economies and are often interpreted as stategic policies. In this respect, the model predicts that oil-poor

economies have an incentive to raise the domestic tax starting from both laissez-faire and efficient allocations because the rent-extraction mechanism guarantees enhanced consumption possibilities. Another empirically plausible result of the model is that oil-exporting economies have a peculiar incentive to subsidize domestic resource use when they exhibit structural gaps in productivity growth rates with respect to oil-importing economies. More generally, the asymmetric welfare effects of national resource taxes suggest that inefficient equilibria may be the outcome of rational strategies pursued by governments involved in political games – an interesting issue that deserves further research.

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A   Appendix

Monopoly rents. Maximizing $\Pi_i(m_i) = \left(P^i_{X(m_i)} - \varsigma P^i_Y\right) \cdot X_i(m_i)$ s.t. (3), we have

$$X_i(m_i) = X_i = \left\{\alpha^2 (v_i L_i)^\beta R_i^\gamma \left[\varsigma(1+b_i)\right]^{-1}\right\}^{\frac{1}{1-\alpha}}, \tag{A.1}$$

$$\Pi_i(m_i) = \Pi_i = (1-\alpha) P^i_X X_i. \tag{A.2}$$

Plugging (A.1) in (3), we have (4). Plugging (A.1) in (1) we also have

$$Y_i = \left(\alpha^2/\varsigma\right)^{\frac{\alpha}{1-\alpha}} \cdot [1+b_i]^{-1} \cdot M_i (v_i L_i)^{\frac{\beta}{1-\alpha}} (R_i)^{\frac{\gamma}{1-\alpha}}. \tag{A.3}$$

Derivation of (9)-(10). The resource extraction problem is to maximize (8) s.t. $\dot{Q}(t) = -R(t)$. The current-value Hamiltonian is $P_R(t) R(t) - \chi(t) R(t)$, where $\chi(t)$ is the multiplier, and optimality conditions read

$$P_R(t) = \chi(t), \tag{A.4}$$

$$\dot{\chi}(t) = r_f(t) \chi(t), \tag{A.5}$$

$$\lim_{t \to \infty} \chi(t) Q(t) e^{-\int_t^\infty r_f(v)dv} = 0, \tag{A.6}$$

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Plugging (A.4) in (A.5), we have (9). Integrating (A.5) and substituting the resulting expression in (A.6), we have $\lim_{t\to\infty} \chi(0) Q(t) = 0$, which implies $\lim_{t\to\infty} Q(t) = 0$. Integrating $\dot{Q}(t) = -R(t)$ between time zero and infinity, and imposing $\lim_{t\to\infty} Q(t) = 0$, we obtain (10).

Zero-profits in the R&D sector. Denoting by V_i the value of each patent, the zero-profit condition is[13]

$$V_i = P_Y^i / [\phi_i (1 + a_i)]. \tag{A.7}$$

The value of each patent satisfies the usual no-arbitrage condition

$$r_i(t) V_i(t) = \Pi_i(t) + \dot{V}_i(t), \tag{A.8}$$

where $r_i(t)$ is the interest rate in country i.

Static consumer problem. Maximizing (11) s.t. (12) for each country $i = f, h$, we obtain

$$c_i^f / c_i^h = \frac{1 - \epsilon}{\epsilon} (P_Y^h / P_Y^f), \tag{A.9}$$

$$P_Y^h c_i^h = \epsilon \cdot E_i^c / L_i \text{ and } P_Y^f c_i^f = (1 - \epsilon) \cdot E_i^c / L_i, \tag{A.10}$$

$$\bar{u}_i = \ln \left\{ \left[\frac{\epsilon}{(P_Y^h)^\epsilon (P_Y^f)^{1-\epsilon}} \left(\frac{1 - \epsilon}{\epsilon} \right)^{1-\epsilon} \right] \cdot E_i^c / L_i \right\}, \tag{A.11}$$

where (A.9) is the first order condition in each country, (A.10) results from plugging (A.9) in (12), and (A.11) is the indirect utility function obtained from substituting (A.10) in the utility index (11). Denoting the term in square brackets in (A.11) as $\omega = \omega(P_Y^h, P_Y^f)$, we can write $\bar{u}_i = \ln[\omega \cdot (E_i^c / L_i)]$.

Dynamic consumer problem: derivation of (14). Individual wealth is a fraction $(1/L_i)$ of the value of the assets in the economy, $V_i M_i$. Defining $n_i \equiv (V_i M_i) / L_i$, the dynamic constraints respectively read

$$\dot{n}_h = r_h n_h + P_L^h - (E_h^c / L_h) - F_h, \tag{A.12}$$

$$\dot{n}_f = r_f n_f + P_L^f - (E_f^c / L_f) - F_f + P_R (R / L_f), \tag{A.13}$$

where $r_i n_i + P_L^i$ is income from assets and labor in country i and $P_R (R / L_f)$ is resource income for each Foreign resident. Agents in country i maximize (13) subject to the relevant constraint ((A.12) or (A.13)), using E_i^c / L_i as control variable. Denoting by λ_i the current-value dynamic multiplier, the optimality conditions imply $L_i / E_i = \lambda_i$ and $\dot{\lambda}_i = \lambda_i (\rho - r_i)$, from which the Keynes-Ramsey rule (14).

Derivation of (18). From (A.2) and (A.7), we have

$$\frac{\Pi_i}{V_i} = \phi_i \frac{(1 + a_i)(1 - \alpha) P_X^i X_i}{P_Y^i} = \varphi_i \cdot \frac{(1 + a_i)(1 - \alpha)\alpha}{1 + b_i}, \tag{A.14}$$

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[13] Aggregate profits of the R&D sector equal $V_i \dot{M}_i - P_Y^i Z_i = V_i \phi_i Z_i (1 + a_i) - P_Y^i Z_i$, so that condition (A.7) maximizes R&D profits for a given marginal productivity ϕ_i. Condition (A.7) can be equivalently obtained assuming free entry in the R&D business (see Barro and Sala-i-Martin (2004)).

where the last term follows from substituting ϕ_i by (6) and $\left(P_X^i M_i X_i\right) / \left(P_Y^i Y_i\right) = \alpha / \left(1 + b_i\right)$ by (3). Equations (A.7) and (6) also imply $V_i = \left(P_Y^i Y_i\right) / \left[\varphi_i \cdot M_i \left(1 + a_i\right)\right]$ so that

$$\hat{V}_i\left(t\right) = \hat{P}_Y^i + \hat{Y}_i - \hat{M}_i. \tag{A.15}$$

Substituting (A.14) and (A.15) in (A.8), we get

$$r_i = \varphi_i \alpha \left(1 - \alpha\right) \left(1 + a_i\right) \left(1 + b_i\right)^{-1} + \hat{P}_Y^i + \hat{Y}_i - \hat{M}_i. \tag{A.16}$$

Time-differentiating (A.3) we obtain

$$\hat{Y}_i = \hat{M}_i + \frac{\beta}{1 - \alpha} \eta_i + \frac{\gamma}{1 - \alpha} \hat{R}_i. \tag{A.17}$$

Plugging (A.17) in (A.16) we obtain equation (18) in the text.

Proof of Proposition 1. Define the expenditure-to-output ratios in country i as

$$\bar{\sigma}_i^c \equiv E_i^c / \left(P_Y^i Y_i\right), \quad \bar{\sigma}_i^d \equiv E_i^d / \left(P_Y^i Y_i\right), \quad \bar{\sigma}_i^x \equiv E_i^x / \left(P_Y^i Y_i\right). \tag{A.18}$$

From (4) we have $\bar{\sigma}_i^x = \left(P_Y^i \varsigma M_i X_i\right) / \left(P_Y^i Y_i\right) = \alpha \left(P_X^i M_i X_i\right) / \left(P_Y^i Y_i\right)$, where we can substitute $\left(P_X^i M_i X_i\right) / \left(P_Y^i Y_i\right) = \alpha / \left(1 + b_i\right)$ from (3) to obtain

$$\bar{\sigma}_i^x = E_i^x / (P_Y^i Y_i) = \alpha^2 \left(1 + b_i\right)^{-1} \text{ for any } i = h, f. \tag{A.19}$$

Now consider Home. Using (A.18), and defining $\tilde{\gamma}_h \equiv \gamma \left(1 + \tau_h\right)^{-1}$, we can rewrite (16) as

$$\bar{\sigma}_h^c + \bar{\sigma}_h^d + \bar{\sigma}_h^x = 1 - \left(P_R R_h\right) / \left(P_Y^h Y_h\right) = 1 - \tilde{\gamma}_h, \tag{A.20}$$

where the last term follows from (2). A standard stability analysis shows that $\bar{\sigma}_h^c$ and $\bar{\sigma}_h^d$ are constant and equal to[14]

$$\bar{\sigma}_h^c = \left(1 - \tilde{\gamma}_h\right) - \frac{\varphi_h \left[\alpha \left(1 - \alpha\right) \left(1 + a_h\right) + \alpha^2\right] - \rho \left(1 + b_h\right)}{\varphi_h \left(1 + b_h\right)}, \tag{A.21}$$

$$\bar{\sigma}_h^d = 1 - \tilde{\gamma}_h - \bar{\sigma}_h^c - \bar{\sigma}_h^x = \frac{\varphi_h \alpha \left(1 - \alpha\right) \left(1 + a_h\right) - \rho \left(1 + b_h\right)}{\varphi_h \left(1 + b_h\right)}. \tag{A.22}$$

in each t. Given definition (A.18), constant values of $\left(\bar{\sigma}_h^c, \bar{\sigma}_h^d, \bar{\sigma}_h^x\right)$ imply that $P_Y^h Y_h$ grows at the same rate as all expenditure shares, $\hat{E}_h^c = \hat{E}_h^d = \hat{E}_h^x$. Since (16) and (2) imply that the ratio

$$E_h / P_Y^h Y_h = \left(1 - \tilde{\gamma}_h\right) \tag{A.23}$$

is constant, the balanced growth rate in Home is determined by the Keynes-Ramsey rule (14):

$$\hat{E}_h = \hat{E}_h^c = \hat{P}_Y^h + \hat{Y}_h = r_h - \rho. \tag{A.24}$$

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[14]Equations (A.21)-(A.22) are derived in the section "Further Mathematical Details" below.

Now use (A.10) to eliminate $P_Y^f c_h^f$ and $P_Y^h c_f^h$ from (15), obtaining

$$P_R R_h + (1 - \epsilon) E_h^c = \epsilon E_f^c. \tag{A.25}$$

Substituting $P_R R_h = \tilde{\gamma}_h P_Y^h Y_h$ from (2), and $E_h^c = \bar{\sigma}_h^c P_Y^h Y_h$ from (A.18), we get

$$E_f^c = \frac{1}{\epsilon} \left[\tilde{\gamma}_h + (1 - \epsilon) \bar{\sigma}_h^c \right] \cdot P_Y^h Y_h, \tag{A.26}$$

where the term in square brackets is constant, implying that $E_f^c / \left(P_Y^h Y_h \right)$ is constant. Since $P_Y^h Y_h$ grows at the same rate as E_h^c by (A.24), we have $\hat{E}_f^c = \hat{E}_h^c$. By the Keynes-Ramsey rules (14), this implies equal interest rates in the two countries, $r_h = r_f$. Imposing $r_h = r_f$ in (19) yields (20). ∎

Proof of Proposition 2. Combining the conditions (2) for Home and Foreign, we obtain

$$\theta(t) = \frac{R_h(t)}{R_f(t)} = \frac{\tilde{\gamma}_h}{\tilde{\gamma}_f} \cdot \frac{P_Y^h(t) Y_h(t)}{P_Y^f(t) Y_f(t)} \text{ in each } t \in [0, \infty), \tag{A.27}$$

where $\tilde{\gamma}_i \equiv \gamma (1 + \tau_i)^{-1}$ is the tax-adjusted resource elasticity in final production. Using the definition $R_h = \theta R_f$ and condition (2) for country $i = f$, constraint (17) implies

$$E_f = P_Y^f Y_f + P_R R_h = P_Y^f Y_f + \theta P_R R_f = P_Y^f Y_f (1 + \tilde{\gamma}_f \theta). \tag{A.28}$$

Recalling definitions (A.18), result (A.28) and the central term in (17) imply $\bar{\sigma}_f^c + \bar{\sigma}_f^x + \bar{\sigma}_f^d = 1 + \tilde{\gamma}_f \theta$, where we can substitute $\bar{\sigma}_f^x = \alpha^2 (1 + b_f)^{-1}$ from (A.19) to obtain

$$\bar{\sigma}_f^d = 1 + \tilde{\gamma}_f \theta - \frac{\alpha^2}{1 + b_f} - \bar{\sigma}_f^c. \tag{A.29}$$

Plugging (A.29) in (B.3) for country $i = f$ we obtain

$$\widehat{\bar{\sigma}_f^c} = \varphi_f \frac{\alpha (1 - \alpha)(1 + a_f)}{1 + b_f} - \varphi_f \left[1 + \tilde{\gamma}_f \theta - \frac{\alpha^2}{1 + b_f} - \bar{\sigma}_f^c \right] - \rho. \tag{A.30}$$

Dividing both sides of (A.26) by $P_Y^f Y_f$ and solving for $\bar{\sigma}_f^c \equiv E_f^c / (P_Y^f Y_f)$, we obtain

$$\bar{\sigma}_f^c = \frac{1}{\epsilon} \left[\tilde{\gamma}_h + (1 - \epsilon) \bar{\sigma}_h^c \right] \cdot \frac{P_Y^h Y_h}{P_Y^f Y_f} = \frac{1}{\epsilon} \left[\tilde{\gamma}_h + (1 - \epsilon) \bar{\sigma}_h^c \right] \cdot \frac{\tilde{\gamma}_f}{\tilde{\gamma}_h} \theta, \tag{A.31}$$

where we have used (A.27) to get the last term. Now define

$$\chi \equiv \frac{1}{\epsilon} + \frac{1 - \epsilon}{\epsilon} \cdot \frac{\bar{\sigma}_h^c}{\tilde{\gamma}_h} > 1. \tag{A.32}$$

Since $\bar{\sigma}_h^c$ is constant by (B.5), χ is also constant and (A.31) implies

$$\bar{\sigma}_f^c = \chi \tilde{\gamma}_f \theta \text{ and } \widehat{\bar{\sigma}_f^c} = \hat{\theta}. \tag{A.33}$$

Substituting the second expression in (A.33) into (A.30) we obtain

$$\hat{\theta} = \varphi_f (\chi - 1) \tilde{\gamma}_f \cdot \theta + \varphi_f \left[\alpha (1 - \alpha)(1 + a_f) + \alpha^2 \right] (1 + b_f)^{-1} - (\varphi_f + \rho), \quad \text{(A.34)}$$

where all terms to the right hand side except θ are constant. Since $\varphi_f (\chi - 1) \tilde{\gamma}_f > 0$, equation (A.34) is globally unstable around the unique stationary point: ruling out by standard arguments explosive dynamics – which would be associated to unbounded dynamics in the propensity to consume in Foreign – we thus have $\theta(t) = \bar{\theta}$ in each $t \in [0, \infty)$, where $\bar{\theta}$ is the steady-state level obtained by imposing $\hat{\theta} = 0$ in (A.34):

$$\bar{\theta} \equiv \frac{(\varphi_f + \rho)(1 + \tau_K^f) - \varphi_f \left[\alpha (1 - \alpha)(1 + a_f) + \alpha^2 \right]}{\tilde{\gamma}_f (\chi - 1) \varphi_f (1 + \tau_K^f)}. \quad \text{(A.35)}$$

From (A.27), a constant θ implies

$$\hat{P}_Y^h - \hat{P}_Y^f = \hat{Y}_f - \hat{Y}_h, \quad \text{(A.36)}$$

where we can substitute (20) and (A.24) to obtain (21). Given $P_R R_h = \tilde{\gamma}_h P_Y^h Y_h$, the Hotelling rule (9) and result (A.24) imply that $P_R R_h$ grows at the rate $r_h - \rho$, so that $\hat{R}_h = -\rho$. A constant θ then implies $\hat{R}_f = -\rho$, which proves (22). From (A.28), a constant θ also implies that E_f grows at the same rate as $P_Y^f Y_f$, which coincides with the growth rate of E_h and $P_Y^h Y_h$ by (A.36) and (A.24). We thus have (23). From (A.33), we can substitute $\bar{\sigma}_f^c = \chi \tilde{\gamma}_f \bar{\theta}$ in (A.29) to obtain $\bar{\sigma}_f^d = 1 - \frac{\alpha^2}{1 + b_f} - (\chi - 1)\tilde{\gamma}_f \bar{\theta}$, where we can eliminate $(\chi - 1)\tilde{\gamma}_f \bar{\theta}$ by means of (A.35) to obtain

$$\bar{\sigma}_f^d = \frac{\varphi_f \alpha (1 - \alpha)(1 + a_f) - \rho(1 + b_f)}{\varphi_f (1 + b_f)}. \quad \text{(A.37)}$$

From (7), both countries exhibit $\hat{M}_i = \varphi_i \bar{\sigma}_i^d$, so that results (A.37) and (A.22) imply (24). ∎

Derivation of (25)-(26)-(27). Defining $I_i \equiv \bar{\sigma}_i^x + \bar{\sigma}_i^d$ and substituting $\bar{\sigma}_i^x$ by (A.19) and $\bar{\sigma}_i^d$ by (A.22)-(A.37), we obtain (25). Substituting the definition $E_f^c = \bar{\sigma}_i^c P_Y^f Y_f$ in (A.26), we have

$$\frac{P_Y^h Y_h}{P_Y^f Y_f} = \frac{\epsilon \bar{\sigma}_f^c}{\tilde{\gamma}_h + (1 - \epsilon) \bar{\sigma}_h^c}. \quad \text{(A.38)}$$

Substituting $\bar{\sigma}_f^c = 1 + \tilde{\gamma}_f \bar{\theta} - \bar{\sigma}_f^x - \bar{\sigma}_f^d$ from (A.29), $\bar{\sigma}_h^c = 1 - \tilde{\gamma}_h - \bar{\sigma}_h^x - \bar{\sigma}_h^d$ from (A.20), and $\frac{P_Y^h Y_h}{P_Y^f Y_f} = \frac{\tilde{\gamma}_f}{\tilde{\gamma}_h} \bar{\theta}$ from (A.27), equation (A.38) yields $\frac{\tilde{\gamma}_f}{\tilde{\gamma}_h} \bar{\theta} = \frac{\epsilon(1 - \bar{\sigma}_f^x - \bar{\sigma}_f^d) + \epsilon \tilde{\gamma}_f \bar{\theta}}{(1 - \epsilon)(1 - \bar{\sigma}_h^x - \bar{\sigma}_h^d) + \epsilon \tilde{\gamma}_h}$, which can be solved for $\bar{\theta}$ to get

$$\bar{\theta} = \frac{\tilde{\gamma}_h}{\tilde{\gamma}_f} \cdot \frac{\epsilon(1 - \bar{\sigma}_f^x - \bar{\sigma}_f^d)}{(1 - \epsilon)(1 - \bar{\sigma}_h^x - \bar{\sigma}_h^d)}. \quad \text{(A.39)}$$

Substituting $I_i \equiv \bar{\sigma}_i^x + \bar{\sigma}_i^d$ in (A.39) and recalling that $\frac{\tilde{\gamma}_h}{\tilde{\gamma}_f} = \frac{1 + \tau_f}{1 + \tau_h}$, we get (26). Plugging (26) in (A.27) yields (27).

Proof of Proposition 3. Results $d\bar{\theta}/d\tau_h < 0$ and $d\bar{\theta}/d\tau_f > 0$ directly follow from (26). Similarly, $\frac{d(P_Y^h Y_h)/(P_Y^f Y_f)}{\partial \tau_h} = \frac{d(P_Y^h Y_h)/(P_Y^f Y_f)}{\partial \tau_f} = 0$ directly follows from (27). The other results reported in Proposition 3 hinge on the closed-form solutions[15]

$$Y_h(t) = (\alpha^2/\varsigma)^{\frac{\alpha}{1-\alpha}} (1+b_h)^{-1} \cdot M_h(0) (v_h(0) L_h)^{\frac{\beta}{1-\alpha}} \left[(\rho Q_0 \bar{\theta})/(1+\bar{\theta})\right]^{\frac{\gamma}{1-\alpha}} \cdot e^{(\Omega_h - \rho)t}, \tag{A.40}$$

$$Y_f(t) = (\alpha^2/\varsigma)^{\frac{\alpha}{1-\alpha}} (1+b_f)^{-1} \cdot M_f(0) (v_f(0) L_f)^{\frac{\beta}{1-\alpha}} \left[(\rho Q_0)/(1+\bar{\theta})\right]^{\frac{\gamma}{1-\alpha}} \cdot e^{(\Omega_f - \rho)t}, \tag{A.41}$$

and

$$Y_h(t)/Y_f(t) = \bar{\theta}^{\frac{\gamma}{1-\alpha}} \cdot \psi_0 \cdot e^{(\Omega_h - \Omega_f)t}, \tag{A.42}$$

$$P_Y^h(t)/P_Y^f(t) = \frac{\epsilon}{1-\epsilon} \cdot \frac{1-I_f}{1-I_h} \cdot \psi_0^{-1} \cdot \bar{\theta}^{-\frac{\gamma}{1-\alpha}} \cdot e^{-(\Omega_h - \Omega_f)t}, \tag{A.43}$$

where we have defined $\psi_0 \equiv \left[\frac{M_h(0)}{M_f(0)} \left(\frac{1+b_f}{1+b_h}\right) \left(\frac{v_h(0)L_h}{v_f(0)L_f}\right)^{\frac{\beta}{1-\alpha}}\right]$. The right hand sides of (A.42)-(A.43) depend on tax rates only through the term θ. Hence, from (A.42) and $d\bar{\theta}/d\tau_h < 0$ we have $d(Y_h/Y_f)/d\tau_h < 0$ and $d(P_Y^h/P_Y^f)/d\tau_h > 0$. Similarly, from (A.43) and $d\bar{\theta}/d\tau_f > 0$, we have $d(Y_h/Y_f)/d\tau_f > 0$ and $d(P_Y^h/P_Y^f)/d\tau_f < 0$.

Derivation of (28). Taking the ratio between (16) and (17) and substituting (2) with $i = h$, we obtain (28).

Proof of Proposition 4. Since $(P_Y^h Y_h)/(P_Y^f Y_f)$ is independent of tax rates by Proposition 3, the right hand side of (28) is related to tax rates only through the term $1 - \tilde{\gamma}_h \equiv 1 - \frac{\gamma}{1+\tau_h}$, which implies $ds_h/d\tau_h > 0$ and $ds_h/d\tau_f = 0$.

Derivation of (34). Substituting the efficient tax rates (31)-(32)-(33) in (26) with I_i defined in (25), we obtain (34). The derivation of (31)-(32)-(33) is reported in the section "Further Mathematical Details" below.

Derivation of (35)-(36). Defining the constant $\bar{\epsilon}_i \equiv (\epsilon/L_i)\left(\frac{1-\epsilon}{\epsilon}\right)^{1-\epsilon}$ and recalling that $E_i^c = \bar{\sigma}_i^c P_Y^i Y_i$ by (A.18), present-value utility (13) reads

$$U_i = \int_0^\infty e^{-\rho t} \cdot \ln\left[\bar{\epsilon}_i \frac{\bar{\sigma}_i^c P_Y^i Y_i}{(P_Y^h)^\epsilon (P_Y^f)^{1-\epsilon}}\right] dt. \tag{A.44}$$

Plugging the respective country indices, we obtain

$$U_h = \int_0^\infty e^{-\rho t} \cdot \ln\left[\bar{\epsilon}_h \left(P_Y^h/P_Y^f\right)^{1-\epsilon} \bar{\sigma}_h^c Y_h\right] dt \text{ and } U_f = \int_0^\infty e^{-\rho t} \cdot \ln\left[\bar{\epsilon}_f \left(P_Y^f/P_Y^h\right)^\epsilon \bar{\sigma}_f^c Y_f\right] dt.$$

Substituting $P_Y^h(t)/P_Y^f(t) = [P_Y^h(0)/P_Y^f(0)]e^{(\Omega_f - \Omega_h)t}$ and $Y_i(t) = Y_i(0)e^{(\Omega_i - \rho)t}$ and

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[15] A detailed derivation of (A.40)-(A.43) is reported in the section "Further Mathematical Details" below.

collecting the terms to isolate the initial values, we can define the constants

$$\varkappa_h \equiv \int_0^\infty e^{-\rho t} \cdot \ln\left[e^{[\Omega_h - \rho + (1-\epsilon)(\Omega_f - \Omega_h)]t}\right] dt + \frac{1}{\rho} \ln \bar{\epsilon}_h,$$

$$\varkappa_f \equiv \int_0^\infty e^{-\rho t} \cdot \ln\left[e^{[\Omega_f - \rho + \epsilon(\Omega_h - \Omega_f)]t}\right] dt + \frac{1}{\rho} \ln \bar{\epsilon}_f,$$

and rewrite U_h and U_f as in (35)-(36).

Derivation of results (37)-(38). Expressions (37)-(38) follow from total-differentiating (35)-(36) with respect to τ_h and τ_f, respectively. The signs of the various terms are obtained as follows. Setting $t = 0$ in (A.40), (A.41) and (A.43), we obtain

$$\frac{d\ln Y_h(0)}{d\tau_h} = \frac{\gamma}{1-\alpha} \cdot \frac{d\ln\left[\bar{\theta}/\left(1+\bar{\theta}\right)\right]}{d\tau_h} = \frac{\gamma}{1-\alpha} \cdot \frac{1}{1+\bar{\theta}} \cdot \frac{d\ln\bar{\theta}}{d\tau_h} < 0, \tag{A.45}$$

$$\frac{d\ln Y_f(0)}{d\tau_f} = \frac{\gamma}{1-\alpha} \cdot \frac{d\ln\left[1/\left(1+\bar{\theta}\right)\right]}{d\tau_f} = -\frac{\gamma}{1-\alpha} \cdot \frac{\bar{\theta}}{1+\bar{\theta}} \cdot \frac{d\ln\bar{\theta}}{d\tau_f} < 0, \tag{A.46}$$

$$\frac{d\ln p_0}{d\tau_h} = -\frac{\gamma}{1-\alpha} \cdot \frac{d\ln\bar{\theta}}{d\tau_h} > 0, \tag{A.47}$$

$$\frac{d\ln p_0}{d\tau_f} = -\frac{\gamma}{1-\alpha} \cdot \frac{d\ln\bar{\theta}}{d\tau_f} < 0, \tag{A.48}$$

where $p_0 \equiv P_Y^h(0)/P_Y^f(0)$. The signs in (A.45)-(A.48) come from $d\bar{\theta}/d\tau_h < 0$ and $d\bar{\theta}/d\tau_f > 0$, as established in Proposition 3, and imply the signs of terms of-trade effects and physical-output effects reported in (37)-(38). As regards consumption-share effects, expression (A.21) implies

$$\frac{d\ln\bar{\sigma}_h^c}{d\tau_h} = \frac{1}{1+\tau_h} \cdot \frac{\tilde{\gamma}_h}{\bar{\sigma}_h^c} = \frac{1}{1+\tau_h} \cdot \frac{\tilde{\gamma}_h}{1 - \tilde{\gamma}_h - I_h} > 0, \tag{A.49}$$

where the last term comes from substituting $I_h = \bar{\sigma}_h^x + \bar{\sigma}_h^d$ in (A.21). In Foreign, equation (A.33) implies

$$\frac{d\ln\bar{\sigma}_f^c}{d\tau_f} = \frac{d\ln\left(\chi\tilde{\gamma}_f\bar{\theta}\right)}{d\tau_f} = \frac{d\ln\chi}{d\tau_f} + \frac{d\ln\left(\tilde{\gamma}_f\bar{\theta}\right)}{d\tau_f} = 0, \tag{A.50}$$

where $d\ln\chi/d\tau_f = 0$ is implied by (A.32) and $d\ln\left(\tilde{\gamma}_f\theta\right)/d\tau_f = 0$ follows from expression (26).[16]

Proof of Proposition 6 (Foreign). Substituting (A.46) and (A.48) in (38), and using $d\ln\bar{\theta}/d\tau_f = (1+\tau_f)$ from (26), we have

$$\rho \cdot \frac{dU_f}{d\tau_f} = \frac{\gamma(1+\tau_f)}{1-\alpha} \cdot \left[\epsilon - \frac{\bar{\theta}}{1+\bar{\theta}}\right], \tag{A.51}$$

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[16]In (A.32), all terms to the right hand side are independent of τ_f, which implies $d\chi/d\tau_f = 0$. In (26), we can multiply both sides by $\tilde{\gamma}_f$ and obtain an expression for $\tilde{\gamma}_f\bar{\theta}$ that is independent of τ_f because the terms $(1+\tau_f)^{-1}$ cancel out, which implies $d\left(\tilde{\gamma}_f\bar{\theta}\right)/d\tau_f = 0$.

the sign of which is determined by the term in square brackets. As $\bar{\theta}$ is monotonously increasing in τ_f by (26), the condition $dU_f/d\tau_f = 0$ is univoquely associated to a Foreign tax $\tau_f^{\max}$ associated to a relative resource use $\bar{\theta}_f^{\max} = \epsilon/(1-\epsilon)$. Condition $dU_f/d\tau_f = 0$ is a maximum because (A.51) implies $dU_f/d\tau_f > 0$ when $\bar{\theta} < \epsilon/(1-\epsilon)$ and $dU_f/d\tau_f < 0$ when $\bar{\theta} > \epsilon/(1-\epsilon)$.

Proof of Proposition 6 (Home). Substituting (A.45), (A.47) and (A.50) in (37), we have

$$\rho \cdot \frac{dU_h}{d\tau_h} = -\frac{\gamma(1-\epsilon)}{1-\alpha} \cdot \frac{d\ln\bar{\theta}}{d\tau_h} + \frac{\gamma}{1-\alpha} \cdot \frac{1}{1+\bar{\theta}} \cdot \frac{d\ln\bar{\theta}}{d\tau_h} + \frac{1}{1+\tau_h} \cdot \frac{\tilde{\gamma}_h}{1-\tilde{\gamma}_h - I_h}.$$

From (26), we have $d\ln\bar{\theta}/d\tau_h = -(1+\tau_h)^{-1}$ and the above expression reduces to

$$\rho \cdot \frac{dU_h}{d\tau_h} = \frac{\gamma}{1+\tau_h} \cdot \left\{ \frac{1}{(1+\tau_h)(1-\tilde{\gamma}_h - I_h)} - \frac{1}{1-\alpha} \cdot \left[\frac{1}{1+\bar{\theta}} - (1-\epsilon) \right] \right\}, \quad \text{(A.52)}$$

the sign of which is determined by the term in curly brackets: defining $\Upsilon^a(\tau_h) \equiv 1/[(1+\tau_h)(1-\tilde{\gamma}_h - I_h)]$ and $\Upsilon^b(\tau_h) \equiv \frac{1}{1-\alpha} \cdot \left[\frac{1}{1+\bar{\theta}} - (1-\epsilon) \right]$, we have

$$\rho \cdot \frac{dU_h}{d\tau_h} = \frac{\gamma}{1+\tau_h} \cdot \left[\Upsilon^a(\tau_h) - \Upsilon^b(\tau_h) \right]. \quad \text{(A.53)}$$

where $\Upsilon^a(\tau_h)$ is strictly decreasing in τ_h and satisfies $\lim_{\tau_h \to \infty} \Upsilon^a(\tau_h) = 0$, while $\Upsilon^b(\tau_h)$ is strictly increasing in τ_h and satisfies $\lim_{\tau_h \to \infty} \Upsilon^b(\tau_h) = \frac{\epsilon}{1-\alpha} > 0$. These results imply that U_h is a hump-shaped function of τ_h achieving a maximum for a unique finite level $\tau_h = \tau_h^{\max}$ which is associated to $\Upsilon^a(\tau_h^{\max}) = \Upsilon^b(\tau_h^{\max}) \to dU_h/d\tau_h = 0$. In particular, consider any level of the Home tax, $\breve{\tau}_R^h$, such that the level of resource use is $\bar{\theta} = \epsilon/(1-\epsilon)$. Then, (A.52) and (A.53) imply $\Upsilon^a(\tau_h) > \Upsilon^b(\tau_h) = 0$, and therefore $dU_h/d\tau_h > 0$. Consequently, the maximum condition $dU_h/d\tau_h = 0$ is necessarily associated to a higher Home resource tax, $\tau_h^{\max} > \breve{\tau}_R^h$, that is, to a lower level of relative resource use, $\theta_h^{\max} < \epsilon/(1-\epsilon)$.

Proof of Proposition 7. By (34), in a symmetric CE-allocation we have $\bar{\theta} = \epsilon/(1-\epsilon)$, in which case result (A.51) implies $dU_f/d\tau_f = 0$ whereas (A.53) implies $dU_h/d\tau_h > 0$.

Proof of Proposition 8. In a laissez-faire equilibrium, $\bar{\theta}$ is given by (30). If $\varphi_h = \varphi_f$ we have $\bar{\theta} = \epsilon/(1-\epsilon)$, in which case $dU_f/d\tau_f = 0$ and $dU_h/d\tau_h > 0$ are proved exactly as in Proposition 7. If $\varphi_h > \varphi_f$, we have $\bar{\theta} > \epsilon/(1-\epsilon)$, which implies $dU_f/d\tau_f < 0$ from (A.51) and $dU_h/d\tau_h > 0$ from (A.52). If $\varphi_h < \varphi_f$, we have $\bar{\theta} < \epsilon/(1-\epsilon)$, which implies $dU_f/d\tau_f > 0$ from (A.51) whereas, from (A.53), the sign of $dU_h/d\tau_h$ is generally ambiguous. ∎

B   Further Mathematical Details

Aggregate constraints: derivation of (16)-(17). The government budget constraint in country i is

$$a_i P_Y^i Z_i = F_i L_i + b_i M_i P_X^i X_i + \tau_i P_R R_i. \quad \text{(B.1)}$$

Equation (16) is derived as follows. Substituting $n_i \equiv (V_i M_i)/L_i$ and (A.8) in (A.12), we obtain

$$V_h \dot{M}_h = \Pi_h M_h + P_L^h L_h - E_h^c - F_h L_h.$$

Plugging $V_i \dot{M}_i = P_Y^i Z_i$ from (5)-(A.7) and $M_i \Pi_i = M_i X_i \left(P_X^i - \varsigma P_Y^i\right)$ from (A.2), we obtain

$$P_Y^h Z_h + E_h^c + P_Y^h \varsigma M_h X_h = M_h P_X^h X_h + P_L^h L_h - F_h L_h$$

Plugging $F_i L_i = a_i P_Y^i Z_i - b_i M_i P_X^i X_i - \tau_i P_R R_i$ from (B.1), we have

$$P_Y^h Z_h \left(1 + a_h\right) + E_h^c + P_Y^h \varsigma M_h X_h = M_h P_X^h X_h \left(1 + b_h\right) + P_L^h L_h + \tau_h P_R R_h$$

From the final sectors' profit-maximizing conditions, we can substitute $P_L^i L_i = \beta P_Y^i Y_i$ and $M_i P_X^i X_i \left(1 + b_i\right) = \alpha P_Y^i Y_i$ to obtain

$$E_h^c + P_Y^h Z_h \left(1 + a_h\right) + P_Y^h \varsigma M_h X_h = \left(\alpha + \beta\right) P_Y^h Y_h + \tau_h P_R R_h,$$

where we can plug $\alpha + \beta = 1 - \gamma$ together with condition (2) to obtain

$$E_h^c + P_Y^h Z_h \left(1 + a_h\right) + P_Y^h \varsigma M_h X_h = P_Y^h Y_h - P_R R_h. \tag{B.2}$$

Substituting $E_h^d \equiv P_Y^h Z_h \left(1 + a_h\right)$ and $E_h^x \equiv P_Y^h \varsigma M_h X_h$ we obtain (16). Repeating the above steps for the Foreign economy starting from constraint (A.13), and recalling that $R - R_f = R_h$, we obtain (17).

Derivation of (A.21)-(A.22). From (14), the growth rate of $\bar{\sigma}_i^c \equiv E_i^c/\left(P_Y^i Y_i\right)$ equals

$$\widehat{\bar{\sigma}_i^c} = r_i\left(t\right) - \rho - \hat{P}_Y^i - \hat{Y}_i = \varphi_i \alpha \left(1 - \alpha\right) \left(1 + a_i\right) \left(1 + b_i\right)^{-1} - \hat{M}_i - \rho,$$

where we have substituted $r_i\left(t\right)$ by (A.16). Plugging $E_i^d \equiv P_Y^i Z_i \left(1 + a_i\right)$ in (7) and using $\bar{\sigma}_i^d \equiv E_i^d/\left(P_Y^i Y_i\right)$, the growth rate of varieties equals $\hat{M}_i = \varphi_i \bar{\sigma}_i^d$, which can be substituted in the above expression to obtain

$$\widehat{\bar{\sigma}_i^c} = \varphi_i \alpha \left(1 - \alpha\right) \left(1 + a_i\right) \left(1 + b_i\right)^{-1} - \varphi_i \bar{\sigma}_i^d - \rho. \tag{B.3}$$

Note that (B.3) is valid *in both economies*. Now consider Home. From (A.20), substitute $\bar{\sigma}_h^d = 1 - \tilde{\gamma}_h - \bar{\sigma}_h^c - \bar{\sigma}_h^x$ in (B.3), and eliminate $\bar{\sigma}_h^x$ by (A.19), to obtain

$$\widehat{\bar{\sigma}_i^c}\left(t\right) = \varphi_h \bar{\sigma}_h^c\left(t\right) + \varphi_h \frac{\alpha \left(1 - \alpha\right) \left(1 + a_h\right) + \alpha^2}{1 + b_h} - \varphi_h \left(1 - \tilde{\gamma}_h\right) - \rho, \tag{B.4}$$

Since $\varphi_h > 0$, equation (B.4) is globally unstable around the unique stationary point: ruling out by standard arguments explosive dynamics in the consumption propensity, we have

$$\bar{\sigma}_h^c = \left(1 - \tilde{\gamma}_h\right) - \frac{\varphi_h \left[\alpha \left(1 - \alpha\right) \left(1 + a_h\right) + \alpha^2\right] - \rho \left(1 + b_h\right)}{\varphi_h \left(1 + b_h\right)} \text{ in each } t. \tag{B.5}$$

From (A.19) and (B.5), constant values of $\bar{\sigma}_h^c$ and $\bar{\sigma}_h^x$ imply a constant $\bar{\sigma}_h^d$ which, from (A.20), equals

$$\bar{\sigma}_h^d = 1 - \tilde{\gamma}_h - \bar{\sigma}_h^c - \bar{\sigma}_h^x = \frac{\varphi_h \alpha (1 - \alpha)(1 + a_h) - \rho(1 + b_h)}{\varphi_h(1 + b_h)}. \tag{B.6}$$

Derivation of (A.40)-(A.43). Equation (A.3) and result (21) imply that physical final output in country i equals

$$Y_i(t) = \frac{(\alpha^2/\varsigma)^{\frac{\alpha}{1-\alpha}}}{1 + b_i} \cdot M_i(0)(v_i(0)L_i)^{\frac{\beta}{1-\alpha}}(R_i(0))^{\frac{\gamma}{1-\alpha}} \cdot e^{(\Omega_i - \rho)t}, \tag{B.7}$$

where $M_i(0)$ and $v_i(0)$ are exogenously given. Initial resource use $R_i(0)$ is determined by the solution of the optimal extraction problem:[17]

$$R_h(0) = \frac{\bar{\theta}}{1 + \bar{\theta}} \rho Q_0 \text{ and } R_f(0) = \frac{1}{1 + \bar{\theta}} \rho Q_0. \tag{B.8}$$

Substituting (B.8) in (B.7) for each $i = h, f$, we obtain (A.40) and (A.41). Taking the ratio between (A.40) and (A.41), and defining $\psi_0 \equiv \left[\frac{M_h(0)}{M_f(0)}\left(\frac{1+b_f}{1+b_h}\right)\left(\frac{v_h(0)L_h}{v_f(0)L_f}\right)^{\frac{\beta}{1-\alpha}}\right]$, we obtain (A.42). Re-writing (A.27) as

$$\frac{P_Y^h(t)}{P_Y^f(t)} = \theta(t) \cdot \frac{1 + \tau_h}{1 + \tau_f} \frac{Y_f(t)}{Y_h(t)},$$

and using (A.42) to eliminate $Y_h(t)/Y_f(t)$, we obtain (A.43).

Conditional efficiency in Home. By definition, the CE-allocation in Home solves

$$\max_{\{E_h^c, E_h^x, E_h^d, R_h\}} \int_0^\infty e^{-\rho t} \cdot \ln((\omega/L_h) \cdot E_h^c)dt \text{ subject to}$$

$$Y_h = M_h X_h^\alpha (v_h L_h)^\beta R_h{}^\gamma,$$

$$E_h^x = P_Y^h \varsigma M_h X_h,$$

$$P_Y^h Y_h = E_h^c + E_h^d + E_h^x + P_R R_h,$$

$$\dot{M}_h = M_h \varphi_h \cdot \left[E_h^d/(P_Y^h Y_h)\right],$$

where $\omega = \omega(P_Y^h, P_Y^f)$ is taken as given and symmetry across varieties is already imposed without any loss of generality. The first constraint is the final-good technology (1), the second is the intermediate-good technology with linear cost, the third is (16), the fourth is the R&D technology (7) with knowledge spillovers taken into account. Recalling

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[17] Since $R = R_h + R_f$ and $\theta = \bar{\theta}$, the intertemporal resource constraint (10) can be written as $Q_0 = \int_0^\infty R_f(t)(1 + \bar{\theta})dt$ and directly integrated to obtain $R_f(0)$ in (B.8), from which $R_h(0)$ can be obtained as $\bar{\theta}R_f(0)$.

that $\sigma_h^d \equiv E_h^d/(P_Y^h Y_h)$ and combining the first three constraints, the problem becomes $\max_{\left\{E_h^c, X_h, \sigma_h^d, R_h\right\}} \int_0^\infty e^{-\rho t} \cdot \ln((\omega/L_h) \cdot E_h^c) dt$ subject to

$$P_Y^h M_h X_h^\alpha (v_h L_h)^\beta R_h^\gamma \left(1 - \sigma_h^d\right) = E_h^c + P_Y^h \varsigma M_h X_h + P_R R_h, \qquad \text{(B.9)}$$

$$\dot{M}_h = M_h \varphi_h \sigma_h^d, \qquad \text{(B.10)}$$

where the controls are $\left\{E_h^c, X_h, \sigma_h^d, R_h\right\}$ and the only state variable is M_h. The current-value Hamiltonian is

$$\ln\left[(\omega_h/L_h) \cdot E_h^c\right] + \mu_h' \cdot M_h \varphi_h \sigma_h^d +$$
$$+ \mu_h'' \cdot \left[P_Y^h M_h X_h^\alpha (v_h L_h)^\beta R_h^\gamma \left(1 - \sigma_h^d\right) - E_h^c - P_Y^h \varsigma M_h X_h - P_R R_h\right]$$

where μ_h' is the dynamic multiplier associated to (B.10) and μ_h'' is the static multiplier attached to (B.9). The optimality conditions read

$$\frac{\partial}{\partial E_h^c} = 0 \rightarrow \qquad \frac{1}{E_h^c} = \mu_h'', \qquad \text{(B.11)}$$

$$\frac{\partial}{\partial X_h} = 0 \rightarrow \qquad \left(1 - \sigma_h^d\right) \alpha P_Y^h Y_h = P_Y^h \varsigma M_h X_h, \qquad \text{(B.12)}$$

$$\frac{\partial}{\partial \sigma_h^d} = 0 \rightarrow \qquad \mu_h' M_h \varphi_h = \mu_h'' P_Y^h Y_h, \qquad \text{(B.13)}$$

$$\frac{\partial}{\partial R_h} = 0 \rightarrow \qquad \left(1 - \sigma_h^d\right) \gamma P_Y^h Y_h = P_R R_h \qquad \text{(B.14)}$$

$$\rho \mu_h' - \dot{\mu}_h' = \frac{\partial}{\partial M_h} \rightarrow \qquad \rho \mu_h' - \dot{\mu}_h' = \mu_h' \varphi_h \sigma_h^d + \mu_h'' P_Y^h \left[\frac{Y_h}{M_h} \left(1 - \sigma_h^d\right) - \varsigma K_h\right], \quad \text{(B.15)}$$

and imply[18]

$$\tilde{E}_h = \left[1 - \gamma \left(1 - \sigma_h^d\right)\right] \cdot P_Y^h Y_h, \qquad \text{(B.16)}$$

$$\tilde{E}_h^x = \alpha \left(1 - \sigma_h^d\right) \cdot P_Y^h Y_h, \qquad \text{(B.17)}$$

$$\tilde{E}_h^c = \beta \left(1 - \sigma_h^d\right) \cdot P_Y^h Y_h, \qquad \text{(B.18)}$$

$$E_h^d = \sigma_h^d \cdot P_Y^h Y_h. \qquad \text{(B.19)}$$

Substituting (B.12) and (B.13) in (B.15) we have

$$\frac{\dot{\mu}_h'}{\mu_h'} = \rho - \varphi_h \left[1 - \alpha \left(1 - \sigma_h^d\right)\right]. \qquad \text{(B.20)}$$

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[18]Plugging (B.14) in constraint (16) we have (B.16). Plugging (B.12) in technology $E_h^k = P_Y^h \varsigma M_h K_h$ yields (B.17). Plugging (B.12) and (B.14) in (B.9) we have (B.18). Equation (B.19) is determined residually by $\tilde{E}_h^d = \tilde{E}_h - \tilde{E}_h^k - \tilde{E}_h^c$.

Time-differentiating (B.13) and using (B.20) we have

$$\frac{\dot{\mu}_h''}{\mu_h''} = \rho - \varphi_h \left(1 - \alpha\right) \left(1 - \sigma_h^d\right) - \frac{\dot{P}_Y^h Y_h}{P_Y^h Y_h},$$

where we can substitute $\mu_h'' = 1/E_h^c$ from (B.11) to obtain

$$\frac{\dot{E}_h^c}{E_h^c} - \frac{\dot{P}_Y^h Y_h}{P_Y^h Y_h} = \varphi_h \left(1 - \alpha\right) \left(1 - \sigma_h^d\right) - \rho. \tag{B.21}$$

From (B.18) we have $\frac{\dot{E}_h^c}{E_h^c} - \frac{\dot{P}_Y^h Y_h}{P_Y^h Y_h} = -\frac{\dot{\sigma}_h^d}{1-\sigma_h^d}$ which can be combined with (B.21) to get

$$\dot{\sigma}_h^d = \rho \left(1 - \sigma_h^d\right) - \varphi_h \left(1 - \alpha\right) \left(1 - \sigma_h^d\right)^2. \tag{B.22}$$

Equation (B.22) is globally unstable around its unique steady state: ruling out explosive dynamics by standard arguments, the conditionally-efficient rate of investment in R&D is

$$\tilde{\sigma}_h^d = \frac{\varphi_h \left(1 - \alpha\right) - \rho}{\varphi_h \left(1 - \alpha\right)} \text{ and } 1 - \tilde{\sigma}_h^d = \frac{\rho}{\varphi_h \left(1 - \alpha\right)} \tag{B.23}$$

in each point in time. Substituting (B.23) in (B.17)-(B.18) we obtain

$$\tilde{\sigma}_h^x = \frac{\alpha \rho}{\varphi_h \left(1 - \alpha\right)} \text{ and } \tilde{\sigma}_h^c = \frac{\beta \rho}{\varphi_h \left(1 - \alpha\right)}. \tag{B.24}$$

Conditional efficiency in Foreign. Following the same preliminary steps of the Home problem, the CE-allocation in Foreign solves

$$\max_{\{E_f^c, X_f, \sigma_f^d, R_h, R_f\}} \int_0^\infty e^{-\rho t} \cdot \ln((\omega/L_f) \cdot E_f^c) dt \text{ subject to}$$

$$P_Y^f M_f X_f^\alpha \left(v_f L_f\right)^\beta R_f^\gamma \left(1 - \sigma_f^d\right) = E_f^c + P_Y^f \varsigma M_f X_f - P_R R_h, \tag{B.25}$$

$$\dot{M}_f = M_f \varphi_f \sigma_f^d, \tag{B.26}$$

$$\dot{Q} = -R_h - R_f \tag{B.27}$$

where (B.25) follows from (17) and, differently from Home, we have the resource constraint (B.27) and also exported resources R_h as an additional control. The state variables are M_f and the resource stock Q. The Hamiltonian is

$$\ln \left[(\omega/L_f) \cdot E_f^c\right] + \mu_f' \cdot M_f \varphi_f \sigma_f^d +$$
$$+ \mu_f'' \cdot \left[P_Y^f M_f X_f^\alpha \left(v_f L_f\right)^\beta R_f^\gamma \left(1 - \sigma_f^d\right) - E_f^c - P_Y^f \varsigma M_f X_f + P_R R_h\right] +$$
$$+ \mu_f''' \cdot \left(-R_h - R_f\right)$$

where μ'_f is the dynamic multiplier associated to (B.26), μ''_h is the Lagrange multiplier attached to (B.25), and μ'''_f is the dynamic multiplier associated to (B.27). The first order conditions read

$$\frac{\partial}{\partial E_f^c} = 0 \rightarrow \qquad \frac{1}{E_f^c} = \mu''_f, \tag{B.28}$$

$$\frac{\partial}{\partial X_f} = 0 \rightarrow \qquad \left(1 - \sigma_f^d\right) \alpha P_Y^f Y_f = P_Y^f \varsigma M_f X_f, \tag{B.29}$$

$$\frac{\partial}{\partial \sigma_f^d} = 0 \rightarrow \qquad \mu'_f M_f \varphi_f = \mu''_f P_Y^f Y_f, \tag{B.30}$$

$$\frac{\partial}{\partial R_h} = 0 \rightarrow \qquad \mu''_f \cdot P_R = \mu'''_f \tag{B.31}$$

$$\frac{\partial}{\partial R_f} = 0 \rightarrow \qquad \mu''_f \cdot \left(1 - \sigma_f^d\right) \gamma P_Y^f Y_f = \mu'''_f R_f, \tag{B.32}$$

$$\rho \mu'_f - \dot{\mu}'_f = \frac{\partial}{\partial M_f} \rightarrow \qquad \rho \mu'_f - \dot{\mu}'_f = \mu'_f \varphi_f \sigma_f^d + \mu''_f P_Y^f \left[\frac{Y_f}{M_f} \left(1 - \sigma_f^d\right) - \varsigma K_f \right], \tag{B.33}$$

$$\rho \mu'''_f - \dot{\mu}'''_f = \frac{\partial}{\partial Q} \rightarrow \qquad \rho \mu'''_f - \dot{\mu}'''_f = 0. \tag{B.34}$$

Notice that, from (B.31)-(B.32) and definition $R_h = \theta R_f$, we have

$$P_R \tilde{R}_f = \left(1 - \sigma_f^d\right) \gamma P_Y^f \tilde{Y}_f, \tag{B.35}$$

$$P_R \tilde{R}_h = \left(1 - \sigma_f^d\right) \gamma \tilde{\theta} \cdot P_Y^f \tilde{Y}_f, \tag{B.36}$$

so that expenditures equal[19]

$$\tilde{E}_f = \left[1 + \left(1 - \tilde{\sigma}_f^d\right) \gamma \tilde{\theta}\right] \cdot P_Y^f \tilde{Y}_f, \tag{B.37}$$

$$\tilde{E}_f^x = \alpha \left(1 - \tilde{\sigma}_f^d\right) \cdot P_Y^f \tilde{Y}_f, \tag{B.38}$$

$$\tilde{E}_f^c = \left(1 - \alpha + \gamma \tilde{\theta}\right) \left(1 - \tilde{\sigma}_f^d\right) \cdot P_Y^f \tilde{Y}_f, \tag{B.39}$$

$$\tilde{E}_f^d = \tilde{\sigma}_f^d \cdot P_Y^f \tilde{Y}_f. \tag{B.40}$$

Before deriving the explicit value of $\tilde{\sigma}_f^d$ we show that the efficient relative resource use $\tilde{\theta}$ is constant over time. From the balanced trade condition (A.25), we have $P_R R_h + (1 - \epsilon) E_h^c = \epsilon E_f^c$ where we can use (B.18) and (B.39) to eliminate E_h^c and E_f^c, respectively, and also use (B.14) to eliminate $P_R R_h$, obtaining

$$\frac{1 - \tilde{\sigma}_h^d}{1 - \tilde{\sigma}_f^d} \cdot \frac{P_Y^h \tilde{Y}_h}{P_Y^f \tilde{Y}_f} = \frac{\epsilon \left(1 - \alpha + \gamma \tilde{\theta}\right)}{\gamma + (1 - \epsilon) \beta}, \tag{B.41}$$

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[19]Plugging (B.36) in (17) yields (B.37). Plugging (B.29) in technology $E_f^k = P_Y^f \varsigma M_f K_f$ yields (B.38). Plugging (B.29) and (B.36) in (B.25) we have (B.39). Equation (B.40) is determined residually by $\tilde{E}_f^d = \tilde{E}_f - \tilde{E}_f^k - \tilde{E}_f^c$.

where tildas denote conditionally-efficient values. Taking the ratio between (B.14) and (B.36) we have

$$\tilde{\theta} = \frac{1 - \tilde{\sigma}_h^d}{1 - \tilde{\sigma}_f^d} \cdot \frac{P_Y^h \tilde{Y}_h}{P_Y^f \tilde{Y}_f}. \tag{B.42}$$

Combining (B.42) with (B.41) we obtain

$$\tilde{\theta} = \frac{\epsilon}{1 - \epsilon} \cdot \frac{1 - \alpha}{\gamma + \beta} = \frac{\epsilon}{1 - \epsilon}. \tag{B.43}$$

Result (B.43) implies that $\tilde{\theta}$ is constant. Now go back to (B.33) and substitute (B.29)-(B.30) to re-write it as

$$\frac{\dot{\mu}_f'}{\mu_f'} = \rho - \varphi_f \left[1 - \alpha \left(1 - \tilde{\sigma}_f^d \right) \right]. \tag{B.44}$$

Time-differentiating (B.30) and substituting (B.28)-(B.26), we obtain

$$\frac{\dot{\mu}_f'}{\mu_f'} = -\frac{\dot{E}_f^c}{E_f^c} + \frac{\dot{P_Y^f Y_f}}{P_Y^f Y_f} - \varphi_f^d \tilde{\sigma}_f^d$$

which can be combined with (B.44) to obtain

$$\frac{\dot{E}_f^c}{E_f^c} - \frac{\dot{P_Y^f Y_f}}{P_Y^f Y_f} = \varphi_f \left[(1 - \alpha) \left(1 - \tilde{\sigma}_f^d \right) \right] - \rho. \tag{B.45}$$

Since $\tilde{\theta}$ is constant by (B.43), time-differentiation of (B.39) yields $\frac{\dot{E}_f^c}{E_f^c} - \frac{\dot{P_Y^f Y_f}}{P_Y^f Y_f} = -\frac{\dot{\sigma}_f^d}{1 - \sigma_f^d}$. Plugging this result in (B.46) we obtain the usual equilibrium relation (see (B.22) above for Home) which can be solved for the steady-state level

$$\tilde{\sigma}_f^d = \frac{\varphi_f (1 - \alpha) - \rho}{\varphi_f (1 - \alpha)} \text{ or } 1 - \tilde{\sigma}_f^d = \frac{\rho}{\varphi_f (1 - \alpha)}. \tag{B.46}$$

Substituting (B.46) in (B.38)-(B.40) we obtain

$$\tilde{\sigma}_f^x = \frac{\alpha \rho}{\varphi_f (1 - \alpha)} \text{ and } \tilde{\sigma}_f^c = \frac{\rho \left(1 - \alpha + \gamma \tilde{\theta} \right)}{\varphi_f (1 - \alpha)}. \tag{B.47}$$

Derivation of (34).   Equation (34) is proved in (B.43).

Derivation of (31)-(33). Efficient taxes are obtained by equalizing efficient and equilibrium values of $(\sigma_i^x, \sigma_i^d, \sigma_i^c)$. First, results (B.24) and (B.47) imply $\tilde{\sigma}_i^x = \frac{\alpha \rho}{\varphi_i (1 - \alpha)}$ in both countries. Imposing the equality between the efficient values $\tilde{\sigma}_i^x$ and the competitive-equilibrium values $\bar{\sigma}_i^x = \frac{\alpha^2}{1 + b_i}$ derived in (A.19), we obtain the efficient tax on intermediates' purchases $\tilde{b}_i$ in (32). Second, results (B.23) and (B.46) imply $\tilde{\sigma}_i^d = \frac{\varphi_i (1 - \alpha) - \rho}{\varphi_i (1 - \alpha)}$ in

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both countries. Imposing the equality between $\tilde{\sigma}_i^d$ and the competitive-equilibrium values $\bar{\sigma}_i^d = \frac{\varphi_i \alpha(1-\alpha)(1+a_i) - \rho(1+b_i)}{\varphi_f(1+b_i)}$ derived in (A.37), and substituting $\tilde{b}_i$ by (32), we obtain the efficient subsidy $\tilde{a}_i$ in (31). Now consider Home: from (B.24) we have $\tilde{\sigma}_h^c = \frac{\beta\rho}{\varphi_h(1-\alpha)}$ whereas (B.6) implies $\bar{\sigma}_h^c = 1 - \tilde{\gamma}_h - \bar{\sigma}_h^d - \bar{\sigma}_h^x$. Setting $\tilde{\sigma}_h^c = \bar{\sigma}_h^c$ and imposing that $\bar{\sigma}_h^x = \tilde{\sigma}_h^x$ and $\bar{\sigma}_h^d = \tilde{\sigma}_h^d$ by virtue of (31)-(32), we obtain

$$\frac{\beta\rho}{\varphi_h(1-\alpha)} = 1 - \tilde{\gamma}_h - \bar{\sigma}_h^d - \bar{\sigma}_h^x = 1 - \tilde{\gamma}_h - \frac{\varphi_h(1-\alpha) - \rho}{\varphi_h(1-\alpha)} - \frac{\alpha\rho}{\varphi_h(1-\alpha)}$$

where the last term follows from $\tilde{\sigma}_h^d$ and $\tilde{\sigma}_h^x$ derived in (B.23) and (B.24). Rearranging terms and solving for $\tilde{\gamma}_h$ we obtain $\tilde{\gamma}_h = \frac{\rho\gamma}{\varphi_h(1-\alpha)}$, which implies the efficient resource tax for Home

$$\tilde{\tau}_h = \frac{\varphi_h(1-\alpha) - \rho}{\rho}. \tag{B.48}$$

The optimal resource tax in Foreign $\tilde{\tau}_f$ then follows from (B.42). Since $1 + \tilde{\tau}_h = 1 - \bar{\sigma}_h^d$ by (B.48), the only way to satisfy $\bar{\theta} = \tilde{\theta}$ in equations (A.27) and (B.42) is to set $1 + \tilde{\tau}_f = 1 - \bar{\sigma}_h^f = \frac{1}{\rho}\varphi_h(1-\alpha)$, which proves (33). It can be easily verified that, residually, (31)-(33) imply $\bar{\sigma}_f^c = \tilde{\sigma}_f^c$.

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