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Abstract

We study the feasibility and efficiency of policy reforms in polarized democracies. We develop a simple election model where *(i)* reforms are costly for voters and politicians and these costs increase with the extent of policy change, and *(ii)* politicians differ in their ability to carry out reforms efficiently. We identify a so-called *Reform Dilemma*, which manifests itself in two variants. From a static perspective, low-reform-ability politicians may be elected, who impose high costs on citizens for each reform step. From a dynamic perspective, incumbents may choose socially undesirable policies to align the social need for reform with their own reform ability and are thus re-elected regardless of their reform ability. In general, both manifestations of the Reform Dilemma are more pronounced when political parties' positions are polarized. Furthermore, the existence of the Reform Dilemma is independent of the exact point in time when the abilities of candidates reveal themselves and become common knowledge.

Keywords: Elections; democracy; costs of reform; political polarization;

JEL Classification: D72, D82

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1 Introduction

Motivation

Within any democracy, changes in the environment, in technologies, in the delivery of services, and in demography demand continuous policy changes. Well-known examples can be found in most policy areas: *(i)* the adjustment of retirement and social care systems whenever a society is aging; *(ii)* the necessity for new financial regulation once it has become apparent that banking systems are too fragile; *(iii)* heightened security after a terrorist attack, such as 9/11; *(iv)* a phase-out of nuclear energy, as in Germany after the Fukushima incident; *(v)* investments in information security once the use of the internet has spread and become crucial to modern societies; *(vi)* winding down the Iraq War after it had became clear that the benefits for the US were lower than predicted.

Policy changes may be desirable because they bring about significant benefits for the citizenry. Unlike maintaining the status quo, however, undertaking policy reforms typically also entails so-called *costs of change*.[1] Policy reforms may involve substantial investments in physical and human capital from the public sector, which is financed by the taxes paid by all citizens. The government may have to invest substantial time and resources to implement the reform, thereby creating opportunity costs for the citizens because politicians cannot spend time on other governmental tasks. Lastly, regulating industries may generate higher wages and prices.

As a general rule, the larger the reforms are, the higher the costs of change. What is more, such costs are not typically exogenous, as able politicians may engineer policy changes more efficiently than less able ones. Able office-holders may be more influential and achievement-oriented, make fewer mistakes, need fewer resources to implement policies, or impose lower costs on the private sector. According to Galasso and Nannicini (2011), Jones and Olken (2005), and Dewan and Myatt (2008), the identity of leading politicians is a determinant for the policies eventually pursued.[2] In the US, in particular, the President's influence on economic growth is, on average, significant, and is highest for high-quality Presidents—see Rohlfs et al. (2015) and the references therein. It thus comes as no surprise that the politicians' ability to carry out policy reforms may influence electoral outcomes.[3]

[1] Throughout the paper, we use the terms "policy change" and "policy reform" as synonyms. We refer to Gersbach et al. (2015) for a detailed description of policy reforms for which the costs of change increase with the extent of the reform.

[2] In general, the impact of leadership capacity on team performance is significant (Day et al., 2004), especially in the face of a disruptive external change and subsequent crisis (Combe and Carrington, 2015).

[3] See Pillai and Williams (1998), Williams et al. (2009), and Williams et al. (2012) for the relationship

Model and results

We augment a standard two-period election model by two features. First, the ideal point of the median voter shifts over time, and adjusting or reforming policies is costly. Second, the costs of reform depend on the office-holder's ability to engineer such changes.

Our main finding is the identification of two sources of inefficiency in democracy. On the one hand, candidates with low reform ability may be elected if the candidates' desire for reform differs from the median voter's and a majority of voters fears that high-reform-ability candidates will undertake excessively large reforms. Low-reform-ability office-holders will nonetheless impose high costs on citizens, even for smaller reforms, as they will carry out these reforms inefficiently. On the other hand, once high-reform-ability candidates have succeeded in entering office, they will choose polar policies that make future reform needs particularly large, thereby making it attractive to re-elect them. Both these weaknesses of elections become more pronounced when political parties are more polarized. Accordingly, we call these two properties the *Reform Dilemma of Polarized Democracies.*

We explore these properties of democratic elections in several variants of the model, and we provide a series of formal results that shed light on how the Reform Dilemma manifests itself both in static and dynamic versions of the model and under different assumptions about the revelation of candidates' reform ability. Specifically, we first show in the static model that the chances for a low-reform-ability candidate to be elected are high in polarized societies, no matter whether his ability is already common knowledge at the first election campaign stage, only after he has entered office, or only revealed towards the end of his first term.

Second, we show in the dynamic version of the model that an office-holder's incentives to align the social need for reform with his own ability by choosing more or less extreme policies occur independently of the moment when the challenger's reform ability reveals itself. Except in Section 6.4, we assume throughout that reform ability becomes common knowledge as soon as it reveals itself. If reform ability reveals itself—and is thus commonly known—particularly late, the policy choice of candidates involves a gambling element, as they do not know whether polar or moderate policy choices will foster their re-election chances.

Our analysis provides further insights. First, a first-order improvement in candidate reform-ability distribution may reduce welfare. Second, we show that incumbents may have incentives to deliberately and irrevocably lower their reform ability in order to obtain re-election. Third,

between a candidate's charisma and voting behavior in US presidential elections, especially in the context of a crisis.

the main thrust of our results is not altered if a candidate's reform ability is also interpreted as pure valence and voters care about the reform ability of politicians per se. Fourth, if a candidate's reform ability is private information (to him) before the end of his first term, such a candidate might strategically exploit this fact, but only if we take a static perspective and not a dynamic one. Fifth, *ceteris paribus*, ideological moderation increases the candidates' election chances. Sixth and last, in both the static and the dynamic versions of the model, we find that earlier revelation of the candidate's reform ability may not enhance welfare measured in utilitarian terms. Especially in polarized societies, early knowledge about the reform ability may be socially undesirable.

The Reform Dilemma in practice

Although we focus on the conceptual understanding of the two manifestations of the Reform Dilemma, empirical observations can be related to the predictions of our results. One such prediction is that more able candidates are elected when the economy performs badly, and thus when large reforms may be needed. Based on several studies, Simonton (2006) estimated the so-called *intellectual brilliance (IB)* of all the Presidents of the US, which can serve as a proxy for their reform ability. Since 1928, the correlation between the Presidents' intellectual brilliance and the unemployment rate when each President was elected for his first term is 0.38.[4] Moreover, when the challenger was able to defeat the incumbent, the latter's IB was lower than the former's, who typically carried out a large reform agenda meant to improve the economy. One example could be 1992, when Bill Clinton (IB=1.0) defeated George H. Bush (IB=-0.3) and the unemployment rate was relatively high, namely 7.4%.[5]

A second prediction of our model is that office-holders may take decisions that are disliked by a majority of their own supporters, let alone by a majority of the whole electorate, in order to present themselves as the remedy to the self-created problem. There is a variety of prominent examples. For the US, it can be argued that the escalation of the Vietnam and Iraq wars by the Republican party, and pro-affirmative actions by the Democrats fall into this category (Glazer et al., 1998). More recently, the Spanish government tried to pass a reform on the abortion opposed by almost 90% of the electorate, but it finally abandoned the reform after presenting itself as the guarantee that there would not be further substantial changes in the

[4]Sources for the unemployment rate: United States Census Bureau (1928-1932) and Bureau of Labor Statistics (1948-2000). For 1929 and 1933, only annual averages of unemployment are available.

[5]For the US, Blinder and Watson (2013) establish that the President's affiliation is a good predictor for the performance of the economy. Our paper suggests a refinement in the sense that such differences might in fact be the consequence of the Republican and Democratic parties systematically differing in their ability to select able candidates.

law. Chancellor Kohl initiated the introduction of the Euro, a decision that was opposed by a majority of Germans at that time. In the subsequent elections, he presented himself as the most able politician to guarantee that the new currency would be stable.

The number of examples of policies undertaken by governments that harm their own constituency more than the electorate as whole, in particular, can be significantly extended. We refer to Saint-Paul et al. (2015) for a full account of this political phenomenon from both an empirical and theoretical viewpoint. To provide further empirical support for our model, let us mention here the following examples from their paper: the liberal policies supported by left-wing parties in many European countries, the support for NAFTA from the Clinton administration, and the reluctance of many left-wing Latin American governments to abandon existing pro-globalisation policies. The implementation of all these policies is consistent with the second prediction of our model.

Organization of the paper

The remainder of the paper is organized as follows: In Section 2 we review the research connected with our article. In Section 3 we outline our baseline set-up. In Section 4 we study and solve a static model of elections. In Section 5 we study and solve a dynamic model of elections. In Section 6 we analyze some extensions of our baseline set-up. In Section 7 we study the welfare implications of the exact timing where the candidates' reform ability is revealed. Section 8 concludes. All the proofs are contained in Appendices A, B, C, and D.

2 Broader Perspective and Relation to the Literature

Our paper is related to several strands of the literature, which enables us to put our results into a broader perspective.

Electoral competition

Our model of candidate competition for winner-take-all elections has features in common with the standard literature on electoral competition. On the one hand, as in Krasa and Polborn (2010a,b), each candidate is both office-motivated and policy-motivated and, additionally, has some exogenous characteristics, which can be broadly interpreted as *valence* (Stokes, 1963). In our model, candidates are ex-ante—but not ex-post—equal regarding their ability to implement policy reforms efficiently.

On the other hand, we consider a one-dimensional policy space on which agents have dissenting preferences. More specifically, voters and candidates (which belong to one of two political parties) have quadratic—and hence single-peaked—preferences over the policies chosen in each of the two periods. Moreover, due to the existence of reform costs, voters and candidates also care about the difference between the policies chosen in two consecutive periods.[6]

Our results show that competition between candidates by means of elections is not sufficient to ensure that socially efficient policies are implemented. To some extent, this is in contrast with Wittman (1989), who argues that political markets tend to yield efficient outcomes when agents are rational and there are no poorly-informed citizens (relative to the politicians).

Costs of reform in policies

To date, few models in the literature have considered the costs associated with change in policies. Glazer et al. (1998) show that when policy changes bring about a large fixed cost, the incumbent will obtain an electoral advantage by choosing a policy beyond his own ideal policy before the election, thereby outcompeting the challenger. Gersbach et al. (2015) consider costs of change that increase with the extent of policy change and where, contrary to this paper, all politicians are alike in terms of reform ability. Our paper adds to this literature on costly reforms by focusing on how differences in politicians' reform ability might lead to inefficiencies in democratic elections.

Prevalence of the status-quo

It has frequently been observed that in democracies the status quo will prevail, even if changes are desired by a majority of voters. Over and above the costs of such changes, there are other frictions in the electoral process that might explain this observation, e.g. the dynamics of internal party politics, the existence of imperfect information, or the importance of reputation.[7] First, policy changes may jeopardize parties' campaign resources (Miller and Schofield, 2003), thereby disincentivizing parties to go for reforms. Second, as already observed by Fernandez and Rodrik (1991) and Gersbach (1993), resistance to reform—which implies a preeminence of the status quo—can also arise whenever some of the winners and losers of a reform cannot be identified beforehand. Third, Tavits (2007) provides evidence that shifts in parties' core beliefs and values—which have a direct impact on policies—harm their parties' electoral support, since

[6]The assumption that utilities are quadratic does not affect our main insights, but it makes their exposition easier. Qualitatively, the same effects are at work when the utility functions are concave.

[7]If qualified majorities are needed for changes, the critical voter is no longer the median voter. Yet, the same problem could arise—i.e., that implemented policies do not match the critical voter's interests.

voters will interpret them as a sign of inconsistency and lack of credibility.

In accordance with the latter hypothesis, elected politicians may face restrictions to reverse policies, especially when voters are retrospectively-looking. In such cases, the influence of a candidate's policy record would be paramount to determining the election outcome and the resulting policy. Forand (2014), for instance, considers a dynamic model where the policies chosen by a politician are persistent throughout his tenure. Unlike the costs of policy reforms considered in our paper, such persistent policies only restrict the incumbent's choices.

Often, both contenders in an election—not just the incumbent—have a policy record that constrains their range of potential policies. This is the case analyzed in Samuelson (1984), where all politicians' action space (upon election) is restricted by their previous political activity (it may include their initial positions in the primaries). Instead of exogenously restricting the policies that an incumbent can choose from his second term on, other papers assume that changes in a candidate's position generate a disutility to all voters. Such ideological adjustment costs may be non-existent (Downs, 1957) or may increase as a convex function of the difference between the extent of ideological adjustment (Bernhardt and Ingerman, 1985; Ingberman, 1989; Enelow and Munger, 1993), and they have been documented empirically in the case of the US Senate (DeBacker, 2015).[8] Ideological adjustment costs differ from costs of change. The latter are a function of the policy shift, regardless of the identity of the office-holders responsible of such policies.

The existence of dynamic links between policies across periods is examined in other research. Chen and Eraslan (2015) study a model where a change in one policy dimension precludes the possibility of working on it again in the near future. Bowen et al. (2014) and Bowen et al. (2015) analyze different political institutions in a dynamic model in which the incumbent will remain in power in the next period with some probability. We focus on the manifestations of the Reform Dilemma that emerge when policies are dynamically linked through costs of change. Lastly, a recent strand of literature investigates models where choices in one period affect utilities in the next period. In this vein, Callander and Raiha (2014) analyze the provision of infrastructures in democracies when policy decisions are durable.

Policy commitment

While, in our model, politicians cannot explicitly commit to a policy position, the costs of reform

[8] While most theoretical insights come from models of elections for executive offices, data are usually obtained from legislative positions.

offer incumbents the possibility of influencing future policy choices—including the incumbent's own—by means of the policy chosen in one period. Models of electoral competition can be divided up according to whether they consider that candidates can commit to policy positions (Hotelling, 1929; Downs, 1957) or not (Austen-Smith and Banks, 1989; Persson et al., 1997; Ashworth, 2012). The latter are models of electoral accountability. Ours can be interpreted as one of the latter models where, however, there exists a mild commitment tool—the costs of reform—that enables policy-makers to influence any subsequent office-holder's policy choice.

Ability or valence of politicians

Different theories aim to explain the level or distribution of elected politicians' competencies. First, Caselli and Morelli (2004) show that when quality is private information and the opportunity costs determined by market wages are low, bad politicians have a comparative advantage in pursuing elective office. Second, Besley and Coate (1997, 1998) point to two alternative explanations why low-ability candidates might win an election: *(i)* if the politicians' ability and political preferences are correlated, voters might want to elect politicians who are ideologically closer to them despite this low quality; *(ii)* coordination problems among voters at the election stage can prevent high-ability candidates from being elected. Third, Bernhardt et al. (2011) show that in a framework where valence is privately known by politicians until the end of their first term in office, the chances of re-election increase with the incumbent's valence as he chooses extreme policies. The fact that high-valence candidates tend to win elections more often than low-valence ones is shown in a number of static models (Ansolabehere and Snyder Jr, 2000; Groseclose, 2001; Aragonés and Palfrey, 2002). The latter models predict a negative correlation between valence and policy extremism.

The Reform Dilemma described in our paper adds to this literature. Our model neither encompasses an adverse selection problem nor does the possibility exist of reaping higher returns from holding office, e.g. by being dishonest. In our model, low-reform-ability office-holders are selected when party polarization is high to ensure that reforms remain comparatively small. Furthermore, low-reform-ability politicians are elected even when the benefits from holding office are very large. Here we differ from Caselli and Morelli (2004). We stress that, unlike Besley and Coate (1997, 1998), *ceteris paribus* voters in our model prefer high-ability politicians to low-ability ones. The reason is that the candidate's reform ability affects the actual implementation of policies. Moreover, since we consider a model with only two candidates, there is no danger of miscoordination. Finally, our (dynamic) model predicts a positive correlation between reform ability and extremism but no relation between re-election chances and the incumbent's

reform ability. In other words, our model features a similar phenomenon to "Only Nixon can go to China", which is based on a politician's ability instead of his ideology. Empirical results are inconclusive with regard to whether there is a positive or a negative relationship between valence and policy extremism (Stone and Simas, 2010).

Persistence of bad governments

Recent contributions have shown that once bad politicians have gained sufficient power bad governments may persist, see e.g. Acemoglu and Robinson (2005), Acemoglu et al. (2009), and Acemoglu et al. (2011). The Reform Dilemma adds to this literature by presenting an alternative perspective on the persistence of bad governments. Low-reform-ability candidates, once in office, will select policies that align social needs for reform with their ability, thereby securing their own re-election. By contrast, high-reform-ability incumbents are forced to choose polar positions that generate a large reform need, as the latter will enhance their re-election chances.

Evolution of voters' and politicians' preferences

One strand in the literature studies the determinants and evolution of the voters' and politicians' preferences, see e.g. Franklin (1984), Adams et al. (2004), Layman et al. (2006), Fiorina and Abrams (2008), Carter and Morrow (2012), or Owens and Pedulla (2014). Voters' preferences tend to fluctuate considerably (see e.g. Durr (1993)) and party competition is essentially a dynamic process where parties constantly adjust their platforms (see e.g. Kollman et al. (1992)). One fundamental issue remains understanding whether citizens' preferences evolve due to the office-holders' actions or, on the contrary, whether the political parties' positions evolve to match the citizens' preferences, as argued by Adams et al. (2004). In our paper, reform requirements can arise from both sources, e.g. when the citizens' preferences shift or when current policies can no longer fulfill the voters' wishes.[9]

3 The Model

3.1 General set-up

We examine a two-period model ($t = 1, 2$) for elections. The population consists of a continuum of citizens of unit measure, each voter being indexed by $i \in [0, 1]$. In each election, every

[9] Although our model can handle both sources, in the case of evolving preferences, the welfare analysis entails major difficulties as it depends on the state of the preferences.

citizen casts a vote to determine the office-holder to whom the society in question delegates policy-making. Policy-makers have different *policy orientations*. We use R and L to denote the pool (or *party*) of *right-wing* and *left-wing* candidates, respectively. At any election date, one candidate from R and one candidate from L compete for office. Throughout the paper, we denote candidates (and policy-makers) by k, k', or k''.

When candidate k, with $k \in R \cup L$, is in office, he chooses a policy from a one-dimensional policy space $\mathcal{I} = [0, 1]$ on which citizens have different preferences.[10] For instance, we can interpret this policy as the usual left/right preference regarding the size and scope of the state in providing public goods and engineering redistribution. We denote the policy choice of office-holder k in a given period t by i_{kt}, with $i_{kt} \in \mathcal{I}$. Alternatively, when there is no confusion possible regarding the identity of the office-holder, we simply denote the policy chosen in period t by i_t. The winning candidate takes office, and the defeated candidate is replaced by another candidate of the same policy orientation at the next election date (if any).

We assume that the voters' ideal policies regarding $\mathcal{I}$ are given by a density function $v(\cdot)$ with support $\mathcal{I}$, which is symmetrically distributed around $\frac{1}{2}$. We denote citizen i's preferred policy by i, with $i \in [0, 1]$. The preferred policy of any right-wing candidate $k \in R$ in $\mathcal{I}$ is denoted by $\mu_k = \mu_R$, and the preferred policy of any left-wing candidate $k \in L$ in $\mathcal{I}$ is $\mu_k = \mu_L$. We assume that $0 \leq \mu_L \leq \mu_R \leq 1$ and

$$\mu_R + \mu_L = 1. \tag{1}$$

That is, candidates of different parties are located symmetrically with respect to the median voter, which we denote as $m = \frac{1}{2}$. Throughout the paper, we have $\pi = \frac{1}{2}(\mu_R - \mu_L) = \mu_R - \frac{1}{2}$ denote the (exogenously given) level of *party polarization*. Hence, π captures the distance between the interests of the two political parties. The symmetry assumption embodied in Equation (1) is not needed for the main thrust of our results, but it simplifies their derivation and presentation. Moreover, it enables us to put the focus only on the candidates' ability regarding the need for reforms, without any other factor that makes candidates different. How the symmetry assumption can be weakened is discussed in Section 6.

In this paper, we put the focus on changes in policies, which we call *policy reforms*. We assume that these reforms are costly for citizens—including candidates—and that these costs increase with the extent of the policy change. Candidates are characterized by their individual ability

[10] Our results can be accommodated to a spatial model provided that there exists a Condorcet-winner (Plott, 1967) and a candidate's reform abilities across dimensions are highly correlated. More general extensions fall beyond the scope of the paper.

to carry out reforms in the policy dimension $\mathcal{I}$. Given an arbitrary candidate $k \in R \cup L$, let c_k denote his *reform ability*.[11] We assume that for every candidate $k \in R \cup L$, his reform ability c_k is drawn independently of other candidates' and his own ideology from a continuous distribution function $F(\cdot)$ with expected value $\bar{C} > 0$ and non-degenerate support contained in $[0, +\infty)$. A candidate's reform ability is revealed to everyone—citizens, the office-holder, other candidates—at a certain point in time before the end of his first term. Except in Section 6.4, we assume that, once an office-holder's reform ability is realized, it becomes common knowledge immediately. This focus on common knowledge about the reform ability enables us to conjecture that the Reform Dilemma in polarized democracies does not depend on the existence of asymmetric information. Throughout the paper, we use $f(x) = F'(x)$ to denote the corresponding density function.

Since we want to study political situations where there is a need for reform triggered by a change in the citizens' and politicians' preferences regarding policy in $\mathcal{I}$, we assume that there is an exogenous *status-quo* policy i_0, with $i_0 \in \mathcal{I}$, which has been chosen in the past. Moreover, since i_0 is exogenous, we assume without loss of generality that the right-wing candidate, say $k \in R$, is initially in office. The interpretation of the above assumptions is as follows: i_0 represents a policy choice taken without connection to the current preferences of both voters and candidates of both parties—which are given by $v(\cdot)$, μ_L, and μ_R respectively. Accordingly, we define the *need-for-reform*

$$\tau = i_0 - \frac{1}{2}. \tag{2}$$

If $\tau = 0$, the status-quo policy matches the median voter's ideal policy, so this policy is the best choice from the point of view of society. Due to the symmetry of $v(\cdot)$, aggregate welfare will coincide with the median voter's welfare. In general, τ captures the distance between the status-quo policy from the ideal policy of the median voter, plus the direction the reform should point to. We stress that our model allows party preferences to suffer from shocks different from the ones affecting the voters' preferences, as no structure is imposed on the parameters beyond Equation (1).

[11] As it will become apparent below, strictly speaking we should refer to $-c_k$ as reform ability, instead of c_k.

3.2 Utilities of voters and candidates

Voters and policy-makers derive utility from the policy choices in $\mathcal{I}$. Suppose $k \in R \cup L$ is in office in period t and chooses $i_{kt} \in \mathcal{I}$. Then voter $i \in [0,1]$ derives utility

$$U_i^{\mathcal{I}}(i_{kt}) = -(i_{kt} - i)^2.$$

Similarly, candidate $k' \in R \cup L$ derives utility

$$U_{k'}^{\mathcal{I}}(i_{kt}) = -(i_{kt} - \mu_{k'})^2.$$

Additionally, when candidate k wins the election, he obtains private benefits $b > 0$ from holding office. The benefits b account for all sources of utility that politicians derive from office beyond policy choices, e.g. wages in office, "ego rents", psychological rewards associated with social status and power, the satisfaction of holding power and leading a governmental branch, as well as additional career opportunities after office-holding (lobbying, diplomacy, public speaking, etc).

A fundamental feature of our model is that policy reforms are costly for voters and candidates, and that these costs increase with the extent of the policy change. More precisely, let $i_{t-1} \in \mathcal{I}$ be the policy chosen in period $t-1$. Then the policy choice in period t, namely $i_t \in \mathcal{I}$, imposes additional costs (or utility losses) on voters and politicians alike, which equal

$$U^{c_k}(i_{t-1}, i_t) = -c_k \cdot (i_{t-1} - i_t)^2, \tag{3}$$

where c_k is specific to the policy-maker k who carries out the reform. That is, the costs of reform are quadratic in the difference between the policies adopted in two consecutive periods, and the extent of such costs depends on the office-holder's identity.[12] In particular, higher c_k impose higher utility losses on everyone, so $-c_k$ can be interpreted as the politician's *quality or reform ability*. The existence of such costs has been discussed in the Introduction. As we have seen, there are several reasons why politicians may differ in their ability to engineer policy reforms. We also note that voters are risk-neutral with respect to uncertainty about the office-holder's reform ability for a given reform. Lastly, given the status-quo policy $i_{t-1} \in \mathcal{I}$, we define the *marginal cost per reform step* in period t for policy $i_t \in \mathcal{I} \setminus \{i_{t-1}\}$ as

$$MC(|i_t - i_{t-1}|, c_k) = -\frac{1}{2} \cdot \frac{\partial U^{c_k}(i_{t-1}, i_t)}{\partial |i_t - i_{t-1}|} = c_k \cdot |i_t - i_{t-1}|. \tag{4}$$

[12] The assumption of quadratic reform costs facilitates the presentation of the results and enables analytical and explicit representation of equilibrium outcomes. The gist of the results would nonetheless be preserved if we were to assume other convex specifications for the costs of reform.

Accordingly, candidates with low reform ability (i.e. with high c_k) impose higher costs per reform step than candidates with high reform ability (i.e. with low c_k). As we shall see below, high reform ability is a double-edged sword. On the one hand, it enables politicians to adapt to unexpected events. On the other, it hinders a politician's capability of committing to small policy changes before elections take place. Lastly, we note that since costs of change depend on the extent of the policy change, we can interpret such costs as a reflection of policy-specific managerial skills.

3.3 Elections, assumptions, and equilibrium concept

Candidates are elected according to the simple majority rule. To break ties, we assume that if both candidates obtain a vote share of 50% the incumbent wins the election.[13] We also assume that, given $F(\cdot)$, the policy-makers' benefits from holding office, b, are sufficiently large to ensure that any policy-maker will prefer being in office to not being in office under any parameter constellation we analyze in the model. If, for instance, $F(\cdot)$ is country-specific, then this assumption simply requires the rewards from elective office to be very large for a given country. As discussed above, these rewards may be both psychological and financial.

We explore different variants depending on the exact timing at which the reform ability of politicians is realized and becomes commonly known. More specifically, we consider three possibilities for the realization of the reform ability of a new candidate. First, we consider the case where this reform ability is commonly-known before his first election, and we call this case *ex ante knowledge about reform ability* (henceforth simply *ex ante*). Thus, voters and the candidate himself know about this reform ability when they vote and when the office-holder makes his policy choice in his first term (if he is elected). Second, we consider the case where the reform ability of a candidate is realized after he has been elected for the first time but before he makes his first-term policy choice. We call this case *ex interim knowledge about reform ability* (henceforth simply *ex interim*). In this case, voters do not know about this reform ability when they vote. Third, we consider the case where the ability of a candidate is only known at the end of his first term, and we call this case *ex post knowledge about reform ability* (henceforth simply *ex post*).[14] Accordingly, neither the voters nor the candidate know about the latter's reform ability when they vote and when he makes his first-term policy choice. Our approach thus enables us to gain some insight on the value of information about the candidates' characteristics

[13] This assumption simplifies the exposition but is not essential.

[14] This is the approach taken by Bernhardt et al. (2011).

in democratic elections. The three possibilities are depicted in Figure 1. In all three cases, the office-holder's reform ability is always common knowledge at the end of his first term in office. This implies that the reform ability of the incumbent is always common knowledge in subsequent elections.

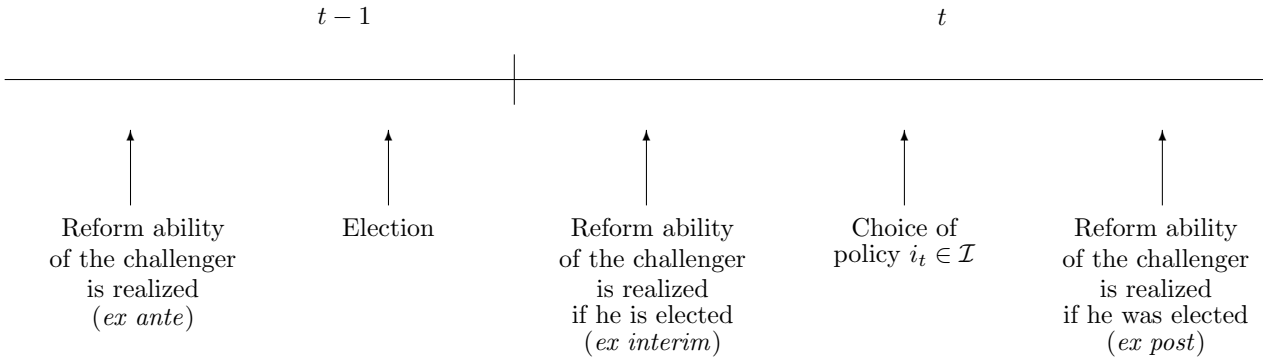

Figure 1: Realization of reform ability—ex ante, ex interim, ex post.

There are different reasons why the timing of the realization of reform ability may vary. For instance, candidates may have already held office in other areas and may be known. In such cases, candidates' reform ability can be inferred from their previous performance as politicians, and the voters know about it at an earlier stage. In other cases, when unknown challengers are entering office, their reform ability may only become known at the end of the first term.

Lastly, we look for subgame perfect Nash equilibria for each of the variants of the election game regarding the different possibilities of the realization of reform ability, both in a static and a dynamic setting. As usual, we assume that voters cast their votes as if they were pivotal to rule out implausible equilibria.

4 A Static Model

In this section we consider a static model and focus on a single term. For this purpose, we take the history before the term under consideration as given. Specifically, let without loss of generality the history be a period $t = 1$ in which the office-holder, $k \in R$, has chosen some policy $i_1 \in \mathcal{I}$ and his reform ability c_k has been realized. Let $k' \in L$ denote the challenger in the election at the end of period $t = 1$. Then we analyze the game, denoted by $\mathcal{G}_1^R$, that starts with the election at the end of this period and continues with the policy choice of the office-holder

in period $t = 2$. In particular, we adapt the definition of (2) and denote by $\tau = i_1 - \frac{1}{2}$ the need-for-reform in this static model. We consider three variations of $\mathcal{G}_1^R$, depending on the moment when the reform ability of the challenger is realized.

4.1 The ex-ante case

We shall only describe in full detail in the main text the ex-ante scenario. The reason is that it is the least complex case and it captures the main features of the Reform Dilemma, which are common to the other two cases (ex-interim and ex-post) as well. The latter are discussed in Section 4.2. The sequence of events in the ex-ante case is summarized in Figure 2. In this figure and all other figures throughout the paper, we use a dashed line to separate what is exogenous from what is endogenous to the corresponding political game.

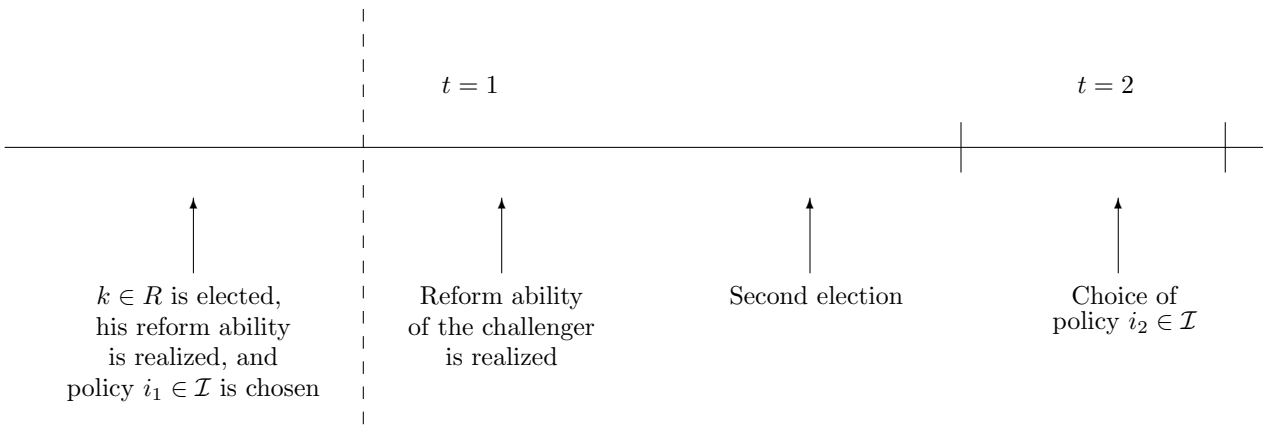

Figure 2: Timeline for the static model—ex-ante.

We work backwards and start with the analysis of the office-holder's behavior in the second period, so we assume that elections in $t = 1$ have already taken place and that candidate $k'' \in \{k, k'\}$ is in office. As there are no further elections, k'' chooses his policy to maximize his instant utility in $t = 2$. We obtain the following result, proved in Appendix A:

Proposition 1

Let $k'' \in R \cup L$ be the office-holder in $t = 2$. The best response of k'' to the status-quo policy $i_1 \in \mathcal{I}$ chosen is given by

$$i_{k''2}(i_1) = \frac{c_{k''}}{c_{k''} + 1} \cdot i_1 + \frac{1}{c_{k''} + 1} \cdot \mu_{k''}. \tag{5}$$

Proposition 1 expresses the intuitive property that the office-holder will choose a policy that is a weighted average between the status quo and his ideal point, with the weights depending on his reform ability.[15] Moreover, (5) can be rewritten as follows:

$$i_{k''2}(i_1) = \begin{cases} \frac{1}{2} + \frac{c_{k''}}{c_{k''}+1} \cdot \tau + \frac{1}{c_{k''}+1} \cdot \pi & \text{if } k'' = k \in R, \\ \frac{1}{2} + \frac{c_{k''}}{c_{k''}+1} \cdot \tau - \frac{1}{c_{k''}+1} \cdot \pi & \text{if } k'' = k' \in L. \end{cases} \tag{6}$$

Therefore, if $\tau > 0$, the lower $c_{k''}$ is, the more moderate will be the policy chosen by $k'' \in R$. Moreover, such a policy becomes more partisan as party polarization, π, increases. Analogous comments apply when either $\tau < 0$ or $k' \in L$ is elected.

Next we focus on the voters' decision in the election at the end of period $t = 1$, in which citizens either select the right-wing incumbent k or the left-wing candidate k'. Since citizens anticipate the policies that candidates will implement if elected, they will vote for the candidate who generates higher utility. We stress that at the time of the election, voters and candidates know both the incumbent's and the challenger's reform ability. We obtain the following result, which is proved in Appendix A:

Proposition 2

Let $k \in R$ be the office-holder in $t = 1$ and $k' \in L$ the challenger in $t = 2$. Let also $i_1 \in \mathcal{I}$ be the status-quo policy chosen in $t = 1$. Then, k is re-elected in equilibrium of $\mathcal{G}_1^R$ if and only if $i_1 \in \mathcal{X}(c_k, c_{k'})$, where

$$\mathcal{X}(c_k, c_{k'}) = \begin{cases} \left[\frac{1}{2} - \pi, \frac{1}{2} + \pi\right] & \textit{if } c_k > c_{k'}, \\ [0, 1] & \textit{if } c_k = c_{k'}, \\ \left[0, \frac{1}{2} - \pi\right] \cup \left[\frac{1}{2} + \pi, 1\right] & \textit{if } c_k < c_{k'}. \end{cases} \tag{7}$$

Therefore, if the incumbent k is less efficient than the challenger k', i.e. $c_k > c_{k'}$, k will be re-elected if and only if $i_1 \in \left[\frac{1}{2} - \pi, \frac{1}{2} + \pi\right] = [\mu_L, \mu_R]$ or, equivalently, if and only if $|\tau| \leq |\pi|$. Thus, inefficient incumbents are re-elected when party polarization is large (relative to the need-for-reform). By contrast, if incumbent k is more efficient than challenger k', i.e. $c_k < c_{k'}$, k will be re-elected if and only if $i_1 \in \left[0, \frac{1}{2} - \pi\right] \cup \left[\frac{1}{2} + \pi, 1\right] = [0, \mu_L] \cup [\mu_R, 1]$ or, equivalently, if

[15] If costs of change are strictly convex—but not quadratic—then the policy chosen by the office-holder k'' in $t = 2$ is not linear in i_1 and $\mu_{k''}$, although it increases in both arguments. Assuming quadratic costs of change simplifies the exposition of the results. However, it does not affect the insights of the upcoming results, which are essentially driven by the fact that the second period office-holder's best response increases in i_1 and $\mu_{k''}$.

and only if $|\tau| \geq |\pi|$. Thus, efficient incumbents are re-elected when party polarization is low (relative to the need-for-reform). Lastly, incumbents are always re-elected if the two candidates' reform ability is the same.[16]

The intuition for Proposition 2 is as follows: When the need-for-reform is small but party polarization is large, selecting an office-holder with high reform ability yields an excessively large reform from the perspective of a majority of voters. Instead, electing an office-holder with low reform ability produces a much smaller reform, yet at a higher cost per reform step. The latter option is preferred by a majority of voters.

4.2 The ex-interim and ex-post cases

A comprehensive analysis of both the ex-interim and ex-post scenarios can be found in Appendix D, which serves as a robustness check of our findings in Section 4.1. Since the analysis of these two informational scenarios is very similar to that of the ex-ante scenario, we will here only stress the differences between the three cases. To this goal, it will come in handy to consider the parameter $\hat{C}$ defined as follows:

$$\frac{\hat{C}}{\hat{C}+1} = \int_0^\infty \frac{c}{c+1} f(c)dc. \tag{8}$$

It can be easily verified that $\hat{C} < \bar{C}$. We analyze both the office-holder's choices and the electorate's behavior.

First, we consider the best response of the office-holder in $t = 2$. In the ex-interim case, it is again given by Proposition 1. The reason is that again, the challenger's reform ability is realized before he makes his policy choice. In the ex-post case, on the other hand, $c_{k'}$ has to be replaced by $\bar{C}$ in Equation (6) to describe the challenger's best response. The reason for this difference stems from the fact that the challenger does not know his reform ability when he chooses the policy in the ex-post case, so that he gambles.

Second, we consider the electorate's behavior in the election at the end of period $t = 1$. In both the ex-interim and the ex post case, the incumbent is re-elected if and only if the status quo policy is within a certain range of policies which depends on whether the incumbent's reform ability is larger or lower than a certain critical value, as in Proposition 2. The critical value of the reform ability that guarantees such re-election, however, varies across scenarios. In the

[16] This feature follows from the tie-breaking rules and from the fact that all terms in the instant utility are quadratic.

ex-interim case, incumbent k is re-elected if and only if $i_1 \in \mathcal{X}(c_k, \hat{C})$, where the latter set is defined as in Equation (7). In the ex-post case, incumbent k is re-elected if and only if $i_1 \in \mathcal{X}(c_k, \bar{C})$. In both cases, the electorate does not know the challenger's reform ability when casting his vote. But in the ex-interim case, unlike in the ex-post case, the electorate anticipates that the challenger will know it before he chooses a policy. This yields the following property: In both scenarios low-reform-ability incumbents are re-elected if and only if the need-for-reform is small relative to policy polarization, i.e., if and only if $|\tau| \leq |\pi|$. In that case, the chances for low-reform-ability incumbents to be re-elected are higher in the ex-interim case compared to the ex-post case. When $|\tau| \geq |\tau|$, on the contrary, these chances are lower.

4.3 The Reform Dilemma: Part one

Proposition 2 and its counterparts for the ex-interim and ex-post cases are manifestations of the first part of the Reform Dilemma. When societies are polarized compared to the need-for-reform, only politicians with low reform ability will be elected or re-elected. These office-holders entail higher costs per reform step. However, as they will undertake only comparatively small reforms, a majority of voters are better off with them than with an office-holder with high reform ability. The reason is that the latter would undertake excessive reforms and that, from the viewpoint of the median voter—who is pivotal in the election—, the status-quo policy is sufficiently moderate for the median voter to prefer to elect the politician who will carry out the smaller policy change. We summarize the first part of the Reform Dilemma in the following result:

Theorem I

From a static perspective, low-reform-ability incumbents are re-elected if and only if the need-for-reform is small relative to policy polarization.

Finally, we note that the informational setting is important only insofar as it determines the exact value of the critical reform ability that enables the incumbent's re-election. Hence, the occurrence of the Reform Dilemma is independent of the point in time when the challenger's reform ability is realized.

5 A Dynamic Model

In this section, we extend the static model and endogenize the policy choice in $t = 1$, namely $i_1 \in \mathcal{I}$, instead of taking this policy choice as given as we did in Section 4. Without loss of generality, we assume that in period $t = 0$, there is some status quo $i_0 \in \mathcal{I}$ and that in period $t = 1$, the office-holder is $k \in R$, whose reform ability is common knowledge. We also let $k' \in L$ denote the challenger in the election at the end of period $t = 1$. We analyze the game, denoted by $\mathcal{G}_2^R$, that starts at the beginning of period $t = 1$. We consider three variations of the game (ex-ante, ex-interim, and ex-post) depending on the exact timing when the reform ability of the challenger, k', is realized.[17] As in the previous section, we solve the ex-ante case in full detail in the main text, where we also explain the differences with regard to the two remaining cases. In either case, we need to compute all subgame perfect Nash equilibria of $\mathcal{G}_2^R$ by backward induction. They are dealt with in full detail in Appendix D. We build on the analysis of Section 4, so we focus only on k's choice in the first period, $i_{k1} \in \mathcal{I}$.

We assume that voters and candidates discount utility in the second period with a common factor β, where $0 < \beta \leq 1$. Let candidate $k'' \in \{k, k'\}$ denote the office-holder in $t = 2$. Then, the expected lifetime utility of voter $i \in [0, 1]$ at the beginning of period $t = 1$ is given by

$$U_i^{exp} = \mathbb{E}\left[U_i^{\mathcal{I}}(i_{k1}) + U^{c_k}(i_0, i_{k1}) + \beta \cdot \left(U_i^{\mathcal{I}}(i_{k''2}) + U^{c_{k''}}(i_{k1}, i_{k''2})\right)\right], \tag{9}$$

where $i_{k''2}$ denotes the policy chosen by k'' in the second period. Similarly, the expected lifetime utility of first-period office-holder $k \in R$ at the beginning of period $t = 1$ is given by

$$\begin{aligned} U_k^{exp}(i_{k1}) = \mathbb{E}[&U_k^{\mathcal{I}}(i_{k1}) + U^{c_k}(i_0, i_{k1}) + \mathcal{I}_k(k) \cdot b \\ &+ \beta \cdot \left(U_k^{\mathcal{I}}(i_{k''2}) + U^{c_{k''}}(i_{k1}, i_{k''2}) + \mathcal{I}_k(k'') \cdot b\right)], \end{aligned} \tag{10}$$

where

$$\mathcal{I}_k(x) = \begin{cases} 1 & \text{if } k = x, \\ 0 & \text{otherwise.} \end{cases}$$

Thus, we shall solve the problem

$$\begin{aligned} \max_{i_{k1}} \quad & U_k^{exp}(i_{k1}) \\ \text{s.t.} \quad & i_{k1} \in \mathcal{I}. \end{aligned} \tag{11}$$

The solution to (11) yields the policy choice of office-holder k in $t = 1$. For our analysis, we need to deal with the three different informational settings separately.

[17] There is another possibility, not considered in this paper, which is that the reform ability of the challenger is realized after the first-period office-holder has made his policy choice.

5.1 The ex-ante case

In this case, the sequence of events is summarized in Figure 3.

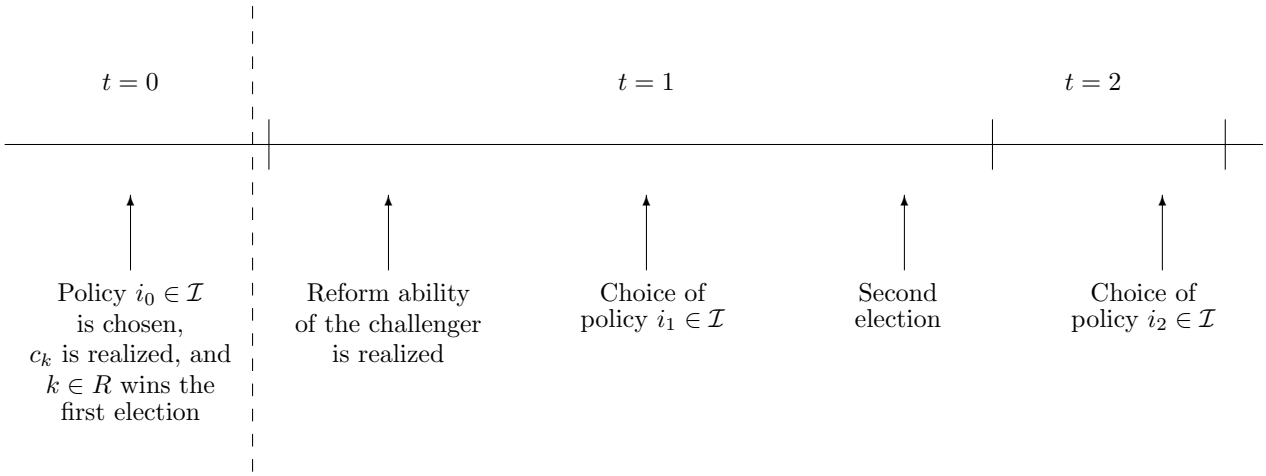

Figure 3: Timeline for the dynamic model—ex-ante.

To describe the incumbent's policy choice in $t = 1$, we let

$$i_{1,*}^{EA}(\tau, \pi, c_k) = \frac{1}{2} + \frac{c_k}{1 + c_k + \beta \cdot \frac{c_k}{c_k+1}} \cdot \tau + \frac{c_k + \beta \cdot \frac{c_k}{c_k+1}}{1 + c_k + \beta \cdot \frac{c_k}{c_k+1}} \cdot \pi. \tag{12}$$

The proof of the next result can be found in Appendix A.

Proposition 3

Let $k \in R$ denote the incumbent in $t = 1$ and $k' \in L$ denote the challenger. Then, in $t = 1$, k chooses the following policy in equilibrium of $\mathcal{G}_2^R$:

(a) When $c_k > c_{k'}$,

$$i_{k1}^{EA} = \min\left\{\frac{1}{2} + \pi, \max\left\{\frac{1}{2} - \pi, i_{1,*}^{EA}(\tau, \pi, c_k)\right\}\right\}.$$

(b) When $c_k = c_{k'}$,

$$i_{k1}^{EA} = i_{1,*}^{EA}(\tau, \pi, c_k).$$

(c) When $c_k < c_{k'}$,

$$i_{k1}^{EA} = \begin{cases} \min\left\{\frac{1}{2} - \pi, i_{1,*}^{EA}(\tau, \pi, c_k)\right\} & \text{if } i_{1,*}^{EA}(\tau, \pi, c_k) \in \left[0, \frac{1}{2}\right), \\ \frac{1}{2} - \pi \text{ or } \frac{1}{2} + \pi & \text{if } i_{1,*}^{EA}(\tau, \pi, c_k) = \frac{1}{2}, \\ \max\left\{\frac{1}{2} + \pi, i_{1,*}^{EA}(\tau, \pi, c_k)\right\} & \text{if } i_{1,*}^{EA}(\tau, \pi, c_k) \in \left(\frac{1}{2}, 1\right]. \end{cases}$$

That is, office-holders with high reform ability tend to choose more extreme policies than low-reform-ability office-holders. With such policies, high-reform-ability office-holders generate a large reform need for the upcoming period, a need that is larger than party polarization, which makes it attractive for a majority of voters to re-elect such high-reform-ability office-holders. This is expressed in Corollary 1 for the case where there is no party polarization, i.e. $\pi \to 0$.[18]

Corollary 1

Let $k \in R$ denote the incumbent in $t = 1$ and $k' \in L$ denote the challenger, and assume that $\pi \to 0$. Then, in $t = 1$, k chooses the following policy in equilibrium of $\mathcal{G}_2^R$:

$$i_{k1}^{EA} = \begin{cases} \frac{1}{2} & \text{if } c_k \geq c_{k'} \\ \frac{1}{2} + \frac{c_k}{1 + c_k + \beta \cdot \frac{c_k}{c_k+1}} \cdot \tau & \text{if } c_k < c_{k'}. \end{cases}$$

According to Corollary 1, in the first period there is convergence *to* the median voter position when the incumbent's reform ability is lower than the challenger's, but there is only convergence *towards* the median voter position when the incumbent's reform ability is higher than the challenger's.

5.2 The ex-interim and ex-post cases

Now we discuss both the ex-interim and ex-post cases and summarize the differences with the ex-ante case.[19] The analysis of the ex-interim case, in particular, is almost analogous to that of the ex-ante case. The only difference is that Proposition 3 has to be modified, with the critical reform ability value being $\hat{C}$ and not $c_{k'}$, k' denoting the challenger. The analysis of the ex-post case, on the other hand, is more intricate. The reason is that the skewness of the reform-ability-distribution now matters. To explore this scenario, for each given $i_0 \in \mathcal{I}$ it will be useful to define

$$\begin{aligned} i_{k1,*}^{EP}(\tau, \pi) = \frac{1}{2} &+ \frac{\bar{C}}{\bar{C} + 1 + \beta \cdot \int_{\bar{C}}^{\infty} \frac{c_k}{1+c_k} f(c_k) dc_k + \beta \cdot F(\bar{C}) \frac{\bar{C}}{\bar{C}+1}} \cdot \tau \\ &+ \frac{1 + \beta \cdot \int_{\bar{C}}^{\infty} \frac{c_k}{1+c_k} f(c_k) dc_k + \beta \cdot F(\bar{C}) \frac{\bar{C}}{\bar{C}+1}}{\bar{C} + 1 + \beta \cdot \int_{\bar{C}}^{\infty} \frac{c_k}{1+c_k} f(c_k) dc_k + \beta \cdot F(\bar{C}) \frac{\bar{C}}{\bar{C}+1}} \cdot \pi, \end{aligned} \tag{13}$$

[18] To prevent our results from being influenced by our tie-breaking rules, we do not impose $\pi = 0$ directly.

[19] We refer to Appendix D for a comprehensive analysis of both the ex-interim and ex-post scenarios.

and

$$i_{k1,**}^{EP}(\tau,\pi) = \frac{1}{2} + \frac{\bar{C}}{\bar{C}+1+\beta\cdot\int_0^{\bar{C}}\frac{c_k}{1+c_k}f(c_k)dc_k+\beta\cdot(1-F(\bar{C}))\frac{\bar{C}}{\bar{C}+1}}\cdot\tau$$
$$+\frac{1+\beta\cdot\int_0^{\bar{C}}\frac{c_k}{1+c_k}f(c_k)dc_k+\beta\cdot(1-F(\bar{C}))\frac{\bar{C}}{\bar{C}+1}}{\bar{C}+1+\beta\cdot\int_0^{\bar{C}}\frac{c_k}{1+c_k}f(c_k)dc_k+\beta\cdot(1-F(\bar{C}))\frac{\bar{C}}{\bar{C}+1}}\cdot\pi. \tag{14}$$

We stress that $i_{k1,*}^{EP}(\tau,\pi)$ and $i_{k1,**}^{EP}(\tau,\pi)$ are functions of τ and π, and hence of i_0. We can now state the following result for the ex-post case, the proof of which can be found in Appendix A:

Proposition 4

In $t=1$, the incumbent, $k\in R$, chooses the following policy in equilibrium of $\mathcal{G}_2^R$:

(a) When $F(\bar{C})<\frac{1}{2}$,

$$i_{k1}^{EP} = \min\left\{\frac{1}{2}+\pi, \max\left\{\frac{1}{2}-\pi, i_{k1,*}^{EP}(\tau,\pi)\right\}\right\}.$$

(b) When $F(\bar{C})>\frac{1}{2}$,

$$i_{k1}^{EP} = \begin{cases} \min\left\{\frac{1}{2}-\pi, i_{k1,**}^{EP}(\tau,\pi)\right\} & \text{if } i_{k1,**}^{EP}(\tau,\pi)\in\left[0,\frac{1}{2}\right), \\ \frac{1}{2}-\pi \text{ or } \frac{1}{2}+\pi & \text{if } i_{k1,**}^{EP}(\tau,\pi)=\frac{1}{2}, \\ \max\left\{\frac{1}{2}+\pi, i_{k1,**}^{EP}(\tau,\pi)\right\} & \text{if } i_{k1,**}^{EP}(\tau,\pi)\in\left(\frac{1}{2},1\right]. \end{cases}$$

(c) When $F(\bar{C})=\frac{1}{2}$,

$$i_{k1}^{EP} = \begin{cases} i_{k1,**}^{EP}(\tau,\pi) & \text{if } i_0\in\left[0,\frac{1}{2}+\pi\right)\cup\left(\frac{1}{2}+\pi,1\right], \\ i & \text{otherwise, for some } i\in\left\{\frac{1}{2}-\pi, i_{k1,*}^{EP}(\tau,\pi), \frac{1}{2}+\pi\right\}. \end{cases}$$

Proposition 4 shows that the incumbent in $t=1$ makes his policy choice dependent on whether $F(\bar{C})$ is smaller or larger than $\frac{1}{2}$.

5.3 The Reform Dilemma: Part two

Proposition 3—together with its counterpart for the ex-interim case—and Proposition 4 are manifestations of the second part of the Reform Dilemma. First, in both the ex-ante and ex-interim case, office-holder k will choose different policies depending on whether his reform ability

is below or above a certain threshold: c'_k, k' being the challenger, and $\bar{C}$ respectively. When k's reform ability is below the threshold, k will choose a policy within the range $\left[\frac{1}{2} - \pi, \frac{1}{2} - \pi\right]$ in the first period. By contrast, if k's reform ability is above the threshold, k will choose a more extreme policy in the first period, i.e. a policy outside the range $\left[\frac{1}{2} - \pi, \frac{1}{2} - \pi\right]$. These policy choices secure re-election.

Second, in the ex-post case, it is the distribution of the candidates' reform ability that determines the incumbent's policy choice. This follows from the following facts: When the distribution is skewed negatively, candidates who are average- or low-skilled in implementing policy reforms will be re-elected. By contrast, if the distribution of candidates' reform ability is skewed positively or not skewed at all, average- or high-skilled candidates will be re-elected.[20] In either case, the policy choices secure re-election.

The property that the incumbent in $t = 1$ will always choose in equilibrium a policy that guarantees re-election regardless of his reform ability yields the second part of the Reform Dilemma. We summarize it in the following result:

Theorem II

From a dynamic perspective, incumbents are re-elected regardless of their reform ability.

The second manifestation of the Reform Dilemma allows us to interpret our dynamic model as one of endogenous incumbency advantage. Once in office, a politician can secure re-election regardless of his reform ability by suitable policy choices. The fact that a government in power is hard to oust is a well-established empirical fact observed in many democracies. In the case of the US, there is a large literature that investigates the existence and causes of the incumbency advantages of congressmen (Alford and Brady, 1989; Gelman and King, 1990; Cox and Katz, 1996; Levitt and Wolfram, 1997)). Erikson et al. (1993) find that governors also have similar advantages when seeking re-election.[21] Our model suggests that models with costly policy reforms are convenient frameworks for politicians to create incumbency advantages.

We conclude this section with two comments. First, for Theorem II to hold it is sufficient that voters do not hold a high-ability incumbent responsible for making the first reform. Second, we have assumed that preferences do not (exogenously) shift from period $t = 1$ to $t = 2$.

[20] Since the ability of a candidate is actually $-c_k$, then it is equivalent to say that the distribution of the candidates' reform ability is negatively (positively) skewed and that the distribution $f(\cdot)$ is positively (negatively) skewed.

[21] In the US, the success rate of the governors that have sought re-election since World War II is about 72%. Indeed, see `http://governors.rutgers.edu/on-governors/us-governors/when-governors-seek-re-election` (retrieved 13 May 2015).

Nevertheless, the Reform Dilemma described in Theorem II would also emerge when such a shift occurred. The argument runs as follows: On the one hand, consider that the shift in preferences is not anticipated. Then, while decisions in $t = 1$ would not change, choices in $t = 2$ would. In Section 6.3 below, we show in our baseline model that if the median voter is not symmetrically located with respect to the two parties, the first manifestation of the Reform Dilemma remains. The analysis therein guarantees that the second manifestation is also preserved when an unexpected shift in preferences occurs, since from a formal viewpoint, the problem is the same in both situations. On the other hand, suppose that the shift is expected, and drawn from a prior distribution. We have already seen that for both the ex-interim and ex-post cases, the incumbent in $t = 1$ is also able to secure re-election despite the fact that he takes his decision under uncertainty with regard to the challenger's reform ability. Moreover, in Section 6.3 we also show that ability and ideology moderation are to some extent orthogonal from any voter's perspective. It thus follows that the incumbent will also be able to secure re-election if uncertainty concerns preferences rather than reform ability.

6 Extensions

In this section, we consider four extensions of our baseline model. First, we assume that politicians can affect their own reform ability while they are in office. Second, we assume that citizens care about the office-holders' reform ability *per se*. Third, we investigate the case where the challenger may be more or less moderate than the incumbent. Fourth, we explore the possibility that the incumbent's reform ability may be only privately known during his first term in office.[22]

6.1 Incentives to become more inefficient

The results in the previous sections have identified the circumstances in which office-holders with low reform ability are re-elected. This raises the question whether in such cases office-holders would like to become less efficient in undertaking reforms, thereby increasing their chances of re-election. To answer this question, we reconsider the dynamic model and extend it by assuming that the incumbent $k \in R$ elected in $t = 1$ can increase (but not decrease) c_k to a certain degree.[23] Increasing c_k by x, with $x \in [0, \bar{C}]$, is costly and is associated with an expected disutility of $-r(x)$, with $r(x) > 0, r'(x) > 0$, $r''(x) < 0$, $r'(0) = 0$, and $\lim_{c_k \to \bar{C}} r'(c_k) = +\infty$.

[22]The proofs of all the results of this section are to be found in Appendix B.

[23]The incumbent cannot affect the reform ability of other candidates.

Since $t = 2$ is the last period, the office-holder in $t = 2$ will never choose to decrease his ability.

Several comments are in order. First, we consider the case where c_k cannot be increased by at least $\bar{C}$. Second, the investments captured by the variable x are best understood in a broad sense, including building up a partisan bureaucracy that blocks large reforms and thus, in turn, reduces an office-holder's ability to undertake reforms. A new office-holder can eliminate such road blocks to reforms by eliminating bureaucracy. Third, $r(x)$ is conceived of as any expected cost associated with being less efficient, for instance lower additional career opportunities after office-holding, as they might be reduced by lower reform ability. Fourth, we stress that there is no signaling problem. Indeed, the agents' ability is not private information, and all agents are ex ante equal with regard to their costs.

We restrict our analysis to the ex-post case. While incentives to lower the reform ability also exist in the ex-ante and ex-interim cases, they are particularly pronounced in the ex post case on which we focus subsequently.[24] The timeline of the modified game, denoted by $\hat{\mathcal{G}}_2^R$, is depicted in Figure 4.

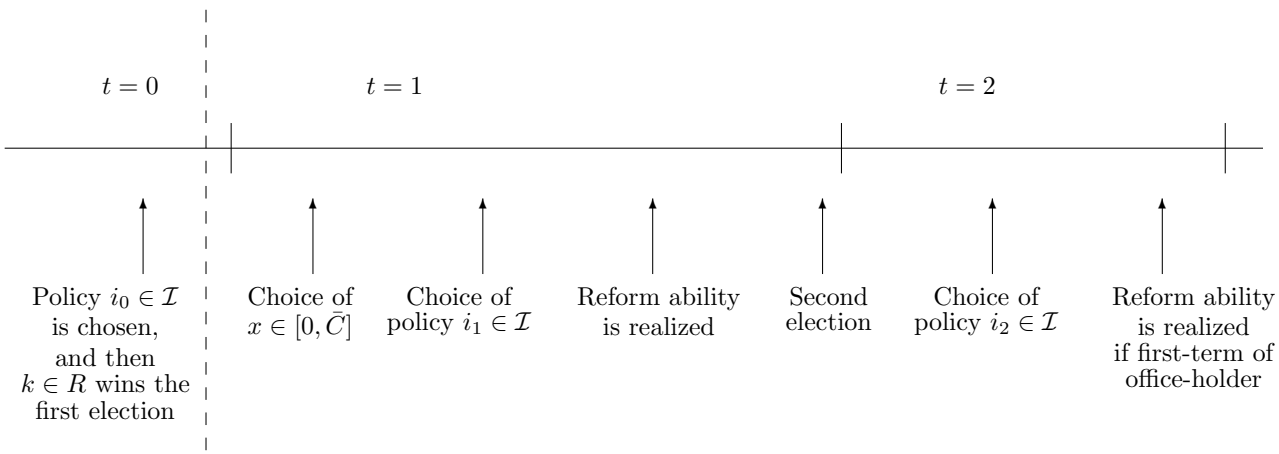

Figure 4: Timeline of events when reform ability can be lowered.

Next we state the main result of this section. It will be convenient to impose the following mild condition:

$$\beta \cdot f(x) > \frac{1+\beta}{b-2} \text{ for all } x \in [0, +\infty). \tag{15}$$

[24] In the ex-ante and ex-interim cases, the magnitude of these incentives the latter cases depends on the precise functional form of $r(x)$.

Assuming condition (15) will guarantee that candidates with all kinds of reform ability can be expected.

Proposition 5

Suppose that (15) holds and that the challenger's reform ability is revealed ex post. Then,

(a) if $F(\bar{C}) < \frac{1}{2}$, the incumbent in $t = 1$ decreases his reform ability in equilibrium of $\hat{\mathcal{G}}_2^R$,

(b) if $F(\bar{C}) \geq \frac{1}{2}$, the incumbent in $t = 1$ does not decrease his reform ability in equilibrium of $\hat{\mathcal{G}}_2^R$ for most parameter constellations.

Accordingly, the incentives for office-holders to become less efficient depend on the skewness of reform ability distribution. To the best of our knowledge, this result establishes a potential new link between ability choices and intrinsic abilities.

6.2 When citizens care about reform ability *per se*

Next we investigate the robustness of the Reform Dilemma when citizens care about the office-holder's reform ability *per se*. There are several reasons why citizens may value being governed by a high-reform-ability politician. For instance, a candidate's reform ability may not only indicate his competency in engineering policy changes, it may also be a proxy for his *valence* (Stokes, 1963) or *character* (Kartik and McAfee, 2007). Voters may value a candidate's character independently of their ideological position and may judge whether the actions he takes in office are in accordance with the character (or valence) assessment. Accordingly, given an office-holder $k \in R \cup L$, we assume that the utility term

$$U^p(c_k) = -\gamma \cdot \frac{c_k}{c_k + 1} \tag{16}$$

is additively added to the instant utilities of all voters (not to the candidates), with $\gamma \geq 0$. This specification of $U^p(c_k)$ allows a transparent analytical solution. The parameter γ thus measures how important it is for the voters that the office-holder's reform ability be high. Note that $U^p(0) = 0$, $(U^p)'(c_k) < 0$, $(U^p)''(c_k) > 0$, and $\lim_{c_k \to +\infty} U^p(c_k) = \gamma$. Consider the following parameter:

$$\pi^\gamma = \sqrt{\max\{0, \pi^2 - \gamma\}}.$$

The results in the previous sections then continue to hold if we replace π with π^γ. The only difference is that, from a static perspective, low-ability candidates are now elected less often for any given need of reform τ. In the limit when $\gamma > \pi^2$, they are never elected.

6.3 Asymmetric positions of candidates

Elections are preceded by primaries in many countries, most notably in the US. While learning the candidates' reform ability may not be possible along the primaries, the candidates' preferred policy is typically made public before one of them is selected to run in the election. A relevant question is then whether from a party's perspective, a moderate challenger is, *ceteris paribus*, a better option than a non-moderate one. We answer this question in the static model only, as in the dynamic one the incumbent will always be able to secure re-election.[25] For this purpose, we drop Equation (1) and define the following parameter:

$$\Phi = \mu_L(1-\mu_L) - \mu_R(1-\mu_R).$$

Observe that $\Phi > 0$ if and only if the challenger's preferred policy μ_R is closer to the median voter's position, viz. $\frac{1}{2}$, than the incumbent's μ_L. Throughout the main body of the paper we have considered $\Phi = 0$. For arbitrary values of Φ, on the other hand, we can directly follow the logic of Proposition 2 and apply it to asymmetric positions of candidates. By doing so we can obtain that incumbent k will be re-elected if and only if the status quo policy i_1 belongs to $\mathcal{X}(c_k, c_{k'})$, where

$$\mathcal{X}(c_k, c_{k'}) = \begin{cases} \left[\frac{1}{2} - \pi(\Phi), \frac{1}{2} + \pi(\Phi)\right] & \text{if } c_k > c_{k'}, \\ [0,1] & \text{if } c_k = c_{k'} \text{ and } \Phi \leq 0 \\ \emptyset & \text{if } c_k = c_{k'} \text{ and } \Phi > 0 \\ \left[0, \frac{1}{2} - \pi(\Phi)\right] \cup \left[\frac{1}{2} + \pi(\Phi), 1\right] & \text{if } c_k < c_{k'}. \end{cases}$$

As in the previous sections, we let $\pi = \mu_R - \frac{1}{2}$. Nevertheless, we slightly abuse notation and, for a given Φ, we have now $\pi(\Phi)$ additionally denote the only positive real solution x to the following equation:[26]

$$x^2 = \pi^2 - \frac{c_k + 1}{c_k - c_{k'}} \cdot \Phi.$$

It is easy to verify that $\pi(0) = \pi$, so that we have generalized Proposition 2. We distinguish three cases. First, assume that the challenger is more able than the incumbent, i.e., $c_k > c_{k'}$. Then, $\frac{\partial \pi(\Phi)}{\partial \Phi} < 0$, so that the set $\mathcal{X}(c_k, c_{k'})$ shrinks as Φ increases. This implies that everything else being equal, more moderate challengers (relative to the incumbent) have better election chances, since they are elected for a wider range of status-quo policies. Second, assume that

[25] In the dynamic model, policy concerns for politicians do not have a first-order effect.

[26] We assume for simplicity that x exists and $x \in \left(-\frac{1}{2}, \frac{1}{2}\right)$. Details are available upon request.

the challenger is less able than the incumbent, i.e., $c_k < c_{k'}$. Then, $\frac{\partial \pi(\Phi)}{\partial \Phi} > 0$, so that the set $\mathcal{X}(c_k, c_{k'})$ shrinks again as Φ increases. Third and last, assume that the challenger is as able as the incumbent, i.e., $c_k = c_{k'}$. Then, increasing Φ either has no effect on re-election chances or prevents the incumbent from being re-elected.

To sum up, in our static model ideological moderation has a beneficial effect on re-election chances regardless of any consideration regarding reform ability. The lattter only influences the magnitude of such a beneficial effect. This downsian feature holds in any informational setting (ex-ante, ex-interim, and ex-post), so that party polarization should not be attributed in accordance with our model solely to differences in the politicians' ability to carry out reforms. The recent increase in party polarization seems to be a consequence of other factors, including geographical changes in partisan alignment (Rohde, 2010; Theriault, 2004)), incumbent-friendly redistricting (Layman et al., 2006), the structure of roll-call voting (Theriault, 2006), changes in the legislative agenda and the strategies of party leaders (Roberts and Smith, 2003) or the design of the leadership selection system (Heberlig et al., 2006).

Lastly, we note that from a formal point of view, assuming $\Phi \neq 0$ is equivalent to assuming that the candidates'/parties' positions are fixed and initially symmetric, but that there is also an exogenous change in the voters' preferences that results in the median voter being different than $\frac{1}{2}$. The analysis in this section has also covered the latter interpretation.

6.4 Asymmetric information

Finally, we study the possibility that there exists asymmetric information between the politicians and the citizens regarding the reform ability of the former. To this end, we dissociate the moment when a politician's reform ability is realized from the moment when it becomes common knowledge. As in the case analyzed in the previous sections, where the reform ability becomes common knowledge immediately after its realization, we consider both a static and a dynamic version of the model. For each of them, we modify the political game—either $\mathcal{G}_1^R$ or $\mathcal{G}_2^R$—accordingly and assume that while the incumbent's reform ability in $t = 1$ is common knowledge, the reform ability of the challenger is private information for him. We denote the modified versions of the games by $\mathcal{GP}_1^R$ and $\mathcal{GP}_2^R$ respectively. Since communicating his own reform ability costs a politician nothing and politicians cannot commit to any policy before they are elected, we are thus dealing with cheap-talk situations. We let $k \in R$ denote the incumbent in $t = 1$. In both the static and the dynamic case we look for subgame perfect Nash equilibria.

First, we consider the static version of the model. In this case, the sequence of events can be summarized as in Figure 5:

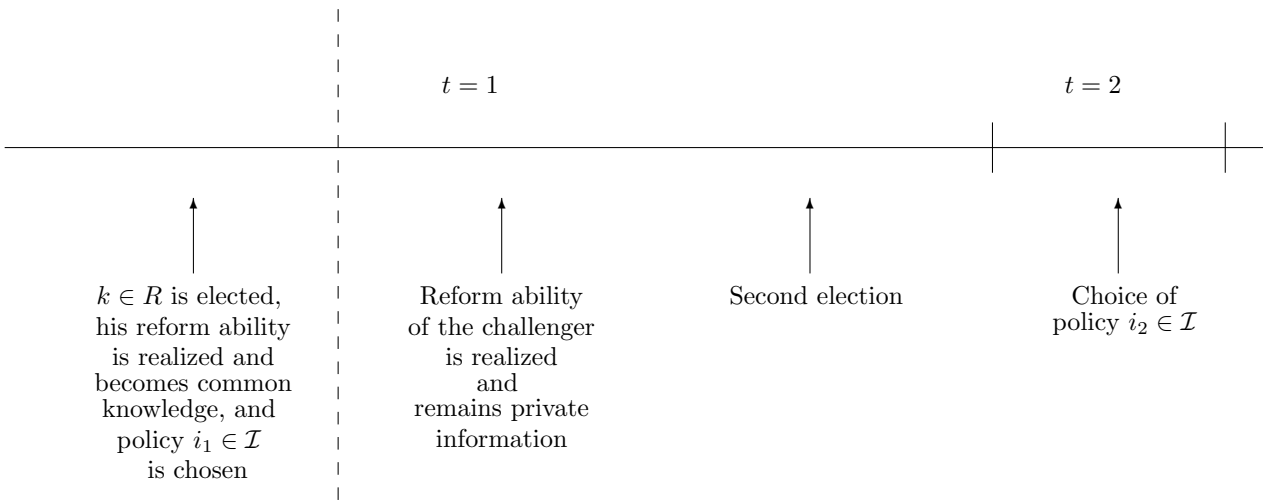

Figure 5: Timeline for the static model with asymmetric information.

It turns out that the game described in Figure 5 can be solved in exactly the same way as in the baseline model for the ex-interim case. In particular, we obtain the following result:

Proposition 6
Let $k \in R$ be the office-holder in $t = 1$. Then, the challenger is elected in equilibrium of $\mathcal{GP}_1^R$ if and only if at least one of the following conditions hold:

(a) $|\pi| \geq |\gamma|$ *and* $c_k \geq \hat{C}$,

(b) $|\pi| \leq |\gamma|$ *and* $c_k \leq \hat{C}$.

According to Proposition 6, if party polarization is high relative to the need-for-reform, i.e. if $|\pi| \geq |\gamma|$, challengers will be elected when the reform ability of the incumbent is sufficiently low, i.e. if $c_k \geq \hat{C}$. In that case, challengers benefit from the fact that their reform ability is private information. By contrast, if party polarization is low relative to the need-for-reform, i.e. if $|\pi| \leq |\gamma|$, challengers can exploit their private information to be re-elected if and only if the incumbet's reform ability is sufficiently high, i.e. if $c_k \leq \hat{C}$. That is, we have identified the circumstances where, from a static perspective, the challenger benefits from his reform ability not being commonly known but only known to him privately.

Second, we consider the dynamic version of the model where challengers' reform ability is private information. In this case, the sequence of events is summarized in Figure 6.

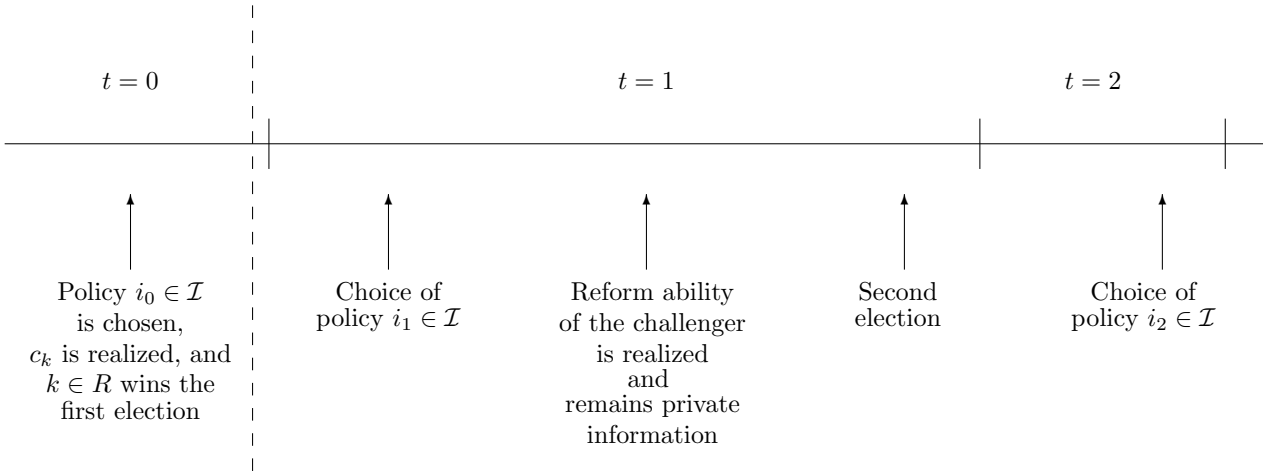

Figure 6: Timeline for the dynamic model with asymmetric information.

Again, the game described in Figure 5 can be solved along the exact same lines as in the baseline dynamic model for the ex-interim case. The result below then follows from the analysis thereof.

Proposition 7

The challenger is never elected in equilibrium of $\mathcal{G}_2^R$.

According to Proposition 7, the possibility for the challenger to exploit the fact that his reform ability is private information is non-existent when we consider a dynamic viewpoint. From this perspective, despite the fact that the challenger's reform ability is only known to the latter, the incumbent—as the only politician able to take an action—is still able to align the social need-for-reform with his reform ability by choosing an appropriate policy, thereby securing his re-election.

7 Welfare and Early Revelation of the Reform Ability

Beyond the identification of the Reform Dilemma, it is instructive to compare the expected welfare achieved by elections under the different informational settings. This will be pursued next, which yields a deeper understanding of the role of public information in elections.[27]

[27]The proofs of all results in this section can be found in Appendix C.

7.1 Welfare in the static model

For the static model, we study the expected utilitarian welfare (or simply *welfare*) under the three different scenarios (ex-ante, ex-interim, and ex-post). We examine welfare in period $t = 2$ from the perspective of period $t = 1$ when no information about the reform ability has been realized yet, given that $k \in R$ is the office-holder and the status-quo policy $i_1 \in \mathcal{I}$ has been chosen. Since the distribution of citizens' preferences is symmetric around $\frac{1}{2}$ and the costs of reform are common to all citizens, addressing expected utilitarian welfare is equivalent to addressing expected welfare of the median voter. Let k'' denote the office-holder in $t = 2$. Welfare can then be defined as follows:

$$W_1 := \mathbb{E}\left[U^{\mathcal{I}}_{\frac{1}{2}}(i_{k''2}) + U^{c_{k''}}(i_1, i_{k''2})\right]. \tag{17}$$

We use W_1^{EA}, W_1^{EI}, and W_1^{EP} to denote welfare in the ex-ante, ex-interim, and ex-post cases, respectively. We also use W_1^{soc} to denote welfare when a social planner could dictate both which candidate should take office and which reform he should undertake.[28] The main result regarding welfare in the static model is:

Proposition 8
For any $\pi \in \left[0, \frac{1}{2}\right]$ and $\tau \in \left(0, \frac{1}{2}\right]$,

(a) If $|\tau| > |\pi|$, then $W_1^{EP} < W_1^{EI} < W_1^{EA} \leq W_1^{soc}$.

(b) If $|\tau| = |\pi|$, then $W_1^{EP} = W_1^{EI} = W_1^{EA} < W_1^{soc}$.

(c) If $|\tau| < |\pi|$, then $W_1^{EI} < W_1^{EP} < W_1^{soc}$ and $W_1^{EI} < W_1^{EA} < W_1^{soc}$.

Three comments are in order. First, elections cannot yield the socially optimal solution, as the Reform Dilemma is operative.[29] In fact, elections yield lower welfare, as low-reform-ability candidates may be elected or re-elected and the extent of the implemented reforms differs from the one desired by the median voter. Second, in non-polarized societies, i.e. in societies where $|\tau| > |\pi|$, the earlier information about reform ability is revealed, the better from a welfare perspective. However, this principle does not apply to polarized societies, i.e. to societies where $|\tau| < |\pi|$. Indeed, when party polarization is larger than the reform need, societies are worse off

[28] A full characterization of the socially optimal solution can be found in Appendix D.
[29] Only in the case where $\pi \to 0$ can elections yield the socially optimal outcome.

when knowledge about the office-holder's quality is realized according to the ex-interim scenario as opposed to both the ex-ante and ex-post scenarios.

The differential impact of information about reform abilities in high- and low-polarized societies can be explained as follows: Welfare is a convex combination of two terms, one which increases when the reform need decreases and another which increases when party polarization decreases. The former term is lower (higher) than the latter, depending on whether the reform need is higher (lower) than party polarization. The weights of both terms in welfare depend on the incumbent's expected reform ability—which varies across the three timing possibilities and depends on whether the reform need is higher or lower than party polarization. In the ex-interim scenario, in particular, the weight is particularly large for the term associated with the reform need. This explains why for $|\tau| < |\pi|$, the ex-interim scenario yields lower welfare than the other scenarios. Third, the following result follows immediately from the proof of Proposition 8:

Corollary 2
For any $\pi \in \left(0, \frac{1}{2}\right)$ and $\tau \in \left(0, \frac{1}{2}\right)$, we have

(i) $\frac{\partial W_1^{EA}}{\partial \pi} < 0$, $\frac{\partial W_1^{EI}}{\partial \pi} < 0$, *and* $\frac{\partial W_1^{EP}}{\partial \pi} < 0$.

(ii) $\frac{\partial W_1^{EA}}{\partial \tau} < 0$, $\frac{\partial W_1^{EI}}{\partial \tau} < 0$, *and* $\frac{\partial W_1^{EP}}{\partial \tau} < 0$.

(iii) $\frac{\partial W_1^{soc}}{\partial \tau} < 0$, *and* $\frac{\partial W_1^{soc}}{\partial \pi} = 0$.

That is, all else being equal, under elections welfare decreases, as either party polarization or the need-for-reform increases. Additionally, changes in party polarization have no effect on the socially optimal solution.

7.2 Welfare in the dynamic model

Next we study the expected utilitarian welfare evaluated before the beginning of period $t = 1$, given that $k \in R$ is the office-holder and $i_0 \in \mathcal{I}$ is the status quo. As in the static model, expected utilitarian welfare coincides with the median voter's expected welfare. Let $k' \in L$ denote the challenger in the elections at the end of period $t = 1$ and $k'' \in \{k, k'\}$ the office-holder in $t = 2$. Then welfare can be defined as

$$W_2 := \mathbb{E}\left[U^{\mathcal{I}}_{\frac{1}{2}}(i_{k1}) + U^{c_k}(i_0, i_{k1}) + \beta \cdot \left(U^{\mathcal{I}}_{\frac{1}{2}}(i_{k''2}) + U^{c_{k''}}(i_{k1}, i_{k''2})\right)\right]. \tag{18}$$

We calculate welfare for each of the three cases considered in this section. Accordingly, we use W_2^{EA}, W_2^{EI}, and W_2^{EP} to denote welfare in the ex-ante, ex-interim, and ex-post reform ability cases, respectively. We also let W_2^{soc} denote the expected welfare generated when a social planner can decide which policies are carried out and which are the politicians that do so.[30] For the sake of analytical tractability, we will focus on the case where party polarization, π, is arbitrarily small. Letting $\pi \to 0$ also facilitates capturing the effects driven by the need-for-reform τ alone. We recall that $\hat{C} < \bar{C}$, where $\hat{C}$ has been defined in (8). The main result regarding welfare in the dynamic model is:

Proposition 9

Let $\tau \in \left(0, \frac{1}{2}\right]$ and $\pi \to 0$, and suppose that $F(\hat{C}) < \frac{\sqrt{5}-1}{2}$. We obtain that

(a) if $F(\bar{C}) < \frac{1}{2}$, then

$$W_2^{EP} < W_2^{EI} < W_2^{EA} < W_2^{soc}.$$

(b) if $F(\bar{C}) \geq \frac{1}{2}$, then

$$W_2^{EI} < W_2^{EA} < W_2^{soc} \quad \text{and} \quad W_2^{EI} < W_2^{EP} < W_2^{soc}.$$

Two comments are in order. First, even if $\pi \to 0$, elections will not yield the social optimum. Second, as in the static model, we cannot unambiguously determine whether earlier revelation of reform abilities is socially preferable to later revelation as a general rule. Moreover, the ranking of welfare for the different scenarios depends on the skewness of the reform ability distribution. The next result follows immediately from the proof of Proposition 9.

Corollary 3

Let $\pi \to 0$. Then, for any $\tau \in \left(0, \frac{1}{2}\right)$,

$$\frac{\partial W_2^{EA}}{\partial \tau} < 0, \ \frac{\partial W_2^{EI}}{\partial \tau} < 0, \ \frac{\partial W_2^{EP}}{\partial \tau} < 0, \text{ and } \frac{\partial W_2^{soc}}{\partial \tau} < 0.$$

That is, all else being equal, welfare decreases as the need-for-reform increases. Consider now that the discount factor, β, is very small. In particular, costs associated with changes become almost unimportant for voters from the perspective of period $t = 1$. Then, it can be proved that elections do not yield the social optimum either. That is, strategic policy choices still matter, even if the future becomes almost irrelevant for the voters.

[30] A full characterization of the socially optimal solution can be found in Appendix D.

We finally analyze further welfare properties of costs of reform by investigating whether a better pool of candidates for office will increase voters' aggregate utility or not. Let $F(\cdot)$ and $G(\cdot)$ be two reform ability distributions with non-degenerate support within $[0, \infty)$. Then, F *stochastically dominates* G if $F(c) \leq G(c)$, i.e., the pool of candidates under G is more attractive than under F. In particular, the expected reform ability is lower for G than for F. For instance, it is easy to verify that given $F(\cdot)$ a uniform distribution on $[0, C_F]$ and $G(\cdot)$ a uniform distribution on $[0, C_G]$, with $C_F, C_G > 0$, then F stochastically dominates G if and only if $C_G \leq C_F$. Additionally, we say that F *strongly stochastically dominates* G if F stochastically dominates G and

$$F\left(\int_0^\infty cf(c)dc\right) \geq G\left(\int_0^\infty cg(c)dc\right). \tag{19}$$

We can prove the following auxiliary result:

Lemma 1

Let $F(\cdot)$ be a uniform distribution on $[0, C_F]$ and $G(\cdot)$ be a uniform distribution on $[0, C_G]$ such that $C_G \leq C_F$. Then F strongly stochastically dominates G.

Intuitively, one might expect welfare to be higher under G than under F if the reform-ability-distribution F stochastically dominates G. Of course, this is true for welfare when a social planner dictates policy decisions. However, this is not true in general for elections, as we see in the next corollary, which follows from Proposition 9. For a given reform ability distribution $F(\cdot)$, we denote welfare as W_2^F.

Proposition 10

Let $F(\cdot)$ and $G(\cdot)$ be two reform ability distributions and $\pi \to 0$. In the ex-interim case, if F strongly stochastically dominates G, we have $W_2^G \leq W_2^F$.[31]

This result is a twist on the result in Bernhardt et al. (2011), where a first-order improvement of candidates' valence distribution benefits all citizens. In our (dynamic) model, a first-order improvement of candidates' reform ability distribution has two conflicting consequences. On the one hand, it reduces the expected per-reform-step costs of policy changes. On the other hand, for a given incumbent whose reform ability is known, such an improvement makes it more likely that his reform quality will be lower than the challenger's (which is unknown at the time of the election). In such cases, an incumbent with low reform ability will choose the median voters' preferred policy to guarantee his re-election, thereby harming the latter's interests in two

[31] For most distributions we have $W_2^G < W_2^F$.

ways: *(i)* by generating excessive policy changes and *(ii)* by reducing the expected incumbent's reform ability in the next term. It turns out that if the first-order improvement of reform ability distribution is such that the mass of citizens whose reform ability is lower than average does not shrink—i.e. so that Equation (19) is satisfied—, the second effect will dominate the first, and thus welfare will diminish.

8 Conclusion

By means of both a static and a dynamic model, we have advanced a so-called Reform Dilemma, at work especially in polarized democracies. This property of elections materializes at time junctures where significant reforms in policy-making are desired to match the voters' preferences and where these reforms are costly for both voters and politicians. The Reform Dilemma manifests itself in two variants, each reproducing some real-world observations in politics. On the one hand, low-reform-ability office-holders are elected if party polarization is large. On the other, to secure re-election, office-holders align the social need-for-reform with their reform ability. In particular, high-reform-ability incumbents choose polar policies.

Our analysis reveals five further insights related with the impact on welfare brought about by elections in the presence of shocks to the politicians' and/or the voters' preferences. First, all else being equal, welfare may be lower in politically polarized societies or when the electorate cares very little about the quality of elected politicians and mostly about actual policies and the associated reform costs. Second, welfare may also depend on the exact time when the politicians' quality becomes public knowledge, but it cannot be unambiguously ascertained in terms of welfare whether it is better for such information to be known earlier or later, especially in polarized societies. This means that the value of ignorance may be positive or negative. Third, elected politicians may have incentives to become less able to undertake reforms, as this may increase their re-election chances. Fourth, ideological moderation never harms the candidates' electoral prospects. Fifth and last, a first-order improvement in the reform ability distribution may not be beneficial for the society.

Extensions beyond the ones already analyzed in the paper can be pursued. For instance, the implications of sudden changes in voters' preferences in systems other than winner-take-all elections deserve scrutiny. This and other issues like how our results extend to a repeated elections in infinite-horizon settings are left for further research.

References

Acemoglu, D., Egorov, G., and Sonin, K. (2009). Political selection and persistence of bad governments. Technical report, National Bureau of Economic Research.

Acemoglu, D. and Robinson, J. A. (2005). *Economic Origins of Dictatorship and Democracy.* Cambridge University Press, London.

Acemoglu, D., Ticchi, D., and Vindigni, A. (2011). Emergence and persistence of inefficient states. *Journal of the European Economic Association*, 9(2):177–208.

Adams, J., Clark, M., Ezrow, L., and Glasgow, G. (2004). Understanding change and stability in party ideologies: Do parties respond to public opinion or to past election results? *British Journal of Political Science*, 34(04):589–610.

Alford, J. R. and Brady, D. W. (1989). Personal and partisan advantage in US congressional elections. In Dodd, L. C. and Oppenheimer, B. I., editors, *Congress Reconsidered.* Praeger, New York, NY.

Ansolabehere, S. and Snyder Jr, J. M. (2000). Valence politics and equilibrium in spatial election models. *Public Choice*, 103(3-4):327–336.

Aragonés, E. and Palfrey, T. R. (2002). Mixed equilibrium in a Downsian model with a favored candidate. *Journal of Economic Theory*, 103(1):131–161.

Ashworth, S. (2012). Electoral accountability: Recent theoretical and empirical work. *Annual Review of Political Science*, 15:183–201.

Austen-Smith, D. and Banks, J. (1989). "Electoral accountability and incumbency" in Peter Ordeshook (ed). *Models of Strategic Choice in Politics,University of Michigan Press, Ann Arbor, MI.*

Bernhardt, D., Câmara, O., and Squintani, F. (2011). Competence and ideology. *Review of Economic Studies*, 78(2):487–522.

Bernhardt, M. D. and Ingerman, D. E. (1985). Candidate reputations and the "incumbency effect". *Journal of Public Economics*, 27(1):47–67.

Besley, T. and Coate, S. (1997). An economic model of representative democracy. *Quarterly Journal of Economics*, 112(1):85–114.

Besley, T. and Coate, S. (1998). Sources of inefficiency in a representative democracy: A dynamic analysis. *American Economic Review*, 88(1):139–156.

Blinder, A. S. and Watson, M. W. (2013). Presidents and the economy: A forensic investigation. *mimeo*.

Bowen, R., Chen, Y., and Eraslan, H. (2014). Mandatory versus discretionary spending. *American Economic Review*, 104(10):2941–2974.

Bowen, T. R., Chen, Y., Eraslan, H., and Zapal, J. (2015). Efficiency of flexible budgetary institutions. *Mimeo*.

Callander, S. and Raiha, D. (2014). Durable policy, political accountability, and active waste. *Mimeo*.

Carter, M. and Morrow, J. (2012). Left, right, left: Income dynamics and the evolving political preferences of forward-looking Bayesian voters. *EOPP*, (2012/034).

Caselli, F. and Morelli, M. (2004). Bad politicians. *Journal of Public Economics*, 88(3):759–782.

Chen, Y. and Eraslan, H. (2015). Dynamic agenda setting. *Mimeo*.

Combe, I. A. and Carrington, D. J. (2015). Leaders' sensemaking under crises: Emerging cognitive consensus over time within management teams. *Leadership Quarterly*, 26(3):307–322.

Cox, G. W. and Katz, J. N. (1996). Why did the incumbency advantage in us house elections grow? *American Journal of Political Science*, 40(2):478–497.

Day, D. V., Gronn, P., and Salas, E. (2004). Leadership capacity in teams. *Leadership Quarterly*, 15(6):857–880.

DeBacker, J. M. (2015). Flip-flopping: Ideological adjustment costs in the United States Senate. *Economic Inquiry*, 53(1):108–128.

Dewan, T. and Myatt, D. P. (2008). The qualities of leadership: Direction, communication, and obfuscation. *American Political Science Review*, 102(03):351–368.

Downs, A. (1957). An economic theory of political action in a democracy. *Journal of Political Economy*, 65(2):135–150.

Durr, R. H. (1993). What moves policy sentiment?. *American Political Science Review*, 87(01):158–170.

Enelow, J. M. and Munger, M. C. (1993). The elements of candidate reputation: The effect of record and credibility on optimal spatial location. *Public choice*, 77(4):757–772.

Erikson, R. S., Wright, G. C., and McIver, J. P. (1993). *Statehouse Democracy: Public Opinion and Policy in the American States*. Cambridge University Press, Cambridge.

Fernandez, R. and Rodrik, D. (1991). Resistance to reform: Status quo bias in the presence of individual-specific uncertainty. *American Economic Review*, 81(5):1146–1155.

Fiorina, M. P. and Abrams, S. J. (2008). Political polarization in the American public. *Annual Review of Political Science*, 11:563–588.

Forand, J. G. (2014). Two-party competition with persistent policies. *Journal of Economic Theory*, 152:64–91.

Franklin, C. H. (1984). Issue preferences, socialization, and the evolution of party identification. *American Journal of Political Science*, 28(3):459–478.

Galasso, V. and Nannicini, T. (2011). Competing on good politicians. *American Political Science Review*, 105(1):79–99.

Gelman, A. and King, G. (1990). Estimating incumbency advantage without bias. *American Journal of Political Science*, 34(4):1142–1164.

Gersbach, H. (1993). Politics and the choice of durability: Comment. *American Economic Review*, 83(3):670–673.

Gersbach, H., Muller, P., and Tejada, O. (2015). Costs of change, political polarization and re-election hurdles. *CER-ETH Working Paper No. 15/222*.

Glazer, A., Gradstein, M., and Konrad, K. A. (1998). The electoral politics of extreme policies. *Economic Journal*, 108(451):1677–1685.

Groseclose, T. (2001). A model of candidate location when one candidate has a valence advantage. *American Journal of Political Science*, 45(4):862–886.

Heberlig, E., Hetherington, M., and Larson, B. (2006). The price of leadership: Campaign money and the polarization of congressional parties. *Journal of Politics*, 68(4):992–1005.

Hotelling, H. (1929). Stability in competition. *Economic Journal*, 39:41–57.

Ingberman, D. E. (1989). Reputational dynamics in spatial competition. *Mathematical and Computer Modelling*, 12(4):479–496.

Jones, B. F. and Olken, B. A. (2005). Do leaders matter? National leadership and growth since World War II. *Quarterly Journal of Economics*, 120(3):835–864.

Kartik, N. and McAfee, R. P. (2007). Signaling character in electoral competition. *American Economic Review*, 97(3):852–870.

Kollman, K., Miller, J. H., and Page, S. E. (1992). Adaptive parties in spatial elections. *American Political Science Review*, 86(04):929–937.

Krasa, S. and Polborn, M. (2010a). The binary policy model. *Journal of Economic Theory*, 145(2):661–688.

Krasa, S. and Polborn, M. K. (2010b). Competition between specialized candidates. *American Political Science Review*, 104(4):745–765.

Layman, G. C., Carsey, T. M., and Horowitz, J. M. (2006). Party polarization in American politics: Characteristics, causes, and consequences. *Annual Review of Political Science*, 9(1):83–110.

Levitt, S. D. and Wolfram, C. D. (1997). Decomposing the sources of incumbency advantage in the US House. *Legislative Studies Quarterly*, 22(1):45–60.

Miller, G. and Schofield, N. (2003). Activists and partisan realignment in the United States. *American Political Science Review*, 97(2):245–260.

Owens, L. A. and Pedulla, D. S. (2014). Material welfare and changing political preferences: The case of support for redistributive social policies. *Social Forces*, 92(3):1087–1113.

Persson, T., Roland, G., and Tabellini, G. (1997). Separation of powers and political accountability. *Quarterly Journal of Economics*, 112(4):1163–1202.

Pillai, R. and Williams, E. A. (1998). Does leadership matter in the political arena? voter perceptions of candidates' transformational and charismatic leadership and the 1996 US President. *Leadership Quarterly*, 9(3):397–416.

Plott, C. R. (1967). A notion of equilibrium and its possibility under majority rule. *American Economic Review*, 57(4):787–806.

Roberts, J. M. and Smith, S. S. (2003). Procedural contexts, party strategy, and conditional party voting in the US House of Representatives, 1971–2000. *American Journal of Political Science*, 47(2):305–317.

Rohde, D. W. (2010). *Parties and leaders in the postreform House*. University of Chicago Press, Chicago, IL.

Rohlfs, C., Sullivan, R., and McNab, R. (2015). Can the president really affect economic growth? Presidential effort and the political business cycle. *Economic Inquiry*, 53(1):240–257.

Saint-Paul, G., Ticchi, D., and Vindigni, A. (2015). A theory of political entrenchment. *Economic Journal*, 126:1238–1263.

Samuelson, L. (1984). Electoral equilibria with restricted strategies. *Public Choice*, 43(3):307–327.

Simonton, D. K. (2006). Presidential IQ, openness, intellectual brilliance, and leadership: Estimates and correlations for 42 US chief executives. *Political Psychology*, 27(4):511–526.

Stokes, D. E. (1963). Spatial models of party competition. *American Political Science Review*, 57(02):368–377.

Stone, W. J. and Simas, E. N. (2010). Candidate valence and ideological positions in US House elections. *American Journal of Political Science*, 54(2):371–388.

Tavits, M. (2007). Principle vs. pragmatism: Policy shifts and political competition. *American Journal of Political Science*, 51(1):151–165.

Theriault, S. M. (2004). The case of the vanishing moderates: Party polarization in the modern congress. In *Annual Meeting of the Midwest Political Science Association*.

Theriault, S. M. (2006). Party polarization in the US congress: Member replacement and member adaptation. *Party Politics*, 12(4):483–503.

Williams, E. A., Pillai, R., Deptula, B., and Lowe, K. B. (2012). The effects of crisis, cynicism about change, and value congruence on perceptions of authentic leadership and attributed charisma in the 2008 presidential election. *Leadership Quarterly*, 23(3):324–341.

Williams, E. A., Pillai, R., Lowe, K. B., Jung, D., and Herst, D. (2009). Crisis, charisma, values, and voting behavior in the 2004 presidential election. *Leadership Quarterly*, 20(2):70–86.

Wittman, D. (1989). Why democracies produce efficient results. *Journal of Political Economy*, 97(6):1395–1424.

Appendix A

Proof of Proposition 1

We stress that when choosing the policy in the second period, office-holder k'' knows his reform ability, $c_{k''}$. Let

$$B(i_{k''2}) = U^{\mathcal{I}}_{\mu_{k''}}(i_{k''2}) + U^{c_{k''}}(i_1, i_{k''2}) = -\left(i_{k''2} - \mu_{c_{k''}}\right)^2 - c_{k''} \cdot (i_1 - i_{k''2})^2 .$$

Hence, k'' solves the following problem:

$$\begin{aligned} \max_{i_{k''2}} \quad & B(i_{k''2}) \\ \text{s.t. } & i_{k''2} \in \mathcal{I}. \end{aligned} \tag{20}$$

It is a matter of simple algebra to verify that

$$\frac{c_{k''}}{c_{k''}+1} \cdot i_1 + \frac{1}{c_{k''}+1} \cdot \mu_{k''}$$

is the unique solution to the problem in (20).

□

Proof of Proposition 2

First, we show that the *decisive voter* in the election is the median voter, $m = \frac{1}{2}$. I.e., we demonstrate that m will vote for the incumbent if and only if a share of citizens of at least 50% votes for the incumbent. For the sake of readability, let $i^*_{R2} := i^*_{k2}(i_1)$ denote the best-response function of the incumbent, k, when he is re-elected and $i^*_{L2} = i^*_{k''2}(i_1)$ denote the best-response function of the challenger, k', when he is elected. Note that $i^*_{k2}(i_1)$ and $i^*_{k'2}(i_1)$ have been defined in Proposition 1. Let $i \in [0,1]$ be an arbitrary voter. According to the tie-breaking rules, i will vote for the incumbent k if and only if

$$U^{\mathcal{I}}_i(i^*_{R2}) + U^{c_k}(i_1, i^*_{R2}) \geq U^{\mathcal{I}}_i(i^*_{L2}) + U^{c_{k'}}(i_1, i^*_{L2}), \tag{21}$$

which can be rewritten as

$$2i \cdot (i^*_{R2} - i^*_{L2}) \geq (i^*_{R2})^2 - (i^*_{L2})^2 + c_k \cdot (i_1 - i^*_{R2})^2 - c_{k'} \cdot (i_1 - i^*_{L2})^2 . \tag{22}$$

Note that the right-hand side of (22) does not depend on i. We distinguish two cases.

- **Case A:** $i^*_{R2} - i^*_{L2} \geq 0$

  From (22) it follows that if a voter i votes for the right-wing candidate, all voters $j \in (i, 1]$ will also vote for him. Similarly, if a voter i votes for the left-wing candidate, all voters $[0, i+\varepsilon]$ will also vote for him, with $\varepsilon > 0$ small enough.

- **Case B:** $i^*_{R2} - i^*_{L2} < 0$

  From (22) it follows that if a voter i votes for the left-wing candidate, all voters $j \in [i-\varepsilon, 1]$ will also vote for him, with $\varepsilon > 0$ small enough. Similarly, if a voter i votes for the right-wing candidate, all voters $[0, i)$ will also vote for him.

That is, in either case m is the decisive voter. From the explicit definitions of i^*_{R2} and i^*_{L2} in Proposition 1, the inequality in (21) when $i = m$ can be rewritten as

$$\begin{aligned}&\left(\frac{1}{2} - \frac{c_{k'}}{c_{k'}+1} i_1 - \frac{1}{c_{k'}+1}\mu_L\right)^2 + c_{k'} \cdot \left(i_1 - \frac{c_{k'}}{c_{k'}+1} i_1 - \frac{1}{c_{k'}+1}\mu_L\right)^2 \\ \geq &\left(\frac{1}{2} - \frac{c_k}{c_k+1} i_1 - \frac{1}{c_k+1}\mu_R\right)^2 + c_k \cdot \left(i_1 - \frac{c_k}{c_k+1} i_1 - \frac{1}{c_k+1}\mu_R\right)^2 . \end{aligned} \tag{23}$$

Given $c \geq 0$, $i_1 \in \mathcal{I}$, and $\mu \in [0,1]$, it follows from straightforward algebraic manipulations that

$$\begin{aligned}&\left(\frac{1}{2} - \frac{c}{c+1} i_1 - \frac{1}{c+1}\mu\right)^2 + c \cdot \left(i_1 - \frac{c}{c+1} i_1 - \frac{1}{c+1}\mu\right)^2 \\ =&\frac{1}{4} - \frac{1}{c+1} \cdot (c \cdot i_1 \cdot (1 - i_1) + \mu \cdot (1 - \mu)) . \end{aligned} \tag{24}$$

We point out that the above expression will be used throughout all appendices in different proofs. In the present proof using (24), we obtain from (23) that

$$\begin{aligned}&\frac{1}{c_k+1} \cdot (c_k \cdot i_1 \cdot (1 - i_1) + \mu_R \cdot (1 - \mu_R)) \\ \geq &\frac{1}{c_{k'}+1} \cdot (c_{k'} \cdot i_1 \cdot (1 - i_1) + \mu_L \cdot (1 - \mu_L)) ,\end{aligned}$$

which, since $\mu_R = 1 - \mu_L$, leads to

$$i_1 \cdot (1 - i_1) \cdot \left(\frac{c_k}{c_k+1} - \frac{c_{k'}}{c_{k'}+1}\right) \geq \mu_R \cdot (1 - \mu_R) \cdot \left(\frac{1}{c_{k'}+1} - \frac{1}{c_k+1}\right),$$

and further to

$$[i_1 \cdot (1 - i_1) - \mu_R \cdot (1 - \mu_R)] \cdot (c_k - c_{k'}) \geq 0.$$

The desired result follows immediately from an inspection of the above inequality.

□

Proof of Proposition 3

We analyze the solution to the maximization problem given in (11). We can use Propositions 1 and 2 for the optimal choice in period $t = 2$ together with the outcome in the second election, and we obtain

$$\begin{aligned}&U_k^{exp}(i_{k1}) = b \\ &- \int_0^\infty c_k (i_0 - i_{k1})^2 f(c_k) dc_k - \int_0^\infty (i_{k1} - \mu_R)^2 f(c_k) dc_k \\ &+ b \cdot \beta \cdot \int_0^\infty \int_0^\infty I(i_{k1}, c_k, c_{k'}) f(c_{k'}) dc_{k'} f(c_k) dc_k \\ &- \beta \cdot \int_0^\infty \int_0^\infty I(i_{k1}, c_k, c_{k'}) \cdot \left(\frac{c_k}{c_k+1} i_{k1} + \frac{1}{c_k+1}\mu_R - \mu_R\right)^2 f(c_{k'}) dc_{k'} f(c_k) dc_k \\ &- \beta \cdot \int_0^\infty \int_0^\infty I(i_{k1}, c_k, c_{k'}) \cdot c_k \left(i_{k1} - \frac{c_k}{c_k+1} i_{k1} - \frac{1}{c_k+1}\mu_R\right)^2 f(c_{k'}) dc_{k'} f(c_k) dc_k \\ &- \beta \cdot \int_0^\infty \int_0^\infty (1 - I(i_{k1}, c_k, c_{k'})) \cdot \left(\frac{c_{k'}}{c_{k'}+1} i_{k1} + \frac{1}{c_{k'}+1}\mu_L - \mu_R\right)^2 f(c_{k'}) dc_{k'} f(c_k) dc_k \\ &- \beta \cdot \int_0^\infty \int_0^\infty (1 - I(i_{k1}, c_k, c_{k'})) c_{k'} \left(i_{k1} - \frac{c_{k'}}{c_{k'}+1} i_{k1} - \frac{1}{c_{k'}+1}\mu_L\right)^2 f(c_{k'}) dc_{k'} f(c_k) dc_k,\end{aligned} \tag{25}$$

where $I(i_{k1}, c_k, c_{k'}) = 1$ if and only if $i_{k1} \in \mathcal{X}(c_k, c_{k'})$ and the set $\mathcal{X}(c_k, c_{k'})$ has been defined in Proposition 2. Because b has been assumed to be very large and c_k and $c_{k'}$ are known when k makes his policy choice, he will choose $i_{k1} \in \mathcal{X}(i_{k1}, c_k, c_{k'})$ for any realization of his reform ability. Note that if $i_{k1} \in \mathcal{X}(i_{k1}, c_k, c_{k'})$, we obtain

$$\begin{aligned}
&U_k^{exp}(i_{k1}) \\
=&b - c_k \cdot (i_0 - i_{k1})^2 - (i_{k1} - \mu_R)^2 + \beta \cdot b \\
-&\beta \cdot \left[\left(\frac{c_k}{c_k+1} i_{k1} + \frac{1}{c_k+1} \mu_R - \mu_R \right)^2 + c_k \cdot \left(\frac{c_k}{c_k+1} i_{k1} + \frac{1}{c_k+1} \mu_R - i_{k1} \right)^2 \right] \\
=&(1+\beta) \cdot b - c_k \cdot (i_0 - i_{k1})^2 - \left(1 + \beta \cdot \frac{c_k}{c_k+1} \right) \cdot (i_{k1} - \mu_R)^2.
\end{aligned}$$

The solution to the problem

$$\begin{aligned}
\max_{i_{k1}} \quad & U_k^{exp}(i_{k1}) \\
\text{s.t. } \; & i_{k1} \in \mathcal{I}
\end{aligned} \tag{26}$$

is then

$$i_{1,*}^{EA}(i_0, c_k) = \frac{c_k}{1 + c_k + \beta \cdot \frac{c_k}{c_k+1}} \cdot i_0 + \frac{1 + \beta \cdot \frac{c_k}{c_k+1}}{1 + c_k + \beta \cdot \frac{c_k}{c_k+1}} \cdot \mu_R. \tag{27}$$

Next we distinguish three cases.

Case I: $c_k > c_{k'}$

In this case, we have $\mathcal{X}(i_{k1}, c_k, c_{k'}) = [1 - \mu_R, \mu_R]$. Hence, k solves the following problem:

$$\begin{aligned}
\max_{i_{k1}} \quad & U_k^{exp}(i_{k1}) \\
\text{s.t. } \; & i_{k1} \in [1 - \mu_R, \mu_R] .
\end{aligned}$$

Given (26) and (27), the solution to the above problem is

$$i_{k1}^{EA} = \min \left\{ \mu_R, \max \left\{ 1 - \mu_R, i_{1,*}^{EA}(i_0, c_k) \right\} \right\} .$$

Case II: $c_k = c_{k'}$

In this case, we have $\mathcal{X}(i_{k1}, c_k, c_{k'}) = [0, 1]$. Hence, given (78) and (79), k chooses

$$i_{k1}^{EA} = i_{1,*}^{EA}(i_0, c_k).$$

Case III: $c_k < c_{k'}$

In this case, we have $\mathcal{X}(i_{k1}, c_k, c_{k'}) = [0, 1 - \mu_R] \cup [\mu_R, 1]$. Hence, k solves the following problem:

$$\begin{aligned}
\max_{i_{k1}} \quad & U_k^{exp}(i_{k1}) \\
\text{s.t. } \; & i_{k1} \in [0, 1 - \mu_R] \cup [\mu_R, 1] .
\end{aligned}$$

Given (78) and (79), the solution to the above problem is

$$i_{k1}^{EA} = \begin{cases} \min\{1-\mu_R, i_{1,*}^{EA}(i_0,c_k)\} & \text{if } i_{1,*}^{EA}(i_0,c_k) \in \left[0,\frac{1}{2}\right), \\ 1-\mu_R \text{ or } \mu_R & \text{if } i_{1,*}^{EA}(i_0,c_k) = \frac{1}{2}, \\ \max\{\mu_R, i_{1,*}^{EA}(i_0,c_k)\} & \text{if } i_{1,*}^{EA}(i_0,c_k) \in \left(\frac{1}{2},1\right]. \end{cases}$$

Finally, the statement of the proposition follows using $\pi = \mu_R - \frac{1}{2}$ and $\tau = i_0 - \frac{1}{2}$.

□

Proof of Proposition 4

We analyze the solution to the maximization problem in (11). With the help of Propositions 15 and 16 (see Appendix D) we can write

$$\begin{aligned} &U_k^{exp}(i_{k1}) = b \\ &- \int_0^\infty c_k(i_0-i_{k1})^2 f(c_k)dc_k - \int_0^\infty (i_{k1}-\mu_R)^2 f(c_k)dc_k + b\cdot\beta\cdot\int_0^\infty I(i_{k1},c_k,\bar{C})f(c_k)dc_k \\ &- \beta\cdot\int_0^\infty I(i_{k1},c_k,\bar{C})\cdot\left(\frac{c_k}{c_k+1}i_{k1}+\frac{1}{c_k+1}\mu_R-\mu_R\right)^2 f(c_k)dc_k \\ &- \beta\cdot\int_0^\infty I(i_{k1},c_k,\bar{C})\cdot c_k\left(i_{k1}-\frac{c_k}{c_k+1}i_{k1}-\frac{1}{c_k+1}\mu_R\right)^2 f(c_k)dc_k \\ &- \beta\cdot\int_0^\infty (1-I(i_{k1},c_k,\bar{C}))\cdot\left[\int_0^\infty\left(\frac{\bar{C}}{\bar{C}+1}i_{k1}+\frac{1-\mu_R}{\bar{C}+1}-\mu_R\right)^2 f(c_{k'})dc_{k'}\right]f(c_k)dc_k \\ &- \beta\cdot\int_0^\infty (1-I(i_{k1},c_k,\bar{C}))\cdot\left[\int_0^\infty c_{k'}\left(i_{k1}-\frac{\bar{C}}{\bar{C}+1}i_{k1}-\frac{1-\mu_R}{\bar{C}+1}\right)^2 f(c_{k'})dc_{k'}\right]f(c_k)dc_k, \end{aligned} \tag{28}$$

where $I(i_{k1},c_k,\bar{C}) = 1$ if and only if $i_{k1} \in \mathcal{X}(c_k,\bar{C})$ and the set $\mathcal{X}(c_k,\bar{C})$ has been defined in Proposition 16 (see Appendix D). Next we define the auxiliary function

$$p(i_{k1}) = \begin{cases} 1-F(\bar{C}) & \text{if } i_{k1} \in [1-\mu_R,\mu_R], \\ F(\bar{C}) & \text{otherwise.} \end{cases}$$

Developing (28), we can write

$$\begin{aligned} &U_k^{exp}(i_{k1}) \\ =&b - \bar{C}(i_0-i_{k1})^2 - (i_{k1}-\mu_R)^2 + \beta\cdot bp(i_{k1}) \\ -&\beta\cdot(i_{k1}-\mu_R)^2\int_0^\infty I(i_{k1},c_k,\bar{C})\frac{c_k}{1+c_k}f(c_k)dc_k \\ -&\beta\cdot(1-p(i_{k1}))\left[\left(\frac{\bar{C}}{\bar{C}+1}i_{k1}+\frac{1}{\bar{C}+1}(1-\mu_R)-\mu_R\right)^2+\frac{\bar{C}}{(\bar{C}+1)^2}\left((1-\mu_R)-i_{k1}\right)^2\right] \\ =&b - \bar{C}(i_0-i_{k1})^2 + \beta\cdot bp(i_{k1}) - (i_{k1}-\mu_R)^2\left(1+\beta\cdot\int_0^\infty I(i_{k1},c_k,\bar{C})\frac{c_k}{1+c_k}f(c_k)dc_k\right) \\ -&\beta\cdot(1-p(i_{k1}))\left[\mu_R^2+\frac{1}{\bar{C}+1}\left(\bar{C}i_{k1}(i_{k1}-2\mu_R)+(1-\mu_R)(1-3\mu_R)\right)\right]. \end{aligned}$$

Next we distinguish two cases.

Case I: $i_{k1} \in [1-\mu_R, \mu_R]$

In this case, we have[32]

$$\begin{aligned}
&U_k^{exp}(i_{k1}) \\
=&b - \bar{C}(i_0 - i_{k1})^2 + \beta \cdot b(1 - F(\bar{C})) - (i_{k1} - \mu_R)^2 \left(1 + \beta \cdot \int_{\bar{C}}^{\infty} \frac{c_k}{1+c_k} f(c_k) dc_k\right) \\
&- \beta \cdot F(\bar{C}) \left[\mu_R^2 + \frac{1}{\bar{C}+1}\left(\bar{C} i_{k1}(i_{k1} - 2\mu_R) + (1-\mu_R)(1-3\mu_R)\right)\right] := h(i_{k1}).
\end{aligned}$$

Then note that

$$\begin{aligned}
\frac{1}{2}\frac{dh(i_{k1})}{di_{k1}} =& -\bar{C}(i_{k1} - i_0) - (i_{k1} - \mu_R)\left(1 + \beta \cdot \int_{\bar{C}}^{\infty} \frac{c_k}{1+c_k} f(c_k) dc_k\right) \\
&- \beta \cdot F(\bar{C}) \frac{\bar{C}}{\bar{C}+1}(i_{k1} - \mu_R).
\end{aligned}$$

Hence the first-order condition is

$$\begin{aligned}
i_{1,*}^{EP}(i_0) =& \frac{\bar{C}}{\bar{C} + 1 + \beta \cdot \int_{\bar{C}}^{\infty} \frac{c_k}{1+c_k} f(c_k) dc_k + \beta \cdot F(\bar{C}) \frac{\bar{C}}{\bar{C}+1}} \cdot i_0 \\
&+ \frac{1 + \beta \cdot \int_{\bar{C}}^{\infty} \frac{c_k}{1+c_k} f(c_k) dc_k + \beta \cdot F(\bar{C}) \frac{\bar{C}}{\bar{C}+1}}{\bar{C} + 1 + \beta \cdot \int_{\bar{C}}^{\infty} \frac{c_k}{1+c_k} f(c_k) dc_k + \beta \cdot F(\bar{C}) \frac{\bar{C}}{\bar{C}+1}} \cdot \mu_R.
\end{aligned}$$

Case II: $i_{k1} \in [0, 1-\mu_R) \cup (\mu_R, 1]$

In this case, we have[33]

$$\begin{aligned}
&U_k^{exp}(i_{k1}) \\
=&b - \bar{C}(i_0 - i_{k1})^2 + \beta \cdot bF(\bar{C}) - (i_{k1} - \mu_R)^2 \left(1 + \beta \cdot \int_{0}^{\bar{C}} \frac{c_k}{1+c_k} f(c_k) dc_k\right) \\
&- \beta \cdot (1 - F(\bar{C})) \left[\mu_R^2 + \frac{1}{\bar{C}+1}\left(\bar{C} i_{k1}(i_{k1} - 2\mu_R) + (1-\mu_R)(1-3\mu_R)\right)\right] := g(i_{k1}).
\end{aligned}$$

Then the first-order condition is

$$\begin{aligned}
i_{1,**}^{EP}(i_0) =& \frac{\bar{C}}{\bar{C} + 1 + \beta \cdot \int_{0}^{\bar{C}} \frac{c_k}{1+c_k} f(c_k) dc_k + \beta \cdot (1 - F(\bar{C})) \frac{\bar{C}}{\bar{C}+1}} \cdot i_0 \\
&+ \frac{1 + \beta \cdot \int_{0}^{\bar{C}} \frac{c_k}{1+c_k} f(c_k) dc_k + \beta \cdot (1 - F(\bar{C})) \frac{\bar{C}}{\bar{C}+1}}{\bar{C} + 1 + \beta \cdot \int_{0}^{\bar{C}} \frac{c_k}{1+c_k} f(c_k) dc_k + \beta \cdot (1 - F(\bar{C})) \frac{\bar{C}}{\bar{C}+1}} \cdot \mu_R.
\end{aligned}$$

We recall that b is very large for each given distribution $F(\cdot)$. Next we distinguish three cases.

Case A: $F(\bar{C}) < \frac{1}{2}$

In this case, we have

$$\max_{i_{k1} \in \mathcal{I}} g(i_{k1}) < \min_{i_{k1} \in \mathcal{I}} h(i_{k1}).$$

[32] We note that $h(i_{k1})$ is defined for all $i_{k1} \in \mathcal{I}$.

[33] We note that $g(i_{k1})$ is defined for all $i_{k1} \in \mathcal{I}$.

First, if $i_0 \in (\mu_R, 1]$, we have $i_{1,*}^{EP}(i_0) \in (\mu_R, 1]$, so

$$\arg\max_{i_{k1} \in \mathcal{I}} U_k^{exp}(i_{k1}) = \{\mu_R\}.$$

Second, if $i_0 \in [1-\mu_R, \mu_R]$, we have $i_{1,*}^{EP}(i_0) \in [1-\mu_R, \mu_R]$, so

$$\arg\max_{i_{k1} \in \mathcal{I}} U_k^{exp}(i_{k1}) = \left\{i_{1,*}^{EP}(i_0)\right\}.$$

Third, if $i_0 \in [0, 1-\mu_R)$, we have $i_{1,*}^{EP}(i_0) \in [0, 1-\mu_R)$, so

$$\arg\max_{i_{k1} \in \mathcal{I}} U_k^{exp}(i_{k1}) = \{1-\mu_R\}.$$

Hence the optimal choice is

$$i_{k1}^{EP} = \min\left\{\mu_R, \max\left\{1-\mu_R, i_{1,*}^{EP}(i_0)\right\}\right\}.$$

Case B: $F(\bar{C}) > \frac{1}{2}$

In this case, we have

$$\max_{i_{k1} \in \mathcal{I}} h(i_1) < \min_{i_{k1} \in \mathcal{I}} g(i_1).$$

First, if $i_0 \in (\mu_R, 1]$, we have $i_{1,**}^{EP}(i_0) \in (\mu_R, 1]$, so

$$\arg\max_{i_{k1} \in \mathcal{I}} U_k^{exp}(i_{k1}) = \left\{i_{1,**}^{EP}(i_0)\right\}.$$

Second, if $i_0 \in [1-\mu_R, \mu_R]$ and $i_{1,*}^{EP}(i_0) \in \left[1-\mu_R, \frac{1}{2}\right)$, we have

$$\arg\max_{i_{k1} \in \mathcal{I}} U_k^{exp}(i_{k1}) = \{1-\mu_R\}.$$

Third, if $i_0 \in [1-\mu_R, \mu_R]$ and $i_{1,*}^{EP}(i_0) = \frac{1}{2}$, we have

$$\arg\max_{i_1 \in \mathcal{I}} U_k^{exp}(i_{k1}) = \{1-\mu_R, \mu_R\}.$$

Fourth, if $i_0 \in [1-\mu_R, \mu_R]$ and $i_{1,*}^{EP}(i_0) \in \left(\frac{1}{2}, \mu_R\right]$, we have

$$\arg\max_{i_1 \in \mathcal{I}} U_k^{exp}(i_{k1}) = \{\mu_R\}.$$

Fifth, if $i_0 \in [0, 1-\mu_R)$, we have $i_{1,**}^{EP}(i_0) \in [0, 1-\mu_R)$, so

$$\arg\max_{i_1 \in \mathcal{I}} U_k^{exp}(i_{k1}) = \left\{i_{1,**}^{EP}(i_0)\right\}.$$

Hence the optimal choice is

$$i_{k1}^{EP} = \begin{cases} \min\{1-\mu_R, i_{1,**}^{EP}(i_0)\} & \text{if } i_{1,**}^{EP}(i_0) \in \left[0, \frac{1}{2}\right), \\ 1-\mu_R \text{ or } \mu_R & \text{if } i_{1,**}^{EP}(i_0) = \frac{1}{2}, \\ \max\{\mu_R, i_{1,**}^{EP}(i_0)\} & \text{if } i_{1,**}^{EP}(i_0) \in \left(\frac{1}{2}, 1\right]. \end{cases}$$

Case C: $F(\bar{C}) = \frac{1}{2}$

In this case, we obtain

$$h(i_{k1}) - g(i_{k1}) = -\beta \cdot (i_{k1} - \mu_R)^2 \left(\int_{\bar{C}}^{\infty} \frac{c_k}{1+c_k} f(c_k) dc_k - \int_0^{\bar{C}} \frac{c_k}{1+c_k} f(c_k) dc_k \right) < 0,$$

where the inequality is explained as follows: $\frac{y}{1+y}$ increases with y,

$$\int_{\bar{C}}^{\infty} f(c_k)dc_k = \int_0^{\bar{C}} f(c_k)dc_k,$$

and

$$\int_{\bar{C}}^{\infty} \frac{c_k}{1+c_k} f(c_k)dc_k > \int_{\bar{C}}^{\infty} \frac{\bar{C}}{1+\bar{C}} f(c_k)dc_k = \int_0^{\bar{C}} \frac{\bar{C}}{1+\bar{C}} f(c_k)dc_k > \int_0^{\bar{C}} \frac{c_k}{1+c_k} f(c_k)dc_k.$$

Then the optimal choice is

$$i_{k1}^{EP} = \begin{cases} i_{1,*}^{EP}(i_0) & \text{if } i_0 \in [0, 1-\mu_R) \cup (\mu_R, 1], \\ i & \text{otherwise, where } i \in \{1-\mu_R, i_{1,*}^{EP}(i_0), \mu_R\}. \end{cases}$$

Finally, the statement of the proposition follows using $\pi = \mu_R - \frac{1}{2}$ and $\tau = i_0 - \frac{1}{2}$.

□

Appendix B

This appendix deals with one extension of our baseline model. More specifically, it contains the proof of Proposition 5. This result analyzes the incentives for incumbents to lower their reform ability.

Proof of Proposition 5

Let $k \in R$ be the incumbent in $t = 1$. Then he faces the following problem:

$$\begin{aligned} \max_{i_{k1},x} \quad & U_k^{exp}(i_{k1}, x) \\ \text{s.t. } & i_{k1} \in \mathcal{I} \text{ and } x \in [0, \bar{C}], \end{aligned} \tag{29}$$

where now

$$\begin{aligned} & U_k^{exp}(i_{k1}, x) = b - r(x) \\ & - \int_0^\infty (c_k + x)(i_0 - i_{k1})^2 f(c_k) dc_k - \int_0^\infty (i_{k1} - \mu_R)^2 f(c_k) dc_k \qquad (30) \\ & + \beta \cdot b \cdot \int_0^\infty I(i_{k1}, c_k + x, \bar{C}) f(c_k) dc_k \\ & - \beta \cdot \int_0^\infty I(i_{k1}, c_k + x, \bar{C}) \left(\frac{c_k + x}{c_k + x + 1} i_{k1} + \frac{1}{c_k + x + 1} \mu_R - \mu_R \right)^2 f(c_k) dc_k \\ & - \beta \cdot \int_0^\infty I(i_{k1}, c_k + x, \bar{C})(c_k + x) \left(\frac{c_k + x}{c_k + x + 1} i_{k1} + \frac{1}{c_k + x + 1} \mu_R - i_{k1} \right)^2 f(c_k) dc_k \\ & - \beta \cdot \int_0^\infty (1 - I(i_{k1}, c_k + x, \bar{C})) \left(\frac{\bar{C}}{\bar{C} + 1} i_{k1} + \frac{1}{\bar{C} + 1} (1 - \mu_R) - \mu_R \right)^2 f(c_k) dc_k \\ & - \beta \cdot \int_0^\infty (1 - I(i_{k1}, c_k + x, \bar{C}))\bar{C} \left(\frac{\bar{C}}{\bar{C} + 1} i_{k1} + \frac{1}{\bar{C} + 1} (1 - \mu_R) - i_{k1} \right)^2 f(c_k) dc_k. \qquad (31) \end{aligned}$$

It will be helpful to define the following auxiliary function:

$$p(i_{k1}, x) = \begin{cases} 1 - F(\bar{C} - x) & \text{if } i_{k1} \in [1 - \mu_R, \mu_R], \\ F(\bar{C} - x) & \text{otherwise.} \end{cases}$$

Developing (30), we can write

$$\begin{aligned} & U_k^{exp}(i_{k1}) \\ =& b - r(x) - (\bar{C} + x)(i_0 - i_{k1})^2 - (i_{k1} - \mu_R)^2 + \beta \cdot bp(i_{k1}, x) \\ & - \beta \cdot (i_{k1} - \mu_R)^2 \int_0^\infty I(i_{k1}, c_k + x, \bar{C}) \frac{c_k + x}{1 + c_k + x} f(c_k) dc_k \\ & - \beta \cdot (1 - p(i_{k1}, x)) \left[\left(\frac{\bar{C}}{\bar{C} + 1} i_{k1} + \frac{1}{\bar{C} + 1} (1 - \mu_R) - \mu_R \right)^2 + \frac{\bar{C}}{(\bar{C} + 1)^2} \left((1 - \mu_R) - i_{k1} \right)^2 \right] \\ =& b - (\bar{C} + x)(i_0 - i_{k1})^2 + \beta \cdot bp(i_{k1}, x) \\ & - (i_{k1} - \mu_R)^2 \left(1 + \beta \cdot \int_0^\infty I(i_{k1}, c_k + x, \bar{C}) \frac{c_k + x}{1 + c_k + x} f(c_k) dc_k \right) \\ & - \beta \cdot (1 - p(i_{k1}, x)) \left[\mu_R^2 + \frac{1}{\bar{C}} \left(\bar{C} i_{k1}(i_{k1} - 2\mu_R) + (1 - \mu_R)(1 - 3\mu_R) \right) \right] - r(x). \end{aligned}$$

Next we distinguish two cases.

Case I: $i_{k1} \in [1-\mu_R, \mu_R]$

In this case, we have

$$\begin{aligned}
&U_k^{exp}(i_{k1}) \\
=&b - (\bar{C}+x)(i_0 - i_{k1})^2 + \beta \cdot b(1 - F(\bar{C}-x)) \\
&- (i_{k1} - \mu_R)^2 \left(1 + \beta \cdot \int_{\bar{C}-x}^{\infty} \frac{c_k + x}{1 + c_k + x} f(c_k) dc_k\right) \\
&- \beta \cdot F(\bar{C}-x) \left[\mu_R^2 + \frac{1}{\bar{C}+1}\left(\bar{C} i_{k1}(i_{k1} - 2\mu_R) + (1-\mu_R)(1-2\mu_R)\right)\right] - r(x) \\
:=&h(i_{k1}, x).
\end{aligned}$$

On the one hand,

$$\begin{aligned}
\frac{1}{2}\frac{\partial h(i_{k1}, x)}{\partial i_{k1}} =& -(\bar{C}+x)(i_{k1} - i_0) - (i_{k1} - \mu_R)\left(1 + \beta \cdot \int_{\bar{C}-x}^{\infty} \frac{c_k + x}{1 + c_k + x} f(c_k) dc_k\right) \\
&- \beta \cdot F(\bar{C}-x) \frac{\bar{C}}{\bar{C}+1}(i_{k1} - \mu_R) = 0,
\end{aligned}$$

which can be rewritten as

$$\begin{aligned}
i_1^*(i_0, x) =& \frac{\bar{C}+x}{\bar{C}+x+1+\beta \cdot \int_{\bar{C}-x}^{\infty} \frac{c_k+x}{1+c_k+x} f(c_k) dc_k + \beta \cdot F(\bar{C}-x)\frac{\bar{C}}{\bar{C}+1}} \cdot i_0 \\
&+ \frac{1+\beta \cdot \int_{\bar{C}-x}^{\infty} \frac{c_k+x}{1+c_k+x} f(c_k) dc_k + \beta \cdot F(\bar{C}-x)\frac{\bar{C}}{\bar{C}+1}}{\bar{C}+x+1+\beta \cdot \int_{\bar{C}-x}^{\infty} \frac{c_k+x}{1+c_k+x} f(c_k) dc_k + \beta \cdot F(\bar{C}-x)\frac{\bar{C}}{\bar{C}+1}} \cdot \mu_R.
\end{aligned}$$

On the other hand, using the Leibniz Rule, we obtain

$$\frac{d}{dx}\left(\int_{\bar{C}-x}^{\infty} \frac{c_k + x}{1 + c_k + x} f(c_k) dc_k\right) = \int_{\bar{C}-x}^{\infty} \left(\frac{1}{1 + c_k + x}\right)^2 f(c_k) dc_k + \frac{\bar{C}}{1+\bar{C}} f(\bar{C}-x).$$

Hence,

$$\begin{aligned}
&\frac{\partial h(i_{k1}, x)}{\partial x} \\
=&\beta \cdot f(\bar{C}-x)\left[b - \frac{\bar{C}}{1+\bar{C}} + \left[\mu_R^2 + \frac{1}{\bar{C}+1}\left(\bar{C} i_{k1}(i_{k1} - 2\mu_R) + (1-\mu_R)(1-2\mu_R)\right)\right]\right] \\
&- (i_0 - i_{k1})^2 - r'(x) - \beta \cdot (i_{k1} - \mu_R)^2 \int_{\bar{C}-x}^{\infty} \left(\frac{1}{1 + c_k + x}\right)^2 f(c_k) dc_k.
\end{aligned}$$

Moreover, if we assume that condition (15) holds, then for each $i_{k1} \in [1-\mu_R, \mu_R]$, we hvae

$$\frac{\partial h(i_{k1}, 0)}{\partial x} > 0 \tag{32}$$

and

$$\lim_{y \to \bar{C}} \frac{\partial h(i_{k1}, y)}{\partial x} = -\infty. \tag{33}$$

Case II: $i_{k1} \in [0,1] \setminus [1-\mu_R, \mu_R]$

In this case, we have

$$\begin{aligned}
&U_k^{exp}(i_{k1}) \\
=&b - (\bar{C}+x)(i_0 - i_{k1})^2 + \beta \cdot bF(\bar{C}-x) \\
&- (i_{k1}-\mu_R)^2 \left(1 + \beta \cdot \int_0^{\bar{C}-x} \frac{c_k + x}{1 + c_k + x} f(c_k) dc_k\right) \\
&- \beta \cdot (1 - F(\bar{C}-x)) \left[\mu_R^2 + \frac{1}{\bar{C}+1}\left(\bar{C} i_{k1}(i_{k1} - 2\mu_R) + (1-\mu_R)(1-2\mu_R)\right)\right] - r(x) \\
:=&g(i_{k1}, x).
\end{aligned}$$

Then,

$$\begin{aligned}
\frac{1}{2}\frac{\partial h(i_{k1}, x)}{\partial i_{k1}} =& -(\bar{C}+x)(i_{k1} - i_0) - (i_{k1} - \mu_R)\left(1 + \beta \cdot \int_0^{\bar{C}-x} \frac{c_k + x}{1 + c_k + x} f(c_k) dc_k\right) \\
&- \beta \cdot (1 - F(\bar{C}-x)) \frac{\bar{C}}{\bar{C}+1}(i_{k1} - \mu_R) = 0,
\end{aligned}$$

which can be rewritten as

$$\begin{aligned}
i_1^{**}(i_0, x) =& \frac{\bar{C}+x}{\bar{C}+x+1+\beta \cdot \int_0^{\bar{C}-x} \frac{c_k+x}{1+c_k+x} f(c_k) dc_k + \beta \cdot (1 - F(\bar{C}-x)) \frac{\bar{C}}{\bar{C}+1}} \cdot i_0 \\
&+ \frac{1+\beta \cdot \int_0^{\bar{C}-x} \frac{c_k+x}{1+c_k+x} f(c_k) dc_k + \beta \cdot (1 - F(\bar{C}-x)) \frac{\bar{C}}{\bar{C}+1}}{\bar{C}+x+1+\beta \cdot \int_0^{\bar{C}-x} \frac{c_k+x}{1+c_k+x} f(c_k) dc_k + \beta \cdot (1 - F(\bar{C}-x)) \frac{\bar{C}}{\bar{C}+1}} \cdot \mu_R.
\end{aligned}$$

Moreover, using the Leibniz Rule, we obtain

$$\frac{d}{dx}\left(\int_0^{\bar{C}-x} \frac{c_k + x}{1 + c_k + x} f(c_k) dc_k\right) = \int_0^{\bar{C}-x} \left(\frac{1}{1 + c_k + x}\right)^2 f(c_k) dc_k - \frac{\bar{C}}{1+\bar{C}} f(\bar{C}-x).$$

Hence,

$$\begin{aligned}
&\frac{\partial h(i_{k1}, x)}{\partial x} \\
=&\beta \cdot f(\bar{C}-x)\left[-b + \frac{\bar{C}}{1+\bar{C}} - \left[\mu_R^2 + \frac{1}{\bar{C}+1}\left(\bar{C} i_{k1}(i_{k1} - 2\mu_R) + (1-\mu_R)(1-2\mu_R)\right)\right]\right] \\
&- (i_0 - i_{k1})^2 - r'(x) - \beta \cdot (i_{k1} - \mu_R)^2 \int_0^{\bar{C}-x} \left(\frac{1}{1 + c_k + x}\right)^2 f(c_k) dc_k < 0. \qquad (34)
\end{aligned}$$

Two comments are in order. On the one hand, recall that b is assumed to be very large for each given distribution $F(\cdot)$. On the other hand, the problem in (29) has at least one solution in $\mathcal{I} \times [0, \bar{C} - \varepsilon]$, with $\varepsilon > 0$ arbitrarily small. We denote any such solution by (i_{k1}^*, x^*). Next we distinguish three cases.

Case A: $F(\bar{C}) < \frac{1}{2}$

In this case, we have for each $x \in [0, \bar{C})$,

$$\max_{i_{k1} \in \mathcal{I}} g(i_1, x) < \min_{i_{k1} \in \mathcal{I}} h(i_1, x).$$

Following the same line of argument as in the proof of Proposition 16 (see Appendix D), we obtain

$$i_{k1}^* = \min\left\{\mu_R, \max\left\{1-\mu_R, i_{k1}^*(i_0, x^*)\right\}\right\}.$$

In particular, $i_{k1}^* \in [1-\mu_R, \mu_R]$. Therefore, from (32) and (33), we deduce that $x^* > 0$.

Case B: $F(\bar{C}) > \frac{1}{2}$

In this case, we have for each $x \in [0, \bar{C})$,

$$\max_{i_{k1}\in\mathcal{I}} h(i_1, x) < \min_{i_{k1}\in\mathcal{I}} g(i_1, x).$$

Following the same line of argument as in the proof of Proposition 16 (see Appendix D), we obtain

$$i_{k1}^* = \begin{cases} \min\{1-\mu_R, i_1^{**}(i_0, x)\} & \text{if } i_1^{**}(i_0, x) \in \left[0, \frac{1}{2}\right), \\ 1-\mu_R \text{ or } \mu_R & \text{if } i_1^{**}(i_0, x) = \frac{1}{2}, \\ \max\{\mu_R, i_1^{**}(i_0, x)\} & \text{if } i_1^{**}(i_0, x) \in \left(\frac{1}{2}, 1\right]. \end{cases}$$

Hence, if $i_1^{**}(i_0, x) \notin [1-\mu_R, \mu_R]$, we obtain from (34) that $x^* = 0$.

Case C: $F(\bar{C}) = \frac{1}{2}$

In this case, we obtain

$$\begin{aligned} &h(i_{k1}, x) - g(i_{k1}, x) \\ =& -\beta\cdot(i_{k1}-\mu_R)^2\left(\int_{\bar{C}-x}^{\infty} \frac{c_k+x}{1+c_k+x} f(c_k)dc_k - \int_0^{\bar{C}+x} \frac{c_k+x}{1+c_k+x} f(c_k)dc_k\right) < 0, \end{aligned}$$

where the inequality is explained as follows: $\frac{y}{1+y}$ increases with y,

$$\int_{\bar{C}-x}^{\infty} f(c_k)dc_k \geq \int_0^{\bar{C}-x} f(c_k)dc_k,$$

and

$$\begin{aligned} \int_{\bar{C}-x}^{\infty} \frac{c_k+x}{1+c_k+x} f(c_k)dc_k &> \int_{\bar{C}-x}^{\infty} \frac{\bar{C}}{1+\bar{C}} f(c_k)dc_k \\ &\geq \int_0^{\bar{C}-x} \frac{\bar{C}}{1+\bar{C}} f(c_k)dc_k > \int_0^{\bar{C}-x} \frac{c_k+x}{1+c_k+x} f(c_k)dc_k. \end{aligned}$$

Following the same line of argument as in the proof of Proposition 16 (see Appendix D), we obtain

$$i_{k1}^* = \begin{cases} i_1^{**}(i_0, x) & \text{if } i_0 \in [0, 1-\mu_R) \cup (\mu_R, 1], \\ i & \text{otherwise, where } i \in \{1-\mu_R, i_1^*(i_0, x), \mu_R\}. \end{cases}$$

Hence, if $i_1^* \notin [1-\mu_R, \mu_R]$, we obtain from (34) that $x^* = 0$. Finally, the statement of the proposition follows using $\pi = \mu_R - \frac{1}{2}$ and $\tau = i_0 - \frac{1}{2}$.

□

Appendix C

In this appendix, we analyze welfare in all three informational settings, and we compare it to the socially optimal solution that a social planner could engineer. We first focus on the static model. Then, we deal with the dynamic model.

8.1 Welfare in the static model

We first characterize the *socially optimal solution* a social planner could engineer. We assume that the social planner can dictate both which candidate should take office and which reform he should undertake. In the following result, we characterize the *socially optimal welfare*, which is the welfare obtained when the social planner chooses the socially optimal solution. The socially optimal welfare will serve as a reference when comparing welfare in equilibrium outcomes across the three different scenarios (ex-ante, ex-interim, and ex-post).

Proposition 11
Let $k \in R$ be the office-holder in $t = 1$, $k' \in L$ the challenger in $t = 2$, and $i_1 \in \mathcal{I}$ the policy chosen in period $t = 1$. Let also

$$c^* = \min\{c_k, c_{k'}\} \tag{35}$$

and

$$\frac{C^{min}}{C^{min}+1} = \left[\int_0^\infty \left(\int_0^{c_k} \frac{c_{k'}}{c_{k'}+1} \cdot f(c_{k'})dc_{k'} + (1-F(c_k)) \cdot \frac{c_k}{c_k+1}\right) f(c_k)dc_k\right]. \tag{36}$$

Then the socially optimal welfare is characterized by:

(i) The social planner always appoints the candidate with the best reform ability c^* as office-holder in $t = 2$.

(ii) The office-holder undertakes the reform

$$i_1(\tau, c^*) = \frac{1}{2} + \frac{c^*}{c^*+1} \cdot \tau. \tag{37}$$

(iii) Welfare is given by

$$W_1^{soc} = -\frac{C^{min}}{1+C^{min}} \cdot \tau^2. \tag{38}$$

Proof:

Let $k \in R$ denote the incumbent and $k' \in L$ denote the challenger in the election that takes place at the end of period $t = 1$. We assume that a social planner knows the candidates' reform ability. Recall that $c^* = \min\{c_k, c_{k'}\}$. Given that the utility associated with reforms, $U^c(i_1, i_2)$, decreases with c, the social planner always chooses the candidate with the best reform ability, i.e., c^*. The social planner then solves the following problem:

$$\begin{aligned} \max_{i_{k''2}} \quad & U^{\mathcal{I}}_{\frac{1}{2}}(i_2) + U^{c^*}(i_1, i_2) \\ \text{s.t.} \quad & i_2 \in \mathcal{I}. \end{aligned} \tag{39}$$

Recall that $\tau = i_1 - \frac{1}{2}$. It is a matter of simple algebra to verify that

$$i_2^*(c^*) = \frac{c^*}{c^*+1} \cdot i_1 + \frac{1}{c^*+1} \cdot \frac{1}{2} = \frac{1}{2} + \frac{c^*}{c^*+1} \cdot \tau$$

is the unique solution to the problem in (39). Then social welfare is

$$\begin{aligned}
W_1^{soc} &= \int_0^\infty \int_0^{c_k} U_{\frac{1}{2}}^{\mathcal{I}}(i_2^*(c_{k'})) + U^{c_k}(i_1, i_2^*(c_{k'})) f(c_{k'}) dc_{k'} f(c_k) dc_k \\
&+ \int_0^\infty \int_{c_k}^\infty U_{\frac{1}{2}}^{\mathcal{I}}(i_2^*(c_k)) + U^{c_{k'}}(i_1, i_2^*(c_k)) f(c_{k'}) dc_{k'} f(c_k) dc_k \\
&= - \int_0^\infty \int_0^{c_k} \left(\frac{1}{2} - \frac{c_{k'}}{c_{k'}+1} \cdot i_1 - \frac{1}{c_{k'}+1} \cdot \frac{1}{2} \right)^2 f(c_{k'}) dc_{k'} f(c_k) dc_k \\
&- \int_0^\infty \int_0^{c_k} c_{k'} \left(i_1 - \frac{c_{k'}}{c_{k'}+1} \cdot i_1 - \frac{1}{c_{k'}+1} \cdot \frac{1}{2} \right)^2 f(c_{k'}) dc_{k'} f(c_k) dc_k \\
&- \int_0^\infty (1 - F(c_k)) \cdot \left(\frac{1}{2} - \frac{c_k}{c_k+1} \cdot i_1 - \frac{1}{c_k+1} \cdot \frac{1}{2} \right)^2 f(c_k) dc_k \\
&- \int_0^\infty (1 - F(c_k)) \cdot \left[c_k \left(i_1 - \frac{c_k}{c_k+1} \cdot i_1 - \frac{1}{c_k+1} \cdot \frac{1}{2} \right)^2 \right] f(c_k) dc_k \\
&= - \left[\int_0^\infty \int_0^{c_k} \frac{c_{k'}}{c_{k'}+1} f(c_{k'}) dc_{k'} f(c_k) dc_k + \int_0^\infty (1 - F(c_k)) \frac{c_k}{c_k+1} f(c_k) dc_k \right] \cdot \tau^2,
\end{aligned}$$

where the last equality follows after some straightforward algebraic manipulations.

□

That is, a social planner does not carry out a full reform as prescribed by τ. Instead, he only performs a partial reform in the direction of the socially desirable policy, with the extent of the policy shift being dependent on the higher reform ability of the two candidates. By doing so, he finds a compromise between the need-for-reform and the extent of the costs associated with carrying out a policy change that maximizes social welfare.

In the following, we provide the proof of the main result regarding welfare in the static model, as stated in Section 7.1 the main body of the paper.

Proof of Proposition 8

Let $k \in R$ denote the incumbent and $k' \in L$ denote the challenger in the election that takes place at the end of period $t = 1$. We distinguish three cases.

Case I: $|\tau| > |\pi|$

First, we consider W_1^{EA}. From Propositions 1 and 2 it follows that the incumbent will be re-elected if and only if $c_k \leq c_{k'}$. For the sake of readability, let $i_{R2}^*(c_k) := i_{k2}^*(i_{k1})$ denote the best-response function of the incumbent and let $i_{L2}^*(c_{k'}) = i_{k''2}^*(i_{k1})$ denote the best-response function of the challenger, where

$i^*_{k2}(i_{k1})$ and $i^*_{k'2}(i_{k1})$ have been defined in Proposition 1. Using (24), we obtain

$$
\begin{aligned}
&W_1^{EA} \\
&= \int_0^\infty \int_0^{c_k} \left[U^{\mathcal{I}}_{\frac{1}{2}}(i^*_{L2}(c_{k'})) + U^{c_{k'}}(i_1, i^*_{L2}(c_{k'})) \right] f(c_{k'}) dc_{k'} f(c_k) dc_k \\
&+ \int_0^\infty \int_{c_k}^\infty \left[U^{\mathcal{I}}_{\frac{1}{2}}(i^*_{R2}(c_k)) + U^{c_k}(i_1, i^*_{R2}(c_k)) \right] f(c_{k'}) dc_{k'} f(c_k) dc_k \\
&= -\int_0^\infty \int_0^{c_k} \left(\frac{1}{2} - \frac{c_{k'}}{c_{k'}+1} i_1 - \frac{1}{c_{k'}+1} \mu_L \right)^2 f(c_{k'}) dc_{k'} f(c_k) dc_k \\
&- \int_0^\infty \int_0^{c_k} c_{k'} \cdot \left(i_1 - \frac{c_{k'}}{c_{k'}+1} i_1 - \frac{1}{c_{k'}+1} \mu_L \right)^2 f(c_{k'}) dc_{k'} f(c_k) dc_k \\
&- \int_0^\infty (1 - F(c_k)) \cdot \left(\frac{1}{2} - \frac{c_k}{c_k+1} i_1 - \frac{1}{c_k+1} \mu_R \right)^2 f(c_k) dc_k \\
&- \int_0^\infty (1 - F(c_k)) \cdot c_k \cdot \left(i_1 - \frac{c_k}{c_k+1} i_1 - \frac{1}{c_k+1} \mu_R \right)^2 f(c_k) dc_k \\
&= -\int_0^\infty \int_0^{c_k} \left[\frac{1}{4} - \frac{1}{c_{k'}+1} \cdot (c_{k'} \cdot i_1(1-i_1) + \mu_R(1-\mu_R)) \right] f(c_{k'}) dc_{k'} f(c_k) dc_k \\
&- \int_0^\infty (1 - F(c_k)) \cdot \left[\frac{1}{4} - \frac{1}{c_k+1} \cdot (c_k \cdot i_1(1-i_1) + \mu_R(1-\mu_R)) \right] f(c_k) dc_k \\
&= -\frac{1}{4} + i_1(1-i_1) \cdot \int_0^\infty \left[\int_0^{c_k} \frac{c_{k'}}{c_{k'}+1} f(c_{k'}) dc_{k'} + (1 - F(c_k)) \cdot \frac{c_k}{c_k+1} \right] f(c_k) dc_k \\
&+ \mu_R(1-\mu_R) \cdot \left[1 - \int_0^\infty \left[\int_0^{c_k} \frac{c_{k'}}{c_{k'}+1} f(c_{k'}) dc_{k'} + (1 - F(c_k)) \cdot \frac{c_k}{c_k+1} \right] f(c_k) dc_k \right].
\end{aligned}
$$

Let

$$
\alpha^{EA} = \int_0^\infty \left[\int_0^{c_k} \frac{c_{k'}}{c_{k'}+1} f(c_{k'}) dc_{k'} + (1 - F(c_k)) \cdot \frac{c_k}{c_k+1} \right] f(c_k) dc_k.
$$

Then we can write

$$
W_1^{EA} = -\frac{1}{4} + \left[\alpha^{EA} \cdot i_1(1-i_1) + (1 - \alpha^{EA}) \cdot \mu_R(1-\mu_R) \right]. \tag{40}
$$

Second, we consider W_1^{EI}. From Propositions 1 and 13 (see Appendix D) it follows that the incumbent will be re-elected if and only if $c_k \leq \hat{C}$. Again, let $i^*_{R2}(c_k) := i^*_{k2}(i_{k1})$ denote the best-response function of the incumbent and let $i^*_{L2}(c_{k'}) = i^*_{k''2}(i_{k1})$ denote the best-response function of the challenger, where

$i^*_{k2}(i_{k1})$ and $i^*_{k'2}(i_{k1})$ have been defined in Proposition 1. Using (24), we obtain

$$
\begin{aligned}
&W_1^{EI} \\
&= \int_{\hat{C}}^{\infty} \int_0^{\infty} \left[U^{\mathcal{I}}_{\frac{1}{2}}(i^*_{L2}(c_{k'})) + U^{c_k}(i_1, i^*_{L2}(c_{k'})) \right] f(c_{k'}) dc_{k'} f(c_k) dc_k \\
&+ \int_0^{\hat{C}} \left[U^{\mathcal{I}}_{\frac{1}{2}}(i^*_{R2}(c_k)) + U^{c_k}(i_1, i^*_{R2}(c_k)) \right] f(c_k) dc_k \\
&= - \int_{\hat{C}}^{\infty} \int_0^{\infty} \left(\frac{1}{2} - \frac{c_{k'}}{c_{k'}+1} i_1 - \frac{1}{c_{k'}+1} \mu_L \right)^2 f(c_{k'}) dc_{k'} f(c_k) dc_k \\
&- \int_{\hat{C}}^{\infty} \int_0^{\infty} c_{k'} \cdot \left(i_1 - \frac{c_{k'}}{c_{k'}+1} i_{k1} - \frac{1}{c_{k'}+1} \mu_L \right)^2 f(c_{k'}) dc_{k'} f(c_k) dc_k \\
&- \int_0^{\hat{C}} \left[\left(\frac{1}{2} - \frac{c_k}{c_k+1} i_1 - \frac{1}{c_k+1} \mu_R \right)^2 + c_k \cdot \left(i_1 - \frac{c_k}{c_k+1} i_1 - \frac{1}{c_k+1} \mu_R \right)^2 \right] f(c_k) dc_k \\
&= - \int_{\hat{C}}^{\infty} \int_0^{\infty} \left[\frac{1}{4} - \frac{1}{c_{k'}+1} \cdot (c_{k'} \cdot i_1 \cdot (1-i_1) + \mu_R \cdot (1-\mu_R)) \right] f(c_{k'}) dc_{k'} f(c_k) dc_k \\
&- \int_0^{\hat{C}} \left[\frac{1}{4} - \frac{1}{c_k+1} \cdot (c_k \cdot i_1 \cdot (1-i_1) + \mu_R \cdot (1-\mu_R)) \right] f(c_k) dc_k \\
&= -\frac{1}{4} + i_1 \cdot (1-i_1) \cdot \left[(1-F(\hat{C})) \cdot \int_0^{\infty} \frac{c_{k'}}{c_{k'}+1} f(c_{k'}) dc_{k'} + \int_0^{\hat{C}} \frac{c_k}{c_k+1} f(c_k) dc_k \right] \\
&+ \mu_R \cdot (1-\mu_R) \cdot \left[(1-F(\hat{C})) \cdot \int_0^{\infty} \frac{1}{c_{k'}+1} f(c_{k'}) dc_{k'} + \int_0^{\hat{C}} \frac{1}{c_k+1} f(c_k) dc_k \right] \\
&= -\frac{1}{4} + i_1 \cdot (1-i_1) \cdot \left[(1-F(\hat{C})) \cdot \int_0^{\infty} \frac{c_{k'}}{c_{k'}+1} f(c_{k'}) dc_{k'} + \int_0^{\hat{C}} \frac{c_k}{c_k+1} f(c_k) dc_k \right] \\
&+ \mu_R \cdot (1-\mu_R) \cdot \left[1 - \left[(1-F(\hat{C})) \cdot \int_0^{\infty} \frac{c_{k'}}{c_{k'}+1} f(c_{k'}) dc_{k'} + \int_0^{\hat{C}} \frac{c_k}{c_k+1} f(c_k) dc_k \right] \right].
\end{aligned}
$$

Let

$$
\alpha^{EI} = (1-F(\hat{C})) \cdot \int_0^{\infty} \frac{c_{k'}}{c_{k'}+1} f(c_{k'}) dc_{k'} + \int_0^{\hat{C}} \frac{c_k}{c_k+1} f(c_k) dc_k.
$$

Then we can write

$$
W_1^{EI} = -\frac{1}{4} + \left[\alpha^{EI} \cdot i_1(1-i_1) + (1-\alpha^{EI}) \cdot \mu_R(1-\mu_R) \right]. \tag{41}
$$

Third, we consider W_1^{EP}. From Propositions 15 and 16 (see Appendix D) it follows that the incumbent will be re-elected if and only if $c_k \leq \bar{C}$. For the sake of readability, let $i^*_{R2}(c_k) := i^*_{k2}(i_{k1})$ denote the best-response function of the incumbent and let $i^*_{L2} = i^*_{k''2}(i_{k1})$ denote the best-response function of the challenger, where $i^*_{k2}(i_{k1})$ and $i^*_{k'2}(i_{k1})$ have been defined in Proposition 15 (see Appendix D).

Using (24), we obtain

$$
\begin{aligned}
&W_1^{EP} \\
&= \int_{\bar{C}}^{\infty} \int_0^{\infty} \left[U_{\frac{1}{2}}^{\mathcal{I}}(i_{L2}^*) + U^{c_k}(i_1, i_{L2}^*) \right] f(c_{k'}) dc_{k'} f(c_k) dc_k \\
&+ \int_0^{\bar{C}} \left[U_{\frac{1}{2}}^{\mathcal{I}}(i_{R2}^*(c_k)) + U^{c_k}(i_1, i_{R2}^*(c_k)) \right] f(c_k) dc_k \\
&= - \int_{\bar{C}}^{\infty} \int_0^{\infty} \left(\frac{1}{2} - \frac{\bar{C}}{\bar{C}+1} i_1 - \frac{1}{\bar{C}+1} \mu_L \right)^2 f(c_{k'}) dc_{k'} f(c_k) dc_k \\
&- \int_{\bar{C}}^{\infty} \int_0^{\infty} c_{k'} \cdot \left(i_1 - \frac{\bar{C}}{\bar{C}+1} i_{k1} - \frac{1}{\bar{C}+1} \mu_L \right)^2 f(c_{k'}) dc_{k'} f(c_k) dc_k \\
&- \int_0^{\bar{C}} \left[\left(\frac{1}{2} - \frac{c_k}{c_k+1} i_1 - \frac{1}{c_k+1} \mu_R \right)^2 + c_k \cdot \left(i_1 - \frac{c_k}{c_k+1} i_1 - \frac{1}{c_k+1} \mu_R \right)^2 \right] f(c_k) dc_k \\
&= - \int_{\bar{C}}^{\infty} \left[\frac{1}{4} - \frac{1}{\bar{C}+1} \cdot \left(\bar{C} \cdot i_1 \cdot (1 - i_1) + \mu_R \cdot (1 - \mu_R) \right) \right] f(c_{k'}) dc_{k'} \\
&- \int_0^{\bar{C}} \left[\frac{1}{4} - \frac{1}{c_k+1} \cdot \left(c_k \cdot i_1 \cdot (1 - i_1) + \mu_R \cdot (1 - \mu_R) \right) \right] f(c_k) dc_k \\
&= -\frac{1}{4} + i_1 \cdot (1 - i_1) \cdot \left[(1 - F(\bar{C})) \cdot \frac{\bar{C}}{\bar{C}+1} + \int_0^{\bar{C}} \frac{c_k}{c_k+1} f(c_k) dc_k \right] \\
&+ \mu_R \cdot (1 - \mu_R) \cdot \left[(1 - F(\bar{C})) \cdot \frac{1}{\bar{C}+1} + \int_0^{\bar{C}} \frac{1}{c_k+1} f(c_k) dc_k \right] \\
&= -\frac{1}{4} + i_1 \cdot (1 - i_1) \cdot \left[(1 - F(\bar{C})) \cdot \frac{\bar{C}}{\bar{C}+1} + \int_0^{\bar{C}} \frac{c_k}{c_k+1} f(c_k) dc_k \right] \\
&+ \mu_R \cdot (1 - \mu_R) \cdot \left[1 - \left[(1 - F(\bar{C})) \cdot \frac{\bar{C}}{\bar{C}+1} + \int_0^{\bar{C}} \frac{c_k}{c_k+1} f(c_k) dc_k \right] \right]
\end{aligned}
$$

Let

$$
\alpha^{EP} = (1 - F(\bar{C})) \cdot \frac{\bar{C}}{\bar{C}+1} + \int_0^{\bar{C}} \frac{c_k}{c_k+1} f(c_k) dc_k.
$$

Then we can write

$$
W_1^{EP} = -\frac{1}{4} + \left[\alpha^{EP} \cdot i_1(1 - i_1) + (1 - \alpha^{EP}) \cdot \mu_R(1 - \mu_R) \right]. \tag{42}
$$

Once W_1^{EA}, W_1^{EI}, and W_1^{EP} have been computed, we next define the auxiliary function $h : [0, 1] \to \mathbb{R}$ as follows:

$$
h(\alpha) = -\frac{1}{4} + [\alpha \cdot i_1(1 - i_1) + (1 - \alpha) \cdot \mu_R(1 - \mu_R)] = -\alpha \cdot \tau^2 - (1 - \alpha) \cdot \pi^2.
$$

Since $|\tau| > |\pi|$,

$$
h(\alpha') \geq h(\alpha'') \Leftrightarrow \alpha' \leq \alpha''. \tag{43}
$$

Also note that $\frac{c}{c+1}$ is strictly increasing in c. Next we compare α^{EA}, α^{EI}, and α^{EP}. On the one hand,

$$\begin{aligned}
\alpha^{EP} - \alpha^{EI} &= (1 - F(\bar{C})) \cdot \frac{\bar{C}}{\bar{C}+1} + \int_0^{\bar{C}} \frac{c_k}{c_k+1} f(c_k) dc_k \\
&\quad - \left[(1 - F(\hat{C})) \cdot \int_0^{\infty} \frac{c_{k'}}{c_{k'}+1} f(c_{k'}) dc_{k'} + \int_0^{\hat{C}} \frac{c_k}{c_k+1} f(c_k) dc_k \right] \\
&= \int_{\hat{C}}^{\bar{C}} \frac{c_k}{c_k+1} f(c_k) dc_k + (1 - F(\bar{C})) \cdot \frac{\bar{C}}{\bar{C}+1} - (1 - F(\hat{C})) \cdot \frac{\hat{C}}{\hat{C}+1} \\
&> \int_{\hat{C}}^{\bar{C}} \frac{c_k}{c_k+1} f(c_k) dc_k - \left(F(\bar{C}) - F(\hat{C}) \right) \cdot \frac{\hat{C}}{\hat{C}+1} > 0, \qquad (44)
\end{aligned}$$

where the second equality follows from (8) and the two strict inequalities hold because $\hat{C} < \bar{C}$. On the other hand,

$$\begin{aligned}
\alpha^{EI} - \alpha^{EA} &= (1 - F(\hat{C})) \cdot \int_0^{\infty} \frac{c_{k'}}{c_{k'}+1} f(c_{k'}) dc_{k'} + \int_0^{\hat{C}} \frac{c_k}{c_k+1} f(c_k) dc_k \\
&\quad - \left[\int_0^{\infty} \left[\int_0^{c_k} \frac{c_{k'}}{c_{k'}+1} f(c_{k'}) dc_{k'} + (1 - F(c_k)) \cdot \frac{c_k}{c_k+1} \right] f(c_k) dc_k \right] \\
&= \int_{\hat{C}}^{\infty} \int_0^{\infty} \frac{c_{k'}}{c_{k'}+1} f(c_{k'}) dc_{k'} f(c_k) dc_k + \int_0^{\hat{C}} \int_0^{\infty} \frac{c_k}{c_k+1} f(c_{k'}) dc_{k'} f(c_k) dc_k \\
&\quad - \left[\int_0^{\infty} \left[\int_0^{c_k} \frac{c_{k'}}{c_{k'}+1} f(c_{k'}) dc_{k'} + (1 - F(c_k)) \cdot \frac{c_k}{c_k+1} \right] f(c_k) dc_k \right] \\
&> \int_{\hat{C}}^{\infty} \int_0^{c_k} \frac{c_{k'}}{c_{k'}+1} f(c_{k'}) dc_{k'} f(c_k) dc_k + \int_{\hat{C}}^{\infty} \int_{c_k}^{\infty} \frac{c_k}{c_k+1} f(c_{k'}) dc_{k'} f(c_k) dc_k \\
&\quad + \int_0^{\hat{C}} \int_0^{c_k} \frac{c_{k'}}{c_{k'}+1} f(c_{k'}) dc_{k'} f(c_k) dc_k + \int_0^{\hat{C}} \int_{c_k}^{\infty} \frac{c_k}{c_k+1} f(c_{k'}) dc_{k'} f(c_k) dc_k \\
&\quad - \left[\int_0^{\infty} \left[\int_0^{c_k} \frac{c_{k'}}{c_{k'}+1} f(c_{k'}) dc_{k'} + (1 - F(c_k)) \cdot \frac{c_k}{c_k+1} \right] f(c_k) dc_k \right] \\
&= 0, \qquad (45)
\end{aligned}$$

where the strict inequality holds because $F(\cdot)$ is non-degenerate. From (43), (44), and (45), it then follows that

$$W^{EP} < W^{EI} < W^{EA}.$$

Finally,

$$\begin{aligned}
W_1^{EA} &= -\frac{1}{4} - i_1(1 - i_1) \cdot \alpha^{EA} - \mu_R(1 - \mu_R) \cdot (1 - \alpha^{EA}) \\
&= -\alpha^{EA} \cdot \left(i_1 - \frac{1}{2} \right)^2 - (1 - \alpha^{EA}) \cdot \left(\mu_R - \frac{1}{2} \right)^2 \\
&= -\alpha^{EA} \cdot \tau^2 - (1 - \alpha^{EA}) \cdot \pi^2 \leq -\alpha^{EA} \cdot \tau^2 = -\frac{C^{min}}{1 + C^{min}} \cdot \tau^2 = W_1^{soc},
\end{aligned}$$

where the penultimate equality holds since

$$\alpha^{EA} = \frac{C^{min}}{1 + C^{min}}.$$

Case II: $|\tau| = |\pi|$

It follows from Proposition 1 and Propositions 13 (see Appendix D) that in the three cases (ex-ante, ex-interim, and ex-post) k is always re-elected, and then he chooses $i_2^*(c_k) = \frac{c_k}{c_k+1} \cdot i_1 + \frac{1}{c_k+1} \cdot \mu_R$. Hence, using (24), we obtain

$$\begin{aligned}
W^{EP} = W^{EI} = W^{EA} &= \int_0^\infty \left[U^{\mathcal{I}}_{\frac{1}{2}}(i^*(c_k)) + U^{c_k}(i_1, i_2^*(c_k)) \right] f(c_k) dc_k \\
&= -\int_0^\infty \left[\left(\frac{1}{2} - \frac{c_k}{c_k+1} i_1 - \frac{1}{c_k+1} \mu_R \right)^2 + \left(i_1 - \frac{c_k}{c_k+1} i_1 - \frac{1}{c_k+1} \mu_R \right)^2 \right] f(c_k) dc_k \\
&= -\int_0^\infty \left[\frac{1}{4} - \frac{1}{c_k+1} \cdot i_1(1-i_1) - \left(1 - \frac{c_k}{c_k+1} \right) \cdot \mu_R(1-\mu_R) \right] \\
&= -\alpha^k \cdot \tau^2 - (1-\alpha^k) \cdot \pi^2 = -\tau^2 < -\frac{C^{min}}{1+C^{min}} \cdot \tau^2 = W_1^{soc},
\end{aligned}$$

where $\alpha^k = \int_0^\infty \frac{c_k}{c_k+1} f(c_k) dc_k$.

Case III: $|\tau| < |\pi|$

First, we consider W_1^{EA}. From Propositions 1 and 2 it follows that the incumbent will be re-elected if and only if $c_k \geq c_{k'}$. For the sake of readability, let $i_{R2}^*(c_k) := i_{k2}^*(i_{k1})$ denote the best-response function of the incumbent and let $i_{L2}^*(c_{k'}) = i_{k''2}^*(i_{k1})$ denote the best-response function of the challenger, where $i_{k2}^*(i_{k1})$ and $i_{k'2}^*(i_{k1})$ have been defined in Proposition 1. Hence, using (24), we obtain

$$\begin{aligned}
&W_1^{EA} \\
&= \int_0^\infty \int_{c_k}^\infty \left[U^{\mathcal{I}}_{\frac{1}{2}}(i_{L2}^*(c_{k'})) + U^{c_{k'}}(i_1, i_{L2}^*(c_{k'})) \right] f(c_{k'}) dc_{k'} f(c_k) dc_k \\
&+ \int_0^\infty \int_0^{c_k} \left[U^{\mathcal{I}}_{\frac{1}{2}}(i_{R2}^*(c_k)) + U^{c_k}(i_1, i_{R2}^*(c_k)) \right] f(c_{k'}) dc_{k'} f(c_k) dc_k \\
&= -\int_0^\infty \int_{c_k}^\infty \left(\frac{1}{2} - \frac{c_{k'}}{c_{k'}+1} i_1 - \frac{1}{c_{k'}+1} \mu_L \right)^2 f(c_{k'}) dc_{k'} f(c_k) dc_k \\
&- \int_0^\infty \int_{c_k}^\infty c_{k'} \cdot \left(i_1 - \frac{c_{k'}}{c_{k'}+1} i_1 - \frac{1}{c_{k'}+1} \mu_L \right)^2 f(c_{k'}) dc_{k'} f(c_k) dc_k \\
&- \int_0^\infty F(c_k) \cdot \left(\frac{1}{2} - \frac{c_k}{c_k+1} i_1 - \frac{1}{c_k+1} \mu_R \right)^2 f(c_k) dc_k \\
&- \int_0^\infty F(c_k) \cdot c_k \cdot \left(i_1 - \frac{c_k}{c_k+1} i_1 - \frac{1}{c_k+1} \mu_R \right)^2 f(c_k) dc_k \\
&= -\int_0^\infty \int_{c_k}^\infty \left[\frac{1}{4} - \frac{1}{c_{k'}+1} \cdot (c_{k'} \cdot i_1(1-i_1) + \mu_R(1-\mu_R)) \right] f(c_{k'}) dc_{k'} f(c_k) dc_k \\
&- \int_0^\infty F(c_k) \cdot \left[\frac{1}{4} - \frac{1}{c_k+1} \cdot (c_k \cdot i_1(1-i_1) + \mu_R(1-\mu_R)) \right] f(c_k) dc_k \\
&= -\frac{1}{4} + i_1(1-i_1) \cdot \int_0^\infty \left[\int_{c_k}^\infty \frac{c_{k'}}{c_{k'}+1} f(c_{k'}) dc_{k'} + F(c_k) \cdot \frac{c_k}{c_k+1} \right] f(c_k) dc_k \\
&+ \mu_R(1-\mu_R) \cdot \left[1 - \int_0^\infty \left[\int_{c_k}^\infty \frac{c_{k'}}{c_{k'}+1} f(c_{k'}) dc_{k'} + F(c_k) \cdot \frac{c_k}{c_k+1} \right] f(c_k) dc_k \right].
\end{aligned}$$

Let

$$\beta^{EA} = \int_0^\infty \left[\int_{c_k}^\infty \frac{c_{k'}}{c_{k'}+1} f(c_{k'}) dc_{k'} + F(c_k) \cdot \frac{c_k}{c_k+1} \right] f(c_k) dc_k.$$

Then we can write

$$W_1^{EA} = -\frac{1}{4} + \left[\beta^{EA} \cdot i_1(1-i_1) + (1-\beta^{EA}) \cdot \mu_R(1-\mu_R)\right]. \tag{46}$$

Second, we consider W_1^{EI}. From Propositions 1 and 13 (see Appendix D) it follows that the incumbent will be re-elected if and only if $c_k \geq \hat{C}$. Again, let $i^*_{R2}(c_k) := i^*_{k2}(i_{k1})$ denote the best-response function of the incumbent and let $i^*_{L2}(c_{k'}) = i^*_{k''2}(i_{k1})$ denote the best-response function of the challenger, where $i^*_{k2}(i_{k1})$ and $i^*_{k'2}(i_{k1})$ have been defined in Proposition 1. Using (24), we obtain

$$\begin{aligned}
&W_1^{EI} \\
&= \int_0^{\hat{C}} \int_0^{\infty} \left[U^{\mathcal{I}}_{\frac{1}{2}}(i^*_{L2}(c_{k'})) + U^{c_k}(i_1, i^*_{L2}(c_{k'}))\right] f(c_{k'})dc_{k'} f(c_k)dc_k \\
&+ \int_{\hat{C}}^{\infty} \left[U^{\mathcal{I}}_{\frac{1}{2}}(i^*_{R2}(c_k)) + U^{c_k}(i_1, i^*_{R2}(c_k))\right] f(c_k)dc_k \\
&= -\int_0^{\hat{C}} \int_0^{\infty} \left(\frac{1}{2} - \frac{c_{k'}}{c_{k'}+1} i_1 - \frac{1}{c_{k'}+1}\mu_L\right)^2 f(c_{k'})dc_{k'} f(c_k)dc_k \\
&- \int_0^{\hat{C}} \int_0^{\infty} c_{k'} \cdot \left(i_1 - \frac{c_{k'}}{c_{k'}+1} i_{k1} - \frac{1}{c_{k'}+1}\mu_L\right)^2 f(c_{k'})dc_{k'} f(c_k)dc_k \\
&- \int_{\hat{C}}^{\infty} \left[\left(\frac{1}{2} - \frac{c_k}{c_k+1} i_1 - \frac{1}{c_k+1}\mu_R\right)^2 + c_k \cdot \left(i_1 - \frac{c_k}{c_k+1} i_1 - \frac{1}{c_k+1}\mu_R\right)^2\right] f(c_k)dc_k \\
&= -\int_0^{\hat{C}} \int_0^{\infty} \left[\frac{1}{4} - \frac{1}{c_{k'}+1} \cdot (c_{k'} \cdot i_1 \cdot (1-i_1) + \mu_R \cdot (1-\mu_R))\right] f(c_{k'})dc_{k'} f(c_k)dc_k \\
&- \int_{\hat{C}}^{\infty} \left[\frac{1}{4} - \frac{1}{c_k+1} \cdot (c_k \cdot i_1 \cdot (1-i_1) + \mu_R \cdot (1-\mu_R))\right] f(c_k)dc_k \\
&= -\frac{1}{4} + i_1 \cdot (1-i_1) \cdot \left[F(\hat{C}) \cdot \int_0^{\infty} \frac{c_{k'}}{c_{k'}+1} f(c_{k'})dc_{k'} + \int_{\hat{C}}^{\infty} \frac{c_k}{c_k+1} f(c_k)dc_k\right] \\
&+ \mu_R \cdot (1-\mu_R) \cdot \left[F(\hat{C}) \cdot \int_0^{\infty} \frac{1}{c_{k'}+1} f(c_{k'})dc_{k'} + \int_{\hat{C}}^{\infty} \frac{1}{c_k+1} f(c_k)dc_k\right] \\
&= -\frac{1}{4} + i_1 \cdot (1-i_1) \cdot \left[F(\hat{C}) \cdot \int_0^{\infty} \frac{c_{k'}}{c_{k'}+1} f(c_{k'})dc_{k'} + \int_{\hat{C}}^{\infty} \frac{c_k}{c_k+1} f(c_k)dc_k\right] \\
&+ \mu_R \cdot (1-\mu_R) \cdot \left[1 - \left[F(\hat{C}) \cdot \int_0^{\infty} \frac{c_{k'}}{c_{k'}+1} f(c_{k'})dc_{k'} + \int_{\hat{C}}^{\infty} \frac{c_k}{c_k+1} f(c_k)dc_k\right]\right].
\end{aligned}$$

Let

$$\beta^{EI} = F(\hat{C}) \cdot \int_0^{\infty} \frac{c_{k'}}{c_{k'}+1} f(c_{k'})dc_{k'} + \int_{\hat{C}}^{\infty} \frac{c_k}{c_k+1} f(c_k)dc_k.$$

Then we can write

$$W_1^{EI} = -\frac{1}{4} + \left[\beta^{EI} \cdot i_1(1-i_1) + (1-\beta^{EI}) \cdot \mu_R(1-\mu_R)\right]. \tag{47}$$

Third, we consider W_1^{EP}. From Propositions 15 and 16 (see Appendix D) it follows that the incumbent will be re-elected if and only if $c_k \geq \bar{C}$. For the sake of readability, let $i^*_{R2}(c_k) := i^*_{k2}(i_{k1})$ denote the best-response function of the incumbent and let $i^*_{L2} = i^*_{k''2}(i_{k1})$ denote the best-response function of the challenger, where $i^*_{k2}(i_{k1})$ and $i^*_{k'2}(i_{k1})$ have been defined in Proposition 15 (see Appendix D).

Using (24), we obtain

$$
\begin{aligned}
&W_1^{EP} \\
&= \int_0^{\bar{C}} \int_0^{\infty} \left[U^{\mathcal{I}}_{\frac{1}{2}}(i^*_{L2}) + U^{c_k}(i_1, i^*_{L2}) \right] f(c_{k'}) dc_{k'} f(c_k) dc_k \\
&+ \int_{\bar{C}}^{\infty} \left[U^{\mathcal{I}}_{\frac{1}{2}}(i^*_{R2}(c_k)) + U^{c_k}(i_1, i^*_{R2}(c_k)) \right] f(c_k) dc_k \\
&= - \int_0^{\bar{C}} \int_0^{\infty} \left(\frac{1}{2} - \frac{\bar{C}}{\bar{C}+1} i_1 - \frac{1}{\bar{C}+1} \mu_L \right)^2 f(c_{k'}) dc_{k'} f(c_k) dc_k \\
&- \int_0^{\bar{C}} \int_0^{\infty} c_{k'} \cdot \left(i_1 - \frac{\bar{C}}{\bar{C}+1} i_{k1} - \frac{1}{\bar{C}+1} \mu_L \right)^2 f(c_{k'}) dc_{k'} f(c_k) dc_k \\
&- \int_{\bar{C}}^{\infty} \left[\left(\frac{1}{2} - \frac{c_k}{c_k+1} i_1 - \frac{1}{c_k+1} \mu_R \right)^2 + c_k \cdot \left(i_1 - \frac{c_k}{c_k+1} i_1 - \frac{1}{c_k+1} \mu_R \right)^2 \right] f(c_k) dc_k \\
&= - \int_0^{\bar{C}} \left[\frac{1}{4} - \frac{1}{\bar{C}+1} \cdot \left(\bar{C} \cdot i_1 \cdot (1 - i_1) + \mu_R \cdot (1 - \mu_R) \right) \right] f(c_{k'}) dc_{k'} \\
&- \int_{\bar{C}}^{\infty} \left[\frac{1}{4} - \frac{1}{c_k+1} \cdot (c_k \cdot i_1 \cdot (1 - i_1) + \mu_R \cdot (1 - \mu_R)) \right] f(c_k) dc_k \\
&= -\frac{1}{4} + i_1 \cdot (1 - i_1) \cdot \left[F(\bar{C}) \cdot \frac{\bar{C}}{\bar{C}+1} + \int_{\bar{C}}^{\infty} \frac{c_k}{c_k+1} f(c_k) dc_k \right] \\
&+ \mu_R \cdot (1 - \mu_R) \cdot \left[F(\bar{C}) \cdot \frac{1}{\bar{C}+1} + \int_{\bar{C}}^{\infty} \frac{1}{c_k+1} f(c_k) dc_k \right] \\
&= -\frac{1}{4} + i_1 \cdot (1 - i_1) \cdot \left[F(\bar{C}) \cdot \frac{\bar{C}}{\bar{C}+1} + \int_{\bar{C}}^{\infty} \frac{c_k}{c_k+1} f(c_k) dc_k \right] \\
&+ \mu_R \cdot (1 - \mu_R) \cdot \left[1 - \left[F(\bar{C}) \cdot \frac{\bar{C}}{\bar{C}+1} + \int_{\bar{C}}^{\infty} \frac{c_k}{c_k+1} f(c_k) dc_k \right] \right].
\end{aligned}
$$

Let

$$
\beta^{EP} = F(\bar{C}) \cdot \frac{\bar{C}}{\bar{C}+1} + \int_{\bar{C}}^{\infty} \frac{c_k}{c_k+1} f(c_k) dc_k.
$$

Then we can write

$$
W_1^{EP} = -\frac{1}{4} + \left[\beta^{EP} \cdot i_1(1 - i_1) + (1 - \beta^{EP}) \cdot \mu_R(1 - \mu_R) \right]. \tag{48}
$$

Once W_1^{EA}, W_1^{EI}, and W_1^{EP} have been computed, we now redefine the auxiliary function $h : [0, 1] \to \mathbb{R}$ as follows:

$$
h(\beta) = -\frac{1}{4} + [\beta \cdot i_1(1 - i_1) + (1 - \beta) \cdot \mu_R(1 - \mu_R)] = -\beta \cdot \tau^2 - (1 - \beta) \cdot \pi^2.
$$

Since $|\tau| < |\pi|$,

$$
h(\beta') \geq h(\beta'') \Leftrightarrow \beta' \geq \beta''. \tag{49}
$$

We stress that $\frac{c}{c+1}$ is strictly increasing in c. Next we compare β^{EA}, β^{EI}, and β^{EP}. On the one hand,

$$\begin{aligned}
\beta^{EP} - \beta^{EI} &= F(\bar{C}) \cdot \frac{\bar{C}}{\bar{C}+1} + \int_{\bar{C}}^{\infty} \frac{c_k}{c_k+1} f(c_k) dc_k \\
&\quad - \left[F(\hat{C}) \cdot \int_0^{\infty} \frac{c_{k'}}{c_{k'}+1} f(c_{k'}) dc_{k'} + \int_{\hat{C}}^{\infty} \frac{c_k}{c_k+1} f(c_k) dc_k \right] \\
&= F(\bar{C}) \cdot \frac{\bar{C}}{\bar{C}+1} - F(\hat{C}) \cdot \frac{\hat{C}}{\hat{C}+1} - \int_{\hat{C}}^{\bar{C}} \frac{c_k}{c_k+1} f(c_k) dc_k \\
&> \Big(F(\bar{C}) - F(\hat{C}) \Big) \cdot \frac{\bar{C}}{\bar{C}+1} - \int_{\hat{C}}^{\bar{C}} \frac{c_k}{c_k+1} f(c_k) dc_k > 0, \qquad (50)
\end{aligned}$$

where the second equality follows from (8) and the two strict inequalities hold because $\hat{C} < \bar{C}$. On the other hand,

$$\begin{aligned}
\beta^{EI} - \beta^{EA} &= F(\hat{C}) \cdot \int_0^{\infty} \frac{c_{k'}}{c_{k'}+1} f(c_{k'}) dc_{k'} + \int_{\hat{C}}^{\infty} \frac{c_k}{c_k+1} f(c_k) dc_k \\
&\quad - \left[\int_0^{\infty} \left[\int_{c_k}^{\infty} \frac{c_{k'}}{c_{k'}+1} f(c_{k'}) dc_{k'} + F(c_k) \cdot \frac{c_k}{c_k+1} \right] f(c_k) dc_k \right] \\
&= \int_0^{\hat{C}} \int_0^{\infty} \frac{c_{k'}}{c_{k'}+1} f(c_{k'}) dc_{k'} f(c_k) dc_k + \int_{\hat{C}}^{\infty} \int_0^{\infty} \frac{c_k}{c_k+1} f(c_{k'}) dc_{k'} f(c_k) dc_k \\
&\quad - \left[\int_0^{\infty} \left[\int_{c_k}^{\infty} \frac{c_{k'}}{c_{k'}+1} f(c_{k'}) dc_{k'} + F(c_k) \cdot \frac{c_k}{c_k+1} \right] f(c_k) dc_k \right] \\
&< \int_0^{\hat{C}} \int_0^{c_k} \frac{c_k}{c_k+1} f(c_{k'}) dc_{k'} f(c_k) dc_k + \int_0^{\hat{C}} \int_{c_k}^{\infty} \frac{c_{k'}}{c_{k'}+1} f(c_{k'}) dc_{k'} f(c_k) dc_k \\
&\quad + \int_{\hat{C}}^{\infty} \int_0^{c_k} \frac{c_k}{c_k+1} f(c_{k'}) dc_{k'} f(c_k) dc_k + \int_{\hat{C}}^{\infty} \int_{c_k}^{\infty} \frac{c_{k'}}{c_{k'}+1} dc_{k'} f(c_k) dc_k \\
&\quad - \left[\int_0^{\infty} \left[\int_{c_k}^{\infty} \frac{c_{k'}}{c_{k'}+1} f(c_{k'}) dc_{k'} + F(c_k) \cdot \frac{c_k}{c_k+1} \right] f(c_k) dc_k \right] \\
&= 0, \qquad (51)
\end{aligned}$$

where the strict inequality holds because $F(\cdot)$ is non-degenerate. From (49), (50), and (51), it follows that

$$W^{EI} < W^{EP} \text{ and } W^{EI} < W^{EA}.$$

Finally,

$$\begin{aligned}
W_1^{EP} &= -\frac{1}{4} + \left[\beta^{EP} \cdot i_1(1-i_1) + (1-\beta^{EP}) \cdot \mu_R(1-\mu_R) \right] \\
&= -\beta^{EP} \cdot \left(i_1 - \frac{1}{2} \right)^2 - (1-\beta^{EP}) \cdot \left(\mu_R - \frac{1}{2} \right)^2 \\
&= -\beta^{EP} \cdot \tau^2 - (1-\beta^{EP}) \cdot \pi^2 < -\beta^{EP} \cdot \tau^2 < -\frac{C^{min}}{1+C^{min}} \cdot \tau^2 = W_1^{soc},
\end{aligned}$$

where the strict inequality holds since

$$
\begin{aligned}
&\beta^{EP} \\
=&(1-F(\bar{C}))\cdot\frac{\bar{C}}{\bar{C}+1}+\int_0^{\bar{C}}\frac{c_k}{c_k+1}f(c_k)dc_k > (1-F(\bar{C}))\cdot\frac{\hat{C}}{\hat{C}+1}+\int_0^{\bar{C}}\frac{c_k}{c_k+1}f(c_k)dc_k \\
=&\int_{\bar{C}}^{\infty}\int_0^{\infty}\frac{c_{k'}}{c_{k'}+1}f(c_{k'})dc_{k'}f(c_k)dc_k+\int_0^{\bar{C}}\int_0^{\infty}\frac{c_k}{c_k+1}f(c_{k'})dc_{k'}f(c_k)dc_k \\
>&\int_0^{\infty}\int_0^{\infty}\frac{\min\{c_k,c_{k'}\}}{\min\{c_k,c_{k'}\}+1}f(c_{k'})dc_{k'}f(c_k)dc_k=\frac{C^{min}}{C^{min}+1}.
\end{aligned}
$$

Similarly,

$$
W_1^{EA}\leq -\beta^{EA}\cdot\tau^2<-\frac{C^{min}}{1+C^{min}}\cdot\tau^2=W_1^{soc},
$$

where the strict inequality holds since

$$
\begin{aligned}
\beta^{EA}&=\int_0^{\infty}\left[\int_{c_k}^{\infty}\frac{c_{k'}}{c_{k'}+1}f(c_{k'})dc_{k'}+F(c_k)\cdot\frac{c_k}{c_k+1}\right]f(c_k)dc_k \\
&=\int_0^{\infty}\int_{c_k}^{\infty}\frac{c_{k'}}{c_{k'}+1}f(c_{k'})dc_{k'}f(c_k)dc_k+\int_0^{\infty}\int_0^{c_k}\frac{c_k}{c_k+1}f(c_{k'})dc_{k'}f(c_k)dc_k \\
&=\int_0^{\infty}\int_0^{\infty}\frac{\max\{c_k,c_{k'}\}}{\max\{c_k,c_{k'}\}+1}f(c_{k'})dc_{k'}f(c_k)dc_k \\
&>\int_0^{\infty}\int_0^{\infty}\frac{\min\{c_k,c_{k'}\}}{\min\{c_k,c_{k'}\}+1}f(c_{k'})dc_{k'}f(c_k)dc_k=\frac{C^{min}}{C^{min}+1}.
\end{aligned}
$$

□

8.2 Welfare in the dynamic model

We start by characterizing the *socially optimal solution* chosen by a social planner who maximizes W_2, knows all candidates' reform ability, and can dictate the policies chosen in periods $t=1$ and $t=2$.

Proposition 12
Let $k\in R$ be the office-holder in $t=1$, $k'\in L$ the challenger in $t=2$, and $i_0\in\mathcal{I}$ the policy chosen in period $t=0$. Recall that $c^*=\min\{c_k,c_{k'}\}$ and let $\hat{C}^{min}$ be such that

$$
\begin{aligned}
&\frac{\hat{C}^{min}}{\hat{C}^{min}+1} \\
=&\int_0^{\infty}\left(\int_0^{c_k}c_k\frac{1+\beta\frac{c_{k'}}{1+c_{k'}}}{1+\beta\frac{c_{k'}}{1+c_{k'}}+c_k}f(c_{k'})dc_{k'}+(1-F(c_k))c_k\frac{1+\beta\frac{c_k}{1+c_k}}{1+\beta\frac{c_k}{1+c_k}+c_k}\right)f(c_k)dc_k.
\end{aligned}
$$

Then the socially optimal welfare is characterized by:

(i) The social planner appoints the candidate with the best reform ability c^* as office-holder in period $t=2$.

(ii) The office-holders in period $t = 1$ and $t = 2$ undertake the reform in two steps by choosing the policies

$$i_1(\tau, c_k, c^*) = \frac{1}{2} + \frac{c_k}{1 + \beta \cdot \frac{c^*}{1+c^*} + c_k} \cdot \tau$$

and

$$i_2(\tau, c_k, c^*) = \frac{1}{2} + \frac{c^*}{1 + c^*} \cdot \frac{c_k}{1 + \beta \cdot \frac{c^*}{1+c^*} + c_k} \cdot \tau$$

respectively.

(iii) Social welfare is given by

$$W_2^{soc} = -\frac{\hat{C}^{min}}{1 + \hat{C}^{min}} \cdot \tau^2. \tag{52}$$

Proof:

Given $c_1, c_2 \in [0, +\infty)$, let us define

$$\begin{aligned} B^{c_1,c_2}(i_1, i_2) &= U^{\mathcal{I}}_{\frac{1}{2}}(i_1) + U^{c_1}(i_0, i_1) + \beta \cdot \left[U^{\mathcal{I}}_{\frac{1}{2}}(i_2) + U^{c_2}(i_1, i_2)\right] \\ &= -\left(\frac{1}{2} - i_1\right)^2 - c_1 (i_0 - i_1)^2 - \beta \cdot \left[\left(\frac{1}{2} - i_2\right)^2 + c_2 (i_1 - i_2)^2\right]. \end{aligned}$$

For any given c_1 and c_2, we next solve the following problem:

$$\begin{aligned} \max_{i_1, i_2} \quad & B^{c_1,c_2}(i_1, i_2) \\ \text{s.t.} \quad & i_1, i_2 \in \mathcal{I}. \end{aligned} \tag{53}$$

The first-order conditions are

$$\frac{1}{2}\frac{\partial B}{\partial i_1} = \left(\frac{1}{2} - i_1\right) + c_1(i_0 - i_1) - \beta \cdot c_2(i_1 - i_2) = 0 \tag{54}$$

and

$$\frac{1}{2}\frac{\partial B}{\partial i_2} = \beta \cdot \left(\frac{1}{2} - i_2\right) + \beta \cdot c_2(i_1 - i_2) = 0. \tag{55}$$

If we solve the system of equations given by (54) and (55), we find

$$\begin{aligned} i_1(c_1, c_2) &= \frac{1 + \beta \cdot \frac{c_2}{1+c_2}}{1 + \beta \cdot \frac{c_2}{1+c_2} + c_1} \cdot \frac{1}{2} + \frac{c_1}{1 + \beta \cdot \frac{c_2}{1+c_2} + c_1} \cdot i_0 \\ &= \frac{1}{2} + \frac{c_1}{1 + \beta \cdot \frac{c_2}{1+c_2} + c_1} \cdot \tau \end{aligned} \tag{56}$$

and

$$\begin{aligned} i_2(c_1, c_2) &= \frac{1}{1 + c_2} \cdot \frac{1}{2} + \frac{c_2}{1 + c_2} \cdot i_1(c_1, c_2) = \frac{1}{2} + \frac{c_2}{1 + c_2} \cdot \left(i_1(c_1, c_2) - \frac{1}{2}\right) \\ &= \frac{1}{2} + \frac{c_2}{1 + c_2} \cdot \frac{c_1}{1 + \beta \cdot \frac{c_2}{1+c_2} + c_1} \cdot \tau. \end{aligned} \tag{57}$$

That is, $i_1(c_1, c_2)$ is a weighted average of i_0 and $\frac{1}{2}$, while $i_2(c_1, c_2)$ is a weighted average of $i_1(c_1, c_2)$ and $\frac{1}{2}$. In particular, $(i_1(c_1, c_2), i_2(c_1, c_2))$ is a candidate for a solution to the problem in (53). Moreover,

$$H_B^{c_1,c_2}(i_1(c_1, c_2), i_2(c_1, c_2)) = \begin{pmatrix} -1 - c_1 - \beta \cdot c_2 & \beta \cdot c_2 \\ \beta \cdot c_2 & -\beta - \beta \cdot c_2 \end{pmatrix},$$

which is negative definite. Therefore $(i_1(c_1, c_2), i_2(c_1, c_2))$ is the solution to the problem in (53). We note that given c_1, c_2, $i_1 = \alpha \cdot \frac{1}{2} + (1 - \alpha) \cdot i_0$, and $i_2 = \delta \cdot \frac{1}{2} + (1 - \delta) \cdot i_1$, we have

$$\begin{aligned} B^{c_1,c_2}(i_1, i_2) &= -\left(i_1 - \frac{1}{2}\right)^2 - c_1 \cdot (i_1 - i_0)^2 - \beta \cdot \left[\left(i_1 - \frac{1}{2}\right)^2 - c_1 \cdot (i_1 - i_0)^2\right] \\ &= -\tau^2 \cdot \left[(1 - \alpha)^2 + c_1\alpha^2 + \beta \cdot (1 - \alpha)^2 \left((1 - \delta)^2 + c_2\delta^2\right)\right]. \end{aligned} \tag{58}$$

Now let

$$\alpha = \frac{1 + \beta \cdot \frac{c_2}{1+c_2}}{1 + \beta \cdot \frac{c_2}{1+c_2} + c_1}$$

and

$$\delta = \frac{1}{1 + c_2}$$

as in (56) and (57) respectively. Then (58) can be rewritten as

$$B^{c_1,c_2}(i_1, i_2) = -\tau^2 \cdot \left[(1 - \alpha)^2 + c_1\alpha^2 + \beta(1 - \alpha)^2 \frac{c_2}{1 + c_2}\right] = -\tau^2 \cdot c_1 \frac{1 + \beta\frac{c_2}{1+c_2}}{1 + \beta\frac{c_2}{1+c_2} + c_1}.$$

Finally, a social planner can select an office-holder—between the incumbent $k \in R$ and the challenger $k' \in L$—to be the candidate who can implement changes in the second period more efficiently. Hence $c_1 = c_k$ and $c_2 = \min\{c_k, c_{k'}\}$. Thus,

$$\begin{aligned} W_2^{soc} &= \int_0^\infty \int_0^{c_k} B^{c_k,c_{k'}}(i_1(c_k, c_{k'}), i_2(c_k, c_{k'}))f(c_k)dc_k f(c_{k'})dc_{k'} \\ &+ \int_0^\infty \int_{c_k}^\infty B^{c_k,c_k}(i_1(c_k, c_k), i_2(c_k, c_k))f(c_k)dc_k f(c_{k'})dc_{k'} \\ &= -\tau^2 \cdot \int_0^\infty \int_0^{c_k} c_k \frac{1 + \beta\frac{c_{k'}}{1+c_{k'}}}{1 + \beta\frac{c_{k'}}{1+c_{k'}} + c_k} f(c_k)dc_k f(c_{k'})dc_{k'} \\ &- \tau^2 \cdot \int_0^\infty \int_{c_k}^\infty c_k \frac{1 + \beta\frac{c_k}{1+c_k}}{1 + \beta\frac{c_k}{1+c_k} + c_k} f(c_k)dc_k f(c_{k'})dc_{k'} \\ &= -\tau^2 \cdot \int_0^\infty \int_0^{c_k} c_k \frac{1 + \beta\frac{c_{k'}}{1+c_{k'}}}{1 + \beta\frac{c_{k'}}{1+c_{k'}} + c_k} f(c_{k'})dc_{k'} f(c_k)dc_k \\ &- \tau^2 \cdot \int_0^\infty (1 - F(c_k))c_k \frac{1 + \beta\frac{c_k}{1+c_k}}{1 + \beta\frac{c_k}{1+c_k} + c_k} f(c_k)dc_k. \end{aligned}$$

□

In the following, we provide the proof of the main result regarding welfare in the dynamic model, as stated in Section 7.2 in the main body of the paper. We also provide the proof for some auxiliary or complementary results.

Proof of Proposition 9

Let $k \in R$ denote the office-holder in $t = 1$, $k' \in L$ the challenger in $t = 2$, and $k'' \in L \cup R$ the office-holder in $t = 2$. We use i_1 and i_2 to denote the equilibrium choices in periods $t = 1$ and $t = 2$ respectively. First, we compute welfare in the ex-ante case, denoted by W_1^{EA}. When $\pi \to 0$, or equivalently when $\mu_R \to \frac{1}{2}$, we have

$$i_{1,*}^{EA}(i_0, c_k) = \frac{c_k}{1 + c_k + \beta \cdot \frac{c_k}{c_k+1}} \cdot i_0 + \frac{c_k + \beta \cdot \frac{c_k}{c_k+1}}{1 + \bar{C} + \beta \cdot \frac{c_k}{c_k+1}} \cdot \frac{1}{2} = \frac{1}{2} + \frac{c_k}{1 + c_k + \beta \cdot \frac{c_k}{c_k+1}} \cdot \tau.$$

From Proposition 3 we obtain

$$i_1 = \begin{cases} \frac{1}{2} & \text{if } c_k > c_{k'} \\ i_{1,*}^{EA} = i_{1,*}^{EA}(i_0, c_k) & \text{if } c_k \leq c_{k'}, \end{cases} \tag{59}$$

and, due to Proposition 2, k is re-elected in any case. Moreover, from Proposition 1, it follows that

$$i_2 = \frac{c_k}{c_k + 1} \cdot i_1 + \frac{1}{c_k + 1} \cdot \frac{1}{2} = \begin{cases} \frac{1}{2} & \text{if } c_k > c_{k'} \\ \frac{c_k}{1+c_k} \cdot i_{1,*}^{EA} + \frac{1}{1+c_k} \cdot \frac{1}{2} & \text{if } c_k \leq c_{k'}. \end{cases} \tag{60}$$

Using (59) and (60), we obtain

$$\begin{aligned}
&W_2^{EA} \\
&= -\int_0^\infty \int_0^{c_k} \left[\left(\frac{1}{2} - \frac{1}{2} \right)^2 + c_k \left(i_0 - \frac{1}{2} \right)^2 \right] f(c_k) dc_k f(c_{k'}) dc_{k'} \\
&- \beta \cdot \int_0^\infty \int_0^{c_k} \left[\left(\left(\frac{1}{2} - \frac{1}{2} \right)^2 + c_k \left(\frac{1}{2} - \frac{1}{2} \right)^2 \right) \right] f(c_k) dc_k f(c_{k'}) dc_{k'} \\
&- \int_0^\infty \int_{c_k}^\infty \left[\left(i_{1,*}^{EA} - \frac{1}{2} \right)^2 + c_k \left(i_0 - i_{1,*}^{EA} \right)^2 \right] f(c_k) dc_k f(c_{k'}) dc_{k'} \\
&- \int_0^\infty \int_{c_k}^\infty \beta \cdot \left[\left(\frac{c_k}{1 + c_k} \cdot i_{1,*}^{EA} + \frac{1}{1 + c_k} \cdot \frac{1}{2} - \frac{1}{2} \right)^2 \right] f(c_k) dc_k f(c_{k'}) dc_{k'} \\
&- \int_0^\infty \int_{c_k}^\infty \beta \cdot \left[c_k \left(\frac{c_k}{1 + c_k} \cdot i_{1,*}^{EA} + \frac{1}{1 + c_k} \cdot \frac{1}{2} - i_{1,*}^{EA} \right)^2 \right] f(c_k) dc_k f(c_{k'}) dc_{k'} \\
&= -\tau^2 \cdot \left[\int_0^\infty F(c_k) c_k f(c_k) dc_k + \int_0^\infty (1 - F(c_k)) \cdot c_k \cdot \frac{1 + \beta \frac{c_k}{c_k+1}}{1 + \beta \frac{c_k}{c_k+1} + c_k} f(c_k) dc_k \right] \\
&= -\tau^2 \cdot \bar{C} + \tau^2 \cdot \left[\int_0^\infty (1 - F(c_k)) \cdot c_k \cdot \frac{c_k}{1 + \beta \frac{c_k}{c_k+1} + c_k} f(c_k) dc_k \right].
\end{aligned} \tag{61}$$

Second, we compute welfare in the ex-interim case. When $\pi \to 0$, or equivalently, when $\mu_R \to \frac{1}{2}$, we have

$$i_{1,*}^{EI}(i_0, c_k) = \frac{c_k}{1 + c_k + \beta \cdot \frac{c_k}{c_k+1}} \cdot i_0 + \frac{c_k + \beta \cdot \frac{c_k}{c_k+1}}{1 + \bar{C} + \beta \cdot \frac{c_k}{c_k+1}} \cdot \frac{1}{2} = \frac{1}{2} + \frac{c_k}{1 + c_k + \beta \cdot \frac{c_k}{c_k+1}} \cdot \tau.$$

From Proposition 14 we obtain

$$i_1 = \begin{cases} \frac{1}{2} & \text{if } c_k > \hat{C} \\ i_{1,*}^{EI} = i_{1,*}^{EI}(i_0, c_k) & \text{if } c_k \leq \hat{C}, \end{cases} \tag{62}$$

and, due to Proposition 13 (see Appendix D), k is re-elected in any case. Moreover, from Proposition 1 it follows that

$$i_2 = \frac{c_k}{c_k+1} \cdot i_1 + \frac{1}{c_k+1} \cdot \frac{1}{2} = \begin{cases} \frac{1}{2} & \text{if } c_k > \hat{C} \\ \frac{c_k}{1+c_k} \cdot i_{1,*}^{EI} + \frac{1}{1+c_k} \cdot \frac{1}{2} & \text{if } c_k \leq \hat{C}. \end{cases} \tag{63}$$

Using (62) and (63), we obtain

$$\begin{aligned}
&W_2^{EI} \\
&= -\int_{\hat{C}}^{\infty} \left[\left(\frac{1}{2} - \frac{1}{2}\right)^2 + c_k \left(i_0 - \frac{1}{2}\right)^2 + \beta \cdot \left(\left(\frac{1}{2} - \frac{1}{2}\right)^2 + c_k \left(\frac{1}{2} - \frac{1}{2}\right)^2 \right) \right] f(c_k) dc_k \\
&- \int_0^{\hat{C}} \left[\left(i_{1,*}^{EI} - \frac{1}{2}\right)^2 + c_k \left(i_0 - i_{1,*}^{EI}\right)^2 \right] f(c_k) dc_k \\
&- \int_0^{\hat{C}} \beta \cdot \left[\left(\frac{c_k}{1+c_k} \cdot i_{1,*}^{EI} + \frac{1}{1+c_k} - \frac{1}{2} \right)^2 + c_k \left(\frac{c_k}{1+c_k} \cdot i_{1,*}^{EI} + \frac{1}{1+c_k} - i_{1,*}^{EI} \right)^2 \right] f(c_k) dc_k \\
&= -\tau^2 \cdot \left[\int_{\hat{C}}^{\infty} c_k f(c_k) dc_k + \int_0^{\hat{C}} c_k \cdot \frac{1 + \beta \frac{c_k}{c_k+1}}{1 + \beta \frac{c_k}{c_k+1} + c_k} f(c_k) dc_k \right] \\
&= -\tau^2 \cdot \bar{C} + \tau^2 \cdot \int_0^{\hat{C}} c_k \cdot \frac{c_k}{1 + \beta \frac{c_k}{c_k+1} + c_k} f(c_k) dc_k.
\end{aligned} \tag{64}$$

Third, we compute welfare in the ex-post case. When $\pi \to 0$, we have

$$\begin{aligned}
i_{k1,**}^{EP} = i_{k1,**}^{EP}(i_0) = &\frac{\bar{C}}{\bar{C} + 1 + \beta \cdot \int_0^{\bar{C}} \frac{c_k}{1+c_k} f(c_k) dc_k + \beta \cdot (1 - F(\bar{C})) \frac{\bar{C}}{\bar{C}+1}} \cdot i_0 \\
&+ \frac{1 + \beta \cdot \int_0^{\bar{C}} \frac{c_k}{1+c_k} f(c_k) dc_k + \beta \cdot (1 - F(\bar{C})) \frac{\bar{C}}{\bar{C}+1}}{\bar{C} + 1 + \beta \cdot \int_0^{\bar{C}} \frac{c_k}{1+c_k} f(c_k) dc_k + \beta \cdot (1 - F(\bar{C})) \frac{\bar{C}}{\bar{C}+1}} \cdot \frac{1}{2}.
\end{aligned}$$

We distinguish two cases, depending on the skewness of the reform ability distribution. On the one hand, let $F(\bar{C}) < \frac{1}{2}$. When $\pi \to 0$, we obtain from Proposition 4 that $i_1 = \frac{1}{2}$. Then, from Proposition 16 (see Appendix D) we know that k is re-elected if and only if $c_k \geq \bar{C}$. Then it follows from Proposition 15 (see Appendix D) that $i_{k''2} = \frac{1}{2}$, no matter who the second office-holder is. As a consequence, we obtain

$$W_2^{EP} = -\bar{C} \cdot \tau^2. \tag{65}$$

On the other hand, let $F(\bar{C}) \geq \frac{1}{2}$. When $\pi \to 0$, we obtain from Proposition 4 that $i_1 = i_{k1}^{**}(i_0)$. Moreover, since $\tau \neq 0$, we know from Proposition 16 (see Appendix D) that k is re-elected if and only if $c_k \leq \bar{C}$. From Proposition 15 (see Appendix D) we also know that if the second office-holder is k, i.e. $k'' = k$, the choice in $t = 2$ is

$$i_{k''2} = \frac{c_k}{1+c_k} \cdot i_{k1,**}^{EP} + \frac{1}{1+c_k} \cdot \frac{1}{2},$$

while if the second office-holder is k', i.e. $k'' = k'$, the choice in $t = 2$ is

$$i_{k''2} = \frac{\bar{C}}{1+\bar{C}} \cdot i_{k1,**}^{EP} + \frac{1}{1+\bar{C}} \cdot \frac{1}{2}.$$

Hence,

$$
\begin{aligned}
W_2^{EP} = &-\int_0^{\infty} c_k \left(i_0 - i_{k1,**}^{EP}\right)^2 f(c_k)dc_k - \int_0^{\infty} \left(i_{k1,**}^{EP} - \frac{1}{2}\right)^2 f(c_k)dc_k \\
&- \beta \cdot \int_0^{\bar{C}} \left(\frac{c_k}{c_k+1} \cdot i_{k1,**}^{EP} + \frac{1}{c_k+1} \cdot \frac{1}{2} - \frac{1}{2}\right)^2 f(c_k)dc_k \\
&- \beta \cdot \int_0^{\bar{C}} c_k \left(i_{k1,**}^{EP} - \frac{c_k}{c_k+1} \cdot i_{k1,**}^{EP} - \frac{1}{c_k+1} \cdot \frac{1}{2}\right)^2 f(c_k)dc_k \\
&- \beta \cdot \int_{\bar{C}}^{\infty} \int_0^{\infty} \left(\frac{\bar{C}}{\bar{C}+1} \cdot i_{k1,**}^{EP} + \frac{1}{\bar{C}+1} \cdot \frac{1}{2} - \frac{1}{2}\right)^2 f(c_{k'})dc_{k'} f(c_k)dc_k \\
&- \beta \cdot \int_{\bar{C}}^{\infty} \int_0^{\infty} \left(i_{k1,**}^{EP} - \frac{\bar{C}}{\bar{C}+1} \cdot i_{k1,**}^{EP} - \frac{1}{\bar{C}+1} \cdot \frac{1}{2}\right)^2 f(c_{k'})dc_{k'} f(c_k)dc_k,
\end{aligned}
$$

which can be simplified to

$$
\begin{aligned}
W_2^{EP} &= -\bar{C} \left(i_0 - i_{k1,**}^{EP}\right)^2 \\
&\quad - \left(i_{k1,**}^{EP} - \frac{1}{2}\right)^2 \left(1 + \beta \cdot \left(\int_0^{\bar{C}} \frac{c_k}{1+c_k} f(c_k)dc_k + (1 - F(\bar{C})) \frac{\bar{C}}{1+\bar{C}}\right)\right) \\
&= -\bar{C} \cdot (i_{k1,**}^{EP} - i_0)^2 - \left(i_{k1,**}^{EP} - \frac{1}{2}\right)^2 \left(1 + \beta \cdot \frac{\hat{C}}{1+\hat{C}}\right) \\
&= -\bar{C} \cdot \tau^2 + \tau^2 \cdot \bar{C} \cdot \frac{\bar{C}}{\bar{C} + 1 + \beta \cdot \frac{\tilde{C}}{\tilde{C}+1}}, \qquad (66)
\end{aligned}
$$

where

$$
\frac{\tilde{C}}{1+\tilde{C}} = \int_0^{\bar{C}} \frac{c_k}{1+c_k} f(c_k)dc_k + (1 - F(\bar{C})) \frac{\bar{C}}{\bar{C}+1}.
$$

We point out that

$$
\tilde{C} < \bar{C}. \qquad (67)
$$

Next we compare welfare in the ex-ante and ex-interim cases. We stress that $c_k \frac{c_k}{1+\beta \frac{c_k}{c_k+1} + c_k}$ is increasing in c_k. Then,

$$
\begin{aligned}
&\frac{1}{\tau^2} \cdot \left(W_2^{EA} - W_2^{EI}\right) \\
=&\int_0^{\infty} (1 - F(c_k)) \cdot c_k \cdot \frac{c_k}{1 + \beta \frac{c_k}{c_k+1} + c_k} f(c_k)dc_k - \int_0^{\hat{C}} c_k \cdot \frac{c_k}{1 + \beta \frac{c_k}{c_k+1} + c_k} f(c_k)dc_k \\
=&\int_{\hat{C}}^{\infty} c_k \cdot \frac{c_k}{1 + \beta \frac{c_k}{c_k+1} + c_k} f(c_k)dc_k - \int_0^{\hat{C}} F(c_k) \cdot c_k \cdot \frac{c_k}{1 + \beta \frac{c_k}{c_k+1} + c_k} f(c_k)dc_k \\
\geq&(1 - F(\hat{C})) \cdot \hat{C} \cdot \frac{\hat{C}}{1 + \beta \frac{\hat{C}}{\hat{C}+1} + \hat{C}} - (F(\hat{C}))^2 \cdot \hat{C} \cdot \frac{\hat{C}}{1 + \beta \frac{\hat{C}}{\hat{C}+1} + \hat{C}} > 0, \qquad (68)
\end{aligned}
$$

where the latter equality holds if $F(\hat{C}) < \frac{\sqrt{5}-1}{2}$.

Finally, we distinguish two cases. First, let $F(\bar{C}) < \frac{1}{2}$. Then, from (61), (64), (65), and (68) we obtain

$$
W_2^{EP} < W_2^{EI} < W_2^{EA} < W_2^{soc},
$$

where the last inequality holds because in the three cases (ex-ante, ex-interim, and ex-post) the incumbent is sometimes elected when he has lower reform ability than the challenger. Second, let $F(\bar{C}) \geq \frac{1}{2}$. Then,

$$\frac{1}{\tau^2} \cdot \left(W_2^{EI} - W_2^{EP}\right) = \int_0^{\hat{C}} c_k \cdot \frac{c_k}{1 + \beta \frac{c_k}{c_k+1} + c_k} f(c_k) dc_k - \bar{C} \cdot \frac{\bar{C}}{\bar{C} + 1 + \beta \cdot \frac{\tilde{C}}{\tilde{C}+1}}$$
$$< \hat{C} \cdot \frac{\hat{C}}{\hat{C} + 1 + \beta \cdot \frac{\hat{C}}{\hat{C}+1}} - \bar{C} \cdot \frac{\bar{C}}{\bar{C} + 1 + \beta \cdot \frac{\bar{C}}{\bar{C}+1}} < 0, \tag{69}$$

where the last inequality holds because $\hat{C} < \bar{C}$, $\tilde{C} < \bar{C}$, and $c \cdot \frac{c}{c+1+\beta \cdot \frac{c}{c+1}}$ is an increasing function in c. From (61), (64), (65), and (69), it follows that

$$W_2^{EI} < W_2^{EA} < W_2^{soc} \quad \text{and} \quad W_2^{EI} < W_2^{EP} < W_2^{soc}.$$

We note that $W_2^{EA} < W_2^{soc}$ and $W_2^{EP} < W_2^{soc}$ because in the two cases (ex-ante and ex-post) the incumbent is sometimes elected when he has lower reform ability than the challenger, while the social planner always appoints the candidate with the best reform ability.

□

We restrict our attention for a moment to the case where the discount factor, β, is very small.

Corollary 4
Let $\tau \in \left(0, \frac{1}{2}\right]$ and $\pi \to 0$. Then,

(a) in the ex-ante case,

$$\lim_{\beta^v \to 0} \frac{W_2^{soc} - W_2^{EA}}{\tau^2} = \int_0^{\infty} F(c_k) \cdot c_k \cdot \frac{c_k}{c_k + 1} f(c_k) dc_k,$$

(b) in the ex-interim case,

$$\lim_{\beta^v \to 0} \frac{W_2^{soc} - W_2^{EI}}{\tau^2} = \int_{\hat{C}}^{\infty} F(c_k) \cdot c_k \cdot \frac{c_k}{c_k + 1} f(c_k) dc_k,$$

(c) in the ex-post case, if $F(\bar{C}) < \frac{1}{2}$,

$$\lim_{\beta^v \to 0} \frac{W_2^{soc} - W_2^{EP}}{\tau^2} = \int_0^{\infty} c_k \cdot \frac{c_k}{c_k + 1} f(c_k) dc_k,$$

while, if $F(\bar{C}) \geq \frac{1}{2}$,

$$\lim_{\beta^v \to 0} \frac{W_2^{soc} - W_2^{EP}}{\tau^2} = \int_0^{\infty} c_k \cdot \frac{c_k}{c_k + 1} f(c_k) dc_k - \bar{C} \cdot \frac{\bar{C}}{\bar{C} + 1}.$$

Proof of Lemma 1

Note that

$$\int_0^{\infty} c f(c) dc = \frac{1}{C_F} \int_0^{C_F} c dc = \frac{C_F}{2}$$

and

$$F\left(\int_0^\infty cf(c)dc\right) = \frac{\frac{C_F}{2}}{C_F} = \frac{1}{2} = G\left(\int_0^\infty cg(c)dc\right).$$

□

Proof of Proposition 10

Recall that $F(\cdot)$ and $G(\cdot)$ are two distributions with non-degenerate support within $[0,\infty)$ such that $F(c) \leq G(c)$, i.e. such that F stochastically dominates G. Integrating by parts, we have for each differentiable and strictly increasing function $h(\cdot)$ and each $C \in [0,\infty]$

$$\begin{aligned}&\int_0^C h(c)f(c)dc - \int_0^C h(c)g(c)dc\\ =&h(C)(F(C)-G(C)) + \int_0^C h'(c)(G(c)-F(c))dc \geq h(C)(F(C)-G(C)).\end{aligned} \tag{70}$$

Let W_F^{EI} (or W_G^{EI}) denote welfare in the ex-interim case when $F(\cdot)$ (or $G(\cdot)$) is the reform ability distribution. Since F dominates G stochastically and by taking $C = \infty$ and $h(c) = \frac{c}{c+1}$ in (70), we have

$$\frac{\hat{C}^F}{\hat{C}^F+1} = \int_0^\infty \frac{c}{c+1}f(c)dc \geq \int_0^\infty \frac{c}{c+1}g(c)dc = \frac{\hat{C}^G}{\hat{C}^G+1},$$

so that $\hat{C}^F \geq \hat{C}^G$. From (64) in the proof of Proposition 9, using (70) with $C = \hat{C}^G$ and $h(c) = c \cdot \frac{c}{1+\beta\frac{c}{c+1}+c}$, we have

$$\begin{aligned}&\frac{1}{\tau^2}\cdot\left(W_F^{EI} - W_G^{EI}\right)\\ =&\int_0^{\hat{C}^F} c_k \cdot \frac{c_k}{1+\beta\frac{c_k}{c_k+1}+c_k}f(c_k)dc_k - \int_0^{\hat{C}^G} c_k \cdot \frac{c_k}{1+\beta\frac{c_k}{c_k+1}+c_k}g(c_k)dc_k\\ =&\int_0^{\hat{C}^G} c_k \cdot \frac{c_k}{1+\beta\frac{c_k}{c_k+1}+c_k}(f(c_k)-g(c_k))dc_k + \int_{\hat{C}^G}^{\hat{C}^F} c_k \cdot \frac{c_k}{1+\beta\frac{c_k}{c_k+1}+c_k}f(c_k)dc_k\\ \geq&\hat{C}^G \cdot \frac{\hat{C}^G}{\hat{C}^G+\beta\frac{\hat{C}^G}{\hat{C}^G+1}+\hat{C}^G}(F(\hat{C}^G)-G(\hat{C}^G)) + \int_{\hat{C}^G}^{\hat{C}^F} c_k \cdot \frac{c_k}{1+\beta\frac{c_k}{c_k+1}+c_k}f(c_k)dc_k\\ \geq&\hat{C}^G \cdot \frac{\hat{C}^G}{\hat{C}^G+\beta\frac{\hat{C}^G}{\hat{C}^G+1}+\hat{C}^G}(F(\hat{C}^F)-G(\hat{C}^G)) \geq 0,\end{aligned}$$

where the first inequality holds by taking $C = \bar{C}^G > 0$ and $h(c) = \frac{c}{1+\beta\frac{c}{c+1}+c}$ in (70) and the last inequality holds if

$$F(\hat{C}^F) - G(\hat{C}^G) \geq 0. \tag{71}$$

Note that (71) holds since F strongly stochastically dominates G.

□

Appendix D

In this appendix we first analyze the ex-interim scenario for the one- and two-period models. Then we do the same for the ex-post scenario.

8.3 The ex-interim case

8.3.1 Static model in the ex-interim case

In this case, the sequence of events is summarized in Figure 7.

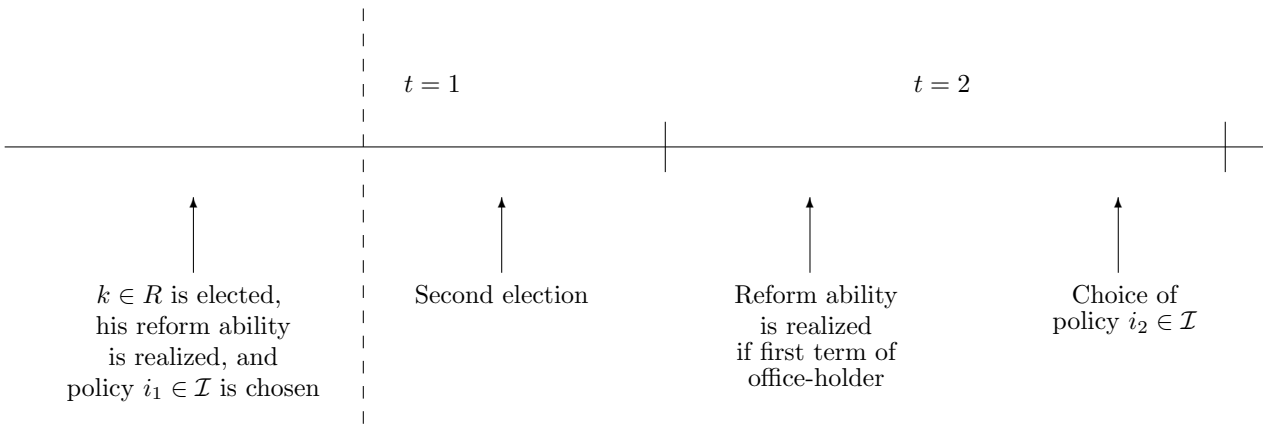

Figure 7: Timeline for the static model—ex-interim.

As in the ex-ante case, we have to start with the analysis of the office-holder's behavior in the second period. However, it is easy to verify that the best response of the office-holder in $t = 2$ is again given by Proposition 1. The reason is that the challenger's reform ability is realized before he makes his policy choice. Consequently, we next focus on the voters' decision in the election of the second period, which differs from the one in the ex-ante case because at the time of the election voters do not know about the challenger's reform ability. For the presentation of our results, it will be useful to introduce a new parameter. Recall that

$$\frac{\hat{C}}{\hat{C}+1} = \int_0^\infty \frac{c}{c+1} f(c)dc. \tag{72}$$

The reform ability value $\hat{C}$ will turn out to be a critical cut-off level for the incumbent's re-election. We first observe that $\hat{C}$ is lower than the expected reform ability $\bar{C}$.

Lemma 2
It holds that $\hat{C} < \bar{C}$.

Proof:

Consider the following real-valued function

$$g(x) = \int_0^\infty \frac{x-c}{c+1} f(c)dc.$$

It is easy to verify that $g(0) < 0$, $\lim_{x\to+\infty} g(x) \geq 0$, and $g'(x) > 0$. Then, it is easy to verify that $\hat{C}$—as introduced in (8)—is the only solution c' to the equation $g(c') = 0$. Moreover, note that for $x > 0$,

$$\begin{aligned} g(x) &= \int_0^\infty \frac{x-c}{c+1} f(c)dc = \int_0^x \frac{x-c}{c+1} f(c)dc - \int_x^\infty \frac{c-x}{c+1} f(c)dc \\ &> \int_0^x \frac{x-c}{x+1} f(c)dc - \int_x^\infty \frac{c-x}{x+1} f(c)dc = \int_0^\infty \frac{x-c}{x+1} f(c)dc = \frac{x-\bar{C}}{x+1}, \end{aligned}$$

implying that $g(\bar{C}) > 0$ or, equivalently, that $\hat{C} < \bar{C}$.

□

Now we are in a position to present the electorate's behavior.

Proposition 13
Let $k \in R$ be the office-holder in $t = 1$ and $k' \in L$ the challenger in $t = 2$. Let also $i_1 \in \mathcal{I}$ be the status-quo policy chosen in period $t = 1$. Then, k is re-elected in equilibrium of $\mathcal{G}_1^R$ if and only if $i_1 \in \mathcal{X}(c_k, \hat{C})$, where

$$\mathcal{X}(c_k, \hat{C}) = \begin{cases} \left[\frac{1}{2} - \pi, \frac{1}{2} + \pi\right] & \text{if } c_k > \hat{C}, \\ [0,1] & \text{if } c_k = \hat{C}, \\ \left[0, \frac{1}{2} - \pi\right] \cup \left[\frac{1}{2} + \pi, 1\right] & \text{if } c_k < \hat{C}. \end{cases}$$

Proof:

First, we show that the median voter, $m = \frac{1}{2}$, is decisive in the election. We stress that voters know the incumbent's reform ability but do not know the ability of the challenger. For the sake of readability, let $i^*_{R2} := i^*_{k2}(i_1)$ denote the best-response function of the incumbent, k, when he is re-elected and let $i^*_{L2}(c'_k) := i^*_{k''2}(i_1)$ denote the best-response function of the challenger, k', when he is elected. Note that $i^*_{k2}(i_1)$ and $i^*_{k'2}(i_1)$ have been defined in Proposition 1. Let $i \in [0,1]$ be an arbitrary voter. According to the tie-breaking rules, i will vote for the incumbent k if and only if

$$U_i^{\mathcal{I}}(i^*_{R2}) + U^{c_k}(i_1, i^*_{R2}) \geq \int_0^\infty \left[U_i^{\mathcal{I}}(i^*_{L2}(c'_k)) + U^{c_{k'}}(i_1, i^*_{L2}(c_{k'}))\right] f(c_{k'})dc_{k'}, \tag{73}$$

which can be rewritten as

$$\begin{aligned} &2i \cdot \int_0^\infty \left(i^*_{R2} - i^*_{L2}(c_{k'})\right) f(c_{k'})dc_{k'} \\ &\geq \int_0^\infty \left[(i^*_{R2})^2 - (i^*_{L2}(c_{k'}))^2 + c_k \cdot (i_1 - i^*_{R2}) - c_{k'} \cdot (i_1 - i^*_{L2}(c_{k'}))^2\right] f(c_{k'})dc_{k'}. \end{aligned} \tag{74}$$

Note that the right-hand side of (74) does not depend on i. We distinguish two cases.

- **Case A:** $\int_0^\infty \left(i^*_{R2} - i^*_{L2}(c_{k'})\right) f(c_{k'})dc_{k'} \geq 0$

  From (74) it follows that if a voter i votes for the right-wing candidate, all voters $j \in (i, 1]$ will also vote for him. Similarly, if a voter i votes for the left-wing candidate, all voters $[0, i+\varepsilon]$ will also vote for him, with $\varepsilon > 0$ small enough.

- **Case B:** $\int_0^\infty \left(i^*_{R2} - i^*_{L2}(c_{k'})\right) f(c_{k'}) dc_{k'} < 0$

  From (74) it follows that if a voter i votes for the left-wing candidate, all voters $j \in [i-\varepsilon, 1]$ will also vote for him, with $\varepsilon > 0$ small enough. Similarly, if a voter i votes for the right-wing candidate, all voters $[0, i)$ will also vote for him.

That is, in either case m is the decisive voter. From the explicit definitions of i^*_{R2} and $i^*_{L2}(c'_k)$ in Proposition 1, the inequality in (73) when $i = m$ can be rewritten as

$$\int_0^\infty \left[\left(\frac{1}{2} - \frac{c_{k'}}{c_{k'}+1} i_1 - \frac{1}{c_{k'}+1}\mu_L\right)^2 + c_{k'} \cdot \left(i_1 - \frac{c_{k'}}{c_{k'}+1} i_1 - \frac{1}{c_{k'}+1}\mu_L\right)^2\right] f(c_{k'}) dc_{k'}$$
$$\geq \left(\frac{1}{2} - \frac{c_k}{c_k+1} i_1 - \frac{1}{c_k+1}\mu_R\right)^2 + c_k \cdot \left(i_1 - \frac{c_k}{c_k+1} i_1 - \frac{1}{c_k+1}\mu_R\right)^2.$$

Equivalently, using (1) and (24), we obtain

$$\frac{1}{c_k+1} \cdot (c_k \cdot i_1 \cdot (1-i_1) + \mu_R \cdot (1-\mu_R))$$
$$\geq \int_0^\infty \left[\frac{1}{c_{k'}+1} \cdot (c_{k'} \cdot i_1 \cdot (1-i_1) + \mu_R \cdot (1-\mu_R))\right] f(c_{k'}) dc_{k'},$$

which leads to

$$i_1 \cdot (1-i_1) \cdot \int_0^\infty \left(\frac{c_k}{c_k+1} - \frac{c_{k'}}{c_{k'}+1}\right) f(c_{k'}) dc_{k'}$$
$$\geq \mu_R \cdot (1-\mu_R) \cdot \int_0^\infty \left(\frac{1}{c_{k'}+1} - \frac{1}{c_k+1}\right) f(c_{k'}) dc_{k'},$$

and further to

$$[i_1 \cdot (1-i_1) - \mu_R \cdot (1-\mu_R)] \cdot \frac{1}{c_k+1} \cdot \int_0^\infty \frac{c_k - c_{k'}}{c_{k'}+1} f(c_{k'}) dc_{k'} \geq 0. \tag{75}$$

Moreover, recall that $\hat{C}$ is such that

$$\int_0^\infty \frac{\hat{C} - c}{c+1} f(c) dc = 0.$$

Then it is easy to verify that (75) holds if and only if

$$[i_1 \cdot (1-i_1) - \mu_R \cdot (1-\mu_R)] \cdot (c_k - \hat{C}) \geq 0.$$

The desired result follows immediately from an inspection of the above inequality.

□

Thus, incumbents whose reform ability c_k is higher than $\hat{C}$ are re-elected when party polarization is large (relative to the need-for-reform). On the reverse, incumbents whose reform ability c_k is lower than $\hat{C}$ are re-elected when party polarization is low (relative to the need-for-reform). Finally, incumbents are always re-elected when the incumbent's reform ability is equal to $\hat{C}$. Proposition 13 adapts Proposition 2 to the ex-interim case. Now the cut-off level $\hat{C}$ is derived from the comparison between the expected policies of low-reform-ability office-holders and the expected policies of high-reform-ability ones at a stage where the challenger's reform ability is still unknown.

8.3.2 Dynamic model in the ex-interim case

In this case, the sequence of events is summarized in Figure 8.

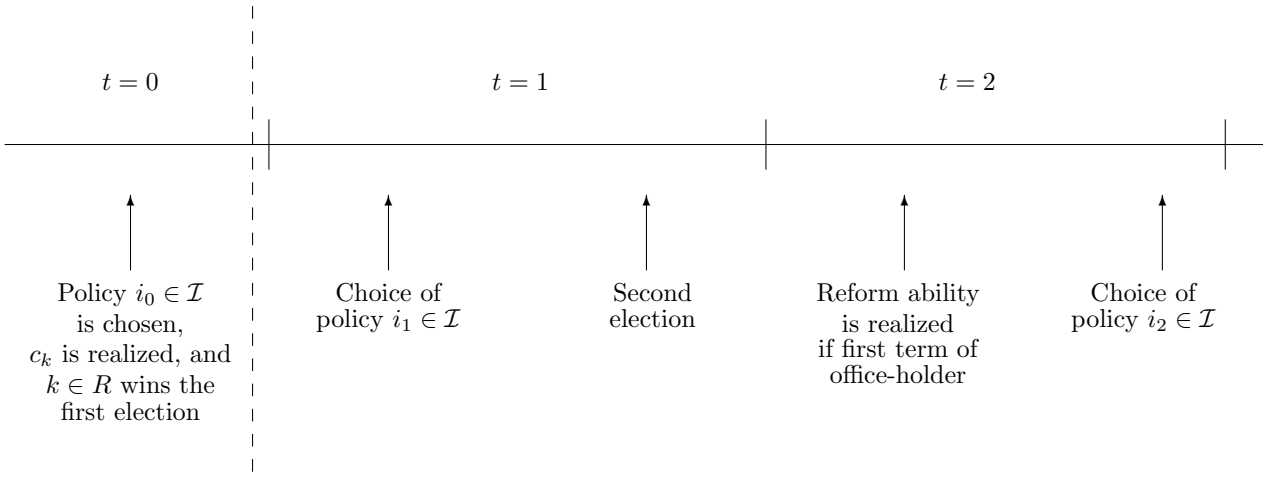

Figure 8: Timeline for the dynamic model—ex-interim.

Next, we describe the policy choice of the incumbent in $t = 1$. For the sake of notation, let

$$i_{1,*}^{EI}(\tau, \pi, c_k) = i_{1,*}^{EA}(\tau, \pi, c_k), \tag{76}$$

where $i_{1,*}^{EA}(\tau, \pi, c_k)$ has been defined in Equation (12).

Proposition 14
Let $k \in R$ denote the incumbent in $t = 1$ and $k' \in L$ the challenger. In $t = 1$, k chooses the following policy in equilibrium of $\mathcal{G}_2^R$:

(a) When $c_k > \hat{C}$,

$$i_{k1}^{EI} = \min\left\{\frac{1}{2} + \pi, \max\left\{\frac{1}{2} - \pi, i_{1,*}^{EI}(\tau, \pi, c_k)\right\}\right\}.$$

(b) When $c_k = \hat{C}$,

$$i_{k1}^{EI} = i_{1,*}^{EI}(\tau, \pi, c_k).$$

(c) When $c_k < \hat{C}$,

$$i_{k1}^{EI} = \begin{cases} \min\left\{\frac{1}{2} - \pi, i_{1,*}^{EI}(\tau, \pi, c_k)\right\} & \text{if } i_{1,*}^{EI}(\tau, \pi, c_k) \in \left[0, \frac{1}{2}\right), \\ \frac{1}{2} - \pi \text{ or } \frac{1}{2} + \pi & \text{if } i_{1,*}^{EI}(\tau, \pi, c_k) = \frac{1}{2}, \\ \max\left\{\frac{1}{2} + \pi, i_{1,*}^{EI}(\tau, \pi, c_k)\right\} & \text{if } i_{1,*}^{EI}(\tau, \pi, c_k) \in \left(\frac{1}{2}, 1\right]. \end{cases}$$

Proof:

We analyze the solution to the maximization problem in (11). Using Propositions 1 and 13 yields the following expression for the expected utility of office-holder k:

$$\begin{aligned}
&U_k^{exp}(i_{k1}) = b \\
&- \int_0^\infty c_k(i_0 - i_{k1})^2 f(c_k)dc_k - \int_0^\infty (i_{k1} - \mu_R)^2 f(c_k)dc_k \\
&+ b \cdot \beta \cdot \int_0^\infty \int_0^\infty I(i_{k1}, c_k, \hat{C}) f(c_{k'})dc_{k'} f(c_k)dc_k \\
&- \beta \cdot \int_0^\infty I(i_{k1}, c_k, \hat{C}) \cdot \left(\frac{c_k}{c_k+1} i_{k1} + \frac{1}{c_k+1}\mu_R - \mu_R \right)^2 f(c_k)dc_k \\
&- \beta \cdot \int_0^\infty I(i_{k1}, c_k, \hat{C}) \cdot c_k \left(i_{k1} - \frac{c_k}{c_k+1} i_{k1} - \frac{1}{c_k+1}\mu_R \right)^2 f(c_k)dc_k \\
&- \beta \cdot \int_0^\infty (1 - I(i_{k1}, c_k, \hat{C})) \cdot \left[\int_0^\infty \left(\frac{c_{k'}}{c_{k'}+1} i_{k1} + \frac{1}{c_{k'}+1}\mu_L - \mu_R \right)^2 f(c_{k'})dc_{k'} \right] f(c_k)dc_k \\
&- \beta \cdot \int_0^\infty (1 - I(i_{k1}, c_k, \hat{C})) \cdot \left[\int_0^\infty c_{k'} \left(i_{k1} - \frac{c_{k'}}{c_{k'}+1} i_{k1} - \frac{1}{c_{k'}+1}\mu_L \right)^2 f(c_{k'})dc_{k'} \right] f(c_k)dc_k,
\end{aligned} \tag{77}$$

where $I(i_{k1}, c_k, \hat{C}) = 1$ if and only if $i_{k1} \in \mathcal{X}(c_k, \hat{C})$ and the set $\mathcal{X}(c_k, \hat{C})$ has been defined in Proposition 2. Because b has been assumed to be very large and c_k is known when k makes his policy choice, he will choose $i_{k1} \in \mathcal{X}(i_{k1}, c_k, \hat{C})$ for any realization of his reform ability. Note that if $i_{k1} \in \mathcal{X}(i_{k1}, c_k, \hat{C})$, we obtain

$$\begin{aligned}
&U_k^{exp}(i_{k1}) \\
=&b - c_k \cdot (i_0 - i_{k1})^2 - \int_0^\infty (i_{k1} - \mu_R)^2 + \beta \cdot b \\
-&\beta \cdot \left[\left(\frac{c_k}{c_k+1} i_{k1} + \frac{1}{c_k+1}\mu_R - \mu_R \right)^2 + c_k \cdot \left(\frac{c_k}{c_k+1} i_{k1} + \frac{1}{c_k+1}\mu_R - i_{k1} \right)^2 \right] \\
=&(1+\beta) \cdot b - c_k \cdot (i_0 - i_{k1})^2 - \left(1 + \beta \cdot \frac{c_k}{c_k+1} \right) \cdot (i_{k1} - \mu_R)^2.
\end{aligned}$$

The solution to the problem

$$\begin{aligned}
\max_{i_{k1}} \quad & U_k^{exp}(i_{k1}) \\
\text{s.t. } \; & i_{k1} \in \mathcal{I}
\end{aligned} \tag{78}$$

is then

$$i_{1,*}^{EI}(i_0, c_k) = \frac{c_k}{1 + c_k + \beta \cdot \frac{c_k}{c_k+1}} \cdot i_0 + \frac{1 + \beta \cdot \frac{c_k}{c_k+1}}{1 + c_k + \beta \cdot \frac{c_k}{c_k+1}} \cdot \mu_R. \tag{79}$$

Next we distinguish three cases.

Case I: $c_k > \hat{C}$

In this case, we have $\mathcal{X}(i_{k1}, c_k, \hat{C}) = [1 - \mu_R, \mu_R]$. Hence, k solves the following problem:

$$\begin{aligned}
\max_{i_{k1}} \quad & U_k^{exp}(i_{k1}) \\
\text{s.t. } \; & i_{k1} \in [1 - \mu_R, \mu_R].
\end{aligned}$$

Given (78) and (79), the solution to the above problem is

$$i_{k1}^{EI} = \min\left\{\mu_R, \max\left\{1-\mu_R, i_{1,*}^{EI}(i_0, c_k)\right\}\right\}.$$

Case II: $c_k = \hat{C}$

In this case, we have $\mathcal{X}(i_{k1}, c_k, \hat{C}) = [0,1]$. Hence, given (78) and (79), k chooses

$$i_{k1}^{EI} = i_{1,*}^{EI}(i_0, c_k).$$

Case III: $c_k < \hat{C}$

In this case, we have $\mathcal{X}(i_{k1}, c_k, \hat{C}) = [0, 1-\mu_R] \cup [\mu_R, 1]$. Hence, k solves the following problem:

$$\begin{aligned} \max_{i_{k1}} \quad & U_k^{exp}(i_{k1}) \\ \text{s.t.} \quad & i_{k1} \in [0, 1-\mu_R] \cup [\mu_R, 1]. \end{aligned}$$

Given (78) and (79), the solution to the above problem is

$$i_{k1}^{EI} = \begin{cases} \min\{1-\mu_R, i_{1,*}^{EI}(i_0, c_k)\} & \text{if } i_{1,*}^{EI}(i_0, c_k) \in \left[0, \frac{1}{2}\right), \\ 1-\mu_R \text{ or } \mu_R & \text{if } i_{1,*}^{EI}(i_0, c_k) = \frac{1}{2}, \\ \max\{\mu_R, i_{1,*}^{EI}(i_0, c_k)\} & \text{if } i_{1,*}^{EI}(i_0, c_k) \in \left(\frac{1}{2}, 1\right]. \end{cases}$$

Finally, the statement of the proposition follows using $\pi = \mu_R - \frac{1}{2}$ and $\tau = i_0 - \frac{1}{2}$.

□

As in the ex-ante case, in equilibrium the incumbent in $t = 1$ will always choose a policy that guarantees his re-election, regardless of his reform ability.

8.4 The ex-post case

8.4.1 Static model in the ex-post case

In this case, the sequence of events can be summarized as in Figure 7.

Again, we start with the analysis of the office-holder's behavior in the second period. Accordingly, we assume that elections in $t = 2$ have already taken place and that candidate $k'' \in \{k, k'\}$ is in office. We stress that in the ex-post scenario the challenger, if elected, does not know about his reform ability when he makes the policy choice. Hence, as there are no further elections, k'' will choose his policy to maximize his expected instant utility in $t = 2$.

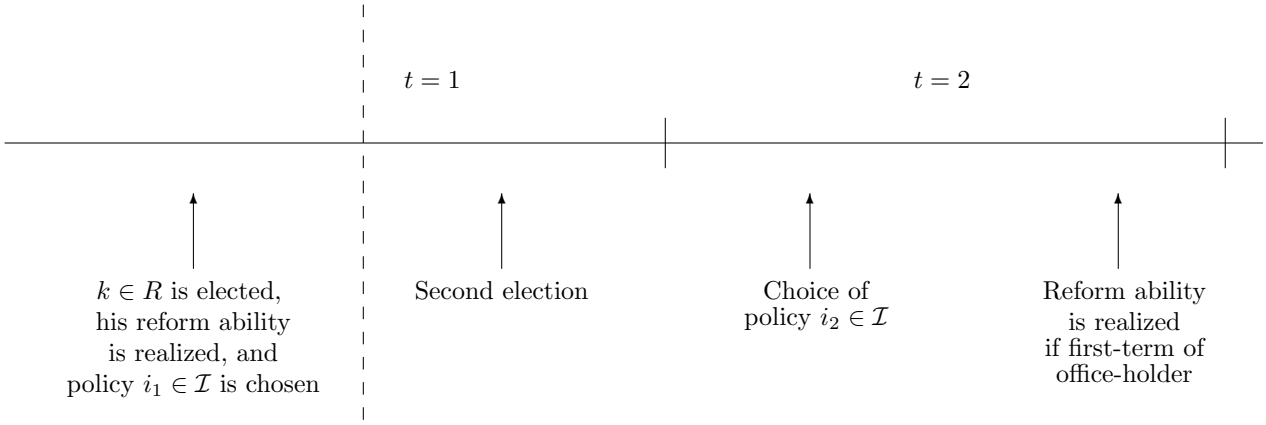

Figure 9: Timeline for the static model—ex-post.

Proposition 15
Let $k'' \in R \cup L$ be the office-holder in $t = 2$. The best response of k'' to the status-quo policy $i_1 \in \mathcal{I}$ chosen in $t = 1$ is given by

$$i_{k''2}(i_1) = \begin{cases} \dfrac{c_{k''}}{c_{k''}+1} \cdot i_1 + \dfrac{1}{c_{k''}+1} \cdot \mu_{k''} & \text{if } k'' = k \in R, \\ \dfrac{\bar{C}}{\bar{C}+1} \cdot i_1 + \dfrac{1}{\bar{C}+1} \cdot \mu_{k''} & \text{if } k'' = k' \in L. \end{cases} \tag{80}$$

Proof:

We only consider the case where the challenger, $k' \in L$, is elected, as for k, it is the same as in Proposition 1. We stress that when choosing the policy in the second period, k' does not know his own reform ability, $c_{k'}$, and only knows that it is drawn from $F(\cdot)$. Let

$$\begin{aligned} B(i_{k'2}) &= \int_0^\infty \left[U^{\mathcal{I}}_{\mu_{k'}}(i_{k'2}) + U^{c_{k'}}(i_1, i_{k'2}) \right] f(c_{k'}) dc_{k'} \\ &= -\left(i_{k'2} - \mu_{k'}\right)^2 - \bar{C} \cdot \left(i_1 - i_{k'2}\right)^2. \end{aligned}$$

Once he is elected, k' solves the following problem:

$$\begin{aligned} \max_{i_{k'2}} \quad & \mathbb{E}[B(i_{k'2})] \\ \text{s.t.} \quad & i_{k'2} \in \mathcal{I}. \end{aligned} \tag{81}$$

Following similar lines to the proof of Proposition 1, it is easy to verify that k'' chooses

$$\frac{\bar{C}}{\bar{C}+1} \cdot i_1 + \frac{1}{\bar{C}+1} \cdot \mu_{k'}$$

as a solution to (81).

□

Proposition 15 is the adaptation of Proposition 1 to the ex-post case. The weights are now determined by the expected reform ability if a new candidate enters office. Note that, similarly to the ex-ante and

the ex-interim cases, (80) can be rewritten as follows:

$$i_{k''2}(i_1) = \begin{cases} \frac{1}{2} + \frac{c_k}{c_k+1} \cdot \tau + \frac{1}{c_k+1} \cdot \pi & \text{if } k'' = k \in R, \\ \frac{1}{2} + \frac{\bar{C}}{\bar{C}+1} \cdot \tau - \frac{1}{\bar{C}+1} \cdot \pi & \text{if } k'' = k' \in L. \end{cases}$$

Accordingly, if $\tau > 0$, the lower c_k is, the more moderate will be the policy chosen by $k \in R$. Moreover, such a policy becomes more partisan as party polarization, π, increases. Analogous comments apply when either $\tau < 0$ or k' is elected.

Next, we focus on the voters' decision at the end of the first period. We stress that at the time of election, voters and candidates only know about the incumbent's, but not the challenger's, reform ability.

Proposition 16
Let $k \in R$ be the office-holder in $t = 1$ and $k' \in L$ the challenger in $t = 2$. Let also $i_{k1} \in \mathcal{I}$ be the policy chosen in period $t = 1$. Then, k is re-elected in equilibrium of $\mathcal{G}_1^R$ if and only if $i_{k1} \in \mathcal{X}(c_k, \bar{C})$, where

$$\mathcal{X}(c_k, \bar{C}) = \begin{cases} \left[\frac{1}{2} - \pi, \frac{1}{2} + \pi\right] & \text{if } c_k > \bar{C}, \\ [0, 1] & \text{if } c_k = \bar{C}, \\ \left[0, \frac{1}{2} - \pi\right] \cup \left[\frac{1}{2} + \pi, 1\right] & \text{if } c_k < \bar{C}. \end{cases}$$

Proof:

First, we show that the median voter, $m = \frac{1}{2}$, is decisive in the election. We stress that voters know the incumbent's reform ability but do not know the challenger's. For the sake of readability, let $i^*_{R2} := i^*_{k2}(i_{k1})$ denote the best-response function of the incumbent, k, when he is re-elected and let $i^*_{L2} := i^*_{k'2}(i_{k1})$ denote the best-response function of the challenger, k', when he is elected. We recall that $i^*_{k2}(i_{k1})$ and $i^*_{k'2}(i_{k1})$ have been defined in Proposition 15. We note that i^*_{L2} does not depend on the actual ability $c_{k''}$, which has not yet been realized. Let $i \in [0, 1]$ be an arbitrary voter. According to the tie-breaking rules, i will vote for the incumbent k if and only if

$$U_i^{\mathcal{I}}(i^*_{R2}) + U^{c_k}(i_{k1}, i^*_{R2}) \geq \int_0^\infty \left[U_i^{\mathcal{I}}(i^*_{L2}(c'_k)) + U^{c_{k'}}(i_{k1}, i^*_{L2}(c_{k'}))\right] f(c_{k'}) dc_{k'},$$

which can be rewritten as

$$2i \cdot (i^*_{R2} - i^*_{L2}) \geq (i^*_{R2})^2 - (i^*_{L2})^2 + c_k \cdot (i_{k1} - i^*_{R2}) - c_{k'} \cdot (i_{k1} - i^*_{L2})^2 .$$

From now on, the proof follows exactly the same lines as in the proof of Proposition 2 by substituting $c_{k'}$ for $\bar{C}$, to conclude that the incumbent, k, will be re-elected if and only if

$$[i_{k1} \cdot (1 - i_{k1}) - \mu_R \cdot (1 - \mu_R)] \cdot (c_k - \bar{C}) \geq 0.$$

The desired result follows immediately from an inspection of the above inequality.

□

Proposition 16 is the counterpart of Proposition 2 and 13 for the ex-post case. Now the critical reform ability threshold is the expected reform ability, because the reform ability of the challenger is known

neither when the election takes place nor when he chooses his policy reform (provided he is elected). Consequently, incumbents who are less efficient than average are re-elected when party polarization is large (relative to the need-for-reform). By contrast, incumbents who are more efficient than average are re-elected when party polarization is low (relative to the need-for-reform). Finally, incumbents who are as efficient at reforms as average candidates are always re-elected.

8.4.2 Dynamic model in the ex-post case

In this case, the sequence of events is summarized in Figure 10.

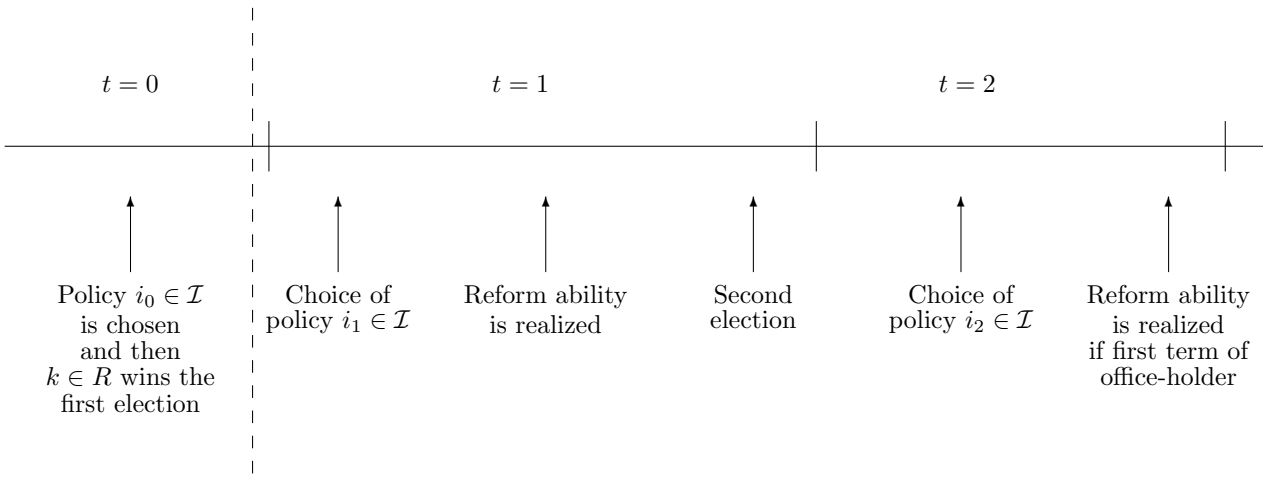

Figure 10: Timeline for the dynamic model—ex-post.

Proposition 4 in the main body of the paper has shown that the incumbent can secure re-election by choosing particular policies in $t = 1$. These policies depend on the skewness of the reform-ability-distribution. We obtain the following corollary:

Corollary 5
Let $i_{k1}^{EP} \in \mathcal{I}$ denote the policy chosen in $t = 1$ in equilibrium of $\mathcal{G}_2^R$ by the incumbent k, with $k \in R$. Then,

(a) if $i_{k1}^{EP} \in \left\{\frac{1}{2} - \pi, \frac{1}{2} + \pi\right\}$, then k is re-elected regardless of his ability.[34]

(b) if $i_{k1}^{EP} \notin \left\{\frac{1}{2} - \pi, \frac{1}{2} + \pi\right\}$ and $F(\bar{C}) < \frac{1}{2}$, then k is re-elected if and only if he is not better-skilled than average, i.e. $c_k \geq \bar{C}$.

(c) if $i_{k1}^{EP} \notin \left\{\frac{1}{2} - \pi, \frac{1}{2} + \pi\right\}$ and $F(\bar{C}) \geq \frac{1}{2}$, then k is re-elected if and only if he is not worse-skilled than average, i.e. $c_k \leq \bar{C}$.

[34] This follows from the tie-breaking rules.

That is, if the incumbent in $t = 1$ chooses a policy different from any of the ideal points of the two political parties, he makes his policy choice dependent on whether $F(\bar{C})$ is smaller or larger than $\frac{1}{2}$. The reason is that if the distribution of candidates' reform ability is negatively skewed, candidates who are average- or low-skilled in efficiently implementing policy reforms will be re-elected. By contrast, if the distribution of candidates' reform ability is positively skewed or not skewed at all, average- or high-skilled candidates will be re-elected.[35]

[35] We note that since the ability of a candidate is actually $-c_k$, then it is equivalent to say that the distribution of the candidates' reform ability is negatively (positively) skewed and that the distribution $f(\cdot)$ is positively (negatively) skewed.

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