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Working Paper 16/252
November 2016

Economics Working Paper Series

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Social Equity Concerns and Differentiated Environmental Taxes

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November 24, 2016

Abstract

This paper examines pollution tax differentiation across industries in light of social equity concerns using theoretical and numerical general equilibrium analyses in an optimal tax framework. We characterize the drivers for non-uniform optimal taxes emanating from the interaction of household heterogeneity with social preferences. Quantitatively assessing the case of price-based CO_2 emissions control in the U.S. economy, we find that optimal carbon taxes differ largely across industries, even when social inequality aversion is low. Our results are robust with respect to the stringency of the environmental target, non-optimal redistribution schemes, and parametric uncertainty in firms' and households' equilibrium tax responses.

Keywords: Differentiated environmental taxes, Carbon pricing, Industries, Heterogeneous households, Social inequality, Optimal taxation, General equilibrium
JEL: H23, Q52, C68

1. Introduction

Controlling pollution or carbon dioxide (CO_2) emissions with market-based regulatory instruments such as taxes or tradable permit systems has been shown to be cost-effective and efficient (Goulder and Parry, 2008; Metcalf, 2009). At the same time, the public acceptance of such policies depends crucially on their distributional consequences (Atkinson and Stiglitz, 1980; Fullerton and Metcalf, 2002). While efficiency and equity are fundamentally linked, the issue of addressing unintended distributional outcomes of market-based environmental policy—among, for example, heterogeneous groups of industries, households, or countries—is often analyzed in isolation from efficiency. A large literature in environmental and public economics elucidates important trade-offs between efficiency and equity (for example, Poterba, 1989; Bovenberg et al., 2005; Bento et al., 2009; Rausch et al., 2010; Sterner, 2012; Fullerton and Monti, 2013) but largely focuses on the issue of how the revenues from efficient market-based regulation, that is based on *uniform* pollution pricing and the principle of equalizing marginal abatement cost in production, can be used to alter incidence outcomes. In an economy with heterogeneous households and social equity concerns, however, efficiency and equity can generally not be separated. This raises two fundamental questions which are of importance public environmental policy. First, is uniform pollution pricing optimal in light of social equity concerns? Second, how is the optimal differentiation of pollution taxes linked to social and private preferences?

We examine these questions using both theoretical and numerical general equilibrium analyses in the context of an optimal taxation framework. Our analysis rests on three premises that reflect fundamental

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constraints for real-world environmental tax policy in most countries. First, while it is in principle possible to address social equity concerns using lump-sum income redistribution, we abstract from such transfers as they seem to be infeasible in practice (Feldstein, 1972). Rather, we focus on the design of environmental policy that—in seeking to address the distributional consequences from environmental taxes—is constrained to using the revenues raised through these taxes. Second, we focus on analyzing revenue-neutral tax policies which do not affect the government budget, i.e. pollution tax revenues are returned back to the economy. Third, we consider pollution tax differentiation among industry sectors and we abstract from direct pollution taxes on private consumption.[3] This is motivated by the observation that pollution is typically taxed at the source which generally lies at the industry level. Taxing pollution downstream at the level of private consumption would raise the issue of accurately determining indirect (embodied) pollution in multiple consumption goods. Moreover, while for some pollutants, such as CO_2 emissions, household emissions constitute a large fraction of economy-wide emissions (e.g., related to private transportation), taxing these sources is politically highly contentious.

We first theoretically study the optimality conditions for industry-differentiated pollution taxes and revenue redistribution to households. Household heterogeneity in preferences and endowments interacts with social equity concerns implying that marginal abatement costs in the absence of social equity concerns are in general not equalized at the social optimum. We identify the motives that affect the sectoral marginal social cost of abatement. Heterogeneity in households' preferences implies a higher marginal social abatement cost for sectors which produce output that is consumed more intensively by households with higher social weights. Marginal social cost of abatement are instead lowered if factor income losses for households with high social weights are relatively small. In addition, the marginal social cost of abatement in a given sector depends on the social value of the pollution tax revenue raised per united of abated pollution. To the extent that these effects vary across sectors, the equalization of sectoral marginal social abatement cost implies that marginal abatement cost in the absence social equity concerns are not equalized, thus requiring differentiated pollution pricing at the social optimum.

We use analytical examples to further examine the conditions under which uniform taxes are not optimal and to study how taxes should be differentiated to improve social welfare. If households have different tastes, then taxes, in light of social equity concerns, should be differentiated according to households' consumption characteristics, in order to shift the burden of taxation towards households with lower social weights. Moreover, pollution taxes should be differentiated to raise revenues for targeted transfers to households with high social weights.

To quantitatively assess the relative importance of the different motives for tax differentiation in an empirical setting, we complement our theoretical analysis with numerical simulations. We develop a numerical framework that casts the problem of optimal sectoral pollution pricing with social equity concerns in the context of a numerical general equilibrium framework that embodies firms' and households' behavioral equilibrium responses to pollution taxes, while at the same time satisfying cross-market and aggregate economy restrictions. More specifically, we focus on the issue of climate policy aimed at reducing CO_2 emissions through price-based market regulation looking at the case of the U.S. economy. To this end, we calibrate the numerical model to observed production, consumption, and aggregate (national) as well as household-level income data for the U.S. economy. We assume that the regulating entity—faced with the problem of designing the optimal carbon pricing policy—is averse to social (income) inequality.

We find that the extent of carbon tax differentiation across industries is substantial, even for relatively low degrees of social inequality aversion (unsurprisingly, the degree of tax differentiation increases with higher social inequality aversion). Optimally differentiated carbon taxes significantly raise the amount of tax revenues collected which are then redistributed to low-income households. Importantly, this largely reduces inequality to levels which would not be attainable under uniform carbon pricing. Our results are robust to varying the stringency of the environmental target and parametric uncertainty with respect to

[3] We use "industry" and "sector" interchangeably throughout this paper.

the equilibrium responses of firms and households to environmental taxes.

Decomposing the different motives for tax differentiation, we find that the deviation from uniform pollution pricing is to a large extent driven by the motive to enhance the amount of tax revenues available for redistribution. Pollution taxes are higher for industries with relatively steep marginal abatement cost as this enables raising higher tax revenues for a given amount of pollution. Hence, we find that efficiency of abatement in production is strongly sacrificed in favor of generating high tax revenues that can be spent on targeted transfers to address equity concerns. While the pattern of sectoral tax differentiation due to the heterogeneity in households' preferences is in line with our theoretical findings, our quantitative analysis reveals that the revenue-raising motive is the main driver for tax differentiation.

Besides examining optimal policies consisting of differentiated pollution taxes and transfers for revenue redistribution, situations in which a non-optimal revenue redistribution scheme is used (for example, per-capita or consumption-based tax rebates) arguably represents the more relevant setting for real-world environmental policy. We thus analyze optimal tax differentiation, and the resulting implications for tax incidence across household income groups, for three non-optimal revenue redistribution schemes commonly employed (or considered) in public environmental policy: per-capita transfers; tax rebates in proportion to income; tax rebates in proportion to "dirty" consumption. While the case for tax differentiation is somewhat diminished for non-optimal transfer schemes, we find that optimal carbon taxes are still strongly differentiated across industries for transfer schemes that allocate a large fraction of tax revenues to low-income households—as it is the case for the per-capita and consumption-based recycling schemes. In terms of incidence impacts by household income groups, optimally differentiated taxes yield more progressive (less regressive) outcomes relative to uniform pollution pricing. In general, we find that the difference in the incidence between uniform and optimally differentiated pollution pricing is larger for transfer schemes which are better suited to address inequality, i.e. distributing larger parts of the revenue to poor households.

This paper contributes to the existing literature in several ways. First, the paper is related to the literature on optimal environmental taxation following the seminal contribution by Pigou (1920) according to which an externality should be priced at its marginal social damage. Subsequent literature has explored a variety of reasons for deviating from this principle. Interactions between environmental taxes and the broader fiscal system as well as the use of tax revenues to reduce pre-existing distortionary taxes have been shown to modify (or generalize) the Pigouvian pricing rule (see, for example, Bovenberg and de Mooij, 1994; Bovenberg and van der Ploeg, 1994; Parry, 1995; Bovenberg and Goulder, 1996). By focusing on efficiency, this literature abstracts from heterogeneous households and social equity concerns. Moreover, it assumes a single polluting sector or, equivalently, uniform emissions pricing across multiple economic activities.

Second, studies concerned with pollution tax differentiation across industries have also focused on efficiency aspects. Boeters (2014) and Landis et al. (2016) find that the cost of climate policy can be lowered by differentiating carbon taxes in light of pre-existing non-environmental tax distortions. In an open economy context and for unilateral environmental policy, international market power, terms-of-trade effects, and emissions leakage have been shown to imply pollution taxes to optimally differ across industries. Hoel (1996) argues that leakage in an incomplete international climate agreement motivates carbon tax differentiation, if import and export tariffs on all goods are not allowed. Böhringer and Rutherford (1997) point to competitiveness arguments based on comparative advantage that can justify exemptions for energy- and export-intensive sectors under otherwise uniform carbon pricing in unilateral climate policy. Böhringer et al. (2014) study optimally differentiated sectoral emission prices, motivated by leakage and terms-of-trade effects, and find that the welfare gains compared to uniform emissions pricing are modest. Similarly, Boeters (2014) finds that market power in export markets gives rise to optimally differentiated carbon taxes across industries. We contribute by examining how optimal pollution taxes should be differentiated across sectors due to social equity concerns.[4]

[4]To isolate the role of social equity concerns, we have deliberately chosen to abstract from the motives for sectorally

Third, we are not the first to investigate the role of equity concerns within an optimal taxation framework. A large body of literature, not directly related to addressing an environmental externality, has established the result that optimal commodity and income taxation are in general affected by household heterogeneity and social preferences (Diamond and Mirrlees, 1971). In a framework that brings together revenue-raising and externality-correcting motives for optimal tax policy, Sandmo (1975) and Cremer et al. (2003) find that the optimal pollution tax rate in an economy with one polluting good depends on social preferences and household characteristics. Equity concerns also motivate non-linear consumption taxes (Cremer et al., 1998, 2003), thus leading to differentiated tax rates among consumers. The issue of differentiating pollution taxes across sectors when social equity concerns are present has, however, not been examined.[5]

Fourth, equity aspects of environmental policy are often studied outside an optimal taxation framework. Here, it is common to assess the distributional outcomes of environmental policy without ranking alternative outcomes based on social desirability or deriving optimal pollution tax rates in light of social preferences (i.e., thus adopting a positive rather than a normative perspective). For example, Poterba (1991) and Fullerton and Heutel (2010) find that energy (gasoline and carbon) taxes are strongly regressive. Fullerton and others have used analytical general equilibrium models building on Harberger (1962) to investigate the incidence of environmental taxes (Fullerton and Heutel, 2007; Fullerton et al., 2012). Bento et al. (2009) find that the incidence of an increase in the gasoline tax depends crucially on how revenues are recycled. In other instances, the amount of revenue collected through the environmental policy has been found to be insufficient to alter incidence in desirable ways. For example, Fullerton and Monti (2013) find that low-wage earners are more impacted by pollution taxes, even when they receive all the tax revenues. Bovenberg et al. (2005) examine the efficiency cost of meeting distributional objectives across industries for emissions taxes. While this segment of the literature has highlighted important trade-offs between efficiency and equity, it has restricted attention to uniform pollution taxes and has not examine the issue of optimal pollution pricing in the presence of social equity concerns.

The remainder of the paper proceeds as follows. Section 2 derives and discusses our theoretical results on optimal pollution pricing across industries. Section 3 presents our quantitative, empirical framework to examine optimal sectoral carbon taxation under social equity concerns in the context of a numerical general equilibrium analysis of the U.S. economy. Section 5 presents and discusses our main findings from the numerical simulations. Section 6 concludes.

2. Theoretical framework and results

2.1. Basic setup

We consider a perfectly competitive and static economy comprising N sectors indexed $n \in \{1, 2, .., N\}$ and H households indexed $h \in \{1, 2, .., H\}$. We assume that the economy is closed (i.e., no international trade) and that there are no pre-existing taxes.[6] Production processes employ constant-returns-to-scale technologies. $M \leq N$ *dirty* sectors indexed by $i \in \{1, ..., M\}$ emit an uniformly dispersed pollutant (such as, for example, CO_2) as part of the production process. Z_i denotes emissions of sector i. Firms maximize profits at given product and factor prices.

Households derive utility from consumption and dis-utility from pollution. We abstract from a labor-leisure choice. The budget of household h, M_h, comprise factor income (F_h) and government transfers

differentiated environmental taxes mentioned above. We leave for future research the investigation of how these motives interact with social equity concerns.

[5]Outside of the optimal taxation framework, Mayeres and Proost (2001) study welfare-improving revenue-neutral marginal tax reforms for an economy with multiple households in the presence of an externality. While they highlight the importance of distributional considerations, their analysis focuses on marginal tax reforms in a setting with only one polluting good.

[6]As mentioned in the literature review above, there exist various motives for tax differentiation related to pre-existing fiscal distortions and international trade. To focus on the implications of social equity concerns for tax differentiation, we deliberately abstract from such effects. We leave for future research the investigation of how these motives for tax differentiation may interact with social equity concerns.

(T_h): $M_h = F_h + T_h$. Households are heterogeneous in two fundamental ways. First, heterogeneous preferences are captured by their respective indirect utility function

$$V_h(\mathbf{p}, M_h, Z) \tag{1}$$

where $\mathbf{p}$ is the vector of consumer prices and $Z := \sum_i Z_i$ is the total amount of pollution. Second, households differ in terms of the level and composition of income which they receive from inelastically supplying factors of production to firms and from government transfers T_h. Households maximize utility by choosing consumption of sectoral goods at given prices and budgets.

The government seeks to maximize social welfare which is given by the following Bergson-Samuelson welfare functional:

$$W = W(V_1(\mathbf{p}, M_1, Z), \ldots, V_h(\mathbf{p}, M_h, Z), \ldots, V_H(\mathbf{p}, M_H, Z)), \tag{2}$$

by choosing a public policy $\{\tau_i, T_h\}$, $\forall i \in \{1, ..., M\}$, $\forall h \in \{1, 2, .., H\}$ consisting of sector-specific pollution taxes and household-specific transfers. τ_i can in general be either input or output taxes chosen such that total emissions do not to exceed a maximum level $\tilde{Z}$. If $\tilde{Z}$ is high enough, the government policy endogenously determines the optimal level of pollution.[7] Household transfers have to be fully financed out of the revenue from pollution taxes, i.e. $\sum_h T_h \leq \sum_i \tau_i Z_i$.

We consider two alternative government problems. In the fully optimal case ($\{\tau_i, T_h\}$), the government chooses optimal taxes and transfers. A constrained-optimal case ($\{\tau_i, T_h := \xi_h \sum_i \tau_i Z_i\}$) considers the situation in which the government can choose optimal pollution taxes but is constrained by an exogenously given transfer scheme for redistributing the pollution tax revenue, represented by fixed redistribution shares $\{\xi_h\}$. Considering the case of a fixed transfer scheme is useful as it enables examining cases beyond the fully optimal case which may reflect the situation of real-world policy for which a given redistribution scheme is already in place (for example, per-capita tax rebates or recycling in proportion to income).[8]

2.1.1. Optimal policies

In the fully optimal case where pollution taxes and transfers can be chosen, the government solves the following problem:

$$\begin{aligned} max_{\{\tau_i \geq 0, T_h \geq 0\}} W \quad s.t. \quad & \sum_h T_h \leq \sum_i \tau_i Z_i \quad (\mu) \\ & \sum_i Z_i \leq \tilde{Z} \qquad (\epsilon) \ , \end{aligned} \tag{3}$$

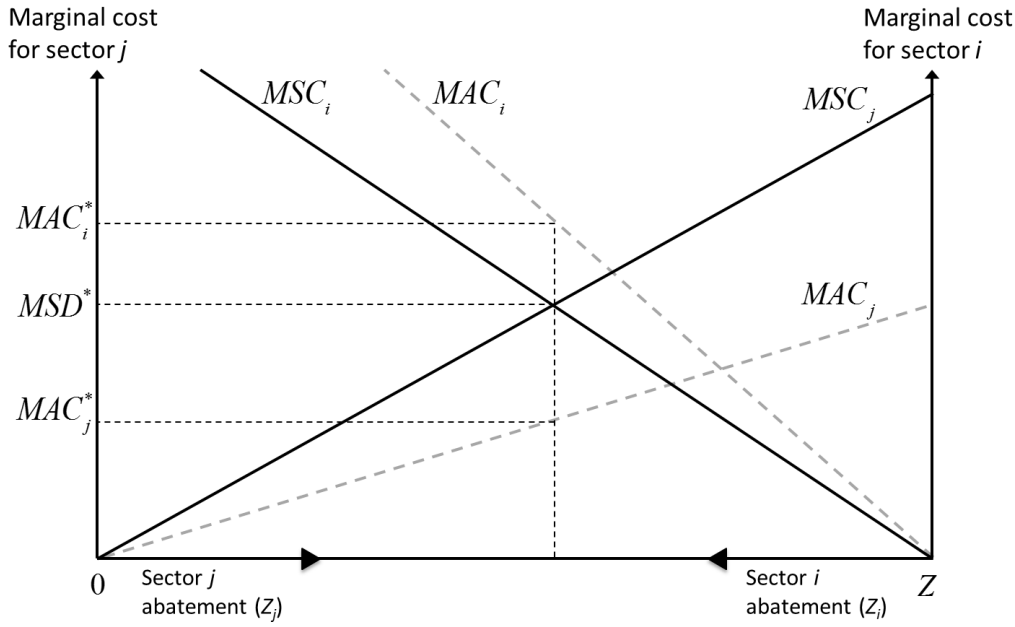
where $\mu \geq 0$ denotes the marginal value of public funds and $\epsilon \geq 0$ is the shadow value of the pollution constraint.

Assuming an interior optimum, the optimality conditions for sectoral pollution taxes are (see Ap-

[7] We introduce a pollution constraint $\tilde{Z}$ to be able to consider non-optimal levels of pollution. This is useful in models that do not explicitly account for dis-utility from pollution, such as the one used for our numerical analysis in Sections 3 and 5.

[8] In addition, optimal tax rates in the fully optimal case can be conveniently analyzed by looking at tax rates in the constrained-optimal case with the appropriate fixed redistribution shares. Denote by $\{\bar{\tau}_i, \bar{T}_h\}$ the unconstrained optimum, and $\{\tau_i^\star\}$ the constrained optimum (with associated transfers $T_h^\star := \xi^h \sum_i \tau_i^\star Z_i^\star$, where $Z_i^\star$ is the pollution emitted by sector i at the constrained optimum). If the fixed transfer scheme implements the optimal redistribution, i.e. $\xi_h = \bar{\xi}_h := \frac{\bar{T}_h}{\sum_{h'} \bar{T}_{h'}}$, then the constrained-optimum coincides with the fully optimal case $\tau_i^\star = \bar{\tau}_i$ and $T_h^\star = \bar{T}_h$. This holds true because, in such a constrained-optimal case, the government optimizes over a subset of taxes and transfers which contains the policy choices that are optimal over the whole set.

Figure 1: Sectoral pollution pricing with and without social equity concerns



Notes: MSC (*MAC*) denote the marginal social cost of abatement taking into account (ignoring) social equity concerns.

pendix A for derivations):[9]

$$MSD = \underbrace{-\underbrace{\frac{\sum_h \sum_n \beta_h X_{nh} \partial_{\tau_i} p_n}{\partial_{\tau_i} Z}}_{\text{=Preference effect}} + \underbrace{\frac{\sum_h \beta_h \partial_{\tau_i} F_h}{\partial_{\tau_i} Z}}_{\text{=Factor income effect}} + \underbrace{\mu \frac{\partial_{\tau_i} \sum_j \tau_j Z_j}{\partial_{\tau_i} Z}}_{\text{=Revenue redistribution effect}}}_{=:MSC_i} \quad \forall\, i \in \{1, \ldots, M\}, \tag{4}$$

where β_h denotes the *direct social marginal utility of income* accruing to household h (Mayeres and Proost, 2001) and is given by $\beta_h := \lambda_h \partial_{V_h} W$, with $\lambda_h := \partial_{M_h} V_h$ denoting the private marginal utility of income.

The *marginal social damage of pollution* is defined as: $MSD := \epsilon - \sum_h \partial_{V_h} W \partial_Z V_h$. Note that *MSD* increases in the shadow value of the pollution constraint (ϵ)[10] and that the terms $\partial_{V_h} W$ ($= \beta_h / \lambda_h$) express the social weighting of the tax-induced change in household h's utility. Also, note that for the case of a uniformly dispersed pollutant such as CO_2—that we use to guide our modeling—*MSD* is independent from the polluting source (sector). The right-hand side (RHS) of Equation (4) is the *marginal social cost of abatement induced by a pollution tax on sector i* (MSC_i). It is thus straightforward to see that the following standard result holds:

Proposition 1. *Optimal pollution taxes equalize the marginal social cost of abatement across sectors (i.e.,* $MSC_i = MSC_j$, $\forall i, j$*).*

Proof. From the conditions in Equation (4), since $MSD = MSC_i$ and $MSD = MSC_j$, $\forall i$ and $\forall j$, it follows that $MSC_i = MSC_j$. □

[9] We employ the short-hand notation $\partial_X \equiv \frac{\partial}{\partial X}$ for partial derivatives.

[10] If the pollution level at the optimum is below the cap or—equivalently—for the case of unconstrained pollution, the shadow value of the pollution constraint is zero; it is positive if the constraint is binding.

Proposition 1 can be viewed as a generalized version of the equi-marginal principle which takes into account the presence of social equity concerns for heterogenous types of households. Figure 1 graphically depicts this situation where at the optimum, sectoral pollution taxes are set such that the marginal social cost of abatement across sectors i and j are equalized (and correspond to the marginal social damage MSD^*).

While this general optimality principle is of course well-known, the central theme of this paper is to understand how social equity concerns modify the rules for optimal sectoral pollution pricing. Is uniform pollution pricing across sectors still optimal if the society is inequality averse? If non-uniform pricing is optimal, in what ways do sectoral pollution taxes have to be differentiated and what determines the magnitude of tax differentiation?

Intuitively, the answer depends on whether and how marginal social cost of abatement differ when social equity concerns are taken into account or left out. When social equity concerns are not considered, we simply refer to marginal social abatement cost as *marginal abatement cost* (*MAC*). As portrayed in Figure 1, if *MSC* and *MAC* differ, then it is likely no longer optimal to uniformly tax pollution in both sectors: equalizing marginal social costs across sectors ($MSC_i = MSC_j$ at MSD^*) then implies that $MAC_i^* \neq MAC_j^*$, thus requiring differentiated sectoral tax rates.

The *MSC* on the RHS of Equation (4) comprise three components that expound how household heterogeneity in preferences and endowments, together with social preferences for equity, determine various effects at play for optimal sectoral pollution taxes.[11]

First, a *Preference effect* captures the effect of consumption choices of households on marginal social cost which are driven by the preferences of heterogeneous households and the interaction with social preferences for equity.[12] The term $\sum_n X_{nh}\partial_{\tau_i} p_n$ captures the marginal impact of the tax in sector i on the consumption bundle of household h. If positive, it implies that a tax increase causes the consumption bundle to become more costly, in turn leading to a positive contribution to MSC_i. Weighting with β_h reflects the fact that the contribution of the *Preference effect* to marginal social cost in sector i is smaller for households who are associated with a lower direct social marginal utility of income.

Second, a *Factor income effect* captures how *MSC* are affected through pollution tax-induced changes in households' factor incomes. Intuitively, if factor income of household h decreases in response to a marginal increase in the pollution tax in sector i, then the marginal social costs are increased. Again, the abatement costs are lower if households which are more negatively impacted through this channel are associated with lower values of β_h.

Third, a *Revenue redistribution effect* reflects the social value of tax revenues raised. If the marginal effect of τ_i on total tax revenue ($\sum_i \tau_i Z_i$) is positive, then overall abatement cost through the tax are reduced. Intuitively, the magnitude of this channel depends on the marginal value of public funds μ.

To see how household heterogeneity and social equity concerns cause the *MSC* to differ from the *MAC*, consider first the case without social concerns, i.e., with equal social weights across households: $\beta_h = \beta, \forall h$. The *Preference effect* is then equal to $\beta \sum_n (X_n \partial_{\tau_i} p_n)/(\partial_{\tau_i} Z)$ and the *Factor income effect* is $\beta \partial_{\tau_i} F/(\partial_{\tau_i} Z)$.

With social equity, i.e. with $\beta_h \neq \beta_{h'}$ for some h, h', the corresponding expressions in general differ as:

$$\sum_h \beta_h X_{nh} \neq (\sum_h \beta_h)(\sum_h X_{nh}) \qquad \text{and} \qquad \sum_h \beta_h F_h \neq (\sum_h \beta_h)(\sum_h F_h)\,.$$

The inequalities above are caused by the interaction between social weights and household heterogene-

[11] Note that the marginal effect of tax increases on pollution is in general expected to be negative (i.e., $\partial_{\tau_i} Z < 0$). Divisions by $\partial_{\tau_i} Z$ normalizes abatement costs by the amount of pollution abated.

[12] It should be noted that the numerator in the first term can also be expressed as follows: $\sum_n \partial_{\tau_i} p_n (\sum_h \beta_h X_{nh})$. The term in brackets is proportional to the *distributional characteristic* of good j as in Mayeres and Proost (2001). Analogous quantities have proven to be of relevance in previous literature, such as, for example, Ahmad and Stern (1984) and Feldstein (1972). In cases where $\partial_{\tau_i} p_n = 1$ for $i = n$ and 0 for $i \neq n$, such as in a partial equilibrium analysis with linear taxes, the distributional characteristic of good i is the only term on the RHS of Equation (4) containing consumption characteristics of households.

ity in consumption and income. The RHS can be interpreted as a social weighting of the different components of the *MAC*. This weighting varies across sectors, as opposed to the case without social equity concerns. Due to the *Preference effect* a sector receives a relatively higher weight if households consuming its output are associated with higher direct social marginal utilities of income. Even if households have identical preferences but differ in terms of factor incomes, the marginal abatement cost in the absence social equity concerns is likely to differ across sectors due to the *Factor income effect*.

In addition, there is another effect related to redistribution of the tax revenues which can cause the marginal social cost to deviate from the marginal abatement cost beyond the interaction between β and heterogeneous household characteristics. The importance of the *Revenue redistribution effect* relative to the other effects on the RHS of Equation (4) depends on the marginal value of public funds μ which in turn depends on the social weighting. This motivates a closer look at the redistributive part of the optimal policy.

The conditions for optimally distribution tax revenues across households are given by (see Appendix A):

$$\underbrace{\mu \geq (1-D_h)^{-1}\Big(\underbrace{\beta_h}_{\substack{\textit{Direct}\\ \textit{effect}}} - \underbrace{\sum_{n,h'} \beta_{h'} X_{nh'} \partial_{T_h} p_n + \sum_{h'} \beta_{h'} \partial_{T_h} F_{h'}}_{\substack{\textit{Indirect consumption and}\\ \textit{factor income effects}}} - \underbrace{MSD\, \partial_{T_h} Z}_{\substack{\textit{Indirect pollution}\\ \textit{effect}}}\Big)}_{\textit{Marginal social benefit of redistributing revenue to household } h} \quad \forall\ h \in \{1, \ldots, H\}, \tag{5}$$

where $D_h = \sum_i \tau_i \partial_{T_h} Z_i$ stands for the increase in revenue caused by the increased spending following an increase in transfers to household h.[13] Equation (5) holds with equality if transfers to household h are positive. Conditions in Equation (5) simply state that the socially optimal redistribution of tax revenues across households is achieved when (1) for each household the marginal social benefit of transfers (RHS) are less or equal to the marginal value of public funds (LHS) and (2) the marginal social benefit of transfers are equalized across all households who receive non-zero transfers.

The first term in parentheses represents a direct effect: the benefit of redistributing tax revenue to household h increases for higher values of the household's direct social marginal utility of income β_h. The remaining terms in Equation (5) are indirect effects which affect all households in the economy. The second and third terms indicate the marginal effect of redistributing tax revenue to household h on prices and returns to factors, and their impact on household consumption patterns and factor incomes, respectively. The fourth term captures the marginal effect on the pollution level, and the resulting damages. Division by the term $1 - D_h$ indicates that, as the household h spends the extra revenue it receives, this will affect tax revenue, which (in the case of increased revenues, i.e., positive D_h) increase the social value of redistribution.

From the conditions in Equation (5), we can determine the pattern of optimal transfers as follows:

Proposition 2. *Assume identical homothetic preferences across households. Then, if household h has a direct social marginal utility of income which is lower than the maximum value (i.e., $\beta_h < max_{\{h'\}}\beta_{h'}$), it receives zero transfers at the optimum (i.e., $T_h = 0$).*

Proof. Since with identical homothetic preferences increasing the income through transfers has the same effect on prices, factor incomes and pollution levels for all households, Equation (5) assume the following form: $\mu \geq \frac{\beta_h + A}{B}$ with $B > 0$. If $\beta_h < max_{\{h'\}}\beta_{h'}$, then $\mu \geq \frac{max_{\{h'\}}\beta_{h'} + A}{B} > \frac{\beta_h + A}{B}$. Since $\mu > \frac{\beta_h + A}{B}$, it follows that $T_h = 0$. □

Proposition 2 implies that if households have identical consumption characteristics, the pollution tax revenues are optimally redistributed to the households with the highest social marginal utility of income.

[13] Note that $D_h < 1$, since the additional tax revenue given to household h results in a less than proportional increase in tax revenues.

As an example, if the direct social marginal utility of income is a decreasing function of income—as in the case of an inequality-averse government—then all the tax income is redistributed to the poorer households. If the tax revenue is sufficient to raise the income of the poorest household to the same level as the second-poorest, then both households have the same direct social marginal utility of income, and both hence receive positive transfers. It also implies that the richest household only receives non-zero transfers if the tax revenue is sufficient to equalize the income of all households.

To the extent that differentiated pollution taxes increase tax revenues and there exist social equity concerns, Proposition 2 suggest that it may be optimal to use non-uniform sectoral polluting pricing to increase the revenues to increase transfers to households with the highest social weighting—thus trading off abatement efficiency in production and social welfare improvements through targeted transfers. Thus, even if households have identical preferences and factor incomes, it may be optimal to differentiate sectoral pollution taxes based on considerations about the optimal redistribution of tax revenues when social weights are differentiated.

2.2. *Optimal sectoral pollution taxes with non-optimal revenue redistribution*

In real-world policy making, the government may not be able to implement optimal transfers, i.e. it may likely be constrained by a given revenue redistribution scheme that is already in place. We thus now consider a situation in which the government problem involves choosing taxes for a fixed and non-optimal revenue redistribution scheme. Moreover, examining such cases enables us to isolate the effect of heterogeneity in private preferences and its interactions with social preferences on optimally differentiated pollution taxes, by considering a specific redistribution scheme that mutes the revenue redistribution effect.

Let $\{\xi_h\}_{h=1}^{H}$ describe a fixed and revenue-neutral transfer scheme which redistributes shares of the total pollution tax revenue to households according to $T_h = \xi_h \sum_i \tau_i Z_i$ with $\sum_h \xi_h = 1$. Given ξ_h, the government chooses sectoral pollution tax rates τ_i to solve the following problem:

$$max_{\{\tau_i \geq 0\}} W \quad s.t. \quad \sum_i Z_i \leq \tilde{Z} \quad (\epsilon) \ . \tag{6}$$

Assuming an interior solution, one can derive similar optimality conditions for sectoral pollution taxes as in equation (3) with μ being replaced by $\sum_h \beta_h \xi_h$ (see Appendix B for the derivations).

Deviations of ξ_h from the optimal transfer scheme, as determined by Proposition 2, affect the relative importance of the revenue redistribution channel for differentiation pollution tax rates. As is borne out by the following proposition, the revenue distribution motive is muted if transfers are proportional to income:

Proposition 3. *Assume that (1) the relative composition of factor income does not vary across households (i.e., $F_{hf} = \phi_h F_f$, where F_{hf} is household h's ownership of factor f, ϕ_h is a fixed share, and F_f is the total factor income of factor f), (2) household preferences are identical and homothetic, and (3) pollution tax revenues are redistributed in proportion to household income (i.e., $\xi_h = M_h / \sum_{h'} M_{h'}$). Then, optimal pollution taxes are uniform across sectors.*

Proof. For equal and homothetic preferences, $MSD = \frac{\sum_h \beta_h \xi_h (\partial_{\tau_i} M - M \sum_n \alpha_n \partial_{\tau_i} p_n)}{\partial_{\tau_i} Z}$ with $\alpha_n := X_{nh}/M_h = X_{nh'}/M_{h'}$ and $M = \sum_h M_h$. The RHS of the above equation is identical to the case of a single household economy with direct social marginal utility of income given by the weighted average of the values for the identical households (i.e., $\beta \equiv \sum_h \beta_h \xi_h$). □

Note that the result of optimal uniform pollution taxes in Proposition (3) even holds when allowing for the presence of social equity concerns. Assumptions (1) and (2) shut off the *Preference effect* and *Factor income effect* in Equation (3). Proposition 3 then essentially states that the *Revenue redistribution effect* is also muted for income-proportional transfers. Intuitively, for income-proportional redistribution the share of each household's factor income in the economy's total is equal to the household's share of

transfers in the total. A given household's income will therefore only be increased if the total amount of pollution tax revenues increases more than the loss in total factor income. The income-proportional redistribution scheme thus invests each household in the trade-off between increased tax revenues and decreased aggregate factor incomes. The result indicates that the increase in tax revenues achieved by differentiating pollution taxes is outweighed by the loss in total factor incomes.

Other redistribution schemes, including the optimal one, are likely to result in differentiated pollution taxes at the optimum due to the revenue redistribution motive. This is because other schemes expose different households unequally to the tax revenue/factor income tradeoff discussed above. For example, a per-capita transfer scheme allows low-income households to benefit more from the increased tax revenues compared to income-proportional redistribution. This creates a social incentive to differentiate taxes to raise revenues, in the measure in which low-income households are given higher social weights compared to high-income households. Importantly, these insights emphasize again the interaction between optimal polluting pricing and the redistributive side of the optimal policy, while illustrating that both aspects of the policy have to be considered at the same time.

2.3. When are differentiated pollution taxes better?

While the results above have allowed us to identify the channels that drive differentiated pollution taxes in the presence of social equity concerns, we now examine in more depth the conditions under which uniform taxes are not optimal and how taxes should be differentiated to improve welfare. To derive results, we need to impose more specific assumptions on the structure of the economy.

We assume two sectors labeled X and Y–both polluting (i.e., $N = M = 2$)–and one factor of production (labor), which is supplied inelastically and mobile across sectors. Labor is subject to a resource constraint: $L_X + L_Y = \bar{L}$. There are two households (i.e., $H = 2$) labeled by A and B. Households have separable utility in pollution and Cobb-Douglas utility in consumption. We model pollution as an input to production, i.e. production functions are $X = X(L_X, Z_X)$ and $Y = Y(L_Y, Z_Y)$ where L_X, L_Y, Z_X and Z_Y are the quantities of labor and pollution used in each sector. The government returns the pollution tax revenues in a lump-sum manner to households according to a given and fixed redistribution scheme (ξ_h).

To analyze when differentiated pollution taxes are better, we consider pollution-neutral tax swaps starting from an initial situation of uniform pollution taxes ($\tau_X = \tau_Y \equiv \tau$). From Proposition 1, we know that if sectoral pollution tax rates are not optimal, then the marginal social cost of abatement differ across sectors. Consider a pollution-neutral tax swap from i to j (i.e., $d\tau_j > 0$, $d\tau_i < 0$ and $dZ = 0$). The following then holds (see Appendix C.1 for derivations):

$$dW = -\partial_{\tau_j} Z(MSC_i^{\xi} - MSC_j^{\xi})d\tau_j\,, \tag{7}$$

where MSC_i^{ξ} denotes the marginal social cost of abatement in sector i given a redistribution scheme ξ. Assuming $\partial_{\tau_j} Z < 0$, it therefore follows that the tax swap is welfare-improving if $MSC_j^{\xi} < MSC_i^{\xi}$ and it is welfare-reducing when the opposite holds. If $MSC_j^{\xi} < MSC_i^{\xi}$, the tax on j should therefore be raised and the tax on i lowered.

To proceed, we need to derive expressions for MSC_i^{ξ} which take into account the equilibrium responses of the economy when marginally changing pollution tax rates. We perform comparative static analysis adopting the standard approach in the literature for analytically solving general equilibrium models by linearization (Harberger, 1962; Fullerton and Heutel, 2007). We then use the local properties of the equilibrium to evaluate Equation (7). Appendix C shows that the condition $MSC_X^{\xi} < MSC_Y^{\xi}$ for

a welfare-improving tax swap from Y to X is equivalent to:[14]

$$\Delta\Bigg\{\Bigg(\frac{\sigma}{\tau L_Y(\delta_X-\delta_Y)}+\frac{\sigma-1}{M}+\sum_h\frac{1-\alpha_h}{1-\gamma}\frac{\xi_h}{M}\Bigg)\underbrace{\sum_h\beta_h M_h\Big(\frac{\alpha_h}{\gamma}-1\Big)}_{\substack{\textit{Social weighting of household}\\ \textit{consumption patterns}}}$$
$$+(1-\sigma)\underbrace{\sum_h\beta_h\Big(\xi_h-\frac{M_h}{M}\Big)}_{\substack{\textit{Social weighting of}\\ \textit{redistribution scheme}}}\Bigg\}<0\,, \tag{8}$$

where $\delta_i := \frac{Z_i}{L_i}$ is the pollution intensity of sector i, total income $M := \sum_h M_h$, σ is the elasticity of substitution in production,[15] $\alpha_h := \frac{p_X X_h}{M_h}$ is the expenditure share of household h on good X, $\gamma := \frac{p_X X}{M}$ is the economy's aggregate expenditure share on good X. $\Delta := \tau L_Y(\delta_X-\delta_Y)[1+\tau L_Y(\delta_X-\delta_Y)\sum_h\frac{1-\alpha_h}{1-\gamma}\frac{\xi_h}{M}]^{-1}$.

The LHS of Equation (8) represents the difference in sectoral *MSC*; it implies that the larger the difference, the larger is the social welfare improvement from the pollution-neutral tax swap. One can now see that the previously identified effects related to social weighting, differences in households' preferences, and redistribution of tax revenues determine whether and to what extent the tax swap moving toward differentiated taxes is welfare-improving.[16]

First, the larger the heterogeneity in households' expenditure shares (i.e., the larger $(\frac{\alpha_h}{\gamma}-1)$), the larger is the difference in sectoral *MSC*. In contrast, if households spend their income in same proportions on different goods (i.e., $\frac{\alpha_h}{\gamma}-1=0$), then the motive for tax differentiation to the *Preference effect* is absent.

Second, the more the revenue redistribution scheme deviates from the income-proportional scheme (i.e., the larger $(\xi_h-\frac{M_h}{M})$), the larger is the difference in sectoral *MSC* and hence the larger is the *Revenue redistribution motive* for tax differentiation. In contrast, if $\xi_h-\frac{M_h}{M}=0$, then the redistribution motive is absent (as implied by Proposition 3).

Third, the effects described in the previous two points are only present if social weighting differs across households. For a given redistribution and household expenditure pattern, the magnitude of the two effects is increasing in the difference of the social weights. If social weighting is equal across households ($\beta_h=\beta$, $\forall h$), then it is easy to see that the LHS of Equation (8) is zero, implying that differentiating taxes does not improve welfare.

Fourth, the relative importance of the *Preference effect* and the *Revenue redistribution effect* is determined by the interaction with production side characteristics (captured by the substitutability σ and differences in pollution intensity across sectors $\delta_X-\delta_Y$) as well as interactions between the two channels (captured by $\sum_h(1-\alpha_h)\xi_h$).

How should sectoral pollution taxes be differentiated to enhance welfare? In answering this question, it is useful to decompose the two effects at play. By assuming income-proportional redistribution, we can first consider the case in which only the *Preference effect* is present. The following proposition clarifies how taxes should be differentiated for given social weights and household consumption patterns:

Proposition 4. *Given initially uniform pollution taxes and income-proportional redistribution (i.e., $\xi_h=\frac{M_h}{M}$), a pollution-neutral tax swap from sector Y to sector X (i.e., $d\tau_Y<0$, $d\tau_X>0$, and $dZ=0$) is*

[14] Assuming $\partial_{\tau_X}Z<0$ and $\partial_{\tau_Y}Z<0$, i.e. pollution is reduced by raising pollution taxes.

[15] For reasons of tractability, we assume here that at the equilibrium point σ is uniform across sectors. This does not rule out that sectors can differ in terms of the factor intensities.

[16] Note that assuming a single factor of production in the economy implies that factor income of different households is affected in the same proportion. The motive for tax differentiation due to affecting factor income differently across households is thus not present in the setup considered in Section 2.3.

welfare-improving if and only if the household with the higher expenditure share on X has a lower direct social marginal utility of income (i.e. $\alpha_h > \alpha_{h'}$ and $\beta_h < \beta_{h'}$).

Proof. See Appendix C.3. □

Proposition 4 implies that if households have different tastes and there exist social equity concerns, then uniform sectoral pollution taxes are not optimal. Taxes should be differentiated according to households' consumption characteristics, in order to shift the burden of taxation towards households with lower direct social marginal utility of income.[17] More specifically, it suggests that the tax rate on a sector whose output is consumed more intensively by households with a higher social weight should be lower compared to other sectors.

To analyze how pollution taxes should be differentiated due to the *Revenue redistribution effect*, we assume that consumption patterns are identical across households and that revenue redistribution is done according to the optimal scheme (following Proposition 2). The following proposition then holds:

Proposition 5. *Assume initially uniform pollution taxes, identical household preferences, unequal social weighting (i.e., $\beta_A \neq \beta_B$), and pollution tax revenues redistributed to the household with the higher β. Then, a pollution-neutral tax swap (i.e., $dZ = 0$) is welfare-improving if and only if it increases the pollution tax revenue $T = \tau_X Z_X + \tau_Y Z_Y$.*

Proof. See Appendix C.4. □

Proposition 5 simply states that for the *Revenue redistribution effect* to drive a deviation from uniform pollution pricing, the tax differentiation has to be such that the pollution tax revenues are increased. If a tax swap reduces tax revenues, it cannot be welfare-improving because less revenues are available to distribute to households with high social weights.

3. Quantitative framework for empirical analysis

To quantitatively assess the scope for optimal pollution pricing and redistribution in an empirical setting, we complement our theoretical analysis with numerical simulations. We develop a numerical framework that casts the problem of optimal sectoral pollution pricing with social equity concerns in the context of a numerical general equilibrium (GE) model that embodies firms' and households' behavioral equilibrium responses to emissions taxes as well as satisfying cross-market and aggregate economy restrictions. Importantly, the numerical model extends the theoretical example presented previously in Section 2.3 by incorporating multiple households, multiple factors of production, multiple polluting sectors including energy-sector detail, intermediate inputs, and CO_2 emissions derived from burning multiple fossil fuels in production and consumption. We calibrate our model to the case of the U.S. economy.

This section (1) describes the general structure of the numerical framework detailing our computational strategy, (2) provides an overview of the numerical GE model, and (3) describes data and calibration.

[17] The result from Proposition 4 that goods which are consumed more intensively by households that contribute less to social welfare should be subject to higher tax rates is analogous to the findings of Diamond and Mirrlees (1971) in the context of optimal commodity taxation.

3.1. General structure and computational strategy

The numerical model maximizes a social welfare function by choosing non-negative carbon taxes for each sector i, τ_i, and non-negative transfers to each household h, T_h, subject to three sets of constraints:

$$\begin{aligned} \max\ & W(V_1(\mathbf{p}, M_1), ..., V_H(\mathbf{p}, M_H)) \\ & \mathbf{p}, \mathbf{x} \in \mathcal{A} \\ & \tilde{Z} \geq \sum_i Z_i \\ & \sum_h T_h = \sum_i \tau_i Z_i \\ & T_h, \tau_i \geq 0\,. \end{aligned} \tag{9}$$

First, $\mathcal{A}$ represents the set of the feasible equilibrium allocations consisting of prices $\mathbf{p}$ and quantities $\mathbf{x}$ derived from the numerical GE component of the model. Second, economy-wide CO_2 emissions as given by the sum of sectoral emissions cannot exceed an exogenously given and fixed target $\tilde{Z}$. Third, the total value of lump-sum transfers to households ($\sum_h T_h$) is equal to the revenues from sectoral carbon taxes ($\sum_i \tau_i Z_i$).

The problem in Equation (9) represents a *Mathematical Program under Equilibrium Constraints* (MPEC), i.e. a bi-level optimization problem which maximizes an objective function subject to a lower-level constraint set that contains an equilibrium problem (Luo et al., 1996). We cast the general equilibrium problem in the lower-level part as a mixed complementarity problem (MCP) (see, for example, Mathiesen, 1985; Rutherford, 1995) solving for primal and dual variables (i.e., quantities and prices). The advantage of this approach is that it naturally accommodates equilibria with corner solutions; here, optimal zero sectoral carbon taxes and household transfers. We solve the MPEC in Equation (9) using the NLPEC solver in the General Algebraic Modeling System (GAMS). The remainder of this section describes in more detail the structure and specification of the general problem laid out in Equation (9).

3.2. Social welfare function

Following Cremer et al. (2003), we consider the case of an inequality-averse government by evaluating policies in light of the following iso-elastic social welfare function:

$$W[V_1(\mathbf{p}, M_1), \ldots, V_H(\mathbf{p}, M_H)] \equiv \frac{1}{1-\eta} \sum_h \pi^h (V_h(\mathbf{p}, \frac{M_h}{\pi^h}))^{1-\eta} \quad \eta \neq 1 \quad \text{and} \quad 0 \leq \eta < \infty\,, \tag{10}$$

where V^h is the indirect utility function of households of type h. Note that we exclude environmental damages or benefits entering households' utility or social welfare. π^h is the population share of household h. η is the "inequality aversion index". The value of η reflects the desired degree of redistribution in the economy: higher values of η mean that the society cares more about equality.

With the social welfare function in Equation (10) and given homothetic household preferences (i.e., $V_h(\mathbf{p}, M_h) = M_h V_h(\mathbf{p}, 1)$), the direct marginal social utility of income for type h is given by:

$$\beta^h = \frac{\pi^h}{M_h^\eta} \left(\frac{V_h(\mathbf{p}, 1)}{\pi^h} \right)^{1-\eta}. \tag{11}$$

For $\eta > 0$, households with lower incomes are associated with a higher direct marginal social utility of income.[18] For $\eta = 0$, the social welfare function is utilitarian with $W = \sum_h M^h V_h(p, 1)$. For this case, if tastes are equal across households (i.e., $V_h(p, 1) = V_{h'}(p, 1)$, $\forall h, h'$), then social weights are uniform: $\beta_h = \beta$, $\forall h$.

[18] Note that $\eta \to \infty$ represents the case of a Rawlsian social welfare function.

3.3. Feasible allocations $\mathcal{A}$ based on general equilibrium model

The set of feasible allocations $\mathcal{A}$ is defined by the equilibrium conditions of the numerical GE model for the U.S. economy. We formulate the GE model as a system of nonlinear inequalities and characterize the economic equilibrium by two classes of conditions: zero-profit and market-clearing. Zero-profit and market-clearing conditions exhibit complementarity with respect to quantities $\mathbf{x}$ and prices $\mathbf{p}$, respectively. We now describe the structure and decisions problems of economic agents (firms and households) to derive the conditions that define $\mathcal{A}$.[19]

We consider a closed and static economy with perfectly competitive output and factor markets. Production of final output in each sector $i \in I$ is characterized by a three-stage process.

At the first stage, final output Y_i is produced using the following constant-returns-to-scale production technology that combines intermediate inputs from other sectors j, M_{ji}, together with a sector-specific composite, V_i:

$$Y_i = \left[\theta_i^Y (\min\{\theta_{1i}^M M_{1i}, \ldots, \theta_{ji}^M M_j i, \ldots, \theta_{Ii}^M M_{Ii}\})^{\frac{\sigma_i - 1}{\sigma_i}} + (1 - \theta_i^Y) V_i^{\frac{\sigma_i - 1}{\sigma_i}}\right]^{\frac{\sigma_i}{\sigma_i - 1}}, \tag{12}$$

where θ_i^Y and θ_{ji}^M are share parameters and $\sigma_i > 0$ denotes the elasticity of input substitution.

At the second stage, the composite input V_i is produced using inputs of capital (K), labor (L), and an sector-specific aggregate of energy inputs (E_i) according to a nested constant-elasticity-of-substitution (CES) function:

$$V_i = \left[\theta_i^V (\theta_i^K K^{\lambda_i} + (1 - \theta_i^K) L^{\lambda_i})^{\frac{(\kappa_i - 1)\lambda_i}{\kappa_i (1 - \lambda_i)}} + (1 - \theta_i^V) E_i^{\frac{\kappa_i - 1}{\kappa_i}}\right]^{\frac{\kappa_i}{\kappa_i - 1}}, \tag{13}$$

where θ_i^K and θ_i^V are share parameters and κ_i and λ_i denote respective elasticity of input substitution parameters. Labor and capital inputs are assumed to be perfectly mobile across sectors.

At the third stage of production in sector i, primary energy inputs R_{ei}, with fossil fuel input $e \in \{Coal, Natural\ gas, Crude\ oil, Refined\ oil\}$ and electricity, B_i, are combined according to the following nested CES function:

$$E_i = \left[\theta_i^E B_i^{\frac{\nu_i - 1}{\nu_i}} + (1 - \theta_i^E)(\sum_e \theta_{ie}^R R_{ie}^{\frac{\mu_i - 1}{\mu_i}})^{\frac{(\nu_i - 1)\mu_i}{\nu_i (\mu_i - 1)}}\right]^{\frac{\nu_i}{\nu_i - 1}}, \tag{14}$$

where θ_i^E and θ_{ei}^R are share parameters and μ_i and ν_i denote respective elasticity of input substitution parameters. Electricity and primary energy inputs are treated in separate nests to distinguish differences in substitution possibilities.

Carbon emissions are modelled as an input into production and are directly associated with using the amount R_{ei} of fossil fuel e in the production of sector i.[20] Given fuel-specific carbon coefficients ϕ_e, the carbon emissions (pollution) caused by burning fuel e in sector i are thus given by

$$Z_i \equiv \sum_e \phi_e R_{ei}\,. \tag{15}$$

Taxing carbon at the rate τ_i would thus increase the cost of using R_{ei} units of fossil fuel by $\tau_i Z_i = \tau_i \sum_e \phi_e R_{ie}$. As energy inputs become more costly following a sectoral carbon price, firms can substitute away by adjusting the input mix at each of the three stages the production process.

[19] Appendix D contains a more detailed exposition of the equilibrium conditions for the numerical general equilibrium model, including the definition of the model parameters employed below.

[20] Our model thus abstracts from process-based carbon emissions. While it would be straightforward to expand the model in this direction, it would not qualitatively affect the insights we derive from our numerical analysis.

Firms producing sectoral outputs maximize profits under perfect competition:

$$\max_{\{M_1 \ldots, M_I, V_i\}} p_i^Y Y_i - p_i^V V_i - \sum_j p_j^M M_{ji} \tag{16}$$

subject to Equations (12)–(14) and taking prices of Y_i, V_i, and M_{ij}, denoted by p_i^Y, p_i^V, and p_i^M, respectively, as given. Optimal cost-minimizing behavior of firms can be summarized by the unit cost function for sector i, denoted by $c_i(\mathbf{p})$.

Households of type $h \in H$ maximize utility from consuming sectoral outputs C_{ih}:[21]

$$\max_{\{C_{1h} \ldots, C_{Ih}\}} U_h = \left[\sum_i \theta_{ih}^U C_{ih}^{\frac{\rho_h - 1}{\rho_h}} \right]^{\frac{\rho_h}{\rho_h - 1}}, \tag{17}$$

where θ_{ih}^U and ρ_h denote share and elasticity of substitution parameters, respectively, subject to a budget constraint:

$$M_h = T_h + r\,\omega_h^K + w\,\omega_h^L, \tag{18}$$

stating that income is derived from transfer income T_h and from inelastically supplying endowments of capital (ω_h^K) and labor (ω_h^L) to firms at respective market prices r and w. Optimal utility-maximizing behavior for households of type h can be summarized by the unit expenditure function $c_h(\mathbf{p})$ which is related to the indirect utility function according to $U_h c_h(\mathbf{p}) \equiv V_h^{-1}(\mathbf{p}, M_h)$. Let p_h^U denote the associated price index for utility.

In equilibrium, the zero-profit conditions for sectoral production and aggregation of consumption goods in household utility determine the equilibrium quantities $\mathbf{q}(\tau_j)$:[22]

$$c_i(\mathbf{p}(\tau_j)) \geq p_i^Y \quad \perp \quad Y_i \geq 0 \;\; \forall i \tag{19}$$

$$c_h(\mathbf{p}(\tau_j)) \geq p_h^U \quad \perp \quad U_h \geq 0 \;\; \forall h\,. \tag{20}$$

The equilibrium formulation is completed by adding market-clearing conditions which determine prices $\mathbf{p}$.[23] The factor markets for capital and labor services, respectively, are in equilibrium if:

$$\sum_h \omega_h^L \geq \sum_i \frac{\partial c_i(\mathbf{p}(\tau_j))}{\partial w} Y_i \quad \perp \quad w \geq 0 \tag{21}$$

$$\sum_h \omega_h^K \geq \sum_i \frac{\partial c_i(\mathbf{p}(\tau_j))}{\partial r} Y_i \quad \perp \quad r \geq 0\,. \tag{22}$$

The sum of intermediate input demands for sectoral output in production and consumption demands by households cannot exceed supply of sectoral output:

$$Y_i \geq \sum_j \frac{\partial c_j(\mathbf{p}(\tau_j))}{\partial p_{ij}^{YE}} Y_i + \sum_h \frac{\partial c_h(\mathbf{p}(\tau_j))}{\partial p_i^Y} U_h \quad \perp \quad p_i^Y \geq 0 \;\; \forall i\,, \tag{23}$$

where p_{ij}^{YE} denotes the carbon tax inclusive price for commodity i use for production in sector j. Finally,

[21] For simplicity, we abstract here from the nesting structure of household utility from consumption, which is expounded in Appendix D.

[22] We use the perpendicular sign $\perp$ to denote the complementarity relation between a function F: $\mathbb{R}^n \longrightarrow \mathbb{R}^n$ and a variable $z \in \mathbb{R}^n$ such that $F(z) \geq 0$, $z \geq 0$, and $z^T F(z) = 0$: $F(z) \geq 0 \perp z \geq 0$.

[23] Applying the envelope theorem, we can derive the demand for a particular commodity used in production (consumption) based on the partial derivative of the unit cost (expenditure) function with respect to the input price.

the market for utility is in equilibrium if:

$$U_h \geq \frac{M_h}{p_h^U} \quad \perp \quad p_h^U \geq 0 \;\; \forall h\,. \tag{24}$$

In summary, conditions (18) to (24) jointly define the set of feasible equilibrium allocations $\mathcal{A}$.[24]

3.4. Data and calibration

This section details the data sources and procedure used to calibrate the multi-sector multi-household model to data for the U.S. economy. First, we briefly explain how production and consumption technologies and the input-output structure of the economy are calibrated based on social accounting matrix data and external estimates about elasticity parameters. Second, we describe the specification of household behavior as well as the benchmark patterns of expenditures and incomes for the heterogeneous households. Third, we discuss our choice of the social inequality aversion parameter η.

3.4.1. Matching social accounting matrix data and choice of substitution elasticities

The calibration of the numerical model follows the standard procedure in applied general equilibrium modeling (see, for example, Rutherford, 1995; Harrison et al., 1997; Böhringer et al., 2016) according to which production and consumption technologies are calibrated to replicate a single-period reference equilibrium consistent with the Social Accounting Matrix (SAM) data for a given year. We use SAM data from the most recent version (version 9) of the database from the Global Trade Analysis Project (GTAP, Aguiar et al., 2016) describing the U.S. economy in the year 2009. Importantly, these data contain detailed information on carbon emissions as well as physical energy flows differentiated by primary and secondary energy carrier.

The ten goods categories shown in Table 1 are an aggregation of the 57 commodities in the GTAP database.[25] The aggregation is guided by the idea to keep sufficient detail with respect to the supply of energy (electricity as well as four primary energy sectors including *Coal*, *Natural gas*, *Crude oil*, *Refined oil products*) and the use of energy in the production of energy-intensive goods and services (such as *Energy-intensive goods* and *Transportation*) as well as other major sectors (*Manufacturing products*, *Services*, and *Agricultural products*). To facilitate calibrating the model as a closed economy without pre-existing tax distortions, we have removed international trade, taxes and transfers, and government spending from the GTAP dataset. Factors of earnings in our dataset comprise capital and labor.

The SAM data allows us to determine the composition of the benchmark economy in terms of sectoral outputs, aggregate consumption, intermediate input demands by sector, labor and capital earnings by sector, and CO_2 emissions (intensity) by sector. Table D.9 in Appendix D provides a list of the parameters. Using these data as anchor points, we then employ estimates from the literature on elasticity of substitution parameters to determine the global properties of the CES functions characterizing sectoral production technologies. Table D.8 in Appendix D lists the parameter values for our cental case.

3.4.2. Parametrization of household heterogeneity

Data on household expenditures (by goods category) and income sources is based on the U.S. Consumer Expenditure Survey (CES) from the U.S. Bureau of Labor Statistics. We divide households into expenditure deciles (i.e., $\pi_h = 0.1$, $\forall h$) and use the data from Rausch et al. (2011) who use the same categories of households and goods.

[24] Note that for reasons of brevity, we have omitted here the equilibrium conditions for a number of price and quantity variables associated with explicitly including the lower levels of sectoral production and household consumption in the equilibrium formulation.

[25] The exact aggregation schemes for sectors and regions and the aggregated benchmark data is available on request from the authors.

Table 1: Household characteristics in benchmark data by expenditure deciles

	Expenditure deciles									
	1	2	3	4	5	6	7	8	9	10
Share of expenditures (in % of economy-wide expenditures)										
	4.1	6.0	7.4	8.3	8.9	9.1	10.7	12.2	13.5	19.8
Expenditure shares by good category (in % of total expenditures for decile)										
Agricultural products	15.2	12.8	12.0	10.7	10.0	8.4	7.8	6.9	4.6	2.9
Coal	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Crude oil	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Energy-intensive goods	0.7	1.2	1.2	1.2	1.4	1.7	2.2	2.9	4.5	5.9
Electricity	1.8	1.7	1.5	1.5	1.4	1.4	1.3	1.2	1.0	0.8
Natural gas	0.2	0.2	0.2	0.2	0.1	0.2	0.1	0.1	0.1	0.1
Manufacturing products	40.5	34.3	32.1	28.4	26.6	22.3	20.8	18.3	12.3	7.8
Refined oil products	0.9	1.0	1.0	1.1	1.1	1.2	1.2	1.2	1.3	1.2
Services	39.5	47.6	50.7	55.5	58.0	63.4	65.1	67.9	74.5	79.7
Transportation	1.1	1.2	1.3	1.4	1.3	1.5	1.5	1.5	1.6	1.5
Income by primary factor (in % of decile income)										
Capital	28.3	27.0	26.6	25.3	25.5	23.4	24.1	26.4	33.2	47.6
Labor	71.7	73.0	73.4	74.7	74.5	76.6	75.9	73.6	66.8	52.4
CO_2 emissions embodied in consumption (as % share of total emissions)[a]										
	4.8	6.7	8.0	8.9	9.3	9.3	10.7	12.0	12.8	17.7

Notes: Data is rounded to one decimal place, i.e. expenditure shares for crude oil and coal are very small but non-zero. [a]To calculate embodied emissions, we adopt the approach described in Böhringer et al. (2016) to recursively solve an input-output version of our numerical general equilibrium model using a diagonalization algorithm.

In order to represent heterogeneous households within the general equilibrium model, household-level and aggregate data must be consistent. For this, we balance household data to match aggregate consumption and income values as follows. First, we calibrate household consumption by multiplying aggregate values by each deciles' expenditure shares in total expenditure on each good. Second, we calibrate household factor incomes by multiplying aggregate income of each factor by decile income shares (i.e., capital and labor) in total income of that factor. Third, differences between calibrated household expenditures and incomes are reconciled by adding income of each factor in proportion the factors' share in aggregate income, thus delivering a very close approximation of the decile's capital-to-labor ratio from the household-level data.

Table 1 shows the household data used in the model. Several dimensions of household heterogeneity are particularly noteworthy here. First, households exhibit a wide variation in income, with the top decile earning more than five times more than the bottom decile. Importantly, the observed disparity of incomes results in differing marginal social utilities of income across households, i.e. β_h in Equation (11). Second, expenditure shares vary mostly monotonically across deciles. For *Agricultural products*, *Electricity*, *Natural gas*, and *Manufacturing products* expenditure shares are higher for lower expenditure deciles whereas for the goods categories *Energy-intensive products*, *Refined oil products*, *Services*, and *Transportation* expenditure shares decline increase with income. For all deciles, *Coal* and *Crude oil* represent negligible shares of household expenditure. Third, the composition of household income in terms of the capital-labor split also varies across deciles, with capital income representing the highest share of income for the two top deciles. Fourth, although expenditure shares for *Electricity* and *Natural gas* are slightly larger for lower deciles as compared to higher deciles, the share of embodied emissions in consumption largely increases with income being more than three times larger when comparing the top to the bottom decile.

Table 2: Optimal sectoral carbon prices (US$/ton CO_2), tax revenues, and change in inequality for different social inequality aversion

	Social inequality aversion		
	$\eta = 0$	$\eta = 0.1$	$\eta = 1.9$
Summary statistics for carbon taxes			
Mean	32.4	41.5	365.7
Standard deviation	0.2	21.3	937.2
Sectoral carbon taxes			
Agricultural products	32.3	27.9	3.9
Coal	32.2	20.9	0.0
Crude oil	32.3	25.2	0.0
Energy-intensive goods	32.3	32.1	0.0
Electricity	32.3	30.0	4.8
Natural gas gas	32.2	23.0	0.0
Manufacturing goods products	32.3	27.4	0.0
Refined oil products	33.3	119.1	690.5
Services	32.3	27.9	0.0
Transportation	32.6	57.2	2904.1
Pollution tax revenues (in billion US$)	107.8	137.9	1214.2
Change in Gini coefficient (in % relative to uniform taxes)[a]	0.0	-1.5	-43.3

Notes: [a]Gini coefficients are based on real household income. The change in the Gini coefficient is computed relative to a policy which restricts pollution taxes to be uniform but allows for optimal transfers.

3.4.3. Choice of social inequality aversion parameter η

Pinning down the inequality aversion parameter η in the iso-elastic social welfare function (10) is fraught with difficulties. Based on estimates for η for the context of France by Cremer et al. (2003), we thus carry out our numerical analysis for low and high values, i.e. $\eta = 0.1$ and $\eta = 1.9$. While higher values for η may be conceivable, we will see that $\eta = 1.9$ already yields strong results in terms of optimally differentiated sectoral carbon taxes as well as optimal redistribution.

4. Simulation results

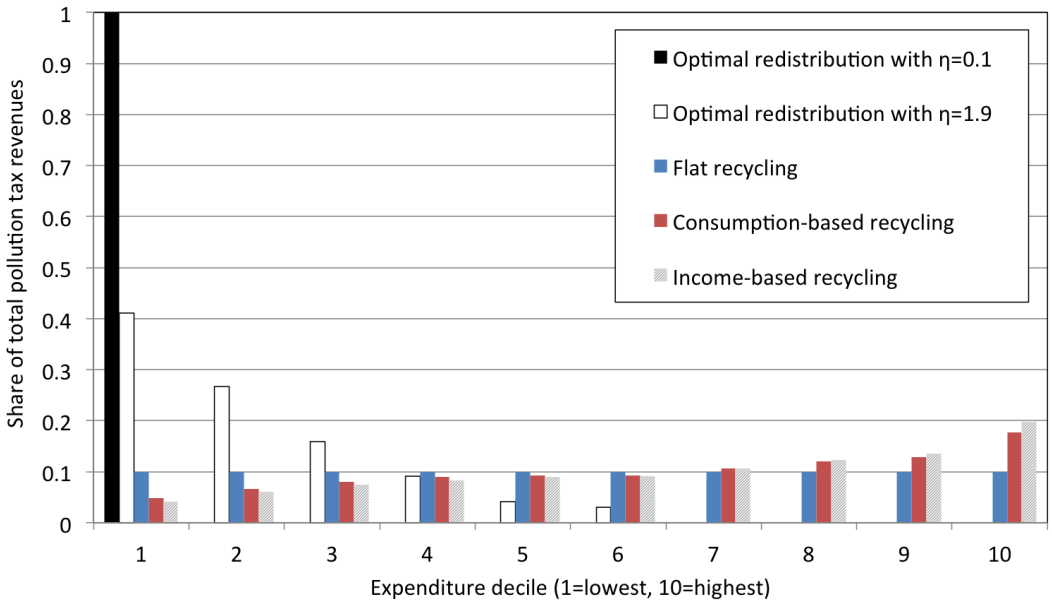
This section presents our results based on numerical simulations using the quantitative model to explore optimal sectoral pollution taxes and redistribution schemes. We start by looking at the optimal case and decompose the magnitude of different channels driving tax differentiation previously identified in the theoretical analysis. We then investigate optimal tax policies for given redistribution schemes and compare incidence impacts across household expenditure groups under uniform and differentiated pollution pricing. We assume throughout this section that pollution is reduced by 20% relative to the benchmark level.

4.1. Optimal policies

Table 2 shows optimal sectoral carbon prices for different degrees of social inequality aversion. Several insights emerge. First, the degree of the social inequality aversion determines the amount of tax differentiation. For $\eta = 0$ sectoral carbon prices are almost identical.[26] The higher η, the more unequal social weights become and the larger is the differentiation of sectoral carbon taxes. It is evident that already a relatively low social inequality aversion ($\eta = 0.1$) brings about a significant deviation from

[26]This case is close to equal social weighting but β_h is not identical across households as there exist differences in households' tastes. See the definition of β_h in Equation (11).

Figure 2: Transfers of pollution tax revenues for optimal and non-optimal redistribution policies



uniform pollution pricing, i.e. the standard deviation of sectoral carbon taxes is 21.3 and the minimum and maximum tax rates are 20.9 and 119.1, respectively.

Second, as social equity concerns become more important (i.e., with increasing η), raising a higher amount of pollution tax revenues is optimal because it allows to a larger extent to implement targeted transfers to households with a higher social weight. Figure 2 shows the transfers (as a share of total revenue) to households by expenditure decile. With social equity concerns, optimal transfers imply higher shares of transfers received by low income households that have a relatively higher β. As the pollution tax revenues increase, sufficient revenue is available to address equity concerns of multiple household groups beyond the lowest expenditure decile. Given that the pollution level is capped, raising large revenues is achieved through generally higher carbon prices, as illustrated by the mean carbon tax.

Third, for high social inequality aversion ($\eta = 1.9$) the optimal pollution tax for some sectors is zero while other sectors face sizable tax rates. The reasons that drive the specific pattern of sectoral pollution taxes are discussed below when decomposing the different motives for tax differentiation. While the very high pollution taxes rates for some sectors reflect that social equity concerns are a powerful driver of tax differentiation, less extreme tax differentiation would be observed when optimal pollution taxes are set for a given, non-optimal redistribution schemes which may reflect constraints on implementing fully optimal transfers in real-world policy settings.

Fourth, the inequality of real household incomes, as measured by the Gini coefficient, decreases under optimally differentiated pollution taxes as compared to uniform pollution pricing. Not surprisingly, this decrease is the larger, the larger is the degree of social inequality aversion. Importantly, differentiated pollution pricing achieves a more equitable (i.e., socially optimal) outcome that cannot be attained under uniform pollution pricing and optimal transfers.

4.2. Decomposing the importance of different motives for tax differentiation

We now examine how much of the differentiation of sectoral pollution taxes as shown in Table 2 is driven by each of the three channels (*Preference effect*, *Revenue redistribution effect*, *Factor income effect*) previously identified. For decomposing these effects, we build on our theoretical results derived in Section 2. More specifically, we use Proposition 3 which states three conditions under which uniform pollution pricing is optimal. Assuming that one of these conditions is not met enables us to isolate the impact through this channel on non-uniform pollution pricing.

Table 3: Optimal sectoral carbon prices (US$/ton CO_2) for different social inequality aversion due to *Preference effect*

	Social inequality aversion		
	$\eta = 0$	$\eta = 0.1$	$\eta = 1.9$
Summary statistics for carbon taxes			
Mean	32.5	32.6	34.5
Standard deviation	0.0	0.4	5.2
Sectoral carbon taxes			
Agricultural products	32.5	31.1	29.4
Coal	32.5	32.9	32.6
Crude oil	32.5	33.0	33.5
Energy-intensive goods	32.5	33.1	39.2
Electricity	32.5	32.4	30.1
Natural gas gas	32.5	32.6	33.9
Manufacturing goods products	32.5	32.1	30.6
Refined oil products	32.5	33.0	34.1
Services	32.5	33.3	42.0
Transportation	32.5	32.9	41.3

4.2.1. Preference effect

To isolate the magnitude of tax differentiation due to the *Preference effect*, we assume income-proportional redistribution (i.e., condition (3) in Proposition 3) and identical composition of factor income across households (i.e., condition (1) in Proposition 3).

Table 3 shows optimal sectoral pollution taxes due to the *Preference effect* for different degrees of social inequality aversion. Several insights emerge. First, the goods consumed intensively by poorer households—who receive a larger social weight β by an inequality-averse government—are taxed at a lower rate compared to goods for which the opposite holds. Looking at the expenditure shares by good category shows that *Services*, *Transportation*, and *Energy-intensive goods* are among the goods that are consumed more intensively by higher expenditure deciles whereas low-income households spend a large share of their income on *Agricultural products*, *Manufacturing products*, and *Electricity* (see Table 1). The sectoral differentiation of tax rates due to the *Preference effect* mostly reflects this pattern of expenditure shares. The intuition from Proposition 4, derived in the context of our analytical example, thus carries over to the more general setting of our numerical framework. In addition, our general equilibrium framework with multiple sectors also picks up the intermediate input-output structure of the economy. Thus, there exists some differentiation between *Coal* and *Crude oil* although there are virtually not consumed directly by households.

Second, while there clearly is a differentiation of sectoral tax rates due to the *Preference effect*, the standard deviation is considerably smaller as compared to the case which includes all channels. This suggests that the other motives are important drivers as well.

Third, the fact that the mean carbon price only increases slightly with a higher social inequality aversion reflects the absence of the revenue redistribution motive.

4.2.2. Revenue redistribution effect

To isolate the *Revenue redistribution effect*, we assume identical household preferences (i.e., condition (2) in Proposition 3) and identical composition of factor income across households (i.e., condition (1) in Proposition 3).

Table 4 shows optimal sectoral pollution taxes due to the *Revenue redistribution effect* for different degrees of social inequality aversion. First, we see that the *Revenue redistribution effect* gives rise to a substantial differentiation of sectoral pollution taxes. As under the *Revenue redistribution effect* the goal is to increase the pollution tax revenues, sectors with relatively steep marginal abatement cost are taxed heavily as for the given cap this drives up carbon prices. We find that CO_2 emissions in *Transportation*

Table 4: Optimal sectoral carbon prices (US$/ton CO_2) and pollution tax revenue for different social inequality aversion due to the *Revenue redistribution effect*

	Social inequality aversion		
	$\eta = 0$	$\eta = 0.1$	$\eta = 1.9$
Summary statistics for carbon taxes			
Mean	32.5	41.0	377.9
Standard deviation	0.0	16.5	1043.9
Sectoral carbon taxes			
Agricultural products	32.5	28.8	0.0
Coal	32.5	20.3	0.0
Crude oil	32.5	24.8	0.0
Energy-intensive goods	32.5	31.0	0.0
Electricity	32.5	29.8	0.6
Natural gas gas	32.5	22.7	0.0
Manufacturing goods products	32.5	27.2	0.0
Refined oil products	32.5	118.9	13.6
Services	32.5	26.7	0.0
Transportation	32.5	56.0	3267.7
Pollution tax revenues raised (billion US$)	107.9	136.2	1254.8

and *Refined oil products* are taxed at relatively high rates. This is driven by the fact that substitution between energy and non-energy inputs is relatively low in the transportation sector and that demand for *Transportation* and *Refined oil products* is relatively inelastic. Thus, to achieve a given emissions reductions through abatement in these sectors can only be achieved with relatively high pollution taxes.

Second, the higher the social inequality aversion, the more pronounced is the differentiation of taxes. For $\eta = 1.9$, the pollution tax revenues are almost entirely raised by taxing *Transportation* and *Refined oil products* only. Here, efficiency in abatement is largely sacrificed in favor of raising high revenues for targeted transfers to enable addressing social inequality concerns. $\eta = 0.1$ represents an intermediate case in which the revenue-raising motive is already present, as is reflected by above-average pollution taxes on *Transportation* and *Refined oil products*. This, however, has to be traded-off against productive efficiency in abatement where the latter is relatively more important given the lower degree of social inequality aversion. Hence, pollution tax rates for other sectors are also substantial for $\eta = 0.1$ and are close to the those in the case with uniform pollution pricing.

Third, when comparing the case in which all three channels are present (Table 2) with the case in which only the *Revenue redistribution effect* is active (Table 4), it is evident that a similar pattern and standard deviation of sectoral pollution taxes emerges. This indicates that the differentiation of sectoral pollution taxes is to a large extent driven by the *Revenue redistribution effect*.

4.2.3. Factor income effect

To isolate the *Factor income effect*, we assume identical household preferences (i.e., condition (2) in Proposition 3) and redistribution in proportion to benchmark income (i.e., an approximation of condition (3) in Proposition 3).[27]

We find that tax differentiation due to the *Factor income effect* is relatively weak. The standard deviation of sectoral pollution taxes for $\eta = 0.1$ and $\eta = 1.9$ is 0.5 and 4.7, respectively. While standard deviations are comparable in magnitude to those under the *Preference effect*, it is important to note that these numbers also pick up parts of the *Revenue redistribution effect* which cannot be cleanly removed.

[27]Unlike for the *Preference effect* and the *Revenue redistribution effect*, the decomposition of the *Factor income effect* is imperfect because in Proposition 3 relaxing the assumption (1) of identical composition of factor income whilst at the same time assuming a fixed and income-proportional redistribution scheme is not possible. Thus, the effect we derive is conflated with the *Revenue redistribution effect*.

Table 5: Optimal differentiation of sectoral pollution taxes for alternative non-optimal and fixed redistribution schemes and different degree of social inequality aversion

	Social inequality aversion	
	$\eta = 0.1$	$\eta = 1.9$
Flat recycling		
Mean pollution tax (US$/ton CO_2)	34.7	132.9
Standard deviation of sectoral pollution taxes	5.0	232.3
Pollution tax revenues raised (billion US$)	115.1	441.4
Consumption-based recycling		
Mean pollution tax (US$/ton CO_2)	33.1	41.7
Standard deviation of sectoral pollution taxes	1.4	21.5
Pollution tax revenues raised (billion US$)	110.1	138.4
Income-based recycling		
Mean pollution tax (US$/ton CO_2)	32.8	36.9
Standard deviation of sectoral pollution taxes	0.7	10.6
Pollution tax revenues raised (billion US$)	109.0	122.6

That indeed the *Revenue redistribution effect* is driving tax differentiation here, can be seen by the fact that *Transportation* and *Refined oil products* are taxed at relatively high rates, in turn largely determining the standard deviation. The fact that the *Factor income effect* is relatively weak is unsurprising given that the composition of factor incomes across household groups in Table 1 is relatively similar.[28]

4.3. Pollution tax differentiation for non-optimal and fixed redistribution schemes

Besides analyzing government policies which comprise optimal pollution taxes *and* optimal transfers, the more relevant situation for real-world environmental policy may be one in which a specific redistribution scheme is already in place or favored over the extreme pattern of transfers under the optimal policy. We therefore now investigate the extent of optimal tax differentiation under the constraint of a given fixed and sub-optimal redistribution scheme. Following Bento et al. (2009), we consider three alternative revenue recycling schemes: "*Flat recycling*" returns revenues in equal amounts to every household; "*Income-based recycling*" returns revenues in proportion to benchmark income; "*Consumption-based recycling*"[29] returns revenues according to each household's share of aggregate dirty consumption in the benchmark.[30]

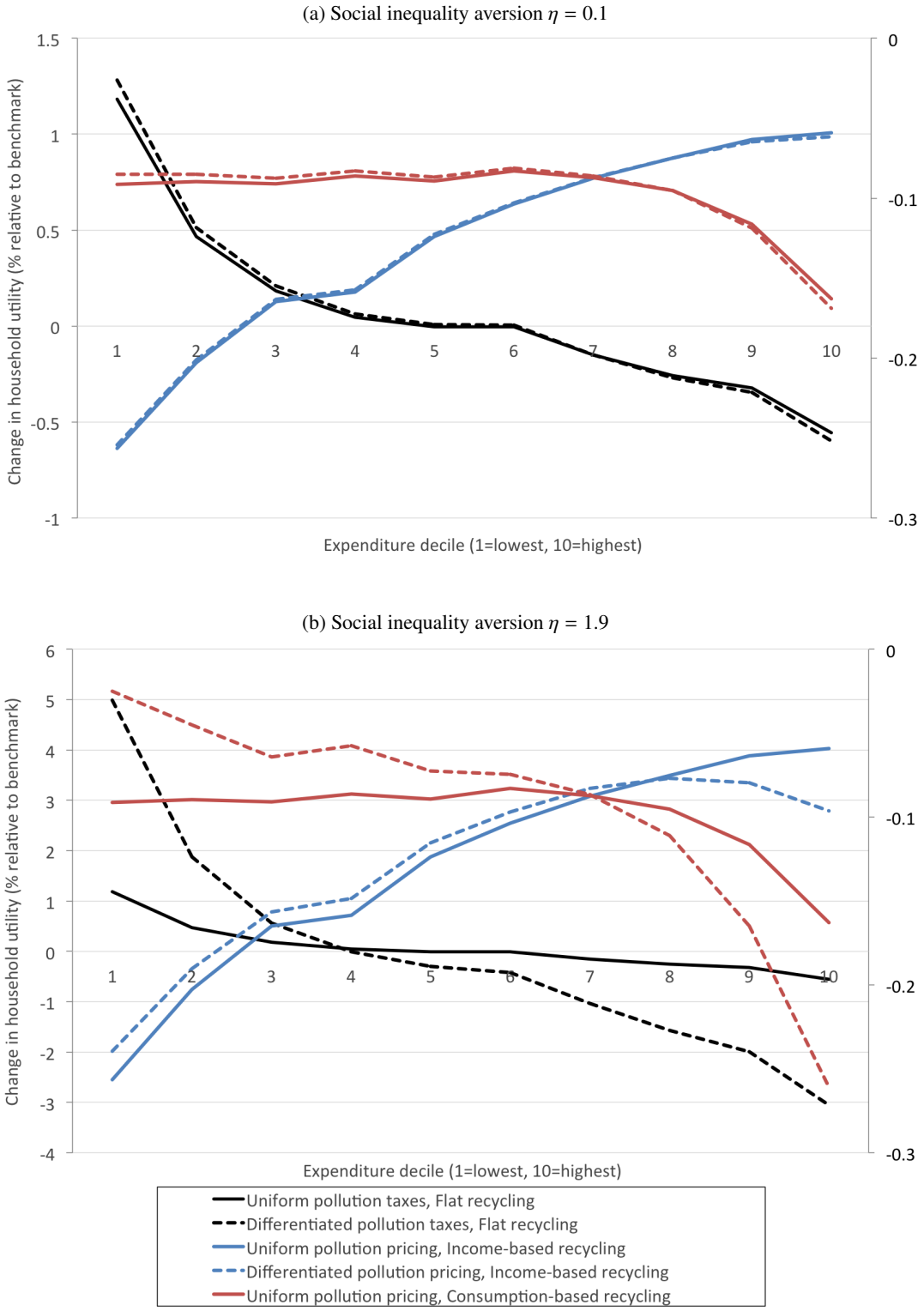
Table 5 reports the optimal differentiation of sectoral pollution taxes for these three redistribution schemes and different degrees of social inequality aversion. First, the differentiation of sectoral pollution taxes is the largest for the *Flat recycling* and the lowest for *Income-based recycling*. The reason is that the transfers associated with *Flat recycling* are the closest to the optimal transfers among the three schemes considered here—as can be seen from Figure 2. Being closer to the optimal transfers means that the *Revenue redistribution effect* is stronger, hence implying a relatively larger tax differentiation to increase tax revenues; this is reflected by the fact that tax revenues as well as the mean carbon tax rate are largest under *Flat recycling*. The relatively low tax differentiation under *Income-based recycling* is due to the fact that this redistribution scheme is closest to a scheme that shuts off the *Revenue redistribution effect*. *Consumption-based recycling* represents an intermediate case reflecting the fact that the consumption of lower expenditure deciles is more carbon-intensive. Hence, as is evident from Figure 2, the distribution

[28]Including government transfers that are unrelated to pollution tax rebates in the analysis may introduce additional heterogeneity on the sources of income side. This, and in general, more heterogeneity with respect to the composition of factor income may thus increase the scope for tax differentiation through this channel.

[29]The "*Consumption-based recycling*" is analogous to the "VMT-based" recycling in Bento et al. (2009).

[30]We define dirty consumption by using a comprehensive measure of embodied CO_2 emissions that captures direct and indirect emissions in household consumption (see Table 1).

Figure 3: Incidence by expenditure decile for non-optimal redistribution schemes: *Flat recycling* (primary axis), *Income-based recycling* and *Consumption-based recycling* (secondary axis)



of transfers across expenditure deciles deviates from the one under the *Income-based recycling* in the direction of *Flat recycling*.

Second, the difference in tax differentiation between the alternative redistribution schemes are magnified with increasing social inequality aversion (i.e., comparing $\eta = 1.9$ to $\eta = 0.1$). As social equity concerns become more important, the increasing difference in the standard deviation between the three schemes reflects their differences in terms of the relative importance of the *Revenue recycling effect*.

Third, despite our finding that these policy-relevant redistribution schemes strongly diminish the *Revenue recycling effect*—which we identified as a main driver for differentiation of sectoral pollution tax in the optimal policy—optimal pollution taxes may still be strongly differentiated (i.e., standard deviation as high as 232.3). This is due to the fact that the *Revenue redistribution effect* can still play an important role if a given redistribution scheme ensures a sufficiently high share of transfers to low-income households (who receive relatively high social weights under an inequality-averse government).

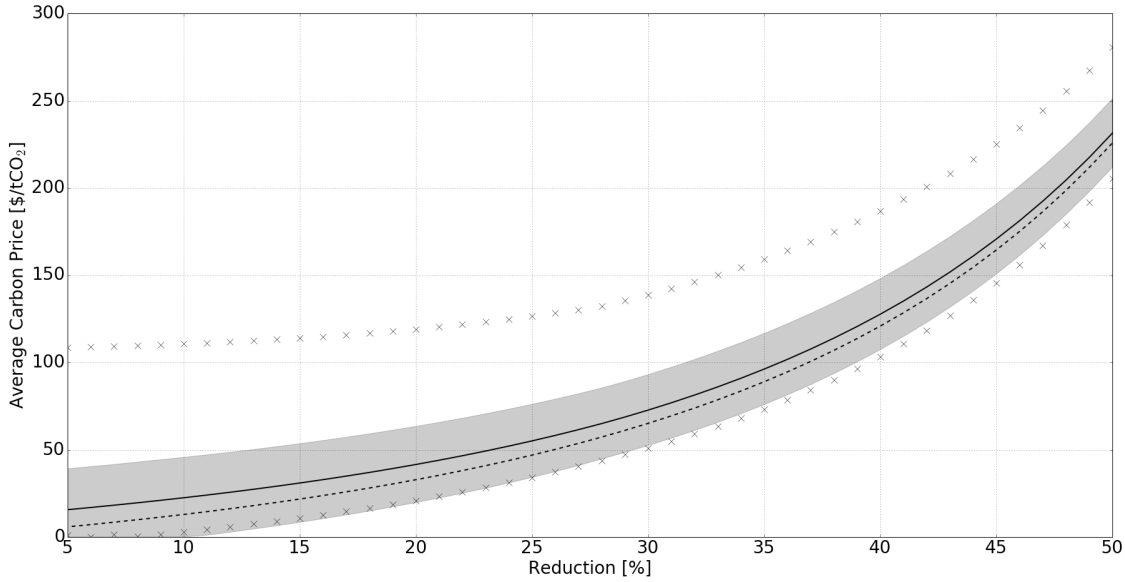
4.4. Incidence impacts of optimally differentiated taxes with non-optimal revenue redistribution

An important and policy-relevant question is to examine how households are affected by implementing pollution control policies that are based on optimally differentiated taxes. For this purpose, we compare the household-level welfare impacts under uniform or differentiated pollution pricing to a no-policy benchmark without pollution taxes. To stay in the space of policies that bear some realism, we investigate this question by continuing to focus on the three recycling schemes considered above.

Figure 3 shows the incidence across expenditure deciles for uniform and differentiated pollution taxes for the alternative revenue recycling schemes; Panel (a) and (b) considers low and high values for social inequality aversion, respectively. First, as documented by the large literature on the distributional impacts of carbon taxation, we find that the way the revenues are recycled can importantly alter the incidence pattern across households (see, for example, Bento et al., 2009; Rausch et al., 2010; Mathur and Morris, 2014; Williams III et al., 2015). In line with the previous literature, we find that for uniform pollution pricing *Flat recycling* yields sharply progressive outcomes. *Income-based recycling* is regressive as poorer households spend a larger fraction of their income on pollution goods while the revenue rebates in proportion to (benchmark) income have a neutral effect. *Consumption-based recycling* represents an intermediate case which we find here to be neutral to somewhat progressive (for higher incomes). Note that these results, obtained under uniform pollution pricing, are not affected by social equity concerns (as the transfer scheme is exogenously fixed and tax differentiation is ruled out).

Based on Figure 3, the following insights emerge when comparing the incidence under differentiated versus uniform sectoral pollution taxes. First, for all three transfer schemes, optimally differentiated pollution taxes lead to an incidence pattern which is more progressive (less regressive) relative to uniform pollution taxes. Allowing for optimally differentiated pollution taxes brings the policy closer to the fully optimal policy with optimal pollution taxes *and* transfers. Deviating from uniform pollution pricing thus allows the government to reduce inequality through the three channels analyzed above (*Preference effect*, *Factor income effect*, and *Revenue redistribution effect*). Second, not surprisingly, the impact on the incidence pattern increases with higher social inequality aversion. Third, how much inequality can be reduced with differentiated pollution taxes depends on the given transfer scheme. In general, transfer schemes which are closer to the optimal one—thus better capturing the *Revenue recycling effect*—are better suited to address inequality thus leading to larger differences in the incidence pattern. Thus, allowing for differentiated pollution taxes in combination with *Flat recycling* has the largest effect on incidence (in line with our findings in Section 4.3). For the same reason, the incidence patterns are affected the least under *Income-based recycling*. Fourth, both under *Consumption-based* and *Income-based* recycling the qualitative pattern of the incidence is altered for parts of the income spectrum. With relatively high inequality aversion (i.e., $\eta = 1.9$), the incidence profile under *Consumption-based recycling* for expenditure deciles 1 to 7 changes from neutral to progressive while under *Income-based recycling* the incidence across the top three expenditure deciles changes from regressive to progressive.

Figure 4: Summary statistics for sectoral carbon prices under uniform and optimally differentiated taxes for alternative environmental targets



Notes: The dotted line shows the carbon tax under uniform emissions pricing to achieve a given reduction target. The solid line shows the emissions-weighted mean of optimally differentiated sectoral carbon taxes and the shaded area characterizes the corresponding emissions-weighted standard deviation; the crosses show the minimum and maximum carbon taxes under optimal tax differentiation, respectively.

5. Robustness checks

How robust is our finding that carbon taxes should be differentiated across sectors in light of social equity concerns, in particular if social inequality aversion is low (i.e., $\eta = 0.1$)? To answer this question, we vary our assumptions along two important dimensions that could potentially affect our results.

First, we consider different environmental targets. While the results presented in Section assume that emissions are reduced by 20%, the degree of carbon tax differentiation in the social optimum may reduce with higher reduction targets. The reason is a simple Laffer curve-type of argument for raising tax revenue: given high reduction targets, it may be optimal to avoid too high carbon taxes for sectors with high MAC as the additional abatement, and hence increases in tax revenues, become very small; rather, to effectively increase revenues it may be optimal to increase carbon taxes on sectors with relatively low MAC, thus resulting in less strongly differentiated carbon taxes across sectors. Second, the parameters governing the equilibrium responses of firms and households in response to carbon pricing behavior are highly uncertain. We use Monte-Carlo analysis to systematically assess how optimal carbon tax differentiation depends on parametric uncertainty.

5.1. Stringency of the environmental target

Figure 4 reports summary statistics (mean, standard deviation, min/max) for sectoral carbon prices under uniform and optimally differentiated taxes for alternative environmental targets ranging from 5% to 50% emissions reduction relative to the benchmark. As expected, both the mean and uniform carbon tax increase with higher reduction targets. Also, as has been shown before, the mean of differentiated carbon taxes always exceeds the level of the carbon tax under uniform pricing as a result of the the revenue-raising motive (i.e., for a given level of pollution, a higher mean carbon tax translates into a higher amount of tax revenues).

Two main insights emerge from Figure 4. First, even for more stringent environmental targets, the size of the optimal tax differentiation is still substantial. For example, for a reduction target as high as 50%, the standard deviation is still about 40 \$/ton CO_2. Second, the degree of tax differentiation in the

Table 6: Sensitivity of average sectoral carbon tax ($/ton CO_2) with respect to uncertainty in firms' and households' equilibrium responses

	Set of sampled elasticity of substitution parameters					
	A: Households		B: Firms		C: Combined	
	Carbon tax policy					
	Optimal	Uniform	Optimal	Uniform	Optimal	Uniform
Mean	38.9 (0.5)	30.5 (0.4)	31.9 (11.3)	25.1 (8.3)	30.5 (10.5)	23.6 (7.2)
Standard deviation	19.7 (0.4)	0 (0)	18.4 (39.7)	0 (0)	17.7 (39.0)	0 (0)

Notes: Numbers in parentheses denote standard deviations of average values across samples. Results shown above assume a 20% emissions reduction target and a low social inequality aversion of $\eta = 0.1$.

optimum, however, decreases somewhat with higher targets—as reflected by a lower standard deviation and a smaller difference between the minimum and maximum sectoral carbon taxes. The explanation is as follows. As marginal abatement costs in each sector become steeper with increasingly stringent targets, further increasing carbon taxes in sectors with relatively high taxes yields only relatively small additional carbon abatement. This means that taxes on sectors with relatively low tax rates also need to be raised. With more stringent targets, the increase in the maximum tax rate is smaller than for the minimum tax rates, in turn implying a lower standard deviation.

5.2. Parametric uncertainty governing equilibrium responses by firms and households

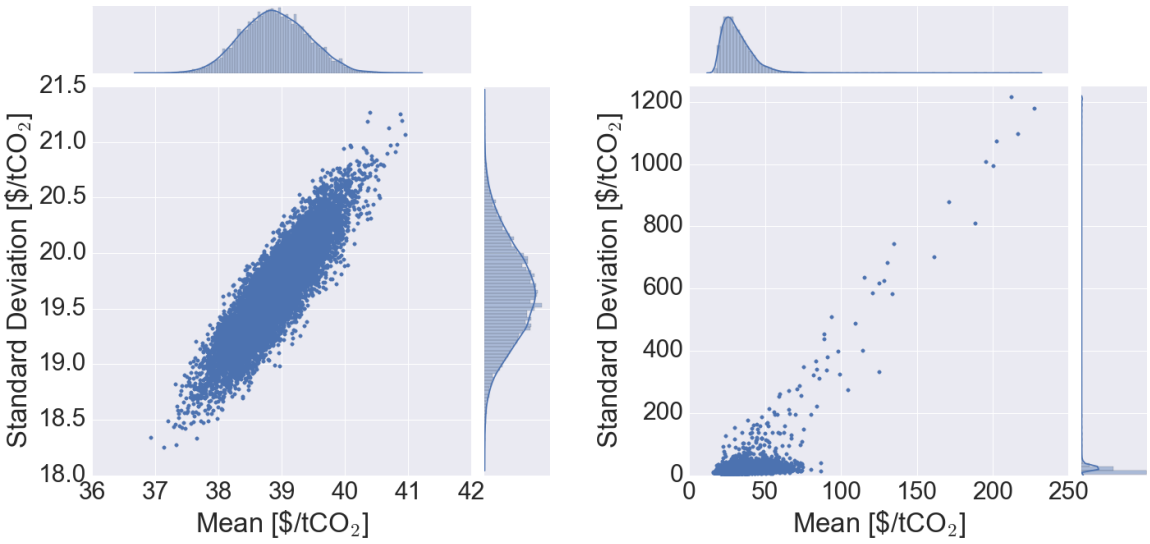
To systematically assess the sensitivity of carbon tax differentiation with respect to firms' and households' equilibrium responses to carbon pricing, we perform Monte-Carlo analysis on the key behavioral parameters in the model, i.e. all substitution elasticities at various levels of sectoral production activities as well as in the nested household utility functions (see Table D.8 in Appendix D). Given the lack of empirical estimates that would characterize the uncertainty for these parameters, we assume that elasticity of substitution parameters are independently and uniformly distributed with support $[0.5c, 1.5c]$ around the respective central case value c. We draw a sample of 10'0000 sets of elasticity parameters. To decompose the influence of parametric uncertainty for households' and firms' behavior, we conduct Monte-Carlo simulations for three samples: sample A varies only substitution parameters in households' utility functions; sample B varies only substitution parameters in production functions; sample C varies substitution parameters for both households and firms.

Importantly, we find that in none of the 30'000 sampled sets of elasticity parameters uniform carbon pricing is optimal. In fact, the smallest standard deviation of sectoral carbon taxes in our sample is 2.7. This underscores that our main finding from the central case simulations that optimal carbon taxes should be differentiated across sectors is highly robust with respect to varying key behavioral parameters.

To further describe the scope for optimal carbon tax differentiation, Table 6 reports, for each of the three samples, the mean and standard deviation of the average of the sectoral carbon tax rates under optimally differentiated and uniform carbon pricing; the standard deviations for both sampled moments are reported in parentheses. Several insights emerge. First, our central case estimates shown in Table 2 in Section 5, with a mean and standard deviation for optimally differentiated sectoral carbon taxes of 41.5 and 21.3, respectively, are quite close to the results shown in Table 6. Our systematic sensitivity analysis thus supports the main conclusion that the degree of the optimal carbon tax differentiation is substantial: for the combined sample C, the standard deviation is 17.7 for an average value of sectoral carbon taxes of 30.5.

Second, increasing the tax revenue has been shown to be a main motive for carbon tax differentiation. The fact that the mean of differentiated carbon taxes is higher than the uniform carbon price in all three samples indicates that this finding is robust. Third, the standard deviations for both sampled moments are substantially larger for sample B as compared to sample A. This suggests that optimal carbon tax differentiation is more sensitive with respect to varying substitution parameters in production as compared to varying the behavioral response of households. As a results, the variation in the combined

Figure 5: Distribution of mean and standard deviation of average of sectoral carbon taxes under optimal emissions pricing for samples A and B



Sample A: Households

Sample B: Firms

sample C is largely driven by the uncertainty with respect to substitution parameters in production. Fourth, when varying production characteristics, the standard deviation varies more than the mean. This indicates that, depending on the specific set of elasticities, the corresponding set of optimal carbon taxes is rather sensitive, while the underlying tax revenue target is comparatively stable.

Lastly, Figure 5 plots the sample distribution of the mean and the standard deviation of the average of sectoral carbon taxes under optimal emissions pricing; it also shows a scatter plot of both summary statistics. Two important observations can be made. First, it is evident that the standard deviation and mean are positively correlated implying that an increase in the mean is therefore associated with an increase the standard deviation. Second, varying substitution parameters in production causes a non-negligible number of outliers to emerge for which optimal carbon taxes are characterized by extremely high means and standard deviations. This indicates that, within a reasonable uncertainty range around our central case parameter values, it is possible to obtain very steep sectoral marginal abatement cost curves, thus enabling extremely high carbon taxes in certain sectors to raise large amounts of tax revenues for redistributive purposes. This once more underscores the importance of the revenue-raising motive for optimal carbon tax differentiation when social equity concerns are present.

6. Concluding remarks

This paper has examined optimal pollution tax differentiation across industries and revenue redistribution in light of social equity concerns using theoretical and numerical general equilibrium analyses. We have illustrated how household heterogeneity in preference and endowments interacts with social equity concerns, thus causing marginal abatement cost in the absence of social equity concerns not to be equalized at the social optimum, thus motivating pollution price differentiation. Relative to a case with uniform pollution pricing, tax rates should be differentiated to increase social welfare by shifting the burden of environmental policy towards households with low social weights, and by increasing the amount of pollution tax revenue for targeted transfers to households with high social weights. In the context of the U.S. economy and focused on price-based CO_2 emissions control, we found that optimal carbon taxes for an inequality-averse government differ strongly across industries, even when social inequality aversion is relatively low. Importantly, our results are robust with respect to varying the stringency of the environmental target and parametric uncertainty related to firms' and households'

equilibrium responses to environmental taxes. The degree of optimal tax differentiation is somewhat diminished for non-optimal revenue redistribution schemes but remains substantial. Relative to uniform carbon pricing, incidence patterns across household income groups for a given redistribution scheme can vary qualitatively when allowing for differentiated taxes.

Our study has a number of implications for policy-making. First, our findings illustrate how pollution taxation, in addition to environmental goals, can serve distributional objectives. Second, social equity concerns should not be considered separately from efficiency aspects of environmental policy. It may indeed be optimal to sacrifice productive efficiency of pollution abatement to shield some households from increases in prices of consumption goods and decreases in returns to factors, as well as to raise pollution tax revenues for targeted transfers to households. Third, in the presence of political constraints on redistribution, the degree of optimal tax differentiation across industries depends on how well the tax rebates approximates the optimal revenue redistribution scheme. Importantly, we find that an inequality-averse government using per-capita tax rebates—which are often perceived as a "fair" and hence politically popular way of redistributing environmental tax revenues—should strongly differentiate carbon taxes across industries.

Our study could be extended in a number of ways. First, the robustness of the insights derived from the quantitative part of our analysis could be further scrutinized by placing our framework into the context of other economies which differ from the U.S. economy in terms of household heterogeneity, distribution of income, composition of industry output, and sectoral differences in pollution intensity. Second, the analysis could be extended to include (optimal and non-optimal) revenue recycling through adjusting pre-existing (distortionary) income taxes. Third, future research could incorporate international market power, terms-of-trade effects, and non-environmental taxes—all of which represent features of real-world economies that have previously been shown to provide a rationale for tax differentiation. While such efficiency-based motives would likely interact with the equity-driven motives studied in this paper, they would, however, not alter our main insight that household heterogeneity and social equity concerns provide strong arguments for differentiating environmental taxes across industries.

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Appendix A. Derivations: optimal taxes and optimal transfers

Consider the maximization problem

$$max_{\{\tau_i, T_h\}} W \quad s.t. \quad \sum_h T_h \leq \sum_i \tau_i Z_i \quad \& \quad \sum_i Z_i \leq \tilde{Z}\,.$$

The Lagrangian is

$$\mathcal{L} = W + \mu\Big(\sum_i \tau_i Z_i - \sum_h T_h\Big) + \epsilon\Big(\tilde{Z} - \sum_i Z_i\Big).$$

The first order conditions are as follows:

$$\partial_{\tau_i}\mathcal{L} \leq 0 \quad \tau_i \geq 0 \quad and \quad \tau_i \partial_{\tau_i}\mathcal{L} = 0 \quad \forall i \tag{A.1}$$

$$\partial_{T_h}\mathcal{L} \leq 0 \quad T_h \geq 0 \quad and \quad T_h \partial_{T_h}\mathcal{L} = 0 \quad \forall h \tag{A.2}$$

$$\partial_\mu \mathcal{L} \geq 0 \quad \mu \geq 0 \quad and \quad \mu\partial_\mu\mathcal{L} = 0$$
$$\partial_\epsilon \mathcal{L} \geq 0 \quad \epsilon \geq 0 \quad and \quad \epsilon\partial_\epsilon\mathcal{L} = 0\,.$$

Start from the optimality conditions for taxes, i.e., Eq. (A.1). Assuming an interior optimum in the tax rates, by use of Roy's identity (i.e., $\partial_{p_i} V_h = -X_{ih}\lambda_h$), and since $\partial_{\tau_i} M_h = \partial_{\tau_i} F_h$, they are equivalent to the following:

$$MSD = -\frac{\sum_h \sum_n \beta_h X_{nh} \partial_{\tau_i} p_n}{\partial_{\tau_i} Z} + \frac{\sum_h \beta_h \partial_{\tau_i} F_h}{\partial_{\tau_i} Z} + \frac{\mu \partial_{\tau_i} \sum_j \tau_j Z_j}{\partial_{\tau_i} Z}\,.$$

Now consider the optimality condition for transfers, i.e., Eq. (A.2). Using the fact that $\partial_{T_{h'}} T_h = 1$ if $h = h'$ and $= 0$ if $h \neq h'$, as well as Roy's identity, in addition to $\sum_i \tau_i \partial_{T_h} Z_i \leq 1$ (which holds since the extra tax revenue given to household h results in a less than proportional increase in tax revenues), this is equivalent to

$$\mu \geq (1 - \sum_i \tau_i \partial_{T_h} Z_i)^{-1} (\beta_h - \sum_{n,h'} \beta_{h'} X_{nh'} \partial_{T_h} p_n + \sum_{h'} \beta_{h'} \partial_{T_h} F_{h'} - MSD\, \partial_{T_h} Z)\,.$$

Appendix B. Derivations: optimal taxes and fixed revenue redistribution scheme

For a given redistribution scheme with fixed shares ξ_h, consider the maximisation problem:

$$max_{\{\tau_i\}} W \quad s.t. \quad \sum_i Z_i \leq \tilde{Z}\,.$$

The Lagrangian is:

$$\mathcal{L} = W + \epsilon\Big(\tilde{Z} - \sum_i Z_i\Big).$$

The optimality condition for taxes (assuming an interior solution) can be rewritten as follows:

$$MSD = -\frac{\sum_h \sum_n \beta_h X_{nh} \partial_{\tau_i} p_n}{\partial_{\tau_i} Z} + \frac{\sum_h \beta_h \partial_{\tau_i} F_h}{\partial_{\tau_i} Z} + \frac{(\sum_h \beta_h \xi_h) \partial_{\tau_i} \sum_j \tau_j Z_j}{\partial_{\tau_i} Z} \tag{B.1}$$

Appendix C. Derivations: pollution-neutral tax swaps

Appendix C.1. Derivation of Equation (7)

A differential, pollution neutral (i.e., $dZ = 0$) change in social welfare can be written as follows, by means of Roy's identity:

$$dW = \sum_h \beta_h [-\sum_n X_{nh} dp_n + dM_h]\,.$$

Express differential quantities in terms of pollution tax rate changes and rearrange, using the fact that (due to pollution neutrality of the tax swap) $d\tau_i = -(\partial_{\tau_j} Z / \partial_{\tau_i} Z) d\tau_j$, thus obtaining:

$$dW = -\partial_{\tau_j} Z \Big(\frac{1}{\partial_{\tau_i} Z} \sum_h \beta_h [-\sum_n X_{nh} \partial_{\tau_i} p_n + \partial_{\tau_i} M_h] - \frac{1}{\partial_{\tau_j} Z} \sum_h \beta_h [-\sum_n X_{nh} \partial_{\tau_j} p_n + \partial_{\tau_j} M_h] \Big) d\tau_j\,.$$

This implies

$$dW = -\partial_{\tau_j} Z (MSC_i^\xi - MSC_j^\xi) d\tau_j\,.$$

Appendix C.2. Derivation of Equation (8)

Total differentiation of the labor delivers the following relationship:[31]

$$\hat{L}_X \frac{L_X}{\bar{L}} + \hat{L}_Y \frac{L_Y}{\bar{L}} = 0\,, \tag{C.1}$$

where hats represent proportional changes, i.e. $\hat{L}_X \equiv \frac{dL_X}{L_X}$. Production levels adjust following changes in the wage rate and in the pollution tax rates, according to the following relationships:

$$\hat{Z}_i - \hat{L}_i = \sigma(\hat{w} - \hat{\tau}_i)\,, \tag{C.2}$$

where σ is the elasticity of substitution in production (which is assumed to be to be uniform across sectors at the equilibrium point, for reasons of tractability). Assuming perfect competition, the following furthermore holds (analogously to Fullerton and Heutel (2007)):

$$\hat{p}_X + \hat{X} = \theta_{XL}(\hat{w} + \hat{L}_X) + \theta_{XZ}(\hat{\tau}_X + \hat{Z}_X) \tag{C.3}$$

$$\hat{p}_Y + \hat{Y} = \theta_{YL}(\hat{w} + \hat{L}_Y) + \theta_{YZ}(\hat{\tau}_Y + \hat{Z}_Y) \tag{C.4}$$

$$\hat{X} = \theta_{XL}\hat{L}_X + \theta_{XZ}\hat{Z}_X \tag{C.5}$$

$$\hat{Y} = \theta_{YL}\hat{L}_Y + \theta_{YZ}\hat{Z}_Y\,, \tag{C.6}$$

where θ_{XL} represents the share of revenue from sector X paid to labor, and the other θ parameters are defined analogously. Household budget: $M_h = w\bar{L}_h + T_h$, where $T_h = \xi_h(\tau_X Z_X + \tau_Y Z_Y)$ is the amount of tax revenue redistributed to household h (where $\xi_A + \xi_B = 1$). Changes in household demand are therefore as follows:

$$\hat{X}_h - \hat{Y}_h = \hat{p}_Y - \hat{p}_X \tag{C.7}$$

$$\hat{X}_h = -\hat{p}_X + \hat{M}_h\,, \tag{C.8}$$

where $\hat{M}_h = \hat{w}\frac{w\bar{L}_h}{M_h} + \frac{\xi_h}{M_h}\tau\Big(Z_X(\hat{\tau}_X + \hat{Z}_X) + Z_Y(\hat{\tau}_Y + \hat{Z}_Y)\Big)$. Finally, total differentiation of the market clearing conditions delivers the following:

$$\hat{X} = \frac{X_A}{X}\hat{X}_A + \frac{X_B}{X}\hat{X}_B \tag{C.9}$$

$$\hat{Y} = \frac{Y_A}{Y}\hat{Y}_A + \frac{Y_B}{Y}\hat{Y}_B\,. \tag{C.10}$$

The above Eqs. (C.1) to (C.10) represent 13 equations in 13 unknowns ($\hat{L}_X$, $\hat{L}_Y$, $\hat{Z}_X$, $\hat{Z}_Y$, $\hat{w}$, $\hat{p}_X$, $\hat{p}_Y$, $\hat{X}$, $\hat{Y}$, $\hat{X}_A$, $\hat{X}_B$, $\hat{Y}_A$, $\hat{Y}_B$) and two exogenous variables ($\hat{\tau}_X$ and $\hat{\tau}_Y$). One equation is redundant, and we choose the wage rate to be the numeraire (hence $\hat{w} = 0$), therefore delivering a square system of 12 equations that we can solve for 12 unknowns, in terms of two exogenous tax rate changes.

To solve the linearized model equations, start by subtracting Eqs. (C.5) and (C.6) from Eqs. (C.3) and (C.4) , thus obtaining:

$$\hat{p}_X = \theta_{XZ}\hat{\tau}_X \tag{C.11}$$

$$\hat{p}_Y = \theta_{YZ}\hat{\tau}_Y\,. \tag{C.12}$$

Insert Eq. (C.8) into Eq. (C.7), then insert the result into Eq. (C.10):

$$\hat{Y} = -\hat{p}_Y + \frac{Y_A}{Y}\hat{M}_A + \frac{Y_B}{Y}\hat{M}_B\,.$$

Insert the explicit expression for the change in the household budget, then insert Eq. (C.2) :

$$\hat{Y} = -\hat{p}_Y + (\frac{Y_A}{Y}\frac{\xi_A}{M_A} + \frac{Y_B}{Y}\frac{\xi_B}{M_B})\Big(\tau Z_X(\hat{L}_X + \hat{\tau}_X(1-\sigma)) + \tau Z_Y(\hat{L}_Y + \hat{\tau}_Y(1-\sigma))\Big). \tag{C.13}$$

Insert Eqs. (C.13) and (C.2) into Eq. (C.4):

$$(\frac{Y_A}{Y}\frac{\xi_A}{M_A} + \frac{Y_B}{Y}\frac{\xi_B}{M_B})\Big(\tau Z_X(\hat{L}_X + \hat{\tau}_X(1-\sigma)) + \tau Z_Y(\hat{L}_Y + \hat{\tau}_Y(1-\sigma))\Big) = \hat{L}_Y + \theta_{YZ}\hat{\tau}_Y(1-\sigma)\,.$$

[31] It should be noted that the following linearized model equations follow closely Fullerton and Heutel (2007) and Rausch and Schwarz (2016).

Now insert Eq. (C.1) and solve for $\hat{L}_Y$:

$$\hat{L}_Y = \frac{\Phi\tau Z_X\hat{\tau}_X(1-\sigma) + (\Phi\tau Z_Y - \theta_{YZ})\hat{\tau}_Y(1-\sigma)}{1-\Phi(\tau Z_Y - \tau Z_X\frac{L_Y}{L_X})},$$

with $\Phi := \frac{Y_A}{Y}\frac{\xi_A}{M_A} + \frac{Y_B}{Y}\frac{\xi_B}{M_B}$. Using Eq. (C.1), we then also find that

$$\hat{L}_X = -\frac{L_Y}{L_X}\frac{\Phi\tau Z_X\hat{\tau}_X(1-\sigma) + (\Phi\tau Z_Y - \theta_{YZ})\hat{\tau}_Y(1-\sigma)}{1-\Phi(\tau Z_Y - \tau Z_X\frac{L_Y}{L_X})}.$$

Eq. (C.2) then implies:

$$\hat{Z}_Y = \hat{\tau}_X\frac{\Phi\tau Z_X(1-\sigma)}{1-\Phi(\tau Z_Y - \tau Z_X\frac{L_Y}{L_X})} + \hat{\tau}_Y\Big(\frac{(\Phi\tau Z_Y - \theta_{YZ})(1-\sigma)}{1-\Phi(\tau Z_Y - \tau Z_X\frac{L_Y}{L_X})} - \sigma\Big). \tag{C.14}$$

and

$$\hat{Z}_X = \hat{\tau}_X\Big(\frac{L_Y}{L_X}\frac{\Phi\tau Z_X(1-\sigma)}{\Phi(\tau Z_Y - \tau Z_X\frac{L_Y}{L_X})-1} - \sigma\Big) + \hat{\tau}_Y\frac{L_Y}{L_X}\frac{(\Phi\tau Z_Y - \theta_{YZ})(1-\sigma)}{\Phi(\tau Z_Y - \tau Z_X\frac{L_Y}{L_X})-1}. \tag{C.15}$$

Now note that the expression for MSC^{ξ} for our model can be written as follows:

$$MSC^{\xi}_X = \frac{\sum_h \beta_h(X_h\partial_{\tau_X}p_X + Y_h\partial_{\tau_X}p_Y - \xi_h(Z_X + \tau_X\partial_{\tau_X}Z_X + \tau_Y\partial_{\tau_X}Z_Y))}{-\partial_{\tau_X}Z}, \tag{C.16}$$

and

$$MSC^{\xi}_Y = \frac{\sum_h \beta_h(X_h\partial_{\tau_Y}p_X + Y_h\partial_{\tau_Y}p_Y - \xi_h(Z_Y + \tau_X\partial_{\tau_Y}Z_X + \tau_Y\partial_{\tau_Y}Z_Y))}{-\partial_{\tau_Y}Z}. \tag{C.17}$$

Based on the solutions to our linearized model equations, we can now calculate the partial derivatives in Eqs. (C.16) and (C.17), as follows. Start by expressing Eqs. (C.11) and (C.12) in term of total differentials:

$$dp_X = \frac{Z_X}{X}d\tau_X, \qquad dp_Y = \frac{Z_Y}{Y}d\tau_Y.$$

Analogously for Eqs. (C.14) and (C.15):

$$dZ_X = d\tau_X\frac{Z_X}{\tau}\Big(\frac{L_Y}{L_X}\frac{\Phi\tau Z_X(1-\sigma)}{\Phi(\tau Z_Y - \tau Z_X\frac{L_Y}{L_X})-1} - \sigma\Big) + d\tau_Y\frac{Z_X}{\tau}\frac{L_Y}{L_X}\frac{(\Phi\tau Z_Y - \theta_{YZ})(1-\sigma)}{\Phi(\tau Z_Y - \tau Z_X\frac{L_Y}{L_X})-1},$$

and

$$dZ_Y = d\tau_X\frac{Z_Y}{\tau}\frac{\Phi\tau Z_X(1-\sigma)}{1-\Phi(\tau Z_Y - \tau Z_X\frac{L_Y}{L_X})} + d\tau_Y\frac{Z_Y}{\tau}\Big(\frac{(\Phi\tau Z_Y - \theta_{YZ})(1-\sigma)}{1-\Phi(\tau Z_Y - \tau Z_X\frac{L_Y}{L_X})} - \sigma\Big).$$

By comparing coefficients with the generic form of the total derivative of the quantities above, we therefore find that, at a market equilibrium with uniform pollution tax rates ($\tau_X = \tau_Y \equiv \tau$) the following holds:

$$\partial_{\tau_X}p_X = \frac{Z_X}{X}, \qquad \partial_{\tau_Y}p_X = 0, \qquad \partial_{\tau_X}p_Y = 0, \qquad \partial_{\tau_Y}p_Y = \frac{Z_Y}{Y},$$

and

$$\partial_{\tau_X}Z_X = \frac{Z_X}{\tau}\Big(\frac{L_Y}{L_X}\frac{\Phi\tau Z_X(1-\sigma)}{\Phi(\tau Z_Y - \tau Z_X\frac{L_Y}{L_X})-1} - \sigma\Big), \qquad \partial_{\tau_Y}Z_X = \frac{Z_X}{\tau}\frac{L_Y}{L_X}\frac{(\Phi\tau Z_Y - \theta_{YZ})(1-\sigma)}{\Phi(\tau Z_Y - \tau Z_X\frac{L_Y}{L_X})-1},$$

$$\partial_{\tau_X}Z_Y = \frac{Z_Y}{\tau}\frac{\Phi\tau Z_X(1-\sigma)}{1-\Phi(\tau Z_Y - \tau Z_X\frac{L_Y}{L_X})}, \qquad \partial_{\tau_Y}Z_Y = \frac{Z_Y}{\tau}\Big(\frac{(\Phi\tau Z_Y - \theta_{YZ})(1-\sigma)}{1-\Phi(\tau Z_Y - \tau Z_X\frac{L_Y}{L_X})} - \sigma\Big).$$

From the above expressions, we can calculate the partial derivatives of total pollution:

$$\partial_{\tau_X}Z = \partial_{\tau_X}Z_X + \partial_{\tau_X}Z_Y = (\frac{Z_X}{\tau}\frac{L_Y}{L_X} - \frac{Z_Y}{\tau})\frac{\Phi\tau Z_X(1-\sigma)}{\Phi(\tau Z_Y - \tau Z_X\frac{L_Y}{L_X})-1} - \frac{Z_X}{\tau}\sigma \tag{C.18}$$

and

$$\partial_{\tau_Y}Z = \partial_{\tau_Y}Z_X + \partial_{\tau_Y}Z_Y = (\frac{Z_X}{\tau}\frac{L_Y}{L_X} - \frac{Z_Y}{\tau})\frac{(\Phi\tau Z_Y - \theta_{YZ})(1-\sigma)}{\Phi(\tau Z_Y - \tau Z_X\frac{L_Y}{L_X})-1} - \frac{Z_Y}{\tau}\sigma \tag{C.19}$$

The expressions for MSC^{ξ} can therefore be expressed as follows:

$$MSC_X = \frac{\sum_h \beta_h \Big(X_h \frac{Z_X}{X} - \xi_h (Z_X + \tau \partial_{\tau_X} Z) \Big)}{-\partial_{\tau_X} Z}, \qquad MSC_Y = \frac{\sum_h \beta_h \Big(Y_h \frac{Z_Y}{Y} - \xi_h (Z_Y + \tau \partial_{\tau_Y} Z) \Big)}{-\partial_{\tau_Y} Z},$$

with $\partial_{\tau_X} Z$ and $\partial_{\tau_Y} Z$ as in Eqs. (C.18) and (C.19), respectively.

Now consider the following case: $MSC_X^{\xi} < MSC_Y^{\xi}$. Assuming $\partial_{\tau_X} Z < 0$ and $\partial_{\tau_Y} Z < 0$, this corresponds to the following:

$$\sum_h \beta_h \Big(Z_X \partial_{\tau_Y} Z (\frac{X_h}{X} - \xi_h) - Z_Y \partial_{\tau_X} Z (\frac{Y_h}{Y} - \xi_h) \Big) > 0 .$$

Insert Eqs. (C.18) and (C.19) and rearrange, using $\frac{X_h}{X} \equiv \frac{\alpha_h M_h}{\gamma M}$, $\frac{Y_h}{Y} \equiv \frac{(1-\alpha_h) M_h}{(1-\gamma)(M)}$ and $p_X X + p_Y Y = M$:

$$\Delta \Big\{ \Big(\frac{\sigma}{\tau L_Y (\delta_X - \delta_Y)} + \frac{\sigma - 1}{M} + \sum_h \frac{1-\alpha_h}{1-\gamma} \frac{\xi_h}{M} \Big) \sum_h \beta_h M_h \Big(\frac{\alpha_h}{\gamma} - 1 \Big) + (1-\sigma) \sum_h \beta_h \Big(\xi_h - \frac{M_h}{M} \Big) \Big\} < 0 .$$

Appendix C.3. Proof of Proposition (4)

Pollution neutrality implies $d\tau_Y = -d\tau_X \partial_{\tau_X} Z / \partial_{\tau_Y} Z$. The requirement that $d\tau_Y < 0$ and $d\tau_X > 0$ thus implies that $\partial_{\tau_X} Z / \partial_{\tau_Y} Z > 0$. We start by showing that $\partial_{\tau_X} Z$ and $\partial_{\tau_Y} Z$ cannot simultaneously be positive. It then follows that both must be negative.

Start by noting that, for the redistribution scheme in Proposition 4, $1 + \Phi\tau(Z_X \frac{L_Y}{L_X} - Z_Y) > 0$ and $\Phi\tau Z_Y - \theta_{YZ} < 0$. Assume on the one hand $\partial_{\tau_X} Z > 0$. This implies $(\frac{Z_X}{\tau} \frac{L_Y}{L_X} - \frac{Z_Y}{\tau})(\sigma - 1) > 0$. It then follows that $\partial_{\tau_Y} Z = (\frac{Z_X}{\tau} \frac{L_Y}{L_X} - \frac{Z_Y}{\tau}) \frac{(\Phi\tau Z_Y - \theta_{YZ})(\sigma-1)}{1+\Phi(\tau Z_X \frac{L_Y}{L_X} - \tau Z_Y)} - \frac{Z_Y}{\tau}\sigma < 0$. Assume on the other hand $\partial_{\tau_Y} Z > 0$. This implies $(\frac{Z_X}{\tau} \frac{L_Y}{L_X} - \frac{Z_Y}{\tau})(\sigma - 1) < 0$, which then implies $\partial_{\tau_X} Z = (\frac{Z_X}{\tau} \frac{L_Y}{L_X} - \frac{Z_Y}{\tau}) \frac{\Phi\tau Z_X(\sigma-1)}{1+\Phi(\tau Z_X \frac{L_Y}{L_X} - \tau Z_Y)} - \frac{Z_X}{\tau}\sigma < 0$.

Since $\partial_{\tau_X} Z$ and $\partial_{\tau_Y} Z$ are both negative, it follows that the tax swap is welfare-improving if and only if $MSC_X^{\xi} < MSC_Y^{\xi}$. For the assumptions in Proposition 4, this is equivalent to $\beta_A(\alpha_A - \gamma) < \beta_B(\alpha_A - \gamma)$ (since the case with Leontief production is excluded by requiring that $\partial_{\tau_X} Z$ and $\partial_{\tau_Y} Z$ are both negative, as can be seen from Eqs. (C.18) and (C.19); furthermore, $M_A(\alpha_A - \gamma) = M_B(\gamma - \alpha_B)$ and $1 + \Big(\frac{Y_A}{Y} \frac{\xi_A}{M_A} + \frac{Y_B}{Y} \frac{\xi_B}{M_B} \Big)(\tau Z_X \frac{L_Y}{L_X} - \tau Z_Y) \equiv 1 + \frac{\tau Z_X}{M_A + M_B} \frac{L_Y}{L_X} - \frac{\tau Z_Y}{M_A + M_B} > 1 - \frac{\tau Z_Y}{M_A + M_B} > 0$). The case with $\alpha_A > \alpha_B$ (equivalent to $\alpha_A > \gamma$) and $\beta_A < \beta_B$ as well as the case $\alpha_B > \alpha_A$ (equivalent to $\alpha_A < \gamma$) and $\beta_B < \beta_A$ satisfy this relation, but no other combination of the relations between αs and βs does. Hence, Proposition 4 follows.

Appendix C.4. Proof of Proposition (5)

Assume without loss of generality that the household with the higher β is A. Pollution neutrality implies $d\tau_Y = -d\tau_X \frac{\partial_{\tau_X} Z}{\partial_{\tau_Y} Z}$. Therefore the requirement that $d\tau_Y < 0$ and $d\tau_X > 0$ implies that $\frac{\partial_{\tau_X} Z}{\partial_{\tau_Y} Z} > 0$. Since $\partial_{\tau_X} Z$ and $\partial_{\tau_Y} Z$ cannot both be positive (analogously to the proof of Proposition 4), it follows that both are negative. The tax swap will therefore be welfare improving if and only if $MSC_X^{\xi} < MSC_Y^{\xi}$. For the above assumptions, this is equivalent to $(Z_X \frac{L_Y}{L_X} - Z_Y)(1 - \sigma) < 0$ (since $1 + \Big(\frac{Y_A}{Y} \frac{\xi_A}{M_A} + \frac{Y_B}{Y} \frac{\xi_B}{M_B} \Big)(\tau Z_X \frac{L_Y}{L_X} - \tau Z_Y) \equiv 1 + \frac{Y_A}{Y} \frac{1}{M_A} (\tau Z_X \frac{L_Y}{L_X} - \tau Z_Y) > 0$). Now note that the change in the total pollution tax revenue T can be expressed as follows: $dT = \frac{d\tau_X}{\partial_{\tau_Y} Z} (Z_X \partial_{\tau_Y} Z - Z_Y \partial_{\tau_X} Z)$. Insert Eqs. (C.18) and (C.19), and substitute $\Phi = (1-\gamma)/p_Y Y$, thus obtaining the following: $dT = (\frac{Z_X}{\tau} \frac{L_Y}{L_X} - \frac{Z_Y}{\tau})(1 - \sigma) \frac{d\tau_X}{\partial_{\tau_Y} Z} \Big(\frac{Z_X \theta_{YZ}}{1 + (1-\gamma) \frac{\tau Z_X}{p_Y Y} \frac{L_Y}{L_X} - (1-\gamma)\theta_{YZ}} \Big)$. It therefore follows that $(Z_X \frac{L_Y}{L_X} - Z_Y)(1 - \sigma) < 0$ is equivalent to $dT > 0$. Hence, Proposition 5 follows.

Appendix D. Equilibrium conditions for numerical general equilibrium model

Tables D.7, D.8, and D.9 define the parameters and variables in the model. Consider first the **unit cost functions** $c_i(\mathbf{p})$ and $c_h(\mathbf{p})$.

The unit cost function for utility of household h is defined as (see Figure D.6):

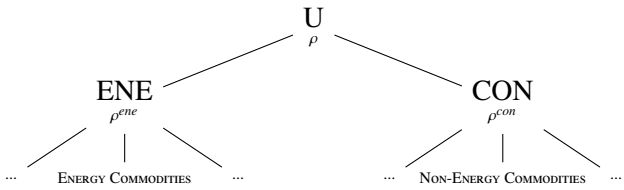
$$c_h := \left[\theta_h^U (c_h^{ENE})^{1-\rho_h} + \left(1-\theta_h^U\right)(c_h^{CON})^{1-\rho_h}\right]^{\frac{1}{1-\rho_h}}$$

where

$$c_h^{UENE} := \left[\sum_{i\in ene} \theta_{ih}^{ENE} PY_i^{1-\rho_h^{ene}}\right]^{\frac{1}{1-\rho_h^{ene}}}$$

$$c_h^{UCON} := \left[\sum_{i\in con} \theta_{ih}^{CON} PY_i^{1-\rho_h^{con}}\right]^{\frac{1}{1-\rho_h^{con}}} ,$$

Figure D.6: Structure of private consumption



The unit cost for production activity i is defined as (see Figure D.7):

$$c_i := \left[\theta_i^Y \sum_{j\in mat} \theta_{ji}^M PYE_{ji}^{1-\sigma_i} + \left(1-\theta_i^Y\right)\left(c_{ir}^V\right)^{1-\sigma_i}\right]^{\frac{1}{1-\sigma_i}}$$

where

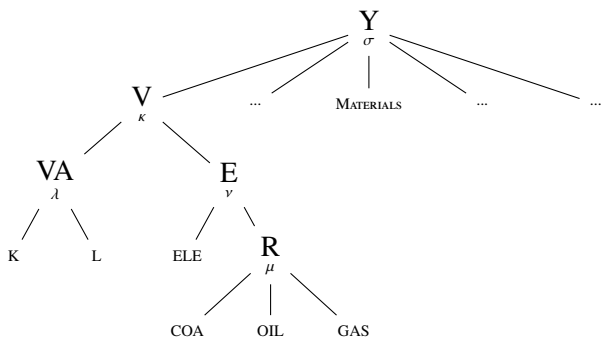
$$c_i^V := \left[\theta_i^V \left(c_i^{VA}\right)^{1-\kappa_i} + \left(1-\theta_i^V\right)\left(c_i^E\right)^{1-\kappa_i}\right]^{\frac{1}{1-\kappa_i}}$$

$$c_{ir}^{VA} := \left[\theta_i^K\ PK^{1-\lambda_i} + \left(1-\theta_i^K\right) PL^{1-\lambda_i}\right]^{\frac{1}{1-\lambda_i}}$$

$$c_i^E := \left[\theta_i^E PYE_{ELE,i}^{1-\nu_i} + \left(1-\theta_i^E\right)\left(c_i^R\right)^{1-\nu_i}\right]^{\frac{1}{1-\nu_i}}$$

$$c_i^R := \left[\sum_{j\in e} \theta_{ei}^R PYE_{ei}^{1-\mu_i}\right]^{\frac{1}{1-\mu_i}} ,$$

Figure D.7: Structure of production



The investment commodity is produced according to a Cobb-Douglas technology:[32]

$$c^I := \prod_i PY_i^{\theta_i^I} .$$

[32]Note that in the main text investment is not mentioned explicitly, as it is not central to our analysis. Households are assumed to consume a fixed amount of investment, which is imputed in proportion to benchmark expenditures.

The model's **zero-profit conditions** are then given by:

$$
\begin{aligned}
c_h &\geq PU_h & \perp \quad U_h &\geq 0 & \forall h \\
c_i &\geq PY_i & \perp \quad Y_i &\geq 0 & \forall i \\
c^I &\geq PI & \perp \quad I &\geq 0 &
\end{aligned}
$$

Using Shepard's lemma, **market clearing equations** become:

$$
\begin{aligned}
Y_i &\geq \sum_j \frac{\partial c_j}{\partial PYE_{ij}} Y_j + \sum_h \frac{\partial c_h}{\partial PY_i} U_h + \frac{\partial c^I}{\partial PY_i} I & \perp \quad PY_i &\geq 0 & \forall i \\
I &\geq \sum_h \bar{i}_h & \perp \quad PI &\geq 0 & \\
\sum_h \omega_h^L &\geq \sum_i \frac{\partial c_i}{\partial PL} Y_i & \perp \quad PL &\geq 0 & \\
\sum_h \omega_h^K &\geq \sum_i \frac{\partial c_i}{\partial PK} Y_i & \perp \quad PK &\geq 0 & \\
U_h &\geq \frac{M_h}{PU_h} & \perp \quad PU_h &\geq 0 & \forall h\,,
\end{aligned}
$$

where PYE_{ij} denotes the carbon tax inclusive price for commodity i employed in sector j. The carbon cost is added to PY_i in proportion to the good's carbon intensity and depending on the carbon tax rate in sector j.

Household income in defined factor income net of investment (savings), plus government transfers:

$$M_h := PL\omega_h^L + PK\omega_h^K - PI\bar{i}_h + T_h\,.$$

Constraints in the optimization problem (9) can be expressed as

$$
\begin{aligned}
\tilde{Z} &\geq \sum_{e,i} \phi_e \frac{\partial c_i}{\partial PYE_{ei}} Y_i \\
\sum_h T_h &= \sum_{e,i} \tau_i \phi_e \frac{\partial c_i}{\partial PYE_{ei}} Y_i
\end{aligned}
$$

Table D.7: Sets, prices, and quantity variables

Symbol	Description
Sets	
$h \in H$	Households
$i \in I$	Commodities
$con \subset I$	Non-energy consumption commodities
$ene \subset I$	Energy consumption commodities
$e \subset I$	Fossil fuel input commodities
$mat \subset I$	Material input commodities
Prices and quantities	
U_h	Private consumption index of household h
M_h	Private income of household h, net of investment
I	Investment consumption index
Y_i	Production index of sector i
PL	Wage rate
PU_h	Consumer price index for household h
PI	Investment consumption price index
PK	Capital rental rate
PY_i	Commodity i output price
T_h	Transfer to household h
τ_i	Carbon tax on sector i emissions

Table D.8: Elasticities of substitution parameters and central case values

Parameter	Description	Value
Production		
σ_i	Top level (materials vs. energy/value-added composite)	0.0
κ_i	Value-added vs. energy composite	0.5
λ_i	Capital vs. labor	0.8
ν_i	Primary energy vs. electricity	1.2
μ_i	Fossil fuels	0.7
Consumption		
ρ_h	Top level (energy vs. non-energy)	0.0
ρ_h^{ene}	Energy commodities	0.7
ρ_h^{con}	Non-energy commodities	0.7

Table D.9: Other model parameters

Symbol	Description
$\bar{i}_h$	Reference investment (savings) level household of h
ω_h^L	Labor endowment of household h
ω_h^K	Capital endowment of household h
θ_h^U	Expenditure share on energy commodities in total expenditure of household h
θ_{ih}^{ENE}	Expenditure share on commodity i in total energy expenditure of household h
θ_{ih}^{CON}	Expenditure share of commodity i in total non-energy expenditure of household h
θ_i^Y	Share of material inputs in top-level production of commodity i
θ_{ji}^M	Share of material input j in total materials cost in production of commodity i
θ_i^V	Share of value-added cost in commodity i value-added/energy composite
θ_i^K	Share of capital cost in commodity i value added composite
θ_i^E	Share of electricity cost in commodity i energy composite
θ_{ei}^R	Share of fossil fuel e cost in commodity i fossil fuel composite
θ_i^I	Expenditure share on commodity i in investment consumption
ϕ_e	Carbon coefficient of fuel e
π_h	Share of household h in total population

ONLINE APPENDIX for "Social Equity Concerns and Differentiated Environmental Taxes" by Jan Abrell, Sebastian Rausch, and Giacomo Schwarz

This section provides a brief documentation of computer codes which can be used to reproduce the quantitative results of the paper using the GAMS (General Algebraic Modeling System) software. All files, including the data, can be downloaded here.[33]

This appendix is structured as follows. We first list the software programs and solvers needed to execute the computer codes. We then describe the folder structure, file names, and functionality of each program in the overall package.

Software prerequisites

The following solvers are needed to solve the GAMS routines:

- *GAMS*: General Algebraic Modeling System. Main software used.
- *PATH*: Solver for GAMS to solve the mixed complementarity problems (MCP) for general equilibrium model.
- *MPSGE*: Higher-level language within GAMS to represent and solve computable general equilibrium (CGE) model as MCP problems.

Folder structure and file names

The folder named "**Model_HH**" contains all model files which are named as follows:

- *calibration.gms*: Routine to assign model parameter values (elasticities and shares)
- *dataload_hh.gms*: Routine to load input data for CGE model.
- *init_MPEC.gms*: Set initial values for CGE model.
- *main_MPEC.gms*: Main file to run CGE model.
- *model_MPEC.gms*: Contains CGE model equations for the MPEC problem in MCP format.
- *model_MPS.gms*: Contains CGE model equations for the MPEC problem in MPSGE format.
- *policies.gms*: Contains parameters/switches to set model options.
- *sample_elasticities.gms*: Routine to assign elasticity parameter values for robustness checks.

The folder named "**Data**" contains data inputs for the CGE model.

- *USA_noTaxes_noTrade_hetero_hh*: Data input file with heterogeneous expenditure and factor income shares across households.
- *USA_noTaxes_noTrade_hetero_sources_hh*: Data input file with heterogeneous factor income shares and homogeneous expenditure shares across households.
- *USA_noTaxes_noTrade_hetero_uses_hh*: Data input file with heterogeneous expenditure shares and homogeneous factor income shares across households.

[33] The electronic version of this document contains a hyperlink. The address is: https://www.ethz.ch/content/dam/ethz/special-interest/mtec/cer-eth/economics-energy-economics-dam/documents/people/srausch/model_files_and_data.zip. For any questions regarding the computer codes, please contact the corresponding author of this paper.

- *USA_noTaxes_noTrade_homo_hh*: Data input file with homogeneous expenditure and factor income shares across households.

The folder named "**Batch**" contains batch files to execute the model runs.

- *USA_Heterogeneous_Optimal.bat*: Batch file to execute Optimal redistribution scenario.
- *USA_Heterogeneous_Uses_Income_Prop.bat*: Batch file to isolate Preference effect.
- *USA_Homogeneous_Optimal.bat*: Batch file to isolate Revenue redistribution effect.
- *USA_Heterogeneous_Sources_Income_Prop.bat*: Batch file to isolate Factor income effect.
- *USA_Heterogeneous_Per_Capita.bat*: Batch file to execute Flat recycling scenario.
- *USA_Heterogeneous_Dirty_Consumption_Prop.bat*: Batch file to execute Consumption-based recycling scenario.
- *USA_Heterogeneous_Income_Prop.bat*: Batch file to execute Income-based recycling scenario.
- *sensitivity_USA_Heterogeneous_carbon_limit.bat*: Batch file to execute robustness check for stringency of the environmental target.
- *sensitivity_USA_Heterogeneous_optimal.bat*: Batch file to execute robustness check for parametric uncertainty of equilibrium responses by firms and households.

The folder named "**Scenarios**" contains scenarios for optimal and for alternative non-optimal revenue redistribution schemes, as well as scenarios for robustness checks. Scenarios are saved in subfolders with the scenario name and contain a *.gms* file with the scenario name, as well as a reporting file (named *report.gms*). The scenario folders are named as follows:

- *Dirty_Consumption_Prop*: Consumption-based recycling scenario.
- *Income_Prop*: Income-based recycling scenario.
- *Optimal*: Optimal redistribution scenario.
- *Per_Capita*: Flat recycling scenario.
- *Optimal_Sensitivity*: Robustness check for parametric uncertainty, for the scenario with optimal redistribution. Note that results are stored here in subfolders, named *hh_random* (Sample A), *sector_random* (Sample B), and *all_random* (Sample C).
- *Target_Sensitivity*: Robustness check for alternative environmental targets. Note that results are stored here in a subfolder named *gdx*.

The folder named "**Results**" stores the results from the model runs based on central case assumptions. Model output from the sensitivity analyses are stored in subfolders of the folder named "**Scenarios**" (see explanations above).

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