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# Household heterogeneity, aggregation, and the distributional impacts of environmental taxes

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**Abstract**

This paper examines how the general equilibrium incidence of an environmental tax depends on the effect of different incomes and preferences of heterogeneous households on aggregate outcomes. We develop a Harberger-type model with general forms of preferences and substitution between capital, labor, and pollution in production that captures the impact of household heterogeneity and interactions with production characteristics on the general equilibrium. We theoretically show that failing to incorporate household heterogeneity can qualitatively affect incidence. We quantitatively illustrate that this aggregation bias can be important for assessing the incidence of a carbon tax, mainly by affecting the returns to factors of production. Our findings are robust to a number of extensions including alternative revenue recycling schemes, pre-existing taxes, non-separable utility in pollution, labor-leisure choice, and multiple commodities.

*Keywords:* Environmental tax incidence, Heterogeneous households, General equilibrium, Aggregation bias, Distributional impacts
*JEL:* H23, Q52

## 1. Introduction

The public acceptance for environmental taxes depends crucially on their distributional consequences. A plethora of applied research in public and environmental economics has investigated the incidence of environmental taxes in various policy settings. Not seldom, however, the empirical evidence whether a specific tax is regressive or not is mixed–even if the incidence of a given tax instrument is analyzed in a similar or identical policy context. Differences arise because the incidence analysis does not consider all relevant channels through which an environmental tax affects market outcomes (see, e.g., Atkinson & Stiglitz (1980) and Fullerton & Metcalf (2002) for a discussion of incidence impacts in the public finance literature).[3] One important channel which is typically omitted by general equilibrium analyses that employ a single, representative household model is the impact of household heterogeneity on the market equilibrium.

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[3]Environmental taxes often appear to be regressive on the "uses side of income" as they affect more heavily the welfare of the poorest households than of the richest ones, since poorer households spend a larger fraction of their income on polluting goods (e.g., energy or electricity). "Sources side of income" impacts can dampen or even offset the regressive incidence on the uses side

Despite the high policy relevance and academic interest for understanding the distributional consequences of price-based pollution controls, an analysis of the effect of household aggregation on tax incidence is lacking.

This paper develops a theoretical Harberger (1962)-type general equilibrium model of the incidence of an environmental tax featuring heterogeneous households, general forms of preferences, differential spending and income patterns, differential factor intensities in production, and general forms of substitution among inputs of capital, labor, and pollution. Its purpose is two-fold. First, we theoretically investigate the implication of the household aggregation problem for the incidence of environmental taxes, i.e., to what extent incidence results derived from a general equilibrium analysis which ignores household heterogeneity are biased. In the absence of identical homothetic preferences for each individual or homothetic preferences and collinear initial endowment vectors (i.e., identical income shares), aggregated preferences depend on the distribution of income (Polemarchakis, 1983).[4] Thus acknowledging heterogeneity in tastes undercuts the representative consumer framework that is used to calculate the general equilibrium effects on output and factor prices (Kortum, 2010). Second, we apply the heterogeneous household model to quantitatively assess how the aggregation bias affects equilibrium outcomes and the incidence of a tax on carbon dioxide ($CO_2$) emissions for the case of the United States. We assess the incidence on the sources and uses side of income, and explore how sensitive results are with respect to key characteristics governing households' and firms' behavior.

Our main finding is that the household aggregation problem can have important implications for assessing the incidence of environmental taxes: basing the analysis on a single, representative household model as opposed to an analysis that integrates household heterogeneity can yield both qualitatively and quantitatively different conclusions. Assuming homothetic preferences, we show that the impact of household heterogeneity on the equilibrium can be characterized by two statistical quantities which capture the degree of household heterogeneity in terms of household preferences and income shares. These metrics provide an intuitive way to express the discrepancy in results obtained under a case with heterogeneous households and a case with identical households. We provide examples of conditions for households' and firms' characteristics under which the aggregation bias does or does not matter. For example, with limited substitutability between inputs of capital, labor, and pollution in production, factor and output price changes can be reversed, in turn yielding qualitatively different incidence results among poor and rich households. Moreover, we find that there exist for any benchmark economy, described by data on production and distributions of consumption and income among households, values of production elasticities such that household aggregation leads to reversed factor price changes. We find that for non-homothetic preferences the burden of an environmental tax on factors of production can be qualitatively different as compared to a case with homothetic preferences.

We quantitatively illustrate that the aggregation bias for empirically motivated cases can be important for assessing the incidence of a carbon tax. As the aggregation bias on welfare is largely caused by the aggregation bias on the returns to factors of production, it mainly affects the sources of income. Additionally,

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to the extent that environmental tax policies affect the returns to factors of production that are disproportionately owned by richer households and used intensively in the production of dirty relative to clean industries (e.g., capital). The regressivity of many environmental taxes on the uses side, including carbon pricing in the context of climate policy, constitutes a serious concern for policymakers and has been investigated extensively in the literature (Poterba, 1991; Metcalf, 1999; Fullerton et al., 2012). Gasoline taxes are generally found to be progressive on the uses side (Sterner, 2012). More recently, work by Fullerton & Heutel (2007), Araar et al. (2011), and Rausch et al. (2011) has also scrutinized the sources side impacts of carbon taxation.

[4] On a more fundamental conceptual level, and not related to the incidence of (environmental) taxation, the aggregation problem for heterogeneous consumers in general equilibrium models has been studied by Ackermann (2002) based on prior work by Rizvi (1994) and Martel (1996).

we find that most of the variation in welfare impacts when altering production and household characteristics is driven by sources side impacts, and may even lead to a reversal of the incidence pattern across households. Our analysis thus points to the importance of including sources of income impacts for tax incidence analysis. We also find that household heterogeneity in the elasticities of substitution in utility magnifies the aggregation bias due to heterogeneity in expenditure and income patterns. In our static model, heterogeneity in income elasticities has a smaller effect compared to heterogeneity in substitution elasticities.

Our findings are robust to a number of extensions including alternative revenue recycling schemes, pre-existing taxes, non-separable utility in pollution, labor-leisure choice, and multiple commodities. Any extension of the model obviously produces quantitatively different results but the point of the paper that household heterogeneity affects equilibrium and hence the incidence of environmental taxes remains. In fact, we argue that the case for the aggregation bias is strengthened rather than weakened.

Our paper builds on a small but growing literature that uses analytical general equilibrium models to study the incidence of environmental taxes. Our model builds on a series of influential papers by Fullerton and others (Fullerton & Heutel, 2007, 2010; Fullerton et al., 2012; Fullerton & Monti, 2013) that extend the Harberger (1962) model and previous theoretical work by Rapanos (1992, 1995) to develop a model which represents pollution as an input along with capital and labor and that allows for general forms of substitution between inputs. We extend the single-consumer model presented in Fullerton & Heutel (2007) to include heterogeneous households. We additionally incorporate non-homothetic preferences. By fully integrating household heterogeneity, our paper also differs from the contributions in Fullerton & Heutel (2010) and Fullerton et al. (2012) that use price impacts derived from the single-consumer model in Fullerton & Heutel (2007) to determine the burdens of a carbon tax using household survey data. Fullerton & Monti (2013) integrate two types of households into an analytical general equilibrium model and investigate the distributional impacts of a pollution tax swap (recycling revenues through a wage tax of low-income workers). They do not, however, study the impact of household heterogeneity on equilibrium outcomes.

Our analysis is also related to the literature that uses computational methods to assess the distributional impacts of environmental taxes. A widespread approach is to employ Input-Output analysis to derive price changes for different consumers goods and then calculate tax burdens for households based on micro-household survey data.[5] Common to these studies is that they adopt a partial equilibrium perspective that does not consider behavioral changes and focuses on the uses sides of the incidence only. A few papers use numerical general equilibrium models with a single, representative consumer to derive price impacts on commodity and factor prices. Metcalf et al. (2008) carry out an analysis of carbon tax proposals and find that a carbon tax is highly regressive but that the regressivity is reduced due to sources side effects to the extent that resource and equity owners bear some fraction of the tax burden. Similarly, Araar et al. (2011) and Dissou & Siddiqui (2014) use price effects to assess the distributional impacts of a carbon tax. None of these studies, however, captures the impact of household heterogeneity on equilibrium outcomes.

Lastly, a few papers integrate heterogeneous households into a numerical general equilibrium framework. For example, Rausch et al. (2010a,b) investigate the incidence of a U.S. carbon tax in a model with nine households representing different income classes and find that the overall impact is neutral to modestly

[5]Examples include Robinson (1985) who studies the distributional burden of industrial abatement in the U.S. economy and Poterba (1991) who focuses on the incidence of U.S. gasoline taxes. Bull et al. (1994); Hassett & Metcalf (2009) compare a tax based on energy content and a tax based on carbon, and Metcalf (1999, 2009) analyze a revenue-neutral package of environmental taxes, including a carbon tax, an increase in motor fuel taxes, and taxes on various stationary source emissions. Dinan & Rogers (2002) assess the efficiency and distributional impacts of a U.S. cap-and-trade program for $CO_2$ emissions, and Mathur & Morris (2014) investigate the distributional effects of a carbon tax in broader U.S. fiscal reform. Other works study the incidence impacts of greenhouse gas emissions pricing policies across household income groups for different countries (e.g., Labandeira & Labeaga (1999) for Spain, Callan et al. (2009) for Ireland, and Jiang & Shao (2014) for China).

progressive due to sources side effects (assuming that government transfers to households are indexed to inflation). Williams III et al. (2015) and Chiroleu-Assouline & Fodha (2014) employ calibrated overlapping generations models to assess the distributional incidence across generations. A major weakness of analyses based on numerical simulation models is, however, their reliance on specific functional forms with limited forms of substitution. In contrast, our paper studies environmental tax incidence in a theoretical setup with general forms of substitution in production and consumption.

The remainder of this paper is organized as follows. Section 2 presents the model. Section 3 derives closed-form expressions to assess the incidence of an environmental tax change, and presents and interprets our theoretical results. Section 4 uses an empirically calibrated version of the model to quantitatively study the aggregation bias. Section 5 provides evidence that the aggregation bias remains relevant when extending the core model in a number of important directions. Section 6 concludes. Appendixes A to C contain additional derivations and proofs for our results.[6]

## 2. Model

We consider a static and closed economy with two sectors and two factors of production. A "clean" good is produced using capital and labor, and a "dirty" good is produced using capital, labor and pollution. Capital and labor are supplied inelastically and are mobile across sectors. The government taxes pollution, returning the revenue lump-sum to households. Our general equilibrium model follows closely Harberger (1962) and Fullerton & Heutel (2007) but differs in two important aspects. First, we introduce heterogeneous households that differ in terms of their preferences and income patterns derived from endowments of capital and labor. Second, we generalize the representation of household behavior by allowing for non-homothetic preferences. Using log-linearization, we analytically solve for first-order changes in equilibrium prices and quantities following an exogenous change in the pollution tax rate. Our model enables us to quantify the general equilibrium incidence of the environmental tax in the context of an economy with no a-priori restrictions placed on the number and characteristics of households.

The clean sector production function $X = X(K_X, L_X)$ and the dirty sector production function $Y = Y(K_Y, L_Y, Z)$ are assumed to exhibit constant returns to scale, where $K_X$, $K_Y$, $L_X$, and $L_Y$ are the quantities of capital and labor used in each sector.[7] The total amounts of factors of production in the economy are exogenously given and fixed: $K_X + K_Y = \bar{K}$ and $L_X + L_Y = \bar{L}$. Totally differentiating the resource constraints yields:

$$\hat{K}_X \frac{K_X}{\bar{K}} + \hat{K}_Y \frac{K_Y}{\bar{K}} = 0 \tag{1}$$

$$\hat{L}_X \frac{L_X}{\bar{L}} + \hat{L}_Y \frac{L_Y}{\bar{L}} = 0\,, \tag{2}$$

where a hat denotes a proportional change, e.g., $\hat{K}_X \equiv dK_X/K_X$. Pollution ($Z$) has no equivalent resource constraint and is a choice of the dirty sector. To ensure a finite use of pollution in equilibrium, we assume a pre-existing positive tax on pollution, $\tau_Z > 0$.

Firms in sector $X$ can substitute between factors in response to changes in the wage rate ($w$) and capital rental rate ($r$) according to an elasticity of substitution in production, $\sigma_X$. Differentiating the definition for

[6] An online appendix provides supplementary analysis on incidence results for alternative revenue recycling schemes as well as derivations and proofs for the model extensions.

[7] Note that the production side of our model is the same as for the single-consumer model of Fullerton & Heutel (2007). In describing production we thus follow closely the model description in Fullerton & Heutel (2007, pp. 574-75).

$\sigma_X$ yields:

$$\hat{K}_X - \hat{L}_X = \sigma_X(\hat{w} - \hat{r})\,. \tag{3}$$

The production decision of firms in sector $Y$ depends additionally on the pollution price they face, which is given by the pollution tax rate $\tau_Z$. We model the choice between the three inputs of capital, labor and pollution by means of the Allen elasticities $e_{ij}$ between inputs $i$ and $j$ (Allen, 1938). The 3×3 matrix of Allen elasticities is symmetric (i.e., $e_{ij} = e_{ji}$), its diagonal entries are less or equal to zero (i.e., $e_{ii} \leq 0$), and at most one of the three independent off-diagonal elements can be negative. Furthermore, $e_{ij}$ is positive whenever inputs $i$ and $j$ are substitutes, and negative whenever they are complements. Totally differentiating input demand functions for sector $Y$, which describe the dirty sector's cost minimization problem, and dividing by the appropriate input level, yields:[8]

$$\hat{K}_Y - \hat{Z} = \theta_{YK}(e_{KK} - e_{ZK})\hat{r} + \theta_{YL}(e_{KL} - e_{ZL})\hat{w} + \theta_{YZ}(e_{KZ} - e_{ZZ})\hat{\tau}_Z \tag{4}$$

$$\hat{L}_Y - \hat{Z} = \theta_{YK}(e_{LK} - e_{ZK})\hat{r} + \theta_{YL}(e_{LL} - e_{ZL})\hat{w} + \theta_{YZ}(e_{LZ} - e_{ZZ})\hat{\tau}_Z\,, \tag{5}$$

where $\theta_{mn}$ is the share of sector $m$'s revenue paid to factor $n$, e.g. $\theta_{XK} = \frac{rK_X}{p_X X}$. Let $p_X$ and $p_Y$ denote output prices for $X$ and $Y$, respectively. Under the assumption of perfect competition, the following expressions hold:

$$\hat{p}_X + \hat{X} = \theta_{XK}(\hat{r} + \hat{K}_X) + \theta_{XL}(\hat{w} + \hat{L}_X) \tag{6}$$

$$\hat{p}_Y + \hat{Y} = \theta_{YK}(\hat{r} + \hat{K}_Y) + \theta_{YL}(\hat{w} + \hat{L}_Y) + \theta_{YZ}(\hat{\tau}_Z + \hat{Z}) \tag{7}$$

$$\hat{X} = \theta_{XK}\hat{K}_X + \theta_{XL}\hat{L}_X \tag{8}$$

$$\hat{Y} = \theta_{YK}\hat{K}_Y + \theta_{YL}\hat{L}_Y + \theta_{YZ}\hat{Z}\,. \tag{9}$$

Households, indexed by $h = \{1, \ldots, H\}$, maximize utility by choosing optimal consumption of goods $X$ and $Y$ subject to an income constraint.[9] Each household inelastically supplies fixed factor endowments $\bar{K}^h$ and $\bar{L}^h$ which satisfy the following relations: $\sum_h \bar{K}^h = \bar{K}$ and $\sum_h \bar{L}^h = \bar{L}$. Income for household $h$ is therefore given by $M^h = w\bar{L}^h + r\bar{K}^h + \xi^h \tau_Z Z$, where $\xi^h$ is the share of the pollution tax revenue redistributed lump-sum to household $h$. Since the tax revenue is returned entirely to households, it follows that $\sum_h \xi^h = 1$.

Following Hicks & Allen (1934), we parameterize non-homothetic consumer preferences for the two goods using the elasticity of substitution between goods $X$ and $Y$ in utility $\sigma^h$, and the income elasticities of demand for goods $X$ and $Y$, denoted by $E^h_{X,M}$ and $E^h_{Y,M}$ respectively.[10] Appendix A derives the following expressions for changes in demand by household $h$ in response to output and factor price changes:

$$\hat{X}^h - \hat{Y}^h = \sigma^h(\hat{p}_Y - \hat{p}_X) + (E^h_{Y,M} - E^h_{X,M})(\alpha^h \hat{p}_X + (1 - \alpha^h)\hat{p}_Y - \hat{M}^h) \tag{10}$$

$$\hat{X}^h = -(\alpha^h E^h_{X,M} + (1 - \alpha^h)\sigma^h)\hat{p}_X - ((1 - \alpha^h)E^h_{X,M} - (1 - \alpha^h)\sigma^h)\hat{p}_Y + E^h_{X,M}\hat{M}^h\,, \tag{11}$$

with $\hat{M}^h = \hat{w}\frac{w\bar{L}^h}{M^h} + \hat{r}\frac{r\bar{K}^h}{M^h} + \frac{\xi^h \tau_Z Z}{M^h}(\hat{\tau}_Z + \hat{Z})$.

[8] Appendix A in Fullerton & Heutel (2007) derives equations (4)-(9).

[9] We assume that pollution, or environmental quality, is separable in utility, thus not influencing the optimal consumption choice. Note that the incidence analysis carried out in this paper focuses on utility derived from market consumption only.

[10] Homothetic preferences are represented by the special case $E^h_{X,M} = E^h_{Y,M} = 1$. In this case the first-order behavior of households can be sufficiently described by $\sigma^h$, as for example in Fullerton & Heutel (2007).

Finally, totally differentiating the market clearing conditions for the two consumption goods, $X = \sum_h X^h$ and $Y = \sum_h Y^h$, yields:

$$\hat{X} = \sum_h \frac{X^h}{X} \hat{X}^h \tag{12}$$

$$\hat{Y} = \sum_h \frac{Y^h}{Y} \hat{Y}^h . \tag{13}$$

Equations (1)–(13) are $11 + 2H$ equations in $11 + 2H$ unknowns ($\hat{K}_X$, $\hat{K}_Y$, $\hat{L}_X$, $\hat{L}_Y$, $\hat{w}$, $\hat{r}$, $\hat{p}_X$, $\hat{X}$, $\hat{p}_Y$, $\hat{Y}$, $\hat{Z}$, $H \times \hat{X}^h$, $H \times \hat{Y}^h$). Following Walras' Law, one of the equilibrium conditions is redundant, thus the effective number of equations is $10 + 2H$. We choose $X$ as the numéraire good, which implies $\hat{p}_X = 0$. The square system of model equations then endogenously determines all the above unknowns as functions of benchmark parameters (characterizing the equilibrium before the tax change), behavioral parameters (elasticities of production and consumption), and the exogenous positive change in the pollution tax ($\hat{\tau}_Z > 0$).

## 3. Analytical results and interpretations

When solving for the model unknowns as functions of the exogenous tax change, we are ultimately interested in the distributional incidence of the environmental tax. Let $v^h$ denote the indirect utility function of household $h$, and $dv^h$ the change in utility from consumption caused by an increase in the pollution tax rate by $d\tau_Z$.[11]

To compare the welfare impacts of an increase in the pollution tax across households, we express utility changes in monetary terms relative to income: $\frac{dv^h}{M^h \partial_{M^h} v^h}$ measures the amount of income which would cause a change in utility equal to $dv^h$ at prices prior to the tax change, expressed relative to the income of household $h$. To isolate the distributional dimension from the economy-wide cost of the tax, we focus on the welfare impact of each household relative to the average welfare change. This ensures that results do not depend on the choice of numéraire. We can then write the welfare impact of household $h$ relative to the average economy-wide monetary loss per unit of income as:[12]

$$\begin{aligned} \Phi^h &\equiv \frac{dv^h}{M^h \partial_{M^h} v^h} - \frac{1}{\sum_{h'} M^{h'}} \sum_{h'} \frac{dv^{h'}}{\partial_{M^{h'}} v^{h'}} \\ &= \underbrace{-(\gamma - \alpha^h)\hat{p}_Y}_{=\textit{Uses of income impact}} + \underbrace{(\theta_L^h - \theta_L)\hat{w} + (\theta_K^h - \theta_K)\hat{r} + (\theta_Z^h - \theta_Z)(\hat{\tau}_Z + \hat{Z})}_{=\textit{Sources of income impacts}} , \end{aligned} \tag{14}$$

[11] Fullerton (2011) provides a taxonomy of six channels of distributional effects of environmental policy. Our analysis is focused on the impacts of environmental taxes caused by higher prices of polluting goods, changes in relative returns to factors like capital and labor and the allocation of pollution tax revenues. It does not consider distributional impacts arising from the benefits from improvements in environmental quality, temporary effects during the transition, and capitalization of all those effects into prices of land, corporate stock, or house values. Also, the uses side in our analysis could be more general if consumption were disaggregated into more than two goods, and the sources side could be extended to represent in more detail the ownership of factors of production (e.g. natural resources, or skilled vs. unskilled labor).

[12] Recall that $p_X$ is the numéraire. Then $dv^h = \partial_{p_Y} v^h dp_Y + \partial_{M^h} v^h dM^h = \partial_{p_Y} v^h p_Y \hat{p}_Y + \partial_M^h v^h (\hat{w} w \bar{L}^h + \hat{r} r \bar{K}^h + \xi^h \tau_Z Z(\hat{\tau}_Z + \hat{Z}))$. Roy's identity (i.e., $\partial_{p_Y} v^h = -Y^h \partial_{M^h} v^h$) then delivers the above equation.

where $\theta_K^h \equiv \frac{r\bar{K}^h}{M^h}$, $\theta_L^h \equiv \frac{w\bar{L}^h}{M^h}$ and $\theta_Z^h \equiv \frac{\xi^h \tau_Z Z}{M^h}$ are the capital and labor income shares of household $h$, and $\theta_K \equiv \frac{r\bar{K}}{p_X X + p_Y Y}$, $\theta_L \equiv \frac{w\bar{L}}{p_X X + p_Y Y}$, $\theta_Z \equiv \frac{\tau_Z Z}{p_X X + p_Y Y}$ and $\gamma \equiv \frac{p_X X}{p_X X + p_Y Y}$ are the value shares of capital, labor, tax revenus and the clean sector in the economy.

The welfare decomposition underlying equation (14) enables an intuitive economic interpretation of the various channels through which household characteristics determine incidence in our analysis. On the one hand, for given changes in goods and factors prices, variation in impacts across households arises for two reasons. First, households differ in how they spend their income. For a given increase in the price of the dirty good ($\hat{p}_Y > 0$), consumers of the dirty good are more negatively impacted as compared to consumers of the clean good. This impact is referred to as the uses of income impact. Second, in a general equilibrium setting, a pollution tax also impacts factor prices. Households which rely heavily on income from the factor whose price falls relative to the other will be adversely impacted compared to the average household. These impacts, together with the impacts arising from the specific tax redistribution scheme, are referred to as sources of income impacts.

Since output and factor price changes are not independent of households' characteristics, two additional and less direct determinants of incidence emerge from the expression (14). First, in an economy with heterogeneous households, output and factor prices are not independent of the distribution of households' consumption profiles and factor endowments across the population; welfare changes for a given household type do not only depend on its own characteristics but also on those of other households in the economy. Second, even in an economy with identical households, the specifics of the household's behavioural response to price and income changes can affect equilibrium outcomes.

Appendix B derives the following general solutions for $\hat{p}_Y$, $\hat{w}$ and $\hat{r}$ following a change in $\tau_Z$:

$$\hat{p}_Y = \frac{(\theta_{YL}\theta_{XK} - \theta_{YK}\theta_{XL})\theta_{YZ}}{D}\left[A(e_{ZZ} - e_{KZ}) - B(e_{ZZ} - e_{LZ}) + (\gamma_K - \gamma_L)(\delta - \sum_h \frac{\phi_Z^h}{\theta_{YZ}})\right]\hat{\tau}_Z + \theta_{YZ}\hat{\tau}_Z \tag{15a}$$

$$\hat{w} = \frac{\theta_{XK}\theta_{YZ}}{D}\left[A(e_{ZZ} - e_{KZ}) - B(e_{ZZ} - e_{LZ}) + (\gamma_K - \gamma_L)(\delta - \sum_h \frac{\phi_Z^h}{\theta_{YZ}})\right]\hat{\tau}_Z \tag{15b}$$

$$\hat{r} = -\frac{\theta_{XL}\theta_{YZ}}{D}\left[A(e_{ZZ} - e_{KZ}) - B(e_{ZZ} - e_{LZ}) + (\gamma_K - \gamma_L)(\delta - \sum_h \frac{\phi_Z^h}{\theta_{YZ}})\right]\hat{\tau}_Z\,, \tag{15c}$$

where $\gamma_K \equiv \frac{K_Y}{K_X}$, $\gamma_L \equiv \frac{L_Y}{L_X}$, $\beta_L \equiv \theta_{XL}\gamma_L + \theta_{YL}$, $\beta_K \equiv \theta_{XK}\gamma_K + \theta_{YK}$, $A \equiv \gamma_L\beta_K + \gamma_K(\beta_L + \theta_{YZ} - \sum_h \phi_Z^h)$, $B \equiv \gamma_K\beta_L + \gamma_L(\beta_K + \theta_{YZ} - \sum_h \phi_Z^h)$, $C \equiv \beta_K + \beta_L + \theta_{YZ} - \sum_h \phi_Z^h$, $D \equiv C\sigma_X + A[\theta_{XK}\theta_{YL}(e_{KL} - e_{ZL}) - \theta_{XL}\theta_{YK}(e_{KK} - e_{ZK})] - B[\theta_{XK}\theta_{YL}(e_{LL} - e_{ZL}) - \theta_{XL}\theta_{YK}(e_{LK} - e_{ZK})] - (\gamma_K - \gamma_L)(\theta_{XK}(\theta_{YL}\delta - \sum_h \phi_L^h) - \theta_{XL}(\theta_{YK}\delta - \sum_h \phi_K^h))$. The remaining expressions depend explicitly on household characteristics: $\phi_L^h \equiv (1 - \frac{\alpha^h}{\gamma})E_{X,M}^h \frac{w\bar{L}^h}{p_Y Y} + \frac{Y^h}{Y}(E_{Y,M}^h - E_{X,M}^h)\frac{w\bar{L}^h}{M^h}$, $\phi_K^h \equiv (1 - \frac{\alpha^h}{\gamma})E_{X,M}^h \frac{r\bar{K}^h}{p_Y Y} + \frac{Y^h}{Y}(E_{Y,M}^h - E_{X,M}^h)\frac{r\bar{K}^h}{M^h}$, $\phi_Z^h \equiv (1 - \frac{\alpha^h}{\gamma})E_{X,M}^h \frac{\xi^h \tau_Z Z}{p_Y Y} + \frac{Y^h}{Y}(E_{Y,M}^h - E_{X,M}^h)\frac{\xi^h \tau_Z Z}{M^h}$ and $\delta \equiv \sum_h \frac{Y^h}{Y}\left(\sigma^h + (\frac{\alpha^h}{\gamma} - 1)(\sigma^h - E_{X,M}^h) + (E_{Y,M}^h - E_{X,M}^h)(1 - \alpha^h)\right)$.[13]

[13] Note that in general $\hat{w} = -\frac{\theta_{XK}}{\theta_{XL}}\hat{r}$. Thus, in order to understand the burden of the change in the pollution tax on the returns to factors of production, it is sufficient to study the change in the returns to capital, keeping in mind that–given our choice of the numéraire good–$\hat{w}$ always has the opposite sign as $\hat{r}$.

While the interpretation of the general solution is limited by its complexity, it is apparent from the analytical expressions above that going beyond a single consumer and introducing multiple, heterogeneous households with non-homothetic preferences into the model in general has a first-order impact on the market equilibrium, and thus on the incidence results following equation (14).

By considering expressions (15a)–(15c) one can identify the following two effects, which have also previously been identified in the context of the Harberger (1962) model. The $(\gamma_K - \gamma_L)(\delta - \sum_h \frac{\phi_Z^h}{\theta_{YZ}})$ term in equations (15b) and (15c) represents the *output effect*: the tax on sector $Y$ reduces output, and consequently depresses the returns to the factor used intensively in the dirty sector. The sign of the output effect follows this intuition only if the denominator $D$ is positive, which in general is not the case, even for identical households and homothetic preferences (Fullerton & Heutel, 2007). Introducing multiple, heterogeneous households and non-homothetic preferences adds another layer of complexity to this indeterminacy, since $\delta - \sum_h \frac{\phi_Z^h}{\theta_{YZ}}$ cannot in general be signed, whereas this expression is positive for identical households with homothetic preferences.[14] The other terms in equations (15b) and (15c) embody the *substitution effects*, which reflect the reaction of firms to factor price changes. Again, while for the case with identical households and homothetic preferences the constants $A$ and $B$ can be signed as positive, this is not the case in our more general model. The substitution effect thus also bears a greater degree of indeterminacy as compared to the Fullerton & Heutel (2007) model.

To better understand the various effects at work, it is necessary to depart from the generality of the above expressions. We therefore consider a series of special cases in which we impose restrictions on household and production characteristics in order to seek definitive results for the changes in prices and returns to factors of production, and therefore better understand the implications for incidence. First, we present a special case for production under which household characteristics have no impact on price changes. Second, we consider cases which allow for full household heterogeneity in terms of preferences and income patterns but where preferences are assumed to be homothetic. Third, the role of non-homothetic preferences is investigated for cases with identical households. These special cases highlight the interaction of production and household characteristics in determining the changes in output and factor prices, and consequently incidence.

### *3.1. Equal factor intensities in production*

Consider first the case in which both industries have the same factor intensities, i.e., both are equally capital and labor intensive. Under this assumption, the price changes derived from a model with heterogeneous households are identical to those derived from a single household model.

**Proposition 1.** *Assume both sectors have the same factor intensities, i.e., $\gamma_K = \gamma_L$. Then, $\hat{p}_Y$, $\hat{w}$ and $\hat{r}$ are independent of household characteristics and depend only on production parameters.*

*Proof.* If $\gamma_K = \gamma_L$, then $A = B = \gamma_K C$. It then follows from (15a)–(15c) that all terms containing household characteristics in the expressions for $\hat{p}_Y$, $\hat{w}$ and $\hat{r}$ cancel out. □

Proposition 1 implies that in the case of equal factor intensities across industries, price changes derived from a single household model with homothetic preferences are sufficient to determine incidence of an environmental tax, even in an economy with different household types. Intuitively, as long as factor intensities are equal, changes in demands for $X$ and $Y$ do not affect *relative* demands for capital and labor, thus

[14] It should be noted that the term $\delta - \sum_h \frac{\phi_Z^h}{\theta_{YZ}}$ is a non-trivial generalization of the expression $(\sigma_U N + J)$ in equation (16) in Fullerton & Monti (2013) from the case of two households, homothetic preferences, and identical $\sigma^h$ among households. This generalization is critical for comparing models with a different degree of household heterogeneity.

implying that relative factor prices are unaffected. Factor price changes in our linearized model are thus determined by the "first-order" response of firms alone, as accounting for "first-order" household behavioral responses in combination with "first-order" firm responses would capture a second-order effect. The sign of factor price changes therefore depends only on production characteristics. Incidence remains in general undetermined, since it depends on how these price changes affect individual households, as determined by their income and expenditure shares.

### *3.2. Heterogeneous households with homothetic preferences*

To provide a clear intuition of the effect of household heterogeneity on the general equilibrium (beyond the case with equal factor intensities in production), we restrict our attention in this section to the case with homothetic preferences. We also consider a specific allocation scheme for the pollution tax revenues, with revenues distributed in proportion to income ($\xi^h = M^h/(p_X X + p_Y Y)$). Since in this case the income shares from pollution are identical across all households (i.e., $\theta_Z^h \equiv \theta_Z$, $\forall h$), one can see from equation (14) that incidence is not affected by the tax revenue. This case therefore allows for an analysis of the incidence of the incidence impacts per se, as given by the changes in consumer prices and returns to factors of production alone.

For homothetic preferences, the heterogeneity of households can be described by the households' population distribution of the three following household characteristics: (i) expenditure shares $\alpha^h$, (ii) income shares $\theta_L^h$, and (iii) elasticities of substitution in utility $\sigma^h$.[15] Accordingly, we can summarize household heterogeneity by the following two quantities. First, we measure the degree in which expenditure and income patterns are correlated. To this end, we define the covariance between the expenditure share of the clean good and the labor income share as:

$$cov(\alpha^h, \theta_L^h) \equiv \sum_h (\alpha^h - \gamma) M^h (\theta_L^h - \theta_L)\,.$$

The covariance is, for example, positive if households who earn an above average share of their income from labor (i.e., $\theta_L^h > \theta_L$) spend an above average share of their income on the clean good (i.e., $\alpha^h > \gamma$).

Second, we quantify the interaction between expenditure shares $\alpha^h$ and substitution elasticities $\sigma^h$ by defining the effective elasticity of substitution between clean and dirty goods in utility as:

$$\rho \equiv \frac{1}{p_Y Y} \sum_h (1 - \alpha^h) M^h \left( \frac{\alpha^h}{\gamma} (\sigma^h - 1) + 1 \right).$$

$\rho$ can be interpreted as a generalized weighted average of the $\sigma^h$'s.[16]

Proposition 2 proves that the two quantities $cov(\alpha^h, \theta_L^h)$ and $\rho$ are indeed sufficient to fully characterize the impact of household heterogeneity on equilibrium prices and the level of pollution. For homothetic preferences, the system of equations (15a)–(15c) characterizing price changes in the general case simplifies to the following expressions, where the expression for $\hat{w}$ has been omitted due to its simple relationship to $\hat{r}$ (see Appendix C.1 for the derivation):

$$\hat{p}_Y = \frac{(\theta_{YL}\theta_{XK} - \theta_{YK}\theta_{XL})\theta_{YZ}}{D_H} \left[ A_H(e_{ZZ} - e_{KZ}) - B_H(e_{ZZ} - e_{LZ}) + (\gamma_K - \gamma_L)\rho \right] \hat{\tau}_Z + \theta_{YZ}\hat{\tau}_Z \tag{16a}$$

$$\hat{r} = -\frac{\theta_{XL}\theta_{YZ}}{D_H} \left[ A_H(e_{ZZ} - e_{KZ}) - B_H(e_{ZZ} - e_{LZ}) + (\gamma_K - \gamma_L)\rho \right] \hat{\tau}_Z\,, \tag{16b}$$

[15] Note that, for given $\xi^h$, a given $\theta_L^h$ uniquely determines $\theta_K^h$.

[16] To see this, consider the case with equal expenditure shares across households, i.e. $\alpha^h = \gamma$, $\forall h$. Then, $\rho = \sum_h M^h \sigma^h / \sum_h M^h$.

where $A_H \equiv \gamma_L\beta_K + \gamma_K(\beta_L + \theta_{YZ})$, $B_H \equiv \gamma_K\beta_L + \gamma_L(\beta_K + \theta_{YZ})$, $C_H \equiv \beta_K + \beta_L + \theta_{YZ}$,
$D_H \equiv C_H\sigma_X + A_H\left(\theta_{XK}\theta_{YL}(e_{KL} - e_{ZL}) - \theta_{XL}\theta_{YK}(e_{KK} - e_{ZK})\right) - B_H\left(\theta_{XK}\theta_{YL}(e_{LL} - e_{ZL}) - \theta_{XL}\theta_{YK}(e_{LK} - e_{ZK})\right)$
$-(\gamma_K - \gamma_L)\rho(\theta_{XK}\theta_{YL} - \theta_{XL}\theta_{YK}) - (\gamma_K - \gamma_L)\frac{cov(\alpha^h, \theta_L^h)}{\gamma_{PY}Y}$. Proposition 2 then follows directly:

**Proposition 2.** *If preferences are homothetic, the impact of household heterogeneity on output and factor price changes in equilibrium only depends on two quantities describing individual households' characteristics: (i) the covariance between the expenditure share of the clean good and the labor income share, $cov(\alpha^h, \theta_L^h)$, and (ii) the effective elasticity of substitution between clean and dirty goods in utility, $\rho$.*

*Proof.* Equations (16a)–(16b). □

Using the quantities $cov(\alpha^h, \theta_L^h)$ and $\rho$, we can now investigate a key question of the paper: under what conditions are price and pollution changes from an economy populated by heterogeneous households with homothetic preferences identical to those derived from an economy with a single representative household? The next proposition describes conditions in terms of household preferences and income patterns under which models with and without household heterogeneity yield identical equilibrium outcomes.

**Proposition 3.** *Assume homothetic preferences and (i) identical expenditure shares ($\alpha^h = \gamma$, $\forall h$) or (ii) identical income shares ($\theta_L^h = \theta_L$, $\forall h$). Then, output and factor price changes are identical to those for a single household characterized by homothetic preferences, clean good expenditure share $\gamma$, and elasticity of substitution between clean and dirty goods in utility equal to the effective elasticity $\rho$.*

*Proof.* Either of the above assumptions (i) and (ii) implies $cov(\alpha^h, \theta_L^h) = 0$. From equations (16a)–(16b) it is then easy to see that price changes are identical to those derived for an economy with a single consumer with homothetic preferences, clean good expenditure share $\gamma$, and elasticity of substitution in utility $\rho$. □

It follows that in the case with homothetic preferences and either identical expenditure shares or identical income shares (or both), households behave in the aggregate as a single representative household characterized by an elasticity of substitution in utility given by $\rho$. In the case with identical expenditure shares, the effective elasticity is equal to the weighted average of the individual households' substitution elasticities: $\rho = \frac{1}{\sum_h M^h}\sum_h M^h\sigma^h$. The resulting aggregate behavior is thus completely independent of patterns of income from capital and labor, and does not depend on the number of households. This, however, no longer holds if households have identical income shares but exhibit heterogeneity on the expenditure side. In the latter case, the value of $\rho$ depends on the interaction between expenditure shares $\alpha^h$ and the substitution elasticities of individual households $\sigma^h$: if households with an above average expenditure share on the dirty good have higher substitution elasticities, the corresponding single household responds in a more price-elastic manner as compared to a case with the same $\sigma^h$'s but $\alpha^h$'s that are identical across households.

Proposition 3 motivates the definition of $\rho$ as well as its interpretation as the "effective" elasticity of substitution between clean and dirty goods: when $cov(\alpha^h, \theta_L^h) = 0$–that is when either the households are identical on the expenditure or the income side (or both)–then in the aggregate, households *effectively* behave like a single household with substitution elasticity $\rho$. While Proposition 3 describes the conditions for household heterogeneity which allow for consumer aggregation, it is clear that consumers in the context of empirical incidence analysis household characteristics most likely violate these conditions. A central question for incidence analysis therefore is to investigate to what extent household heterogeneity can affect output and factor price changes.

**Proposition 4.** *Assume different factor intensities (i.e., $\gamma_K \neq \gamma_L$) and correlated income and consumption patterns (i.e., $cov(\alpha^h, \theta_L^h) \neq 0$). Assume homothetic, unit-elastic preferences (i.e., $\sigma^h = 1, \forall h$). Then, for any observed consumption and production decisions before the tax change, there exist production elasticities*

*(i.e., $\sigma_X$ and $e_{ij}$) such that the relative burden on factors of production is of opposite sign compared to the single-consumer model based on the same production data.*

*Proof.* See Appendix C.2. □

Proposition 4 proves that in the presence of heterogeneous households the sources of income impacts from a pollution tax not only differ quantitatively but can yield qualitatively different results when relying on factor price changes derived from a single-household model. Importantly, the possibility of reversed factor price changes does not depend on a particular distribution of households' characteristics as long as the covariance between income and expenditure patterns is non-zero. $cov(\alpha^h, \theta_L^h) \neq 0$ seems to be the empirically relevant case since $cov(\alpha^h, \theta_L^h) = 0$ describes the case in which households are identical or their consumption and income patterns are completely uncorrelated. Proposition 4 thus highlights how the incidence of environmental taxes among heterogeneous households may be qualitatively affected by the impact of household heterogeneity on equilibrium outcomes.

To further illustrate the range of (differing) equilibrium outcomes which depend on the nature and degree of household heterogeneity, we provide an example for a special case of our simple economy.

**Proposition 5.** *Assume homothetic, unit-elastic preferences (i.e., $\sigma^h = 1$), Leontief technologies in clean and dirty good production (i.e., $\sigma_X = e_{ij} = 0$), and that the dirty sector is relatively capital-intensive (i.e., $\gamma_K > \gamma_L$). Then, the following holds:*[17]

*(i) if consumers are identical on the sources or uses side of income, or both: $\hat{p}_Y = 0$, $\hat{w} > 0$, and $\hat{r} < 0$.*

*(ii) If labor ownership and clean good consumption have a negative covariance, then $\hat{p}_Y > 0$, $\hat{w} > 0$ and $\hat{r} < 0$.*

*(iii) If labor ownership and clean good consumption have a positive covariance, then $\hat{p}_Y < 0$, $\hat{w} > 0$, $\hat{r} < 0$ if the covariance is* low *(i.e., $D_{H,1} > 0$), and $\hat{p}_Y > 0$, $\hat{w} < 0$, $\hat{r} > 0$ if the covariance is* high *(i.e., $D_{H,1} < 0$).*

*Proof.* Given the above assumptions, price changes assume the following form:

$$\hat{p}_Y = -\frac{cov(\alpha^h, \theta_L^h)}{D_{H,1}\gamma p_Y Y}\theta_{YZ}\hat{\tau}_Z \tag{17a}$$

$$\hat{r} = -\frac{\theta_{XL}\theta_{YZ}}{D_{H,1}}\hat{\tau}_Z\,, \tag{17b}$$

where $D_{H,1} \equiv (\theta_{XL}\theta_{YK} - \theta_{XK}\theta_{YL}) - \frac{cov(\alpha^h, \theta_L^h)}{\gamma p_Y Y}$. □

Proposition 5 illustrates that, depending on assumptions about heterogeneity of households' expenditure and income patterns, almost any combination of $\hat{p}_Y \gtrless 0$, $\hat{w} \gtrless 0$, $\hat{r} \gtrless 0$ may arise. This suggests that a pollution tax change can lead to qualitatively different incidence results on the uses and sources side of income. Lastly, note that one can easily show that for a model with a single household and Leontief production, $\hat{p}_Y = 0$. Hence, Proposition 5 provides cases in which price changes derived from an economy with heterogeneous households cannot arise in a single-consumer economy with the same production characteristics. This additionally supports our argument that consistently integrating household heterogeneity in general equilibrium analyses is important.

[17] Note that for the case where the dirty sector is relatively labor-intensive (i.e., $\gamma_K < \gamma_L$), the sign of all the results in Proposition 5 is the opposite.

*3.3. Identical households with non-homothetic preferences*

Our results have so far proven that household heterogeneity can have a qualitative impact on the market equilibrium following an increase in a pollution tax, with implications for incidence. We now abstract from household heterogeneity in order to focus on the effect of non-homothetic preferences on the equilibrium.

As the following special case illustrates, accounting for non-homothetic preferences can also qualitatively affect price changes in equilibrium. Assume that all cross-price elasticities have the same positive value $c$: $\sigma^h = \sigma_X = e_{KL} = e_{KZ} = e_{LZ} \equiv c > 0$. Price changes are then of the following form:

$$\hat{p}_Y = -\frac{\theta_{XK}\theta_{XL}\gamma\theta_{YZ}}{D_{ID}}[(\gamma_K - \gamma_L)^2(E_{Y,M} - E_{X,M})]\hat{\tau}_Z + \theta_{YZ}\hat{\tau}_Z \tag{18a}$$

$$\hat{r} = -\frac{\theta_{XL}\theta_{YZ}}{D_{ID}}[(\gamma_K - \gamma_L)(E_{Y,M} - E_{X,M})(1-\gamma)]\hat{\tau}_Z\,, \tag{18b}$$

where $E^h_{X,M} \equiv E_{X,M}$ and $E^h_{Y,M} \equiv E_{Y,M}$ $\forall h$, $D_{ID} \equiv C_{ID} + A_{ID}\theta_{XL} + B_{ID}\theta_{XK} + (\gamma_K - \gamma_L)^2\theta_{XK}\theta_{XL}\frac{\gamma}{1-\gamma}$, $A_{ID} \equiv \gamma_L\beta_K + \gamma_K(\beta_L + \theta_{YZ} + (E_{X,M} - E_{Y,M})\frac{\tau_Z Z}{p_X X + p_Y Y})$, $B_{ID} \equiv \gamma_K\beta_L + \gamma_L(\beta_K + \theta_{YZ} + (E_{X,M} - E_{Y,M})\frac{\tau_Z Z}{p_X X + p_Y Y})$, $C_{ID} \equiv \beta_K + \beta_L + \theta_{YZ} + (E_{X,M} - E_{Y,M})\frac{\tau_Z Z}{p_X X + p_Y Y}$.

In order to determine the sign of the above price changes, we define the following *Condition 1*: $D_{ID} > 0$. Condition 1 holds if the expenditure share on the clean good increase with income ($E_{X,M} > E_{Y,M}$). It also holds when the clean good expenditure share decreases with income ($E_{Y,M} > E_{X,M}$), but the difference between the income elasticities is not too large. We can then prove that a wide range of possible combinations of output and factor price changes are possible in this special case, depending on the preference parameters.

**Proposition 6.** *Assume identical households and equal cross-price elasticities ($\sigma^h = \sigma_X = e_{KL} = e_{KZ} = e_{LZ} \equiv c > 0$). Then, the following holds:*

*(i) If preferences are homothetic, then $\hat{p}_Y = \theta_{YZ}\hat{\tau}_Z$, and $\hat{w} = \hat{r} = 0$.*

*(ii) Assume that the dirty sector is relatively capital-intensive (i.e. $\gamma_K > \gamma_L$).*[18]

*(a) If Condition 1 holds, then for $E_{Y,M} > E_{X,M}$: $\hat{p}_Y < \theta_{YZ}\hat{\tau}_Z$, $\hat{w} > 0$ and $\hat{r} < 0$, and for $E_{Y,M} < E_{X,M}$: $\hat{p}_Y > \theta_{YZ}\hat{\tau}_Z$, $\hat{w} < 0$ and $\hat{r} > 0$.*

*(b) If Condition 1 does not hold, then for $E_{Y,M} > E_{X,M}$: $\hat{p}_Y > \theta_{YZ}\hat{\tau}_Z$, $\hat{w} < 0$ and $\hat{r} > 0$, and for $E_{Y,M} < E_{X,M}$: $\hat{p}_Y < \theta_{YZ}\hat{\tau}_Z$, $\hat{w} > 0$ and $\hat{r} < 0$.*

*Proof.* Equations (18a)–(18b). For (i): use $E_{Y,M} = E_{X,M}$. □

We have therefore illustrated that there exist cases where the relative burden on factors of production depends on the interaction between production characteristics and the income elasticities of demand for the clean and the dirty goods. It follows that, by extending the Fullerton & Heutel (2007) model to incorporate household heterogeneity and non-homothetic preferences, we have added two dimensions that can both qualitatively alter the economy's reaction to an exogenous increase in the pollution tax. Both features are therefore in general significant for incidence.

[18] Note that for the case with $\gamma_K < \gamma_L$, the results for $\hat{w}$ and $\hat{r}$ are of opposite signs to the analogous expressions in Proposition 6 (ii). The results for $\hat{p}_Y$ remain unchanged, as long as factor intensities differ ($\gamma_K \neq \gamma_L$).

Table 1: Household expenditures on clean and dirty goods and household income by source for annual expenditure deciles (in % of total expenditure for a given household group)

| Expenditure | Income sources | | Expenditures by commodity | |
|---|---|---|---|---|
| decile $h$ | Labor | Capital | Clean | Dirty |
| 1 | 42.8 | 13.5 | 85.5 | 14.5 |
| 2 | 74.5 | 13.8 | 84.8 | 15.2 |
| 3 | 86.3 | 16.2 | 85.4 | 14.6 |
| 4 | 103.5 | 18.0 | 86.1 | 13.9 |
| 5 | 108.8 | 20.4 | 86.8 | 13.2 |
| 6 | 114.4 | 29.4 | 87.7 | 12.3 |
| 7 | 118.8 | 31.2 | 88.5 | 11.5 |
| 8 | 120.0 | 38.4 | 89.2 | 10.8 |
| 9 | 124.6 | 45.1 | 90.7 | 9.3 |
| 10 | 93.4 | 54.7 | 94.1 | 5.9 |

*Notes:* Household data is based on the "Consumer Expenditure Survey" (CEX) data as shown in Fullerton & Heutel (2010).

## 4. Numerical analysis

In this section, we apply the heterogeneous household model to quantitatively assess how the aggregation bias affects equilibrium outcomes and the incidence of a tax on carbon dioxide ($CO_2$) emissions for the case of the United States. We assess the incidence on the sources and uses side of income, and explore how sensitive results are with respect to key characteristics governing households' and firms' behavior.

### *4.1. Data and calibration*

In order to situate our study in the context of the literature, we calibrate our model to data used previously for a two-sector general equilibrium environmental tax incidence analysis. For this purpose, we chose the production and consumption data of Fullerton & Heutel (2010). They aggregate a data set of the U.S. economy to a '"dirty" and a "clean" sector, where the dirty sector comprises the highly $CO_2$-intensive industries (electricity generation, transportation and petroleum refining). As in Fullerton & Heutel (2010) we assume an initial and pre-existing carbon tax of \$15 per metric ton of $CO_2$. Our comparative-static analysis considers a 100% increase in the carbon tax.

All prices in the benchmark are normalised to one, and quantities are normalised such that the total value of the economy is equal to one, i.e., $p_X X + p_Y Y = 1$. Calibrated values for outputs and inputs are then as follows: $X = 0.929$, $L_X = 0.579$, $L_Y = 0.029$, $K_X = 0.350$, $K_Y = 0.037$, and $Z = 0.005$. Households are grouped by annual expenditure deciles,[19] and data for expenditures by clean and dirty goods as well as capital and labor income are shown in Table 1. Note that our analysis abstracts from government transfers.

Incorporating heterogeneous households in a calibrated general equilibrium model of the U.S. economy requires that—at the aggregate level—data describing household consumption and income are consistent with the production data on output by sector and aggregate, economy-wide factor income. To reconcile data sources, we adjust the household data to be consistent with aggregate production data while preserving

[19]It is well-known in the literature on tax incidence that absent a fully dynamic framework, categorizing households by expenditure deciles is a better proxy for lifetime income as compared to a ranking based on annual income deciles (see, for example, Poterba, 1991; Fullerton & Heutel, 2010).

the relative characteristics of household expenditures across expenditure deciles. More specifically, data adjustments for each expenditure decile are as follows. First, we scale income to mach expenditure while keeping fixed the decile's capital-to-labor ratio. Second, we scale the capital ownership of all deciles by a common factor in order for aggregate household income by factor to match production side data, whilst preserving the relative capital ownership amongst deciles. Third, we perform an analogous scaling for consumption of the dirty good. This procedure yields consistent household and production data which is used to calibrate the general equilibrium model.

For our central case parametrization of production elasticities we follow Fullerton & Heutel (2010) assuming $\sigma_X = 1$, $e_{KL} = 0.1$, $e_{KZ} = 0.2$, and $e_{LZ} = -0.1$. This implies that capital is a better substitute for pollution than labor. For the single household model, Fullerton & Heutel (2010) assume that the elasticity of substitution between the clean and the dirty good in utility is unity, and that preferences are homothetic. Our central case is based on analogous assumptions for each household group, i.e., $\sigma^h = 1$ and $E^h_{X,M} = E^h_{Y,M} = 1$, $\forall h$. Note that while these parameter choices reflect central case assumptions, we perform extensive sensitivity analysis to check for the size of the aggregation bias and the incidence patterns from increases in the pollution tax.

### *4.2. Size of the aggregation bias and implications for incidence analysis*

From the theoretical analysis we know that heterogeneous households and non-homothetic preferences can have a significant effect on price changes following an increase in the pollution tax. We now measure the aggregation bias introduced by modeling an economy comprising heterogeneous households as an economy with a single representative household. We first compute the price changes following a change in the pollution tax from the heterogeneous household model with expenditure and income patterns calibrated based on the data shown in Table 1. These price changes are then compared with price changes derived from a model calibrated to the same aggregate data but with a single representative household.[20]

Biased price changes translate into biased welfare results. To quantify this bias, we define the "Welfare Aggregation Bias", $\Gamma$, as:

$$\Gamma = \Omega^{-1} \sum_h \frac{M^h}{\sum_{h'} M^{h'}} \left| \Phi^h - \Phi^h_{Aggregate} \right| , \tag{19}$$

where $h$ and $h'$ are indexes for expenditure deciles and $\Phi^h$ is the household-level welfare impact as given by equation (14). $\Phi^h_{Aggregate}$ is also derived from equation (14) but uses instead price changes which are derived from the model with a single household representing aggregate demand.[21] Dividing by $\Omega \equiv \sum_h \frac{M^h}{\sum_{h'} M^{h'}} \left|\Phi^h\right|$ expresses the aggregation bias as a share relative to the average welfare impact across households.

$\Gamma$ yields a measure of the average difference in welfare impacts derived under the consistent approach and the generally biased representative household approach. $\Gamma$ is greater or equal to zero as it is defined as the weighted average of the absolute value of the difference between $\Phi^h$ and $\Phi^h_{Aggregate}$. If $\Gamma = 0$, the welfare results derived under the two approaches are identical. If $\Gamma > 0$, then there is a bias on the household-level welfare impacts when employing the representative household approach, and therefore the pattern of incidence will in general be biased.

[20] To focus on the incidence effects due to goods and factor price changes only, we here assume that the pollution tax revenue is redistributed in proportion to income. We consider alternative revenue recycling schemes in Section 5.

[21] This aggregate household is assumed to be characterized by an elasticity of substitution in utility between clean and dirty consumption and income elasticities that are given by the expenditure-weighted average of the elasticities of individual deciles, i.e., $\sigma_{Aggregate} = \frac{1}{\sum_{h'} M^{h'}} \sum_h M^h \sigma^h$ and $E^{Aggregate}_{X/Y,M} = \frac{1}{\sum_{h'} M^{h'}} \sum_h M^h E^h_{X/Y,M}$.

Table 2: Price changes and welfare aggregation bias for alternative assumptions about household heterogeneity and production characteristics

| | Aggregate household model | Heterogeneous household model | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $cov_{Base}$ | | $cov_{Low}$ | | | | $cov_{High}$ | | | |
| | | $\rho_{Base}$ | | $\rho_{Low}$ | | $\rho_{High}$ | | $\rho_{Low}$ | | $\rho_{High}$ | |
| | $\hat{r}$ | $\hat{r}$ | $\Gamma$ | $\hat{r}$ | $\Gamma$ | $\hat{r}$ | $\Gamma$ | $\hat{r}$ | $\Gamma$ | $\hat{r}$ | $\Gamma$ |
| *Substitutability between capital and labor in the clean sector* | | | | | | | | | | | |
| $\sigma_X = 1.5$ | -0.08 | -0.08 | 0.0 | -0.07 | 1.4 | -0.09 | 1.4 | -0.05 | 3.2 | -0.11 | 3.4 |
| $\sigma_X = 1$ | -0.12 | -0.12 | 0.0 | -0.10 | 2.2 | -0.13 | 2.3 | -0.08 | 5.0 | -0.16 | 5.3 |
| $\sigma_X = 0.5$ | -0.23 | -0.23 | 0.2 | -0.21 | 5.1 | -0.26 | 5.5 | -0.15 | 10.6 | -0.31 | 12.4 |
| *Substitutability between capital, labor, and pollution in the dirty sector* | | | | | | | | | | | |
| $e_{K/LZ} = \pm 0.5$ | 0.11 | 0.11 | 0.0 | 0.13 | 1.6 | 0.10 | 1.7 | 0.15 | 3.9 | 0.07 | 4.3 |
| $e_{K/LZ} = \mp 0.5$ | -0.58 | -0.58 | 0.6 | -0.57 | 5.4 | -0.59 | 5.1 | -0.54 | 9.7 | -0.62 | 10.2 |

*Notes:* $\hat{r}$ is expressed as the percentage change relative to the price level before the pollution tax increase. Price changes for the dirty good are virtually identical across the cases shown here and are hence not shown. $\Gamma$ is expressed as a percentage share.

Given the considerable uncertainty surrounding both the household survey data as well as household and production side parameters, we investigate a range of alternative cases around our central case assumptions which are based on observed data for the U.S. economy and parameter assumptions from the literature (see Section 4.1). First, "$cov_{Low}$" and "$cov_{High}$" represent cases where the covariance measure is respectively halved and doubled relative to the central case "$cov_{Base}$", representing cases where there is respectively less and more heterogeneity in expenditure and income shares among households. Second, we consider different assumptions with respect to higher-order properties of households' utility functions by introducing heterogeneity in the price and income elasticities of demand across households. A case labeled "$\rho_{Low}$" and "$\rho_{High}$" assumes that poorer households in lower expenditure deciles are described by a smaller and larger elasticity of substitution between clean and dirty goods relative to the richer households, respectively. We interact different cases regarding household characteristics with alternative assumptions about the production side, i.e., cases which differ with respect to the substitutability between capital and labor in the clean sector ($\sigma_X$) and between capital, labor, and pollution in the dirty sector ($e_{K/LZ}$). Table 2 reports the aggregation bias in terms of both price changes and welfare for these cases. The following key insights emerge.

First, comparing price changes from the aggregate household and heterogeneous household models, the aggregation bias on the returns to capital is larger than on the price of the dirty good; the aggregation bias for $\hat{r}$, i.e., the percentage difference between price changes, can be up to 38% (for "$cov_{High}$", "$rho_{High}$", and $\sigma_X = 1.5$) whereas for $\hat{p}_Y$ it is negligible for all cases. The reason is that $\hat{p}_Y$ is dominated by the "direct" cost pass-through effect which is represented by the term $\theta_{YZ}\hat{\tau}_Z$ in equation (15a) (see also Fullerton & Heutel, 2010). The output and substitution effects arising in general equilibrium are only a fraction of the total change in $\hat{p}_Y$ but fully determine $\hat{r}$ and $\hat{w}$ (see first line of equation (15a) and equations (15b) and (15c)). As the cost pass-through is independent of household characteristics, the aggregation bias manifests itself only through the general equilibrium effects which explains why the relative impact of the aggregation bias for $\hat{p}_Y$ is smaller than for the factor price changes.

Second, the aggregation bias on prices for $\rho_{Base}$ (which corresponds to $\sigma^h = 1, \forall h$) is small compared to the other cases. This translates into a smaller welfare aggregation bias $\Gamma$. When substitution elasticities are identical across households, for a given increase in the price of the dirty good, households all substitute the same percentage of dirty good consumption with clean consumption. Abstracting from changes in income, it then follows that the aggregate change in consumption is the same as for a representative household with the same substitution elasticity. The numerical results show that in this case other effects that may depend on household heterogeneity are not of particular significance.

Third, we find that, for a given covariance between income and expenditure patterns, the returns to capital are decreasing in the effective elasticity $\rho$. Intuitively, the reaction of aggregate demand to an increase of the price of the dirty good is disproportionately affected by the households that consume the dirty good more intensively. For $\rho_{High}$, these households' demand is more price elastic than the average demand, hence aggregate demand will react more elastically to an increase in the price of the dirty good as compared to the single consumer. This in turn depresses demand for the dirty good more, leading to a decrease in both the price of the dirty good and the returns to the factor which is used intensively in the dirty industry, i.e. capital. An analogous explanation holds true for the $\rho_{Low}$ case.

Fourth, the changes in the return to capital are increasing in the absolute value of the covariance for $\rho_{Low}$, and decreasing in the absolute value of the covariance for $\rho_{High}$. A higher covariance means that households consuming an above-average share of the dirty good consume even more. This in turn magnifies the above-mentioned impact of the effective elasticity $\rho$ on the determination of equilibrium price changes. Finally, we find that the aggregation bias is not much affected by introducing heterogeneity in the income elasticities of consumption (which we therefore do not show in Table 2). This points to the fact that heterogeneity in price effects dominates heterogeneity in income effects in determining aggregate consumption behavior.

In summary, we find that the effect of the aggregation bias for the empirically motivated cases shown in Table 2 is non-negligible, especially for changes in returns to factors of production. Household heterogeneity in the elasticities of substitution in utility magnifies the aggregation bias due to heterogeneity in expenditure and income patterns. In our static model, heterogeneity in income elasticities has a smaller effect compared to heterogeneity in substitution elasticities.

Lastly, Table 3 presents selected cases for which the aggregation bias is sufficiently large to cause incidence patterns to be qualitatively different, changing the incidence shape from "U" to inverted "U" and reversing the sign of the welfare impact for some households. The wide variation in welfare impacts across deciles in these cases emphasizes the fact that within the range of possible values of household and production parameters there exist equilibria in which the economy is particularly sensitive to an increase in the pollution tax. Although these cases are relatively "distant" to our central case assumptions, they illustrate the pitfalls in assessing distributional impacts of an environmental tax in a model with a single, representative consumer.

### *4.3. Applying the heterogeneous household model: distributional impacts of a U.S. carbon tax*

We now use our calibrated model to assess the incidence of a U.S. carbon tax. Importantly, we maintain our assumption that the carbon tax revenue is recycled in proportion to income thereby abstracting from differential impacts among households due to revenue recycling. This allows us to focus on the relative importance of channels for incidence which are affected by the household aggregation bias, i.e. consumer and factor price changes.[22]

[22]Our analysis should thus not be interpreted as a comprehensive assessment of a specific U.S. carbon tax policy proposal with specific provisions for revenue recycling. Of course, as documented by the large literature on the distributional impacts of carbon

Table 3: Selected cases for which welfare aggregation bias is "large", i.e. incidence results across household groups differ qualitatively due to the aggregation bias

| Expenditure | Case1 | | Case 2 | | Case 3 | |
|---|---|---|---|---|---|---|
| decile | $\Phi^h$ | $\Phi^h_{Aggregate}$ | $\Phi^h$ | $\Phi^h_{Aggregate}$ | $\Phi^h$ | $\Phi^h_{Aggregate}$ |
| 1 | -0.15 | -0.21 | 0.16 | 0.48 | 0.56 | -0.67 |
| 2 | 0.21 | -0.36 | 3.06 | 5.95 | 5.35 | -6.03 |
| 3 | 0.23 | -0.32 | 3.01 | 5.83 | 5.23 | -5.89 |
| 4 | 0.31 | -0.30 | 3.37 | 6.48 | 5.77 | -6.49 |
| 5 | 0.29 | -0.25 | 3.05 | 5.84 | 5.19 | -5.83 |
| 6 | 0.12 | -0.15 | 1.45 | 2.79 | 2.50 | -2.81 |
| 7 | 0.14 | -0.10 | 1.34 | 2.56 | 2.26 | -2.54 |
| 8 | 0.01 | -0.02 | 0.16 | 0.30 | 0.27 | -0.31 |
| 9 | -0.03 | 0.09 | -0.63 | -1.23 | -1.11 | 1.26 |
| 10 | -0.36 | 0.40 | -4.16 | -8.02 | -7.15 | 8.04 |

*Notes:* Cases are defined as follows. Case 1: $\sigma_X = 0$, $\sigma^h = 2$, for $h = 1, \ldots, 5$, $\sigma^h = 0$, for $h = 6, \ldots, 10$, $e_{KL} = 0.1$, $e_{KZ} = 0.5$, and $e_{LZ} = 0.4$. Case 2: Leontief production, $\sigma^h$ as for $\rho_{low}$, $E^h_Y = 2$, for $h = 1, \ldots, 7$, and $E^h_Y = 0$, for $h = 8, \ldots, 10$. Case 3 corresponds to the case in Proposition 4: $\sigma_X = 0$, $\sigma^h = 1$, $e_{KL} = -0.145$, $e_{KZ} = e_{LZ} = 0$, and $E^h_Y = 1$.

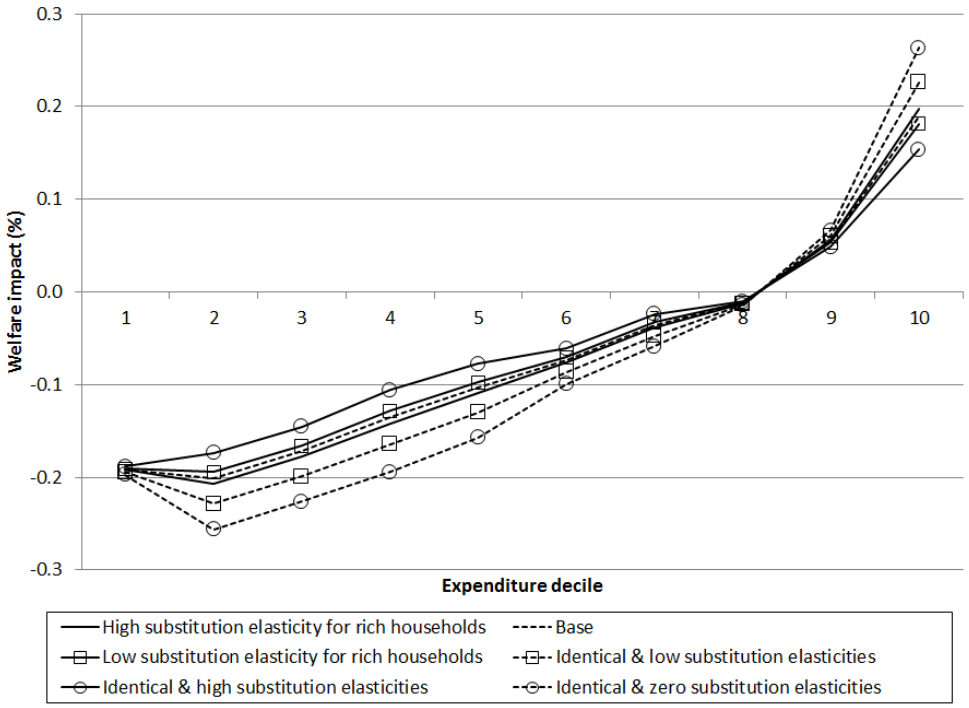
We explore the robustness of the incidence result through "piecemeal" sensitivity analysis by varying household and production elasticities. For each case, we identify the relative importance of uses and sources effects of income. Figure 1a displays welfare impacts for a range of cases which vary household characteristics around the base case. We assume different values for $\sigma^h$, the elasticity of substitution in utility between clean and dirty goods. For "low" and "high" substitution cases for rich households, we set $\sigma^h$ for different household groups as in $\rho_{High}$ and $\rho_{Low}$, respectively. For cases with identical "zero", "low", and "high" substitution elasticities the following values are assumed, respectively: $\sigma^h = 0$, $\sigma^h = 0.5$, and $\sigma^h = 1.5$, $\forall h$. In all cases, household expenditure and income shares are left unchanged.

From Figure 1a it is evident that a carbon tax is regressive in the base case, and that this result is robust to varying household characteristics. Even if households are more able to substitute away from the taxed dirty good, as reflected by high $\sigma^h$'s, the carbon tax puts disproportionately large burdens on households in lower expenditure deciles. The incidence is slightly more regressive for low values of $\sigma^h$ as compared to cases with high values for $\sigma^h$. This is driven by the fact that for relatively low $\sigma^h$'s, the burden from higher prices for the dirty good is borne to a larger extent by consumers, hence falling more heavily on those household groups that spend a relatively large fraction of their income on the dirty good. At the same time, as consumers are less able to substitute away from the dirty good, the reduction in the dirty sector output, $Y$, is relatively smaller, hence the return to capital, the factor used intensively in the production of $Y$, decreases by less. This explains why the welfare losses on the sources side of richer households with

taxation, the way the revenues are recycled can importantly alter the the incidence pattern across households (see, for example, Bento et al., 2009; Rausch et al., 2010a; Mathur & Morris, 2014; Williams III et al., 2015). To illustrate this point in the context of our model, an online appendix contains supplementary analysis which considers two additional revenue recycling schemes. A first case assumes that the revenue is distributed in proportion to the consumption of the dirty good reflecting concerns about offsetting adverse impacts for poorer households. The resulting incidence pattern looks more neutral when compared to Figure 1. A second case considers distributing the carbon revenue equally among households on a per capita basis, resulting in a sharply progressive outcome.

Figure 1: Welfare impacts ($\Phi^h$) of increased pollution tax across annual expenditure deciles

(a) Alternative assumptions about household characteristics




(b) Alternative assumptions about production characteristics

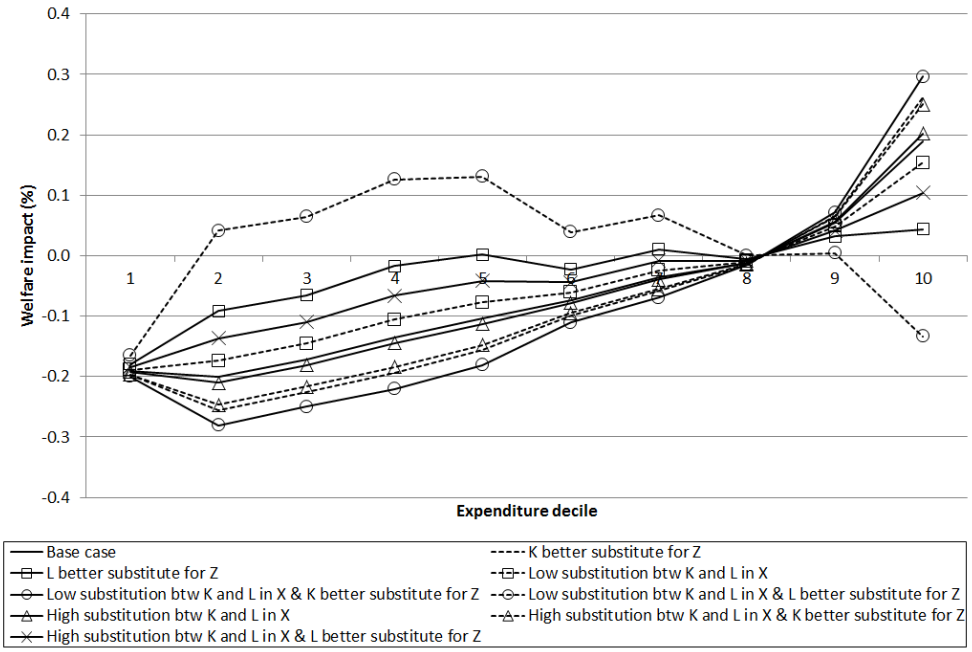


Table 4: Household welfare impacts ($\Phi^h$) by expenditure decile (in %) by uses and sources side of income for alternative household characteristics

| Expenditure | Uses side | Sources side | | | | | |
|---|---|---|---|---|---|---|---|
| Decile | *All cases*[a] | Central case ($\sigma^h = 1$) | $\rho_{low}$ | $\rho_{high}$ | $\sigma^h = 1.5$ | $\sigma^h = .5$ | $\sigma^h = 0$ |
| 1 | -0.19 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 2 | -0.23 | 0.03 | 0.02 | 0.03 | 0.05 | 0.00 | -0.03 |
| 3 | -0.20 | 0.03 | 0.02 | 0.03 | 0.05 | 0.00 | -0.03 |
| 4 | -0.16 | 0.03 | 0.02 | 0.04 | 0.06 | 0.00 | -0.03 |
| 5 | -0.13 | 0.03 | 0.02 | 0.03 | 0.05 | 0.00 | -0.03 |
| 6 | -0.09 | 0.01 | 0.01 | 0.02 | 0.02 | 0.00 | -0.01 |
| 7 | -0.05 | 0.01 | 0.01 | 0.01 | 0.02 | 0.00 | -0.01 |
| 8 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 9 | 0.06 | -0.01 | 0.00 | -0.01 | -0.01 | 0.00 | 0.01 |
| 10 | 0.23 | -0.04 | -0.03 | -0.04 | -0.07 | 0.00 | 0.04 |

*Notes:* Cases shown in columns are identical to cases in Figure 1a. [a]Uses side impacts are virtually identical for all the cases, hence only one column is shown.

relatively high capital income shares (i.e., deciles 9 and 10) get smaller as $\sigma^h$ decreases. For $\sigma^h = 0$ rich households experience gains, relative to the average household, on both the uses and sources side.

Figure 1b displays welfare impacts for a range of cases which vary production characteristics around our base case assumptions. Cases shown vary either the elasticity of substitution between capital and labor in clean production, $\sigma_X$ (halving and doubling the value from the base case), the substitutability between capital and labor vis-à-vis pollution, or a combination of the two. The case "$K$ better substitute for $Z$" assumes $e_{KZ} = 0.5$ $e_{LZ} = -0.5$, and the case "$L$ better substitute for $Z$" assumes $e_{KZ} = -0.5$ $e_{LZ} = 0.5$.

The following insights emerge from Figure 1b. First, while for the majority of cases the carbon tax is found to be regressive, there is considerable variation in welfare impacts depending on production parameters. Second, the pattern of distributional impacts depends largely on the substitutability of inputs in the production of the dirty good. If capital is a better substitute for pollution than labor, then the carbon tax is regressive, due to the regressivity of both the uses and the sources of income incidence. On the sources of income side, as the burden on factor prices falls on labor rather than capital, poorer households with high labor income shares experience large welfare losses, while richer households with high capital income shares experience larger relative gains. In contrast, the carbon tax is less regressive and can even in some cases be inversely U-shaped if labor is a relatively good substitute for pollution vis-à-vis capital, due to the progressivity of the sources of income incidence when the burden falls on capital rather than on labor. Third, higher values of $\sigma_X$ imply flatter incidence curves, since this dampens the burden on the returns to the factors of production.

For the cases shown in Figure 1, Tables 4 and 5 decompose welfare impacts into uses and sources side impacts. For the range of household and production characteristics that we consider, we find that uses side effects are markedly regressive and that there is relatively little variation in the size of uses side impacts for a given household group. The sources side impacts on the other hand tend to be mostly neutral or progressive, driven by the fact that burdens mostly fall on capital, and are much more sensitive to behavioural parameters

Table 5: Household welfare impacts ($\Phi^h$) by expenditure decile (in %) by uses and sources side of income for alternative production characteristics

| Expenditure | Uses side | Sources side | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Decile | *All cases*[a] | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| 1 | -0.19 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.02 | 0.00 | 0.00 | 0.01 |
| 2 | -0.23 | 0.03 | -0.03 | 0.13 | 0.05 | -0.05 | 0.26 | 0.02 | -0.02 | 0.09 |
| 3 | -0.20 | 0.03 | -0.02 | 0.13 | 0.05 | -0.05 | 0.26 | 0.02 | -0.02 | 0.09 |
| 4 | -0.16 | 0.03 | -0.03 | 0.14 | 0.06 | -0.05 | 0.28 | 0.02 | -0.02 | 0.10 |
| 5 | -0.13 | 0.03 | -0.03 | 0.13 | 0.05 | -0.05 | 0.26 | 0.02 | -0.02 | 0.09 |
| 6 | -0.09 | 0.01 | -0.01 | 0.06 | 0.02 | -0.02 | 0.12 | 0.01 | -0.01 | 0.04 |
| 7 | -0.05 | 0.01 | -0.01 | 0.06 | 0.02 | -0.02 | 0.11 | 0.01 | -0.01 | 0.04 |
| 8 | -0.01 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 |
| 9 | 0.06 | -0.01 | 0.01 | -0.03 | -0.01 | 0.01 | -0.05 | 0.00 | 0.00 | -0.02 |
| 10 | 0.23 | -0.04 | 0.03 | -0.18 | -0.07 | 0.07 | -0.35 | -0.02 | 0.02 | -0.12 |

*Notes*: Cases shown in columns are identical to cases in Figure 1b. [a]Uses side impacts are virtually identical for all the cases, hence only one column is shown. Columns are defined as follows: (1)=central case, (2)=K better substitute for Z ($e_{KZ} = 0.5$ and $e_{LZ} = -0.5$), (3)=L better substitute for Z ($e_{KZ} = -0.5$ and $e_{LZ} = 0.5$), (4)=Low substitution between $K$ and $L$ in sector $X$ ($\sigma_X = 0.5$), (5)=Low substitution between $K$ and $L$ in sector $X$ and K better substitute for Z, (6)=Low substitution between $K$ and $L$ in sector $X$ and L better substitute for Z (7)=High substitution between $K$ and $L$ in sector $X$ ($\sigma_X = 1.5$), (8)=X more price elastic and K better substitute for Z, (9)=High substitution between $K$ and $L$ in sector $X$ and L better substitute for Z.

as compared to the uses side impacts.[23]

To summarize, while we find evidence that a carbon tax itself–i.e., ignoring differential impacts among households from revenue recycling–can be regressive, sensitivity analysis on production and household characteristics illustrates that other incidence outcomes (inverted U shape and progressive across the top five expenditure deciles) may be possible. As the aggregation bias on welfare is largely caused by the aggregation bias on the returns to factors of production, it mainly affects the sources of income. We also find that most of the variation in welfare impacts is driven by sources side impacts. Our analysis thus points to the importance of including sources of income impacts for tax incidence analysis.

## 5. Extensions

In this section, we extend our analysis in a number of directions going beyond the stylized setup of our core model to check for the robustness of our results. As one would expect, any extension of the model produces different quantitative results. The point of the paper, however, that household heterogeneity affects equilibrium and hence the incidence of environmental taxes remains. In fact, we find that the case for the aggregation bias is strengthened rather than weakened since extending the analysis creates additional dimensions along which households may differ. Alongside the effects previously identified for our core model these extensions introduce new channels through which household heterogeneity affects the general

[23]Note that the small variation in impacts for the first and eighth expenditure deciles reflects that these households have a capital-labor ratio which is similar to the sample's average. Hence, the sources side impacts relative to the average are small for these two deciles.

equilibrium. In turn, we find that in general these channels affect the results. We briefly summarize the main findings for each extension here, while the detailed analysis is documented in the online appendix.

### *5.1. Alternative revenue recycling schemes*

Our analysis so far has assumed that the environmental tax revenue is distributed in a way that abstracts from differential impacts among households, i.e. in proportion to income. Redistributing the tax revenue in a non-neutral manner introduces an additional channel of heterogeneity on the sources of income side. This could potentially affect how household heterogeneity impacts equilibrium outcomes. We consider two alternative ways of recycling the carbon tax revenue: a first case assumes distribution in proportion to dirty good consumption and a second case assumes that the revenue is distributed on an equal per capita basis. We find that price changes for both $\hat{r}$ and $\hat{p}_Y$ are very similar among alternative revenue recycling cases indicating that the impact of household heterogeneity on the equilibrium outcome is largely independent of the way the environmental tax revenue is redistributed.

### *5.2. Pre-existing, non-environmental taxes*

Accounting for pre-existing taxes on capital and labor in the benchmark, analogous to Fullerton & Heutel (2007), modifies the production cost shares now including tax payments ($\theta_{YK} \equiv \frac{r(1+\tau_K)K_Y}{p_Y Y}$, and similarly for $\theta_{YL}$, $\theta_{XK}$ and $\theta_{XL}$) as well as the households' income constraints now including tax revenues as new sources of income. As long as the revenue from capital and labor taxes is also distributed in proportion to income, there is no additional effect of household heterogeneity on price changes as heterogeneity in terms of both uses and sources side is unchanged. In this case, all Propositions 1–6 remain valid. Distributing capital and labor tax revenue in a non-neutral way will introduce additional heterogeneity on the sources side. In this case, Propositions 1 and 6 still hold true and price changes for $\hat{r}$ and $\hat{p}_Y$ are quantitatively similar (analogously to our findings in Section 5.1).

### *5.3. Non-separable utility in pollution*

With non-separable utility, consumption of clean and dirty goods in general depends on the level of pollution: $X^h = X^h(p_X, p_Y, M^h, Z)$ and $Y^h = Y^h(p_X, p_Y, M^h, Z)$. The change in the pollution level following a pollution tax increase can thus affect the equilibrium behavior of households. Aggregate economy outcomes therefore now depend on the household-level responses to changes in pollution as well as the interaction with other household characteristics. This introduces an additional dimension of heterogeneity to the extent that households have different preferences about pollution. This effect can be captured by introducing a new quantity that describes the interaction between expenditure patterns and pollution elasticities (similar to the effective elasticity of substitution between clean and dirty goods in utility $\rho$). All Propositions 1–6 can then be straightforwardly extended to account for the new pollution channel whilst maintaining the effects previously shown. In general, the overall effect of the impact of household heterogeneity on equilibrium outcomes may lead to a smaller or larger aggregation bias compared to the case with separable utility in pollution.

### *5.4. Labor-leisure choice*

An important dimension along which households can differ is their valuation of leisure time resulting in differences with respect to the elasticity of labor supply. Incorporating endogenous labor supply significantly enhances the complexity of studying the impact of household heterogeneity of equilibrium outcomes as it affects both how income is earned and spent. To keep the theoretical analysis tractable, we restrict our attention to Cobb-Douglas utility and assume that in the benchmark households dedicate an equal fraction

of their productive time to leisure. We find that results are mainly similar with new parameters summarizing the additional channels of household heterogeneity as well as the aggregate impact of labor-leisure choice on the general equilibrium. Proposition 1 is identical. Proposition 2 is analogous accounting in addition for interactions between leisure choice and expenditure and income patterns. Proposition 3 is analogous with the presence of a term that reflects the impact of average expenditure share of leisure on aggregate outcomes. Propositions 4 and 5 are analogous, too. For the special case of Cobb-Douglas utility, we thus find that the effect of household heterogeneity is similar to the case without labor-leisure choice; where it differs it can be understood in terms of additional terms reflecting interactions between the various types of heterogeneity (i.e., labor-leisure choice, expenditures and income patterns). Whether or not the aggregation bias is quantitatively smaller or larger would depend on the specific parametrization.

### *5.5. More than two sectors*

Closely based on Fullerton & Heutel (2007), our analysis assumed a highly aggregated sectoral representation which is also in line with much of the literature following Harberger (1962). Including more sectors can obviously affect the aggregation bias as it enables representing household heterogeneity along more dimensions. With a finer sectoral resolution, it is, for example, conceivable that poorer households may have higher expenditure shares on some dirty goods and lower expenditure shares on some others when compared to richer households. The problem is further compounded by the possibility that different polluting goods may be produced with different capital and labor intensities, interacting with the sources of income incidence. As the aggregation bias is determined by the interaction between household and production side characteristics, the impact of going from two to multiple sectors on the aggregation bias is thus in general not clear-cut. For a special case, one can nevertheless show that the aggregation bias remains important for assessing the incidence of environmental taxes in a setting which includes an arbitrary number of sectors. Analogous to Proposition 5 with Leontief technologies, we find that the value of the covariance between the ownership of labor and consumption of each dirty good across households can reverse the sign of the factor price changes.

## 6. Conclusion

This paper has theoretically and quantitatively examined how the incidence of an environmental tax depends on how different incomes and preferences of heterogeneous households affect aggregate equilibrium outcomes. To this end, we have developed a simple theoretical Harberger-type model that allows for heterogeneous households, general forms of preferences, differential spending and income patterns, differential factor intensities in production, and general forms of substitution among inputs of capital, labor and pollution.

We have shown that ignoring the household aggregation problem can have important implications for analyzing the incidence of environmental taxes. Our theoretical analysis provides an intuitive way to characterize the degree of household heterogeneity and the impact of heterogeneity on equilibrium outcomes. We have provided conditions under which the household aggregation bias is large and incidence results vary substantially and can be reversed depending on the distribution of households' expenditure and income shares. We have also characterized conditions for which the household aggregation problem is muted. We have calibrated our model based on empirical parameter values to quantitatively assess the household aggregation problem for the example of a U.S. carbon tax. We find that the magnitude of the aggregation bias is non-negligible and that incidence patterns for household income groups may even be affected qualitatively, changing the incidence from "U" to an inverted "U" shape and reversing the sign of the welfare impact for some households. We find that most of the variation in welfare impacts is driven by sources

side impacts. As the aggregation bias on welfare is largely caused by the aggregation bias on the returns to factors of production, it mainly affects the sources of income. Our analysis thus points to the importance of including sources of income impacts for tax incidence analysis. Finally, our findings are robust to extending our model in a number of directions, including alternative revenue recycling schemes, pre-existing taxes, non-separable utility in pollution, labor-leisure choice, and multiple commodities. In fact, we find that the case for the aggregation bias is strengthened rather than weakened.

Beyond the model extensions considered here, and based on the rich literature that followed the original Harberger (1962) article, the analysis could be extended in many additional ways allowing, for example, for imperfect factor mobility, increasing returns to scale, capital accumulation and economic growth, international trade in goods and factors, other factors of production, intermediate inputs, and government transfers. Any such addition to this model would indeed affect the quantitative results, but they are studied elsewhere, and they would not affect the point of this paper that household heterogeneity affects the general equilibrium incidence of environmental taxation.

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## Appendix A. Derivation of equations (10) and (11)

Consider the household demand functions $X = X(p_X, p_Y, M)$ and $Y = Y(p_X, p_Y, M)$, where the household index $h$ is omitted for simplicity. Define the income elasticities of demand of good $X$ and $Y$ as $E_{X,M} = \frac{M}{X}\frac{\partial X}{\partial M}$ and $E_{Y,M} = \frac{M}{Y}\frac{\partial Y}{\partial M}$, respectively. Let $E_{X,p_X} = -\frac{p_X}{X}\frac{\partial X}{\partial p_X}$ and $E_{Y,p_X} = -\frac{p_X}{Y}\frac{\partial Y}{\partial p_X}$ denote the respective own price elasticities of demand. As shown in Hicks & Allen (1934), at the equilibrium solution the following conditions hold: $E_{X,p_X} = \alpha E_{X,M} + (1-\alpha)\sigma$, $E_{Y,p_X} = \alpha E_{Y,M} - \alpha\sigma$, $E_{X,p_Y} = (1-\alpha)E_{X,M} - (1-\alpha)\sigma$, $E_{Y,p_Y} = (1-\alpha)E_{Y,M} + \alpha\sigma$, where $\sigma$ is the elasticity of substitution between clean and dirty consumption in utility.

Using these four conditions, changes in household $h$'s demand for good $X$ and $Y$ given changes in the prices of goods and factor prices can be expressed, respectively, as:

$$\begin{aligned}\hat{X}^h &= \frac{1}{X^h}(p_X\partial_{p_X}X^h\hat{p}_X + p_Y\partial_{p_Y}X^h\hat{p}_Y + M^h\partial_{M^h}X^h\hat{M}^h)\\ &= -E^h_{X,p_X}\hat{p}_X - E^h_{X,p_Y}\hat{p}_Y + E^h_{X,M}\hat{M}^h\\ &= -(\alpha E^h_{X,M} + (1-\alpha)\sigma^h)\hat{p}_X - ((1-\alpha)E^h_{X,M} - (1-\alpha)\sigma^h)\hat{p}_Y + E^h_{X,M}\hat{M}^h\,, \end{aligned} \tag{A.1}$$

and

$$\begin{aligned}\hat{Y}^h &= \frac{1}{Y^h}(p_X\partial_{p_X}\hat{p}_X + p_Y\partial_{p_Y}\hat{p}_Y + M^h\partial_{M^h}\hat{M}^h)\\ &= -E^h_{Y,p_X}\hat{p}_X - E^h_{Y,p_Y}\hat{p}_Y + E^h_{Y,M}\hat{M}^h\\ &= -(\alpha E^h_{Y,M} - \alpha\sigma^h)\hat{p}_X - ((1-\alpha)E^h_{Y,M} + \alpha\sigma^h)\hat{p}_Y + E^h_{Y,M}\hat{M}^h\,. \end{aligned} \tag{A.2}$$

## Appendix B. Derivation of price and pollution changes in general solution (equations (15a)–(15c))

Subtract (8) from (6) and (9) from (7), to obtain:

$$\hat{p}_X = \theta_{XK}\hat{r} + \theta_{XL}\hat{w} \tag{B.1}$$

$$\hat{p}_Y = \theta_{YK}\hat{r} + \theta_{YL}\hat{w} + \theta_{YZ}\hat{\tau}_Z\,. \tag{B.2}$$

Substitute (12) and (13) into (8) and (9):

$$\sum_h \frac{X^h}{X}\hat{X}^h = \theta_{XK}\hat{K}_X + \theta_{XL}\hat{L}_X \tag{B.3}$$

$$\sum_h \frac{Y^h}{Y}\hat{Y}^h = \theta_{YK}\hat{K}_Y + \theta_{YL}\hat{L}_Y + \theta_{YZ}\hat{Z}\,. \tag{B.4}$$

Solve (10) for $\hat{Y}^h$ and insert the result into (B.4). Rearrange to obtain:

$$\frac{1}{Y}\sum_h Y^h(\sigma^h(\hat{p}_Y - \hat{p}_X) + (E^h_{Y,M} - E^h_{X,M})(\alpha^h\hat{p}_X + (1-\alpha^h)\hat{p}_Y - \hat{M}^h)) =$$

$$\sum_h \frac{Y^h}{Y}\hat{X}^h - \theta_{YK}\hat{K}_Y - \theta_{YL}\hat{L}_Y - \theta_{YZ}\hat{Z}\,.$$

From (B.3), insert the following on the right-hand side of the equality: $+0 = \theta_{XK}\hat{K}_X + \theta_{XL}\hat{L}_X - \sum_h \frac{X^h}{X}\hat{X}^h$ and use the fact that $X$ is chosen to be the numéraire, thus yielding:

$$\frac{1}{Y}\sum_h Y^h(\sigma^h(\hat{p}_Y) + (E^h_{Y,M} - E^h_{X,M})((1-\alpha^h)\hat{p}_Y - \hat{M}^h)) =$$

$$\sum_h \frac{M^h}{p_Y Y}(1 - \frac{\alpha^h}{\gamma})\hat{X}^h + \theta_{XK}\hat{K}_X + \theta_{XL}\hat{L}_X - \theta_{YK}\hat{K}_Y - \theta_{YL}\hat{L}_Y - \theta_{YZ}\hat{Z}\,. \tag{B.5}$$

Eliminate $\hat{X}^h$ from equation (B.5) by using equation (11), then insert the explicit expression for the budget change $\hat{M}^h$:

$$\hat{p}_Y\delta = \sum_h \phi^h_L\hat{w} + \sum_h \phi^h_K\hat{r} + \sum_h \phi^h_Z\hat{\tau}_Z$$

$$+\theta_{XK}\hat{K}_X + \theta_{XL}\hat{L}_X - \theta_{YK}\hat{K}_Y - \theta_{YL}\hat{L}_Y + (\sum_h \phi^h_Z - \theta_{YZ})\hat{Z}\,. \tag{B.6}$$

Next, solve equations (1) and (2) for $\hat{K}_X$ and $\hat{L}_X$, and insert them into (B.6). Furthermore, insert equation (B.2) to eliminate $\hat{p}_Y$, thus obtaining:

$$(\sum_h \phi^h_Z - \theta_{YZ})\hat{Z} = (\delta\theta_{YK} - \sum_h \phi^h_K)\hat{r} + (\delta\theta_{YL} - \sum_h \phi^h_L)\hat{w} + (\delta\theta_{YZ} - \sum_h \phi^h_Z)\hat{\tau}_Z$$

$$+\hat{K}_Y(\theta_{XK}\gamma_K + \theta_{YK}) + \hat{L}_Y(\theta_{XL}\gamma_L + \theta_{YL})\,. \tag{B.7}$$

Solve equations (4) and (5) for $\hat{K}_Y$ and $\hat{L}_Y$, and insert them into equation (B.7). This yields:

$$-C\hat{Z} = (-\sum_h \phi^h_K + \theta_{YK}(\delta + \beta_K(e_{KK} - e_{ZK}) + \beta_L(e_{LK} - e_{ZK})))\hat{r}$$

$$+(-\sum_h \phi^h_L + \theta_{YL}(\delta + \beta_K(e_{KL} - e_{ZL}) + \beta_L(e_{LL} - e_{ZL})))\hat{w}$$

$$+(-\sum_h \phi^h_Z + \theta_{YZ}(\delta + \beta_K(e_{KZ} - e_{ZZ}) + \beta_L(e_{LZ} - e_{ZZ})))\hat{\tau}_Z\,. \tag{B.8}$$

Next eliminate $\hat{Z}$. To achieve this, substitute equations (1) and (2) into equation (3), obtaining:

$$-\gamma_K\hat{K}_Y + \gamma_L\hat{L}_Y = \sigma_X(\hat{w} - \hat{r})\,. \tag{B.9}$$

Substituting equations (4) and (5) into (B.9) yields:

$$\sigma_X(\hat{w} - \hat{r}) = (\gamma_L - \gamma_K)\hat{Z} + \theta_{YK}(\gamma_L(e_{LK} - e_{ZK})\hat{r} - \gamma_K(e_{KK} - e_{ZK}))\hat{r}$$

$$\theta_{YL}(\gamma_L(e_{LL} - e_{ZL})\hat{w} - \gamma_K(e_{KL} - e_{ZL}))\hat{w} +$$

$$\theta_{YZ}(\gamma_L(e_{LZ} - e_{ZZ})\hat{\tau}_Z - \gamma_K(e_{KZ} - e_{ZZ}))\hat{\tau}_Z\,. \tag{B.10}$$

Now solve equation (B.10) for $\hat{Z}$ and equate to equation (B.8):

$$\Big((\gamma_K - \gamma_L)(-\sum_h \phi^h_K + \theta_{YK}\delta) + C\sigma_X + \theta_{YK}[-A(e_{KK} - e_{ZK}) + B(e_{LK} - e_{ZK})]\Big)\hat{r}$$

$$+\Big((\gamma_K - \gamma_L)(-\sum_h \phi^h_L + \theta_{YL}\delta) - C\sigma_X + \theta_{YL}[-A(e_{KL} - e_{ZL}) + B(e_{LL} - e_{ZL})]\Big)\hat{w}$$

$$= \Big((\gamma_L - \gamma_K)(-\sum_h \phi^h_Z + \theta_{YZ}\delta) + \theta_{YZ}[-A(e_{ZZ} - e_{KZ}) + B(e_{ZZ} - e_{LZ})]\Big)\hat{\tau}_Z \tag{B.11}$$

Equations (B.1) and (B.11) are two equations in two unknowns, $\hat{r}$ and $\hat{w}$. Solve (B.1) for $\hat{w}$ and substitute into (B.11), solving for $\hat{r}$. Inserting $\hat{r}$ into (B.1) and (B.2) then delivers $\hat{w}$ and $\hat{p}_Y$, respectively. These expressions correspond to (15a)–(15c).

## Appendix C. Special cases and proofs

### *Appendix C.1. Derivation of equations (16a)–(16b)*

In the case of homothetic preference, $E^h_{X,M} = E^h_{Y,M} = 1$. We can therefore simplify some of the terms that reflect the heterogeneity of preferences in (15a)–(15c) as follows:

$$\sum_h \phi^h_Z = \sum_h (1 - \frac{\alpha^h}{\gamma})\xi^h \frac{\tau_Z Z}{p_Y Y} = \frac{\tau_Z Z}{p_X X p_Y Y} \sum_h (\gamma - \alpha^h) M^h = 0\,,$$

$$\sum_h \phi^h_L = \sum_h (1 - \frac{\alpha^h}{\gamma})\frac{w\bar{L}^h}{p_Y Y} = \frac{1}{\gamma p_Y Y} \sum_h (\gamma - \alpha^h) M^h \theta^h_L = -\frac{cov(\alpha^h, \theta^h_L)}{\gamma p_Y Y}\,,$$

$$\sum_h \phi^h_K = \sum_h (1 - \frac{\alpha^h}{\gamma})\frac{r\bar{K}^h}{p_Y Y} = \sum_h (1 - \frac{\alpha^h}{\gamma})\frac{M^h - w\bar{L}^h - \xi^h \tau_Z Z}{p_Y Y} = \frac{cov(\alpha^h, \theta^h_L)}{\gamma p_Y Y}\,,$$

$$\delta \equiv \rho := \frac{1}{p_Y Y} \sum_h (1 - \alpha^h) M^h \Big( \frac{\alpha^h}{\gamma}(\sigma^h - 1) + 1 \Big) \geq \frac{1}{\gamma p_Y Y} \sum_h (1 - \alpha^h) M^h (\gamma - \alpha^h) = \frac{1}{\gamma p_Y Y} \sum_h M^h (\gamma - \alpha^h)^2 \geq 0\,.$$

Inserting these simplified expressions into the system of equations (15a)–(15c) delivers (16a)–(16b).

### *Appendix C.2. Proof of Proposition 4*

If preferences are homothetic and unit-elastic, the change in returns to capital is given by:

$$\hat{r} = -\frac{\theta_{XL}\theta_{YZ}}{D_{H,2}}[A_H(e_{ZZ} - e_{KZ}) - B_H(e_{ZZ} - e_{LZ}) + (\gamma_K - \gamma_L)]\hat{\tau}_Z \tag{C.2}$$

where $D_{H,2} = C_H\sigma_X + e_{KL}[A_H\theta_{YL} + B_H\theta_{YK}] + e_{LZ}[B_H\theta_{XK}(\theta_{YZ} + \theta_{YL}) - A_H\theta_{XK}\theta_{YL}] + e_{KZ}[A_H\theta_{XL}(\theta_{YZ} + \theta_{YK}) - B_H\theta_{XL}\theta_{YK}] + (\gamma_K - \gamma_L)(\theta_{XL}\theta_{YK} - \theta_{XK}\theta_{YL}) - (\gamma_K - \gamma_L)\frac{1}{p_Y Y_\gamma} cov(\alpha^h, \theta^h_L)$.

Since income and expenditure patterns are assumed to be correlated, the last term in $D_{H,2}$—which is the only term depending on household characteristics other than the aggregate ones—is non-zero. Note that on the other hand, for a single consumer, this term equals zero. It thus follows that one can choose Allen elasticities such that the sign is reversed when setting the last term to zero, i.e., when considering the model with a single consumer. An example of such a choice would be $\sigma_X = e_{KZ} = e_{LZ} = 0$ and $-[A_H\theta_{YL} + B_H\theta_{YK}]e_{KL} \in \Big(min\{(\gamma_K - \gamma_L)(\theta_{XL}\theta_{YK} - \theta_{XK}\theta_{YL}) - (\gamma_K - \gamma_L)\frac{1}{p_Y Y_\gamma} cov(\alpha^h, \theta^h_L), (\gamma_K - \gamma_L)(\theta_{XL}\theta_{YK} - \theta_{XK}\theta_{YL})\}, max\{(\gamma_K - \gamma_L)(\theta_{XL}\theta_{YK} - \theta_{XK}\theta_{YL}) - (\gamma_K - \gamma_L)\frac{1}{p_Y Y_\gamma} cov(\alpha^h, \theta^h_L), (\gamma_K - \gamma_L)(\theta_{XL}\theta_{YK} - \theta_{XK}\theta_{YL})\}\Big)$. As the numerator in (C.2) depends only on aggregate household characteristics, its value will be identical in both the heterogeneous and the single consumer case. It thus follows—for the given choice of Allen elasticities—that the signs of $\hat{w}$ and of $\hat{r}$ are reversed as compared to the model with a single household with homothetic preferences.

# Online appendix for "Household heterogeneity, aggregation, and the distributional impacts of environmental taxes"

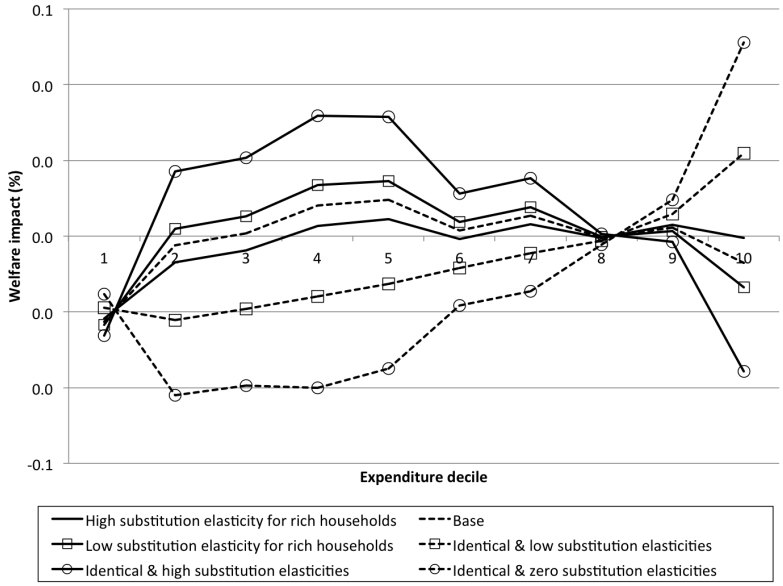
This online appendix contains supplementary analysis (1) for Section 4.3 exploring the distributional impacts among households under alternative revenue recycling schemes and (2) for Section 5 providing additional propositions that are analogous to Propositions 1–6 for multiple extension of the core model (pre-existing taxes on capital and labor, non-separable utility in pollution, labor-leisure choice, an arbitrary number of commodities, and alternative revenue recycling schemes).

## 1. Incidence for alternative carbon tax revenue recycling schemes

This section provides supplementary results for Section 4.3 in the main text focusing on incidence for a U.S. carbon tax for two alternative revenue recycling schemes. A first case assumes that households receive the revenue in proportion to their consumption of the dirty good reflecting concerns about offsetting adverse impacts for poor households (see Figure A.2). A second case considers equally distributing the carbon revenue on a per capita basis (see Figure A.3).

Figure A.2: Welfare impacts ($\Phi^h$) of increased pollution tax across annual expenditure deciles; revenues allocated in proportion to dirty good consumption

(a) Alternative assumptions about household characteristics




(b) Alternative assumptions about production characteristics

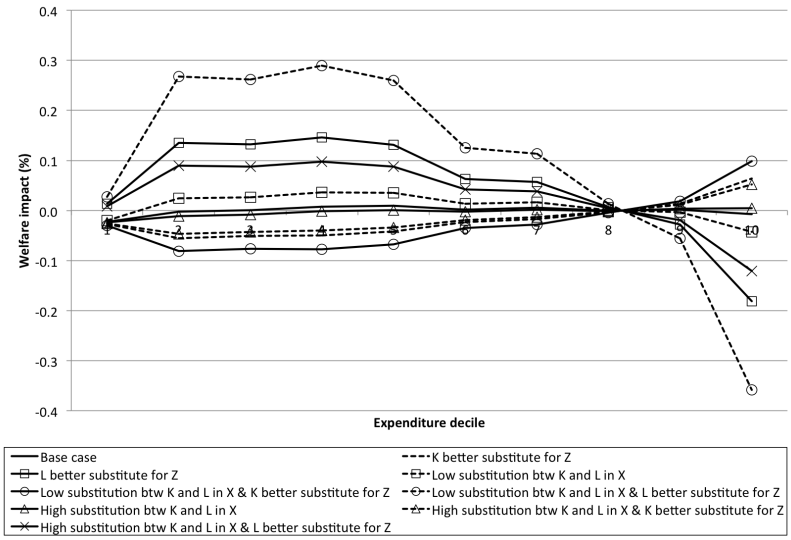
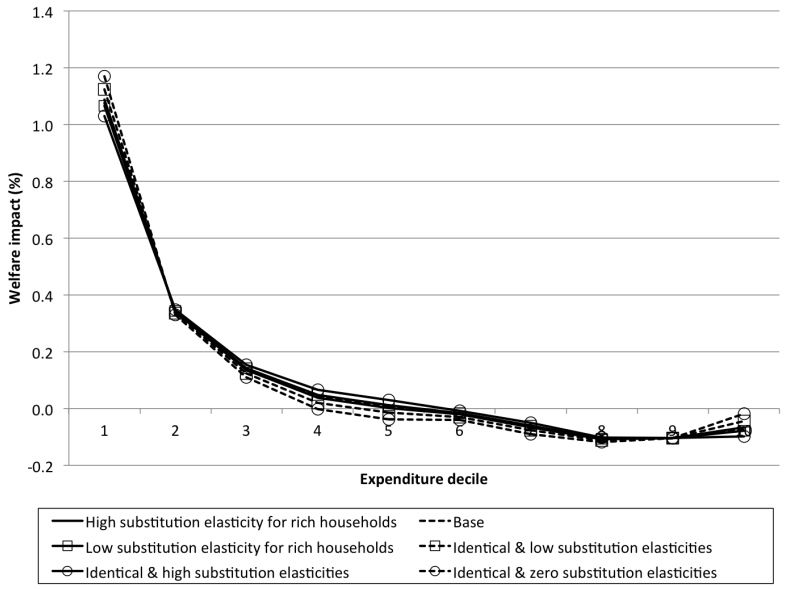
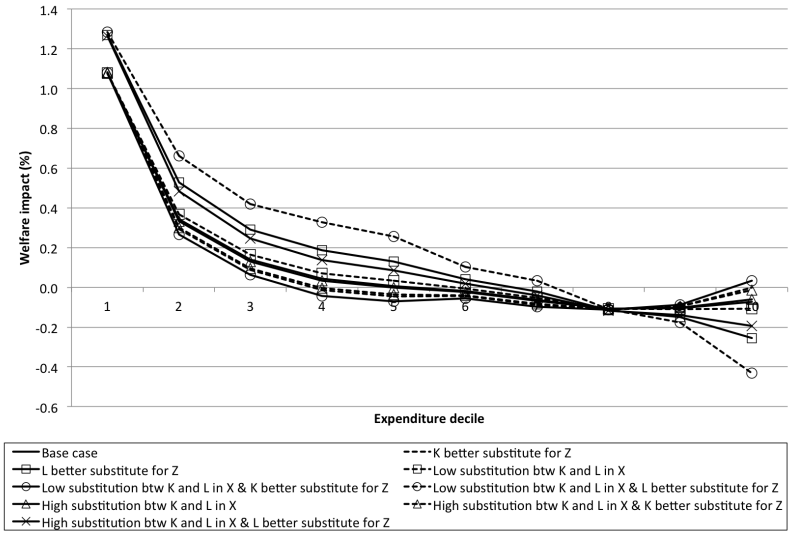


Figure A.3: Welfare impacts ($\Phi^h$) of increased pollution tax across annual expenditure deciles; revenues allocated on per-capita basis

(a) Alternative assumptions about household characteristics




(b) Alternative assumptions about production characteristics



## 2. Extensions

### 2.1. Alternative revenue recycling schemes

Table B.6 reports price changes for alternative revenue recycling schemes. As is evident the price changes for both $\hat{r}$ and $\hat{p}_Y$ are very similar among alternative revenue recycling cases indicating that the impact of household heterogeneity on the equilibrium outcome is largely independent of the way the environmental tax revenue is redistributed.

Table B.6: Price changes for alternative revenue recycling schemes

| $cov_{Base}$ | | $cov_{Low}$ | | | | $cov_{High}$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| $\rho_{Base}$ | | $\rho_{Low}$ | | $\rho_{High}$ | | $\rho_{Low}$ | | $\rho_{High}$ | |
| $\hat{p}_Y$ | $\hat{r}$ | $\hat{p}_Y$ | $\hat{r}$ | $\hat{p}_Y$ | $\hat{r}$ | $\hat{p}_Y$ | $\hat{r}$ | $\hat{p}_Y$ | $\hat{r}$ |
| *Redistribution proportional to income* | | | | | | | | | |
| 7.20 | -0.12 | 7.20 | -0.10 | 7.19 | -0.13 | 7.21 | -0.08 | 7.18 | -0.16 |
| *Redistribution proportional to dirty good consumption* | | | | | | | | | |
| 7.20 | -0.12 | 7.20 | -0.10 | 7.19 | -0.13 | 7.21 | -0.07 | 7.19 | -0.16 |
| *Redistribution on per capita basis* | | | | | | | | | |
| 7.20 | -0.12 | 7.20 | -0.10 | 7.19 | -0.13 | 7.21 | -0.07 | 7.19 | -0.16 |

*Notes:* $\hat{r}$ and $\hat{p}_Y$ are expressed as the percentage change relative to the price level before the pollution tax increase. The results in the table are based on the central case assumptions for production side characteristics.

### 2.2. Pre-existing, non-environmental taxes

Accounting for pre-existing taxes on capital and labor in the benchmark, analogous to Fullerton & Heutel (2007), modifies the cost shares now including tax payments ($\theta_{YK} \equiv \frac{r(1+\tau_K)\bar{K}_Y}{p_Y Y}$, and similarly for $\theta_{YL}$, $\theta_{XK}$ and $\theta_{XL}$) as well as the households' income constraint now including tax revenues as new sources of income:

$$M^h = w\bar{L}^h + r\bar{K}^h + \xi^h \tau_Z Z + \xi^h_K \tau_K K + \xi^h_L \tau_L L\,,$$

where $\tau_K$ and $\tau_L$ denote the *ad valorem* tax rate on capital and labor, respectively, and $\xi^h$, $\xi^h_K$, and $\xi^h_L$ are the shares of total revenue from pollution, capital and labor taxes redistributed to household $h$, respectively.

We find that as long as the revenue from capital and labor taxes is also distributed in proportion to income, there is no additional effect of household heterogeneity on price changes as heterogeneity in terms of both uses and sources side is unchanged. In this case, all Propositions 1–6 remain valid. Distributing capital and labor tax revenue in a non-neutral way will introduce additional heterogeneity on the sources side. In this case, Propositions 1 and 6 still hold true and price changes for $\hat{r}$ and $\hat{p}_Y$ are quantitatively similar (analogously to Section 5.1).

This can be seen as follows. The budget change following a change in the pollution tax is now given by:

$$\hat{M}^h = \hat{w}\frac{w\bar{L}^h}{M^h} + \hat{r}\frac{r\bar{K}^h}{M^h} + \frac{\xi^h \tau_Z Z}{M^h}(\hat{\tau}_Z + \hat{Z}) + \frac{\xi^h_K r\tau_K K}{M^h}(\hat{r} + \hat{K}) + \frac{\xi^h_L w\tau_L L}{M^h}(\hat{w} + \hat{L})\,.$$

Since the total amounts of capital and labor in the economy are assumed to be exogenously given and fixed, it follows that $\hat{K} = 0$ and $\hat{L} = 0$. Hence:

$$\hat{M}^h = \hat{w}\left(\frac{w(\bar{L}^h + \xi^h_L \tau_L L)}{M^h}\right) + \hat{r}\left(\frac{r(\bar{K}^h + \xi^h_K \tau_K K)}{M^h}\right) + \frac{\xi^h \tau_Z Z}{M^h}(\hat{\tau}_Z + \hat{Z})\,.$$

This expression is formally identical to the budget change without capital and labor taxes, with $\bar{L}^h$ replaced by $\bar{L}^h + \xi^h_L \tau_L L$ and $\bar{K}^h$ replaced by $r(\bar{K}^h + \xi^h_K \tau_K K)$. It therefore follows that the model results are identical to (15a)-(15c), with the following changes:

$$\phi^h_L \to \phi^h_{OT,L} \equiv (1 - \frac{\alpha^h}{\gamma})E^h_{X,M}\frac{w(\bar{L}^h + \xi^h_L \tau_L L)}{p_Y Y} + \frac{Y^h}{Y}(E^h_{Y,M} - E^h_{X,M})\frac{w(\bar{L}^h + \xi^h_L \tau_L L)}{M^h} \tag{1.OT}$$

$$\phi_K^h \to \phi_{OT,K}^h \equiv (1 - \frac{\alpha^h}{\gamma})E_{X,M}^h \frac{r(\bar{K}^h + \xi_K^h \tau_K K)}{p_Y Y} + \frac{Y^h}{Y}(E_{Y,M}^h - E_{X,M}^h)\frac{r(\bar{K}^h + \xi_K^h \tau_K K)}{M^h}\,. \tag{2.OT}$$

From the above considerations, it is straightforward to derive the following propositions which are analogous to Propositions 1–6 in the paper. We use the label "OT" to enable comparison between original propositions and the propositions based on the model with pre-existing, other taxes.

**Proposition 1.OT.** *Assume non-zero ad valorem taxes on capital and labor inputs in production. Then Proposition 1 holds.*

*Proof.* Analogous to the proof of Proposition 1. □

**Proposition 2-5.OT.** *Assume non-zero ad-valorem taxes on capital and labor inputs in production. Assume that all tax revenue is redistributed in proportion to benchmark income: $\xi^h = \xi_K^h = \xi_L^h \equiv \frac{M^h}{p_X X + p_Y Y}$. Then Propositions 2–5 hold.*

*Proof.* With $\phi_{OT,L}^h$ as in (1.OT) and $\phi_{OT,K}^h$ as in (2.OT) it follows that $\sum_h \phi_{OT,L}^h = \sum_h (1 - \frac{\alpha^h}{\gamma})\frac{w\bar{L}^h}{p_Y Y}$ and $\sum_h \phi_{OT,K}^h = \sum_h (1 - \frac{\alpha^h}{\gamma})\frac{r\bar{K}^h}{p_Y Y}$. These expressions are identical to the case with homothetic preferences and zero taxes on capital and labor. Hence the price changes are also identical. □

**Proposition 6.OT.** *Assume non-zero ad valorem taxes on capital and labor inputs in production. Then Proposition 6 holds.*

*Proof.* Since for identical households the $\phi_{OT}^h$ expressions are zero, it follows that the taxes on capital and labor have no impact on the results. □

## *2.3. Non-separable utility in pollution*

With non-separable utility, consumption of clean and dirty goods in general depends on the level of pollution: $X^h = X^h(p_X, p_Y, M^h, Z)$ and $Y^h = Y^h(p_X, p_Y, M^h, Z)$. Equations (A.1) and (A.2) then become

$$\hat{X}^h = -(\alpha E_{X,M}^h + (1-\alpha)\sigma^h)\hat{p}_X - ((1-\alpha)E_{X,M}^h - (1-\alpha)\sigma^h)\hat{p}_Y + E_{X,M}^h \hat{M}^h + E_{X,Z}^h \hat{Z}$$

$$\hat{Y}^h = -(\alpha E_{Y,M}^h - \alpha\sigma^h)\hat{p}_X - ((1-\alpha)E_{Y,M}^h + \alpha\sigma^h)\hat{p}_Y + E_{Y,M}^h \hat{M}^h + E_{Y,Z}^h \hat{Z}\,,$$

where $E_{X,Z}^h \equiv \frac{Z}{X^h}\partial_Z X^h$ and $E_{Y,Z}^h \equiv \frac{Z}{Y^h}\partial_Z Y^h$ can, respectively, be interpreted as the pollution elasticity of clean and dirty consumption. Equations (10) and (11) can then be written as:

$$\hat{X}^h - \hat{Y}^h = \sigma^h(\hat{p}_Y - \hat{p}_X) + (E_{Y,M}^h - E_{X,M}^h)(\alpha^h \hat{p}_X + (1-\alpha^h)\hat{p}_Y - \hat{M}^h) + (E_{X,Z}^h - E_{Y,Z}^h)\hat{Z} \tag{1.NS}$$

$$\hat{X}^h = -(\alpha E_{X,M}^h + (1-\alpha)\sigma^h)\hat{p}_X - ((1-\alpha)E_{X,M}^h - (1-\alpha)\sigma^h)\hat{p}_Y + E_{X,M}^h \hat{M}^h + E_{X,Z}^h \hat{Z}\,. \tag{2.NS}$$

In order analyze our propositions, we first need to derive the price changes for the case with non-separable utility in pollution. They turn out to be identical to those for separable preferences, up to the coefficients $A$, $B$ and $C$, which become

$$A \to A_{NS} \equiv A - \gamma_K \sum_h \phi_{NS}^h \qquad B \to B_{NS} \equiv B - \gamma_L \sum_h \phi_{NS}^h \qquad C \to C_{NS} \equiv C - \sum_h \phi_{NS}^h\,, \tag{3.NS}$$

with $\phi_{NS}^h = \frac{M^h}{p_Y Y}(1 - \frac{\alpha^h}{\gamma})E_{X,Z}^h + \frac{Y^h}{Y}(E_{Y,Z}^h - E_{X,Z}^h)$. The next subsection derives this result.

### *2.3.1. Derivation of price changes for non-separable utility in pollution*

Solve (1.NS) for $\hat{Y}^h$ and insert the result into (B.4). Rearrange to obtain:

$$\frac{1}{Y}\sum_h Y^h(\sigma^h(\hat{p}_Y - \hat{p}_X) + (E_{Y,M}^h - E_{X,M}^h)(\alpha^h \hat{p}_X + (1-\alpha^h)\hat{p}_Y - \hat{M}^h) + (E_{X,Z}^h - E_{Y,Z}^h)\hat{Z}) =$$

$$\sum_h \frac{Y^h}{Y}\hat{X}^h - \theta_{YK}\hat{K}_Y - \theta_{YL}\hat{L}_Y - \theta_{YZ}\hat{Z}\,.$$

From (B.3) insert the following on the right-hand side of the equality: $+0 = \theta_{XK}\hat{K}_X + \theta_{XL}\hat{L}_X - \sum_h \frac{X^h}{X}\hat{X}^h$ and use the fact that $X$ is the numéraire, thus yielding:

$$\frac{1}{Y}\sum_h Y^h(\sigma^h(\hat{p}_Y) + (E_{Y,M}^h - E_{X,M}^h)((1-\alpha^h)\hat{p}_Y - \hat{M}^h) + (E_{X,Z}^h - E_{Y,Z}^h)\hat{Z}) =$$

$$\sum_h \frac{M^h}{p_Y Y}(1 - \frac{\alpha^h}{\gamma})\hat{X}^h + \theta_{XK}\hat{K}_X + \theta_{XL}\hat{L}_X - \theta_{YK}\hat{K}_Y - \theta_{YL}\hat{L}_Y - \theta_{YZ}\hat{Z}\,. \tag{4.NS}$$

Eliminate $\hat{X}^h$ from equation (4.NS) by using equation (2.NS), then insert the explicit expression for the budget change $\hat{M}^h$:

$$\hat{p}_Y\delta = \sum_h \phi_L^h\hat{w} + \sum_h \phi_K^h\hat{r} + \sum_h \phi_Z^h\hat{\tau}_Z\,,$$
$$+\,\theta_{XK}\hat{K}_X + \theta_{XL}\hat{L}_X - \theta_{YK}\hat{K}_Y - \theta_{YL}\hat{L}_Y + \Big(\sum_h(\phi_{NS}^h + \phi_Z^h) - \theta_{YZ}\Big)\hat{Z}\,. \tag{5.NS}$$

with $\phi_{NS}^h = \frac{M^h}{p_Y Y}(1-\frac{\alpha^h}{\gamma})E_{X,Z}^h + \frac{\gamma^h}{Y}(E_{Y,Z}^h - E_{X,Z}^h)$. Next, solve equations (1) and (2) for $\hat{K}_X$ and $\hat{L}_X$, and insert them into (5.NS). Furthermore, insert equation (B.2) to eliminate $\hat{p}_Y$, thus obtaining:

$$(\sum_h(\phi_{NS}^h + \phi_Z^h) - \theta_{YZ})\hat{Z} = (\delta\theta_{YK} - \sum_h \phi_K^h)\hat{r} + (\delta\theta_{YL} - \sum_h \phi_L^h)\hat{w} + (\delta\theta_{YZ} - \sum_h \phi_Z^h)\hat{\tau}_Z$$
$$+\,\hat{K}_Y(\theta_{XK}\gamma_K + \theta_{YK}) + \hat{L}_Y(\theta_{XL}\gamma_L + \theta_{YL})\,. \tag{6.NS}$$

Solve equations (4) and (5) for $\hat{K}_Y$ and $\hat{L}_Y$, and insert them into equation (6.NS). This yields:

$$-C_{NS}\hat{Z} = (-\sum_h \phi_K^h + \theta_{YK}(\delta + \beta_K(e_{KK} - e_{ZK}) + \beta_L(e_{LK} - e_{ZK})))\hat{r}$$
$$+\,(-\sum_h \phi_L^h + \theta_{YL}(\delta + \beta_K(e_{KL} - e_{ZL}) + \beta_L(e_{LL} - e_{ZL})))\hat{w}$$
$$+\,(-\sum_h \phi_Z^h + \theta_{YZ}(\delta + \beta_K(e_{KZ} - e_{ZZ}) + \beta_L(e_{LZ} - e_{ZZ})))\hat{\tau}_Z\,, \tag{7.NS}$$

with $C_{NS} = \beta_K + \beta_L + \theta_{YZ} - \sum_h(\phi_Z^h + \phi_{NS}^h)$. Next eliminate $\hat{Z}$. To achieve this, substitute equations (1) and (2) into (3), obtaining:

$$-\gamma_K\hat{K}_Y + \gamma_L\hat{L}_Y = \sigma_X(\hat{w} - \hat{r})\,. \tag{8.NS}$$

Substituting equations (4) and (5) into (8.NS) yields:

$$\sigma_X(\hat{w} - \hat{r}) = (\gamma_L - \gamma_K)\hat{Z} + \theta_{YK}(\gamma_L(e_{LK} - e_{ZK})\hat{r} - \gamma_K(e_{KK} - e_{ZK}))\hat{r}$$
$$\theta_{YL}(\gamma_L(e_{LL} - e_{ZL})\hat{w} - \gamma_K(e_{KL} - e_{ZL}))\hat{w} +$$
$$\theta_{YZ}(\gamma_L(e_{LZ} - e_{ZZ})\hat{\tau}_Z - \gamma_K(e_{KZ} - e_{ZZ}))\hat{\tau}_Z\,. \tag{9.NS}$$

Now solve equation (9.NS) for $\hat{Z}$ and equate to equation (7.NS):

$$\Big((\gamma_K - \gamma_L)(-\sum_h \phi_K^h + \theta_{YK}\delta) + C_{NS}\sigma_X + \theta_{YK}[-A_{NS}(e_{KK} - e_{ZK}) + B_{NS}(e_{LK} - e_{ZK})]\Big)\hat{r}$$
$$+\,\Big((\gamma_K - \gamma_L)(-\sum_h \phi_L^h + \theta_{YL}\delta) - C_{NS}\sigma_X + \theta_{YL}[-A_{NS}(e_{KL} - e_{ZL}) + B_{NS}(e_{LL} - e_{ZL})]\Big)\hat{w}$$
$$=\,\Big((\gamma_L - \gamma_K)(-\sum_h \phi_Z^h + \theta_{YZ}\delta) + \theta_{YZ}[-A_{NS}(e_{ZZ} - e_{KZ}) + B_{NS}(e_{ZZ} - e_{LZ})]\Big)\hat{\tau}_Z\,, \tag{10.NS}$$

with $A_{NS} \equiv -(\gamma_K - \gamma_L)\beta_K + C_{NS}\gamma_K = A_P - \gamma_K\sum_h \phi_{NS}^h$ and $B_{NS} \equiv (\gamma_K - \gamma_L)\beta_L + C_{NS}\gamma_L = B_P - \gamma_L\sum_h \phi_{NS}^h$. (10.NS) is formally identical to (B.11), with the coefficients $A$, $B$ and $C$ replaced by $A_{NS}$, $B_{NS}$ and $C_{NS}$. It therefore follows that price changes will also be identical up to the value of these coefficients.

### *2.3.2. Results*

As can be seen from above, the change in the pollution level following a pollution tax increase can affect price changes and hence the equilibrium behavior of households. This introduces an additional dimension of heterogeneity to the extent that households have different preferences about pollution.

From the above considerations, it is straightforward to derive the following propositions which are analogous to Propositions 1–6 in the paper. We use the label "NS" to enable comparison between the original propositions and the propositions based on the model with non-separable pollution.

Equal factor intensities in production

**Proposition 1.NS.** *Assume non-separable utility from pollution. Then Proposition 1 holds.*

*Proof.* If $\gamma_K = \gamma_L$, then from the proof of Proposition 1, we know that it then follows that $A = B = \gamma_K C$. This implies that $A - \gamma_K \sum_h \phi^h_{NS} = B - \gamma_L \sum_h \phi^h_{NS} = \gamma_K(C - \sum_h \phi^h_{NS})$ which in turn is equivalent to $A_{NS} = B_{NS} = \gamma_K C_{NS}$. It then follows that all the terms containing household characteristics in the expressions for the price changes drop out. □

## Heterogeneous households with homothetic preferences

In this paragraph, assume that the pollution tax revenue is returned to households in proportion to income. Now define the *effective pollution elasticity of clean consumption* $\Xi \equiv \sum_h \frac{X^h}{X} E^h_{X,Z}$. It then follows that:

$$\sum_h \phi^h_{NS} = -\frac{1}{1-\gamma}\Xi,$$

using the fact that, from the budget constraint, the following holds: $E^h_{Y,Z} = -\frac{\alpha^h}{1-\alpha^h} E^h_{X,Z}$. Using this, the analogues to Propositions 2–5 hold.

**Proposition 2.NS.** *Assume non-separable utility from pollution. It then follows that, in addition to $cov(\alpha^h, \theta^h_L)$ and $\rho$, Proposition 2 is extended to include the effective pollution elasticity of clean consumption, $\Xi$.*

*Proof.* The proof is analogous to the one for Proposition 2 accounting for the new term $\Xi$. □

**Proposition 3.NS.** *Assume non-separable utility from pollution. Then, in addition to $\gamma$ and $\rho$, the single household with homothetic preferences in Proposition 3 is also characterized by a pollution elasticity of clean consumption equal to the effective elasticity $\Xi$.*

*Proof.* For the assumptions in Proposition 3 it is straightforward to see that price changes are identical to those derived for an economy with a single consumer with homothetic preferences, clean good expenditure share $\gamma$, elasticity of substitution in utility $\rho$, and pollution elasticity of clean consumption $\Xi$. □

**Proposition 4.NS.** *Assume non-separable utility from pollution. Then the analogue of Proposition (4) holds, with the single consumer being characterized by a pollution elasticity of clean consumption given by the effective pollution elasticity $\Xi$.*

*Proof.* The proof of Proposition 4.NS carries through analogously to the one for Proposition (4). However, since now $A_{NS,H}\theta_{YL} + B_{NS,H}\theta_{YK}$ could in principle be equal to zero, it is necessary to additionally show that the other two coefficients multiplying the $e$'s in $D_{NS,H,2}$ cannot also be zero. It then follows that, if $A_{NS,H}\theta_{YL} + B_{NS,H}\theta_{YK} = 0$, then one can construct the example analogously, based on $e_{LZ}$ or $e_{KZ}$. To show this, assume that $A_{NS,H}\theta_{YL} + B_{NS,H}\theta_{YK} = 0$. It follows that $B_{NS,H}\theta_{XK}(\theta_{YZ}+\theta_{YL}) - A_{NS,H}\theta_{XK}\theta_{YL} = \theta_{XK}B_{NS,H}$ and $A_{NS,H}\theta_{XL}(\theta_{YZ}+\theta_{YK}) - B_{NS,H}\theta_{XL}\theta_{YK} = \theta_{XL}A_{NS,H}$, therefore in order to be both zero, the following must hold: $A_{NS,H} = 0$ and $B_{NS,H} = 0$. This in turn implies $A_H = \gamma_K \sum_h \phi^h_{NS}$ and $B_H = \gamma_L \sum_h \phi^h_{NS}$, which in turn implies $\frac{A_H}{\gamma_K} = \frac{B_H}{\gamma_L}$. Inserting the explicit expressions for $A_H$ and $B_H$ delivers: $\gamma_L\gamma_L\beta_K + \gamma_L\gamma_K\beta_L = \gamma_K\gamma_K\beta_L + \gamma_K\gamma_L\beta_K \Leftrightarrow (\gamma_L - \gamma_K)\frac{\beta_K}{\gamma_K} = (\gamma_K - \gamma_L)\frac{\beta_L}{\gamma_L} \Leftrightarrow (\gamma_L - \gamma_K)(\theta_{XK} + \frac{\theta_{YK}}{\gamma_K}) = (\gamma_K - \gamma_L)(\theta_{XL} + \frac{\theta_{YL}}{\gamma_L})$. Since we are assuming that $\gamma_K \neq \gamma_L$, it follows that the last equality is equivalent to $(\theta_{XK} + \frac{\theta_{YK}}{\gamma_K}) = -(\theta_{XL} + \frac{\theta_{YL}}{\gamma_L})$, which is a contraction. □

**Proposition 5.NS.** *Assume non-separable utility from pollution. Then Proposition 5 holds.*

*Proof.* Since only $A$, $B$ and $C$ are affected by this model extension, and since they all multiply with elasticities that are zero, it follows that price changes in this special case are identical to those in the original model. □

## Identical households with non-homothetic preferences

**Proposition 6.NS.** *Assume non-separable utility from pollution. Then Proposition (6) holds, with the coefficients in Condition 1 are generalised as follows: $A_{ID} \to A_{ID} + \frac{\gamma_K}{1-\gamma}E_{X,Z}$, $B_{ID} \to B_{ID} + \frac{\gamma_L}{1-\gamma}E_{X,Z}$ and $C_{ID} \to C_{ID} + \frac{1}{1-\gamma}E_{X,Z}$.*

*Proof.* This follows from equation (3.NS), using the fact that $E_{Y,Z} = -\frac{\gamma}{1-\gamma}E_{X,Z}$. □

Proposition 6.NS illustrates that while extending the model to allow for non-separability of utility in pollution can affect the quantitative parameter values at which the model behavior switches, it does not change the qualitative behavior of the model.

## 2.4. Labor-leisure choice

An important dimension along which households can differ is their valuation of leisure time resulting in differences with respect to the elasticity of labor supply. Incorporating endogenous labor supply significantly enhances the complexity of studying the impact of household heterogeneity of equilibrium outcomes as it affects both how income is earned and spent. To keep the theoretical analysis tractable, we restrict our attention to Cobb-Douglas utility:

$$U^h(X^h, Y^h, l^h) = X^{\eta_X^h} Y^{\eta_Y^h} l^{1-\eta_X^h-\eta_Y^h},$$

where income is given by $M^h = w(T^h - l^h) + r\bar{K}^h + \tau_Z Z\xi^h$. $T^h$ represents household $h$'s endowment of productive time.[24] We further assume that in the benchmark, households dedicate an equal fraction of their productive time to leisure: $\frac{l^h}{T^h} \equiv \mathcal{L}$, $\forall h$.

Using the first-order conditions, the demand functions are:

$$X^h = \frac{\eta_X^h}{p_X}(wT^h + r\bar{K}^h + \xi^h\tau_Z Z) \qquad Y^h = \frac{\eta_Y^h}{p_Y}(wT^h + r\bar{K}^h + \xi^h\tau_Z Z) \qquad l^h = \frac{1-\eta_X^h-\eta_Y^h}{w}(wT^h + r\bar{K}^h + \xi^h\tau_Z Z)\,.$$

It then follows that

$$\hat{X}^h = -\hat{p}_X + \hat{w}\frac{wT^h}{M^h + wl^h} + \hat{r}\frac{r\bar{K}^h}{M^h + wl^h} + \frac{\xi^h\tau_Z Z}{M^h + wl^h}(\hat{\tau}_Z + \hat{Z}) \tag{1.LL}$$

$$\hat{Y}^h = -\hat{p}_Y + \hat{w}\frac{wT^h}{M^h + wl^h} + \hat{r}\frac{r\bar{K}^h}{M^h + wl^h} + \frac{\xi^h\tau_Z Z}{M^h + wl^h}(\hat{\tau}_Z + \hat{Z})$$

$$\hat{l}^h = -\hat{w} + \hat{w}\frac{wT^h}{M^h + wl^h} + \hat{r}\frac{r\bar{K}^h}{M^h + wl^h} + \frac{\xi^h\tau_Z Z}{M^h + wl^h}(\hat{\tau}_Z + \hat{Z})\,. \tag{2.LL}$$

We now need to modify our model equations as follows: (11) is replaced by (1.LL), and (2) is replaced by:

$$\hat{L}_X\frac{L_X}{L} + \hat{L}_Y\frac{L_Y}{L} = -\frac{1}{L}\sum_h l^h\hat{l}^h\,. \tag{3.LL}$$

In order analyze our propositions, we first need to derive the price changes for the case with labor-leisure choice. They turn out to be identical to those for a model with exogenous labor supply, up to the value of the $\phi$ parameters, which are extended as follows:

$$\phi_K^h \to \phi_{LL,K}^h \equiv \left(1 - \frac{wl^h}{wT^h + r\bar{K}^h + \tau_Z Z\xi^h}\right)\phi_K^h - \frac{wl^h}{wT^h + r\bar{K}^h + \tau_Z Z\xi^h}\frac{r\bar{K}^h}{p_X X}$$

$$\phi_L^h \to \phi_{LL,L}^h \equiv \left(1 - \frac{wl^h}{wT^h + r\bar{K}^h + \tau_Z Z\xi^h}\right)\frac{wT^h}{w(T^h - l^h)}\phi_L^h + \frac{wl^h}{p_X X}\frac{r\bar{K}^h + \tau_Z Z\xi^h}{wT^h + r\bar{K}^h + \tau_Z Z\xi^h}$$

$$\phi_Z^h \to \phi_{LL,Z}^h \equiv \left(1 - \frac{wl^h}{wT^h + r\bar{K}^h + \tau_Z Z\xi^h}\right)\phi_Z^h - \frac{wl^h}{p_X X}\frac{\xi^h\tau_Z Z}{wT^h + r\bar{K}^h + \tau_Z Z\xi^h}\,.$$

The next subsection derives these results.

### 2.4.1. Derivations for labor-leisure choice

Price changes

Up until equation (B.5), the derivation is analogous, yielding the following expression:

$$\hat{p}_Y = \sum_h \frac{M^h}{p_Y Y}(1 - \frac{\alpha^h}{\gamma})\hat{X}^h + \theta_{XK}\hat{K}_X + \theta_{XL}\hat{L}_X - \theta_{YK}\hat{K}_Y - \theta_{YL}\hat{L}_Y - \theta_{YZ}\hat{Z}\,. \tag{4.LL}$$

Eliminate $\hat{X}^h$ from equation (4.LL) by using equation (1.LL) :

$$\hat{p}_Y = \hat{w}\sum_h \frac{M^h}{M^h + wl^h}\frac{wT}{wL^h}\phi_L^h + \hat{r}\sum_h \frac{M^h}{M^h + wl^h}\phi_K^h + \hat{\tau}_Z\sum_h \frac{M^h}{M^h + wl^h}\phi_Z^h$$

$$+ \theta_{XK}\hat{K}_X + \theta_{XL}\hat{L}_X - \theta_{YK}\hat{K}_Y - \theta_{YL}\hat{L}_Y + \hat{Z}\Big(\sum_h \frac{M^h}{M^h + wl^h}\phi_Z^h - \theta_{YZ}\Big), \tag{5.LL}$$

[24] Differences in $T^h$ could be viewed as reflecting differences in labor productivity across households.

where $L^h = T^h - l^h$ and the $\phi$s are evaluated for homothetic preferences. Next, solve equations (1) and (3.LL) for $\hat{K}_X$ and $\hat{L}_X$, and insert them into (5.LL). Furthermore, insert equation (B.2) to eliminate $\hat{p}_Y$, thus obtaining:

$$\Big(\sum_h \frac{M^h}{M^h+wl^h}\phi_Z^h - \theta_{YZ}\Big)\hat{Z} = \Big(\theta_{YK} - \sum_h \frac{M^h}{M^h+wl^h}\phi_K^h\Big)\hat{r} + \Big(\theta_{YL} - \sum_h \frac{M^h}{M^h+wl^h}\frac{wT^h}{wL^h}\phi_L^h\Big)\hat{w} + \Big(\theta_{YZ} - \sum_h \frac{M^h}{M^h+wl^h}\phi_Z^h\Big)\hat{\tau}_Z$$
$$+ \hat{K}_Y(\theta_{XK}\gamma_K + \theta_{YK}) + \hat{L}_Y(\theta_{XL}\gamma_L + \theta_{YL}) + \frac{\theta_{XL}}{L_X}\sum_h l^h\hat{l}^h\,. \tag{6.LL}$$

Now eliminate $\hat{l}^h$ by substituting (2.LL) into (6.LL) :

$$\Big(\sum_h \phi_{LL,Z}^h - \theta_{YZ}\Big)\hat{Z} = \Big(\theta_{YK} - \sum_h \phi_{LL,K}^h\Big)\hat{r} + \Big(\theta_{YL} - \sum_h \phi_{LL,L}^h\Big)\hat{w} + \Big(\theta_{YZ} - \sum_h \phi_{LL,Z}^h\Big)\hat{\tau}_Z +$$
$$+ \hat{K}_Y(\theta_{XK}\gamma_K + \theta_{YK}) + \hat{L}_Y(\theta_{XL}\gamma_L + \theta_{YL})\,, \tag{7.LL}$$

where $\phi_{LL,K}^h \equiv \frac{M^h}{M^h+wl^h}\phi_K^h - \frac{wl^h}{p_XX}\frac{r\bar{K}^h}{M^h+wl^h}$, $\phi_{LL,L}^h \equiv \frac{M^h}{M^h+wl^h}\frac{wT^h}{wL^h}\phi_L^h + \frac{wl^h}{p_XX}\frac{r\bar{K}^h+\tau_ZZ\xi^h}{M^h+wl^h}$ and $\phi_{LL,Z}^h \equiv \frac{M^h}{M^h+wl^h}\phi_Z^h - \frac{wl^h}{p_XX}\frac{\xi^h\tau_ZZ}{M^h+wl^h}$. Equation (7.LL) is formally identical to (B.7), with the exception of the $\phi$s. It therefore follows that the resulting price changes are also formally identical to 15a–(15c) up to the value of the parameters $\phi$.

## Household parameters

Using the fact that $cov(\alpha^h,\beta^h) = \sum_h M^h\alpha^h\beta^h - M\alpha\beta$ and $\sum_h M^h\alpha^h\beta^h\gamma^h = cov(\alpha^h,\beta^h,\gamma^h) + \alpha cov(\beta^h,\gamma^h) + \beta cov(\alpha^h,\gamma^h) + \gamma cov(\alpha^h,\beta^h) + M\alpha\beta\gamma$.

$$\sum_h \phi_{LL,L}^h = \sum_h (1-\lambda^h)\frac{wT}{w(T-l^h)}\Big(1-\frac{\alpha^h}{\gamma}\Big)\frac{w\bar{L}^h}{p_YY} + \frac{cov(\lambda^h,\theta_K^h)}{p_XX} + \frac{\lambda\theta_K}{\gamma} + \frac{\tau_ZZ}{p_XX}\lambda =$$
$$\frac{1}{1-\mathcal{L}}\frac{1}{\gamma p_YY}\sum_h M^h\theta_L^h\lambda^h\alpha^h - \frac{1}{1-\mathcal{L}}\frac{cov(\theta_L^h,\alpha^h)}{\gamma p_YY} - \frac{1}{1-\mathcal{L}}\frac{cov(\theta_L^h,\lambda^h)}{p_YY} + \frac{cov(\lambda^h,\theta_K^h)}{p_XX}$$
$$-\frac{1}{1-\mathcal{L}}\frac{(p_XX+p_YY)}{p_YY}\theta_L\lambda + \frac{\lambda}{\gamma}(1-\theta_L)\,,$$

hence

$$\sum_h \phi_{LL,L}^h = \frac{cov(\theta_L^h,\lambda^h,\alpha^h) + \theta_L cov(\lambda^h,\alpha^h) + (\lambda-1)cov(\theta_L^h,\alpha^h)}{(1-\mathcal{L})\gamma p_YY} - \frac{cov(\lambda^h,\theta_L^h)}{p_XX} - \frac{\lambda(\theta_L-1)}{\gamma}\,.$$

Analogously:

$$\sum_h \phi_{LL,K}^h = \frac{-cov(\theta_L^h,\lambda^h,\alpha^h) + \theta_K cov(\lambda^h,\alpha^h) - (\lambda-1)cov(\theta_L^h,\alpha^h)}{\gamma p_YY} + \frac{cov(\lambda^h,\theta_L^h)}{p_XX} - \frac{\lambda\theta_K}{\gamma}\,.$$

Using the above expressions consider the $\phi$ parameters as they appear in the expressions for the price changes:

$$\sum_h \phi_{LL,Z}^h = \frac{\tau_ZZ}{p_XXp_YY}cov(\lambda^h,\alpha^h) - \frac{\tau_ZZ}{p_XX}\lambda \tag{8.LL}$$

and

$$\theta_{XL}\sum_h \phi_{LL,K}^h - \theta_{XK}\sum_h \phi_{LL,L}^h =$$
$$-\frac{1}{\gamma p_YY}\frac{1}{(1-\mathcal{L})}\Big((1-\mathcal{L}\theta_{XL})cov(\theta_L^h,\lambda^h,\alpha^h) + (\theta_K(\mathcal{L}\theta_{XL}-1) + \theta_{XK}(1-\theta_Z))cov(\lambda^h,\alpha^h) + (\lambda-1)(1-\mathcal{L}\theta_{XL})cov(\theta_L^h,\alpha^h)\Big)$$
$$+\frac{cov(\lambda^h,\theta_L^h)}{p_XX} - \frac{\lambda}{\gamma}(\theta_K + \theta_Z\theta_{XK})\,, \tag{9.LL}$$

where $\lambda^h \equiv \frac{wl^h}{wT^h+r\bar{K}^h+\tau_ZZ\xi^h}$, $\lambda \equiv \frac{1}{\sum_{h'}M^{h'}}\sum_h \lambda^hM^h = \frac{1}{p_XX+p_YY}\sum_h \lambda^hM^h$, and the covariance of three variables is defined analogously to the definition for two variables in our paper. Note that, in the following, we will refer to $\lambda^h$ as household $h$'s *expenditure share on leisure*.

### 2.4.2. Results

We find that results are mainly similar with new parameters summarizing the additional channels of household heterogeneity as well as the aggregate impact of labor-leisure choice on the general equilibrium. Proposition 1 is identical, Proposition 3 is analogous with presence of a term that reflects the impact of average expenditure share of leisure on aggregate outcomes, and Proposition 2 is analogous accounting in addition for interactions between leisure choice and expenditure and income patterns. Propositions 4 and 5 are analogous, too. For the special case of Cobb-Douglas utility, we thus find that effect of household heterogeneity is similar to the case without labor-leisure choice; where it differs it can be understood in terms of additional terms reflecting interactions between the various types of heterogeneity (labor-leisure choice, expenditures and income patterns). Whether or not the aggregation bias is quantitatively smaller or larger depends on specific parametrization. The following subsection provide detailed analysis supporting the above statements. We use the label "LL" to enable comparison between the original propositions and the propositions based on the model with leisure.

#### Equal factor intensities in production

**Proposition 1.LL.** *Assume the model with labor-leisure choice and Cobb-Douglas utility. Then Proposition (1) holds.*

*Proof.* If $\gamma_K = \gamma_L$, then from the proof of Proposition (1), we know that it then follows that $A = B = \gamma_K C$. This implies that $A_{LL} = B_{LL} = \gamma_K C_{LL}$. It then follows that all the terms containing household characteristics in the expressions for the price changes drop out. □

#### Heterogeneous households with homothetic preferences

**Proposition 2.LL.** *Assume the model with labor-leisure choice, equal benchmark share of leisure time across households ( $\frac{l^h}{L^h} = \mathcal{L}$, $\forall h$ ), and Cobb-Douglas utility. Then, in addition to $cov(\alpha^h, \theta_L^h)$ and $\rho$, Proposition 2 is extended to include $cov(\alpha^h, \lambda^h)$, $cov(\theta_L^h, \lambda^h)$ and $cov(\theta_L^h, \lambda^h, \alpha^h)$.*

*Proof.* Equations (8.LL) and (9.LL). □

**Proposition 3.LL.** *Assume the model with labor-leisure choice, equal benchmark share of leisure time across households ( $\frac{l^h}{L^h} = \mathcal{L}$, $\forall h$ ), and Cobb-Douglas utility. If income shares are identical across households ($\theta_L^h = \theta_L$, $\forall h$ ), then output and factor price changes are identical to those for a single household characterized by Cobb-Douglas preferences, clean good expenditure share $\gamma$, an elasticity of substitution between clean and dirty goods in utility equal to the effective elasticity $\rho$, and expenditure share on leisure given by the income-weighted average of the shares across households, $\lambda$.*

*Proof.* Equations (8.LL) and (9.LL). Consider furthermore the following: $\lambda^h = \frac{wl^h}{M^h + wl^h} = \frac{wl^h}{M^h}\frac{M^h}{M^h + wl^h} = \frac{wl^h}{M^h}(1-\lambda^h) = \frac{\mathcal{L}}{1-\mathcal{L}}\frac{wL^h}{M^h}(1-\lambda^h)$ using $l^h = \frac{\mathcal{L}}{1-\mathcal{L}}L^h$. Rewrite the above equality, therefore obtaining: $\lambda^h = \frac{\theta_L^h}{(\frac{1-\mathcal{L}}{\mathcal{L}} + \theta_L^h)}$. It therefore follows that, in the case where labor income shares are identical across households ($\theta_L^h = \theta_L$, $\forall h$), the same holds for the $\lambda^h$s, thus implying $cov(\lambda^h, \alpha^h) = 0$. □

**Proposition 4.LL.** *Assume the model with labor-leisure choice, equal benchmark leisure time across households ( $\frac{l^h}{L^h} = \mathcal{L}$, $\forall h$ ), and Cobb-Douglas utility. Assume different factor intensities (i.e., $\gamma_K \neq \gamma_L$), constant expenditure shares across households (i.e., $\alpha^h = \gamma$, $\forall h$) and non-zero covariance between labor income shares and expenditure shares on leisure (i.e., $cov(\lambda^h, \theta_L^h) \neq 0$). Then, for any observed consumption and production decisions before the tax change, there exist production elasticities (i.e., $\sigma_X$ and $e_{ij}$) such that the relative burden on factors of production is opposite compared to the model with a single consumer, coupled to the same production side data and characterized by an expenditure share on leisure given by the income-weighted average of shares across households, $\lambda$.*

*Proof.* For the above assumptions, the change in the return on capital is given by:

$$\hat{r} = -\frac{\theta_{XL}\theta_{YZ}}{D_{LL,1}}\left[A_{LL,1}(e_{ZZ} - e_{KZ}) - B_{LL,1}(e_{ZZ} - e_{LZ}) + (\gamma_K - \gamma_L)(1 + \frac{1-\gamma}{\gamma}\lambda)\right]\hat{\tau}_Z\,,$$

where $A_{LL,1} = \gamma_L\beta_K + \gamma_K(\beta_L + \theta_{YZ} + \frac{\tau_Z Z}{p_X X}\lambda)$, $B_{LL,1} = \gamma_K\beta_L + \gamma_L(\beta_K + \theta_{YZ} + \frac{\tau_Z Z}{p_X X}\lambda)$, $C_{LL,1} = \beta_K + \beta_L + \theta_{YZ} + \frac{\tau_Z Z}{p_X X}\lambda$, $D_{LL,1} = C_{LL,1}\sigma_X + e_{KL}[A_{LL,1}\theta_{YL} + B_{LL,1}\theta_{YK}] + e_{LZ}[B_{LL,1}\theta_{XK}(\theta_{YL} + \theta_{YZ}) - A_{LL,1}\theta_{XK}\theta_{YL}] + e_{KZ}[A_{LL,1}\theta_{XL}(\theta_{YK} + \theta_{YZ}) - B_{LL,1}\theta_{XL}\theta_{YK}] + (\gamma_K - \gamma_L)(\theta_{XL}\theta_{YK} - \theta_{XK}\theta_{YL} + \frac{\lambda}{\gamma}(\theta_K + \theta_Z\theta_{XK})) - (\gamma_K - \gamma_L)\frac{cov(\lambda^h, \theta_L^h)}{p_X X}$. Analogously to the proof of Proposition 4, one parameter choice that leads to the reversal of factor price changes between the heterogeneous household model and the single household model is the following: $\sigma_X = e_{KZ} = e_{LZ} = 0$ and $-[A_{LL,1}\theta_{YL} + B_{LL,1}\theta_{YK}]e_{KL} \in \Big(min[(\gamma_K - \gamma_L)(\theta_{XL}\theta_{YK} - \theta_{XK}\theta_{YL} + \frac{\lambda}{\gamma}(\theta_K + \theta_Z\theta_{XK})) - (\gamma_K - \gamma_L)\frac{cov(\lambda^h, \theta_L^h)}{p_X X}, (\gamma_K - \gamma_L)(\theta_{XL}\theta_{YK} - \theta_{XK}\theta_{YL} +$

$\frac{\lambda}{\gamma}(\theta_K + \theta_Z\theta_{XK}))], max[(\gamma_K - \gamma_L)(\theta_{XL}\theta_{YK} - \theta_{XK}\theta_{YL} + \frac{\lambda}{\gamma}(\theta_K + \theta_Z\theta_{XK})) - (\gamma_K - \gamma_L)\frac{cov(\lambda^h, \theta_L^h)}{p_X X}, (\gamma_K - \gamma_L)(\theta_{XL}\theta_{YK} - \theta_{XK}\theta_{YL} + \frac{\lambda}{\gamma}(\theta_K + \theta_Z\theta_{XK}))]\Big)$. □

**Proposition 5.LL.** *Assume the model with labor-leisure choice, equal benchmark leisure time across households ( $\frac{l^h}{L^h} = \mathcal{L}$, $\forall h$ ), and Cobb-Douglas utility. Assume Leontief technologies in clean and dirty good production (i.e., $\sigma_X = e_{ij} = 0$), and that the dirty sector is relatively more capital intensive (i.e., $\gamma_K > \gamma_L$), such that the following holds: $(\theta_{YL}\theta_{XK} - \theta_{YK}\theta_{XL})\frac{p_Y Y}{p_X X} + \frac{1}{\gamma}(\theta_K + \theta_Z\theta_{XK}) = 0$. Then:*

- *(i) if consumers are identical on the sources and uses side of income: $\hat{p}_Y = 0$, $\hat{w} > 0$ and $\hat{r} < 0$.*
- *(ii) if consumers are identical on the uses side of income, and the $\theta_L^h$ and $\lambda^h$s have* low *covariance (i.e., $D_{LL} > 0$), then $\hat{p}_Y < 0$, $\hat{w} > 0$ and $\hat{r} < 0$.*
- *(iii) if consumers are identical on the uses side of income, and the $\theta_L^h$ and $\lambda^h$s have* high *covariance (i.e., $D_{LL} < 0$), then $\hat{p}_Y > 0$, $\hat{w} < 0$ and $\hat{r} > 0$.*

*Proof.* Price changes assume the following form:

$$\hat{p}_Y = \frac{\theta_{YZ}}{D_{LL}}\Big((\theta_{YL}\theta_{XK} - \theta_{YK}\theta_{XL})\frac{1-\gamma}{\gamma}\lambda + \frac{\lambda}{\gamma}(\theta_K + \theta_Z\theta_{XK}) - \frac{cov(\lambda^h, \theta_L^h)}{p_X X}\Big)\hat{\tau}_Z$$

$$\hat{r} = -\frac{\theta_{XL}\theta_{YZ}}{D_{LL}}(1 + \frac{p_Y Y}{p_X X}\lambda)\hat{\tau}_Z\,,$$

where $D_{LL} = (\theta_{XL}\theta_{YK} - \theta_{XK}\theta_{YL}) + \frac{\lambda}{\gamma}(\theta_K + \theta_Z\theta_{XK}) - \frac{cov(\lambda^h, \theta_L^h)}{p_X X}$. □

### *2.4.3. More than two sectors*

Our analysis so far assumed a highly aggregated sectoral representation. Including more sectors can obviously affect the aggregation bias as it enables representing household heterogeneity along more dimensions. With a finer sectoral resolution, it is, for example, conceivable that poorer households have higher expenditure shares on some dirty goods and lower expenditure shares on some others when compared to richer households. The problem is further compounded by the possibility that different polluting goods are likely to be produced with different capital and labor intensities. As the aggregation bias is determined by the interaction between household and production side characteristics, the impact of going from two to multiple sectors on the aggregation bias is thus in general not clear-cut.

We show for a special case with Leontief technologies in production that the aggregation bias can still be important for assessing the incidence of environmental taxes in a setting which includes an arbitrary number of of dirty sectors $J$, denoted by the index $j$. Analogous to Proposition 5 with Leontief technologies, we find that the covariance between the ownership of labor and consumption of each dirty good across households can reverse the sign of the factor price changes. We use the label "MC" to enable comparison between the original propositions and the propositions based on the model with multiple polluting commodities.

**Proposition 5.MC.** *Assume Cobb-Douglas preferences and Leontief technologies in clean and dirty production sectors. Assume furthermore that each dirty sector $j$ is more capital-intensive than the economy-wide average (i.e., $\frac{K_{Y_j}}{L_{Y_j}} > \frac{\bar{K}}{\bar{L}}$, $\forall j$), and that every dirty sector $j$ is more capital intensive than the clean sector (i.e., $\frac{K_{Y_j}}{L_{Y_j}} > \frac{K_X}{L_X}$, $\forall j$). Then, the following holds:*

- *(i) If consumers are identical on the sources or uses side of income, or both: $\hat{w} > 0$ and $\hat{r} < 0$.*
- *(ii) If labor ownership and dirty good consumption (for each dirty good $j$) have a positive covariance, then $\hat{w} > 0$ and $\hat{r} < 0$.*
- *(iii) If labor ownership and dirty good consumption (for each dirty good $j$) has a negative covariance, then $\hat{w} > 0$ and $\hat{r} < 0$ if covariance is* low *(i.e. $D_J > 0$), and $\hat{w} < 0$, $\hat{r} > 0$ if covariance is* high *(i.e. $D_J < 0$).*

*Proof.* For $J \geq 1$, we derive in the subsection "Derivations" below the following expression for the rental rate of capital:

$$\hat{r} = -\frac{\theta_{XL}\sum_{n=1}^{J}\theta_{Y_n Z_n}\Big(\frac{K_{Y_n}}{\bar{K}} - \frac{L_{Y_n}}{\bar{L}}\Big)}{D_J}\hat{\tau}_Z\,,$$

where $D_J \equiv \sum_{n=1}^{J}\Big(\frac{K_{Y_n}}{\bar{K}} - \frac{L_{Y_n}}{\bar{L}}\Big)\Big(\theta_{XL}\theta_{Y_n K_n} - \theta_{XK}\theta_{Y_n L_n} - cov(\frac{\alpha^h}{p_X X} - \frac{\alpha_n^h}{p_{Y_n} Y_n}, \theta_L^h)\Big)$. For $J = 1$, this is identical to the case considered in Proposition 5. Consider the above equation for $\hat{r}$, bearing in mind that if labor ownership and dirty good consumption have a positive covariance for each good $j$, then the labor ownership and clean good consumption have negative covariance, since $\alpha^h = 1 - \sum_j \alpha_j^h$.

Furthermore $\theta_{XL}\theta_{Y_jK_j} - \theta_{XK}\theta_{Y_jL_j} = \theta_{XL}\theta_{XK}(\frac{\theta_{Y_jK_j}}{\theta_{XK}} - \frac{\theta_{Y_jL_j}}{\theta_{XL}}) = \theta_{XL}\theta_{XK}\frac{p_XX}{p_YY}(\frac{K_{Y_j}}{K_X} - \frac{L_{Y_j}}{L_X})$. This expression is positive if every dirty sector is more capital intensive than the clean sector. □

Derivations

The equilibrium conditions (1)–(13) for the model with $J$ dirty sectors and one clear sector are given by:

$$\hat{K}_X\frac{K_X}{\bar{K}} + \sum_l \hat{K}_{Y_l}\frac{K_{Y_l}}{\bar{K}} = 0 \tag{1.MC}$$

$$\hat{L}_X\frac{L_X}{\bar{L}} + \sum_j \hat{L}_{Y_l}\frac{L_{Y_l}}{\bar{L}} = 0 \tag{2.MC}$$

$$\hat{K}_X - \hat{L}_X = 0 \tag{3.MC}$$

$$\hat{K}_{Y_j} - \hat{Z}_j = 0 \quad \forall j \tag{4.MC}$$

$$\hat{L}_{Y_j} - \hat{Z}_j = 0 \quad \forall j \tag{5.MC}$$

$$\hat{p}_X + \hat{X} = \theta_{XK}(\hat{r} + \hat{K}_X) + \theta_{XL}(\hat{w} + \hat{L}_X) \tag{6.MC}$$

$$\hat{p}_{Y_j} + \hat{Y}_j = \theta_{Y_jK_j}(\hat{r} + \hat{K}_{Y_j}) + \theta_{Y_jL_j}(\hat{w} + \hat{L}_{Y_j}) + \theta_{Y_jZ_j}(\hat{\tau}_Z + \hat{Z}_j) \quad \forall j \tag{7.MC}$$

$$\hat{X} = \theta_{XK}\hat{K}_X + \theta_{XL}\hat{L}_X \tag{8.MC}$$

$$\hat{Y}_j = \theta_{Y_jK_j}\hat{K}_{Y_j} + \theta_{Y_jL_j}\hat{L}_{Y_j} + \theta_{Y_jZ_j}\hat{Z}_j \quad \forall j \tag{9.MC}$$

$$\hat{X}^h - \hat{Y}_j^h = \hat{p}_{Y_j} \quad \forall h, j \tag{10.MC}$$

$$\hat{X}^h = \hat{M}^h \quad \forall h \tag{11.MC}$$

$$\hat{X} = \sum_h \frac{X^h}{X}\hat{X}^h \tag{12.MC}$$

$$\hat{Y}_j = \sum_h \frac{Y_j^h}{Y_j}\hat{Y}_j^{\,h} \quad \forall j, \tag{13.MC}$$

with $\hat{M}^h = \hat{w}\frac{w\bar{L}^h}{M^h} + \hat{r}\frac{r\bar{K}^h}{M^h} + \frac{\tau_Z Z}{p_X X + p_Y Y}(\hat{\tau}_Z + \frac{\sum_j Z_j\hat{Z}_j}{Z})$. Equations (1.MC)–(13.MC) are $6 + 5J + H + JH$ equations in $6 + 5J + H + JH$ unknowns ($\hat{K}_X$, $J \times \hat{K}_{Y_j}$, $\hat{L}_X$, $J \times \hat{L}_{Y_j}$, $\hat{w}$, $\hat{r}$, $\hat{X}$, $p_x$, $J \times \hat{p}_{Y_j}$, $J \times \hat{Y}_j$, $J \times \hat{Z}_j$, $H \times \hat{X}^h$, $J \times H \times \hat{Y}_j^h$). Following Walras' Law, one of the equilibrium conditions is redundant, thus the effective number of equations is $5 + 5J + H + JH$. We choose $X$ as the numéraire good, thus delivering a square system of equations. The equilibrium solutions are therefore fully determined as functions of the exogenous tax increase $\hat{\tau}_Z > 0$.

In order to derive the factor price changes, start by subtracting (8.MC) from (6.MC) an (9.MC) from (7.MC):

$$0 = \theta_{XK}\hat{r} + \theta_{XL}\hat{w} \tag{14.MC}$$

$$\hat{p}_{Y_j} = \theta_{Y_jK_j}\hat{r} + \theta_{Y_jL_j}\hat{w} + \theta_{Y_jZ_j}\hat{\tau}_Z \quad \forall j\,. \tag{15.MC}$$

Substitute (12.MC) into (8.MC) and (13.MC) into (9.MC):

$$\sum_h \frac{X^h}{X}\hat{X}^h = \theta_{XK}\hat{K}_X + \theta_{XL}\hat{L}_X \tag{16.MC}$$

$$\sum_h \frac{Y_j^h}{Y_j}\hat{Y}_j^{\,h} = \theta_{Y_jK_j}\hat{K}_{Y_j} + \theta_{Y_jL_j}\hat{L}_{Y_j} + \theta_{Y_jZ_j}\hat{Z}_j \quad \forall j\,. \tag{17.MC}$$

Solve (10.MC) for $Y_j^h$ and insert into (17.MC):

$$\frac{1}{Y_j}\sum_h Y_j^h\hat{p}_{Y_j} = \frac{1}{Y_j}\sum_h Y_j^h\hat{X}^h - \theta_{Y_jK_j}\hat{K}_{Y_j} - \theta_{Y_jL_j}\hat{L}_{Y_j} - \theta_{Y_jZ_j}\hat{Z}_j \quad \forall j\,. \tag{18.MC}$$

From (16.MC) insert the following on the right-hand side of the equality: $0 = \theta_{XK}\hat{K}_X + \theta_{XL}\hat{L}_X - \sum_h \frac{X^h}{X}\hat{X}^h$ and use the fact that $\frac{1}{Y_j}\sum_h Y_j^h\hat{p}_{Y_j} = \hat{p}_{Y_j}$, thus yielding:

$$\hat{p}_{Y_j} = \sum_h(\frac{Y_j^h}{Y_j} - \frac{X^h}{X})\hat{X}^h + \theta_{XK}\hat{K}_X + \theta_{XL}\hat{L}_X - \theta_{Y_jK_j}\hat{K}_{Y_j} - \theta_{Y_jL_j}\hat{L}_{Y_j} - \theta_{Y_jZ_j}\hat{Z}_j \quad \forall j\,. \tag{19.MC}$$

Eliminate $\hat{X}^h$ from equation (19.MC) by using equation (11.MC), then insert the explicit form of the budget change $\hat{M}^h$:

$$\hat{p}_{Y_j} = \hat{w} \sum_h \phi^h_{Lj} + \hat{r} \sum_h \phi^h_{Kj} + \theta_{XK}\hat{K}_X + \theta_{XL}\hat{L}_X - \theta_{Y_jK_j}\hat{K}_{Y_j} - \theta_{Y_jL_j}\hat{L}_{Y_j} - \theta_{Y_jZ_j}\hat{Z}_j \quad \forall j\,, \tag{20.MC}$$

where $\phi^h_{Lj} = (\frac{Y^h_j}{Y_j} - \frac{X^h}{X})\frac{w\bar{L}^h}{M^h}$, $\phi^h_{Kj} = (\frac{Y^h_j}{Y_j} - \frac{X^h}{X})\frac{r\bar{K}^h}{M^h}$ and using the fact that $\sum_h(\frac{Y^h_j}{Y_j} - \frac{X^h}{X})\frac{\tau_Z Z}{p_X X + p_Y Y} = 0$. Now solve equations (1.MC) and (2.MC) for $\hat{K}_X$ and $\hat{L}_X$ and insert them into equation (20.MC). Furthermore, insert equation (15.MC) to eliminate $\hat{p}_{Y_j}$, thus obtaining:

$$-\theta_{Y_jZ_j}\hat{Z}_j = (\theta_{Y_jK_j} - \sum_h \phi^h_{Kj})\hat{r} + (\theta_{Y_jL_j} - \sum_h \phi^h_{Lj})\hat{w} + \theta_{Y_jZ_j}\hat{\tau}_Z + \hat{K}_{Y_j}\theta_{Y_jK_j} + \sum_l \hat{K}_{Y_l}\theta_{XK}\frac{K_{Y_l}}{K_X} + \hat{L}_{Y_j}\theta_{Y_jL_j} + \sum_l \hat{L}_{Y_l}\theta_{XL}\frac{L_{Y_l}}{L_X} \quad \forall j\,. \tag{21.MC}$$

Solve equations (4.MC) and (5.MC) for $\hat{K}_{Y_j}$ and $\hat{L}_{Y_j}$, and insert them into equation (21.MC). This yields:

$$-\hat{Z}_j - \sum_l(\theta_{XK}\frac{K_{Y_l}}{K_X} + \theta_{XL}\frac{L_{Y_l}}{L_X})\hat{Z}_l = (-\sum_h \phi^h_{Kj} + \theta_{Y_jK_j})\hat{r} + (-\sum_h \phi^h_{Lj} + \theta_{Y_jL_j})\hat{w} + \theta_{Y_jZ_j}\hat{\tau}_Z \quad \forall j\,. \tag{22.MC}$$

Next eliminate the $\hat{Z}$s. To achieve this, substitute (1.MC) and (2.MC) into (3.MC), obtaining:

$$-\sum_l \frac{K_{Y_l}}{K_X}\hat{K}_{Y_l} + \sum_l \frac{L_{Y_l}}{L_X}\hat{L}_{Y_l} = 0\,. \tag{23.MC}$$

Substituting equations (4.MC) and (5.MC) into (23.MC) yields:

$$\sum_l(-\frac{K_{Y_l}}{K_X} + \frac{L_{Y_l}}{L_X})\hat{Z}_l = 0\,. \tag{24.MC}$$

Now insert (24.MC) into (22.MC):

$$\hat{r}(\theta_{Y_jK_j} - \sum_h \phi^h_{Kj}) + \hat{w}(\theta_{Y_jL_j} - \sum_h \phi^h_{Lj}) + \hat{\tau}_Z\theta_{Y_jZ_j} = -\sum_l \frac{K_{Y_l}}{K_X}\hat{Z}_l - \hat{Z}_j \quad \forall j\,. \tag{25.MC}$$

Now combine the above $J$ equations in (25.MC) in order to be able to solve for the factor price changes. To do so, multiply each equation by an unknown parameter $A_j$ and sum over $j$:

$$\hat{r}\sum_j(A_j\theta_{Y_jK_j} - A_j\sum_h \phi^h_{Kj}) + \hat{w}\sum_j(A_j\theta_{Y_jL_j} - A_j\sum_h \phi^h_{Lj}) + \hat{\tau}_Z\sum_j A_j\theta_{Y_jZ_j} = -\sum_l((\sum_j A_j)\frac{K_{Y_l}}{K_X} + A_l)\hat{Z}_l\,. \tag{26.MC}$$

It therefore follows that if the right-hand side is zero, then the $\hat{Z}$s drop out of (26.MC). As an ansatz, require the following, which will then make the right-hand side of (26.MC) zero due to (24.MC):

$$(\sum_l A_l)\frac{K_{Y_j}}{K_X} + A_j = \frac{K_{Y_j}}{K_X} - \frac{L_{Y_j}}{L_X} \quad \forall j\,. \tag{27.MC.D}$$

In order to solve for the set of $A$s that satisfies (27.MC.D), sum (27.MC.D) over $j$, and relabel indices to obtain the following (using the notation $K_Y \equiv \sum_l K_{Y_l}$ and analogous notation for the other aggregate variables):

$$(\sum_l A_l) = \frac{K_X}{\bar{K}}(\frac{K_Y}{K_X} - \frac{L_Y}{L_X})\,. \tag{28.MC}$$

Insert (28.MC) back into (27.MC.D), and solve for $A_j$:

$$A_j = \frac{K_{Y_j}}{K_X} - \frac{L_{Y_j}}{L_X} - \frac{K_{Y_j}}{\bar{K}}(\frac{K_Y}{K_X} - \frac{L_Y}{L_X}) = \frac{\bar{L}}{L_X}\Big(\frac{K_{Y_j}}{\bar{K}} - \frac{L_{Y_j}}{\bar{L}}\Big) \quad \forall j\,. \tag{29.MC}$$

For the $A$ coefficients as in (29.MC), (26.MC) then becomes:

$$\Big(\sum_j A_j(\theta_{Y_jK_j} - \sum_h \phi^h_{Kj})\Big)\hat{r} + \Big(\sum_j A_j(\theta_{Y_jL_j} - \sum_h \phi^h_{Lj})\Big)\hat{w} = -\hat{\tau}_Z\sum_j A_j\theta_{Y_jZ_j}\,. \tag{30.MC}$$

Solve (14.MC) for $\hat{w}$ and substitute into (30.MC), thus obtaining:

$$\hat{r} = -\frac{\theta_{XL}\sum_j A_j \theta_{Y_j Z_j}}{\sum_j A_j\Big(\theta_{XL}\theta_{Y_j K_j} - \theta_{XK}\theta_{Y_j L_j} - \theta_{XL}\sum_h \phi^h_{Kj} + \theta_{XK}\sum_h \phi^h_{Lj}\Big)}\hat{\tau}_Z\,. \tag{31.MC}$$

This then delivers the expression for $\hat{r}$, using the fact that

$$-\theta_{XL}\sum_h \phi^h_{Kj} + \theta_{XK}\sum_h \phi^h_{Lj} = \sum_h\Big((\frac{Y^h_j}{Y_j} - \frac{X^h}{X})(\theta^h_L - \theta_L)\Big) = \frac{1}{p_{Y_j}Y_j}cov(\alpha^h_j, \theta^h_L) - \frac{1}{p_X X}cov(\alpha^h, \theta^h_L)\,. \tag{32.MC}$$

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