Effectiveness of renewable energy subsidies in a CO2 intensive electricity system

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Effectiveness of renewable energy subsidies in a CO₂ intensive electricity system

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Can subsidies to renewable energy effectively internalise CO₂ costs in electricity production? Under current policy design it only matters that the replaced energy is dirty, but not how dirty it is. We use a modified peak-load pricing model, including variable renewable generators and the external costs of carbon, to examine the way in which a unit subsidy to variable renewables cannot restore first best optimum. In our model, electricity is generated using a combination of three technology types: two dispatchable, thermal, and CO₂ emitting technologies, differing in their emission intensity, and a non-dispatchable renewable technology. We show that available wind capacity is never idle, and derive equations determining optimal installed capacities for all technologies. We then describe the mechanism by which a subsidy that does not discriminate between dirty energies fails to restore first best. Our analysis highlights the importance of a carbon price: even one below the social cost of carbon could have a corrective effect on the merit order of fossil fuels and improve the effectiveness of a subsidy.

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1 INTRODUCTION

“Its progress of late, however, has been less than stellar: Despite its aggressive deployment of wind turbines and solar panels, the carbon intensity of California’s economy — measured by the CO₂ emissions per unit of economic product — declined by only 26.6 percent between 2000 and 2014. That put it in 28th place. In New York, which came in seventh, carbon intensity declined 35.4 percent.”

— New York Times, 17 January 2017¹

“European lawmakers have backed measures that would substantially raise the European Union’s clean-energy ambitions. By 2030, more than one-third of energy consumed in the EU should be from renewable sources such as wind and solar power, the European Parliament says — up from the existing target of just over one-quarter.”

— Nature News, 17 January 2018²

The potential costs of climate change are a strong incentive to reduce the amount of CO₂ emitted by the electricity sector, one of the largest single sources of carbon emissions (Ritchie and Roser, 2018). Variable and carbon free renewable generators are one of the key technologies with which the electricity sector can reduce emissions while continuing to produce similar levels of electrical energy. However, the costs of climate change brought about by these carbon emissions are external to the operation of the sector; without policy intervention, the social cost of carbon will not bring about more investment in renewables. Policy makers, aware of this social cost of carbon, have responded by instituting a variety of measures to encourage the expansion of renewable energy.

These measures fall into two broad categories: subsidies and taxes. Subsidies are more widespread, even if economic theory might predict that of the two, taxes are more effective at internalising the social cost of carbon. Nevertheless, policy makers continue to make use of subsidies to support renewables, thereby aiming to reduce the CO₂ intensity of the electricity sector; subsidies are simply the politically palatable policy. Given the extensive presence of subsidies to renewables in electricity systems, we would like to understand how they affect the carbon abatement possibilities offered by renewables.

We ask whether subsidies to renewable energy effectively internalise the CO₂ costs from electricity production. To answer this question, we propose a highly stylised model of the electricity sector. This model serves as the basis to describe the mechanism through which a subsidy to renewables — the clean technology — acts on the carbon emissions, produced by a separate, dirty technology.

Our core message is that the social planner’s merit order in electricity generation is a strong determinant of a subsidy’s effectiveness in internalising the cost of carbon. Unfortunately a subsidy has no influence over this merit order. Its effectiveness could be increased by a price on carbon, even if this price is not

equivalent to the social cost of carbon. Using our model, we describe how this occurs in the electricity sector.

The electricity sector includes both clean and carbon emitting generators. While electricity is homogeneous in consumption, it is not in production\(^3\). If electricity was completely homogenous, we could expect clean renewables to completely replace the carbon emitting fossil fuels. Unfortunately, as a result of limited electricity storage\(^4\), the variability of renewable generators and consumer preferences for uninterrupted supply of electricity, electricity generated from renewables cannot simply replace electricity generated from thermal generators on a one-to-one basis. The clean, variable renewables and the dirty fossil fuel generators may have to co-exist in the same system.

We use a theoretical model inspired by the peak-load pricing literature to characterise how exactly a subsidy fails to achieve the first best environmental gains, in a power system comprising of one variable and multiple dispatchable generation technologies. Even though economics intuits that a subsidy will be inferior to a tax in reducing the external cost of CO\(_2\), it is important to understand the mechanism by which this occurs. We anticipate that this analysis can contribute to the broader thinking on environmental policy.

In a model with just two generating technologies, a subsidy to the clean technology could be found to be the perfect converse of an environmental tax on the polluting technology; we are interested in the more realistic case of how effectively a subsidy replaces an environmental tax in a system with renewables and multiple other technologies. Results are shown for both the social planner’s optimum, where external costs are considered, and the decentralised equilibrium under perfect competition, where external costs are ignored. The social planner’s optimum serves as the benchmark against which we compare the decentralised equilibrium with and without a subsidy to variable renewable energy.

Our model combines multiple generation technologies with a characterisation of their external costs and the variability of renewables; to the best of our knowledge, existing literature does not make use of this approach. Apart from presenting such a model, our main contribution is to describe the failure mode of a policy that aims to displace carbon emissions by subsidising renewables. This failure is driven by the inability of a subsidy to distinguish between which carbon-generating technology it helps displace. The failure is exacerbated when the least carbon-intensive fossil generator is displaced, an event we label the “merit order effect” in this paper. Our main message is that in the absence of carbon prices, subsidies alone do not constitute an effective environmental policy in the electricity sector. Our model shows that a subsidy fails to capture the full range of carbon abatement possibilities. It also fails to replicate the first best,

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\(^3\)Electrical energy produces the same energy services, whenever and wherever it is used; if this was the only criterion, electricity could be labelled a homogeneous good. However, as a result of the constraints in generation, transportation, storage and use, electricity is heterogeneous. Hirth et al. (2016) ascribe three dimensions to heterogeneity: over time, between locations and across the lead time between contract and delivery.

\(^4\)While we recognise that electricity storage will probably play a more significant role in the future, in most areas the viable storage technologies lag the development of renewables. As we would like to discuss the effectiveness of renewable support policies currently being deployed, we assume no storage in our model.
as defined by the social planner’s solution to our model.

The failure of a subsidy to internalise carbon costs is to be expected, according to economic theory and recent literature; even so, it is important to understand the mechanism by which a subsidy might fall short. Subsidies are still the dominant policy to support decarbonisation, despite the growth in the number of carbon pricing systems around the world. Given this extensive presence of subsidies to renewables, our precise description of how subsidies — which may be implemented for a variety of reasons — affect the carbon abatement possibilities offered by renewables, contributes to the discussion on environmental policy and market failures.

Our model exploits the peak-load pricing literature, first developed by Crew and Kleindorfer (1976) and subsequently Chao (1983) and Kleindorfer and Fernando (1993). This class of models aims to determine the optimal price for a good — such as electrical energy — which is produced using a variety of technologies, in a setting where demand varies from period to period. The development of this literature is reviewed by Crew et al. (1995). To characterise the variability of renewable energy generators, we exploit the contributions of Ambec and Crampes (2012) — who introduce the concept of two states representing environmental conditions under which renewables can or cannot operate — and Chao (2011) — who describes how the available capacity of a variable renewable generator can be represented by a stochastic variable. This latter representation of variability has been expanded by Andor and Voss (2016), who derive conditions under which policies supporting renewables are welfare increasing. Their model examines the capacity installation and energy output of a single electricity generator that can produce either external costs or external benefits: from the perspective of a variable renewable generator, these externalities could be avoided CO₂ emissions or the need for more flexible conventional generators, respectively. Another recent paper using a peak-load pricing approach is Helm and Mier (2016), who characterise an efficient diffusion path of renewables, again with an explicit characterisation of intermittency.

The peak-load pricing literature is only one of several that examine how to best expand renewable energy in the electricity sector. Borenstein (2012) provides a qualitative review of the challenges we face in using renewables to decarbonise the electricity sector. Our research question has also been approached using other types of economic models. Abrell et al. (2018) use a theoretically grounded numerical model to assess the effectiveness of various renewable support policies in the face of heterogeneous renewable generators. Gerlagh and van der Zwaan (2006) use a computable general equilibrium model to compare the effectiveness of five policy instruments in promoting carbon capture and storage technologies; they find that a subsidy is the most expensive policy to successfully achieve a set of climate goals, while a tax is the most cost-effective. Böhringer and Rosendahl (2010) examine the effect of tradeable CO₂ and renewables quotas on the electricity system, but with no explicit characterisation of the electricity system’s particular mechanics. They find that using both of these policies simultaneously will decrease the total emissions but not by reducing emissions from the dirtiest CO₂ emitting technology; this parallels our result of the merit order effect. Fischer (2010) focuses on the impacts of a re-
newable portfolio standard on the electricity system and highlights the failure of renewable energy policies to distinguish between the dirty technologies being displaced.

While this paper, and the literature we contribute to, is completely theoretical, it is worth noting the parallels to recent empirical work evaluating the effectiveness of specific policy support schemes to support renewable energy. In a series of papers Kaffine et al. quantify the emissions savings per additional MWh of wind power, and examines how the variability of renewables affects the quantity of carbon abated (Kaffine et al., 2013; Kaffine and McBee, 2017). Abrell et al. (2017) estimate the cost of reducing a ton of CO$_2$ through subsidies to wind and solar generators, while Cullen (2013) concludes that support for wind power in the US is justified when the social cost of carbon is at least USD42 per ton$^5$.

We begin the analysis by describing the model setup in section 2. We then solve for the social planner’s optimum in section 3. This comprises of two cases — expensive natural gas and cheap natural gas — a ramification of the assumptions we make concerning the relative cost of coal and natural gas — both of these serve as benchmarks when evaluating the effect of policies on the decentralised equilibrium. Following the social planner’s optimum, we solve the decentralised equilibrium in section 4. The effectiveness of subsidy as environmental policy is analysed in section 5. We close with a discussion of the model’s implications, section 6.

2 Model Setup

In our model, electricity can be generated using a combination of three types of technology: two thermal — i.e. CO$_2$ emitting — technologies, which we represent by natural gas and coal, and a renewable technology, which we represent by wind. The two CO$_2$ emitting technologies differ in cost and emission intensity. Each of these three generators produces the same type of electrical energy, with the caveat that while output from the thermal generators can be determined, that of the wind generator depends on wind speed, a stochastic quantity. Given that the thermal technologies can adjust, or dispatch, their electricity production at the request of the operator, we will refer to the thermal technologies as being dispatchable, and to the renewable technology as being non-dispatchable. The use of three technologies is not accidental: the problem we want to analyse only occurs in a system with multiple carbon emitting generators of varying carbon intensity$^6$. The variation of carbon intensity is a key characteristic of our model.

Utility, $U(q)$, is derived from consuming electrical energy $q$. We assume that utility is an increasing concave function, $MU(\cdot) > 0$, $MU'(\cdot) < 0$. Marginal utility needs to be sufficiently high for all three technologies to materialise in the system. As we described in the introduction, the problem we analyse does not occur in the presence of a single fossil fuel generator. For the purposes of our

$^5$The papers of Kaffine et al. & Cullen use detailed grid operation and emission data for the ERCOT power grid in Texas, USA. Abrell et al. focus on Germany and Spain.

$^6$Our model can accommodate more than three technologies, however the key messages from the analysis would be the same.
We exclude marginal utilities for which fewer than three technologies are installed, by imposing the interiority assumption appropriate to each case.

Although each technology — wind $w$, coal $c$ or natural gas $g$ — differs in some characteristics, the quantities of electricity they each produce $q_i > 0$, $i \in \{w,c,g\}$ are substitutes as far as utility is concerned. The subscript $i$ denotes the full set of technologies, while we use subscript $f$ when specifically considering the subset of fossil based generators. It costs $b_f > 0$, $f \in \{c,g\}$ to operate the generators and produce each unit of energy from fossil fuels; wind energy has no operating costs.

As the variation of carbon intensity across fossil fuel generators is key to our model, we require an explicit characterisation of the external cost of carbon, $e_f > 0$. In our setup it can be added to each technology’s operating cost. Therefore, a social planner’s cost is the sum of the operating cost and the external cost of carbon for each technology, $b_f + e_f$. The emission cost of coal is always higher than the emission cost of natural gas, $e_c > e_g$. This is a physical reality that always holds: natural gas of a particular energy content contains approximately half the carbon as coal of the same energy content (E.I.A., 2016).

The quantity of energy $q_i$ can be produced up to capacity $K_i > 0$, reflecting capacity constraints and our assumption of no storage. Each generator is installed at an increasing convex cost which is a function of the total capacity installed in the system, in our case $\frac{1}{2} \beta_i K_i^2$, $\beta_i > 0$. Power plants become increasingly expensive to build: as the best plants are built out, the remaining potential locations will prove ever more expensive to prepare; for instance, they may have poorer links to the natural gas or coal supply, or to the power grid. In our analysis, the costs are considered over the whole lifetime of the plants, so we can consider quantity costs $b_f$ and investment costs $\beta_i$ simultaneously, even though they represent slightly different physical quantities & their original units differ.

As renewable energy depends upon environmental conditions, it is unavailable on occasion. This unavailability is a fundamental characteristic of the wind generators in our model; it drives the heterogeneity of electricity, a characteristic we discussed in the introduction. We introduce the availability of wind $\alpha \in [0,1]$, a stochastic variable that helps distinguish between the wind capacity that is installed, $K_w$, and the fraction of this capacity that is available to produce electricity, $\alpha K_w$. The probability that $\alpha$ takes a particular value is drawn from a probability distribution function $f(\alpha)$. If $\alpha = 0$, no wind capacity is available, although some is installed; if $\alpha = 1$, all installed wind capacity is available.8 In contrast to wind, we consider the coal and natural gas capacities to be fully available, i.e. their installed and available capacities always coincide.

The standard approach in the literature to solve this type of model separates the production of electricity into two stages: the decision on how much capacity

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7There are two competing schools of thought when it comes to the evolution of marginal costs for generation capacity. In the one, as more of a particular technology is installed, the learning effects dominate to drive down the cost of the technology. The other school of thought is that the evolution generation investment cost is dominated by the availability of a scarce resource — for example the high quality sites we mention here — so costs will increase in capacity. In this we follow the work of others in the field, such as Abrell et al. (2018)

8The dispatch of energy occurs for a particular value of $\alpha$, while capacity is built prior to dispatch, based on a distribution of $\alpha$. 

to build, and the decision on how much energy to produce with this installed capacity. We name these the investment and dispatch stages respectively. This division reflects the constraints placed upon the design of a system: the quantity of electricity dispatched can only be produced after the requisite generation capacity has been installed. Counterintuitively, the dispatch stage is solved first, yielding the equilibrium quantity of electricity that is generated; this result is used in the investment stage to solve for the quantity of capacity required to produce this electrical energy. We refer to this process as a backward solving, stage-wise approach.

In the dispatch stage and for a given value of \( \alpha \), welfare is the utility derived from consuming electricity, \( U(q(\alpha)) \), less the cost of producing said quantity:

\[
S_q(q(\alpha)) = U(q(\alpha)) - \sum_f b_f q_f(\alpha) \quad \forall f \in \{c, g\}
\]

where \( q(\alpha) = \sum_i q_i(\alpha) \) \( \forall i \in \{w, c, g\} \)

\[
\bar{\mathcal{M}}U(\cdot) > 0 \quad \bar{\mathcal{M}}U'(\cdot) < 0
\]

The quantity of energy calculated in this stage feeds into the investment stage, which we solve to obtain the complete results of the model.

The investment stage considers the total expected welfare. This is gained by consuming the quantity of electricity — determined in the dispatch stage — generated from the available capacity, less the cost of installing said capacity. The subscript \( K \) denotes that this is the capacity stage.

\[
S_K = \mathbb{E}[S_q(q(\alpha))] - \sum_i \frac{1}{2} \beta_i K_i^2 \quad \forall i \in \{w, c, g\}
\]

The solution to this problem will vary depending on the relative costs of the fossil fuel generators — amongst other things. However, as the two fossil fuels are identical in all ways other than cost, the solution of the case in which natural gas is costlier to operate than coal will be similar in form to the solution of the case in which natural gas is cheaper to operate than coal. To simplify the presentation of our model, we will only consider the cases in which natural gas is costlier than coal \( (b_c < b_g) \). We choose this case, because historically the cost of operating natural gas plants has been higher than that of coal plants. The model could also be solved with the opposite assumption. Nevertheless, in the interest of having a straightforward presentation of our results, we do not consider the case of coal being costlier to operate than natural gas.

With the model’s ingredients in mind, we can move on to the next steps of our analysis. We begin with the social planner’s problem, followed by the decentralised equilibrium. The social planner’s solution consists of two cases, which serve as benchmarks against which we can assess how the decentralised equilibrium does not replicate first best. Finally, we will examine what occurs when a subsidy to wind is used in the decentralised equilibrium as a policy measure.
3 Social planner’s solution

The first step of the solution is the dispatch stage. We use the results as inputs in the investment stage, according to the backward solving, stage-wise solution approach we discussed in the setup of the model. As presented in the previous section, given capacities $K_i$ and a measure of availability for wind $\alpha$, the social planner’s problem is the utility gained from a quantity of electricity, less the cost of producing said electricity. The production of electricity is constrained by the available capacity of each technology, which in the case of wind is limited by the factor $\alpha$. The problem reads:

$$\max_{q_i(a) > 0} \left\{ S_q := U(q(a)) - \sum_f (b_f + e_f)q_f(a) \right\} \quad \forall f \in \{c, g\}$$

such that

$$q(a) = \sum_i q_i(a) \quad \forall i \in \{w, c, g\}$$

$$K_f \geq q_f(a) \quad \forall f \in \{c, g\}$$

$$\alpha K_w \geq q_w(a)$$

Recall that electricity quantities from different technologies are perfect substitutes in terms of utility. The Lagrangian of this stage is:

$$\mathcal{L}_q = U(\sum_i q_i(a)) - \sum_f (b_f + e_f)q_f(a) + \sum_f \lambda_f(a)\left[K_f - q_f(a)\right] + \lambda_w(a)\left[\alpha K_w - q_w(a)\right]$$

(3.1)

We obtain the first order conditions from the Lagrangian’s partial derivatives, the superscript $SP$ denotes that we are in the social planner’s solution:

$$\frac{\partial \mathcal{L}_q}{\partial q_w(a)} = 0 \Rightarrow M U(\sum_i q_i^{SP}(a)) - \lambda_w^{SP}(a) = 0 \quad (3.2)$$

$$\frac{\partial \mathcal{L}_q}{\partial q_c(a)} = 0 \Rightarrow M U(\sum_i q_i^{SP}(a)) - (b_c + e_c) - \lambda_c^{SP}(a) = 0 \quad (3.3)$$

$$\frac{\partial \mathcal{L}_q}{\partial q_g(a)} = 0 \Rightarrow M U(\sum_i q_i^{SP}(a)) - (b_g + e_g) - \lambda_g^{SP}(a) = 0 \quad (3.4)$$

These first order conditions tell us a few things about the amount of electricity generated by each technology, which we discuss later in this section; their results will feed into the investment stage of the problem, from which we obtain our complete results. However to do this, we have to consider how solving the model is affected by our assumptions on the relative levels of the operating and external costs.

The social planner’s problem depends on the sum of operating and external costs, $b_f + e_f$. As we have made no assumption on the level of these parameters, our assumption on the operating costs of fossil fuels, $b_c < b_g$, and the fact that natural gas is less polluting than coal, $e_g < e_c$, mean the social planner faces one of two cases. In the case of expensive natural gas, the social planner’s cost of natural gas is higher than that of coal: $b_c + e_c < b_g + e_g$. In the case of cheap natural gas, the social planner’s cost of natural gas is lower than that of coal:
Both of these cases will serve as benchmarks to assess how effectively a subsidy restores first best in the decentralised equilibrium, which we do in section 5. To continue solving the social planner’s problem, we need to apply our assumption about the relative operating costs of natural gas and coal. We begin with the expensive natural gas case, and continue with the cheap natural gas case; the steps taken to solve each case are similar, so in the cheap natural gas case, we will keep the analysis concise at the points that are similar.

### 3.1 Expensive natural gas

When the social planner’s operating costs of coal are lower than those of natural gas, \( b_c + e_c < b_g + e_g \), the interior solution assumption is that the marginal utility from the fully used technologies — wind and coal — is greater than the social cost of operating coal, i.e. \( MU(K_w + K_c) > b_g + e_g \). In this part of the social planner’s problem, coal is always fully used because it is cheaper than natural gas. Note that capacities are exogenous at this stage of the problem. The Lagrange multiplier for wind is always positive in the relation controlling the dispatch of wind (3.4), so at least some electrical energy will be generated from wind.

Natural gas is the most expensive technology available for use; the amount used is driven by the available capacity of wind \( aK_w \). This in turn depends on the threshold value of \( \alpha \). When \( \alpha \) is above the threshold \( \alpha_1 \), natural gas will only be partially used, as there is a large amount of wind energy being generated. When \( \alpha \) falls below this threshold, natural gas capacity is fully used. The threshold \( \alpha_1 \) is determined from (3.4), the relation derived from the marginal technology in the current case:

\[
MU(aK_w + K_c + K_g) \geq b_g + e_g
\]

The threshold then is:

\[
\alpha_1 = \frac{MU^{-1}(b_g + e_g) - K_c - K_g}{K_w}, \quad \alpha_1 \in (0, 1)
\]

By definition, the threshold \( \alpha_1 \) is the highest value of \( \alpha \) for which all three technologies fully use their installed capacity, i.e. \( MU(a_1K_w + K_c + K_g) = b_g + e_g \). The intuition behind this threshold can also be seen in Figure 1. If the marginal technology, in this case natural gas, becomes more expensive, that is if \( b_g + e_g \) increases, then the threshold decreases. As a result, natural gas will be partially used for more values of \( \alpha \).

When \( \alpha \) falls below this threshold, there is insufficient available wind capacity, increasing the shadow price of all technologies. The quantity of electricity consumed is then the sum of available capacities, \( aK_w + K_c + K_g \). Above the threshold, there is excess capacity of the marginal technology, in this case natural gas. The quantity of electricity consumed is the same, irrespective of the value of \( \alpha \), and equal to \( MU^{-1}(b_g + e_g) \). This process is illustrated in Figure 1; the left hand figure for the situation when \( \alpha \) falls below the threshold, the right hand figure for values of \( \alpha \) above the threshold.

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\( ^9 \)We will not consider the case where social costs for coal and natural gas are identical, \( b_c + e_c = b_g + e_g \), because then the two technologies collapse in one.
We summarise our observations on the use of the capacities of wind, coal and natural gas in three Lemmata. Superscript \textit{ex} denotes the expensive natural gas case. The order in which technologies are used depends on their operating cost: given zero operating cost for wind, its available capacity is always completely used, as described by Lemma 3.1. Lemma 3.2 describes how coal is always fully used; in this case, coal is cheaper than natural gas by assumption, and will be the next technology in the merit order. Finally, Lemma 3.3 shows that natural gas is employed to the necessary extent, depending on the value of $\alpha$.

**Lemma 3.1.** The quantity of wind dispatched will always equal the available capacity.

**Proof.** Since $\mathcal{MU}(\sum_{i} q_{i}^{\text{ex}}(\alpha)) > 0$, the Lagrange multiplier will always be positive in (3.2), $\lambda_{ex}^{w} > 0$. This implies that the capacity constraint for wind always holds: $q_{ex}^{w}(\alpha) = \alpha K_{w}$. \hfill \qed

**Lemma 3.2.** Coal is always fully used, $q_{ex}^{c} = K_{c}$.

**Proof.** As we assume that the sum of coal’s operating and carbon external costs are lower than those of natural gas, $b_{c} + e_{c} < b_{g} + e_{g}$, we conclude that $q_{ex}^{c}(\alpha) = K_{c}$. \hfill \qed

**Lemma 3.3.** Natural gas is either fully or partially used, depending on the value of the availability factor $\alpha$, as described in

$$q_{ex}^{g}(\alpha) = \begin{cases} K_{g}, & \alpha \leq \alpha_{1} \\ \mathcal{MU}^{-1}(b_{g} + e_{g}) - aK_{w} - K_{c} < K_{g}, & \alpha > \alpha_{1} \end{cases}$$

where $\alpha_{1}$ is defined as

$$\alpha_{1}(K_{w}, K_{c}, K_{g}) \equiv \frac{\mathcal{MU}^{-1}(b_{g} + e_{g}) - K_{c} - K_{g}}{K_{w}}, \quad \alpha_{1} \in (0, 1)$$

**Proof.** Building again on the assumption that coal is cheaper than natural gas, $b_{c} + e_{c} < b_{g} + e_{g}$, we conclude that $q_{ex}^{g}(\alpha) \leq K_{g}$. Depending on the wind availability, natural gas can either be fully used, i.e. $q_{ex}^{g} = K_{g}$, or it can serve the remaining demand, $q_{ex}^{g}(\alpha) = \mathcal{MU}^{-1}(b_{g} + e_{g}) - aK_{w} - K_{c} < K_{g}$. The amount of natural gas capacity we use depends on the value $\alpha$ takes relative to a threshold $\alpha_{1}$.
α₁; below it there is insufficient wind and all three technologies must be fully used, while above it, wind and coal are fully used, and natural gas is only partially used. The threshold α₁ is defined as the highest value of α for which all technologies are fully used.

We now take the output quantities from the dispatch stage and use them to solve the investment stage problem, from which we obtain the optimal capacity of each technology. The expected welfare from the dispatch stage depends on the availability of wind capacity, αₖₚ. As discussed in the model setup, the dependence of wind generation on a stochastic phenomenon means that the full installed capacity Kₚ is only available under specific wind conditions; under other conditions there may be insufficient wind to fully operate the installed capacity. To represent this, we use the stochastic variable α. The probability of each value of α occurring is drawn from a probability distribution function f(α).

We denote that this is the investment stage utility by S_K. Hence, this stage’s problem maximises the optimal expected welfare from the dispatch stage minus the costs of investment for the capacities:

\[
\max_{K_i > 0} \left\{ S_K := \int_0^{1} S^\text{ex}_q(\alpha K_w, K_c, K_g, \alpha) f(\alpha) d\alpha - \sum_i \frac{1}{2} \beta_i K_i^2 \right\} \quad \forall i \in \{w, c, g\}.
\]

As previously discussed, we will not deal with the cases which have fewer than three generation technologies. For these, the problem of a subsidy to renewables for environmental reasons is not interesting: if wind replaces a single fossil fuel emitting technology, it is possible to correctly value a subsidy to wind with regard to the external cost of carbon emitted by the single fossil fuel generator.

The investment stage Lagrangian becomes a little more complex as a result of the threshold wind availability α₁. The integral of the expected welfare is split into two parts, representing the expected welfare from the quantity of energy below this threshold, and the expected welfare above this threshold. These two are not the same, due to the different utilisation of natural gas capacity, according to Lemma 3.3.

\[
L_K = \int_0^{\alpha_1(K_w, K_c, K_g)} \left[ U(\alpha K_w + K_c + K_g) - \sum_f (b_f + e_f) K_f \right] f(\alpha) d\alpha + \\
+ \int_{\alpha_1(K_w, K_c, K_g)}^1 \left[ U(\mathcal{M} \mathcal{U}^{-1}(b_g + e_g)) - (b_c + e_c) K_c - \\
-(b_g + e_g)(\mathcal{M} \mathcal{U}^{-1}(b_g + e_g) - \alpha K_w - K_c) \right] f(\alpha) d\alpha \\
- \sum_i \frac{1}{2} \beta_i K_i^2
\]

We derive the first order conditions by applying the Leibniz integral rule to the Lagrangian and assuming a continuous uniform distribution for the availability
factor of wind $0 \leq \alpha \leq 1$, that is $f(\alpha) = 1$:

\[
\frac{\partial L_K}{\partial K_w} = 0 \implies \alpha_1(K_{ex}^{cw}, K_{ex}^{cc}, K_g^{cx}) \int_0^1 \mathcal{MU}(\alpha K_w^{cx} + K_c^{cx} + K_g^{cx}) d\alpha - \left( b_g + e_g \right) \left( \mathcal{MU}^{-1}(b_g + e_g) - \frac{K_c^{cx} - K_g^{cx}}{K_w^{cx}} \right)^2 - \beta_w K_w^{cx} + \frac{b_g + e_g}{2} = 0
\]

\[
\frac{\partial L_K}{\partial K_c} = 0 \implies \alpha_1(K_{ex}^{cw}, K_{ex}^{cc}, K_g^{cx}) \int_0^1 \mathcal{MU}(\alpha K_w^{cx} + K_c^{cx} + K_g^{cx}) d\alpha - \left( b_g + e_g \right) \frac{\mathcal{MU}^{-1}(b_g + e_g) - \frac{K_c^{cx} - K_g^{cx}}{K_w^{cx}}}{K_w^{cx}} - (b_c + e_c) - \beta_c K_c^{cx} + b_g + e_g = 0
\]

\[
\frac{\partial L_K}{\partial K_g} = 0 \implies \alpha_1(K_{ex}^{cw}, K_{ex}^{cc}, K_g^{cx}) \int_0^1 \mathcal{MU}(\alpha K_w^{cx} + K_c^{cx} + K_g^{cx}) d\alpha - \left( b_g + e_g \right) \frac{\mathcal{MU}^{-1}(b_g + e_g) - \frac{K_c^{cx} - K_g^{cx}}{K_w^{cx}}}{K_w^{cx}} - \beta_g K_g^{cx} = 0
\]

This system of three equations determines the optimal installed capacities for the three technologies. These capacities satisfy the assumption ensuring an interior solution to our problem, as long as the utility function $U$ is such that $\mathcal{MU}(K_w^{cx} + K_c^{cx}) > b_g + e_g$.

Combining and re-arranging equations for the capacity of coal (3.7) and natural gas (3.8), we obtain a condition for the equilibrium.

**Condition 1.** The total costs for the two fossil fuel technologies must be equal in the equilibrium:

\[ b_g + e_g + \beta_g K_g^{cx} = b_c + e_c + \beta_c K_c^{cx} \]

Unfortunately the form of the first order conditions for the capacity stage — (3.6), (3.7), (3.8) — does not lend itself to easy interpretation of the solution to the social planner's problem. To make the problem tractable, we make a simplifying assumption about the properties of $\alpha$: instead of allowing it to take any value in an interval, it can only take two discrete values on this interval: $a_L \leq \alpha_1$ or $a_H > \alpha_1$, i.e. $\alpha$ is now a discrete stochastic variable. There is an equal probability that each value occurs $Pr(\alpha = a_L) = Pr(\alpha = a_H) = \frac{1}{2}$, and that gives us the probability mass function. We justify this assumption by arguing that the key difference between values is whether they lie above or below the threshold $\alpha_1$, identified in Lemma 3.3. Above it, every value has the
same effect on our problem: natural gas is only partially used, while below it, natural gas is fully used. Collapsing all values below or above the threshold to discrete values does not change our result, but does make it more tractable. The Lagrangian for capacity (3.5) can be re-written as,

\[
\mathcal{L}_K = \frac{1}{2} \left[ U(\alpha_L K_w + K_c + K_g^s) - \sum_f (b_f + e_f) K_f \right] + \\
+ \frac{1}{2} \left[ U(\mathcal{M}u^{-1}(b_g^s + e_g^s)) - (b_c + e_c^s) K_c - \\
- (b_g^s + e_g^s)(\mathcal{M}u^{-1}(b_g^s + e_g^s) - \alpha_H K_w - K_c) \right] - \\
\sum_i \frac{1}{2} \beta_i K_i^2
\]

(3.9)
and the first order conditions (3.6), (3.7) and (3.8), re-computed as:

\[
\mathcal{M}u(\alpha_L K_w^c + K_c^c + K_g^c) = \frac{2\beta_w K_w^c}{\alpha_L} - \frac{\alpha_H}{\alpha_L} (b_g^s + e_g^s) 
\]

(3.10)
\[
\mathcal{M}u(\alpha_L K_w^c + K_c^c + K_g^c) = 2\beta_c K_c^c + 2(b_c + e_c) - (b_g^s + e_g^s)
\]

(3.11)
\[
\mathcal{M}u(\alpha_L K_w^c + K_c^c + K_g^c) = 2\beta_g K_g^c + b_g^s + e_g^s
\]

(3.12)
It can be easily verified that Condition 1 still holds by combining (3.11) and (3.12). The system of equations determining \(K_w^c\), \(K_c^c\) and \(K_g^c\) is:

\[
\mathcal{M}u\left(\frac{\alpha_L ((\alpha_H + \alpha_L)(b_g^s + e_g^s) + 2\alpha_L \beta_g K_g^c)}{2\beta_w} + \\
\frac{\beta_c K_c^c + b_g^s + e_g^s - (b_c + e_c)}{\beta_c} + K_g^c\right) = 2\beta_g K_g^c + b_g^s + e_g
\]

(3.13)
\[
K_w^c = \frac{(\alpha_H + \alpha_L)(b_g^s + e_g^s) + 2\alpha_L \beta_g K_g^c}{2\beta_w}
\]

(3.14)
\[
K_c^c = \frac{\beta_c K_c^c + b_g^s + e_g^s - (b_c + e_c)}{\beta_c}
\]

(3.15)
These three equations provide us with the benchmark to assess whether the application of a subsidy to the decentralised equilibrium in section 4 replicates the social planner solution. These equations also allow us to draw some initial conclusions: the high and low values of \(\alpha\) have a first order effect only on the optimal capacity of wind, while they have a second order effect on the thermal capacities, through \(K_w^c\).

Before moving to the social planner problem in the case of cheap natural gas, it is worth discussing a special case of the wind availability \(\alpha\), and how this affects both the wind and the fossil fuel technologies. If \(\alpha_L\) took its lower possible value, \(\alpha_L = 0\), then there would be a decoupling between wind and thermal capacities in the system of equations (3.13), (3.14) and (3.15). Below the threshold \(\alpha_1\) there is now no wind capacity available, and thermal capacities compete with each other in supplying electricity; their equilibrium capacities are determined only by their relative costs. Wind capacity is only available when \(\alpha\) is above the threshold \(\alpha_1\), where it competes with the marginal technology, in this case natural gas.
3.2 Cheap natural gas

We now consider the case where the social planner’s cost of natural gas is lower than that of coal, \( b_g + e_g < b_c + e_c \). The switch from natural gas being the most expensive to the cheapest reverses the merit order of the two thermal technologies. Even though the operating cost of coal is lower, when its level is close enough to that of natural gas, combining it with its carbon external cost could mean that the social planner considers it more expensive than natural gas. Natural gas becomes the technology which is always fully used, while coal serves the remaining demand; the two thermal technologies have exchanged roles compared to the expensive natural gas case. Note that the condition to ensure interior solutions now reads \( \mathcal{MU}(K_w + K_g) > b_c + e_c \).

The first order conditions (3.2), (3.3), (3.4) give the solution to the dispatch stage; these apply for both the case of expensive natural gas, dealt with previously, and the case that we are about to analyse. While Lemma 3.1, which describes the amount of wind generated, still holds as is, Lemmata 3.2 and 3.3, describing coal and natural gas generation respectively, essentially switch positions: natural gas is now fully used, so behaves as coal did in the case of expensive natural gas, while coal is the marginal technology, whose use depends on the realised value of \( \alpha \). The value of the threshold can be found similarly to the expensive natural gas case to be:

\[
\alpha_2 \equiv \frac{\mathcal{MU}^{-1}(b_c + e_c) - K_c - K_g}{K_w}, \quad \alpha_2 \in (0, 1)
\]

Once again, a more expensive marginal technology results in a lower threshold \( \alpha_2 \), which in turn leads to coal being partially used for more realised values of \( \alpha \). These changes are illustrated in Figure 2; which can be contrasted to Figure 1, illustrating the dispatch stage solution for the case of expensive natural gas. The two solutions are identical in form, and differ only in the order with which the three technologies are dispatched. Wind is again the first to be used, generating the available capacity \( \alpha K_w \), now followed by the cheap(er) natural gas and the coal, which is now the marginal technology. Regarding the total quantity of electricity consumed, it is equal to \( \alpha K_w + K_g + K_c \) when \( \alpha \leq \alpha_2 \), and equal to \( \mathcal{MU}^{-1}(b_c + e_c) \) when \( \alpha > \alpha_2 \).

We also restate the Lemmata to reflect the new merit order of the thermal technologies.

**Lemma 3.4** (Restating Lemma 3.2 for cheap natural gas). Assuming natural gas is cheaper than coal, \( b_g + e_g < b_c + e_c \), we can conclude that natural gas is always fully used, \( q_{ch}^g = K_g \).

**Lemma 3.5** (Restating Lemma 3.3 for cheap natural gas). When \( b_g + e_g < b_c + e_c \), we can conclude that

\[
q_{ch}^c(\alpha) = \begin{cases} 
K_c, & \alpha \leq \alpha_2 \\
\mathcal{MU}^{-1}(b_c + e_c) - \alpha K_w - K_g < K_c, & \alpha > \alpha_2
\end{cases}
\]

where \( \alpha_2 \) is defined as

\[
\alpha_2(K_w, K_c, K_g) \equiv \frac{\mathcal{MU}^{-1}(b_c + e_c) - K_c - K_g}{K_w}, \quad \alpha_2 \in (0, 1)
\]
Figure 2: Social planner dispatch solution for cheap natural gas

As in the expensive natural gas case, we simplify $\alpha$ by assuming it takes two values, above and below the threshold $\alpha_2$, i.e. $0 < \alpha_L < \alpha < \alpha_H < 1$. After applying our simplifying assumptions about $\alpha$, the Lagrangian for the capacity stage can be written as:

$$L_K = \frac{1}{2} \left[ U(\alpha_LK_w + K_c + K_g) - \sum_f (b_f + e_f)K_f \right] +$$

$$+ \frac{1}{2} \left[ U(\mathcal{MU}^{-1}(b_c + e_c)) - (b_g + e_g)K_g - (b_c + e_c)(\mathcal{MU}^{-1}(b_c + e_c) - \alpha_H K_w - K_g) \right] -$$

$$- \sum_i \frac{1}{2} \beta_i K_i^2$$

(3.16)

**Condition 1**, referring to the requirement that operating cost of the two fossil technologies are equal, still holds, hence we can modify the first order conditions (3.10), (3.11), (3.12) to derive the capacities for the cheap natural gas case, denoted by the superscript $ch$:

$$\mathcal{MU}(\alpha_L K_w^{ch} + K_c^{ch} + K_g^{ch}) = \frac{2\beta_w K_w^{ch}}{\alpha_L} - \frac{\alpha_H}{\alpha_L} (b_c + e_c)$$

(3.17)

$$\mathcal{MU}(\alpha_L K_w^{ch} + K_c^{ch} + K_g^{ch}) = \frac{2\beta_g K_g^{ch}}{\alpha_L} + 2(b_g + e_g) - (b_c + e_c)$$

(3.18)

$$\mathcal{MU}(\alpha_L K_w^{ch} + K_c^{ch} + K_g^{ch}) = \frac{2\beta_c K_c^{ch}}{\alpha_L} + b_c + e_c$$

(3.19)

These first order conditions can be rewritten as:

$$\mathcal{MU} \left( \frac{\alpha_L((\alpha_H + \alpha_L)(b_c + e_c) + 2\alpha_L \beta_c K_c^{ch})}{2\beta_w} + K_c^{ch} \right) + K_c^{ch}$$

$$+ \frac{\beta_c K_c^{ch} + b_c + e_c - (b_g + e_g)}{\beta_g} = 2\beta_c K_c^{ch} + b_c + e_c$$

(3.20)

$$K_w^{ch} = \frac{(\alpha_H + \alpha_L)(b_c + e_c) + 2\alpha_L \beta_c K_c^{ch}}{2\beta_w}$$

(3.21)

$$K_g^{ch} = \frac{\beta_c K_c^{ch} + b_c + e_c - (b_g + e_g)}{\beta_g}$$

(3.22)
Recall that in order for $K_{ch}^w$, $K_{ch}^g$ and $K_{ch}^c$ to be positive, we have already assumed a high enough utility function that results in $\mathcal{MU}(K_{ch}^w + K_{ch}^g) > b_c + e_c$.

The comments on the extreme value of $\alpha_L$, in the previous case of expensive natural gas, still hold: if $\alpha_L$ becomes zero, wind optimal capacity, $K_{ch}^w$, only depends on its own investment cost and the operating cost of the coal, which is now the marginal technology. In the complete absence of wind, when $\alpha = \alpha_L = 0$, thermal capacities are decided according to their relative costs. Additionally, the values of $\alpha$ directly affect only the wind capacity, and indirectly the thermal capacities.

The key difference between the two cases of the social planner, namely expensive natural gas and cheap natural gas, is the order of the fossil fuels in the supply curve. In both cases wind, being the cheapest generator to operate, is used to the extent permitted by $\alpha$. However, while natural gas is the marginal technology in the expensive natural gas case, this role is taken on by coal in the cheap natural gas case, due to the difference in operating cost being smaller than the difference in the cost of the carbon externality, $b_g - b_c < e_c - e_g$. This change in the merit order has some interesting implications, when the policy measure will be introduced later in our analysis.

Equations (3.20), (3.21) and (3.22) complete the analysis for the solution planner’s planner. Together with the respective equations from the expensive natural gas case, they are what the decentralised equilibrium ideally produces. However as we will see, this does not occur. We next analyse which generation technologies are used when the external cost of carbon is not considered, and apply a policy to the decentralised equilibrium to correct this. We will then be able to compare the decentralised equilibrium with a policy to the two cases of the social planner’s solution.

## 4 Decentralised equilibrium problem

The decentralised equilibrium models a market under perfect competition; this market structure is often chosen as a starting point for policy analysis, since it allows for the possible effects on the generation technologies to be clearly understood\(^\text{10}\). We assume representative producers for each of the three types of generation technology — i.e. wind $w$, coal $c$, and gas $g$ — and a representative consumer who gains utility $\mathcal{U}$ from consuming electrical energy. These representative agents only face their private costs and benefits and ignore the external carbon costs of the two thermal technologies; compared to the social planner’s problem, the thermal technology agents only face operating costs $b_c$ and $b_g$. We continue with our simplifying assumption that coal is cheaper to operate than natural gas, $b_c < b_g$.\(^\text{11}\) In contrast to the social planner’s solution, in the decentralised equilibrium problem the external carbon costs are not taken

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\(^\text{10}\)A number of papers do examine how the electricity markets operate under other market structures, for example an oligopolistic market structure in Borenstein et al. (1999), Borenstein (2002), or Joskow and Tirole (2007).

\(^\text{11}\)As we discussed in the model setup, this is done to have a straightforward presentation of the results, and the solution would have a similar structure if natural gas was cheaper to operate than coal.
into account; we do not have to consider what happens when the external carbon costs make coal the most socially expensive technology. To ensure we have an interior solution to our problem so all three technologies are installed, we again assume that $MU(K_w + K_c) > b_g + e_g > b_g$ for all capacities, including the equilibrium ones.

Wind availability is the key determinant for prices and quantities of consumed electricity. We can again define a threshold $\alpha_3$ below which all three technologies are fully used, but above which the marginal technology — in this case natural gas — is only partially used:

$$\alpha_3 \equiv \frac{MU^{-1}(b_g) - K_c - K_g}{K_w}, \quad \alpha_3 \in (0, 1)$$

where $K_w$, $K_c$ and $K_g$ are the decentralised equilibrium’s installed capacities.

As when solving the social planner’s problem, we make the simplifying assumption that $\alpha$ can only take two discrete values with equal probabilities, $\alpha_L$ and $\alpha_H$, below and above the threshold $\alpha_3$ respectively. We illustrate the two outcomes in Figure 3. Below the threshold, when $\alpha = \alpha_L \leq \alpha_3$, the quantity of electricity consumed is the sum of the available wind capacity and the fully used installed thermal capacities: $q_L \equiv \alpha_L K_w + K_c + K_g$. The price all producers receive and the consumer pays is denoted by $p_L$ when wind availability is low, hence the subscript $L$. Above the threshold, when $\alpha = \alpha_H > \alpha_3$, the quantity of electricity consumed is the intersection of the marginal utility curve with the operating costs of natural gas, i.e. $MU^{-1}(b_g)$. The technology mix now fully uses the available wind, $\alpha_H K_w$, and installed coal, $K_c$, while the rest of the energy consumed is covered by natural gas. The price that clears the market is the marginal cost of the last technology that is employed, $p_H = b_g$.

As we now have the quantity of energy consumed for each realised value of $\alpha$, we can formulate and specify equilibrium installed capacities for each of the three technologies. This follows the two-stage, backward solving approach of starting with the dispatch stage, then feeding the results back into the investment stage. The producers maximise profits, less their operating and investment costs, given the probability density function for $\alpha$. When electricity is abundant, the
natural gas producer’s revenues and cost cancel each other out, since natural gas is the marginal technology in the mix. The maximisation problems for the wind, coal and natural gas producers respectively are:

\[
\max_{K_w>0} \left\{ \frac{1}{2} p_L \alpha L K_w + \frac{1}{2} b_g \alpha H K_w - \frac{1}{2} \beta_w K_w^2 \right\}
\]

\[
\max_{K_c>0} \left\{ \frac{1}{2} p_L K_c + \frac{1}{2} b_g K_c - b_c K_c - \frac{1}{2} \beta_c K_c^2 \right\}
\]

\[
\max_{K_g>0} \left\{ \frac{1}{2} p_L K_g - \frac{1}{2} b_g K_g - \frac{1}{2} \beta_g K_g^2 \right\}
\]

The representative consumer maximises utility, less the cost of paying for electricity:

\[
\max_{q_L>0} \left\{ \frac{1}{2} U(q_L) + \frac{1}{2} U \left( \mathcal{M}U^{-1}(b_g) \right) - \frac{1}{2} p_L q_L - \frac{1}{2} b_g \mathcal{M}U^{-1}(b_g) \right\}
\]

The first order conditions of the producers of wind, coal and natural gas respectively, are:

\[
\frac{1}{2} \left[ p_L^* \alpha L + b_g \alpha H \right] = \beta_w K_w^*
\]

\[
\frac{1}{2} \left[ p_L^* + b_g \right] - b_c = \beta_c K_c^*
\]

\[
\frac{1}{2} \left[ p_L^* - b_g \right] = \beta_g K_g^*
\]

while the first order condition of the consumer is:

\[
\mathcal{M}U(q_L^*) = p_L^*
\]

where \( q_L^* \equiv \alpha_L K_w^* + K_c^* + K_g^* \), as we have seen in the dispatch stage of the decentralised equilibrium.

The first order conditions for wind, coal and natural gas, (4.1), (4.2) and (4.3), are consistent with the economic theory: the wind producer is compensated on a weighted average of high and low prices that is based on the availability values of the wind resource, while the coal producer receives the average of the high and low prices less operating costs. The natural gas producer only makes a positive profit when electrical energy is scarce, i.e. when wind is below the threshold value \( \alpha = \alpha_L \) and prices are high. Investment costs influence the installed capacities of all three technologies; these costs need to be compensated.

The equilibrium installed capacities are given by the system of first order conditions for the three representative producers and the consumer, in (4.1), (4.2), (4.3) and (4.4). We re-write this as:

\[
\mathcal{M}U(\alpha_L K_w^* + K_c^* + K_g^*) = \frac{2\beta_w K_w^*}{\alpha_L} - \frac{\alpha_H}{\alpha_L} b_g
\]

\[
\mathcal{M}U(\alpha_L K_w^* + K_c^* + K_g^*) = 2\beta_c K_c^* + 2b_c - b_g
\]

\[
\mathcal{M}U(\alpha_L K_w^* + K_c^* + K_g^*) = 2\beta_g K_g^* + b_g
\]
To obtain open form solutions for all technologies, we re-arrange this system further:

\[
\begin{align*}
M \mathcal{U} & \left( \frac{\alpha_L ((\alpha_H + \alpha_L) b_g + 2\alpha_L \beta_g K_g^w)}{2\beta_w} + \frac{\beta_g K_g^w + b_g - b_c}{\beta_c} + K_g^w \right) = 2\beta_g K_g^w + b_g \\
K_g^w &= \frac{(\alpha_H + \alpha_L) b_g + 2\alpha_L \beta_g K_g^w}{2\beta_w} \\
K_c^e &= \frac{\beta_g K_g^w + b_g - b_c}{\beta_c}
\end{align*}
\]  
(4.8) (4.9) (4.10)

Similarly to the expensive natural gas case, the equilibrium quantities are equal to the available capacity for wind, \(q_g^w = \alpha K_g^w\), and the total capacity for coal, \(q_c^e = K_c^e\). Natural gas's equilibrium quantity \(q_g^w\) depends on whether \(\alpha\) is below or above the threshold \(\alpha_3\). Below it, \(\alpha = \alpha_L < \alpha_3\), the quantity is equal to the total capacity, \(q_g^w = K_g^w\). Above the threshold, \(\alpha = \alpha_H > \alpha_3\), the equilibrium quantity is the energy demand that the wind and coal cannot serve, \(q_g^w = M \mathcal{U}^{-1}(b_g) - \alpha_H K_g^w - K_c^e\).

The decentralised equilibrium fails to replicate the social planner's solutions. The number of market failures varies depending on which thermal technology is most expensive for the social planner; the two cases that we have labelled expensive natural gas and cheap natural gas. This can be confirmed by comparing the decentralised equilibrium's equations for optimal capacities, (4.8), (4.9), (4.10), to the systems of equations for optimal capacities in the expensive natural gas case, (3.13), (3.14), (3.15), and the cheap natural gas case, (3.20), (3.21), (3.22).

Compared to the expensive natural gas case, the market failure is one of sub-optimal investments in capacity. The cost levels differ between the social planner's case and the decentralised equilibrium — the latter ignores the external cost of carbon — and that leads to the decentralised equilibrium not replicating the social planner's solution. However, the order of technologies in the dispatch curve remains the same, so there is only one failure.

The more interesting market failures occur in comparison to the cheap natural gas case. Here, the external costs of carbon alter the order in which thermal technologies are dispatched. Compared to the social planner's solution, the market not only installs sub-optimal amounts of generator capacity, it also dispatches thermal technologies in the wrong order. While the social planner fully uses natural gas and serves the remaining demand with coal, the decentralised equilibrium has coal being fully used and the natural gas covering the difference. Therefore, in addition to the market failure of sub-optimal investments, there is a second market failure of wrong merit order and higher usage of the more polluting thermal technology.
5 Effect of a Subsidy for Wind Energy

Subsidies are a well-established policy for supporting renewable energy generators. However, they may be ineffective at restoring the first best, as defined by the social planner. In this section, we examine whether such a subsidy is effective at correcting the failures we identified in the decentralised equilibrium, in section 4.

In our model, variable renewables are supported through a subsidy; under the assumed scheme, a policy maker fully commits to offering a payment in addition to the market price for each unit of electrical energy from a wind generator; this is known as a feed-in premium scheme\(^{12}\). We modify the decentralised equilibrium to reflect this payment: in addition to the market price the wind producer is paid a premium \(\sigma\). The optimisation problem becomes:

\[
\max_{K_w > 0} \left\{ \frac{1}{2} p_L a_L K_w + \frac{1}{2} b_g a_H K_w + \frac{1}{2} \sigma (a_L + a_H) K_w - \frac{1}{2} \beta_w K_w^2 \right\}
\]

The subsidy does not directly affect the thermal producers, so their maximisation problem remains identical to the decentralised equilibrium:

coal producer \[
\max_{K_c > 0} \left\{ \frac{1}{2} p_L K_c + \frac{1}{2} b_g K_c - b_c K_c - \frac{1}{2} \beta_c K_c^2 \right\}
\]

natural gas producer \[
\max_{K_g > 0} \left\{ \frac{1}{2} p_L K_g - \frac{1}{2} b_g K_g - \frac{1}{2} \beta_h K_g^2 \right\}
\]
as does the maximisation problem of the representative consumer,

\[
\max_{q_L > 0} \left\{ \frac{1}{2} U(q_L) + \frac{1}{2} U\left( MU^{-1}(b_g) \right) - \frac{1}{2} p_L q_L - \frac{1}{2} b_g MU^{-1}(b_g) \right\}
\]

The first order condition for wind is modified by the subsidy,

\[
\frac{1}{2} \left[ (p_L + \sigma) a_L + (b_g + \sigma) a_H \right] = \beta_w \tilde{K}_w \quad (5.1)
\]

while those of coal and natural gas remain the same:

\[
\frac{1}{2} \left[ \tilde{p}_L + b_g \right] - b_c = \beta_c \tilde{K}_c \quad (5.2)
\]

\[
\frac{1}{2} \left[ \tilde{p}_L - b_g \right] = \beta_g \tilde{K}_g \quad (5.3)
\]

Finally, the first order condition of the representative consumer also remains unchanged:

\[
MU(q_L) = \tilde{p}_L \quad (5.4)
\]

where \(q_L = a_L \tilde{K}_w + \tilde{K}_c + \tilde{K}_g\), similar to the decentralised equilibrium without subsidy.

\(^{12}\)Such a policy is not equivalent to a Pigouvian tax that forces the CO2 emitter to internalise the cost of carbon.
Combining the four first order conditions, we specify the system of equations controlling the installed capacities of all technologies:

\[ \mathcal{M}U(\alpha L \tilde{K}_w + \tilde{K}_c + \tilde{K}_g) = \frac{2 \beta w \tilde{K}_w}{\alpha L} - \frac{\alpha H}{\alpha L} b_g - \left( \frac{\alpha H}{\alpha L} + 1 \right) \sigma \]

\[ \mathcal{M}U(\alpha L \tilde{K}_w + \tilde{K}_c + \tilde{K}_g) = 2 \beta w \tilde{K}_w + 2b_c - b_g \]

\[ \mathcal{M}U(\alpha L \tilde{K}_w + \tilde{K}_c + \tilde{K}_g) = 2 \beta g \tilde{K}_g + b_g \]

Re-arranging these we obtain:

\[ \mathcal{M}U \left( \frac{\alpha L ((a_H + \alpha L)(b_g + \sigma) + 2 \alpha L \beta g \tilde{K}_g)}{2 \beta w} + \frac{\beta g \tilde{K}_g + b_g - b_c}{\beta c} + \tilde{K}_g \right) = 2 \beta g \tilde{K}_g + b_g \quad (5.5) \]

\[ \tilde{K}_w = \frac{(a_H + \alpha L)(b_g + \sigma) + 2 \alpha L \beta g \tilde{K}_g}{2 \beta w} \quad (5.6) \]

\[ \tilde{K}_c = \frac{\beta g \tilde{K}_g + b_g - b_c}{\beta c} \quad (5.7) \]

These relations show that the installed capacities of all technologies depend on the policy \( \sigma \): the subsidy affects the capacity stage of our model, even though it is awarded via that quantity of energy generated by the renewable technology. Although installed capacities for all technologies can be manipulated somewhat by \( \sigma \), there is no subsidy that could replicate the optimal solution, as defined by the social planner’s solution. The subsidy either fails to replicate the optimal investments of the expensive natural gas case, or fails to replicate both the optimal merit order and optimal investments of the cheap natural gas case.

We verify this failure by comparing the system of equations that give the capacities of natural gas, wind and coal in the decentralised equilibrium in the presence of a subsidy, (5.5), (5.6) (5.7), to the respective systems of equations for equilibrium capacities in the expensive natural gas case, (3.13), (3.14), (3.15), and the respective equations for the cheap natural gas case, (3.20), (3.21), (3.22). For example, we could try to obtain the optimal wind capacity with the subsidy, by setting it to the price of carbon of the marginal technology, natural gas: \( \sigma = e_g \). Although this would make the equation controlling wind capacity, (5.6), identical to the equation controlling wind capacity in the social planner’s case:

\[ (3.14) \Rightarrow K_{w}^{ex} = \frac{(a_H + \alpha L)(b_g + e_g) + 2 \alpha L \beta g K_g^{ex}}{2 \beta w} \]

we would still fail to replicate the complete expensive natural gas case of the social planner. There is no way the decentralised equilibrium equation for natural gas capacity, (5.5), can include the carbon externality costs of both fossil fuel technologies, \( e_g \) & \( e_c \); it can never be made identical to the corresponding
equation for natural gas in the social planner's problem,

\[(3.13) \Rightarrow MU \left( \frac{a_L((\alpha_H + \alpha_L)(b_g + e_g) + 2a_L\beta_gK^{cx}_g)}{2\beta_w} + \frac{\beta_gK^{cx}_g + b_g + e_g - (b_c + e_c)\beta_c}{\beta_c} + K^{cx}_g \right) = 2\beta_gK^{cx}_g + b_g + e_g.\]

The absence of a policy measure in the equations controlling the capacity of coal and natural gas, (5.5), (5.7), makes the subsidy unable to correct the market failure of sub-optimal investments, let alone the two market failures — the incorrect merit order and sub-optimal investments — combined.

Apart from the capacity, the subsidy affects the price of electricity when there is insufficient wind. According to the economic theory (Fischer, 2010), a subsidy to variable renewables should decrease the market price when electricity is scarce. To verify this for our model, we take the derivatives of the first order conditions for wind, coal, natural gas, and the consumer with respect to the subsidy \(\sigma\); i.e. (5.1), (5.2), (5.3) and (5.4) respectively. These are:

\[
\frac{dK_w}{d\sigma} = \frac{1}{2\beta_w} \left[ a_L \left( \frac{d\tilde{p}_L}{d\sigma} + 1 \right) + \alpha_H \right] \quad (5.8)
\]

\[
\frac{dK_c}{d\sigma} = \frac{1}{2\beta_c} \frac{d\tilde{p}_L}{d\sigma} \quad (5.9)
\]

\[
\frac{dK_g}{d\sigma} = \frac{1}{2\beta_g} \frac{d\tilde{p}_L}{d\sigma} \quad (5.10)
\]

\[
\left[ \frac{d\tilde{K}_w}{d\sigma} + \frac{d\tilde{K}_c}{d\sigma} + \frac{d\tilde{K}_g}{d\sigma} \right] MU' = \frac{d\tilde{p}_L}{d\sigma} \quad (5.11)
\]

Solving for the effect the subsidy has on \(\tilde{p}_L\), we obtain the following relation:

\[
\frac{d\tilde{p}_L}{d\sigma} = -\frac{a_L(a_L + \alpha_H)\beta_c\beta_g}{\alpha_L^2\beta_c\beta_g + \beta_w\beta_g + \beta_w\beta_c - \frac{2\beta_w\beta_g}{MU'}} \quad (5.12)
\]

The representative consumer's utility is increasing and concave by assumption, therefore \(\frac{d\tilde{p}_L}{d\sigma} < 0\) and a positive subsidy to variable renewables forces the price under scarcity, \(\tilde{p}_L\), to fall. Recall that when electricity is abundant — above the threshold — the price of electricity is constant and equal to the operating cost of natural gas, \(\tilde{p}_H = b_g\).

Next, we examine the effect the subsidy has on \(\tilde{K}_w\), starting from the installed wind capacity. We combine the derivative of the first order condition for wind with respect to the subsidy, (5.8), and the relation between the high price \(\tilde{p}_L\) and the subsidy, (5.12), to obtain:

\[
\frac{d\tilde{K}_w}{d\sigma} = \frac{1}{2\alpha_L^2\beta_c\beta_g + \beta_w\beta_g + \beta_w\beta_c - \frac{2\beta_w\beta_g}{MU'}} (a_L + \alpha_H)(\beta_c + \beta_g - \frac{2\beta_c\beta_g}{MU'}) > 0
\]

Two effects influence the installed wind capacity: the subsidy, which acts directly to increase \(\tilde{K}_w\), and the indirect effect of \(\tilde{p}_L\), which decreases under the subsidy’s influence, and in turn decreases the equilibrium wind capacity. Overall
the direct effect dominates, so wind capacity \( \hat{K}_w \) rises with an increase in the subsidy, \( \sigma \). The thermal generators’ capacities decrease under the indirect effect the subsidy exerts through the price, something we can see from (5.9) and (5.10). This indirect effect decreases the capacities when the subsidy \( \sigma \) rises, i.e.

\[
\frac{d\hat{K}_c}{d\sigma} < 0, \quad \frac{d\hat{K}_g}{d\sigma} < 0.
\]

A subsidy to the wind producer can change the energy mix, although it cannot restore first best capacities. Nevertheless, the subsidy reduces the carbon intensity of the system, compared to the decentralised equilibrium without a subsidy. Although this causes a certain amount of decarbonisation, it is not the optimal amount that could be achieved. To understand this decarbonisation we consider what happens when wind is scarce, \( \alpha = \alpha_L \), and when wind is plentiful, \( \alpha = \alpha_H \). When wind is scarce, the quantity of electricity consumed is \( \hat{q}_L = \alpha_L \hat{K}_w + \hat{K}_c + \hat{K}_g \). We have already proven that capacity of wind is greater under the subsidy than without it in the decentralised equilibrium, \( \hat{K}_w > K_w^* \), while the capacities of the thermal generators are lower with the subsidy than without, \( \hat{K}_c < K_c^* \) and \( \hat{K}_g < K_g^* \). When wind is abundant, the quantity of electricity is \( \hat{q}_H = MU^{-1}(b_g) \), the same as in the absence of a subsidy. However, the energy mix to produce this quantity of electricity has changed, and it consists of more wind and less carbon; the remaining demand is also served by the less carbon intensive technology, natural gas. Moreover, when natural gas is the cheaper technology for the social planner, the preferred technology is crowded out as the subsidy cannot restore the correct merit order.

The mechanism by which a per unit subsidy to the wind producer affects the electricity system highlights the need for a correct carbon price alongside supporting renewables. Sufficiently high carbon prices would ensure that the merit order is correct. The fact that thermal technologies have different carbon contents makes prevents a subsidy from replicating optimal capacities.

6 Conclusion

The risk of climate change is a significant driver for increasing the amount of CO₂ free renewable energy in the electricity system. Subsidies are the more prevalent policy to support this growth in renewable energy. In this paper, we examine the mechanisms through which a subsidy fails to restore first best, i.e. installing and using the capacities of each technology which result in the optimal merit order and CO₂ emissions. We characterise the optimal solution; this serves as a benchmark for analysing the effectiveness of a subsidy to renewables. This analysis aims to contribute to the recent literature on the effectiveness of various renewable energy support policies in the electricity sector.

Our analysis emphasises two characteristics of electricity generation: the variable output of renewable energy generators, and the heterogeneous carbon intensity of the fossil fuel based generators that the renewables displace. We show that the subsidy either fails to displace the optimal amount of CO₂ intense thermal technology, or fails to displace the “correct” thermal technology, as determined by the social planner’s solution. Whether one failure or the other occurs depends solely on conditions that are exogenous to the subsidy, i.e. the relat-
tive investment and operating costs of each carbon emitting generator. This last point underlines the importance of a carbon price: even if it isn't high enough to fully internalise the external cost of carbon, its corrective effect on the merit order of the fossil fuels would still mean the most polluting fossil fuel is being displaced.

It is worth pointing out a few key limitations of our analysis. Our model only examines a market under perfect competition. The existence of market power by some of the producer agents might change our results in ways that are unpredictable by our model, particularly if each agent is not associated with a single technology, as we assume in our setup. Our analysis does not consider other benefits of the subsidies to renewables; even if subsidies are not the most effective environmental policy, they may be implemented for other benefits that they provide, for instance supporting a nascent wind turbine or solar photovoltaic industry. Finally, our analysis does not include nuclear power, another large scale and carbon free method of electricity generation.

Subsidies alone are not an effective measure, if renewable energy is being supported as a way to reduce carbon emissions. Subsidies either do not completely exploit the abatement opportunities that exist — by displacing natural gas instead of the more polluting coal — or they cannot displace the optimal amount of fossil fuel based generation.

Having described the relation between clean renewables and the dirty thermal generators, we observe that for an environmental policy that uses subsidies, carbon prices are still helpful. Even if this price is lower than the social cost of carbon, it could ensure that the most polluting generation technology is also the most expensive for the market. This is integral to a climate policy that uses subsidies to internalise carbon costs.

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