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By Sebastian Rausch and Hagen Schwerin*

We develop a general equilibrium growth theory of vintage capital and energy use in businesses and households to measure the response of energy use to energy-saving technological change. Both investment-specific technological progress and a higher energy price save energy by increasing energy efficiency, yet investment-specific technological progress spurs while a higher energy price depresses energy use. Calibration of the model's balanced growth path to U.S. post-WWII data shows that higher energy efficiency increased rather than reduced energy use. Investment-specific technological progress enhanced energy use by more than the increase in the energy price reduced it. Both neutral and investment-specific technological changes were major determinants of observed growth in energy use. (JEL D13, E23, O30, O41, Q43)

I. Introduction

Increases in resource efficiency are widely viewed as reducing resource use. Energy efficiency improvements can thus help to address three major challenges related to fossil fuels: limiting carbon dioxide emissions to mitigate climate change, lowering "local" air pollution to yield health benefits, and enhancing the security of energy supply. This standard thinking is correct when the amount of services produced with energy is viewed as being fixed. In his book *The Coal Question* Jevons (1865, p. 141) maintained, however, that "*It is the very economy of its [coal's] use which leads to its extensive consumption.*"[1] The potential resource savings from improved efficiency can be diminished—or even overcompensated—by a rebound effect on resource demand that arises because higher efficiency lowers the price for energy services and increases real income stimulating energy service demand.

Although the efficiency paradox (rebound overcompensating savings) raises doubt about the role of energy efficiency for addressing the challenges of fossil fuel use, surprisingly little is known about how *aggregate* energy use has responded to energy efficiency improvements.[2] This paper takes an empirical macroeconomic perspec-

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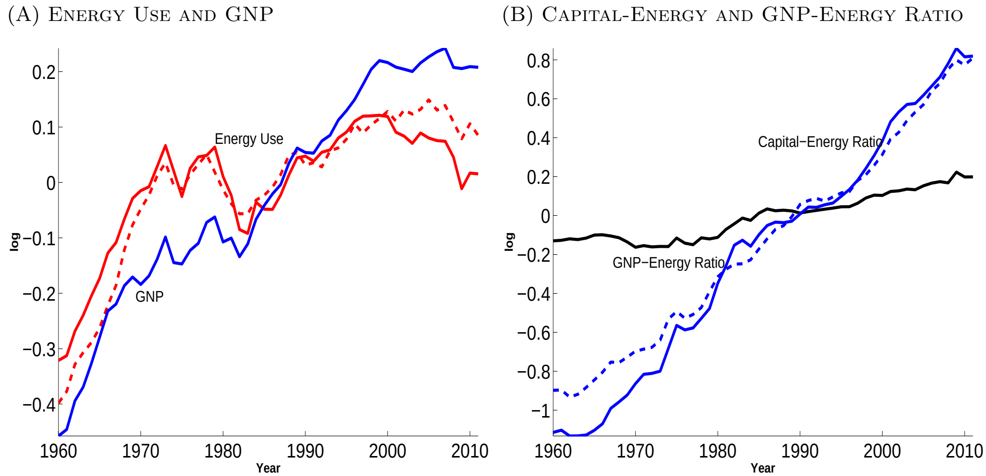
[1] British coal consumption soared following the deployment of James Watt's improved coal-fired steam engine. Jevons' concern was the sustainability of coal use which, in his view, was intimately linked to England's economic prosperity.

[2] The literature on energy rebound, as reviewed in Gillingham, Rapson, and Wagner (2014) and Greening, Greene, and Difiglio (2000), predominantly focuses on the empirical investigation of rebounds for household- and firm-level energy consumption or theoretical micro-economic analyses (Binswanger, 2001; Borenstein, 2015). The few papers that adopt a macroeconomic perspective are largely concerned with


(A) Energy Use and GNP

(B) Capital-Energy and GNP-Energy Ratio

Fig 1. Energy Use, GNP, and Relative Energy Use (Businesses: Solid Curve, Households: Dashed Curve) U.S. Data 1960-2011: (A) Energy Use and GNP; (B) Capital-Energy and GNP-Energy Ratio

Notes: Energy use is a composite of primary fossil energy, biomass energy, and electricity. GNP is gross national product less gross housing product and energy value added. The series are relative to available hours for work. Household capital-energy ratio is real durable consumption goods relative to household energy use. Business capital-energy ratio is equipment capital stock relative to business energy use. GNP-energy ratio is GNP divided by business and household energy use. The series are demeaned and in logarithms. See the Appendix A for greater detail.

tive to examine the response of energy use to energy efficiency change in the long run.

Aggregate energy use in the U.S. has increased over the post-WWII period (see Figure 1A) while at the same time energy use has declined relative to output and relative to capital suggesting an increase in energy efficiency (see Figure 1B). An empirical investigation of the efficiency paradox faces two main issues. First, it requires defining and measuring energy efficiency and understanding its impact on energy use. As the paradox involves comparing energy use under alternative energy efficiency paths, one needs to specify the sources underlying energy efficiency in a way that can be measured empirically and, importantly, can be varied in a counterfactual analysis. Second, it is important to control for factors other than energy efficiency that influence energy use. For example, Figure 1A shows that GNP has largely co-moved with aggregate energy use suggesting a positive impact of economic growth on energy use.

We develop a general equilibrium growth theory of vintage capital and energy use which enables identifying changes in energy efficiency for businesses and households. Our theory rests on four main assumptions. First, the production of energy services requires combining capital—for example, machines, vehicles, electric appliances, and heating systems for buildings—with energy. Energy efficiency is defined

the question which functional forms for production in neoclassical growth theory can potentially yield a rebound in excess of one hundred percent of energy savings (Khazzoom, 1980; Brookes, 1990; Saunders, 1992).

as the output of energy services per unit of energy used and is positively related to the capital-energy ratio. Lower capital prices or higher energy prices (relative to consumption) are energy-saving because they increase the capital-energy ratio and thus increase energy efficiency. Second, we posit neutral technological change in a standard growth framework that is augmented with exogenous investment-specific technology (Greenwood, Hercowitz, and Krusell, 1997) and exogenous energy prices. Third, energy services can be produced with capital varieties that differ in terms of their capital-energy ratio. To account for inertia in adjusting energy efficiency, we adopt a putty-clay formulation in which the capital-energy ratio can only be chosen for new capital vintages (Atkeson and Kehoe, 1999). Fourth, as a large fraction (about one half) of economy-wide energy use occurs in the household sector, we extend the household production model of Greenwood and Hercowitz (1991) to include household energy services and energy use.

To evaluate the efficiency paradox, we compare alternative long-run equilibrium paths of our model. One path corresponds to the calibrated equilibrium explaining observed U.S. energy and output growth for the 1960-2011 period. Viewing energy-saving technological change as exogenous enables us to analyze the counterfactual path of energy services that would have occurred without energy-saving technological change. Without the response in energy services to energy-saving technological change, improved energy efficiency would have yielded savings of energy use relative to the equilibrium without energy-saving technological change. Accordingly, we can measure energy rebound as the difference in energy use between the situation with savings and the equilibrium path calibrated to observed energy use.

We find evidence for a rebound of energy use to increased energy services of 102 percent of savings of energy use to increased energy efficiency (rebound rate of 1.02). The reason is that investment-specific technological progress (lower capital prices) enhanced energy use by more than higher energy prices reduced energy use (controlling for neutral technological change in producing output). Responsible for the impacts of energy efficiency and services on energy use are both the relative magnitude of the changes in capital and energy prices and the calibrated response of output with respect to energy services. We find that both effects have led to significant growth in energy services per output in the market sector.

Our framework enables us to measure the macroeconomic evolution of energy efficiency. Energy efficiency in businesses and households has risen on average by about three percent per year over the 1960-2011 period. With a contribution of 86 percent, the rise is largely explained by the decline in the price for capital used in the production of energy services, while a higher energy price accounts for 14 percent. We then find the relative contributions of the different types of technological change to the observed growth in energy use. Equilibrium growth accounting shows that neutral technological change and a lower capital price are the main drivers of growth in energy use contributing with 92 and 80 percent. The increase in the energy price accounts for -72 percent.

Our paper is related to the literature in several ways. Energy rebound calculations have been made with product-specific energy efficiency improvements and no underpinning of their source (Greening, Greene, and Difiglio, 2000; Gillingham, Rapson, and Wagner, 2014). Efficiency gains in delivering capital-energy services have been well documented for specific services. For example, lighting has substantially increased in lumen per watt (Nordhaus, 1996), and automobiles have experienced increases in fuel economy controlling for size and power (Knittel, 2011). At the same time, data show an increase in the services of illumination and miles driven with automobiles. We contribute with identifying sources for the change in energy efficiency and energy use in a structural model at the economy-wide level. We show the income and substitution effects on energy use from exogenous technological change determining rebound relative to savings.

Linking technological change and energy efficiency is obviously not new. Popp (2002) and Newell, Jaffe, and Stavins (1999) studied product innovation in energy efficiency in response to energy price at sectoral level. We set up a model of induced energy efficiency from technological change that helps to find the technological gap between energy efficiency of new vintages and average practice over time. To measure the technological gap, we compute a distribution of energy efficiency and services over past vintages of capital for each date in the sample period, assuming the steady-state distribution at the first year of the sample period and exploiting the laws of motion for capital and energy that are relevant for economic behavior with aggregate data on capital and energy. Hassler, Krusell, and Olovsson (2012) analyzed how capital and energy efficiency, measured as coefficients in a production function, evolved. They postulated directed research effort as the main determinant. We contribute an alternative theory in which factor efficiency of new vintages responds to energy-saving technological change.

We are not the first to consider energy use with investment-specific technological change. Díaz and Puch (2013) used investment-specific technology to study the fluctuations in the energy expenditure, capital-energy ratio, and capital-output ratio in the business sector but do not investigate energy rebound. Several papers analyze macroeconomic effects of household production. Greenwood and Hercowitz (1991), Benhabib, Rogerson, and Wright (1991), and McGrattan, Rogerson, and Wright (1997) examined the role of household production for the business cycle. Gomme and Rupert (2007) considered investment-specific technological change with household production in a business cycle analysis. We contribute with an analysis of the role of household production for long-run energy use.

The remainder of the paper is organized as follows. Section II describes the model. Section III describes the data and model calibration. Section IV presents and discusses our main results on the role of energy-saving and neutral technological change for energy use. Section V reports findings from alternative counterfactual experiments to analyze the energy rebound. Section VI provides a number of robustness checks. Section VII concludes. Additional appendices describe the construction of the data set and present equilibrium conditions of our model.

II. Model

We study a two-sector growth model with energy use. Each sector uses two specific capital stocks, one putty-clay with energy (energy-using capital), and one putty-putty with labor (nonenergy-using capital). The model features balanced growth with an increasing capital-energy ratio which enables representing long-run increases in energy efficiency.

A. The Economic Environment

We consider an infinite-horizon discrete-time economy inhabited by a continuum of households.

Preferences.—All households have preferences over market output, c_M, and non-market or home output, c_N, expressed by the expected utility

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t U(c_{M,t}, c_{N,t})\right]$$

with the discount factor $\beta \in (0,1)$ and the expectation operator $\mathbb{E}_0$. The instantaneous utility function

$$U(c_M, c_N) = \frac{1}{1-\gamma}\left[{c_M}^{\xi}{c_N}^{1-\xi}\right]^{1-\gamma}, \quad \gamma > 0,\, \xi \in (0,1),$$

assumes a constant elasticity of intertemporal substitution, $1/\gamma$, and a distribution parameter ξ. In each period, households are endowed with one unit of time which they supply labor to the production of market and home goods.[3] To ease notation, we suppress the time index whenever no ambiguity arises.

Production.—Output is produced with inputs of capital, energy, and labor. The use of k_s units of non-energy capital, x_M units of energy services, and ℓ units of labor, yields output of the market consumption good according to:

$$y = G(k_s, x_M, z_M\ell) = {k_s}^{\alpha_M\gamma_M}{x_M}^{\alpha_M(1-\gamma_M)}(z_M\ell)^{1-\alpha_M}, \tag{1}$$

with the share parameters $0 < \alpha_M, \gamma_M < 1$ and exogenous labor efficiency $z_M > 0$. In our model, nonenergy-using capital is not essential to formulate energy-saving technological change; a broad base of capital including equipment and structures is, however, required for the empirical analysis of the efficiency paradox.

Energy services are produced with capital varieties and energy. The varieties are described by the capital intensity of energy $v \in V = (0, \infty)$. The number of capital units $k_e(v)$ thus require the number of units of energy $k_e(v)/v$. The intensity can be chosen at the date of investment in capital and remains fixed thereafter. As variety-specific services $k_e(v)f(v)/v$ are produced with the efficiency of the factor capital $f(v)/v$, where $f(v) = v^{\varepsilon_M}$, $0 < \varepsilon_M < 1$, market services produced are

$$x_M = \int_{v\in V}\frac{1}{v}k_e(v)f(v)dv.$$

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[3] Leisure can be viewed as essentially productive or inherently desirable, consistent with the view that households have preferences for leisure.

Capital that requires less energy (has higher v) therefore is less productive (exhibits lower $f(v)/v$). We restrict attention to the essential putty-clay assumption that the capital intensity of energy of any vintage is determined once. (Section VI.C explores the implications of a chosen utilization of capital.) Capital and energy use need to be substitutable at some point of time for energy efficiency to be a choice. Energy efficiency is defined as the efficiency of the factor energy in producing energy services, $f(v)$. Aggregate energy use in producing market services is then given by

$$u_M = \int_{v \in V} \frac{1}{v} k_e(v) dv.$$

In a symmetric way, households combine k_r units of nonenergy-using capital, x_N units of energy services, and $(1-\ell)$ units of labor to produce the home consumption good

$$c_N = H(k_r, x_N, z_N(1-\ell)) = {k_r}^{\alpha_N \gamma_N} {x_N}^{\alpha_N(1-\gamma_N)} (z_N(1-\ell))^{1-\alpha_N}, \tag{2}$$

with the share parameters $0 < \alpha_N, \gamma_N < 1$ and exogenous labor efficiency $z_N > 0$. Labor efficiency can be unequal in the business and household sector. Households use varieties of capital goods $k_d(v)$ and energy $k_d(v)/v$ to produce the home energy services x_N. Production of households energy services is given by

$$x_N = \int_{v \in V} \frac{1}{v} k_d(v) h(v) dv,$$

with type-v energy efficiency $h(v) = v^{\varepsilon_N}$, $0 < \varepsilon_N < 1$. Aggregate energy use in household production is

$$u_N = \int_{v \in V} \frac{1}{v} k_d(v) dv.$$

The Laws of Motion.—The laws of motion of capital are informed by the data. The energy-using capital goods types $k_d(v)$ and $k_e(v)$ (to be measured as durable consumption goods and private nonresidential equipment capital) each depreciate at the rate δ_j for $j \in \{d, e\}$. Over the 1960-2011 period, the price of durable consumption goods and the price of equipment capital relative to nondurable consumption and services (not to be confused with the model's energy services) have drastically decreased. Nonenergy-using capital stocks k_r and k_s (to be measured as residential and nonresidential structures capital) depreciate at the rate δ_j for $j \in \{r, s\}$. Capital price change is nearly absent for structures in the data. Accordingly, any new unit of structures stock is created one-to-one using the market consumption good,

$$k_r{}' - (1-\delta_r) k_r = i_r, \tag{3}$$

$$k_s{}' - (1-\delta_s) k_s = i_s, \tag{4}$$

where prime denotes the next period's value. Technological change making replacement equipment capital or durable consumption goods less expensive in terms of market output is accounted for in the evolution of the type-specific capital stocks. With the number of new capital units per unit of market output foregone q_i for $i \in \{M, N\}$,

$$k_d{}'(v) - (1-\delta_d)k_d(v) = q_N i_d(v) \geq 0, \tag{5}$$

$$k_e{}'(v) - (1-\delta_e)k_e(v) = q_M i_e(v) \geq 0, \tag{6}$$

all $v \in (0, \infty)$. An increase in q_i over time implies a declining unit cost of capital $(1/q_i)$ for $i \in \{M, N\}$. Investment in capital of each type in each sector is irreversible, and hence nonnegative.

The Resource Constraint.—The market good can be used for consumption, investment in capital stock $i_d(v)$ and $i_e(v)$ all $v \in (0, \infty)$, and i_r and i_s, and energy purchases $p(u_M + u_N)$,

$$c_M + \int_{v \in V} [i_d(v) + i_e(v)]dv + i_r + i_s + p(u_M + u_N) = y, \tag{7}$$

where the exogenous import price for energy p determines the cost of energy as the units of market output traded for one unit of energy. Market and home production functions have the Cobb-Douglas form, and hence a unitary elasticity of substitution.[4]

Model Mechanism.—On an intuitive level, the key model mechanism with respect to energy efficiency and use can be described as follows. A lower capital price $1/q_M$ and a higher energy price p increase energy efficiency $f(v)$ in the market given the production function of services. Likewise, a lower capital price $1/q_N$ and a higher energy price p increase energy efficiency $h(v)$ in households. As resources used to invest and acquire energy are produced with investment goods and energy and investment goods are used at decreasing returns to scale in the market $(\alpha_M < 1)$, a lower capital price $1/q_M$ increases energy use $(u_M + u_N)$. A higher energy price, however, reduces energy use $(u_M + u_N)$. One can easily see these features in our dynamic putty-clay setting in which one incurs the cost of investment today and of energy tomorrow. As we plausibly model household energy use requiring market output, the price for energy-using capital $1/q_N$ has no role for aggregate energy use yet affects the capital-energy ratio in households.

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[4] Within the constant elasticity of substitution (CES) family of production functions, complementary inputs in home production require that household durable goods services in our model increase at the same rate as household labor efficiency along a balanced growth path. This precludes that the marginal product of investment in durable goods services $q_N f(v)/v$ with v chosen capital intensity of energy by households changes along a balanced growth path, implying that the rates of change in investment-specific productivity q_N and the energy price p are connected, which would restrict counterfactuals.

B. Equilibrium

To quantitatively examine the response of energy use to energy efficiency change, we find values for the model parameters through equilibrium conditions. We thus now turn to studying the equilibrium behaviour of firms and households.

We analyze a competitive equilibrium with finitely many endogenous state variables inspired by Atkeson and Kehoe (1999). To calibrate the model to long-run growth of output and energy in the data, a government is included in the decentralized economy to measure the effects of income taxation on the returns to investment in physical capital and the use of time for work. Equilibrium is presented in a recursive way as in Greenwood, Hercowitz, and Krusell (1997).

The Aggregate State of the World.—Households own in total the number of equity shares s. Both households and firms supply the stocks of structures capital, services, and energy use to themselves.[5] Households and firms take expectations with respect to the evolution of the exogenous technology given by the energy price p, investment-specific productivity $q = (q_M, q_N)$, and labor efficiency $z = (z_M, z_N)$. The aggregate state of the world $\epsilon = (k, x, u, s, \omega)$ is comprised of aggregate structures $k = (k_s, k_r)$, energy services $x = (x_M, x_N)$, energy use $u = (u_M, u_N)$, equity shares s, and technology $\omega = (1/p, q, z)$ subject to some evolution of technology. All agents take the motion of the endogenous aggregate state, i.e., $(k', x', u', s') = \Phi(\epsilon) \equiv (K_M(\epsilon), K_N(\epsilon), X_M(\epsilon), X_N(\epsilon), U_M(\epsilon), U_N(\epsilon), S(\epsilon))$, as exogenously given. The decision problems of households and firms are formulated with individual quantities including of the energy requirement of new capital e_M and e_N.

Dividend and Prices.—Households receive the profit of firms in the form of a dividend on equity shares. The equilibrium dividend d, equity share price ψ, and wage rate w all are functions of the aggregate state ϵ. Here, $d = D(\epsilon)$, $\psi = \Psi(\epsilon)$, and $w = W(\epsilon)$.

Government.—A government returns taxes on total dividends ds and labor income $w\ell$ to households in the form of a lump-sum payment $\tau = T(\epsilon)$. The dividend tax rate is τ_d. The labor income tax rate equals τ_w. This gives the government budget constraint

$$\tau_d ds + \tau_w w\ell = \tau.$$

The Decision Problem of Households.—The goal of a representative household is solving the problem

$$\text{P(1)} \quad \begin{aligned} V(k_r, x_N, u_N, s, \epsilon) = \max_{c_M, e_N, v_N, \ell, k_r{}', x_N{}', u_N{}', s'} & \{U(c_M, H(k_r, x_N, z_N(1-\ell))) \\ & + \beta \mathbb{E}[V(k_r{}', x_N{}', u_N{}', s', \epsilon')]\} \end{aligned}$$

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[5] Firm ownership of the capital that firms use avoids a market for services used by firms and households purchasing energy used by firms.

subject to the budget constraint

$$\begin{aligned} &c_M + {k_r}' - (1-\delta_r)k_r + e_N v_N/q_N + pu_N + \Psi(\epsilon)s' \\ &\quad = [\Psi(\epsilon) + (1-\tau_d)D(\epsilon)]s + (1-\tau_w)W(\epsilon)\ell + T(\epsilon), \end{aligned}$$

the laws of motion of home energy services and use,

$$ {x_N}' - (1-\delta_d)x_N = e_N h(v_N), \tag{8}$$

$$ {u_N}' - (1-\delta_d)u_N = e_N, \tag{9}$$

and $(k', x', u', s') = \Phi(\epsilon)$. The number of equity shares in the firm are indeterminate, so they can take any positive value.

The Decision Problem of Firms.—A representative firm on the unit interval seeks to solve the problem

$$\text{P(2)} \qquad \begin{aligned} Q(k_s, x_M, u_M, \epsilon) = \max_{e_M, v_M, \tilde{\ell}, {k_s}', {x_M}', {u_M}'} &\Bigg\{ G(k_s, x_M, z_M\tilde{\ell}) - W(\epsilon)\tilde{\ell} \\ &- [{k_s}' - (1-\delta_s)k_s] - e_M v_M/q_M - pu_M \\ &+ \mathbb{E}\bigg[\frac{\Psi(\epsilon)}{\Psi(\epsilon') + D(\epsilon')} Q({k_s}', {x_M}', {u_M}', \epsilon')\bigg]\Bigg\} \end{aligned}$$

subject to the laws of motion of market energy services and use,

$$ {x_M}' - (1-\delta_e)x_M = e_M f(v_M), \tag{10}$$

$$ {u_M}' - (1-\delta_e)u_M = e_M, \tag{11}$$

and $(k', x', u', s') = \Phi(\epsilon)$. To simplify notation, define $A_i(\epsilon) \equiv (K_i(\epsilon), X_i(\epsilon), U_i(\epsilon))$, $i \in \{M, N\}$.

Definition of Equilibrium.—An equilibrium is a set of allocation functions for aggregate physical assets $(A_M(\epsilon), A_N(\epsilon))$, the financial asset $S(\epsilon)$, and the quantities $C(\epsilon)$, $D(\epsilon)$, $E_M(\epsilon)$, $E_N(\epsilon)$, $V_M(\epsilon)$, $V_N(\epsilon)$, and $L(\epsilon)$, and pricing and transfer functions $\Psi(\epsilon)$, $W(\epsilon)$, and $T(\epsilon)$, and an aggregate law of motion for endogenous states $\Phi(\epsilon)$ such that:

(i) Households solve problem P(1), taking as given the aggregate state of the world ϵ, the allocation rule D, the pricing functions Ψ and W, and transfer function T, and $\Phi(\epsilon)$, so that individual quantities are $c_M = C(\epsilon)$, $e_N = E_N(\epsilon)$, $v_N = V_N(\epsilon)$, $\ell = L(\epsilon)$, $({k_r}', {x_N}', {u_N}') = A_N(\epsilon)$, and $s' = S(\epsilon) = 1$ (individual states equal the functional value).

(ii) Firms solve problem P(2), taking as given the aggregate state of the world ϵ, and the functions D, Ψ, W, and $\Phi(\epsilon)$, such that individual quantities are $e_M = E_M(\epsilon)$, $v_M = V_M(\epsilon)$, $\tilde{\ell} = L(\epsilon)$, $({k_s}', {x_M}', {u_M}') = A_M(\epsilon)$, and firm and financial asset value are balanced, $Q(k_s, x_M, u_M, \epsilon) = (\Psi[\epsilon] + D[\epsilon])s$ (individual value on the left side, aggregate numbers on the right side).

(iii) Aggregate services and energy use evolve according to (8), (9), (10), and

(11). The resource constraint of the market consumption good holds in every period, i.e.,

$$c_M + e_M v_M/q_M + e_N v_N/q_N + i_r + i_s + p(u_M + u_N) = G(k_s, x_M, z_M \ell),$$

where

$$i_r = k_r{}' - (1-\delta_r)k_r, \quad i_s = k_s{}' - (1-\delta_s)k_s,$$

and (e_M, v_M) and (e_N, v_N) govern (8), (9), (10), and (11).

We have reduced the economy in equilibrium in two ways. First, we have posited the choice of one type of putty-clay capital at each date. Conveniently, the model predicts that only one type v_N and one type of v_M are chosen at each date, among all available types on the continuum V. The property follows from the strict concavity of the production function of services in capital and energy (see Atkeson and Kehoe (1999) for a proof). Second, we have transformed the law of motion of vintage capital into laws of motions of service and energy stock. Given initial states and fully utilized capital units, a realization of equilibrium thus has the same allocation as a realization of an equilibrium with type-specific capital.[6]

C. *Balanced Growth*

To find an equilibrium allocation consistent with observed long-run growth of energy and output, we now analyze a deterministic steady-state equilibrium path.

Growth Rates.—The resource constraint (7) dictates that consumption, investment, and output grow at the same rate, denoted by g. Investment-specific technological change expressed by

$$\gamma_M = \text{gross rate of change in}\, q_M,$$
$$\gamma_N = \text{gross rate of change in}\, q_N,$$

then implies that physical capital k_e and k_d built from investment grow at the rates $g\gamma_{q_M}$ and $g\gamma_{q_N}$.

The energy expenditure $p(u_M + u_N)/y$ is constant, so that energy use is proportional to the ratio of output and energy price. The growth rate of energy use—u_M and u_N—thus equals $g\gamma_{1/p}$ with

$$\gamma_{1/p} = \text{inverse of gross rate of change in}\, p.$$

We now derive the growth rate of the capital intensity of energy important for our empirical analysis. The laws of motion of energy (9) and (11) follow from

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[6] A continuum of capital types allows smooth substitution of energy and capital in the long-run. An equilibrium with such a continuum has infinitely many endogenous state variables. The reduction of the state space used is only valid, if capital is fully utilized. In Atkeson and Kehoe (1999), capital is efficiently fully utilized, because the capital intensity of energy is constant in the long-run and the energy price does not fluctuate too much. Here, capital units with relatively small capital intensity would be efficiently underutilized if utilization could be chosen, because new vintage capital intensity of energy is growing. We discuss the validity of our assumption of fully utilized capital for evaluating the efficiency paradox in Section VI.

the putty-clay combination of capital and energy. As investment occurs in exactly one type of the capital intensity of energy at each date, the type of new vintages changes at the rate capital relative to energy changes—equal to the ratio of the growth rate of investment-specific technology and the inverse of the rate of change in the energy price:

$$\phi_i \equiv \gamma_{q_i}/\gamma_{1/p}. \tag{12}$$

The laws of motion of services (8) and (10) then imply that energy services x_i grow at the rate $g{\gamma_{q_i}}^{\varepsilon_i}{\gamma_{1/p}}^{1-\varepsilon_i}$. We need to derive the growth rate of market output g to express services change solely in terms of technological change. Inserting the growth rate of structures, g, and services yields a condition on the growth rate of labor efficiency z_M—i.e., γ_{z_M}:

$$g = \gamma_{z_M}[{\gamma_{q_M}}^{a\varepsilon_M}{\gamma_{1/p}}^{a(1-\varepsilon_M)}] \tag{13}$$

where $a \equiv \alpha_M(1-\gamma_M)/(1-\alpha_M)$ denotes the elasticity of output change with respect to services change.

For each set of rates of change in the energy price $1/\gamma_{1/p}$, investment-specific productivity $(\gamma_{q_M}, \gamma_{q_N})$, and market and household output, there are rates of labor efficiency change $(\gamma_{z_M}, \gamma_{z_N})$ consistent with a deterministic balanced growth path.

Equilibrium Behavior.—To obtain parameter values, we use necessary conditions for solutions to the households' and firms' problems P(1) and P(2) that hold along a balanced growth path. Next we report these conditions (which we derived from conditions given in Appendix C). Entering some conditions is the after-tax rate of return on equity $R' \equiv [\psi' + (1-\tau_d)d']/\psi$.[7]

The deterministic balanced growth path analogues to the Euler equations of nonresidential structures, services, and energy use in market production are

$$1 = ([1-\tau_d]/[R-\tau_d g])\left\{\alpha_M\gamma_M\,(y/k_s) + 1 - \delta_s\right\}, \tag{14}$$

(15)

$${\gamma_{q_M}}^{\varepsilon_M}{\gamma_{1/p}}^{1-\varepsilon_M} = ([1-\tau_d]/[R-\tau_d g])\left\{\alpha_M\varepsilon_M(1-\gamma_M)\left[\underbrace{\left(\frac{q_M y}{x_M}\right)\frac{f(v)}{v}}_{\psi_M}\right] + 1 - \delta_e\right\},$$

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[7] The relationship of the pretax return rate on equity and the after-tax return rate on equity on a deterministic balanced growth path,

$$\tau_d g + (1-\tau_d)\quad \underbrace{[\psi' + d']/\psi}_{\text{Pretax Return Rate}} \quad = \quad \underbrace{R'}_{\text{After-tax Return Rate}}\quad,$$

can be derived utilizing that the return rates are constant and dividends increase at the same rate as output.

$$(16) \qquad \gamma_{1/p} = ([1-\tau_d]/[R-\tau_d g]) \left\{ \frac{\varepsilon_M}{1-\varepsilon_M} \left[\underbrace{\left(\frac{q_M y}{u_M}\right) \frac{1}{v}}_{\theta_M} \right] (pu_M/y) + 1 - \delta_e \right\},$$

and the laws of motion of these stocks (4), (10), and (11), can be rewritten as

$$(17) \qquad i_s/y = [g - (1-\delta_s)]/(y/k_s),$$

$$(18) \qquad \int_{v\in V} [i_e(v)/y]dv = [g\gamma_{q_M}{}^{\varepsilon_M}\gamma_{1/p}{}^{1-\varepsilon_M} - (1-\delta_e)]/\psi_M,$$

$$(19) \qquad \int_{v\in V} [i_e(v)/y]dv = [g\gamma_{1/p} - (1-\delta_e)]/\theta_M,$$

with the capital intensity of energy chosen by firms v.

Along the balanced growth path, the capital intensity of energy of new vintage capital grows at the same rate as the average capital intensity.[8] The output-capital ratio times the ratio of the new to average capital-service ratio $\psi_M = (q_M y/\int_{v\in V} k_e(v)dv) \times (\int_{v\in V} k_e(v)dv/x_M)/(v/f(v))$, is constant, $\psi_M = \psi_M{}'$, where capital corresponds to energy-using capital. Likewise, the output-capital ratio times the relative capital-energy ratio $\theta_M = (q_M y/\int_{v\in V} k_e(v)dv) \times (\int_{v\in V} k_e(v)dv/u_M)/v$, is constant, $\theta_M = \theta_M{}'$. New vintages' and average capital-service ratios change at the same rate, as do new vintages' and average capital-energy ratios. As the output-capital ratio $q_M y/\int_{v\in V} k_e(v)dv$ is constant, ψ_M and θ_M each equal the output-capital ratio times a constant. The ratio $\psi_M/\theta_M = f(v)/(x_M/u_M)$ expresses the *technological gap* between energy efficiency in new vintages and average practice.

The deterministic balanced growth path analogues to the Euler equations of residential capital, services, energy use, and equity holdings by households are

$$(20) \qquad 1 = (1/R)\left\{\alpha_N \gamma_N (y/k_r)(c_N/y)\frac{U_2}{U_1} + 1 - \delta_r\right\},$$

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[8] The distributions of capital and energy over capital age are thus fixed, which we exploit below to measure energy efficiency as a technology residual.

(21)
$$\gamma_{q_N}{}^{\varepsilon_N}\gamma_{1/p}{}^{1-\varepsilon_N} = (1/R)\left\{\alpha_N\varepsilon_N(1-\gamma_N)\left[\underbrace{\left(\frac{q_N y}{x_N}\right)\frac{h(v)}{v}}_{\psi_N}\right](c_N/y)\frac{U_2}{U_1} + 1 - \delta_d\right\},$$

(22)
$$\gamma_{1/p} = (1/R)\left\{\frac{\varepsilon_N}{1-\varepsilon_N}\left[\underbrace{\left(\frac{q_N y}{u_N}\right)\frac{1}{v}}_{\theta_N}\right](pu_N/y) + 1 - \delta_d\right\},$$

(23)
$$R = g^{\gamma}/\beta,$$

using the marginal utility of consumption U_i, the derivative of U with respect to its ith argument, and the laws of motion of the stocks of residential capital, services, and energy (3), (8), and (9), imply that

(24)
$$i_r/y = [g - (1-\delta_r)]/(y/k_r),$$

(25)
$$\int_{v\in V}[i_d(v)/y]dv = [g\gamma_{q_N}{}^{\varepsilon_N}\gamma_{1/p}{}^{1-\varepsilon_N} - (1-\delta_d)]/\psi_N,$$

(26)
$$\int_{v\in V}[i_d(v)/y]dv = [g\gamma_{1/p} - (1-\delta_d)]/\theta_N,$$

with the capital intensity of energy chosen by households v.

Like in the market sector, new vintages' and average capital-service and capital-energy ratios in the household sector change at the same rate so that the corresponding relative ratios are constant, where capital corresponds to energy-using capital. The output-capital ratio $q_N y/\int_{v\in V} k_d(v)dv$ is constant. Then the output-capital ratio times the relative capital-service ratio ψ_N and the output-capital ratio times the relative capital-energy ratio θ_N are constant. The ratio $\psi_N/\theta_N = h(v)/(x_N/u_N)$ expresses the *technological gap* between energy efficiency in new vintages and average practice.

Remaining are conditions for the allocation of time to market and home production, substitution of market and home consumption goods, and the use of market output:

$$(1-\tau_w)(1-\alpha_M)[(1-\ell)/\ell]=(1-\alpha_N)(c_N/y)\frac{U_2}{U_1}, \tag{27}$$

$$\frac{U_2}{U_1}=[(1-\xi)/\xi]\left(\frac{c_M/y}{c_N/y}\right), \tag{28}$$

$$c_M/y+\int_{v\in V}[i_e(v)/y]dv+i_s/y+\int_{v\in V}[i_d(v)/y]dv+i_r/y+pu_M/y+pu_N/y=1. \tag{29}$$

The necessary condition governing the allocation of time (27) uses the marginal rate of substitution of market and home consumption goods U_2/U_1 defined by (28). The resource constraint (stated in the equilibrium condition [iii]) can be rewritten as (29).

The conditions from P(1) determine households' stocks of residential capital, service from durable consumption goods and energy use, and the financial asset, in addition to the use of time given dividends d and the wage rate w. The conditions from P(2) equate the wage rate to the marginal product of time directed to market production and leave the firm with profits to pay for investments, and pay out dividends. Equation (27) combines the efficiency condition of labor demand and supply.

It is important to note that the households' and firms' problems P(1) and P(2) along with the resource constraint can be rewritten in terms of detrended variables in such a way that there is a unique stationary point in equilibrium. This allows us to calibrate the model on a balanced growth path reflecting average observed growth of energy use.

III. Data, Calibration, and Model Fit

This section describes the data, model calibration, and how energy-saving and neutral technological change is measured using the calibrated model.

A. Description of Data

We give here a brief overview of the data sources and the main issues involved in constructing our dataset. Appendices A and B provide more detail. The variables in our dataset are energy use and prices, capital stocks, investment and prices, the fraction of time worked in the market, and output produced in the market. We use annual time series data for the U.S. economy for the 1960-2011 period.[9]

Energy use and energy deflators are constructed from the *Energy Information Administration*'s Annual Energy Review 2011 and the State Energy Data System

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[9] Energy price data are only available from 1960 thus determining the first year of our sample. Using on hours worked for 1947-2011 based on Cociuba, Prescott, and Ueberfeldt (2012) determines the last year of the sample.

1960-2012. To obtain energy prices, values for an energy deflator are expressed relative to the common deflator. The common deflator is a weighted average of the deflators for nondurable consumption and services (not to be confused with the model's services) in the NIPA.

The prices of investment in capital are inferred from published deflators for investment and consumption in the *Bureau of Economic Analysis*' National Income and Product Accounts (NIPA) and the producer durable equipment price index of Gordon (1990). We extend Gordon's (1990) equipment investment deflator 1947-1983 until 2011 using an aggregative approach which incorporates the NIPA equipment investment deflator relative to a common deflator in the NIPA. The equipment price ($1/q_M$) and the durable consumption goods price ($1/q_N$) are measured by the deflator for investment in equipment capital and durable consumption goods relative to the common deflator (for nondurable consumption and services). Figure 2 reveals a salient decline in the price of equipment capital ($1/q_M$) and durable consumption goods ($1/q_N$) and a relatively moderate increase in the price for energy (p) in the long-run.

To measure investment in capital of market sector, we use expenditures on private nonresidential equipment and structures investment from the NIPA. To measure investment in capital of the household sector, we use expenditure on durable consumption goods and investment in residential structures from the NIPA. To measure capital stocks of household and market production, we use the corresponding net stock of fixed assets from the *Bureau of Economic Analysis*' Fixed Asset Accounts (FAA). Aggregate market output corresponds to gross national product less gross housing product and energy value added in the NIPA. Investment, capital, output, and energy use are expressed per hours of work. The hours represent the available time for work by the noninstitutional population aged 16-64 from the *Bureau of Labor Statistics* and include hours worked by military personnel consistent with assuming that government military expenditure is consumed by households.

B. *Benchmark Calibration*

To continue, we now find values for the parameters of the model. We set in advance based upon a priori information: γ for preferences, δ_e δ_d, δ_r, and δ_s for the depreciation of capital, γ_{q_M}, γ_{q_N}, $\gamma_{1/p}$, for the capital and energy price changes, and τ_w for government policy. We set the values of the remaining parameters so that some moments from the model match their counterparts in the data consistent with the model behavioral equations in equilibrium in Section II.C: β, σ, and ξ for preferences, α_M, ε_M, γ_M, α_N, ε_N, and γ_N for production, and τ_d for government policy. Along with parameter values we obtain stationary values of ratios of endogenous economic variables.

A Priori Calibration.—Based on a-priori information, we assign the following parameter values:

Parameter	δ_e	δ_s	δ_d	δ_r	γ_{q_M}	γ_{q_N}	$\gamma_{1/p}$	τ_w	γ
Value	0.139	0.039	0.232	0.033	1.031	1.032	0.995	0.4	1

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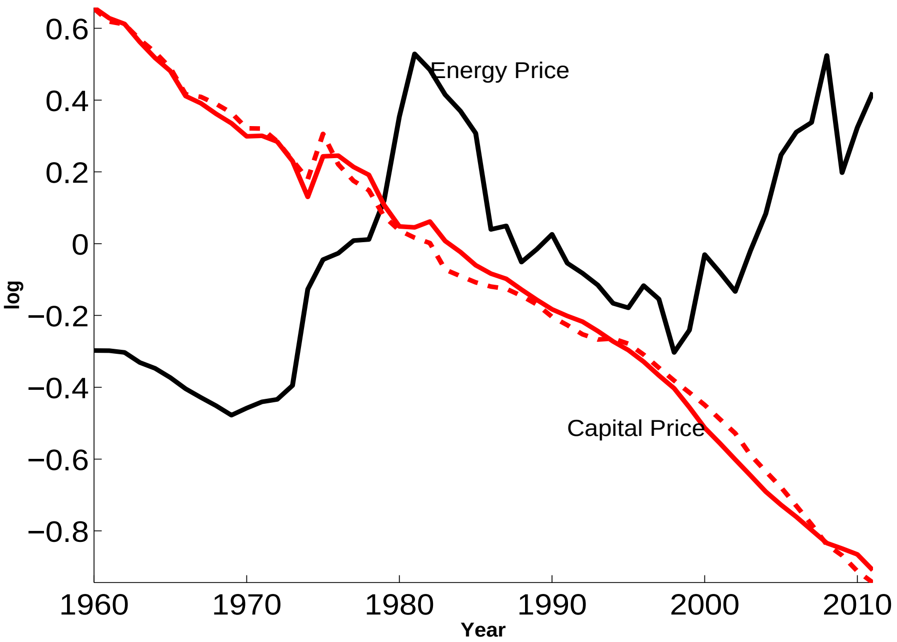

Fig 2. Energy Price and Capital Price Businesses (Solid Curve) and Households (Dashed Curve) U.S. Data 1960-2011

Notes: Energy price is the ratio of the deflator for primary fossil energy, biomass energy, and electricity, to the consumption (nondurable consumption and services) deflator. Household capital price is the deflator of durable consumption goods divided by the consumption deflator. Business capital price is the deflator for nonresidential equipment investment divided by the consumption deflator. The series are demeaned and in logarithms. See the Appendix A for further detail.

The rates of capital depreciation are computed from the data as follows. The depreciation rate δ_j equals the sample average of $[1-({k_j}'-q_ji_j)/k_j]$, $j \in \{e,d,r,s\}$. We determine time series for q_ji_j, k_j, and ${k_j}'$ by adjusting investment and capital stock data by prices and hours.[10]

The rates of energy-saving technological change are found as follows. First, we measure the investment-specific technologies q_M and q_N as the ratio of the deflator for consumption relative to the deflator for new capital equipment and durable consumption goods.[11] Second, we determine the inverse of the long-run change in the energy price, $\gamma_{1/p}$. Energy expenditure relative to market output $p(u_M+u_N)/y$ is constant along a balanced growth path. We thus measure $\gamma_{1/p}$ as

[10] We use the relative capital prices implied by the NIPA deflators for new capital goods. Specifically, we weight nonresidential equipment and structures investment with their price levels and divide by hours to form the series for new capital q_ei_e and q_si_s. The corresponding current-cost capital stocks are divided by the one-period lagged price of investment goods and hours to obtain a series for k_e and k_s. See Appendix B for the reason behind this procedure. Analogously, we weight durable consumption goods and residential structures investment by their relative prices and divide by hours to form the series for capital additions q_di_d and q_ri_r. The corresponding current-cost capital stocks are normalized by these prices and hours with one period lag, yielding a series for k_d and k_r. The q_j's here thus differ from the investment-specific technologies adjusting the NIPA deflators.

[11] The relative deflators are based on the producer durable equipment price index (corresponding to a deflator) in Gordon (1990) 1960-1983 and an autoregressive econometric model that adjusts the NIPA deflators for new capital goods 1984-2011 reported in Appendix A.

the average growth rate of the energy intensity of market output $(u_M + u_N)/y$.[12]

The effects of energy-saving technological changes on energy use in the long-run do not depend on the values we set for γ and τ_w (as these values do not affect the values for α_M, γ_M, and ε_M). We set the marginal tax rate on labor following Greenwood, Hercowitz, and Krusell (1997) ($\tau_w = 0.4$) and assume a unitary elasticity of intertemporal substitution of consumption ($\gamma = 1$).[13]

Firm Equilibrium Conditions.—To calibrate parameters in the firms' equilibrium conditions governing usiness energy use, we use the following empirical moments. First and second, the mean of investment relative to GNP (net of gross housing product) is 7.0 percent related to nonresidential equipment and 3.1 percent to structures. Third, energy expenditure by businesses pu_M relative to the value of market goods given by GNP (net of housing product and energy value added) (y) has been on average 3.4 percent. Fourth, GNP (net of housing product and energy value added) has grown on average by 1.3 percent over the sample period. Fifth, the labor share of market income often used is 0.7, while one minus the labor share of market income in the model is α_M. Sixth, the mean return on capital was four percent (McGrattan and Prescott, 2003).[14]

These moments determine the expressions $pu_M/y = 0.034$, $\int_{v \in V} [i_e(v)/y] dv = 0.070$, $i_s/y = 0.031$, $g = 1.013$, $\alpha_M = 0.3$, and $R = 1.04$, which we insert into equations (14)-(19) to obtain:

Parameter	γ_M	ε_M	τ_d
Value	0.44	0.81	0.84
Non-targeted moment	(y/k_s)	ψ_M	θ_M
Value	1.68	2.54	2.11

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Household Equilibrium Conditions.—To calibrate parameters in the households' equilibrium conditions governing energy use, we use the following empirical moments. First and second, the mean of the investment-GNP ratio is 9.6 percent related to durable consumption goods and 5.2 percent to residential structures. Third, energy expenditure by households pu_N relative to market output on average has been 2.7 percent. Fourth, total working hours on average represent the fraction 0.236 of available time for work.[15] We thus obtain $pu_N/y = 0.027$,

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[12] Real gross national product (GNP) less gross housing value and energy value added per hour available for work on average has grown by about 1.3 percent per year. Energy use per available hour for work on average has increased by 0.8 percent per year. This yields an average annual increase in the energy price of 0.5 percent.

[13] In the limit, as $\gamma \to 1$, the instantaneous utility function becomes $U(c_M, c_N) = \xi \ln c_M + (1-\xi) \ln c_N$.

[14] McGrattan and Prescott (2003) report the NIPA 1929-2008 mean return on capital of 4 percent. This estimate is close to the quarterly US data 1954-2008 annual mean after-tax return on private capital of 3.93 in Gomme, Ravikumar, and Rupert (2011) and the annual US data 1959-1996 mean after-tax return on capital for the nonfinancial corporate sector of 3.9 percent in Poterba (1998). The balanced growth path after-tax return on equity is held equal to the observed mean after-tax return on capital. McGrattan and Prescott (2003) report the US data 1880-2002 mean after-tax return on equity of 5.4 percent. They argue that it is an upper bound, because their calculation excludes "capital-gains taxes, brokerage costs, and possibly higher pre-1980 diversification costs" than accounted for (McGrattan and Prescott, 2003, 394).

[15] We use data on population size and total working hours from the Bureau of Labor Statistics as

$\int_{v \in V}[i_d(v)/y]dv = 0.096$, $i_r/y = 0.052$, and $\ell = 0.236$ which we use in equations (20)-(29) to yield:

Parameter	β	ξ	α_N	γ_N	ε_N
Value	0.97	0.30	0.14	0.38	0.80
Non-targeted moment	$(c_N/y)U_2/U_1$	c_M/y	(y/k_r)	ψ_N	θ_N
Value	1.57	0.69	0.89	2.81	2.49

C. Productivity Measurements

The purpose of this section is twofold. First, we describe how we measure technology residuals for model use and validation (where residual means combining model and data). We need to compute residual capital intensity of energy to (1) validate the use of relationships of the model along a balanced growth path for energy efficiency growth with counterfactual energy-saving technological change, and (2) find residual labor efficiency. The validation requires to compute also the residual technological gap of the capital intensity of energy between new vintages and average practice. We need to compute residual Harrod-neutral productivity, or labor efficiency, z_M to (1') use the sample period average of z_M to evaluate the change in energy use (to compute rebound and savings and find the relative contribution of energy-saving and neutral technological change to observed change in energy use), and (2') validate the use of balanced growth relationships in (1'). Second, we document how we obtain residual energy efficiency and the technological gap of energy efficiency between new vintages and average practice to reveal if energy efficiency has moved up or down.

(a) *Model Use and Validation.*—To compute the capital intensity of energy of new capital vintages, we insert the data on capital and energy into the left sides of Equations (5), (6), (9) and (11)—summed over capital intensity types. This yields an expression for the right sides. The measured capital-energy ratio v of new household durable consumption goods then equals the quotient of the right side of the law of motion of capital (5) and the right side of the law of motion of energy (9). Similarly, the capital-energy ratio v of new business equipment involves the laws of motion (6) and (11). We thus utilize that the agents in the model invest in capital with a single capital intensity of energy at each date.

The technological gap of the capital intensity of energy between new vintages and average practice can be found by relating the capital intensity of energy of new vintages and average practice. The capital intensity of average practice equals capital relative to energy, $\int_{v \in V} k_e(v)dv/u_M$ and $\int_{v \in V} k_d(v)dv/u_N$.

Labor efficiency in the market can be found given the calibrated production function of market output. To back out a value for the labor efficiency z_M, the values for structures, energy services, and observed working hours are plugged into the production function and observed output is used. Appendix D contains a plot of z_M. To find services over the sample period, we thus construct the time

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provided in Cociuba, Prescott, and Ueberfeldt (2012).

sequence of the distribution of energy by capital vintage, and use it to relate to energy efficiency by vintage. The age distribution of (energy-using) capital and energy over time expresses the distribution of capital and energy over types of the capital intensity of energy, as agents invest in only one type of the capital intensity of energy (for each sector). We distribute the capital and energy use at the initial sample date over types according to the steady-state age distribution of capital and energy with truncation at age of 200 years. We then track the age distribution of capital and energy over the sample period with the same truncation.[16] To find energy services, we compute the energy efficiency of new capital vintages. Energy services over the sample period are found by summing energy times the energy efficiency over vintages for each date (energy $k_e(v)/v$ and energy efficiency $f(v)$, and energy $k_d(v)/v$ and energy efficiency $h(v)$).

(b) *Model Revelation.*—Energy efficiency of new vintages can be constructed from the capital intensity of energy of new capital vintages—$f(v)$ in the market sector and $h(v)$ in the household sector (as used to find energy services). The technological gap of energy efficiency between new vintages and average practice can be found analogously to the technological gap of the capital intensity by relating the energy efficiency of new vintage capital and average practice. Energy efficiency of average practice can be derived from energy (measured in the data) and energy services (measured using data and model). We define energy efficiency of average practice as energy services divided by energy use, x_M/u_M and x_N/u_N.

D. Model Fit

Before turning to the application of the model to analyze the efficiency paradox, we first examine the model's fit to data on energy efficiency change and the effect of labor efficiency on energy use. A good fit validates the use of balanced growth relationships in our analysis.

Energy Efficiency.—As empirical measures of aggregate energy efficiency are not available, we evaluate the fit to energy efficiency with growth of the average capital intensity of energy. The model's balanced growth path predicts that the capital intensity of new vintages increases by about 3.7 percent per year in the household and market sector (ϕ_N and ϕ_M). This provides a good fit with the corresponding growth rates in the data of 3.4 and 3.9 percent. Regarding the technological gap, we also obtain a good fit.[17] We can thus use the balanced growth rate of energy

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[16] On a balanced growth path, the distribution of equipment capital over age is given by the summands in $\int_{v\in V} k_e{}'(v)dv = q_M i_e(v)\{\sum_{\tau=1}^{T}[(1-\delta_e)/g\gamma_{q_M}]^{\tau-1} + [(1-\delta_e)/g\gamma_{q_M}]^{T} + \ldots\}$, where $q_M i_e(v)[(1-\delta_e)/g\gamma_{q_M}]^{\tau-1}$ denotes the equipment capital aged τ periods from the date of its construction until the date with aggregate equipment capital $\int_{v\in V} k_e{}'(v)dv$ and $i_e(v)$ denotes investment in equipment capital one period ago for some $T \geq 1$. With balanced growth, the distribution of energy over age is given by the summands in $u_M{}' = (q_M i_e(v)/v)\{\sum_{\tau=1}^{T}[(1-\delta_e/g\gamma_{1/p}]^{\tau-1} + [(1-\delta_e))/g\gamma_{1/p}]^{T} + \ldots\}$, where $(q_M i_e(v)/v)[(1-\delta_e)/g\gamma_{1/p}]^{\tau-1}$ denotes the energy requirement of age-τ equipment capital at the date with aggregate energy use $u_M{}'$. The distribution of durable consumption goods and their energy requirement along a balanced growth path follow analogously.

[17] The stationary values of the technological gap of the capital intensity of energy in the market and household production can be found by rewriting the laws of motion of capital (5) and (6). Inserting calibrated balanced growth path values, the technological gap stays the same over time at 16 percent

efficiency ${\phi_i}^{\varepsilon_i}$ to reasonably well account for the long-run change in energy efficiency in sector $i \in \{M, N\}$.[18]

Energy Use.—The balanced growth relationships are suitable for discussing the effects of energy-saving technological change on energy use if the calibrated model predicts well empirical growth of energy use controlling for neutral productivity and neutral technological change is measured well. Energy relative to Harrod-neutral technology on the model's balanced growth path changes annually grows by 0.063 percent. The corresponding figure for the energy use in the data relative to the Harrod-neutral productivity residual is, similarly, 0.105 percent. We can thus be confident that the model balanced growth relationships of energy use control the rate of change in energy use for neutral technological change in the long-run on our sample period.

For using the time series average of measured labor efficiency or the calibrated balanced growth path value of the rate of change in labor efficiency for counterfactual energy use change (to compute rebound and savings and perform growth accounting of energy use to neutral technological change), the time series of residual labor efficiency z_M should fluctuate around the path with the growth rate of z_M on the calibrated balanced growth path. The value for γ_{z_M} in the residual time series of 1.0069 is close to the value for γ_{z_M} on the calibrated balanced growth path of 1.0072. The sample period average of residual labor efficiency or the calibrated balanced growth path rate of labor efficiency can then be used to predict reasonably well the change in energy use with counterfactual rates of energy-saving technological change.

IV. The Results

We now present our results which are based on the calibrated model. We first describe the evolution of energy efficiency in the household and business sectors. We then evaluate the energy efficiency paradox and examine the relative importance of technological change for changes in energy efficiency and energy use.

A. Evolution of Energy Efficiency

Figure 3 portrays the energy efficiency of new vintages in business equipment (solid) and household durables (dashed) as identified by our model. Four main insights emerge. First, energy efficiency of household and market production has on average increased by 2.7 and 2.9 percent per year. Second, business energy efficiency has often lagged household energy efficiency by one to two years. Third, business energy efficiency has fluctuated more than household energy efficiency (the

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for consumer durable goods and 25 percent for business equipment capital. The residual technological gap can be deduced using the laws of motion of equipment capital (5) and (6) and energy (9) and (11) and data on according aggregate variables. These empirical time series have no time trend, as a Dickey-Fuller test rejects trend-stationarity at the 99 percent significance level. The empirical series fluctuated around a mean of 11 and 27 percent in the household and market production being close to the calibrated steady-state value.

[18] Capital intensity growth at the rates of ϕ_M and ϕ_N implies energy efficiency growth at the rates of $(\phi_M)^{\varepsilon_M}$ and $(\phi_N)^{\varepsilon_N}$.

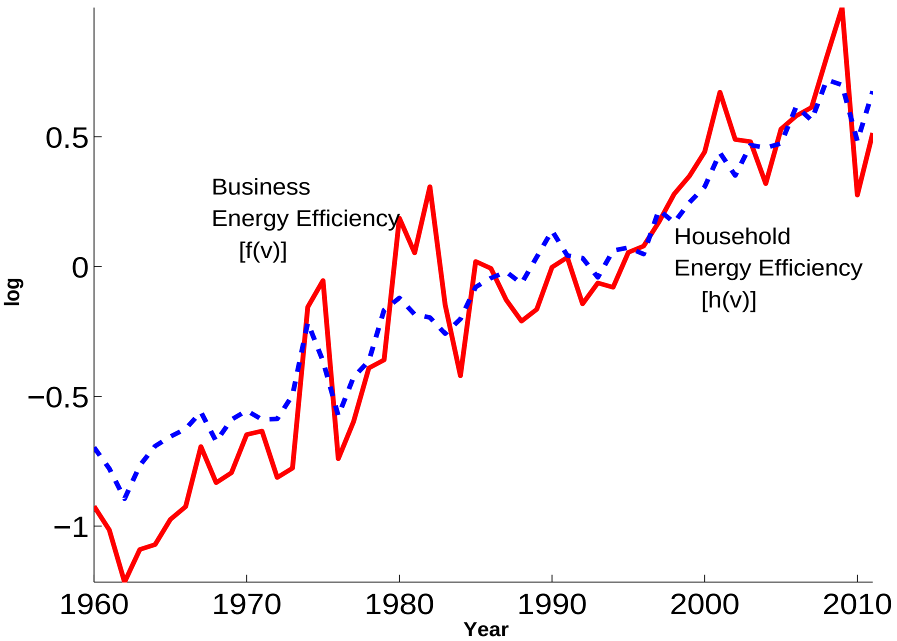

Fig 3. Energy Efficiency of New Vintages 1960-2011

Notes: Energy efficiency denotes the energy efficiency of the youngest business equipment vintage, $f(v)$, and of the youngest durable consumption goods vintage, $h(v)$. The series are demeaned (as the model identifies energy efficiency up to scaling by a constant) and in natural logarithms.

variance of the former is about twice as large as the one for the latter in new energy-using capital goods). This suggests that businesses responded more strongly to energy price shocks.[19] Fourth, further evidence for the greater response in business equipment compared to household durables can be found from the technological gap between the energy efficiency of new vintages and average practice that Figure 4 shows. Businesses significantly adjust energy efficiency of new equipment capital vintages above average practice—several times by more than 60 percent higher and up to almost twice as large as under average practice. In contrast, households install new consumer durable goods which are at most 32 percent more energy-efficient than the average stock of consumer durable goods. New equipment goods were on average relatively more efficient than new durable consumption goods when compared to the average efficiency of energy-use capital at a given point in time.

B. Increased Energy Efficiency, Higher Long-run Energy Use?

Over the sample period 1960-2011, two salient features emerge from the data and our calibrated model: (1) economy-wide energy use (measured per available

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[19] An intuitive explanation behind this result is as follows. Energy efficiency embedded in capital adjusts more strongly to changes in energy cost when the capital is expected to be used longer. Business equipment capital goods are used longer on average as they have depreciated more slowly relative to household durables ($\delta_e = 0.139$ smaller than $\delta_d = 0.232$).

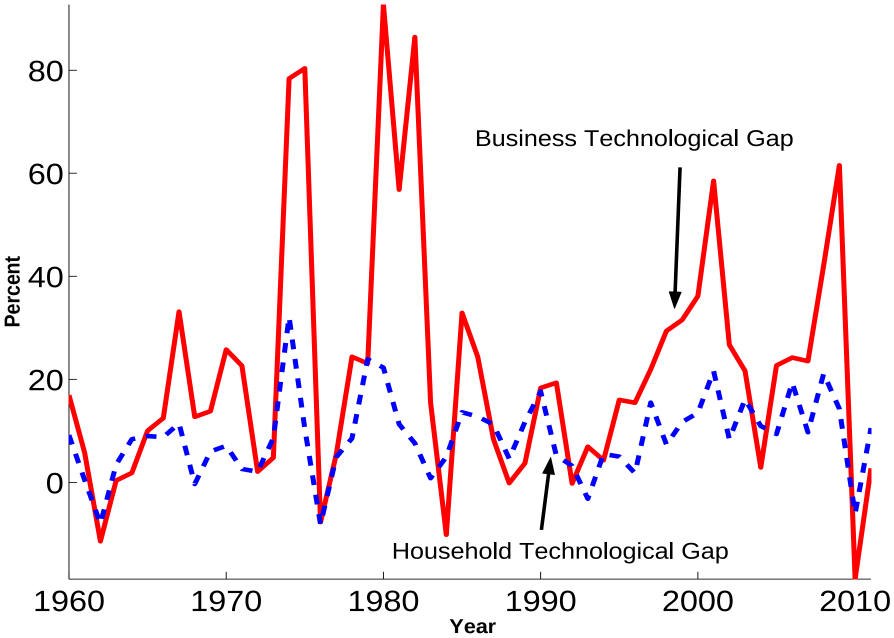

Fig 4. Technological gap between the energy efficiency of new vintages and average practice 1960-2011

Notes: Technological gap denotes the energy efficiency of the youngest vintage relative to the average energy efficiency. Average energy efficiency equals aggregate services divided by aggregate energy use by all business equipment vintages, x_M/u_M, and by all durable consumption goods, x_N/u_N.

working hour) has grown as shown in Figure 1A and (2) energy efficiency has improved steadily as shown in Figure 3. But how did improvements in energy efficiency affect the change in energy use?

At the centre of the energy efficiency paradox is the positive effect of technological progress (lower capital prices over time) on energy use induced by a lower price for energy services and increased real income. A complete picture of technological change, however, includes the negative effect of an increase in the energy price over time on energy use. Before we can discuss the change in the price for energy services based on empirical rates of technological change, we need to find the causal effects on energy use from technological change in our structural model.

Technological change can help saving energy as follows. A more rapidly declining capital price $1/q_i$ (greater γ_{q_i}) or a more rapidly increasing energy price p (lower $\gamma_{1/p}$) increase *energy efficiency growth* in the corresponding sector $i \in \{M, N\}$, recalling that (12) yields the rate of change in energy efficiency $\phi_i^{\varepsilon_i} = (\gamma_{q_i}/\gamma_{1/p})^{\varepsilon_i}$. With higher energy efficiency, a given unit of energy service output can be produced using less energy because energy service output is equal to the product of energy use and energy efficiency. In the aggregate, the change in energy use thus equals the change in energy service output divided by the change in energy efficiency:

$$\underbrace{[\gamma_{1/p} g/\gamma_{z_M}]\gamma_{z_M}}_{\text{Growth rate of energy use}} = [\ \underbrace{\phi_i^{\varepsilon_i}}_{\text{Growth rate of energy efficiency}}\ ]^{-1} \times \underbrace{g\gamma_{q_i}{}^{\varepsilon_i}\gamma_{1/p}{}^{1-\varepsilon_i}}_{\text{Growth rate of services}}. \tag{30}$$

In equation (30) we have disentangled the change of energy use into effects from energy-saving technological change (in brackets) and neutral technological change (the solo term after the brackets).

To examine the efficiency paradox, we now analyze the energy use requirement in producing energy services with energy efficiency which equation (30) inherits. We have to separate two effects of the impact of energy-saving technological change on energy use. "Energy savings" should measure how energy use responded to energy efficiency had energy service demand be unaffected by the increase in energy efficiency. "Energy rebound" should measure how energy use responded to a change in the energy service demand for a given energy efficiency improvement.

More formally, we are interested in decomposing the change in energy use between the calibrated equilibrium with energy efficiency improvement (i.e., our calibrated balanced growth path, denoted by $E = 1$), and the counterfactual equilibrium with no energy efficiency improvement (i.e., another balanced growth path, denoted by $E = 0$). To reflect the sources of energy efficiency change, the paths are characterized by empirical and no energy-saving technological change. Let $\vartheta_i(E)$ and $\eta_i(E)$ denote the gross rate of change in energy services and the gross rate of change in energy efficiency in equilibrium $E \in \{0, 1\}$ and sector $i \in \{M, N\}$.[20] The change in energy use in sector i between equilibria $E = 1$ and $E = 0$ can then be decomposed as follows:

$$\underbrace{\frac{1}{\eta_i(1)}\vartheta_i(1) - \frac{1}{\eta_i(0)}\vartheta_i(0)}_{\text{Energy use}} = \underbrace{\frac{\vartheta_i(1) - \vartheta_i(0)}{\eta_i(1)}}_{\substack{\text{Energy rebound } \mathcal{R}_i \\ \text{(Effect of energy services} \\ \text{on energy use)}}} - \underbrace{\left[\frac{1}{\eta_i(0)} - \frac{1}{\eta_i(1)}\right]\vartheta_i(0)}_{\substack{\text{Energy savings } \mathcal{S}_i \\ \text{(Effect of energy efficiency} \\ \text{on energy use)}}}.$$

Energy savings express the difference in the change of energy use between hypothesized counterfactual energy services change and measured energy efficiency change [$\eta_i(1)$ and $\vartheta_i(0)$] and the equilibrium with counterfactual services and energy efficiency change [$\eta_i(0)$ and $\vartheta_i(0)$], thus giving the response of energy use to energy efficiency change. Energy rebound describes the difference in the change of energy use between the equilibrium with measured energy efficiency and energy services change [$\eta_i(1)$ and $\vartheta_i(1)$] and hypothesized measured energy efficiency change and counterfactual energy services change [$\eta_i(1)$ and $\vartheta_i(0)$], thus giving the response of energy use to energy services change.[21]

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[20] To tackle the question of how energy efficiency affected energy use on average, using the balanced growth relationships is a better way than simulating equilibrium paths (given balanced growth approximates observed average growth of energy use).

[21] Energy demanded at given prices of capital and energy in equilibrium situations $E = 0$ or $E = 1$ can

Using equations (13) and (30), the change in energy services and energy efficiency $\vartheta_i(E)$ and $\eta_i(E)$ can be written in terms of different technological changes so that

$$\mathcal{R}_i = (\tilde{g} - \phi_i^{-\varepsilon_i})\gamma_{z_M}, \quad \mathcal{S}_i = (1 - \phi_i^{-\varepsilon_i})\gamma_{z_M}, \tag{31}$$

where $\tilde{g} \equiv \gamma_{1/p} g / \gamma_{z_M}$ denotes the growth rate of energy use when neutral technological change was absent (the term corresponding to energy-saving technological change in brackets in (30)).[22] We control for neutral technological change by keeping it constant, thus keeping γ_{z_M} constant. The rate of change $\tilde{g}$ is relevant for energy services change in both the household and market sectors as it appears in $\vartheta_i(1)$ for both sectors. This reflects that energy used by both firms and households is paid for with market output.

Energy savings are positive, $\mathcal{S}_i > 0$, if energy efficiency improves over time, $\phi_i^{\varepsilon_i} > 1$. The response of energy use to energy efficiency change is captured by the sign of $(1 - \phi_i^{-\varepsilon_i})$, which measures the difference in the annual rate of change in energy use between energy efficiency change based on counterfactual rates of energy-saving technological change (1) and empirical rates $(\phi_i^{-\varepsilon_i})$. As the model identifies improved energy efficiency, positive energy savings are empirically relevant. A positive (negative) energy rebound then counteracts (reinforces) energy savings. Rebound is positive, $\mathcal{R}_i > 0$, if energy-saving technological change induces a positive effect on energy use through stimulating energy services, precisely increases energy services, $\phi_i^{\varepsilon_i}\tilde{g} > 1$. Rebound is negative, $\mathcal{R}_i < 0$, if energy services contract, $\phi_i^{\varepsilon_i}\tilde{g} < 1$. The response of energy use to energy services change is captured by the sign of $(\tilde{g} - \phi_i^{-\varepsilon_i})$, which measures the difference in the annual rate of change in energy use between energy services change based on empirical rates of energy-saving technological change $(\tilde{g})$ and counterfactual rates $(\phi_i^{-\varepsilon_i})$. The response of energy services induced by energy-saving technological change can be so strong that energy use increases because of energy-saving technological change, $\tilde{g} > 1$. Importantly, then rebound exceeds savings, $\mathcal{R}_i > \mathcal{S}_i$. While the magnitudes of rebound and savings depend on neutral technological change, it is important to understand that the ratio of rebound to savings is unaffected by the effect of neutral technological change on energy use controlled for (i.e. held constant).

Results and Interpretation.—We can now find rebound and savings of energy use and the energy rebound rate ρ_i defined as the ratio of rebound to savings,

$$\rho_i \equiv \mathcal{R}_i \times \frac{1}{\mathcal{S}_i},$$

by sector through evaluating (31). Table 1 shows the terms giving rebound and savings as average annual rates of change, which we now interpret.

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be thought of as uncompensated (Marshallian) energy demand. Energy demanded at measured change in energy efficiency and counterfactual change in services can be interpreted as compensated (Hicksian) energy demand.

[22] Specifically, $\eta_i(1) = \phi_i^{\varepsilon_i}$, $\eta_i(0) = 1$, $\vartheta_i(1) = \phi_i^{\varepsilon_i}[\tilde{g}\gamma_{z_M}]$, and $\vartheta_i(0) = \gamma_{z_M}$. Note that for the counterfactual equilibrium with no energy-saving technological change ($E = 0$), we have used 1 in place of γ_{q_i} and $\gamma_{1/p}$ (rather than introduce new symbols, we let γ_{q_i} and $\gamma_{1/p}$ equal their calibrated values, which $\tilde{g}$ uses).

Table 1

Rebound and savings of energy use (in terms of average annual percentage change)

	Businesses $(i = M)$	Households $(i = N)$
Energy rebound $\mathcal{R}_i$	$2.92\ (0.79 + 2.13)$	$2.97\ (0.79 + 2.18)$
With effect of energy-saving technological change on energy services ($\tilde{g}\gamma_{z_M}$)	0.79	0.79
Without effect of energy-saving technological change on energy services ($\phi_i^{-\varepsilon_i}\gamma_{z_M}$)	−2.13	−2.18
Energy savings $\mathcal{S}_i$	$2.85\ (0.72 + 2.13)$	$2.90\ (0.72 + 2.18)$
Without effect of energy-saving technological change on energy efficiency (γ_{z_M})	0.70	0.70
With effect of energy-saving technological change on energy efficiency ($\phi_i^{-\varepsilon_i}\gamma_{z_M}$)	−2.13	−2.18
Energy rebound rate $\mathcal{R}_i/\mathcal{S}_i$	1.0245	1.0241

Notes: Energy rebound measures the effect of energy service demand controlling for energy efficiency. Energy savings measures the effect of energy efficiency controlling for energy service demand. Calculations are based on equation (31) using the gross rates of change $\gamma_{q_M} = 1.031$, $\gamma_{q_N} = 1.032$, $\gamma_{1/p} = 0.995$ plugged into $\phi_M = \gamma_{q_M}/\gamma_{1/p}$ and $\phi_N = \gamma_{q_N}/\gamma_{1/p}$, and $\tilde{g}\gamma_{z_M} = g\gamma_{1/p}$ with $g = 1.013$, subject to $\varepsilon_M = 0.81$, $\varepsilon_N = 0.80$, and $\gamma_{z_M} = 1.0072$ that are derived from the calibrated model. The values imply $\tilde{g} = 1.0007$.

Focusing first on energy savings, business and household energy use would have increased by 0.7 percent per year in the absence of energy-saving technological change and given neutral technological change (with no effect from energy efficiency). Business (household) energy use would have declined by 2.13 (2.18) percent per year with the energy efficiency improvements as identified by our model, if one controls for the impact of energy-saving technological change on energy service demand (with effect from energy efficiency). The energy savings $\mathcal{S}_i$ from energy efficiency improvements for business (household) energy use are thus 2.85 (2.90) percent per year. This confirms the energy savings hypothesis which states that energy efficiency improvements bring about reductions in energy use.

Reporting now energy rebound, the decline in energy use is correct when the rates of change in energy services are held being fixed (with no effect from energy services). But they cannot be held fixed. The energy savings are counteracted by the increase in energy services induced by energy-saving technological change, which in turn requires greater energy use. Business and household energy use grow by 0.77 per year with this effect from energy services, controlling for the effect of energy efficiency on energy use (with effect from energy services), leading to an energy rebound $\mathcal{R}_i$ from energy services response of 2.92 (2.97) per year for businesses (households).

We thus find evidence for energy rebound in excess of energy savings with an economy-wide rebound rate of 1.02 [by weighting $\rho_M = 2.92/2.85$ for businesses and $\rho_N = 2.97/2.90$ for households with any weighting scheme].[23] It seems unsur-

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[23] An economy-wide rebound rate can be calculated as a weighted average of sectoral rebound rates ρ_M

prising that the sectoral rebound rates are positive but the striking result is that the rebound rates are much closer to one than to zero; in fact they are greater than one. Energy demanded thus "backfired" in response to energy-saving technological change: higher energy efficiency has increased, not reduced energy use—thus providing a confirmation of the energy efficiency paradox.

What explains that energy rebound exceeded energy savings? Rebound exceeds savings if energy use increases because of energy-saving technological change, i.e., $\tilde{g} > 1$. To see why $\tilde{g} > 1$, it is instructive to investigate the different channels through which energy-saving technological change affects energy use with the balanced growth representation:

$$\tilde{g} = \underbrace{\gamma_{q_M}{}^{a\varepsilon_M}}_{\substack{\text{Cross-price}\\\text{income effect}}} \underbrace{\gamma_{1/p}{}^{a(1-\varepsilon_M)}}_{\substack{\text{Own-price}\\\text{income effect}}} \underbrace{\gamma_{1/p}}_{\substack{\text{Own-price}\\\text{substitution effect}}} .$$

A more rapidly declining equipment capital price (greater γ_{q_M}) stimulates growth in energy use—the cross-price effect, as the construction of new investment goods requires output produced with investment goods at diminishing returns to scale. A more rapidly growing energy price (smaller $\gamma_{1/p}$) depresses the rate of change in energy use—the own price income and substitution effects, as the acquisition of energy requires output produced with energy. An income effect here designates an effect on market output. A substitution effect impacts energy use and does not affect market output. Hence, the following reasons emerge for an energy rebound rate in excess of one:

(1) *Strong decline in the capital price in the market relative to the increase in the energy price, as depicted by Figure 2.* Energy use has experienced a relatively strong cross-price income effect from investment-specific technological change that has dominated the negative own-price income and substitution effects following higher energy prices. We find that the relative contribution of each of these three effects to the growth in energy use (had neutral technological change been absent) was: 822 percent for the *cross-price income effect*, −31 percent for the *own-price income effect*, and −691 percent for the *own-price substitution effect.*

(2) *Growth in output is relatively elastic with respect to growth in services.* The cross-price income effect from investment-specific technological change is magnified if output change is relatively elastic with respect to services change, which is expressed by the exponent $a = \alpha_M(1-\gamma_M)/(1-\alpha_M)$. We derive $a = 0.24$. Importantly, this "large" value reflects the importance of energy services relative to other production factors (labor and capital structures) in producing output. If energy services comprised only a small income share of output (equal to $\alpha_M(1-\gamma_M)$), then intuitively the income effect on energy use induced by energy-saving technological change would be much smaller,

and ρ_N. As the sectoral rebound rates turn out to be quite similar, the specific weighting scheme does not play a role for the value of the economy-wide rebound rate.

in turn implying a lower rebound rate.

We can now discuss the development of the price for energy services. Energy growth because of energy-saving technological change requires that energy services grow relative to output in the market in the equilibrium with empirical energy-saving technological changes.[24] The implicit price for energy services in the market equal to the marginal product of output with respect to services $(\alpha_M(1-\gamma_M)y/x_M)$, declines whenever energy services grow relative to output. The implicit price for energy services thus declines in the equilibrium subject to empirical energy-saving technological changes.

C. *What Drives Long-run Growth in Energy Efficiency and Energy Use?*

We now examine the relative importance of energy-saving and Harrod-neutral technological change for the identified energy efficiency improvements and the observed growth in energy use through equilibrium growth accounting. In general, technological change has an income effect (IE) or a substitution effect (SE) on the growth of energy use.

Using the relationships (13) and (30), we can decompose the gross rate of change of services as follows:

$$g{\gamma_{q_i}}^{\varepsilon_i}{\gamma_{1/p}}^{1-\varepsilon_i} = \underbrace{\overbrace{{\gamma_{q_M}}^{a\varepsilon_M}}^{\text{IE}}\overbrace{{\gamma_{1/p}}^{a(1-\varepsilon_M)}}^{\text{IE}}\overbrace{\gamma_{1/p}}^{\text{SE}}\overbrace{\gamma_{z_M}}^{\text{IE}}}_{\substack{\text{Growth rate of}\\ \text{energy use}\,(=\tilde{g}\gamma_{z_M})}}\underbrace{\overbrace{{\gamma_{q_i}}^{\varepsilon_i}}^{\text{SE}}\overbrace{{\gamma_{1/p}}^{-\varepsilon_i}}^{\text{SE}}}_{\substack{\text{Growth rate of}\\ \text{energy efficiency}}}, \quad i \in \{M, N\}\,.$$

The energy price change (denoted by $1/p$) affects the change in energy use through a SE and IE. Equipment-specific technological change in the market sector (denoted by q_M) affects energy use only through an IE. Harrod-neutral technological change in the market sector (denoted by z_M) affects change in energy use through an IE. Note that energy is paid in units of market output; hence IEs appearing in the growth rate of energy use are those from the market sector.

Sources of Long-run Energy Efficiency Improvements.—We now quantify the relative importance of technological changes for the identified energy efficiency improvement. In the market sector, energy efficiency would have increased on average by 2.53 percent per year if energy prices had not trended downward or upward. Energy efficiency would have grown on average by 0.4 percent per year if equipment-specific technological change had been absent. The contribution of equipment-specific technological change to energy efficiency growth in the market sector is thus 85.5 percent [=2.53/(2.53 + 0.43)], see Table 2. In household production, the corresponding numbers are 2.53 percent and 0.42 percent implying a contribution by equipment-specific technological change to energy efficiency

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[24] The rate of change of services demanded per output unit, ${\gamma_{q_M}}^{\varepsilon_M}{\gamma_{1/p}}^{1-\varepsilon_M}$, exceeds unity. The net income effect is thus positive. With empirical rates of change in the technologies pinning down capital and energy prices, services grow because capital grows, though the efficiency of capital in producing services dwindles.

Table 2

Contributions to Growth (in percent)

Energy efficiency^{a}	
Equipment-specific technological change (SE)	85.5 / 85.8
Energy price change (SE)	14.5 / 14.2
Energy use	
Equipment-specific technological change (IE)	80
Energy price change (IE & SE)	-72
Harrod-neutral technological change (IE)	92

Notes: The growth accounting is based on equation (IV.C). IE and SE denotes an income and substitution effect, respectively. ^{a}The first and second number refers to the business and household sector, respectively.

growth of 85.8 percent. The large role of equipment-specific technological change for energy efficiency growth is unsurprising given the drastic decline in the capital price and the moderate increase in the energy price over our sample period.

Sources of Long-run Growth in Energy Use.—We now quantify the relative importance of technological changes for the observed energy growth. Energy use per available working hour would have decreased on average by 0.54 percent per year if the nonresidential equipment price and Harrod-neutral technology had been constant in the long-run. This number corresponds to the sum of the SE and IE of energy price on energy demand. Energy use would have increased by 0.60 percent if the energy price and Harrod-neutral technology had been constant. Energy use would have grown by 0.69 percent if the nonresidential equipment and energy prices had been constant. Equipment-specific technological change thus contributed about 80 percent to energy-per-hour growth $[=0.60/(0.60-0.54+0.69)]$, see Table 2. Higher energy prices contributed about -72 percent to energy-per-hour growth $[=-0.54/(0.60-0.54+0.69)]$. The remaining contribution by Harrod-neutral technological change with 92 percent is the relatively most important source for growth in energy use.

Importantly, we can see that energy use would have grown if only energy-saving technological change had occurred, because the IE of equipment-specific technological change outweighed the combined IE and SE of the energy price change on energy demanded.

V. Alternative Counterfactual Viewpoints

A more narrow view of the energy efficiency paradox would control for changes in the energy price. To implement this view, we now vary investment-specific technological change, controlling for the energy price change. The empirical energy price change then affects the change in energy services in the equilibrium with counterfactual technological change. Energy rebound and savings are calculated using the formula in (31) now based on new counterfactual changes of energy efficiency and services $\tilde{\eta}_i(0)$ and $\tilde{\vartheta}_i(0)$ that are determined by the rates of change of the inverse of the energy price ($\lambda_{1/p}$), and business equipment-specific technology

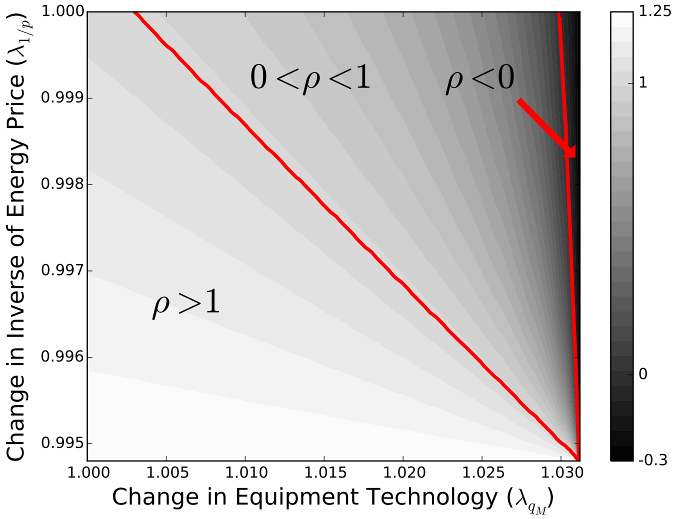

Fig 5. Energy Rebound Rate ρ 1960-2011: Alternative Counterfactual Viewpoints about Energy-Saving Technological Change

Notes: The counterfactual average annual growth rate of equipment-specific technology is on the x-axis. The inverse of the counterfactual average annual rate of change in the energy price is on the y-axis. The counterfactual growth rates of equipment-specific technology $(\lambda_{q_M}, \lambda_{q_N})$ are positively related through $\ln \lambda_{q_N} = \ln \lambda_{q_M} \ln \gamma_{q_N} / \ln \gamma_{q_M}$. The diagonal red curve shows cases for $\rho = 1$; the red curve close to the right edge of the box shows cases for $\rho = 0$.

(λ_{q_M}), and durable goods-specific technology (λ_{q_N}).[25]

The rebound rate to either source of energy-saving technological change is 1.25 (narrow view of the energy efficiency paradox) and -0.30 (energy price rebound). By construction, the rebound rate controlling for energy price change is larger than unity and the energy price rebound rate is smaller than zero. Despite the a priori known ranges for these rebound rates, the magnitudes are far less clear. On average over the period of the sample, for each percent potential savings of energy use, equipment-specific technological change has counteracted with a rebound of energy use of 1.25 percent (controlling for energy price change). In contrast, the energy price increase saved energy at the rate of -0.30 percent of the hypothetical savings (controlling for investment-specific technological change). The positive rebound related to investment-embodied technological progress thus was much stronger than the negative rebound from higher energy prices.

The two cases analyzed above represent counterfactual cases holding one type of energy-saving technological change at its empirical value. These cases bound the three-dimensional space of counterfactual technology for capital price change in each sector and energy price change. The space is spanned by the values corre-

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[25] The respective expressions for energy efficiency and services change for $E = 0$ are then given by $\tilde{\eta}_i(0) = (\lambda_{q_i}/\lambda_{1/p})^{\varepsilon_i}$ and $\tilde{\vartheta}_i(0) = (\lambda_{q_i}/\lambda_{1/p})^{\varepsilon_i} \lambda_{1/p} \gamma_{z_M} [\lambda_{q_M}{}^{a\varepsilon_M} \lambda_{1/p}{}^{a(1-\varepsilon_M)}]$. The rebound rate controlling for energy price change obtains for $\lambda_{1/p} = \gamma_{1/p}$, $\lambda_{q_M} = 1$, $\lambda_{q_N} = 1$. The rebound relative to the energy savings controlling for capital price change uses $\lambda_{1/p} = 1$, $\lambda_{q_M} = \gamma_{q_M}$, $\lambda_{q_N} = \gamma_{q_N}$. The rebound rates of the market and household sectors are weighted with the mean of the empirical portion of sectoral energy on the sample period.

sponding to no change (the respective gross rate of change equal one) and empirical rates of change. Figure 5 shows the contour set of the economy-wide energy rebound rate on a two-dimensional subspace of this space as an example. To attain this subspace, we fix a positive relationship between sectoral equipment-specific technological changes. It contains the point of empirical rates of changes for equipment-specific technological and energy price change at the bottom-right corner. The bottom-left corner depicts the case of the narrow view of the energy efficiency paradox when controlling for energy price change, the top-right corner depicts the case of the energy price rebound when equipment-specific technological change is held at the empirical value. Note that the axes show the *counterfactual* rates of changes.

Figure 5 bears out the following insights (for the chosen example fixing a positive relationship between sectoral equipment-specific technological changes). First, the economy-wide rebound rate will be the larger (smaller) the farther away the counterfactual rate of change of equipment-specific technology (energy price) lies from its empirical value.[26] Second, the variation in the distance between the contour levels of the rebound rate implies that the rebound rate increases more than proportionally when moving the counterfactual rate of change away from the empirical value for the equipment-specific technology. The income effect from equipment-specific technological progress thus turns out to be increasingly important as counterfactual and empirical values for this type of energy-saving technological change are more distant from one another. This underlines the importance of considering changes in income through energy-saving technological progress.

Importantly, Figure 5 makes clear that the size of the rebound rate depends on the design of the counterfactual of energy-saving technological change. Each design of the counterfactual case yields a distinct rebound rate which needs to be interpreted accordingly, even though different designs can yield the same value of the rebound rate. The case relevant to the energy efficiency paradox, or energy rebound hypothesis, that greater energy efficiency leads to greater energy use—in line with the conjecture by Jevons (1865)—involves no energy-saving technological change which is depicted in the top-left corner exhibiting an energy rebound rate in excess of one.

VI. Robustness Checks

This section examines the sensitivity of the rebound rate with respect to the calibration choice of the elasticity of intertemporal substitution of consumption, the time discount factor, and the labor share of income, and illuminates if assuming full utilization of capital significantly affects our estimation of the energy rebound rate.

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[26] The maximum and minimum rebound rates are reached with controlling for energy price change and for capital price change separately.

A. *Elasticity of Intertemporal Substitution and Time Preference*

We explore alternative values for the elasticity of substitution of intertemporal consumption $1/\gamma$ and the discount factor β. To this end, we adapt the calibration of the model to exogenously set the preference parameters (β, γ). With this calibration strategy we discard the information on the mean annual return on equity used in the benchmark calibration.[27] The after-tax return rate R becomes lower when each household becomes more patient (higher β) or prefers a more smooth intertemporal allocation of consumption (lower γ) given growth of market output (see equation (23)). Thus, greater patience or greater desire to smooth consumption, through decreasing the interest rate R, decreases the capital intensity elasticity of energy efficiency ε_i (see equation (22)), and thereby increases the rebound rate in the household sector, ρ_N. The values for (β, γ) do not affect the calibrated parameter values for market production, and thereby have no impact on the rebound rate in the market sector, ρ_M.

Changing the after-tax interest rate from the benchmark case of $R = 1.04$ to $R \in \{1.02, 1.07\}$ has only a negligible effect on the economy-wide energy rebound rate; in terms of the contour plot shown in Figure 5 there is virtually no difference. The effect is small because in these cases the elasticity of energy efficiency with respect to the capital-energy ratio ($\epsilon_N \in \{0.79, 0.82\}$) only slightly deviates from the benchmark case ($\epsilon_N = 0.80$).

B. *Labor Share of Income*

With a labor share of income of 70 percent, we have erred optimistically for a low energy rebound relative to savings. According to Cooley and Prescott (1995), the labor share of income is upwardly biased with standard disaggregation of income flows in the NIPA. A labor share as low as 60 percent increases the energy rebound to 112 percent of energy savings (rebound rate of 1.12), as the elasticity of output change with respect to services change in the market increases to 0.35.[28]

C. *Optimal Utilization of Capital*

We assumed that equipment capital and durable consumption goods are always fully utilized. Assuming full utilization enables to measure energy efficiency and back out a neutral technology residual for growth accounting without knowing the utilization rate of capital in the data. A model with chosen utilization of capital would predict that capital units with relatively low energy efficiency would be idle. This could therefore in principle affect the estimation of the energy rebound rate. Assuming full utilization is a good approximation, if the scrapping age of capital is large.

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[27] Changing one of (β, γ) allows to keep using the benchmark mean annual return without any effect on the rebound rate.

[28] Weighting the rebound rates in the business and household sectors of 1.120 and 1.126 with their portion of the energy use in the first year of the sample yields an economy-wide rebound rate equal to 1.123. The values for the parameters in the elasticity $\alpha_M(1-\gamma_M)/(1-\alpha_M)$ are $\alpha_M = 0.4$ and $\gamma_M = 0.48$.

For each vintage, we can check the economic incentive for underutilization given our calibrated model. We find that all capital units installed after 1976 would be fully utilized on the remainder of the sample period if utilization could be chosen. Equipment capital and durable goods depreciate so fast that only a small fraction of capital units would be underutilized—representing only up to a maximum of less than 0.1 percent for a given date over our sample period in both the market and household sector.[29] Full utilization of capital is thus an innocuous assumption in our context.

VII. Conclusions

This paper has quantitatively examined the response of energy use to technological change using a general equilibrium growth model of vintage capital and energy in businesses and households. The empirically observed decline of capital price (for business equipment and durable consumption goods) and higher energy prices can explain the long-run increase in the capital-energy ratio (in market and household production). As we have postulated that the capital-energy ratio and energy efficiency are intimately linked, these price changes are sources of long-run improvement in energy efficiency we measure. Long-run neutral technological change is predicted well by the balanced growth path.

We have established evidence for energy rebound having been 102 percent of energy savings in the U.S. economy over the 1960-2011 period on average per year. The potential reduction in energy use from energy efficiency improvements has been reversed by the rebound effect macroeconomically because a lower price for new equipment capital enhanced energy use more than a higher energy price reduced energy use. Our analysis thus supports the energy efficiency paradox, or the energy rebound hypothesis—as conjectured by Jevons (1865) in his book *The Coal Question*.

Three major conclusions can be drawn. First, the energy efficiency paradox encapsulates the role of factors leading to changes in the efficiency of energy in producing energy services. The size of the energy rebound relative to energy savings is influenced by factors determining the costs of energy reducing energy use. A higher energy price leads to greater efficiency and smaller energy use whereas a lower energy price implies smaller energy efficiency and greater energy use. Second, that the evolution of sources for energy efficiency growth, investment-specific technological and energy price change, has increased macroeconomic energy use casts strong doubts on the ability of energy-saving technological change to help address the challenges of fossil fuels related to climate, local pollution, and energy security.

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[29] The cutoff level of energy efficiency below which underutilization is rational in market production $f(v^*)$ and in household production $h(v^*)$ is given by $F_2(k_s, x_M, z_M\ell) = p/f(v^*)$ and $H_2(k_r, x_N, z_N(1-\ell))U_2/U_1 = p/h(v^*)$. These two conditions are analogous to Equation (15) in Atkeson and Kehoe (1999). The conditions can be developed into $\alpha_M(1-\gamma_M)y/x_M = p/f(v^*)$ and $\alpha_N(1-\gamma_N)(y/x_N)(c_N/y)U_2/U_1 = p/h(v^*)$. Clearly, the left side decreases over time during balanced growth with capital intensity increase so that the cutoff level increases over time. Capital may be mothballed and utilized later when the energy price has decreased sufficiently, if the energy price fluctuates around the deterministic balanced growth path. See Gilchrist and Williams (2000) and Wei (2013) for simulated models with a discrete number of capital goods which are mothballed.

Third, our structural approach to the efficiency paradox can (and we think should) inform also the analysis of microeconomic and product-specific technology-induced rebound effects.

Appendix A: Data

This appendix describes the construction of the data series used in the paper. We begin with the energy data from the Annual Energy Review 2011—*AER*, and State Energy Data System 1960-2012—*SEDS*, and continue with the equipment price series of Gordon (1990), and data on income, expenditure, and prices in the National Income and Product Accounts—NIPA, and on capital stock in the Fixed Asset Accounts—FAA.[30]

Energy Price, Energy Use

The energy price p is the ratio of the deflator for energy $\tilde{p}$ to the consumption deflator P_c. The deflator for energy uses the aggregate deflators p_M and p_N and real energy use u_M and u_N for the sectors M and N. We obtain the deflator for energy by weighting the sectoral deflators with their corresponding share of sectoral real energy use, $\tilde{p} = (p_M u_M + p_N u_N)/(u_M + u_N)$. The deflator for sectoral energy utilizes specific energy deflators and quantities, and aggregates their information. Let the deflator for energy type i (for some primary energy not used for electricity production or electricity) at date t be $P_{i,t}$. Let the corresponding quantity be $E_{i,t}$. The deflator for energy in sector $j \in \{M, N\}$ is the Laspeyres constant price index $p_{j,t} = \sum_{i \in \mathcal{I}_j} P_{i,t} E_{i,t} / \sum_{i \in \mathcal{I}_j} P_{i,b} E_{i,t}$ with the set $\mathcal{I}_j$ and base period year 2009 indexed by b. The deflator $P_{i,b}$ is the mean price relative to the mean price of one type of energy (coal used in the Industrial sector). We obtain real energy use as $u_{j,t} = \sum_{i \in \mathcal{I}_j} P_{i,b} E_{i,t}$ for $j \in \{M, N\}$.

The deflator $P_{i,t}$ corresponds to the date-t price in an institutional sector i in the data. We use the prices for biomass, coal, petroleum, and natural gas in *AER* Table 3.1 and *SEDS* various tables. We use the electricity retail price. The corresponding quantities of energy use, described by $E_{i,t}$, are measured as the Commercial, Industrial, Transportation sectors' energy consumption and electricity consumption from *AER* Tables 2.1a-d, 8.4a, and 8.4c.[31] We adjust electricity consumption for production in different sectors. Electricity produced in the Commercial and Industrial sectors is added to electricity consumed by these sectors which is produced in the Electricity sector. We sort the resulting Commercial and Industrial sector energy use to market energy use, Residential sector energy use to household energy use, and Electricity sector sale to the Commercial, Industrial, and Residential sectors in this manner to market and household energy use. We account each a share of Transportation energy use to the market and household energy use. To obtain

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[30] The current releases of the energy data used are available at http://www.eia.gov/totalenergy/data/annual/ and http://www.eia.gov/state/seds/seds-data-complete.cfm. The national account data used are taken from http://www.bea.gov/iTable/index.cfm.

[31] Hassler, Krusell, and Olovsson (2012) form a Laspeyres price index with the average relative price among energy resources. An alternative to the average relative price among energy resources is a base price in some period as in Atkeson and Kehoe (1999). Both these routes produce very similar indexes.

the share of transportation energy used in the market and household sectors, we use Highway Statistics on motor gasoline.[32] The energy price and quantity series include renewable energy through biomass and electricity.

The consumption deflator P_c is the Törnqvist index of nondurables consumption and nonhousing services (not to be confused with the services in the model).

Equipment and Durable Consumption Goods Prices

We measure the investment-specific technology q_M and q_N (the inverse of equipment and durable goods prices) as the ratio of the deflator for consumption relative to the deflator for new private nonresidential equipment capital and nondurable consumption goods. For the period 1960-1983, the investment-specific technology q_M is the common deflator P_c relative to Gordon's (1990) producer durable equipment (PDE) price index for business equipment (available for the period 1947-1983), q^{PDE}. For the period 1984-2011, the investment-specific technology q_M is the forecast of the dependent variable q^{PDE} in the autoregressive model

$$q_t{}^{PDE} - aq_{t-1}{}^{PDE} = b(q_{M,t}{}^{NIPA} - (c/b)q_{M,t-1}{}^{NIPA}) + u_t,$$

where the independent variable $q_M{}^{NIPA}$ is measured as the common deflator relative to the NIPA deflator for private nonresidential equipment investment, and u is a normally distributed error term. Column 1 in Table A1 presents the estimated values of the coefficients a, b, and c along with their T-statistics. A specification with constant term, appearing in Column 2 in Table A1, is ruled out because the constant is insignificantly different from zero at the five-percent significance level. We solve forward the econometric model to obtain the forecast for the period 1984-2011 which we append to the observed series of relative prices q^{PDE} to form the vector of adjusted relative prices q_M.

To form the NIPA equipment investment deflator, we account for heterogeneous prices for equipment investment in capital involved and not involved in energy production. The NIPA equipment investment deflator is formed by using the NIPA's chain-weighting procedure to combine the price index for aggregate private nonresidential equipment investment in Table 1.1.4, the indexes for equipment investment in capital used for energy production in Detailed Accounts Tables, and the associated current-value flows (see below for a description of components deducted).[33]

We compute the investment-specific technology of durable consumption goods q_N by ratio splicing the NIPA deflator for durable consumption goods with the nonresidential equipment deflator relative to the NIPA deflator for private nonresidential equipment and applying the common deflator, $q_N = q_N{}^{NIPA} q_M / q_M{}^{NIPA}$. The common deflator relative to the deflator for durable consumption goods forms $q_N{}^{NIPA}$. The NIPA deflator for durable consumption goods is taken from Table

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[32] The series of prices $p_{M,t}$ and $p_{N,t}$ are very similar to each other.

[33] We use the HP-filtered series of the equipment investment price index formed after deduction of energy sector investment to express $q_M{}^{NIPA}$ (with low smoothness parameter of 6.25), because the index fluctuates more than the aggregate equipment investment price index. We use the HP-filtered series of the corresponding consumer durables price index to express $q_N{}^{NIPA}$ for consistency. Investment before and after deduction of energy sector investment fluctuates about the same.

TABLE A1
q-REGRESSIONS, OLS METHOD, 1947-1983

	Dependent Variable Gordon's PDE Deflator (${q_t}^{PDE}$)	
Regressor	(1)	(2)
${q_{t-1}}^{PDE}$	0.952	0.945
	(23.30)	(31.25)
${q_{M,t}}^{NIPA}$	0.631	0.650
	(3.76)	(2.69)
${q_{M,t-1}}^{NIPA}$	0.572	0.574
	(3.86)	(3.94)
Constant	No	Yes
		-0.011
		(0.21)
R^2	0.979	0.982
Observations	36	36

Note: T-statistics based on White heteroskedasticity-robust standard errors are in parentheses. The independent variables are the one-period lagged ratio of the consumption deflator to the PDE Deflator by Gordon (1990), and the current and one-period lagged ratio relative to the deflator for private nonresidential equipment investment computed using price indexes from the NIPA (U.S. Bureau of Economic Analysis, 2014b, Table 1.1.4 and Detailed Accounts Tables).

1.1.4.

It is important that we develop a method to find the relative price of equipment investment and durable consumption goods as we use deflators from the NIPA published after the 2004 revisions.[34] Cummins and Violante (2002) and Pakko (2002) adjusted disaggregated NIPA deflators published before the revisions.

Table A2 shows the average annual growth rates of investment-specific technology in the NIPA data from the BEA and adjusted series.

Nonresidential and Residential Structures Prices

We adjust the investment-specific technology of nonresidential and residential structures to $q_s = q_r = 1$ (and thus obtain the nonresidential and residential structures prices equal to 1) over the whole sample. In the NIPA data of the 1960-2011 period, the deflator for nonresidential and residential structures relative to the common deflator increased by less than one percent on average per year, as Table A2 shows. In contrast, Gort, Greenwood, and Rupert (1999) estimate one percent growth in productivity creating structures relative to market consumption 1959-1996. As an intermediate value, we choose constant investment-specific technology in structures, implying the deflator for structures is the same as the common

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[34] Krusell, Ohanian, Ríos-Rull, and Violante (2000) use forecasts from extrapolating all but the price of the computer component of equipment. The use of computer price indexes is majorly responsible for the steeper increase in q 1980-1992 compared to 1963-1979 in their sample. Computer equipment is not central to energy use, so we omit such detailed consideration. Krusell, Ohanian, Ríos-Rull, and Violante (2000) estimate a production function in structures, equipment, and unskilled and skilled labor, using value share equations.

Table A2
Growth Rates of Investment-specific Technology

	Businesses		Households	
	Equipment Investment	Nonresidential Structures Investment	Durable Consumption Goods	Residential Structures Investment
Data	1960-2011	1960-2011	1960-2011	1960-2011
BEA	1.027	0.995	1.026	0.998
Adjusted	1.031	1	1.032	1

Note: Average annual growth rate of the nondurable consumption and nonhousing services deflator relative to the respective investment or durable consumption goods deflator.

deflator.

Hours of Work

To measure work time ℓ, we divide hours of work by the available time for work. Available time for work is assumed 16 hours per day for 365 days per year for each member of the noninstitutional population aged 16-64.

Hours of work are taken from Cociuba, Prescott, and Ueberfeldt (2012), representing the time used for work by the noninstitutional population aged 16-64, based on data from the *Bureau of Labor Statistics*.

Output

Market output y is GNP less gross housing value added and value added in energy resource and electricity production, per hours available for work. Residential housing services are not produced in the model. Energy resources and electricity are produced in the model requiring the market output. To deduct value added in the energy sector, we include the series of the Oil and Gas Extraction sector and series based on Nuclear fuel, Steam engines, and Electric transmission and distribution accounted for by the Utilities sector.[35]

GNP is taken from the NIPA Table 1.1.5. The series for energy sector value added come from Detailed Sectoral Accounts.

Investment

To measure investment in the market sector i_e and i_s, the investment goods that energy resource and electricity production use need to be subtracted from the gross value of investment in the data. Equipment investment i_e is formed by nonresidential equipment investment less series of the Oil and Gas Extraction sector and Nuclear fuel, Steam engines, and Electric transmission and distribution.

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[35] To account energy sector value added portions of the Utilities sector, we use the mean of equipment capital and structures of the corresponding capital series in the Utilities sector in each period. A wider form of energy sector variables, containing in addition the series of the Mining, Except Oil and Gas and Support Activities for Mining sectors, and the remainder series of the Utilities sector, might be too wide, because these two mining sectors do not only mine coal, and the Utilities sector also includes water services.

Structures investment i_s is measured using nonresidential structures investment less the series of the Oil and Gas Extraction sector and Electric accounted for by the Utilities sector. We express the amounts after adjustment for housing and energy sector investment relative to the consumption deflator and hours available for work. To measure real investment in capital used in household production i_d and i_r, we normalize durable consumption goods and residential structures investment by the consumption deflator and hours.

We use private nonresidential equipment investment, nonresidential structures investment, durable consumption goods and residential structures investment from the NIPA Table 1.1.5. The energy sector investment series come from Detailed Sectoral Accounts.

Capital Stock

The value of capital used in energy resource and electricity production must be deducted from the value of private nonresidential capital in the data to measure capital used in the market in the model. We choose the same categories as for adjusting private nonresidential equipment. Capital is equal to the ratio of current-cost capital stock less the energy sector series to the one-period lagged investment deflator (based on the price index for unadjusted investment and chain-type quantity indexes of the energy series) and one-period lagged hours. See Appendix B. The base year for the deflator is 2009. The constant-price capital stocks k_j (for $j \in \{e, d, r, s\}$) are computed by solving their laws of motion forward starting with the initial value based on the adjusted price of capital obtained (as described in Appendix B) and employing the computed average depreciation rates.

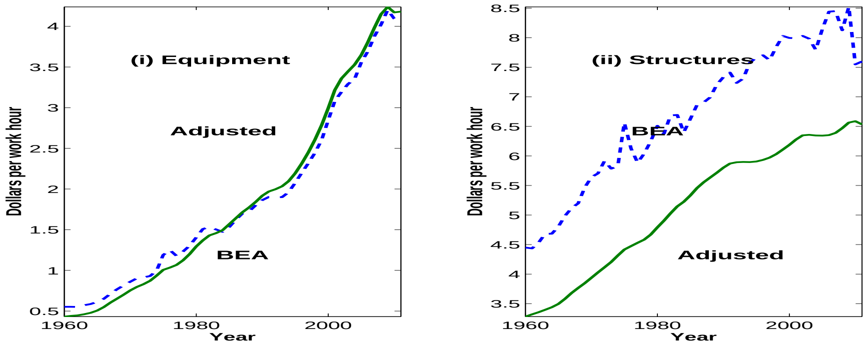
Figure A1C plots both the real nonresidential capital stock from the NIPA data and the real stock used here. Figure A1D shows the corresponding series for household capital. The NIPA values corresponding to investment-specific productivity q_s and q_r have declined, so that less capital is measured at the initial date using the adjusted series.

We use the current-cost capital stock from FAA Table 1.1. The price index for unadjusted investment comes from the NIPA Table 1.1.4.

Appendix B: Measurement of Real-Cost Capital Stock

In this appendix, we describe our method of measuring real capital stock involving the one-period lagged price of investment in the data, and derive physical depreciation of capital per available working hour. The service at date t of vintage v will be denoted by $i_v(t)$. All other variables have a subscript for the date. In a model with geometric depreciation of physical capital at rate $(1-\psi)$, the date-t price of vintage $v \leq t-1$ equals ψ^{t-v-1} times the price of the most recent vintage P_{t-1}. At the beginning of period t, the current-cost capital stock then is $K_t \equiv P_{t-1}(i_{t-1}(t) + \psi i_{t-2}(t) + \psi^2 i_{t-3}(t) + \ldots)$. Thus, we divide the reported contemporaneous current-cost capital stock, K_t, by the one-period lagged price of investment goods, P_{t-1}, to form the real-cost capital stock ${k_t}^{real} \equiv K_t/P_{t-1}$. In addition, we measure current-cost investment $I_t = P_t i_t(t+1)$ in the data

(C) Equipment and Nonresidential Structures



(D) Durable Consumption Goods and Residential Structures

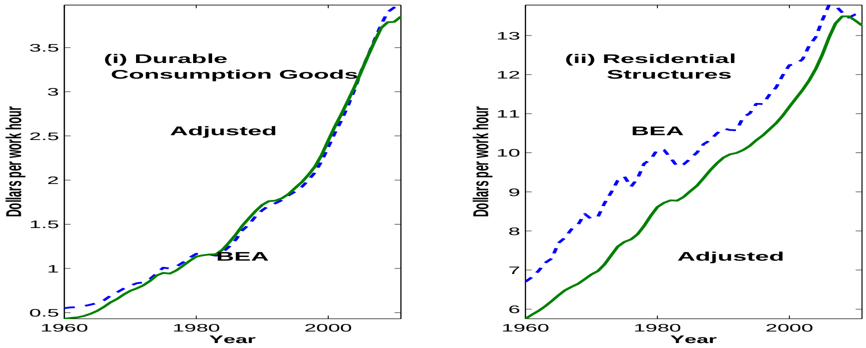


Fig A1. Capital Stock 1960-2011: (A) Equipment and Nonresidential Structures; (B) Durable Consumption Goods and Residential Structures

and define real investment as $i_t{}^{real} \equiv (1/P_{c,t})I_t$. Shifting forward the identity $i_{t-1}(t) + \psi i_{t-2}(t) + \psi^2 i_{t-3}(t) + \ldots = \psi(i_{t-2}(t) + \psi i_{t-3}(t) + \ldots) + i_{t-1}(t)$ by one period implies the law of motion of the real-cost capital stock,

$$k_{t+1}{}^{real} = \psi k_t{}^{real} + \underbrace{(P_{c,t}/P_t)}_{q_t} i_t{}^{real}.$$

This procedure to measure economic depreciation of real capital stock in terms of consumption units, $k_{t+1}{}^{real}/(P_{c,t}/P_t)$, by accounting for investment-specific technology in the NIPA data differs from Greenwood, Hercowitz, and Krusell (1997).

To consistently measure inputs and outputs per hour available for work, we obtain real-cost capital stock per hour as $K_{t+1}/(P_tH_t)$ and real investment per hour as $I_t/(P_{c,t}H_t)$ using hours available for work H, so that

$$K_{t+1}/P_tH_t = \underbrace{\psi(H_{t-1}/H_t)}_{1-\delta} K_t/(P_{t-1}H_{t-1}) + (P_{c,t}/P_t)I_t/(P_{c,t}H_t)$$

with depreciation adjusted for hour growth. The procedure outlined here is useful to measure k_t in the model by ${k_t}^{real}/H_{t-1}$ and i_t in the model by ${i_t}^{real}/H_t$.

Appendix C: Equilibrium

This appendix provides first-order necessary conditions for an equilibrium. First we find conditions governing investment and the capital intensity of energy of new investment goods. Investment in equipment capital $i_e(v) > 0$ equates the marginal net benefit from services and energy expenditure to the marginal cost of investment. Therefore, $q_M f \mu_x - q_M \mu_u = v$, using the Lagrange multipliers μ_x on (10) and μ_u on (11). The chosen capital intensity of energy in the business sector v balances the marginal benefit of reduced energy expenditure and the marginal cost of less productive capital, $[f - f_1 v]\mu_x = \mu_u$. Then $q_M \mu_x = 1/f_1$ and $q_M \mu_u = [f - f_1 v]/f_1$. In the household sector, the choice of $i_d(v) > 0$ and $v \in V$ requires two analogue conditions. The Lagrange multipliers φ_x on (8) and φ_u on (9) are thus determined by $q_N \varphi_x = U_1/h_1$ and $q_N \varphi_u = U_1[h - h_1 v]/h_1$. We utilize these conditions to express the Euler equations.

The allocation rules for c_M, c_N, ${k_s}'$, ${x_M}'$, ${u_M}'$, ${k_r}'$, ${x_N}'$, ${u_N}'$, ℓ, and d, and pricing rule for ψ, in an equilibrium are implicit functions in the following system of equations. The Euler equations associated with the stocks of nonresidential structures, services from equipment capital and energy, and energy used by firms are:

$$1 = \mathbb{E}\left[\frac{\psi}{\psi' + d'}\left(G_1({k_s}', {x_M}', {z_M}'\ell') + 1 - \delta_s\right)\right] \tag{C1}$$

$$\frac{1}{q_M f_1(v)} = \mathbb{E}\left[\frac{\psi}{\psi' + d'}\left(G_2({k_s}', {x_M}', {z_M}'\ell') + (1 - \delta_e)\frac{1}{{q_M}' f_1(v')}\right)\right] \tag{C2}$$

$$\frac{f(v) - f_1(v)v}{q_M f_1(v)} = \mathbb{E}\left[\frac{\psi}{\psi' + d'}\left(p' + (1 - \delta_e)\frac{f(v') - f_1(v')v'}{{q_M}' f_1(v')}\right)\right] \tag{C3}$$

where v and v' denote the intensity of energy capital chosen by firms. The Euler equations associated with the stocks of residential structures, services from durable consumption goods and energy, energy used by households, and equity shares are:

$$\begin{aligned} U_1(c_M, c_N) = \beta\mathbb{E}\Bigg[U_1({c_M}', {c_N}')\Bigg(&H_1({k_r}', {x_N}', {z_N}'(1 - \ell')) \\ &\times \frac{U_2({c_M}', {c_N}')}{U_1({c_M}', {c_N}')} + 1 - \delta_r\Bigg)\Bigg] \end{aligned} \tag{C4}$$

$$\text{(C5)}\quad \frac{U_1(c_M, c_N)}{q_N h_1(v)} = \beta \mathbb{E}\Bigg[U_1(c_M{}', c_N{}')\Bigg(H_2(k_r{}', x_N{}', z_N{}'(1-\ell')) \times \frac{U_2(c_M{}', c_N{}')}{U_1(c_M{}', c_N{}')} + (1-\delta_d)\frac{1}{q_N{}' h_1(v')}\Bigg)\Bigg],$$

$$\text{(C6)}\quad U_1(c_M, c_N)\frac{h(v) - h_1(v)v}{q_N h_1(v)} = \beta \mathbb{E}\Bigg[U_1(c_M{}', c_N{}')\Bigg(p' + (1-\delta_d)\frac{h(v') - h_1(v')v'}{q_N{}' h_1(v')}\Bigg)\Bigg],$$

$$\text{(C7)}\quad \psi U_1(c_M, c_N) = \beta \mathbb{E}[(\psi' + (1-\tau_d)d')U_1(c_M{}', c_N{}')]$$

where v and v' denote the capital intensity of energy chosen by households. The efficiency condition of labor demand and supply is

$$\text{(C8)}\quad z_M(1-\tau_w)G_3(k_s, x_M, z_M \ell) = z_N H_3(k_r, x_N, z_N(1-\ell))\frac{U_2(c_M, c_N)}{U_1(c_M, c_N)}.$$

Together with the asset balancing condition (given in the equilibrium condition [ii]), the resource constraint (given in the equilibrium condition [iii]), and the laws of motions (3), (8), and (9), as well as (4), (10), and (11), the equations (C1)-(C8) define the allocation rules.

Appendix D: Harrod-neutral Technology

Figure A2 displays the Harrod-neutral technology, or labor efficiency, that we computed as residual.

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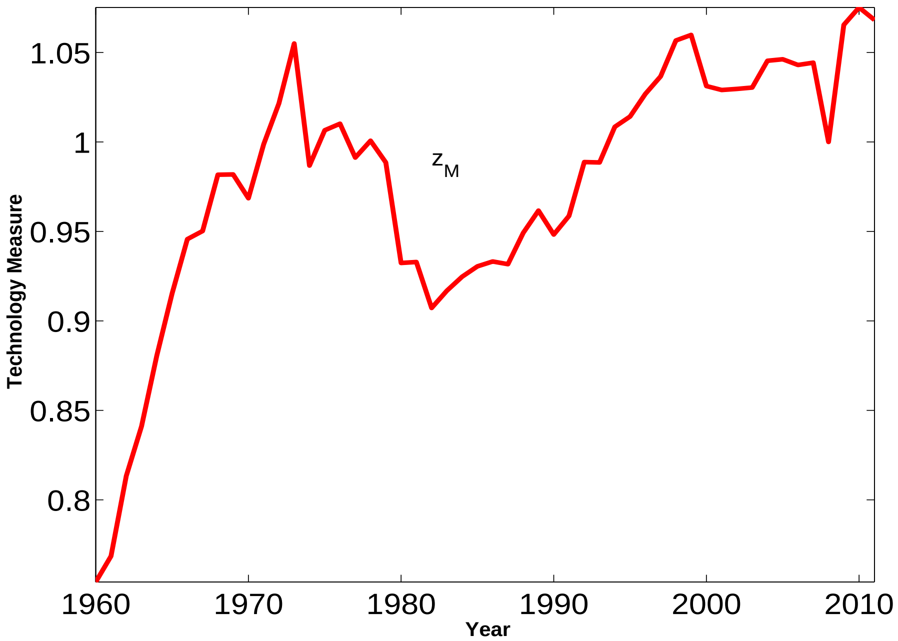

Fig A2. Harrod-neutral technology 1960-2011

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