Informal Credit Markets, Common-pool Resources and Education

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Informal Credit Markets, Common-pool Resources and Education

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Abstract

The paper analyses the effect of interest rate changes on education and child labor in an economy with a high-skilled sector, a low-skilled sector and fragmented credit markets. The high-skilled sector takes educated labor as input. The low-skilled sector takes unskilled labor, physical capital and natural common-pool resources as inputs. Credit supply consists of (a) loans with collateral in form of productive investments in the low-skilled sector and (b) higher-priced loans without collateral. Lower interest rates increase the net present value of the returns to education. They also reduce costs of capital investment in current production. This increases labor productivity and the opportunity costs of education. Overuse of the common-pool resource can reverse this productivity effect. We show that the overall effect of interest rate changes on education depends on the initial wealth of the household, resource use and the credit market segment that is subject to improvements.

Keywords: Informal Credit Markets, Education, Common-pool Resources, Child Labor, Fragmented Credit Markets, Common-pool Externality

JEL Classification: D13, D91, J24, O16, Q20

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1 Introduction

This paper studies the impact of credit market improvements on education and child labor in the setting of a dual economy with an urban, high-skilled sector with high incomes and a rural, low-skilled sector with low incomes. Many households in rural areas of developing countries depend on natural resources either directly for food, energy or construction material or as a source of income (Angelsen et al. 2014). Property rights over these resources are often not defined or only weakly enforced, leading to resource overuse and low resource incomes (Stavins 2011). Better income alternatives may require investment in education, but many children contribute to the household’s income instead of going to school.\(^1\) Poor households may not be able to afford the income shortfall during schooling, which perpetuates rural poverty and environmental degradation.

Credit markets could help to overcome the initial income shortfall if children go to school. They could therefore help the poor rural households to escape the poverty trap (Noack et al. 2015). However, if loans are available in poor rural areas, they will be often informal and characterized by low borrowing limits (Baland and Robinson 2000; Banerjee and Duflo 2005; Ranjan 2001) and high and differentiated interest rates—yearly interest rates of 40-80% per year are common (Banerjee and Duflo 2010). To deal with low borrowing limits, households often take out several loans simultaneously.\(^2\) Lower interest rates are usually available for loans connected to business investments, as the investment can serve as collateral to reduce the default probability. Education usually does not serve as collateral, because it is difficult to appropriate in case of default.

To enhance the situation of the rural poor, one focus has been on credit market improvements, e.g. through the introduction and expansion of micro finance. Micro finance loans often come with comparably lower interest rates (see e.g. Riekhof (2014)). Still, in a world full of market failures, this may have unintended side effects. While credit market reforms

\(^1\)One tenth of children aged 5-17 were involved in child labor as of 2012 (Diallo et al. 2013). ‘Child labor [...] excludes all children working legally in accordance with ILO Conventions Nos. 138 and 182.’ page vii, Diallo et al. (2013). For some children, working may have no influence on their education, but for some children, it has a negative impact (Dumas 2015).

\(^2\)Guirkinger (2008) and Riekhof (2014) document that households often have several loans with different interest rates simultaneously.
are very likely to improve the life of the people that are currently making the decisions—household heads, grown-up family members —, they may induce decisions with a negative impact on future generations. One example is a shift away from education. In this paper, we thus ask how an exogenous interest rate reduction impacts education when business investments and education are not treated equally to fulfill collateral requirements. We especially take into account that—in poor, rural areas—credit markets are fragmented such that households may have several loans with differing interest rates, and that externalities from resource use may prevail.

The basic mechanisms that drive results are as follows. Lower interest rates reduce the households discounting, which increases the net present value of education and more time is devoted to education. Menon (2010) and Dumas (2013) show that this may not hold when loans are tied to a productive investment and labor markets are imperfect.\(^3\) When loans are tied to a productive investment, a reduction in the interest rate increases investment into capital. Then, labor could become so scarce relatively that children work instead of going to school. This ‘productivity’ mechanism counteracts the ‘intertemporal’ (discounting) mechanism described above.

Additional effects occur when different loans are used simultaneously and when resource use-rights are not defined. With several loans at different interest rates, it is not obvious when the ‘productivity’ mechanism or the ‘intertemporal’ mechanism occur. Further, without use-rights for the natural resource, resource externalities prevail: the harvest of one households reduces the use possibilities of the other households. This may impact labor productivity. The reason is as follows. If a whole village obtains access to cheaper credits that are tied to productive investments, villagers may increase their investments to such a degree that the productivity of the natural resource becomes the limiting factor. Labor may not become relatively scarce and the opportunity costs of education may decline.

In this paper, we examine how common-pool resources and the simultaneous use of different loan types influence the impact of an interest rate change on education. We incorporate common-pool resources and a stylized informal credit market in an otherwise standard two-

\(^3\)In general, this may not hold when different investments with differing time horizons are present.
sector/two-period household decision model. In the first period, the household can either allocate time towards income generation, using low-skilled labor and physical capital, or towards education. With education, the household can work in the high-skilled sector in the second period. The stylized credit market includes a high interest rate for unsecured borrowing (‘unsecured loan’), an intermediate interest rate for borrowing secured by a sizeable collateral in the form of a business investment in the low-skilled sector (‘secured loan’), and a low interest rate for saving. The behaviour on the credit markets turns out to be related to initial wealth. To analyze the role of fragmented credit markets, we analyze this set-up before we introduce the common-pool resource in the low-skilled sector. The set-up with physical capital, labor and common-pool resources as inputs in the low-skilled sector, compared to labor and human capital as inputs in the high-skilled sector, is a stylized representation of the differences between rural, resource-based work and urban, employment-based work. As it turns out, the results from the model without common-pool resource corresponds to the case of a natural resource under perfect property rights.

Our results differentiate the ambiguous effect of an interest rate change in a secured loan on education. We show that if the household has an additional, higher interest rate loan, a decrease in the interest rate from the secured loan reduces education. In this case, only labor productivity is affected. When strong common pool externalities are present, a decrease in the interest rate of the secured loan leads to more education. It does not matter whether the household also has an unsecured loan. The reason is that common pool externalities reduce the labor productivity effect. Our results may help explaining in more detail the mixed or insignificant impact of increased access to credit on education and to better predict the impact of credit market policies for education.

Our paper relates to the child labor literature that evaluates the impact of moving from absent credit markets to perfect credit markets on child labor (Baland and Robinson 2000; Bommier and Dubois 2004; Ranjan 1999). Some set-ups also consider an investment ‘loan’ repaid within the period during which it was taken out (Menon 2010; Wydick 1999). Dumas

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4Some studies find a positive effect of better access to credit on education (Beegle et al. 2006; Dehejia and Gatti 2005), while others find a negative effect (Augsburg et al. 2015; Hazarika and Sarangi 2008; Islam and Choe 2013; Maldonado and Gonzlez-Vega 2008). Further studies report mixed results (Shimamura and Lastarria-Cornhiel 2010; Tarozzi et al. 2015).
(2013) and Jafarey and Lahiri (2002) study a gradual reduction of market imperfections and its effect on child labor, but they focus on labor market imperfections as well as on the effectiveness of trade sanctions for different grades of internationally integrated credit markets, respectively. Dumas (2015) examines how rainfall shocks impact child labor for different market imperfections. Our model is closest to the set-up in Lochner and Monge-Naranjo (2011), who present the only two-period model with a fragmented credit market that includes two different loan types. They consider borrowing and college attendance in the United States. In their paper and in contrast to our set-up, college students can borrow against a higher future income generated by education.\textsuperscript{5}

The remainder of the paper is organized as follows. Section 2 introduces the model framework without common-pool externalities and focuses on the effects of fragmented credit markets. Section 3 presents the results. Section 4 extends the previous analysis to the case with common-pool externalities. Section 5 concludes.

\section{The Model}

The model consists of two periods and represents an economy with a fragmented credit market as well as a high-skilled and a low-skilled production sector. In each period, a representative household inelastically supplies one unit of labor consisting of the labor supply of all household members. The household allocates labor between low-skilled production and education in the first period and between low- and high-skilled production in the second period. We assume that education is a prerequisite for working in the high-skilled sector and that wages are higher in the high-skilled sector. Then, the share of labor allocated to education in the first period equals the share of time allocated to high-skilled production in the second period. The time-share allocated to low-skilled production is thus also the same in both periods. We assume continuous time-shares to keep the model simple, they are intended to represent the different household members. The following subsections describe

\textsuperscript{5}Other approaches to child labor have a different focus and do not include credit markets at all, e.g. Basu and Van (1998), Ranjan (2001) and Bell and Gersbach (2009). Recently, Edmonds (2008) gave an overview of child labor literature, and Blume and Breyer (2011) did so for child labor and micro-finance.
production, markets and the household in more detail.

2.1 Production

Production in the low-skilled sector uses physical capital $k$ and labor $l$.\(^6\) The low-skilled sector produces output $h(l,k)$ using a strictly increasing and strictly concave technology that satisfies the Inada Conditions. Production in the high-skilled sector uses educated labor $b$ as input, with the production technology $w(b)$ and $b = 1 - l$. This accounts for education, 1 – $l$ in the first period, and labor time, 1 – $l$ in the second period. Income $w(b)$ includes returns to education. The production technology $w$ is also strictly increasing and strictly concave, and satisfies the Inada Conditions. Demand for outputs is perfectly elastic at constant prices. Labor markets are non-existent for low-skilled labor.\(^7\)

2.2 Credits

To capture the fragmented nature of informal credit markets and the role of collateral in form of a productive investment, we introduce two loan types. The ‘secured loan’ $v$ at an intermediate interest factor $\tau$ is constrained by the investment in physical capital. The ‘unsecured’ loan $z$ reflects the possibility for a second, unconstrained, but higher-priced loan. In other words, credit supply is perfectly elastic at the interest factor $\tau$ as long as the capital investment serves as collateral. For a loan that exceeds the capital investment, credit supply is perfectly elastic at the higher interest factor $\iota$. The set-up results in a discontinuity in credit supply that is typical for informal credit markets and which is modeled similarly in e.g. Bell et al. (1997). For completeness, we also include saving. Thus, the household chooses the amount $s \geq 0$ it saves at a fixed interest factor $\phi$.

\(^6\)We assume that capital investment only affects low skilled-work, as education and high-skilled work are often associated with labor reallocation from rural production to wage work or other types of urban production (Beegle et al. 2011; Taylor and Yunez-Naude 2000). This labor reallocation makes returns to education independent from local capital investments.

\(^7\)While the non-existence of labor markets is indeed a strong (and unrealistic) assumption, our results hold qualitatively as long as additional outside labor cannot satisfy the additional demand due to an increase in labor productivity related to a change in interest rates.
the amount it borrows $v \geq 0$, constrained by the collateral $k$ according to

$$v \leq k$$  \hspace{1cm} (1)

at the fixed interest factor $\tau$ and the amount $z \geq 0$ it borrows at the fixed interest factor $\iota$, with

$$1 < \phi < \tau < \iota < \infty.$$  \hspace{1cm} (2)

The analysis directly uses interest factors instead of interest rates to facilitate the interpretation of the results in terms of one period with several years (as e.g. in Lochner and Monge-Naranjo (2011)).

We disregard that the household may start lending money as (a) the lending of money requires its own infrastructure and (b) the focus of the paper is on indebted households.

**2.3 The Representative Household**

The household is endowed with exogenously given wealth $\kappa \geq 0$ and with one unit of labor per period. It can allocate labor towards low-skilled production or education in the first period. Following Jafarey and Lahiri (2002) and Bhalotra and Heady (2003), the household acts as a single decision-maker without intra-household bargaining on time allocation. The household head makes the decision at the beginning of the first period when children are too young to decide. Schooling could be interpreted as an investment such that there is a trade-off between child labor and schooling. The model set-up with two-periods implies that the household head neither takes the children’s adult life nor the life of future generations into account when deciding. This may be justified, as old-age security considerations may be more important than altruism (Cigno 1992).

The household can invest in physical capital $k$ for the low-skilled activity at the beginning of the first period. Capital lasts until the end of the second period without depreciation and depreciates completely thereafter.

The household has strictly convex, strictly monotone and homothetic preferences. Prefer-
ences can thus be represented by a strictly quasi-concave and linear homogeneous utility function \( u(c_1, c_2) \) that is strictly increasing in consumption in period one, \( c_1 \), and period two, \( c_2 \), and that satisfies the Inada Conditions. All incomes, capital, loans, savings, and initial wealth are measured in units of the consumption good.

The household maximizes utility \( u(c_1, c_2) \) subject to the constraints

\[
\begin{align*}
    c_1 + k + s & \leq h(l, k) + \kappa + v + z, \\
    c_2 + \tau v + \iota z & \leq h(l, k) + w(1 - l) + \phi s, \\
    v & \leq k,
\end{align*}
\]

with non-negative variables \( k, v, l, s, z, c_1 \) and \( c_2 \). The corresponding Lagrangian is

\[
y(c_1, c_2, l, k, v, z, s, \lambda_1, \lambda_2, \lambda_3) = u(c_1, c_2) + \lambda_1[h(l, k) + \kappa + v + z - c_1 - k - s] \\
+ \lambda_2[h(l, k) + w(1 - l) + \phi s - c_2 - \tau v - \iota z] \\
+ \lambda_3(k - v),
\]

with shadow prices for consumption \( \lambda_1 \) and \( \lambda_2 \) in the first and second period, respectively, and shadow price \( \lambda_3 \) for the capital constraint of the secured loan. The next section derives the optimal solution.

### 2.4 Optimality Conditions

In the following, we omit the arguments of the functions, writing \( h \) instead of \( h(l, k) \) and so on. Furthermore, subscripts denote partial derivatives. For marginal utilities, it is \( u_1 \) for \( \partial u / \partial c_1 \) and \( u_2 \) for \( \partial u / \partial c_2 \). To explicitly take corner solutions into account and include that households may not have two loans or may not save, we use the Kuhn-Tucker Optimality
Conditions. They read

\begin{align*}
    u_1 - \lambda_1 &= 0, \\
    u_2 - \lambda_2 &= 0, \\
    \lambda_1 h_l + \lambda_2 (h_l + w_l) &= 0, \\
    \lambda_1 (h_k - 1) + \lambda_2 h_k + \lambda_3 &= 0, \\
    \lambda_1 - \lambda_2 \tau - \lambda_3 &\leq 0, \\
    \lambda_1 - \lambda_2 \iota &\leq 0, \\
    -\lambda_1 + \lambda_2 \phi &\leq 0, \\
    h + v + z + \kappa - c^1 - s - k &\geq 0, \\
    h + w + s \phi - c^2 - v \tau - z \iota &\geq 0, \\
    k - v &\geq 0,
\end{align*}

The equality of the first four conditions follows from the Inada Conditions.

We define the household’s consumption discount factor as

\[ f := \frac{u_1}{u_2} = \frac{\lambda_1}{\lambda_2}. \]

It displays the price, in terms of period-two goods, for an extra unit of a period-one good the household is willing to pay to shift a marginal income unit between periods. It equals the relevant market interest factor if the household is not credit-constrained. If the household has a high discount factor, it will borrow even at high interest costs. If the same household had a lower discount factor, it may not borrow, but may save.

The household’s possible activity on the credit market can be classified into five five credit regimes that follow from the Kuhn-Tucker Conditions (see Appendix A). These regimes—termed according to their main characteristic—can be ordered according to the household’s discount factor (from high to low) and are characterized as follows:

1. **Two loans (TL) regime**: The household exhausts the secured loan and takes out
an additional loan \((z > 0, v = k, s = 0, f = \iota \text{ and } \lambda_3 \lambda_2 = \iota - \tau)\).

2. **Exhausted loan (EL) regime:** The household exhausts the secured loan, but does not take an additional loan \((z = 0, v = k, s = 0, \tau \leq f \leq \iota \text{ and } \lambda_3 \lambda_2 = f - \tau)\).

3. **One loan (OL) regime:** The household takes out a secured loan, but does not exhaust it \((z = 0, k > v > 0, s = 0, f = \tau \text{ and } \lambda_3 = 0)\).

4. **No credit market activities (NO) regime:** The household neither borrows nor lends \((z = 0, v = 0, s = 0, \phi \leq f \leq \tau \text{ and } \lambda_3 = 0)\).

5. **Saving (SA) regime:** The household saves \((z = 0, v = 0, s > 0, f = \phi \text{ and } \lambda_3 = 0)\).

The discount factor, and thus the optimal credit regime, depend on the household’s initial wealth. All other household characteristics—like child ability—are kept constant, as the model considers the decision of the same household for different possible initial wealth levels.\(^8\) Of course, one could vary other household’s characteristics besides the initial wealth level, but the focus here is on the role of the credit market arrangements, which are closely connected to wealth. Figure 1 shows how the initial wealth level (horizontal axis) relates to the household’s discount factor that may equal the prevailing market interest factor (vertical axis, the proof is given in Appendix B.). It also shows the different credit regimes for a calibrated model.

It depends on the parameter combinations whether all five possible credit regimes can occur for positive and finite initial wealth levels. In what follows, the analysis relates to (a) the credit regimes, i.e. to the possible optimal credit market activity of the household, given its initial wealth, and to (b) how the household’s decisions differ depending on the regimes. If no secured loan is available, the set-up reduces to TL-NO-SA, with only one loan with interest factor \(\iota\) in the former TL-regime.

Optimal capital investment and labor allocation depend on the subjective discount factor—

\(^8\)No functional forms need to be assumed.
and thus on initial wealth—as depicted by the first-order conditions

\[ h_l(1 + f) - w_{l-1} = 0, \]
\[ h_k(1 + f) - \tau = 0 \quad \text{for the TL-, EL- and OL-regimes, and} \]
\[ h_k(1 + f) - f = 0 \quad \text{for the NO- and SA-regimes.} \] (7)

The condition for optimal labor allocation applies in all credit-regimes. Labor is allocated to equate the discounted marginal returns to labor in low-skilled production with returns to labor in high-skilled production. The first first-order condition for capital investment only applies in the TL-, EL- and OL-regimes. The second only applies in the NO- and SA-regimes. The discount factor \( f \) in each regime is as depicted in the regime’s description. The first-order conditions for capital equate marginal discounted returns of capital in low-skilled production with the marginal costs of capital. Capital investment and labor allocation differ between the credit regimes, as marginal capital costs differ.

3 Interest Factors, Education and Labor Allocation

This section analyzes the impact of credit market improvements, in the form of lower interest factors, on the household’s time allocation between low-skilled labor and education.
First, some definitions that ease interpretation are introduced. We then sketch the calculations and present results. Results differ according to regimes such that the mechanisms that explain differences are characterized before a final interpretation is given. We then discuss how the inclusion of different simultaneous loans into the model changes results compared to the situation when only one loan is considered.

Since education is defined as $1 - l$, changes in education and low-skilled work, $l$, have opposite signs. We introduce $\alpha := h_l l / h$ as the output elasticity of labor in low-skilled production and $1 - \alpha := h_k k / h$ as the output elasticity of capital in low-skilled production. Further, we define the elasticity of substitution between capital and labor in low-skilled production by

$$\sigma := \frac{d \ln(l/k)}{d \ln(h_k/h_l)},$$

and the intertemporal elasticity of substitution in consumption by

$$\eta := \frac{d \ln(c_2/c_1)}{d \ln(u_1/u_2)}.$$

In principal, the elasticities do not need to be constant. They would be constant for e.g. a Cobb-Douglas production function in the low-skilled sector, with $\sigma = 1$ and $h(l, k) = l^\alpha k^{1 - \alpha}$.

To determine the direction of change in low-skilled work due to a change in interest factors, we take the total differential of the first-order conditions (7) and reduce it to

$$\text{sgn} \left( \frac{d\bar{v}}{di} \right) = \text{sgn}( -y_{li} y_{kk}^* + y_{ki} y_{lk}^* ),$$

with $\bar{v}$ denoting the time allocated towards low-skilled work in credit regime $j \in \{TL, EL, OL, NO, SA\}$. The interest factors are given by $i \in \{\phi, \tau, \iota\}$, and $y^*$ denotes the maximized Lagrangian with first derivatives $y_l^*$ and $y_k^*$. Appendix C provides a detailed derivation of the expression.

Table 1 summarizes the results for $d\bar{v}/di$. They differ with the credit regime and the affected interest factor (see Appendix D for a detailed derivation). Both results, more or
Table 1: Low-Skilled Work and Interest Factor Changes.

<table>
<thead>
<tr>
<th>Credit Regime</th>
<th>sgn (dl/dι)</th>
<th>sgn (dl/dτ)</th>
<th>sgn (dl/dφ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TL</td>
<td>&gt; 0</td>
<td>&lt; 0</td>
<td>= 0</td>
</tr>
<tr>
<td>EL</td>
<td>= 0</td>
<td>= sgn \left( \frac{f + \frac{2}{1 + f}}{1 + f} \right) - 1 - \frac{\log J}{\log J} \right)</td>
<td>= 0</td>
</tr>
<tr>
<td>OL</td>
<td>= 0</td>
<td>= sgn (τ - \frac{1 - \alpha}{\alpha})</td>
<td>= 0</td>
</tr>
<tr>
<td>NO</td>
<td>= 0</td>
<td>= 0</td>
<td>= 0</td>
</tr>
<tr>
<td>SA</td>
<td>= 0</td>
<td>= 0</td>
<td>= sgn \left( \phi - \frac{1 - \alpha}{\alpha} \right)</td>
</tr>
</tbody>
</table>

The proofs are in Appendix D.

less education due to a decrease in an interest factor, are possible. Certain parameter constellations make one or the other more likely.

The following mechanisms determine the results. A change in an interest factor may affect the costs of capital as well as discounting. The former leads to a change in production patterns, while the latter may affect both production and consumption:

The productivity effect \((-h_{lk})\) captures a change in production patterns due to a change in capital costs. Increasing capital costs reduce capital investment in the low-skilled sector, which reduces low-skilled labor productivity. Capital investment adjusts to changes in its market price \(\tau\) (in the TL-, EL-, OL-regimes) or opportunity costs \(\phi\) (in the SA-regime). If capital costs \(\tau\) or opportunity costs of capital \(\phi\) increase, capital investment, and thus low-skilled labor productivity, decrease. The productivity effect decreases low-skilled labor and increases education for an increase in capital costs.

The intertemporal effect \((h_k h_{lk} - h_l h_{kk} = d \log(h_l/h_k)/d \log(k/l))\) captures the effect of changes in subjective discounting on production patterns. If the subjective discount factor increases, future income is devaluated. Production factors are re-allocated to increase low-skilled income, which already occurs in the first period. The effect on \(l\) is determined via the inverse of the elasticity of substitution between capital and labor in low-skilled work. Higher capital costs decrease capital use, and labor increases to counteract the effect. The intertemporal effect increases low-skilled labor and decreases education for an increase in the discount factor.

The credit-constraint effect depicts a change in consumption pattern due to a change
in discounting. It only occurs if credit-constraints bind and intertemporal consumption allocation via credit markets is limited. The household uses labor reallocation to smooth consumption over time. Two counteracting sub-effects arise, both related to a change in the discount factor \( f \). The first sub-effect \( (f_\tau < 0) \) concerns higher capital costs, which imply that a larger amount has to be repaid in the second period. If the household has a relatively lower period-two income due to higher capital costs, the price \( f \) the household is willing to pay for an extra unit of a period-one good in terms of period-two goods decreases. However, higher capital costs imply a smaller loan amount, such that less has to be repaid in the second period. Thus, the second sub-effect is \(-f_k h_l > 0\), with a minus because the effect of \( \tau \) on \( k \) is negative.

Now consider the prevalence of the described effects for the different combinations of interest factor changes and credit regimes to explain the results in Table 1. In the TL-regime, the household discounts with \( f = \iota \) and faces capital costs \( \tau \). Using (8) to evaluate the impact of an increase in the unsecured interest factor \( \iota \) on time allocation in the TL-regime yields\(^9\)

\[
\text{sgn} \left( \frac{dI_{TL}}{d\iota} \right) = \text{sgn} \left( \left( h_k h_{lk} - h_l h_{kk} \right) (1 + \iota) \right) = \text{sgn} \left( \frac{(1 + \iota)}{\sigma} \right).
\]

Low-skilled work increases and education declines. This is a standard result, which is often stated in terms of lower interest rates that lead to an increase in education. A change in the unsecured interest factor \( \iota \) does not affect capital costs \( \tau \) (no productivity effect), but it affects discounting. Consumption patterns are not affected because the household is not credit-constrained (no credit-constraint effect), but production patterns are affected. Thus, the intertemporal effect determines the overall change. The time allocations in all other credit market regimes are unaffected by a change of \( \iota \) because capital costs and subjective discounting are independent of \( \iota \) in all but the TL-regime.

If, instead, the secured interest factor \( \tau \) in the TL-regime is increased, the labor allocation

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\(^9\)One could also omit \( 1 + \iota \) as it does not affect the sign. We kept the term to keep effects comparable with results in the EL-regime.
changes according to
\[
\text{sgn} \left( \frac{dl^{EL}}{d\tau} \right) = \text{sgn} \left( \frac{<0}{\text{productivity effect}} - h_{lk}(1 + \iota) \right)
\]
and low-skilled work decreases while education increases. The standard result does not hold anymore. Discounting is not affected and neither an intertemporal nor a credit-constraint effect occur. Since the capital costs $\tau$ are affected, the productivity effect is present and determines the overall change.

All three effects occur if the secured interest factor $\tau$ changes in the EL-regime. The household is credit-constrained and the secured interest factor depicts the capital costs.

The secured interest factor is also part of the subjective discount factor. Using (8) yields

\[
\text{sgn} \left( \frac{dl^{EL}}{d\tau} \right) = \text{sgn} \left( \frac{<0}{\text{productivity effect}} - h_{lk}(1 + \iota) \right)
\]

which becomes

\[
\text{sgn} \left( \frac{dl^{EL}}{d\tau} \right) = \text{sgn} \left( \frac{f + \frac{c_2}{f}}{\frac{1 + f}{1 + f} - 1 - \frac{\eta c_2}{fh}} \right)
\]

after some calculations (see Appendix D for details). Expression (10) is positive for a small elasticity of intertemporal substitution $\eta$ or a large elasticity of substitution in low-skilled production $\sigma$. It is negative for a large $\eta$ and a small $\sigma$. A small elasticity of substitution between capital and labor in low-skilled production implies a low substitutability between input factors: if capital were reduced due to an increase in $\tau$, it would be costly to compensate capital with low-skilled work. In this case, the productivity effect dominates and a higher interest factor leads to less low-skilled work and more education, so that the standard result does not hold. Furthermore, $\text{sgn} \left( \frac{dl^{EL}}{d\tau} \right) < 0$ for better developed credit markets with lower interest factors and $\text{sgn} \left( \frac{dl^{EL}}{d\tau} \right) > 0$ for less developed credit markets with higher interest factors (see Appendix F). A decrease in education due to a decrease in the interest factor $\tau$, i.e. the non-standard result, is more likely to occur in
economies with less developed credit markets.

In the OL-regime, the secured interest factor $\tau$ represents the discount factor as well as capital costs, and

$$\text{sgn} \left( \frac{dl^{OL}}{d\tau} \right) = \text{sgn} \left( \frac{h_k h_{lk} - h_l h_{lk}}{(1 + \tau)} \right)$$

which becomes

$$\text{sgn} \left( \frac{dl^{OL}}{d\tau} \right) = \text{sgn} \left( \tau - \frac{1 - \alpha}{\alpha} \right)$$

after rearrangement and substituting in the output elasticities of capital and labor.\textsuperscript{10} Since the household is not credit-constrained, only the intertemporal and the productivity effect occur. Both effects draw towards different directions, such that the change of low-skilled work is ambiguous. This is the usual argument put forward to explain mixed results. The increase of the secured interest factor leads to less education and more low-skilled work if the intertemporal effect dominates the productivity effect. This standard result is always obtained if $\alpha \geq 0.5$ (based on Equation (12) and $\tau > 1; 1/(1 + \tau) < 0.5$). Still, empirical estimates find that $\alpha \in [0.2, 0.8]\textsuperscript{11}$, such that for lower output elasticities, the effect of a higher interest factor on low-skilled labor is negative for $\alpha < 1/(1 + \tau)$ and positive for $\alpha > 1/(1 + \tau)$. As before, the decrease of the secured interest factor is more likely to decrease education—the non-standard result—on better developed credit markets with lower interest factors, given the same output elasticity of labor.

An increase in the secured interest factor in the NO- and the SA-regimes has no effect on time allocation, as it neither affects the subjective discount factor nor the capita costs in these regimes. The same applies for changes of the saving interest factor and time allocation in the TL-, EL-, OL- and NO-regimes. The effect of an increase in the interest factor for saving on time allocation in the SA-regime is equivalent to the effect of an increase in the

\textsuperscript{10} A detailed derivation is given in Appendix D

\textsuperscript{11} Mundlak (2001, 2005) report estimates of 0.25 - 0.45 from agriculture, including the share of land which accounts for 20-50%. Estimates in the range 0.6 - 0.8 are from labor shares of the fishery in Norway (Hannesson 2007) and aggregate production in the US (Acemoglu 2008, p. 57).
interest factor for borrowing secured by a collateral in the OL-regime. The household is not credit-constrained, and the interest factor on saving $\phi$ represents the discount factor as well as the opportunity costs of capital.

Summing up, $\partial l/\partial i > 0$ depicts the standard result that a lower interest rate leads to less child labor and more education. This result holds when the loan is not tied to investments or when labor markets are perfect. The ambiguous outcome of $\partial l/\partial \tau$ represents the case when the household only has an investment loan. We additionally show that $\partial l/\partial \tau < 0$ holds when the household has an additional loan. In the next section, we discuss how results change when common-pool externalities are also taken into account.

4 Common-pool Externalities

In this section, we additionally consider a common-pool resource externality. To do so, we consider a common-pool resource with poorly defined property rights. For well-defined and enforced property rights, the results are as before. Low-skilled production now depends on a regenerative resource, but high-skilled production remains resource-independent. A fixed number of potential resource users has unrestricted access to the resource and the number of resource users is sufficiently large to avoid strategic behavior. Low-skilled production of the representative household is given by $\bar{h}(l, k, x) = h(l, k)x$, with ‘effort’ $h(l, k)$ and the resource stock $x$. The multiplicative combination of effort and the resource stock is a common assumption in resource economics (Clark 2005; Conrad 2010; Hannesson 1983). Aggregate harvest reduces the stock size of the resource, such that $x$ is a function of aggregate labor $L$ and aggregate capital $K$ with $x_K < 0$ and $x_L < 0$. Due to the large number of resource users, each harvester neglects its individual effect on the resource stock, and the first-order conditions become

$$h_l(1 + f)x - w_{1-l} = 0,$$

$$h_k(1 + f)x - \tau = 0 \quad \text{for the TL-, EL- and OL- regimes and}$$

$$h_k(1 + f)x - f = 0 \quad \text{for the NO- and SA-regimes.}$$
Resource dynamics differ largely in their speed. Forest growth can be very slow whereas the dynamics of rangelands or some fish species are relatively fast compared to a human lifetime. We suppose a fast growing resource for our model and assume that the resource reaches its steady state immediately at the beginning of each period for a given level of aggregate harvesting effort. The resource stock will be the same in both periods because \( l \) and \( k \) do not differ between periods. To simplify the analysis further, we assume identical harvesters of mass one such that \( L = l \) and \( K = k \). We define the stock elasticity that measures the response of the steady state resource stock to harvesting effort as

\[
\varepsilon = -\frac{xh}{x}. \tag{14}
\]

The stock elasticity is a measure for the common-pool externality and increases with the impact of the harvest on the steady state stock size. The stock elasticity is high for resources with low reproduction rates and low density dependent mortality rates such as the ones for whales. It is low for resources with low depletion rates—such as fertile soil—or with high reproduction rates and high density dependent mortality rates—such as the ones of some fish species. There is no common-pool externality for \( \varepsilon = 0 \) and the results are as in Table 1.

The direction of the overall effect of interest factor changes on time allocation is still determined by (8), but with the first-order conditions (13) instead of (7). Although the individual harvester neglects his impact on the resource, his marginal productivity of capital and labor in (13) is still affected by changes in resource abundance. He thus takes into account that the resource reacts to effort changes resulting from interest factors changes. Table 2 summarizes the change in low-skilled work in response to interest factor changes for the different credit regimes. Appendix E derives the results.

The results of Table 2 differ from the results of Table 1 because of the resource effect \((-h_lx_k)\). There is a minus because the effect of \( \tau \) or \( \phi \) on \( k \) is negative. The overall effect is positive. The resource effect reflects the impact of changes in the resource stock size on marginal labor productivity in low-skilled production, and represents the negative
externality of harvesting. If capital investment in the harvesting sector decreases as a response to rising capital costs, the resource stock recovers and marginal labor productivity in low-skilled production increases. The resource effect therefore draws in the opposite direction than the productivity effect and only occurs when the productivity effect occurs. This implies that the resource effect only occurs in the TL-, EL- and OL-regime as a response to a change in the secured interest factor, as well as in the SA-regime as a response to a change in the savings interest factor. All other cases are unaffected by the introduction of a common-pool externality.

If the secured interest factor changes in the TL-regime, the direction of change in low-skilled work is determined by

$$\text{sgn} \left( \frac{dI_{TL}}{d\tau} \right) = \text{sgn} \left( -h_l x(1 + \iota) - h_l x_k(1 + \iota) \right),$$

which one can rearrange to

$$\text{sgn} \left( \frac{dI_{TL}}{d\tau} \right) = \text{sgn} \left( \varepsilon - \frac{1}{\sigma} \right).$$

The sign of $dI_{TL}/d\tau$ depends on the size of the two elasticities. A large stock elasticity implies that the resource recovers strongly after a reduction in total harvesting capital, which increases the marginal productivity of labor and increases the time allocated to low-skilled labor. A large substitution elasticity in the low-skilled sector implies that capital can easily be substituted by labor, which also increases the time that is allocated to low-skilled production. If both elasticities are below one, the sign is negative. If both elasticities are above one, it is positive, and the standard result holds. Given empirical estimates of $\varepsilon \in [0.2, 1.5]$ (see Appendix G for details) and $\sigma \in [0.5, 1]$ (Arrow et al. 1961; Klump et al. 2007), the sign is more likely to be negative than in the case without resource externality. The direction of change in labor allocated to low-skilled production in the EL-regime as a
response to a change of the secured interest factor is determined by

\[
\text{sgn} \left( \frac{dl}{d\tau} \right) = \begin{cases} 
>0 & \text{inter-temporal effect} \\
<0 & \text{productivity effect} \\
>0 & \text{resource effect}
\end{cases}
\]

which is equivalent to

\[
\text{sgn} \left( \frac{dl}{d\tau} \right) = \text{sgn} \left( \left( \frac{\tau}{(1 - \alpha)(1 + \tau)} - 1 \right) + \varepsilon \sigma \right).
\]

The expression is positive if the product of the substitution elasticity in low-skilled production \(\sigma\) and the stock elasticity \(\varepsilon\) is above 0.5 (see Figure 2 in Appendix H). The common-pool externality reduces the number of \(\tau - \alpha\) - combinations for which the non-standard effect occurs. As before and independent from \(\sigma\varepsilon\), the standard result holds if \(\alpha \geq 0.5\).

Results in the SA-regime are the same as in the OL-regime, with \(\phi\) instead of \(\tau\), as before.
Table 2: Low-Skilled Work and Interest Factor Changes with Common-pool Externalities.

<table>
<thead>
<tr>
<th>Regime</th>
<th>$\text{sgn}\ (dl/d\iota)$</th>
<th>$\text{sgn}\ (dl/d\tau)$</th>
<th>$\text{sgn}\ (dl/d\phi)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TL</td>
<td>$&gt; 0$</td>
<td>$=\text{sgn}\ (-1 + \varepsilon\sigma)$</td>
<td>$=0$</td>
</tr>
<tr>
<td>EL</td>
<td>$=0$</td>
<td>$=\text{sgn}\left(\sigma\left(f + \frac{\eta_2}{1+f}\right) - 1 - \eta_2 f h x + \varepsilon\sigma\left(\frac{\eta_2}{f h x} + \frac{1-\eta_2}{1+f}\right)\right)$</td>
<td>$=0$</td>
</tr>
<tr>
<td>OL</td>
<td>$=0$</td>
<td>$=\text{sgn}\left(\tau - \frac{1-\alpha}{\alpha} + \varepsilon\sigma(1+\tau)(1-\alpha)\right)$</td>
<td>$=0$</td>
</tr>
<tr>
<td>NO</td>
<td>$=0$</td>
<td>$=0$</td>
<td>$=0$</td>
</tr>
<tr>
<td>SA</td>
<td>$=0$</td>
<td>$=0$</td>
<td>$=\text{sgn}\left(\phi - \frac{1-\alpha}{\alpha} + \varepsilon\sigma(1+\phi)(1-\alpha)\right)$</td>
</tr>
</tbody>
</table>

The proofs are in Appendix E.
Our results with common pool externalities may—at first sight—suggest that common-pool regulation leading to an internalization of the externalities have an undesirable side effect for credit market policies: reducing common-pool externalities reduces the scope for credit market improvements that do not lower education. As it turns out, this is not the case. If one considers credit market improvements in the case of a regulated resource, the relevant case to consider is a situation when credit market improvements lead to a desired increase in harvest that is not allowed due to binding regulations. If regulations do not bind after credit market improvements, the results from Table 2 still apply.\textsuperscript{12} As the regulation implies that the resource stock, and therefore also the harvest, is kept constant, the only way the household can react is by changing the input composition of labor and capital. Two effects may occur. First, if relative prices change, the capital-labor input ratio changes. Lower capital costs lead to more capital and, to keep the harvest constant, less labor. This means more education. Second, a constraint effect may increase capital to increase the secured loan and thus present day consumption. Again, more capital leads to more education, because more capital means less unskilled labor to keep harvest constant.

Overall, a strong resource externality leads to a strong recovery of the resource as capital is withdrawn from resource harvesting, which has a positive effect on labor productivity in the low-skilled sector. Thus, a large common-pool externality, i.e. a large $\varepsilon$, leads to an increase of low-skilled labor with increasing interest factors. Compared to a situation without common-pool externality—in which $\partial l/\partial \tau < 0$ for a household with an additional loan and in which the outcome is ambiguous if the household had no additional loan—it is $\partial l/\partial \tau > 0$ for strong common pool externalities for different loan combinations. In other words, credit market reforms that lower interest factors are more likely to increase education in the presence of common-pool externalities.

\textsuperscript{12}It does not matter whether regulations bind before the credit market improvements.
5 Conclusion

Credit market improvements in developing economies are important, as they enable especially the poor to make investments to increase income. Many loans relate to business improvements and are tied to a productive investment. The loans tied to a productive investment usually come with lower interest rates, as the investments count as collateral. Education usually does not count as a collateral, because it is difficult to appropriate in case of default. This difference in treatment may lead to situations in which lower interest rates decrease education. From a long-run perspective and a social point of view, lower educational levels may not be warranted.

In general, the effect of a lowered interest rate of loans tied to a productive investment is ambiguous. For lower interest rates, an intertemporal effect increases the net present value of education. At the same time, labor productivity is increased by the investment in the business, making education less attractive. We add to this results by taking the empirical observations into account that (a) informal credit markets are fragment and households often have several loans with differing interest rates and that (b) many poor households rely on harvesting common-pool resources to generate income. Thereby, we identify situations in which one of the described mechanisms dominates. This allows us to better predict how a change in an interest rate impacts education.

When a household has a loan tied to a productive investment and an additional loan that is not tied to a productive investment and therefore comes with a higher interest rate, the intertemporal effect is governed by the high interest rate, while the labor productivity effect is related to the lower interest rate. Lowering the lower interest rate further increases investment and thus labor productivity, but has no impact on the net present value of education. Overall, education will go down. For the case of common-pool resources without properly defined and enforced use rights, a lower interest rate is more likely to increase education. The reason is that resource degradation negatively impacts labor productivity. When natural resource become degraded, outside options become even more important.

For policy design, our results suggest the following. If no educational externalities are
present, interest rate reductions that decrease capital costs and increase investment in low-skilled production improve the welfare of the current generation. The policy may, however, negatively affect future generations if higher capital investments increase child labor and decrease education. Credit market improvements are unequivocally beneficial only if their negative effect on subjective discounting outweighs the positive effects on child labor productivity, and education increases. Policies like the introduction of use rights, an improved labor market and additional incentives for education—or at least better compatibility of education and child labor as a first step—are important to complement credit market improvements.

References


Appendix

A Derivation of the Five Credit Regimes

From the Kuhn-Tucker Conditions (5) and definition (6), it follows that

\[
\begin{align*}
(A) & \quad v = 0 \text{ or } f = \tau + \frac{\lambda_3}{\lambda_2}, \\
(B) & \quad z = 0 \text{ or } f = \iota, \\
(C) & \quad s = 0 \text{ or } f = \phi, \\
(D) & \quad \lambda_3 = 0 \text{ or } v = k.
\end{align*}
\]

Table 3 lists the 16 possible combinations between A, B, C and D and shows that only five combinations (TL, EL, OL, NO and SA) do not lead to contradictions.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>Credit regime</th>
<th>Contradictions</th>
</tr>
</thead>
<tbody>
<tr>
<td>v = 0</td>
<td>f = \tau + \lambda_3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>w = 0</td>
<td>f = \iota</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>s = 0</td>
<td>f = \phi</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>\lambda_3 = 0</td>
<td>v = k</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

R1: \( f = \iota \) and \( f = \phi \) (B=1 and C=1) is not possible since \( \iota \neq \phi \).
R2: \( f = \tau + \lambda_3 \) and \( f = \phi \) (A=1 and C=1) is not possible since \( \tau > \phi \) and \( \lambda_3 \geq 0 \).
R3: \( \lambda_3 = 0 \) and \( f = \iota \) (B=1 and D=0) is not possible since \( \lambda_3 > 0 \) is needed to fulfill \( \iota = f \leq \tau + \lambda_3 \) and \( \iota > \tau \).
R4: \( v = 0 \) and \( v = k \) is not optimal because it implies \( k = 0 \), which is ruled out by the Inada Conditions.
B Relation between Household’s Discount Factor, Credit Market Regimes and Initial Wealth Level

We show that \( f(\kappa) \) is a non-increasing continuous function in \( \mathbb{R}_{+0} \). Define \( Z(\kappa) = \sup_{c_1,c_2,q} \{ u(c_1,c_2) \mid c_1-g_1(q) \leq \kappa, c_2-g_2(q) \leq 0, q \geq 0 \} \) with the vector \( q \) of an arbitrary dimension. Furthermore, \( g_1 = h + v + z - k - s \) and \( g_2 = h + w + \phi s - \tau v - \iota z \), both concave, such that \( Z \) is also concave. Take \( f(\kappa) := \frac{u_1}{u_2} \), evaluated at \((c_1^*,c_2^*)\), for which the supremum is attained.

Consider \( \tilde{Z}(\kappa) = u(c_1,c_2^*) \) with \( c_1 = \kappa + g_1(q^*) \). Then, \( Z(\kappa) \geq \tilde{Z}(\kappa) \) with equality for \( \kappa = \kappa^* \), and \( \tilde{Z}'(\kappa^*) = u_1(c_1^*,c_2^*) \). Hence, \( Z_+(\kappa^*) \geq u_1(c_1^*,c_2^*) \leq Z_-(\kappa^*) \). Furthermore, as \( Z \) is concave, \( Z_-(\kappa^*) \geq Z_+(\kappa^*) \). Therefore,

\[
Z_+(\kappa^*) = Z_-(\kappa^*) = u_1(c_1^*,c_2^*).
\]

\( \frac{u_1}{u_2} \) is increasing in \( u_1 \) due to linear homogeneity of \( u \) such that it is non-increasing in \( \kappa \). The rest follows from the credit regimes’ definition.

C Derivation of Equation (8)

To determine the sign of \( dl/di \) with \( i \in \{\iota, \tau, \phi\} \), we use the implicit function theorem and Cramer’s Rule, i.e. \( dl/di = |H^{j,i}|/|H^j| \) for \( j \in \{TL, EL, OL, NO, SA\} \) with the Hessian Matrix\(^{13} \)

\( H^j \). We have

\[
H^j = \begin{bmatrix}
y_{li}^* & y_{l,k}^* \\
y_{kl}^* & y_{kk}^*
\end{bmatrix}
\quad \text{and} \quad
H^{j,i} = \begin{bmatrix}
-y_{li}^* & y_{l,k}^* \\
-y_{kl}^* & y_{kk}^*
\end{bmatrix}
\]

with \( i \in \{\iota, \tau, \phi\}, j \in \{TL, EL, OL, NO, SA\} \) and the Lagrangian at the optimum \( y^* \) (see equation (4)). A locally unambiguously defined optimum implies \( |H^j| > 0 \)\(^{14} \) such that the denominator only influences the size of the effect but not the direction, and \( |H^{j,i}| \) calculated in equation (8) determines the sign.

As example, take \( j = TL \) and \( i = \iota: y^* = u(c_1,c_2) + \lambda_1(h(l,k) + \kappa + z - c_1) + \lambda_2(h(l,k) + w(1-l) - c_2 - \iota z) \) with \( c_1,c_2,l,k \) and \( z \) being chosen optimally. Then, \( y_{li}^* = \lambda_1 h_l(l,k) + \lambda_2 (h_l(l,k) + w_l(1-l)) = 0 \) (based on optimality conditions). Insert \( \lambda_1/\lambda_2 = \iota \) and then take second derivatives.

D Proofs for Table 1

We use (8) and (7) to determine the direction of change of labor reallocation to low-skilled work as a response of rising interest factors.

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\(^{13}\)One can also think of it as the Jacobian of the first order conditions.

\(^{14}\)The objective function is concave if and only if the Hessian Matrix is semi definite (\(|H^j| \geq 0 \)). We assume a locally unambiguously defined optimum. Then, \(|H^j| = 0 \) occurs with probability zero.
Proof of $dl^j/d\eta$:

**TL:** For $j = TL$ and $f = \iota$:

$$\text{sgn} \left( \frac{dl^{TL}}{d\iota} \right) = \text{sgn}((h_k h_lk - h_l h_{kk})(1 + \iota)). \quad (16)$$

For a linear homogenous production function, the ratio of the marginal productivities is only a function of the factor input ratio such that

$$\frac{d(h_k/h_l)}{d(k/l)} \frac{k/l}{h_k/h_l} = \frac{d(h_k/h_l)}{dk} \frac{k/l}{h_k/h_l}.$$ 

The inverse elasticity of substitution between capital and labor in low-skilled production, $\sigma$, can therefore be represented by

$$\frac{1}{\sigma} = k \left( \frac{h_{lk}}{h_l} - \frac{h_{kk}}{h_k} \right) = l \left( \frac{h_{lk}}{h_k} - \frac{h_{ll}}{h_l} \right). \quad (17)$$

Using the Euler Equation $h_k k + h_l l = h$ and its first derivative with respect to $k$ yields

$$\frac{1}{\sigma} = \frac{h_{lk} h}{h_{lk} h_k}.$$ 

Multiplying both sides of (16) with $k/[(1 + \iota)h_k h_l]$ and using (17) gives

$$\text{sgn} \left( \frac{dl^{TL}}{d\iota} \right) = \text{sgn} \left( \frac{1}{\sigma} \right).$$

**EL, OL, NO, SA:** For $j \in \{EL, OL, NO, SA\}$ and $f = \iota$, $y^*_{hl} = y^*_{lk} = 0$ such that $dl^j/d\iota = 0$.

Proof of $dl^j/d\tau$:

**TL:** For $j = TL$ and $f = \iota$:

$$\text{sgn} \left( \frac{dl^{TL}}{d\tau} \right) = \text{sgn}(-h_k(1 + \iota)).$$

**EL:** For $j = EL$ and $f = \tau + \lambda_3/\lambda_2$:

$$\text{sgn} \left( \frac{dl^{EL}}{d\tau} \right) = \text{sgn} \left( f_r(h_k h_{lk} - h_l h_{kk})(1 + f) - h_{lk}(1 + f) - f_k h_l \right). \quad (18)$$

The inverse elasticity of intertemporal substitution in consumption can be expressed by

$$\frac{1}{\eta} = \frac{d \ln(u_1/u_2)}{d \ln(c_2/c_1)} = \frac{f c_2 c_2}{f} = \frac{f c_1 c_1}{f} = -\frac{u_{22} c_2}{u_2} = -\frac{u_{11} c_1}{u_1}.$$

(19)
The derivation of this expression is analogous to (17).

Calculate

\[ f_k = \frac{u_2 u_{11} h_k - u_1 u_{22} (h_k - \tau)}{u_2} = \frac{u_{11} h_k u_{1c1}}{u_2} - \frac{u_1 u_{22} (h_k - \tau) c_2}{u_2 u_2 c_2}. \]

Inserting the elasticity of intertemporal substitution and \( f = \frac{u_1}{u_2} \) gives

\[ f_k = -f \frac{h_k c_2}{\eta c_2} + f \frac{h_k - \tau}{\eta c_2}. \]

Replacing \( h_k - \tau \) by \( -f h_k \) from (7) yields

\[ f_k = -f \frac{h_k c_2}{\eta c_2} - f \frac{f h_k}{\eta c_2}. \]

Inserting \( f_{c2} = f/(\eta c_2) \) from (19) yields

\[ f_k = -f c_2 h_k (\frac{c_2}{c_1} + f). \]

The derivative of the subjective discount factor with respect to the interest factor \( \tau \) can be expressed as

\[ f_\tau = \frac{u_1 u_{22} k}{u_2} = -f c_2 k. \]

Multiply (18) by \( 1/(h_l h_k) \), use the elasticity of substitution (17) and insert \( f_k = -f c_2 h_k (c_2/c_1 + f) \) and \( f_\tau = -f c_2 k \) to obtain

\[
\text{sgn} \left( \frac{dl}{d\tau} \right) = \text{sgn} \left( f_\tau \left( \frac{h_{lk}}{h_l} - \frac{h_{kk}}{h_k} \right)(1 + f) - \frac{f_{c2}}{h_{lk}} (1 + f) - f_k \right).
\]

\[
= \text{sgn} \left( -f c_2 \frac{h_{lk}}{h_l} (1 + f) - \frac{h_{lk}}{h_l} (1 + f) + f_{c2} \frac{c_2}{c_1} (1 + f) \right)
\]

\[
= \text{sgn} \left( -f c_2 \frac{c_2}{c_1} (1 + f) - \frac{1 + f}{h_{lk}} + f_{c2} (f + \frac{c_2}{c_1}) \right).
\]

Multiply with \( \sigma/(f_{c2}(1 + f)) \) and re-arrange to obtain

\[
\text{sgn} \left( \frac{dl}{d\tau} \right) = \text{sgn} \left( -\frac{1}{h_{lk}} + \frac{\sigma f + c_2^2}{1 + f} \right) = \text{sgn} \left( \frac{\sigma f + c_2^2}{1 + f} - 1 - \frac{\eta c_2}{hf} \right).
\]

**OL**: For \( j = OL \) and \( f = \tau \):

\[
\text{sgn} \left( \frac{dl^{OL}}{d\tau} \right) = \text{sgn} \left( (h_k h_{lk} - h_l h_{kk})(1 + \tau) - h_{lk}(1 + \tau) \right). \tag{20}
\]

Multiplying (20) with \( ((1 + \tau)h_{lk} h_k)^{-1} \) leads to

\[
\text{sgn} \left( \frac{dl}{d\tau} \right) = \text{sgn} \left( 1 - \frac{h_{lk} h_{kk}}{h_{lk} h_k} - \frac{1}{h_k} \right).
\]

31
Note that the first-order condition for capital in (7) leads to
\[ \frac{1}{h_k} = \frac{1}{\tau} + 1. \] (21)

Now insert the first derivation of the Euler Equation of the production function (see (17)) and (21) to obtain
\[ \text{sgn} \left( \frac{dl}{d\tau} \right) = \text{sgn} \left( \frac{h_l}{kh_k} - \frac{1}{\tau} \right) = \text{sgn} \left( \frac{kh_k}{lh_l} \right) = \text{sgn} \left( \tau - \frac{1 - \alpha}{\alpha} \right). \]

**NO, SA:** For \( j \in \{NO, SA\} \) and \( f = \tau \), \( y^*_{l\tau} = y^*_{k\tau} = 0 \) such that \( dl/d\tau = 0 \).

**Proof of \( dl^j/d\phi \):**

**TL, EL, OL, NO:** For \( j \in \{TL, EL, OL, NO\} \) and \( f = \phi \), \( y^*_{l\phi} = y^*_{k\phi} = 0 \) such that \( dl^j/d\phi = 0 \).

**SA:** The derivation of \( dl^{SA}/d\phi \) is equivalent to \( dl^{OL}/d\tau \), i.e.
\[ \text{sgn} \left( \frac{dl}{d\phi} \right) = \text{sgn} \left( \frac{1 - \alpha}{\alpha} \right). \]

**E Proofs for Table 2**

We use (8) and (13) to determine the direction of change of labor reallocation to low-skilled work as a response of rising interest factors.

**Proof of \( dl^j/d\iota \):**

**TL:** For \( j = TL \) and \( f = \iota \):
\[ \text{sgn} \left( \frac{dl^{TL}}{d\iota} \right) = \text{sgn} \left( h_k x (h_k x + h_l x_k) (1 + \iota) - h_l x (h_k k x + h_l x_k) (1 + \iota) \right) = \text{sgn} \left( (h_k x h_k x - h_l x h_k x_k) (1 + \iota) + (h_k x h_l x_k - h_l x h_k x_k) (1 + \iota) \right) = \text{sgn} \left( (h_k x h_k x - h_l x h_k x_k) (1 + \iota) \right), \]
which is equivalent to
\[ \text{sgn} \left( \frac{dl^{TL}}{d\iota} \right) = \text{sgn} \left( \frac{1}{\sigma} \right). \]

**EL, OL, NO, SA:** For \( j \in \{EL, OL, NO, SA\} \) and \( f = \iota \), \( y^*_{l\iota} = y^*_{k\iota} = 0 \) such that \( dl^j/d\iota = 0 \).
Proof of $dl^j/d\tau$:

**TL:** For $j=\text{TL}$ and $f = \tau$:

$$\text{sgn} \left( \frac{dl^\text{TL}}{d\tau} \right) = \text{sgn}(-h_k x (1 + \iota) - h_l x_k (1 + \iota)).$$

Multiplication with $k/[(1 + \iota)h_l x]$ gives

$$\text{sgn} \left( \frac{dl^\text{TL}}{d\tau} \right) = \text{sgn} \left( -h_k \frac{k}{h_l} - x_k \frac{k}{x} \right).$$

Expansion by $hh_k/(hh_k)$ yields

$$\text{sgn} \left( \frac{dl^\text{TL}}{d\tau} \right) = \text{sgn} \left( -h_k \frac{k}{h_l} h_k - x_k \frac{k}{x} h_k \right).$$

Replacing $x_k$ by $x_h h_k$, multiplying by $h/(hh_k)$ and using the elasticities (17) and (14) gives

$$\text{sgn} \left( \frac{dl^\text{TL}}{d\tau} \right) = \text{sgn} \left( h_k \frac{k}{h_l} \left( -hh_k \frac{h}{h_l} - x_h \frac{h}{x} \right) \right) = \text{sgn} \left( \frac{-1}{\sigma} + \varepsilon \right).$$

**EL:** For $j=\text{EL}$ and $f = \tau + \lambda_3/\lambda_2$:

$$\text{sgn} \left( \frac{dl^\text{EL}}{d\tau} \right) = \text{sgn}(-f_t h_l x((1 + f)(h_k x + h_k x_k) + f_k h_k x) + (h_k x f_t - 1)((1 + f) h_l x + h_l x_k) + f_k h_l x)$$

$$= \text{sgn}(-f_t h_l x(f_t h_k x - f_t h_k x_k) - f_t h_l x f_k h_k x + h_k x f_t h_l x - (1 + f) h_l x_k - h_l x_k - f_k h_l x)$$

$$= \text{sgn}(-f_t x^2(1 + f)) (h_k h_l h_k) - h_k h_l) - (1 + f) h_l x_k - (1 + f) h_l x_k - f_k h_l x). \quad (22)$$

Calculate

$$f_t = \frac{k u_2 u_1}{u_2^2} = -\frac{k f}{\eta c_2} \quad (23)$$

and

$$f_k = \frac{u_2 u_1}{u_2^2} (h_k x + h_k x_k) - u_1 u_2 (h_k x + h_k x_k - \tau)$$

$$= \frac{u_1 (h_k x + h_k x_k) u_1 c_1}{u_2^2} - \frac{f u_2 (h_k x + h_k x_k - \tau) c_2}{u_2 c_2}$$

$$= -\frac{f}{\eta} \frac{(h_k x + h_k x_k) h_k}{c_1} + \frac{f}{\eta} \frac{(h_k x + h_k x_k - \tau) c_2}{c_2}$$

$$= \frac{f h_k (x + h_k x)}{\eta} \left( -\frac{1}{c_1} + \frac{1}{c_2} \right) - \frac{f \tau}{\eta c_2} \quad (24)$$

Multiply (22) with $(h_l h_k x^2(1 + f))^{-1}$, rearrange (17) to get $h_k h_l h_k = -hh_k/k$ and (13) to obtain $h_k x(1 + f) = \tau$, and insert both expressions together with (24) and (23) into (22), replace
\[ x_k \text{ by } x_h h_k, \text{ and rearrange (22) to obtain} \]

\[ \text{sgn} \left( \frac{dl^E \left( d\tau \right)}{dx_k} \right) = \text{sgn} \left( \frac{f_x h h_k}{h_l h_k} - \frac{h_k}{h_l h_k x} - \frac{x_k}{h_k x^2} - \frac{f_k}{(1 + f) h_k x} \right) \]

\[ = \text{sgn} \left( -\frac{f_x h h_k}{\eta c_2 h h_k} - \frac{h_k}{h_l h_k x} - \frac{x_k}{h_k x^2} - \frac{f_k}{\eta(1 + f) h_k x} \right) (x + h x_h) \left( -\frac{1}{c_1} - \frac{1}{c_2} \right) + \frac{f \tau}{\eta c_2(1 + f) h_k x} \]

\[ = \text{sgn} \left( -\frac{f}{\eta c_2} \sigma + \frac{1}{\sigma h x} + \frac{\varepsilon}{h x} - \frac{f(1 - \varepsilon)}{(1 + f) h x} \right) \left( \frac{1}{c_1} - \frac{1}{c_2} \right) + \frac{f}{\eta c_2} \]

\[ = \text{sgn} \left( \frac{f}{\eta c_2} \left( \frac{1}{\sigma} - \frac{1}{1 + f} \right) \left( \frac{1}{c_1} - \frac{1}{c_2} \right) + 1 \right) - \frac{\tau}{\sigma} \varepsilon. \]  

(25)

Multiply with \( \sigma \eta c_2 / f \) and 'isolate' \( \varepsilon \) to obtain

\[ \text{sgn} \left( \sigma \left( \frac{f + \frac{\varepsilon}{\sigma}}{1 + f} \right) - \frac{\eta c_2}{fh x} - 1 + \varepsilon \sigma \left( \frac{\eta c_2}{fh x} + \frac{1 - \frac{\varepsilon}{c_1}}{1 + f} \right) \right). \]

OL: For \( j = EL \) and \( f = \tau \)

\[ \text{sgn} \left( \frac{dl^{OL} \left( d\tau \right)}{dx_k} \right) = \text{sgn} \left( -h_l x (h_{kk} x + h_k x_k)(1 + \tau) + (h_k x - 1)(h_l k x + h_l x_k)(1 + \tau) \right) \]

\[ = \text{sgn} \left( -h_l x h_{kk} x - h_l x h_k x_k + h_k x h_l k x + h_k x h_l x_k - h_l k x - h_l x_k \right)(1 + \tau) \]

\[ = \text{sgn} \left( -h_l x h_{kk} x + h_k x h_l k x - h_l k x - h_l x_k \right)(1 + \tau) \]

\[ = \text{sgn} \left( -x^2 (h_l h_{kk} x - h_l h_k x_k) + h_l k x + h_l x_k \right)(1 + \tau) \].

Then use the relationship \( h_{kk} h_l - h_l h_{kk} = -h_{lk} / k \) from (17) and the elasticity of substitution, insert \( x_k = x_h h_k \) and divide by \( h_l x, h_k x \) and \( (1 + \tau) \) to obtain

\[ \text{sgn} \left( \frac{dl^{OL} \left( d\tau \right)}{dx_k} \right) = \text{sgn} \left( \frac{h_l h_k}{h_k h_l} \left( \frac{h_{kk} x}{h_l h_k x} - \frac{x_k}{h_k x^2} \right) \right) = \text{sgn} \left( \sigma^{-1} \left( \frac{h_l k x}{h_k x} - \frac{x_k}{h_k x^2} \right) \right) \]

\[ = \text{sgn} \left( \sigma^{-1} \left( \frac{h_l x}{h_k x} - 1 \right) - \frac{x_k h_k h}{h_k x^2} \right) = \text{sgn} \left( \sigma^{-1} \left( \frac{h_l x}{h_k x} - 1 \right) + \varepsilon \right). \]

Using the output elasticity of capital \( 1 - \alpha \) and the relation \( h_k x = \tau / (1 + \tau) \) from (13) yields

\[ \text{sgn} \left( \frac{dl^{OL} \left( d\tau \right)}{dx_k} \right) = \text{sgn} \left( \frac{\sigma}{\sigma} \left( \frac{\tau}{(1 - \alpha)(1 + \tau)} - 1 \right) + \varepsilon \right). \]

Multiply with \( \sigma \) to attain

\[ \text{sgn} \left( \frac{dl^{OL} \left( d\tau \right)}{dx_k} \right) = \text{sgn} \left( \frac{\tau}{(1 - \alpha)(1 + \tau)} - 1 + \varepsilon \sigma \right). \]
NO, SA: For \( j \in \{NO, SA\} \) and \( f = \tau, y^*_\tau = y^*_k = 0 \) such that \( dl^j/d\tau = 0 \).

Proof of \( dl^j/d\phi \):

TL, EL, OL, NO: For \( j \in \{TL, EL, OL, NO\} \) and \( f = \phi, y^*_\phi = y^*_k = 0 \) such that \( dl^j/d\phi = 0 \).

SA: The derivation of \( dl^{SA}/d\phi \) is equivalent to \( dl^{OL}/d\tau \), i.e.

\[
\text{sgn}(dl^{SA}/d\phi) = \text{sgn}(\phi/[\sigma(1 - \alpha)(1 + \phi)] - \varepsilon).
\]

F Note on \( \text{sgn} \left( \frac{dl^{EL}}{d\tau} \right) \)

Assuming a constant intertemporal elasticity of substitution with a utility function of the form \( u(c) = c^{1-\eta}/(1 - \eta) \), one can replace \( c_2/c_1 \) by \( f^{1/\eta} \) in (10). One can also replace \( c_2/h \) by \( (1 + \kappa/h)f^{1/\eta} \). Then, equation (10) can be written as

\[
\text{sgn} \left( \frac{dl^{EL}}{d\tau} \right) = \text{sgn} \left( \sigma \left( \frac{f + f^{1/\eta}}{1 + f} \right) - 1 - \frac{\eta}{f}(1 + \kappa/h)f^{1/\eta} \right).
\]

Initial wealth over income in the low-skilled sector can be seen as some kind of poverty measure which is low for poor households and zero for the poorest households. Empirical estimates suggest \( \eta \in [0.3, 0.8] \) (Attanasio and Weber 1993, 1995; Ogaki and Reinhart 1998) and \( \sigma \in [0.5, 1] \) (Arrow et al. 1961; Klump et al. 2007). \( \text{sgn} \left( \frac{dl^{EL}}{d\tau} \right) < 0 \) for lower interest factors and \( \text{sgn} \left( \frac{dl^{EL}}{d\tau} \right) > 0 \) for higher interest factors.

G Calculation of the Stock Elasticity

To calculate the stock elasticity, \( \varepsilon \), we assume a logistic growth function for the resource (Clark 2005), where the steady state resource stock is given by \( \bar{x} = \kappa \left( 1 - \frac{h}{\rho} \right) \). The parameters \( \rho \) and \( \kappa \) are the intrinsic growth rate and carrying capacity, respectively. The first derivative of the steady state resource stock with respect to effort, \( h \), is \( \bar{x}_h = -\kappa/\rho \). The relation of effort to harvesting is \( h(l, k) = \bar{h}(l, k, x)/x \), a common assumption in resource economics (Clark 2005; Perman et al. 2011). Using the steady state stock and its derivative in equation (14) yields

\[
\varepsilon = \left( \frac{\rho}{h} - 1 \right)^{-1}.
\]

We use fish species to estimate the stock elasticity as they are fast growing and reach their steady state quickly in comparison with forest, for example. Current harvests and stock sizes may thus be close to the steady state values. Inserting the current stock size, the current harvest and the intrinsic growth rate from Quaas et al. (2012) and Noack et al. (2015) into (27) results in \( \varepsilon \in [0.2, 1.5] \).
The Role of the Resource Externality in the OL-Regime

Figure 2 illustrates how \( \text{sgn}\left(\frac{dl^{OL}}{d\tau}\right) \) depends on \( \tau \), \( \alpha \) and different combinations of the substitution elasticity in low-skilled production \( \sigma \) and the stock elasticity \( \varepsilon \). The lines show the \( \alpha-\tau \)-combinations for different values of \( \varepsilon \sigma \) for which \( \text{sgn}\left(\frac{dl^{OL}}{d\tau}\right) = 0 \). Combinations above the lines mean \( \text{sgn}\left(\frac{dl^{OL}}{d\tau}\right) > 0 \) and below the lines \( \text{sgn}\left(\frac{dl^{OL}}{d\tau}\right) < 0 \).

Figure 2: Effect of a Change in \( \tau \) on Labor Allocation in the OL-Regime for Different \( \alpha-\tau \) Combinations.

d\( l^{OL}/d\tau < 0 \) left of the respective line, \( dl^{OL}/d\tau > 0 \) right of the respective line, given \( \sigma \varepsilon \).
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