CER-ETH – Center of Economic Research at ETH Zurich

Formal and Real Power in General Equilibrium

H. Gersbach and H. Haller

Working Paper 15/212
March 2015

Economics Working Paper Series

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Formal and Real Power in General Equilibrium*

Hans Gersbach† and Hans Haller‡

This version: January 2015

Abstract

We integrate individual power in groups into general equilibrium models with endogenous group formation. We distinguish between formal power (the say in group decisions) and real power (utility gain from being in groups). Their values will be determined as part of the equilibrium. We find that higher formal power does not necessarily translate into higher equilibrium utility or higher real power. One reason is that induced price changes may offset the group member's increased influence. A second reason is that the group may dissolve when a group member gains too much influence, because other members can exercise the option to leave. We also show that maximal real power can be compatible with Pareto efficiency. We further identify circumstances when changes of formal power in one group do not impact on other groups. Finally, we establish existence of competitive equilibria, including equilibria where some individual enjoys real power.

Keywords: group formation, competitive markets, power, exit

JEL: D41, D50, D60.

*A precursor has appeared as "Exit and Power in General Equilibrium", CESifo Working Paper, No. 2369. We thank Elias Aptus, Clive Bell, Pierre-André Chiappori, Egbert Dierker, Bryan Ellickson, Theresa Fahrenberger, Volker Hahn, Martin Hellwig, Benny Moldovanu, Martin Scheffel, Klaus Schmidt, Bill Zame, and seminar participants in Bielefeld, Bonn, Basel, Berlin, Heidelberg and UCLA for helpful comments.

†CER-ETH – Center of Economic Research at ETH Zurich, Zürichbergstrasse 18, 8092 Zurich, Switzerland, hgersbach@ethz.ch.

‡Department of Economics, Virginia Polytechnic Institute and State University, Blacksburg, VA 24061-0316, USA, haller@vt.edu.

1 Introduction

Formal and Real Power

We are going to examine how individual power in groups can be integrated into a general equilibrium model. We consider a finite pure exchange economy with multi-member groups. While we stick to the suggestive term “group”, a broader interpretation as socio-economic group or simply group would be appropriate in many instances. Groups are endogenously formed: An allocation consists of a partition of the consumer population into groups and an allocation of commodities to consumers. Group members have individual preferences. They make efficient collective consumption decisions based on their individual preferences, where different groups may use different collective decision mechanisms. Moreover, at the going prices, individuals may decide to leave a multi-member group and become singles. In equilibrium, no individual wants to exercise this exit option.

Groups operate within a competitive market environment. Therefore, neither groups nor deserting members have any market power. Nonetheless, individuals can exert power in multi-member groups. The departure from the traditional model of pure exchange enables us to examine individual power in such groups. We focus on two notions of power: formal and real power. The former refers to the say in group decisions captured, e.g., by the weight in a group welfare function or by the relative bargaining power in Nash bargained group decisions. Real power refers to the additional utility an individual can achieve in a group in comparison to the stand-alone utility as a single person.

Main Results

First, we introduce competitive equilibria with free exit which serve as the basic concept for our investigation of power. Next, we define real power in groups in such an equilibrium and identify instances of its absence as well as its presence. Third, we identify circumstances in which the presence of maximal real power is compatible with Pareto efficiency. We conclude that high real power per se is no indication of social inefficiency. Fourth, we introduce formal power and illustrate that higher formal power does not necessarily translate into higher equilibrium utility or higher real power because groups may dissolve or relative price changes may offset higher formal power.

Fifth, we identify conditions under which changes of formal power in one group do not affect other groups in society and, thus, power spill-overs are absent. Finally, we establish existence of competitive equilibria with free exit, including equilibria where some individual enjoys real power. Overall, our approach enables us to make a first albeit moderate step toward the study of endogenous power, its determinants and consequences in general equilibrium.

Related Work

The notion of power can have very different meanings in economics. Concepts such as market power, veto power, agenda setting power, voting power, bargaining power, and power indices are well known.

Here we develop a framework to define power in a general equilibrium context. Our approach is in the tradition of cooperative models of groups as recently surveyed by Apps and Rees (2009). In this tradition, we integrate collective rationality of groups into a general equilibrium model. We start from Gersbach and Haller (2011) and develop a suitable framework to examine the role of individual power. Our paper is also related to the influential work of Hirschman (1970) who has considered the comparative efficiency of the exit and voice options as mechanisms of recuperation. Our analysis suggests that the exit option limits power as long as externalities in groups are sufficiently small. Our notions of real and formal power can be viewed as a parallel to formal and real authority in organizations (Aghion and Tirole (1997) and Rajan and Zingales (1998)). The former refers to the right to decide and the latter to effective control over decisions. In our model, formal power captures the say in collective decisions and real power captures the increase of an individual's welfare resulting from such decisions in a competitive environment. A key feature — distinguishing this paper from the literature (e.g. Aghion and Tirole (1997), Gersbach and Haller (2009)) — is the assumption of price-dependent outside options. This not only yields new phenomena, but also allows to study the relationship between power and Pareto optimality in a general equilibrium setting.

Finally, two core ingredients of our equilibrium concept — price taking and free exit — have a long history in the literature on group formation. Well-known contributions on local public good economies and group formation games (e.g. Guesnerie and Oddou (1981), Greenberg and Weber (1986), Konishi, Le Breton and Weber (1997, 1998)) have developed non-cooperative equilibrium concepts and characterized equilibrium existence and properties with these ingredients, also including stronger stability conditions. We focus on group formation when the group takes collective decisions and may trade in commodity markets with other groups as an entity.

Outline

The paper is organized as follows. In the next two sections, we introduce the formal framework and define an equilibrium with free exit. Section 4 explores the presence of real power and the relationship between real power and Pareto efficiency. In section 5, we study the relationship between formal and real power and their equilibrium implications. In section 6, we consider the special case of quasi-linear utilities. Section 7 deals with existence of competitive equilibria with free exit. Section 8 concludes. More elaborate proofs are collected in an appendix.

2 Consumer Characteristics and Allocations

In this section, we describe the basic structure of the model: consumers, group structures, commodities, endowments, allocations, preferences, and optimality.

Consumers and Group Structures. We consider a finite population of **consumers**, represented by a set $I = \{1, \ldots, N\}$. A generic consumer is denoted by i or j. $\mathcal{H} = \{h \subseteq I : h \neq \emptyset\}$ constitutes the set of all potential groups. A generic group is denoted by h or h'. A single-person group formed by individual i is denoted by $\{i\}$. The population I will be partitioned into groups. That is, there exists a partition P of I into non-empty subsets which represent groups. We call any such partition P a **group structure in** I. If P consists of H groups, we frequently label them $h = 1, \ldots, H$, provided this causes no confusion.

We treat the group structure as an object of endogenous choice: Groups are endogenously formed so that some group structure P is ultimately realized. Consequently, our **consumer allocation space** is $\mathcal{P}$, the set of all group structures in I. For $P \in \mathcal{P}$ and $i \in I$, let $P(i)$ denote the unique element of P to which i belongs.

Commodities. There exists a finite number $\ell \geq 1$ of commodities. Each commodity is formally treated as a private good, possibly with externalities in consumption. Each consumer $i \in I$ has a consumption set $X_i = \mathbb{R}^\ell_+$ so that the **commodity allocation space** is $\mathcal{X} \equiv \prod_{i \in I} X_i$. Generic elements of $\mathcal{X}$ are denoted $\mathbf{x} = (x_i)_{i \in I}$, $\mathbf{y} = (y_i)_{i \in I}$. Commodities are denoted by superscripts $k = 1, \ldots, \ell$. For a group $h \in \mathcal{H}$, set $\mathcal{X}_h = \prod_{i \in h} X_i$, the consumption set for group h. $\mathcal{X}_h$ has generic elements $\mathbf{x_h} = (x_i)_{i \in h}$. If $\mathbf{x} = (x_i)_{i \in I} \in \mathcal{X}$ is a commodity allocation, then consumption for group h is $\mathbf{x_h} = (x_i)_{i \in h}$, the restriction of $\mathbf{x} = (x_i)_{i \in I}$ to h .

Endowments. For a group $h \in \mathcal{H}$, its **endowment** is a commodity bundle $\omega_h \in \mathbb{R}^\ell$ given by the sum of the endowments of all participating individuals: $\omega_h = \sum_{i \in h} \omega_{\{i\}}$. The **social endowment** is given by

$$\omega_S \equiv \sum_{h \in P} \omega_h = \sum_{i \in I} \omega_{\{i\}}. \tag{1}$$

Allocations. An **allocation** is a pair $(\mathbf{x}; P) \in \mathcal{X} \times \mathcal{P}$ specifying the consumption bundle and group membership of each consumer. We call an allocation $(\mathbf{x}; P) \in \mathcal{X} \times \mathcal{P}$ **feasible**, if

$$\sum_{i \in I} x_i \leq \omega_S. \tag{2}$$

Consumer Preferences. In principle, a consumer might have preferences on the allocation space $\mathcal{X} \times \mathcal{P}$ and care about each and every detail of an allocation. For individual $i \in I$, we assume that i has preferences on $\mathcal{X} \times \mathcal{P}$ represented by a utility function $\mathcal{U}_i : \mathcal{X} \times \mathcal{P} \to \mathbb{R}$.

In the following, we shall make the **general assumption** that an individual does not care about the features of an allocation beyond the boundaries of his own group. If a particular group structure is given, he is indifferent about the affiliation and consumption of individuals not belonging to his own group. Condition HSP below is a formal expression of this assumption. To formally represent such **group-specific preferences**, let us define $\mathcal{H}_i \equiv \{h \subseteq I | i \in h\}$ for $i \in I$. $\mathcal{H}_i$ is the set of potential groups of which i would be a member. Let us further denote $\mathcal{X}^* = \bigcup_{h \in \mathcal{H}} \mathcal{X}_h$ and define $\mathcal{A}_i = \{(\mathbf{x_h}; h) \in \mathcal{X}^* \times \mathcal{H} : h \in \mathcal{H}_i, \mathbf{x_h} \in \mathcal{X}_h\}$ for $i \in I$.

(HSP) Group-Specific Preferences:

There exist $U_i : \mathcal{A}_i \to \mathbb{R}$, $i \in I$, such that
$\mathcal{U}_i(\mathbf{x}; P) = U_i(\mathbf{x_h}; h)$ for $i \in h$, $h \in P$, $(\mathbf{x}; P) \in \mathcal{X} \times \mathcal{P}$.

The general assumption HSP is justifiable on the grounds that we want to design a model where multi-member groups play a significant allocative role. A more detailed justification of this assumption is given in Gersbach and Haller (2011). HSP still admits a lot of flexibility. For example, it permits various types of consumption externalities. Later on, we shall exploit the occurrence of pure group externalities that depend solely on the persons belonging to a group and not on consumption.

(PGE) Pure Group Externalities:
For each consumer i, there exist functions $U_i^c : X_i \to \mathbb{R}$ and $U_i^g : \mathcal{H}_i \to \mathbb{R}$ such that $U_i(\mathbf{x_h}; h) = U_i^c(x_i) + U_i^g(h)$ for all $(\mathbf{x_h}; h) \in \mathcal{A}_i$.

PGE assumes that one can additively separate the pure consumption effect $U_i^c(x_i)$ from the pure group effect $U_i^g(h)$. A very special case is the absence of externalities, corresponding to $U_i^g \equiv 0$.

Optimality. An allocation determines the welfare of each and every consumer. We say that an allocation $(\mathbf{x}; P)$ is **fully Pareto optimal** if $(\mathbf{x}; P)$ is feasible and there is no feasible allocation $(\mathbf{x}'; P')$ that satisfies $(\mathcal{U}_i(\mathbf{x}'; P'))_{i\in I} > (\mathcal{U}_i(\mathbf{x}; P))_{i\in I}$.[1] Denote by $\mathcal{M}^*$ the set of fully Pareto optimal allocations. If all utility functions are continuous in consumption, $\mathcal{M}^*$ is not empty (Gersbach and Haller (2001)). We denote by $\mathcal{P}^* \subseteq \mathcal{P}$ the set of all potentially optimal group structures, i.e., $P \in \mathcal{P}^*$ if and only if there exists $\mathbf{x} \in \mathcal{X}$ such that $(\mathbf{x}; P) \in \mathcal{M}^*$.

In the special case of pure group externalities (PGE) with utility representations of the form $U_i(\mathbf{x_h}; h) = U_i^c(x_i) + U_i^g(h)$ for all $i \in I$, $(\mathbf{x_h}; h) \in \mathcal{A}_i$, we frequently employ the concept of **optimal group structure based solely on group preferences**. Namely in that special case, we can define a partial order $\succsim$ on $\mathcal{P}$ by $P' \succsim P$ for $P, P' \in \mathcal{P}$ if and only if $U_i^c(P'(i)) \geq U_i^c(P(i))$ for all $i \in I$. Let $\mathcal{P}_*$ denote the set of maximal elements of the partial order $\succsim$ on $\mathcal{P}$. The elements of $\mathcal{P}_*$ are the optimal group structures based solely on group preferences. Since $\mathcal{P}$ is finite, $\mathcal{P}_* \neq \emptyset$. Let us collect a few elementary facts for later reference.

[1]The notation "$>$" means in this context that $\mathcal{U}_i(\mathbf{x}'; P') \geq \mathcal{U}_i(\mathbf{x}; P)$ for all $i \in I$ and $\mathcal{U}_i(\mathbf{x}'; P') > \mathcal{U}_i(\mathbf{x}; P)$ for at least one $i \in I$.

Fact 1 *Suppose PGE holds. Then:*

(i) *For any $P \in \mathcal{P}$, there exists $P' \in \mathcal{P}_*$ with $P' \succsim P$.*

(ii) *For any $P \in \mathcal{P} \setminus \mathcal{P}_*$, there exists $P' \in \mathcal{P}_*$ with $P' \succ P$.*

(iii) $\mathcal{P}^* \subseteq \mathcal{P}_*$.

(iv) *If $\mathcal{P}_*$ is a singleton, say $\mathcal{P}_* = \{P'\}$, and $\mathbf{x}$ is a Pareto optimal allocation of the pure exchange economy given by $(X_i, U_i^c, \omega_{\{i\}})_{i \in I}$, then $(\mathbf{x}; P') \in \mathcal{M}^*$ and $\mathcal{P}^* = \mathcal{P}_* = \{P'\}$.*

(i) and (ii) follow from the finiteness of $\mathcal{P}$. Regarding (iii), if $P \in \mathcal{P}^*$, then there exists a feasible $\mathbf{x}$ with $(\mathbf{x}; P) \in \mathcal{M}^*$. In case $P \notin \mathcal{P}_*$, there exists $P' \in \mathcal{P}_*$ with $P' \succ P$, by (ii). But then $(\mathcal{U}_i(\mathbf{x}; P'))_{i \in I} > (\mathcal{U}_i(\mathbf{x}; P))_{i \in I}$, contrary to $(\mathbf{x}; P) \in \mathcal{M}^*$. This shows (iii). (iv) follows immediately from the definitions. Notice that $\mathcal{P}^* = \mathcal{P}_*$ does not hold in general. For instance, $\mathcal{P}^* = \emptyset$ can occur while always $\mathcal{P}_* \neq \emptyset$.

3 Equilibrium

There are several conceivable ways to formulate an equilibrium state of a model with variable group structure. We follow Gersbach and Haller (2011) and employ the concept of a competitive equilibrium with free exit. We consider a group $h \in \mathcal{H}$ and a price system $p \in \mathbb{R}^\ell_+$. For $\mathbf{x_h} = (x_i)_{i \in h} \in \mathcal{X}_h$, $p \cdot \left(\sum_{i \in h} x_i\right)$ denotes the expenditure of group h on consumption plan $\mathbf{x_h}$ at the price system p. Then h's **budget set** is defined as

$$B_h(p) = \{\mathbf{x_h} \in \mathcal{X}_h \ : \ p \cdot \left(\textstyle\sum_{i \in h} x_i\right) \leq p \cdot \omega_h\}.$$

We next define the **efficient budget set** $EB_h(p)$ as the set of $\mathbf{x_h} \in B_h(p)$ with the property that there is no $\mathbf{y_h} \in B_h(p)$ such that $(U_i(\mathbf{y_h}; h))_{i \in h} > (U_i(\mathbf{x_h}; h))_{i \in h}$.

Further we define a **state** of the economy as a triple $(p, \mathbf{x}; P)$ such that $p \in \mathbb{R}^\ell_+$ is a price system and $(\mathbf{x}; P) \in \mathcal{X} \times \mathcal{P}$ is an allocation. I.e., $\mathbf{x} = (x_i)_{i \in I}$ is an allocation of commodities and P is an allocation of consumers (a group structure, a partition of the population into groups). A state $(p, \mathbf{x}; P)$ is a **competitive equilibrium with free exit (CEFE)** if it satisfies the following conditions:

1. $\mathbf{x_h} \in EB_h(p)$ for all $h \in P$.

2. $\sum_{i \in I} x_i = \omega_S$.

3. There are no $h \in P$, $i \in h$ and $y_i \in B_{\{i\}}(p)$ such that $U_i(y_i; \{i\}) > U_i(\mathbf{x_h}; h)$.

Condition 1 reflects collective rationality. Efficient choice by the group refers to the individual consumption and welfare of its members, not merely to the aggregate consumption bundle of the group. Condition 2 requires market clearing. Conditions 1 and 2 alone define a **competitive equilibrium** $(p, \mathbf{x})$, given group structure P, discussed and studied in Haller (2000) and Gersbach and Haller (2001).

In addition, we impose condition 3 that no individual wants to leave a group and participate as a one-member group in the market at the going equilibrium prices. Condition 3 constitutes an individual rationality or voluntary participation (membership) constraint. Conditions 1 to 3 together define a **competitive equilibrium with free exit**.

4 Real Power

Informally, a person enjoys real power in a group, if the person's utility exceeds her stand-alone value, the utility she can achieve as a single individual. For a formal definition, set $V_i^0(p) = \sup \{U_i(y_i; \{i\}) | \, y_i \in B_{\{i\}}(p)\}$ for $i \in I$ and $p \in \mathbb{R}_+^\ell$. If $V_i^0(p) < \infty$, we call $V_i^0(p)$ i's **stand-alone value** at the price system p.

Lemma 1 *If $(p, \mathbf{x}; P)$ is a CEFE, then $V_i^0(p) < \infty$ for all $i \in I$ and $U_i(\mathbf{x_h}; h) \geq V_i^0(p)$ for $h \in P$, $i \in h$.*

Proof. For all $h \in P$, $i \in h$ and $y_i \in B_{\{i\}}(p)$, $U_i(y_i; \{i\}) \leq U_i(\mathbf{x_h}; h)$ by equilibrium condition 3. Hence $V_i^0(p) \leq U_i(\mathbf{x_h}; h) < \infty$. ■

Definition 1 (Real Power) *Let $(p, \mathbf{x}; P)$ be a CEFE and $h \in P$, $i \in h$. We say that individual i enjoys real power in the CEFE $(p, \mathbf{x}; P)$ if $U_i(\mathbf{x_h}; h) > V_i^0(p)$ and measure i's real power by the difference $U_i(\mathbf{x_h}; h) - V_i^0(p)$.*

4.1 Absence and Presence of Real Power

Any discussion of real power in the present context ought to begin with the neutrality theorem of Gersbach and Haller (2011, Proposition 1). The neutrality or no-power theorem states that in the absence of any externalities, individuals cannot achieve higher utility levels by participating in groups rather than acting and trading individually — which renders the notion of power within groups obsolete. For real power to exist, there has to be some advantage, some positive externality in group formation.[2] We are going to show that the existence of externalities can indeed create real power for individuals in a group. To this end, we formulate the concept of strong large group advantage, which postulates that given his own consumption, a consumer always fares better as member of the particular multi-person group than as a single consumer, regardless of the consumption of other group members.

Definition 2 *Let* $h \in \mathcal{H}$.

> **Strong large group advantage** *prevails in group* h *if*
> $U_i(\mathbf{x_h}; h) > U_i(x_i; \{i\})$ *for all* $i \in h$ *and all* $\mathbf{x_h} = (x_j)_{j \in h} \in \mathcal{X}_h$.

Strong large group advantage in group h implies $|h| > 1$.

Proposition 1 (Presence of Real Power) *Let* $(p, \mathbf{x}; P)$ *be a CEFE. Suppose for some group* $h \in P$*, strong large group advantage prevails in group* h *and* $V_i^0(p) = \max\{U_i(y_i; \{i\}) | \, y_i \in B_{\{i\}}(p)\}$ *holds for every consumer* $i \in h$*. Then there exists a member* $i \in h$ *who enjoys real power.*

Proof. Let $(p, \mathbf{x}; P)$ be a CEFE. Further let $h \in P$ such that strong large group advantage prevails in group h and $V_i^0(p) = \max\{U_i(y_i; \{i\}) | \, y_i \in B_{\{i\}}(p)\}$ holds for every consumer $i \in h$. By Lemma 1, $U_i(\mathbf{x_h}; h) \geq V_i^0(p)$ for all $i \in h$. If none of the members of h enjoys real power, then $U_i(\mathbf{x_h}; h) = V_i^0(p)$ for all $i \in h$. By assumption, there exists $y_i \in B_{\{i\}}(p)$ such that $V_i^0(p) = U_i(y_i; \{i\})$ for $i \in h$. Let $\mathbf{y_h} = (y_i)_{i \in h}$. Then $\mathbf{y_h} \in B_h(p) \subseteq \mathcal{X}_h$ and strong large group advantage implies $U_i(\mathbf{y_h}; h) > U_i(y_i; \{i\}) = V_i^0(p) = U_i(\mathbf{x_h}; h)$ for each $i \in h$, contradicting $\mathbf{x_h} \in EB_h(p)$. Hence to the contrary, some member of h must enjoy real power. ∎

[2] Another advantage of group formation could be group production. For instance, in a reduced form of group production, group formation could simply augment the initial endowment with resources: The collective endowment of a multi-member group could exceed the sum of the individual endowments of group members, which constitutes a positive endowment externality in the taxonomy of the seminal paper by Gori and Villanacci (2011) on this subject.

The condition $V_i^0(p) = \max\{U_i(y_i;\{i\}) | \, y_i \in B_{\{i\}}(p)\}$ is satisfied whenever $p \gg 0$ (so that $B_{\{i\}}(p)$ is compact) and $U_i(y_i;\{i\})$ is continuous in y_i, but may not hold otherwise. In addition to Proposition 1, we are going to demonstrate the occurrence of real power in Example 1 below. The proposition shows that given a multi-member group h which is formed in a particular CEFE, at least one member of group h enjoys real power if the strong large group advantage condition holds for the consumers in h. Such is the case in Example 1.

Example 1. Let $\ell = 2$, $I = \{1,2,3\}$. Preferences exhibit pure group externalities, that is, they are represented by $U_i(\mathbf{x_h};h) = U_i^c(x_i) + U_i^g(h) = U_i^c(x_i^1, x_i^2) + U_i^g(h)$ for $h \in \mathcal{H}_i$, $\mathbf{x_h} \in \mathcal{X}_h$ where x_i^k denotes the quantity of good k ($k = 1,2$) consumed by individual i. $U_i^g(h)$ captures the pure group externality contributing to the utility of individual i. Specifically, we assume

$$\begin{array}{rcll} U_i^c(x_i^1, x_i^2) & = & \ln x_i^i & \text{for } i = 1,2; \\ U_3^c(x_3^1, x_3^2) & = & \frac{1}{2}\ln x_3^1 + \frac{1}{2}\ln x_3^2; & \\ & & & \\ U_i^g(h) & = & \left\{ \begin{array}{l} \ln 2 \\ 0 \end{array} \right. & \begin{array}{l} \text{for } h = \{1,2\}, i = 1,2; \\ \text{otherwise.} \end{array} \end{array}$$

We further assume the individual endowments

$$\omega_1 = (0, 1/2)\,, \omega_2 = (0, 1/2)\,, \omega_3 = (1, 0)\,.$$

In this example, $(p^*, \mathbf{x}^*; P^*)$ with $p^* = (1,1)$, $P^* = \{\{1,2\},\{3\}\}$, $x_1^* = (1/2, 0)$, $x_2^* = (0, 1/2)$, $x_3^* = (1/2, 1/2)$ constitutes a CEFE. Notice that (x_1^*, x_2^*) maximizes $U_1 + U_2$ on the budget set of group $\{1,2\}$, hence $(x_1^*, x_2^*) \in EB_{\{1,2\}}(p^*)$. Both members of group $\{1,2\}$ enjoy real power $\ln 2$ in the particular equilibrium. However, it is not necessarily the case that both members of the two-person group enjoy real power in a CEFE. To see this, consider instead the state $(p^{**}, \mathbf{x}^{**}; P^*)$ with $p^{**} = (1, 2/3)$, $P^* = \{\{1,2\},\{3\}\}$, $x_1^{**} = (1/2, 0)$, $x_2^{**} = (0, 1/4)$, $x_3^{**} = (1/2, 3/4)$. Notice that (x_1^{**}, x_2^{**}) maximizes U_1 on the budget set of group $\{1,2\}$ subject to the further constraint $U_2 = V_2^0(p^{**})$. To be precise, $U_2^c(x_2^{**}) + U_2^g(\{1,2\}) = U_2^c(\omega_2) = V_2^0(p^{**}) = -\ln 2$ and $p^{**}(\omega_1 + \omega_2) - p^{**}x_2^{**}$ is spent on member 1's consumption of good 1. To increase the utility of either member, the consumption of the other member would have to be reduced. It follows that $(x_1^{**}, x_2^{**}) \in EB_{\{1,2\}}(p^{**})$ and $(p^{**}, \mathbf{x}^{**}; P^*)$ is a CEFE where consumer 1 enjoys real power $\ln 3$ and consumer 2 has no real power. □□

4.2 Real Power and Pareto Otimality

In this subsection, we are going to examine competitive equilibria with free exit and to look specifically for Pareto optimality and manifestation of power. Prima facie, manifestation of other notions of power appears to be detrimental to efficiency. For instance, market power is frequently — but not always — associated with inefficiency. In our model, however, consumers and groups operate in a perfectly competitive market environment. Therefore, market power does not exist and thus cannot be the source of any inefficiency. Inefficient equilibrium allocations in our model, to the extent that they exist, result from frictions in the interaction of three allocation mechanisms, each operating at a particular level of aggregation: (a) Individual decisions are made to join or leave groups. (b) Collective decisions within groups determine the consumption plans of group members. (c) Competitive exchange across groups achieves a feasible allocation of resources.

For our inquiry into power in general equilibrium, the most pertinent question is how real power and equilibrium efficiency are related. First of all, CEFE allocations can be inefficient. In Example 1, let $P^0 = \{\{1\}, \{2\}, \{3\}\}$ denote the group structure consisting of all singletons. The state $(p^*, \mathbf{x}^*; P^0)$ is a CEFE. The equilibrium allocation $(\mathbf{x}^*; P^0)$ is weakly dominated by (Pareto inferior to) the allocation $(\mathbf{x}^*; P^*)$ of the CEFE $(p^*, \mathbf{x}^*; P^*)$, since ceteris paribus the utilities in the two-person group $\{1, 2\}$ are strictly higher while the third individual obtains the same utility. Hence CEFE may even be Pareto-ranked.

Second, the presence of real power is consistent with Pareto optimality. In the CEFE $(p^*, \mathbf{x}^*; P^*)$ of Example 1, consumers 1 and 2 enjoy real power and the equilibrium allocation is Pareto-optimal. Third, absence of real power is consistent with Pareto optimality as well. E.g., in the absence of any externalities, competitive equilibria with free exit (with any group structure) exist under standard assumptions. Then the first welfare theorem holds and none of the consumers enjoys real power.

We conclude that while the presence of real power is compatible with Pareto optimality, the two properties are not closely related in general. But under certain circumstances, when strong large group advantage prevails, Pareto optimality requires real power:

Proposition 2 *Let $(p, \mathbf{x}; P)$ be a CEFE. Suppose that*

(i) *strong large group advantage prevails in all groups $h \in \mathcal{H}$ with $|h| = 2$;*

(ii) *strong large group advantage prevails in all groups $h \in P$ with $|h| > 1$;*

(iii) *the allocation $(\mathbf{x}; P)$ is Pareto-optimal and*

(iv) $V_i^0(p) = \max\{U_i(y_i; \{i\}) | \, y_i \in B_{\{i\}}(p)\}$ *holds for all consumers $i \in I$.*

Then there exists a consumer $i \in I$ who enjoys real power in the CEFE $(p, \mathbf{x}; P)$.

Proof. Let $(p, \mathbf{x}; P)$ be a CEFE such that (i) — (iv) hold. Then for (i) and (iii) to hold, there can be at most one consumer whose group in P is a singleton. For if $P = \{\ldots, \{i\}, \{j\}\}$ with $i, j \in I$, $i \neq j$, then because of strong large group advantage in group $h' = \{i, j\}$, i and j are better off and nobody is worse off at the feasible allocation $(\mathbf{x}; P')$ than at $(\mathbf{x}; P)$, where $P' = P \setminus \{\{i\}, \{j\}\} \cup \{\{i, j\}\}$. Since $(\mathbf{x}; P)$ is Pareto-optimal, this cannot be the case and, therefore, P contains at most one singleton. It follows, since $N \geq 2$, that P includes at least one group h with $|h| \geq 2$. By (ii) and (iv), such a multi-person group satisfies the hypothesis of Proposition 1 and, consequently, has a member who enjoys real power. ■

Finally, we provide a sufficient condition for the existence of CEFE where maximal real power and Pareto efficiency coexist. Maximal real power in a CEFE is realized if there is no other CEFE in which real power is higher for some individuals in their respective groups and not less for any individual.

Proposition 3 *Suppose pure group externalities, that is* $U_i(\mathbf{x_h}; h) = U_i^c(x_i) + U_i^g(h)$ *for* $\mathbf{x_h} \in \mathcal{X}_h, h \in \mathcal{H}$. *If*

(i) $(p, \mathbf{x})$ *is a competitive equilibrium of the pure exchange economy represented by* $(U_i^c, \omega_{\{i\}})_{i \in I}$,

(ii) *all* U_i^c, $i \in I$, *satisfy local non-satiation, and*

(iii) P^* *is the unique optimal group structure based solely on group preferences represented by* $U_i^g, i \in I$,

then

(iii) *the state* $(p, \mathbf{x}; P^*)$ *is a fully Pareto optimal CEFE and*

(iv) *there does not exist another CEFE in which real power is higher for some individuals in their respective groups and not less for any individual.*

The proof is given in the appendix. We illustrate Proposition 3 by means of an example.

Example 2. We can again use lead Example 1. Clearly, the group structure $P^* = \{\{1, 2\}, \{3\}\}$ is the unique optimal group structure based solely on group preferences. The competitive equilibrium based solely on $U_i^c(x_i)$, where all individuals are singles, was given by $p^0 = (1, 1)$, $x_1^0 = \left(\frac{1}{2}, 0\right)$, $x_2^0 = \left(0, \frac{1}{2}\right)$ and $x_3^0 = \left(\frac{1}{2}, \frac{1}{2}\right)$. Hence, real power, denoted by ρ_i^0, in the equilibrium $(p^0, \mathbf{x}^0, P^*)$ is given as:

$$\begin{aligned} \rho_1^0 &= \ln\frac{1}{2} + \ln v_1 - \ln\frac{1}{2} = \ln v_1 \\ \rho_2^0 &= \ln\frac{1}{2} + \ln v_2 - \ln\frac{1}{2} = \ln v_2 \\ \rho_3^0 &= 0 \end{aligned}$$

In any other CEFE that can generate real power we must have $P = P^*$. As discussed in section 4, equilibria with $P^* = \{\{1, 2\}, \{3\}\}$ have equilibrium prices $p^* = \left(1, \frac{1}{2\alpha}\right)$ with α denoting the utilitarian weight of individual 1 in the group $\{1, 2\}$ and $\alpha \in [\frac{1}{2v_1}, 1 - \frac{1}{2v_2}]$. Hence, such equilibria generate real power of:

$$\begin{aligned} \rho_1 &= \ln\frac{1}{2} + \ln v_1 - \ln\left(\frac{1}{4\alpha}\right) \\ \rho_2 &= \ln(1 - \alpha) + \ln v_2 - \ln\frac{1}{2} \\ \rho_3 &= 0 \end{aligned}$$

Hence, compared to $(p^0, \mathbf{x}^0, P^*)$ if ρ_1 is larger than ρ_1^0 $\left(\text{i.e. if } \alpha > \frac{1}{2}\right)$, then ρ_2 is smaller than ρ_2^0. If on the other hand ρ_2 is larger than ρ_2^0 (which corresponds to $\alpha < \frac{1}{2}$), then ρ_1 is smaller than ρ_1^0. Confirming Proposition 3, it is impossible that real power in a CEFE is larger for some individuals and not less for others than in $(p^0, \mathbf{x}^0, P^*)$. □□

We remark that in Examples 1 and 2, $(\mathbf{x}^*; P^*) = ((1/2, 0), (0, 1/2), (1/2, 1/2)); \{\{1, 2\}, \{3\}\})$ as well as $(\mathbf{x}; \{I\}) = ((0, 0), (0, 0), (1, 1)); \{I\})$ are full Pareto optima and, thus, P^* is not the only optimal group structure, although P^* is the unique optimal group structure based solely on group preferences. This seems at odds with Fact 1 (iii). Notice, however, that Fact 1 (iii) is derived under the assumption of utility functions $U_i^c : X_i \to \mathbb{R}$, $i \in I$, which is violated by $\ln x_i^i$. If need be, we could set $U_i^c(x_i^1, x_i^2) = \ln(\varepsilon + x_i^i)$ for $i \in \{1, 2\}, (x_i^1, x_i^2) \in X_i$. This would yield $\mathcal{P}_* = \mathcal{P}^* = \{P^*\}$ and result in the same conclusions otherwise, but render the calculations more complicated.

5 Formal and Real Power

We have seen that the existence of externalities can create real power for agents in a group. In Examples 1 and 2, this has been achieved with positive group externalities. The next example is a generalization of Example 1. It serves to introduce the concepts of formal power and to explore the relationship between formal and real power.

Example 3. The primitive data are the same as in Example 1, except for more general group externalities of the form $U_i^g(h) = \ln v_i$, with $v_i \geq 1$, for $h = \{1, 2\}$, $i = 1, 2$. The variables v_1 and v_2 stand for the extent of group externalities that individual 1 and 2 experience when they live together.

`Equilibria.` Commodity prices are normalized so that $p_1 = 1$. Then there exists a unique competitive equilibrium $(p^0, \mathbf{x^0}; P^0)$, given the group structure
$P^0 = \big\{\{1\}, \{2\}, \{3\}\big\}$:

$$p^0 = (1, 1),\ x_1^0 = (1/2, 0),\ x_2^0 = (0, 1/2),\ x_3^0 = (1/2, 1/2).$$

Like in Example 1, we obtain the CEFE with group structure $P^* = \big\{\{1, 2\}, \{3\}\big\}$ as the equilibria of the form $(p^*, \mathbf{x}^*; P^*)$ with

$$p^* = (1, 1/(2\alpha)),\ x_1^* = (1/2, 0),\ x_2^* = (0, 1-\alpha),\ x_3^* = (1/2, \alpha).$$

The non-exit conditions in group $\{1,2\}$ require $\alpha \in [\underline{\alpha}, \overline{\alpha}] = [\frac{1}{2v_1}, 1 - \frac{1}{2v_2}]$. □□

5.1 Utilitarian Social Welfare Maximization

Continuing with the analysis of Example 3, let us now assume a price system $p = (1, p_2)$ and further assume that group $h = \{1,2\}$ maximizes a utilitarian social welfare function

$$\begin{aligned} W_h &= \alpha\, U_1 + (1-\alpha)\, U_2 \\ &= \alpha \ln x_1^1 + (1-\alpha) \ln x_2^2 + \alpha \ln v_1 + (1-\alpha) \ln v_2, \end{aligned}$$

subject to the budget constraint $x_1^1 + p_2 x_2^2 = p_2$, where $0 < \alpha < 1$. The parameter α can be interpreted as the weight of individual 1 in group h. Similarly, $1-\alpha$ is the weight of individual 2. Solving the welfare maximization problem for group h yields a bundle in h's efficient budget set given by $x_1^1 = p_2\alpha$, $x_2^2 = 1-\alpha$, and $x_1^2 = x_2^1 = 0$.

The group externalities do not affect excess demand of group $h = \{1,2\}$. The excess demand vectors of the groups h and $h' = \{3\}$, denoted by z_h and $z_{h'}$, are given by

$$\begin{aligned} z_h &= (\alpha p_2, -\alpha), \\ z_{h'} &= \left(-\frac{1}{2}, \frac{1}{2p_2}\right). \end{aligned}$$

A market equilibrium without exit considerations $(p^*, \mathbf{x}^*)$, with given group structure P^*, would require

$$p^* = (1, 1/(2\alpha)),\ x_1^* = (1/2, 0),\ x_2^* = (0, 1-\alpha),\ x_3^* = (1/2, \alpha).$$

Hence the parameter α used previously corresponds to utilitarian welfare weights or Pareto weights α and $1-\alpha$ in group h in the equilibrium $(p^*, \mathbf{x}^*)$ with given group structure P^*: Every CEFE with group structure P^* is obtained as an equilibrium where group h maximizes its utilitarian welfare with the corresponding welfare weights. We further obtain

Fact 2 $\partial\underline{\alpha}/\partial v_1 < 0$ *and* $\partial\overline{\alpha}/\partial v_2 > 0$.

An increase in the positive externality of individual 1 decreases the lower bound on his weight in the group welfare function, since he is prepared to sacrifice more consumption in order to stay in the group. If $v_1 = 1$ we obtain $\underline{\alpha} = \frac{1}{2}$ and thus individual 1 has at least the same weight as individual 2. The opposite effects occur for $\overline{\alpha}$ when v_2 increases. $1 - \overline{\alpha}$ and thus the lower bound of the weight of individual 2 declines. For $v_2 = 1$ we have $\overline{\alpha} = \frac{1}{2}$.

5.2 Concepts of Power

At this stage it is useful to distinguish between different notions of power in a group. We follow closely the setup of the example and assume that the two-person group h maximizes a utilitarian welfare function. Then we can distinguish between two different concepts of power in a competitive equilibrium with free exit:

- **Formal power**, the say in a group decision expressed either by the weight of an individual's utility in the group welfare function or the weight of an individual in the Nash-bargaining decision of a group.
- **Real power**, the additional utility an individual can achieve in a group in comparison with exit.

To discuss the two notions of power, we consider Nash-bargained consumption choices in group $h = \{1, 2\}$ in Example 3. Let for $i = 1, 2$, $x_i^0(p_2)$ denote consumer i's individual demand at the price system $(1, p_2)$ when forming a single-person group. Let us consider the possibility that for every price p_2, group h maximizes the Nash product

$$\begin{aligned} N_h &= \left\{U_1^c(x_1) + U_1^g(h) - U_1^c\left(x_1^0(p_2)\right)\right\}^{\beta} \cdot \left\{U_2^c(x_2) + U_2^g(h) - U_2^c\left(x_2^0(p_2)\right)\right\}^{1-\beta} \\ &= \left\{\ln(x_1^1 \cdot v_1) - \ln\left(\frac{1}{2}p_2\right)\right\}^{\beta} \cdot \left\{\ln\left(x_2^2 \cdot v_2\right) - \ln\frac{1}{2}\right\}^{1-\beta} \end{aligned}$$

where β and $1 - \beta$, respectively, denotes the relative bargaining power of individual 1 and 2, respectively. Note that the conflict outcomes for group $h = \{1, 2\}$ are the outside options available at the price p_2. The outside option values (stand-alone values) amount to $V_1^0(p) = U_1^c(x_1^0(p_2)) = \ln(\frac{1}{2}p_2)$ for individual 1, and $V_2^0(p) = U_2^c(x_2^0(p_2)) = \ln(\frac{1}{2})$ for the second individual. Using the group budget constraint $x_1^1 = p_2 - p_2 x_2^2$, the first-order condition (w.r.t. x_2^2) for maximizing N_h amounts to:

$$\beta \cdot \frac{x_2^2}{1 - x_2^2} = (1 - \beta) \cdot \frac{\ln\left((1 - x_2^2) \cdot 2v_1\right)}{\ln\left(x_2^2 \cdot 2v_2\right)} \tag{3}$$

This is an implicit equation for x_2^2. Now suppose the same allocation is obtained in a competitive equilibrium with free exit where the group maximizes its utilitarian welfare function W_h, with respective weights α and $1-\alpha$. Then we have $x_2^2 = 1-\alpha$ and thus equation (3) becomes an implicit equation for β:

$$\frac{\beta}{1-\beta} = \frac{\alpha}{1-\alpha}\frac{\ln(\alpha\, 2v_1)}{\ln\big((1-\alpha)2v_2\big)} \tag{4}$$

For each value of $\alpha \in [\underline{\alpha}, \overline{\alpha}]$, equation (4) defines a unique $\beta(\alpha) \in [0,1]$, the relative bargaining power of individual 1 that yields the same group decision as the group's utilitarian welfare maximum with weights α and $1-\alpha$. Hence by definition of $\beta(\alpha)$, the weight α in W_h and the weight $\beta = \beta(\alpha)$ in N_h lead to the same allocation for group h. We obtain the following properties for $\beta(\alpha)$:

Fact 3 $\beta\left(\frac{1}{2v_1}\right) = 0,\ \beta\left(1-\frac{1}{2v_2}\right) = 1,\ \frac{\partial\beta}{\partial\alpha} > 0.$

5.3 Relationship between Formal and Real Power

Continuing the examination of power in the context of Example 3, we note that a higher weight in the group welfare function translates into higher relative bargaining power, as long as α is in the range $\left[\frac{1}{2v_1}, 1-\frac{1}{2v_2}\right]$, for which the competitive equilibrium with free exit involving group h exists. The maximal utilities of the individuals are given by

$$U_1 = \begin{cases} \ln\frac{1}{2} + \ln v_1 & \text{if } \alpha \in \left[\frac{1}{2v_1}, 1-\frac{1}{2v_2}\right], \\ \ln\frac{1}{2} & \text{if } \alpha \notin \left[\frac{1}{2v_1}, 1-\frac{1}{2v_2}\right], \end{cases}$$

$$U_2 = \begin{cases} \ln(1-\alpha) + \ln v_2 & \text{if } \alpha \in \left[\frac{1}{2v_1}, 1-\frac{1}{2v_2}\right], \\ \ln\frac{1}{2} & \text{if } \alpha \notin \left[\frac{1}{2v_1}, 1-\frac{1}{2v_2}\right], \end{cases}$$

where we have assumed that $(p^*, \mathbf{x}^*; P^*)$ prevails for $\alpha \in \left[\frac{1}{2v_1}, 1-\frac{1}{2v_2}\right]$, while only $(p^0, \mathbf{x^0}; P^0)$ can occur for $\alpha \notin \left[\frac{1}{2v_1}, 1-\frac{1}{2v_2}\right]$. Note that for $\alpha \in \left[\frac{1}{2v_1}, 1-\frac{1}{2v_2}\right]$, the stand-alone utilities achieved by exit are $\ln\left(\frac{1}{4\alpha}\right)$ for individual 1 and $\ln\left(\frac{1}{2}\right)$ for individual 2.

We obtain the following comparisons of utilitarian, bargaining and real power:

CASE 1. If $\alpha < \frac{1}{2v_1}$ and thus agent 2 has a large weight $1 - \alpha$, formation of group h is impossible in equilibrium, since individual 1 is better off as a single since $\ln\left(\frac{1}{2}\right) + \ln v_1 < \ln\left(\frac{1}{4\alpha}\right)$. Moreover, no real power or bargaining power exists.

CASE 2. If $\frac{1}{2v_1} \leq \alpha \leq 1 - \frac{1}{2v_2}$, formation of group h is possible. Higher utilitarian power for an individual translates into higher bargaining power. But only for individual 2 does higher formal bargaining power $1 - \alpha$ or $1 - \beta(\alpha)$ yield higher utility $\ln(1 - \alpha) + \ln v_2$. Consequently, the consumer's real power increases as well, since her stand-alone utility remains constant at $\ln \frac{1}{2}$. For individual 1, an increase in utilitarian and bargaining power yields a negative price effect, since the market value of the endowments of group h decreases. For this individual, the negative price effect and an increase in his bargaining power offset each other exactly and his utility remains constant. Nevertheless, the individual's real power increases in α. The reason is that the individual's stand-alone utility decreases, since $V_i^0(p_2) = \ln(\frac{1}{2}p_2) = \ln(\frac{1}{4\alpha})$.

CASE 3. If $\alpha > 1 - \frac{1}{2v_2}$, group h does not form, as individual 2 is better off as a single and thus no bargaining or real power exists.

Overall, we observe that higher formal power may not translate into higher real power because groups dissolve, like in CASE 1 and CASE 3. Also, higher formal power may not yield higher equilibrium utility because adverse price effects outweigh the higher weight in group decisions, like for consumer 1 in Examples 1 and 3.

5.4 Origins of Power

Example 3 further demonstrates that if an individual is neither an externality generator nor an externality receiver, his real power in a two-person group is zero. Namely, if $v_1 = v_2 = 1$, then group $\{1, 2\}$ only forms in a CEFE when $\alpha = 1/2$ and both consumers have zero real power.

However, the same is no longer true in larger groups. Let us continue the example, but modify utility in the following way.

Example 4. Let again $\ell = 2$, $I = \{1,2,3\}$ and preferences be represented by

$$U_1(x_1^1, x_1^2; h) = \begin{cases} x_1^1 + v_1 & \text{in case } h = \{1,2\} \text{ or } h = \{1,2,3\}, \\ x_1^1 & \text{in all other cases;} \end{cases}$$

$$U_2(x_2^1, x_2^2; h) = \begin{cases} x_2^2 + v_2 & \text{in case } h = \{1,2\} \text{ or } h = \{1,2,3\}\ , \\ x_2^2 & \text{in all other cases;} \end{cases}$$

$$U_3(x_3^1, x_3^2; h) = \frac{1}{2}x_3^1 + \frac{1}{2}x_3^2.$$

We assume $\frac{1}{2} \geq v_1 \geq 0$ and $\frac{1}{2} \geq v_2 \geq 0$. There exists an equilibrium with free exit $(p, \mathbf{x}; P)$ with group structure $P = \big\{\{1,2,3\}\big\}$, namely:

$$p = (1,1),\ x_1 = (1/2 - v_1, 0),\ x_2 = (0, 1/2 - v_2),\ x_3 = (v_1 + 1/2, v_2 + 1/2).$$

Note that the allocation is an efficient choice of the sole group $h = \{1,2,3\}$ at the going prices. Moreover, markets clear. And neither individual 1 nor individual 2 can gain utility by leaving the group. Although individual $i = 3$ is neither an externality generator nor an externality receiver, he has real power whereas individuals 1 and 2 do not have any real power. In fact, he extracts all the surplus generated by the favorable externalities which individuals 1 and 2 generate by living together. His only contribution is that he does not destroy the externalities the other individuals in the group generate and receive when he is part of the group.

Fact 4 *Suppose that an individual is neither an externality generator nor an externality receiver. Then his real power can be positive if he belongs to a group with more than two members.*

Of course, there are other equilibria with free exit in the above example where all three individuals have real power or in which the third individual is powerless. □□

Larger group size may be conducive to more real power, but only if larger group size gives rise to more positive externalities.

Example 5. Let $\ell = 2$, $I = \{1,2,3\}$. The individual endowments are as in Examples 1 to 3. Group externalities are similar to Example 3 and of the form

$U_i^g(h) = \ln v_i$, with $v_i > 1$, for $h = \{1,2\}$ or I, $i = 1,2$ and for $h = I$, $i = 3$;
$U_i^g(h) = 0$ otherwise.

We find

Fact 5 *There exists a CEFE with group structure $\{I\}$ where consumer 1 has more real power than in any CEFE with group structure $P^* = \{\{1,2\},\{3\}\}$.*

Fact 5 is proven in the appendix. Here the positive externality received by consumer 3 creates additional surplus that can benefit consumer 1. In contrast, in Example 3, there is no surplus to be shared in group I and consumer 1 has no power when this group is formed. □□

6 Quasi-linear Preferences and Spillovers

In this section, we consider the special case of quasi-linear preferences. This case allows to further illustrate our main propositions. It also serves to illustrate to what extent changes of formal power in the form of bargaining power in one group affect other groups. Such an effect is called **power spillover**.

6.1 Setup

We examine a society where $n = N/2 > 1$ two-member groups will be formed. Group $h \in \{1, \ldots, n\}$ has members $h1$ and $h2$, called the first member and the second member. This group structure is denoted by $\widehat{P}$. There are $\ell > 1$ goods. The consumption of good k $(k = 1, \ldots, \ell)$ by individual hi $(i = 1, 2)$ is denoted by x_{hi}^k. The vector $x_{hi} = \left(x_{hi}^1, \ldots, x_{hi}^\ell\right)$ denotes the consumption of group member hi. Each group h is endowed with $\omega_h = \left(\omega_h^1, \ldots, \omega_h^\ell\right)$. The two members of group h have quasi-linear utility representations, given by

$$U_{h1}(\mathbf{x_h}; h) = V_{h1}\left(x_{h1}^1, \ldots, x_{h1}^{\ell-1}\right) + x_{h1}^\ell + v_1 \tag{5}$$

$$U_{h2}(\mathbf{x_h}; h) = V_{h2}\left(x_{h2}^1, \ldots, x_{h2}^{\ell-1}\right) + x_{h2}^\ell + v_2 \tag{6}$$

where V_{hi} is assumed to be strictly concave and strictly increasing in $(x_{hi}^1, \ldots, x_{hi}^{\ell-1})$. The parameters $v_1 > 0$ and $v_2 > 0$, which are the same in all two-person groups, capture the group externalities that individuals $h1$ and $h2$ experience when living together. When a person is single, he or she has the same utility function with respect to consumption as in a two-person group, but does not experience group externalities. Living together with the same type of individual[3] or in a group with more than two individuals is assumed to exert negative group externalities on everybody. Hence, such groups will never be formed in a CEFE. For $p \gg 0$, group h maximizes

$$S_h = \left\{U_{h1}(\mathbf{x_h}; h) - U_{h1}\left(x^0_{h1}(p); \{h1\}\right)\right\}^{\beta_h} \left\{U_{h2}(\mathbf{x_h}; h) - U_{h2}\left(x^0_{h2}(p); \{h2\}\right)\right\}^{1-\beta_h} \quad (7)$$

subject to the constraints $\mathbf{x_h} \in B_h(p)$, $U_{h1}(\mathbf{x_h}; h) - U_{h1}\left(x^0_{h1}(p); \{h1\}\right) \geq 0$ and $U_{h2}(\mathbf{x_h}; h) - U_{h2}\left(x^0_{h2}(p); \{h2\}\right) \geq 0$, where $0 < \beta_h < 1$ is the bargaining power of individual $h1$ in group h. The functions $x^0_{h1}(p)$ and $x^0_{h2}(p)$ denote consumer $h1$'s and $h2$'s individual demand at the price system p when they are singles.

6.2 Equilibria

For the group structure $\widehat{P}$, we denote equilibrium values by $\widehat{x}^k_{hi}$, equilibrium utilities by $\widehat{U}_{hi}$ and $\widehat{V}_{hi}$, real power by $\widehat{\rho}_{hi}$, and the equilibrium prices by $\widehat{p}$. Then $(\widehat{p}, \widehat{\mathbf{x}}; \widehat{P})$ is a CEFE. In the following, we assume that for any array of bargaining power parameters $(\beta_1, \ldots, \beta_n)$ under consideration: (a) Every group member consumes a non-negative amount of the numéraire good ℓ in the CEFE. (b) For the given group structure $\widehat{P}$ and array $(\beta_1, \ldots, \beta_n)$, the economy has a unique equilibrium $(p, \mathbf{x})$, up to price normalization. We obtain:

Proposition 4

(i) $\dfrac{\partial \widehat{p}}{\partial \beta_h} = 0.$

(ii) $\dfrac{\partial \widehat{x}^k_{h1}}{\partial \beta_h} = \dfrac{\partial \widehat{x}^k_{h2}}{\partial \beta_h} = 0 \;\; \forall k = 1, \ldots, \ell - 1.$

(iii) $\dfrac{\partial \widehat{x}^\ell_{h1}}{\partial \beta_h} > 0, \dfrac{\partial \widehat{x}^\ell_{h2}}{\partial \beta_h} < 0.$

(iv) $\widehat{\rho}_{h1}/\widehat{\rho}_{h2} = \beta_h/(1 - \beta_h).$

(v) *Suppose that groups are homogeneous with respect to utility representations* ($V_{h1} =$

[3] That is, if for instance members $h1$ and $g1$ of two different groups $g \in \widehat{P}$ and $h \in \widehat{P}$ formed a new two-person group, both members would experience negative group externalities.

V_1, $V_{h2} = V_2$) *and endowments with* $\omega_h = \overline{\omega}$, $\forall h = 1, \ldots, n$. *Then:*

$$\begin{aligned}\widehat{x}^{\ell}_{h1} = \beta_h \overline{\omega}^{\ell} \quad &+ \quad \beta_h \left\{ \widehat{V}_{h2} + v_2 - U_{h2}\left(x^0_{h2}(\widehat{p}); \{h2\}\right)\right\} \\ &- \quad (1-\beta_h)\left\{ \widehat{V}_{h1} + v_1 - U_{h1}\left(x^0_{h1}(\widehat{p}); \{h1\}\right)\right\}, \end{aligned} \tag{8}$$

$$\begin{aligned}\widehat{x}^{\ell}_{h2} = (1-\beta_h) \overline{\omega}^{\ell} \quad &+ \quad (1-\beta_h) \left\{ \widehat{V}_{h1} + v_1 - U_{h1}\left(x^0_{h1}(\widehat{p}); \{h1\}\right)\right\} \\ &- \quad \beta_h\left\{ \widehat{V}_{h2} + v_2 - U_{h2}\left(x^0_{h2}(\widehat{p}); \{h2\}\right)\right\}. \end{aligned} \tag{9}$$

The proof is given in the appendix.

6.3 Discussion

Proposition 4 has several interesting implications which we explore in this subsection.

a) ***Power Shifts and Power Spillovers***

According to Proposition 4 (iv), relative real power and relative bargaining power of a group member coincide. As Proposition 4 further illustrates, a change of bargaining power in group h only influences the distribution of the numéraire in group h. Consumption of the first $\ell-1$ commodities is not affected. Moreover, consumption in other groups is not affected either because $\widehat{p}$ remains the same. We formulate this observation as

Corollary 1 (Absence of Power Spillovers) *A change of* β_h *in a particular group* h *has no impact on individuals in other groups.*

The corollary states that there are no power spillovers in the case of quasi-linear preferences as bargaining power changes do not affect prices.

b) ***Manipulability***

Corollary 1 also means that a group h cannot manipulate outcomes and possibly improve the utility of all group members at the expense of outsiders by misrepresenting internal bargaining power. Using different bargaining power only within a particular group is merely redistributing utility within this group. Makowski, Ostroy and Segal

(1999) have comprehensively characterized continuous, efficient and anonymous incentive compatible mechanisms and have shown that such mechanisms must be perfectly competitive. Quasi-linear preferences are one of the examples that can allow for incentive compatible mechanisms or perfect competition. Our investigation shows that with quasi-linear preferences and the exit option a multi-person group has no incentive to misrepresent the internal bargaining power.

c) *Pareto Optimality*

Since $v_1 > 0$ and $v_2 > 0$, $\widehat{U}_{h1} - U_{h1}\left(x^0_{h1}(\widehat{p}); \{h1\}\right) > 0$ and $\widehat{U}_{h2} - U_{h2}\left(x^0_{h2}(\widehat{p}); \{h2\}\right) > 0$ hold when group h maximizes the Nash product (7). Hence $(p, \mathbf{x}, \widehat{P})$ is a CEFE where all consumers enjoy real power. But is $(\mathbf{x}; \widehat{P})$ also fully Pareto optimal? The answer is in the affirmative if $\widehat{P}$ is the unique optimal group structure based solely on group preferences. Namely, then $U^g_i(\widehat{P}(i)) \geq U^g_i(P'(i))$ for any group structure P' and $i \in I$, as a consequence of Fact 1 (i). By Proposition 1 of Haller (2000), $\mathbf{x}$ is Pareto optimal given the group structure $\widehat{P}$. Suppose $(\mathbf{x}; \widehat{P})$ is not fully Pareto optimal. Then there exists a feasible allocation $(\mathbf{x}', P')$ such that $(\mathcal{U}_i(\mathbf{x}'; P'))_{i \in I} > (\mathcal{U}_i(\mathbf{x}; \widehat{P}))_{i \in I}$. But then $U^g_i(\widehat{P}(i)) \geq U^g_i(P'(i))$ for all i yields $(\mathcal{U}_i(\mathbf{x}'; \widehat{P}))_{i \in I} \geq (\mathcal{U}_i(\mathbf{x}'; P'))_{i \in I} > (\mathcal{U}_i(\mathbf{x}; \widehat{P}))_{i \in I}$, contradicting the fact that $\mathbf{x}$ is Pareto optimal given the group structure $\widehat{P}$.

6.4 The Impact of Group Externalities

In previous sections, we stressed the role of group externalities. Closer inspection of the proof of Proposition 4 (v) demonstrates that a change of v_1 or v_2 only affects the distribution of the numéraire good within groups as long as the exit conditions are met and we obtain:

Corollary 2
Suppose that groups are homogeneous with respect to utility representations and endowments with $\omega_h = \overline{\omega}$, $\forall h = 1, \ldots, n$. Then:

(i) $\dfrac{\partial \widehat{x}^\ell_{h1}}{\partial v_1} < 0, \quad \dfrac{\partial \widehat{x}^\ell_{h1}}{\partial v_2} > 0;$

(ii) $\dfrac{\partial \widehat{x}^\ell_{h2}}{\partial v_2} < 0, \quad \dfrac{\partial \widehat{x}^\ell_{h2}}{\partial v_1} > 0;$

(iii) $\dfrac{\partial \widehat{U}_{hi}}{\partial v_i} > 0, \quad \dfrac{\partial \widehat{U}_{hi}}{\partial v_j} > 0, \quad i \neq j.$

Hence, if individual $h1$ gains relatively more from living in group h, i.e., when v_1 increases, he receives less of the numéraire good. But the net effect on utility is positive. Since equilibrium prices are not affected and thus stand-alone utilities remain the same, real power of both individuals increases when v_1 becomes larger.

7 Existence of CEFE

We finally identify circumstances when CEFE exist. Proposition 2 of Gersbach and Haller (2011) presents sufficient conditions for the existence of a non-trivial CEFE $(p; \mathbf{x}; P)$ with a group $h \in P$ and thresholds $\delta_i(p) \geq 0, i \in h$, such that $|h| \geq 2$ and $U_i(\mathbf{x_h}; h) - V_i^0(p) \geq \delta_i(p)$ for $i \in h$. Individual i enjoys real power in $(p; \mathbf{x}; P)$ in case $U_i(\mathbf{x_h}; h) - V_i^0(p) > \delta_i(p)$ or $\delta_i(p) > 0$.

More specific premises yield more specific conclusions:

Proposition 5 *Suppose pure group externalities, that is* $U_i(\mathbf{x_h}; h) = U_i^c(x_i) + U_i^g(h)$ *for all* $h \in \mathcal{H}, \mathbf{x_h} \in \mathcal{X}_h$. *If*

(i) $(p, \mathbf{x})$ *is a competitive equilibrium of the pure exchange economy represented by* $(U_i^c, \omega_{\{i\}})_{i \in I}$,

(ii) *all* U_i^c, $i \in I$, *satisfy local non-satiation,*

(iii) P *is an optimal group structure solely based on group preferences,*

(iv) *strong large group advantage prevails in all groups* $h \in \mathcal{H}$ *with* $|h| = 2$,

(v) *strong large group advantage prevails in all groups* $h \in P$ *with* $|h| > 1$,

(vi) $V_i^0(p) = \max\{U_i(y_i; \{i\}) | \, y_i \in B_{\{i\}}(p)\}$ *holds for all consumers* $i \in I$,

then $(p, \mathbf{x}; P)$ *is a CEFE where some consumer enjoys real power.*

Observe that conditions (iv) and (v) can be expressed in terms of the functions U_i^g.

Proof. Because of pure group externalities, (i), (ii) and (v), $(p, \mathbf{x}; P)$ is a CEFE as exit cannot increase utility. By an argument similar to the one given in the proof of Proposition 2, pure group externalities, (iii) and (iv) imply that P contains at least one multi-member group h. By (v) and (vi), such a multi-member group satisfies the hypothesis of Proposition 1 and, consequently, has a member who enjoys real power in the CEFE $(p, \mathbf{x}; P)$. ■

8 Conclusion

In this paper we have presented a framework that allows to integrate formal and real power into general equilibrium models. We aim at taking a first step toward a study of causes and consequences of power in market economies. There are many desirable extensions, such as allowing agents to generate positive externalities for others endogenously by being friendly or helpful. Moreover, an intriguing question is whether wealthier individuals have higher formal or real power. The analysis of this and other questions related to power must await future research.

9 Appendix

Proof of Proposition 3

Recall that for $i \in I$ and $P \in \mathcal{P}$, $P(i)$ denotes the group to which i belongs in the group structure P.

First, we show that $(\mathbf{x}; P^*)$ is a fully Pareto optimal allocation. For suppose not. Then there exists a feasible allocation $(\mathbf{y}; P)$ that Pareto dominates $(\mathbf{x}; P^*)$: $U_i(y_i; P(i)) > U_i(x_i; P^*(i))$ for some $i \in I$ and $U_i(y_i; P(i)) \geq U_i(x_i; P^*(i))$ for all $i \in I$. Since $(p, \mathbf{x})$ is a competitive equilibrium of the pure exchange economy $(U_i^c, \omega_{\{i\}})_{i \in I}$ and consumers are locally non-satiated, $\mathbf{x}$ is a Pareto optimal allocation of the pure exchange economy. Therefore, if it is the case that $U_i^c(y_i) > U_i^c(x_i)$ for some i, then there exists $j \neq i$ with $U_j^c(y_j) < U_j^c(x_j)$ and, consequently, $U_j^g(P(j)) > U_j^g(P^*(j))$ (because $(\mathbf{y}; P)$ Pareto dominates $(\mathbf{x}; P^*)$). If it is the case that $U_i^c(y_i) \leq U_i^c(x_i)$ for all i, then $U_j^g(P(j)) > U_j^g(P^*(j))$ for some j. In any case, $U_j^g(P(j)) > U_j^g(P^*(j))$ for some j. But then, by Fact 1 (i) there exists an optimal group structure P' based solely on group preferences such that $U_j^g(P'(j)) \geq U_j^g(P(j)) > U_j^g(P^*(j))$ and, consequently, $P' \neq P^*$, contradicting (iii).

Second, we show that $(p, \mathbf{x}; P^*)$ is a CEFE. Because of pure group externalities, (i) and (ii), the first two conditions for a CEFE hold. Moreover for $i \in I$:

(a) Since $(p, \mathbf{x})$ is a competitive equilibrium of the pure exchange economy $(U_i^c, \omega_{\{i\}})_{i \in I}$, x_i is an optimal consumption bundle in i's budget set.

(b) Since P^* is the unique optimal group structure solely based on group preferences, $U_i^g(\{i\}) \leq U_i^g(P^*(i))$, by Fact 1 (i).

Hence i cannot fare better as a one-person group. Thus the third condition for a CEFE holds as well.

Third, we show that there does not exist another CEFE in which real power is higher for some individuals in their respective groups and not lower for any individual. Namely, suppose that there exists a CEFE $(p', \mathbf{y}; P')$ in which real power is higher for some individuals and not lower for any individual than in $(p, \mathbf{x}; P^*)$. Hence,

$$U_i^c(y_i) + U_i^g\left(P'(i)\right) - V_i^0(p') \geq U_i^c(x_i) + U_i^g\left(P^*(i)\right) - V_i^0(p)$$

for all i, with strict inequality for some i. As P^* is the unique optimal group structure based solely on group preferences, we obtain $U_i^g(P'(i)) \leq U_i^g(P^*(i))$ for all i by Fact 1 (i). Moreover, $U_i^c(x_i) = V_i^0(p)$ holds because of (i). Therefore,

$$U_i^c(y_i) - V_i^0(p') \geq U_i^c(x_i) - V_i^0(p) = 0$$

for all individuals i, with strict inequality for some individual. Now pick $j \in I$ with $U_j^c(y_j) > V_j^0(p')$. If for some $i \in P'(j)$, $p'y_i < p'\omega_{\{i\}}$, then local non-satiation implies $V_i^0(p') > U_i^c(y_i)$, contradicting $U_i^c(y_i) \geq V_i^0(p')$. Therefore, $p'y_i \geq p'\omega_{\{i\}}$ for all $i \in P'(j)$. Since $\mathbf{y}_{P'(j)} \in EB_{P'(j)}(p')$, this implies $p'y_i = p'\omega_{\{i\}}$ and, consequently, $U_i^c(y_i) \leq V_i^0(p')$ for all $i \in P'(j)$. Thus a contradiction to $U_j^c(y_j) > V_j^0(p')$ results. Hence, to the contrary, it cannot be the case that in the CEFE $(p', \mathbf{y}; P')$, real power is higher for some individuals and not lower for any individual than in $(p, \mathbf{x}; P^*)$. ■

Proof of Fact 5

STEP 1: We first determine equilibrium quantities and examine the non-exit conditions in case $P = \{I\}$. Given the price system $p = (1, p_2)$, group $\{1, 2, 3\}$ solves

$$\max_{(x_1^1, x_2^2, x_3^1, x_3^2)} \left[\alpha_1(\ln x_1^1 + \ln v_1) + \alpha_2(\ln x_2^2 + \ln v_2) + \frac{1}{2}(1 - \alpha_1 - \alpha_2)(\ln x_3^1 + \ln x_3^2 + 2 \ln v_3) \right]$$

subject to $x_1^1 + x_3^1 + p_2 x_2^2 + p_2 x_3^2 = 1 + p_2$. This yields

$$\begin{aligned} &x_1^1 = \alpha_1 \cdot (1 + p_2), x_2^2 = \alpha_2 \cdot (1 + p_2)/p_2, \\ &x_3^1 = (1 - \alpha_1 - \alpha_2) \cdot (1 + p_2)/2, x_3^2 = (1 - \alpha_1 - \alpha_2) \cdot (1 + p_2)/(2p_2). \end{aligned}$$

The stand-alone demands are:

$$x_1^{10} = \frac{1}{2} p_2, x_2^{20} = \frac{1}{2}, x_3^{10} = \frac{1}{2}, x_3^{20} = \frac{1}{2p_2}.$$

The market clearing price is given by $1 + p_2^* = \dfrac{2}{1 + \alpha_1 - \alpha_2}$.

Then the non-exit condition for the first individual is:

$$\ln\left(\alpha_1\left(p_2^* + 1\right)\right) + \ln v_1 \geq \ln\left(\frac{1}{2} p_2^*\right),$$

which is equivalent to $\alpha_1 \geq \frac{p_2^*}{2(p_2^*+1)v_1}$ and $\alpha_1 \geq \frac{1+\alpha_2}{4v_1+1}$.

Similarly, for the second individual we obtain

$$\ln\left(\alpha_2 \frac{p_2^* + 1}{p_2^*}\right) + \ln v_2 \geq \ln\frac{1}{2}$$

or $\alpha_2 \geq \frac{p_2^*}{(p_2^*+1)2v_2}$ which is equivalent to $\alpha_2 \geq \frac{1-\alpha_1}{4v_2-1}$.

Finally, the third individual's non-exit condition amounts to

$$\frac{1}{2}\left\{\ln\frac{1}{2}+\ln\frac{1}{2p_2^*}\right\} \leq \frac{1}{2}\ln\left((1-\alpha_1-\alpha_2)\frac{p_2^*+1}{2}\right)+\frac{1}{2}\ln\left((1-\alpha_1-\alpha_2)\frac{p_2^*+1}{2p_2^*}\right)+\ln v_3.$$

It implies $(1-\alpha_1-\alpha_2)(p_2^*+1) \geq 1/v_3$ or $1-\alpha_1-\alpha_2 \geq (1+\alpha_1-\alpha_2)/(2v_3)$ or $\alpha_1 \leq \frac{2v_3-1}{2v_3+1}\cdot(1-\alpha_2)$.

STEP 2: We next examine whether real power of individual 1 can be equal or higher under the group structure $P = \{\{1,2\},\{3\}\}$ than under $P^* = \{I\}$. That is, we examine whether it is possible to delineate parameter values such that $\rho_1 \geq \max\{\rho_1^*\}$, where $\rho_1 = \ln\frac{1}{2}+\ln v_1+\ln(4\alpha)$ for $\alpha \in [\frac{1}{2}, 1-\frac{1}{2v_2}]$ (from Example 2) and ρ_1^* is 1's real power in the above equilibrium. When choosing the maximal $\alpha = 1-\frac{1}{2v_2}$, the inequality is equivalent to

$$\ln\left(\frac{4v_2-2}{2v_2}\right) \geq \max\left\{\ln\left(\frac{2\alpha_1(1+p_2^*)}{p_2^*}\right)\right\}$$

which implies

$$\frac{4v_2-2}{2v_2} \geq \max\left\{\frac{4\alpha_1}{1-\alpha_1+\alpha_2}\right\}.$$

Observe that with $\alpha_1 = \alpha_2 = 1/4$, all three non-exit conditions are satisfied and individual 1 enjoys real power $\rho_1^* = \ln v_1 > 0$ in the corresponding CEFE. Therefore, in order to maximize ρ_1^*, it suffices to solve the following problem:

$$\max_{\alpha_1,\alpha_2\in[0,1]}\left\{\frac{4\alpha_1}{1-\alpha_1+\alpha_2}\right\}$$

s.t.

$$\begin{aligned}\alpha_2 &\geq \frac{1-\alpha_1}{4v_2-1},\\ \alpha_1 &\leq \frac{2v_3-1}{2v_3+1}\cdot(1-\alpha_2).\end{aligned}$$

It follows that the optimal solution for α_2 is given by $\alpha_2 = \frac{1-\alpha_1}{4v_2-1}$ since $\frac{\partial\rho_1^*}{\partial\alpha_2} < 0, \frac{\partial\rho_1^*}{\partial\alpha_1} > 0$ and the right-hand side of the last constraint is monotonically decreasing in α_2. Hence our problem is reduced to

$$\max_{\alpha_1\in[0,1]}\left\{\frac{(4v_2-1)\alpha_1}{v_2(1-\alpha_1)}\right\}$$

where the constraint amounts to

$$(2v_3 + 1)\alpha_1 \leq (2v_3 - 1) \cdot \left(1 - \frac{1 - \alpha_1}{4v_2 - 1}\right)$$

which leads to

$$\alpha_1 \leq \frac{(2v_2 - 1)(2v_3 - 1)}{2(v_2 - v_3 + 2v_3 v_2)} < 1.$$

Hence, maximization of ρ_1^* is obtained by

$$\alpha_1 = \frac{(2v_2 - 1)(2v_3 - 1)}{2(v_2 - v_3 + 2v_3 v_2)}.$$

Now based on the foregoing transformation, the inequality $\rho_1 \geq \max\{\rho_1^*\}$ leads to

$$\frac{4v_2 - 2}{2v_2} \geq \frac{(4v_2 - 1)\alpha_1}{v_2(1 - \alpha_1)}$$

which implies $\alpha_1 \leq \frac{2v_2 - 1}{6v_2 - 2}$. Hence, $\widehat{\rho_1} \geq \max\{\rho_1^*\}$ yields

$$\frac{(2v_2 - 1)(2v_3 - 1)}{2(v_2 - v_3 + 2v_2 v_3)} \leq \frac{2v_2 - 1}{6v_2 - 2}$$

which implies $4v_3 v_2 - 4v_2 - v_3 \leq -1$ or $4v_2(v_3 - 1) \leq v_3 - 1$.

Since $v_3 > 1$, the latter implies $4v_2 \leq 1$ which contradicts $v_2 \geq 1$ and, therefore, $\widehat{\rho_1} < \max\{\rho_1^*\}$ has to hold.

Hence, there are no parameter constellations such that the maximal real power of individual 1 in a CEFE with $P = I$ is strictly smaller than $\widehat{\rho_1}$.

■

Notice that in case $v_3 = 1$, we are back to Example 2 and $\widehat{\rho_1} = \max\{\rho_1^*\}$. In fact, in case $v_3 = 1$, the foregoing proof shows that $\widehat{\rho_1} > \max\{\rho_1^*\}$ would yield $0 = 4v_2(v_3 - 1) < v_3 - 1 = 0$ and, thus, $0 < 0$; therefore, $\widehat{\rho_1} \leq \max\{\rho_1^*\}$ has to hold.

Proof of Proposition 4

Good ℓ serves as a numéraire so that the price system assumes the form $(p_1, \ldots, p_{\ell-1}, 1)$. We are focusing on interior solutions regarding all commodities, including the numéraire good.[4] Let us consider then the first-order conditions for maximizing $\ln S_h$ in group h, subject to h's budget constraint:

$$\begin{aligned}
\beta_h \frac{1}{U_{h1} - U_{h1}(x^0_{h1}(p); \{h1\})} \frac{\partial V_{h1}}{\partial x^k_{h1}} - \lambda_h p_k &= 0, \quad k = 1, \ldots, \ell - 1; \\
\beta_h \frac{1}{U_{h1} - U_{h1}(x^0_{h1}(p); \{h1\})} - \lambda_h &= 0; \\
(1 - \beta_h) \frac{1}{U_{h2} - U_{h2}(x^0_{h2}(p); \{h2\})} \frac{\partial V_{h2}}{\partial x^k_{h2}} - \lambda_h p_k &= 0, \quad k = 1, \ldots, \ell - 1; \\
(1 - \beta_h) \frac{1}{U_{h2} - U_{h2}(x^0_{h2}(p); \{h2\})} - \lambda_h &= 0.
\end{aligned}$$

Therefore:

$$\lambda_h = \beta_h \frac{1}{U_{h1} - U_{h1}(x^0_{h1}(p); \{h1\})} = (1 - \beta_h) \frac{1}{U_{h2} - U_{h2}(x^0_{h2}(p); \{h2\})}. \tag{10}$$

$$\frac{\partial V_{h1}}{\partial x^k_{h1}} = \frac{\partial V_{h2}}{\partial x^k_{h2}} = p_k, \quad k = 1, \ldots, \ell - 1. \tag{11}$$

Equation (11) implies that the demand of group h for commodities $k = 1, \ldots, \ell - 1$ is independent of the bargaining power β_h and $1 - \beta_h$ of individual $h1$ and $h2$, respectively. Since the V_{hi} are strictly concave and strictly increasing, the budget of the particular group h is exhausted. It follows that h's total demand for commodity ℓ is independent of β_h as well. Therefore, aggregate demand and, thus, equilibria in commodity markets do not depend on internal bargaining power of groups. As a consequence, changes of bargaining power in group h have no effect on equilibrium prices. This establishes points (i) and (ii).

However, a shift of the power in groups affects the distribution of the numéraire good in group h. Using the notation for the equilibria we have from equation (10):

$$\frac{\beta_h}{\widehat{V}_{h1} + \widehat{x}^\ell_{h1} + v_1 - U_{h1}(x^0_{h1}(\widehat{p}); \{h1\})} = \frac{1 - \beta_h}{\widehat{V}_{h2} + \widehat{x}^\ell_{h2} + v_2 - U_{h2}(x^0_{h2}(\widehat{p}); \{h2\})} \tag{12}$$

Since $\widehat{V}_{h1}, v_1, U_{h1}(x^0_{h1}(\widehat{p}); \{h1\})$ and $\widehat{V}_{h2}, v_2, U_{h2}(x^0_{h2}(\widehat{p}); \{h2\})$ are independent of β_h and $\widehat{x}^\ell_{h1} + \widehat{x}^\ell_{h2}$ does not depend on β_h either, we obtain the third point (iii):

[4] This is guaranteed if the endowments of all individuals with the numéraire good is sufficiently large. The assumption allows us to work with the entire set of first-order conditions.

$$\frac{\partial \widehat{x}^{\ell}_{h1}}{\partial \beta_h} > 0, \quad \frac{\partial \widehat{x}^{\ell}_{h2}}{\partial \beta_h} < 0$$

Furthermore, (12) is tantamount to (iv).

If groups are completely homogeneous with respect to U_{hi} and w_h, a group equilibrium does not involve any positive net trades, again using the fact that differences in β_h have no effect on aggregate excess demand. Therefore, $\widehat{x}^{\ell}_{h1} + \widehat{x}^{\ell}_{h2} = \omega^{\ell}_h$ and via equation (12) we obtain (v).

■

References

Apps, P., Rees, R.: *Public Economics and the Group.* Cambridge University Press: Cambridge, UK, 2009.

Aghion, P., Tirole, J.: "Formal and Real Authority in Organizations", *Journal of Political Economy 105* (1997), 1-5.

Becker, G.S.: "Nobel Lecture: The Economic Way of Looking at Behavior", *Journal of Political Economy 101* (1993), 385-409.

Dufwenberg, M., Heidhues, P., Kirchsteiger, G., Riedel, F., Sobel, J., "Other-Regarding Preferences in General Equilibrium," *Review of Economic Studies* 78 (2011), 613-639.

Gersbach, H., Haller, H: "Collective Decisions and Competitive Markets," *Review of Economic Studies 68* (2001), 347-368.

Gersbach, H., Haller, H.: "Bargaining Power and Equilibrium Consumption," *Social Choice and Welfare* 33(4) (2009), 665-690.

Gersbach, H., Haller, H.: "Competitive Markets, Collective Decisions and Group Formation," *Journal of Economic Theory* 146 (2011), 275-299.

Gori, M., Villanacci, A.: "A Bargaining Model in General Equilibrium," *Economic Theory* 46 (2011), 327-375.

Greenberg, J., Weber, S.: "Strong Tiebout Equilibrium under Restricted Preferences Domain," *Journal of Economic Theory* 38 (1986), 101-117.

Guesnerie, R., Oddou, C.: "Second Best Taxation as a Game," *Journal of Economic Theory* 25 (1981), 67-91.

Haller, H.: "Group Decisions and Equilibrium Efficiency", *International Economic Review 41(4)*, (2000), 835-847.

Hirschman, A.O.: *Exit, Voice, and Loyality*, Harvard University Press, Cambridge, Massachusetts, 1970.

Konishi, H., Le Breton, M., Weber, S.: "Pure Strategy Nash Equilibrium in a Group Formation Game with Positive Externalities," *Games and Economic Behavior 21* (1997), 161-182.

Konishi, H., Le Breton, M., Weber, S.: "Equilibrium in a Finite Local Public Goods Economy," *Journal of Economic Theory 79* (1998), 224-244.

Makowski, L., Ostroy, J. M., Segal, U.: "Efficient Incentive Compatible Economies are Perfectly Competitive," *Journal of Economic Theory 85* (1999), 169-225.

Rajan, R., Zingales, L.: "Power in a Theory of the Firm" *Quarterly Journal of Economics 113* (1998), 387-432.

Working Papers of the Center of Economic Research at ETH Zurich

(PDF-files of the Working Papers can be downloaded at www.cer.ethz.ch/research).

15/212 H. Gersbach and H. Haller
Formal and Real Power in General Equilibrium

15/211 L. Bretschger and J. C. Mollet
Prices vs. equity in international climate policy: A broad perspective

15/210 M. Filippini and F. Heimsch
The regional impact of a CO2 tax on gasoline demand: a spatial econometric approach

15/209 H. Gersbach and K. Wickramage
Balanced Voting

15/208 A. Alberini and C. Towe
Information v. Energy Efficiency Incentives: Evidence from Residential Electricity Consumption in Maryland

14/207 A. Bommier
A Dual Approach to Ambiguity Aversion

14/206 H. Gersbach, U. Schetter and M. T. Schneider
Taxation, Innovation, and Entrepreneurship

14/205 A. Alberini and A. Bigano
How Effective Are Energy-Efficiency Incentive Programs? Evidence from Italian Homeowners

14/204 D. Harenberg and A. Ludwig
Social Security in an Analytically Tractable Overlapping Generations Model with Aggregate and Idiosyncratic Risk

14/203 A. Bommier, L. Bretschger and F. Le Grand
Existence of Equilibria in Exhaustible Resource Markets with Economies of Scale and Inventories

14/202 L. Bretschger and A. Vinogradova
Growth and Mitigation Policies with Uncertain Climate Damage

14/201 L. Bretschger and L. Zhang
Carbon policy in a high-growth economy: The case of China

14/200 N. Boogen, S. Datta and M. Filippini
Going beyond tradition: Estimating residential electricity demand using an appliance index and energy services

14/199 V. Britz and H. Gersbach
Experimentation in Democratic Mechanisms

14/198 M. Filippini and E. Tosetti
Stochastic Frontier Models for Long Panel Data Sets: Measurement of the Underlying Energy Efficiency for the OECD Countries

14/197 M. Filippini and W. Greene
Persistent and Transient Productive Inefficiency: A Maximum Simulated Likelihood Approach

14/196 V. Britz, P. J.-J. Herings and A. Predtetchinski
Equilibrium Delay and Non-existence of Equilibrium in Unanimity Bargaining Games

14/195 H. Gersbach, M. T. Schneider and O. Tejada
Coalition-Preclusion Contracts and Moderate Policies

14/194 A. Bommier
Mortality Decline, Impatience and Aggregate Wealth Accumulation with Risk-Sensitive Preferences

14/193 D. Harenberg and A. Ludwig
Social Security and the Interactions Between Aggregate and Idiosyncratic Risk

14/192 W. Mimra, A. Rasch and C. Waibel
Second Opinions in Markets for Expert Services: Experimental Evidence

14/191 G. Meunier and J-P. Nicolai
Higher Costs for Higher Profits: A General Assessment and an Application to Environmental Regulations

14/190 A. Alberini, M. Bareit and M. Filippini
Does the Swiss Car Market Reward Fuel Efficient Cars? Evidence from Hedonic Pricing Regressions, Matching and a Regression Discontinuity Design

14/189 J-P. Nicolai and J. Zamorano
"Windfall profits 2.0" during the third phase of the EU-ETS

13/188 S. Hector
Accounting for Different Uncertainties: Implications for Climate Investments

13/187 J-P. Nicolai
Delaying the pollution cap determination when firms invest?

13/186 C. Christin, J-P. Nicolai and J. Pouyet
Pollution Permits, Imperfect Competition and Abatement Technologies

13/185 L. Bretschger and S. Valente
International Resource Tax Policies Beyond Rent Extraction

13/184 L. Bretschger and C. Karydas
Optimum Growth and Carbon Policies with Lags in the Climate System

13/183 M. Filippini and L. Zhang
Measurement of the "Underlying energy efficiency" in Chinese provinces

13/182 J. G. Becker and H. Gersbach
A Theory of Threshold Contracts

13/181 M. Filippini and L. C. Hunt
'Underlying Energy Efficiency' in the US

13/180 O. Tejada
Complements and Substitutes in Generalized Multisided Assignment Economies

13/179 A. Rasch and C. Waibel
What drives fraud in a credence goods market? Evidence from a field study

13/178 B. Lanz and S. Rausch
Cap-and-Trade Climate Policy, Free Allowances, and Price-Regulated Firms

13/177 H. Karle
Creating Attachment through Advertising: Loss Aversion and Pre-Purchase Information

13/176 W. Mimra, A. Rasch and C. Waibel
Price competition and reputation in credence goods markets: Experimental evidence

13/175 H. Gersbach and M. T. Schneider
On the Global Supply of Basic Research

13/174 B. Lanz, T. F. Rutherford and J. E. Tilton
Subglobal climate agreements and energy-intensive activities: An evaluation of carbon leakage in the copper industry

13/173 L. Blazquez, F. Heimsch and M. Filippini
The Economic Crisis and Residential Electricity Consumption in Spanish Provinces: A Spatial Econometric Analysis

13/172 A. Bommier and F. Le Grand
A Robust Approach to Risk Aversion

13/171 P. J. Agrell, M. Farsi, M. Filippini and M. Koller
Unobserved heterogeneous effects in the cost efficiency analysis of electricity distribution systems