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Incentive Pay for Policy-makers?*

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Abstract

We study how to efficiently motivate policy-makers to solve political multi-task problems. Political multi-task problems typically have outcomes that are difficult to measure. Moreover, there are conflicts among citizens about optimal policies and the agents have the power to tax the citizens to invest in better outcomes of some tasks. We develop a political agency model with two tasks and only one measurable outcome. In such an environment, policy-makers choose socially inefficient public good levels and expropriate minorities. A judicious combination of constitutional limits on taxation and incentive pay for policy-makers is second-best. Incentive pay is conditional on the public good level.

Keywords: incentive contracts, politicians, multi-task problems

JEL Classification: D72; D82

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1 Introduction and Motivation

Motivation

As a society we are interested in *good policy-making.* Good policy-making depends on how aligned policy-maker's preferences are with the public's preferences. Therefore, to achieve good policy-making, it is important to understand first how incentives can align politicians' preferences with the public's preferences and second, what attracts politicians to office.

As to the first question, think of the policy-maker as an agent and of citizens as principals in a political principal-agent problem. In a political principal-agent problem, there is a conflict of interest between the agent and the principal, similar to the standard principal-agent problem. As an additional complication, however, the society consists of heterogeneous citizens. Contrary to the standard problem, the principals themselves have diverse and often conflicting interests. For example, not all the age groups in the society prefer the same retirement reform plan.

Another difference to the standard principal-agent problem is that the policy-maker has the power to tax the principals to finance government activities. Hence, the agent's budget is determined by the agent himself. Moreover, the policy-maker can tax particular groups more than others and treat heterogeneous principals asymmetrically.

In addition, we need to take into account that a policy-maker has many tasks. Besides using his power to tax people to finance social security and social insurance, the policy-maker provides a variety of public goods such as physical safety, health services, education, or public infrastructure. In such "political multi-task problems", it is difficult to precisely measure the output of each task in the private sector. For instance, the output change from investments in public health services delivery or social insurance are difficult or impossible to capture by a single figure, while the output of other tasks is easier to measure. Examples are reduction of CO_2 emissions or public debt, or the construction of a bridge.

As discussed before, political multi-task problems involve additional aspects compared to those in the private sector. In a democracy, the standard solution to political multi-task problems is repeated elections, leading to dismissal or reappointments of incumbents. Elections make office-holders accountable to citizens for both the outputs of all tasks and the level and mode of financing government activities. However, given output measurement issues and the aforementioned characteristics of political multi-task problems, reelection incentives alone fail to ensure *good policy-making* and efficient policy choices.[1]

Understanding the characteristics of the political multi-task problem is a first step towards understanding how we can better incentivize policy-makers by using tools other than elections. This leads us to the second question. The policy-makers are driven by their preferences over power, public image, altruism, public good concerns and private consumption. While power, public image, altruism, and public good concerns are specific to a political agent, private consumption is a common interest of both non-political and political agents. In the private sector, the agent's interest in private consumption is the main reason for offering higher salaries to the agent to mitigate moral hazard and adverse selection problems. The question whether this holds for elected policy-makers as well has been recently addressed in the literature. While the theoretical models deliver ambiguous results[2], several empirical analyses show the positive impact of higher pay on the politicians' quality.[3]

In this paper, we first study the inefficiencies that arise when office-holders face political multi-task problems, highlighting the trade-off between providing public goods at the cost of taxing citizens' private good. We then show how these inefficiencies can be alleviated by traditional instruments such as constitutional limits on taxation and protection from governmental extortion. Second, we explore whether and how adding incentive contracts on tasks whose output is verifiable can improve welfare. Such incentive contracts make the policy-maker's pay and thus his consumption dependent on the output of particular tasks. Still, whether policy-makers are or remain in office is solely determined by elections. Thus, the dual mechanism—incentive contracts on particular tasks and elections—is compatible with the rules of liberal democracies. Our aim is to explore whether it is welfare-improving to use the dual mechanism in politics.

Model and Results

We consider an economy with a private and a public good. All citizens have the same preferences over the two goods, but they have heterogeneous initial endowments of private good. In our model the multi-task policy-maker imposes wealth taxes on citizens based on their initial endowments and uses the tax revenue to finance the public good. In other words, he chooses (i) the budget size and (ii) the budget allocation among citizens. A share of citizens participates in policy-making

[1] See for example Barro (1973), Ferejohn (1986) and Maskin and Tirole (2004).

[2] See e.g. Besley (2004), Messner and Polborn (2004), Poutvaara and Takalo (2007) and Mattozzi and Merlo (2008).

[3] See e.g. Ferraz and Finan (2009) and Kotakorpi and Poutvaara (2011) for empirical papers.

and the size of this group relative to the population is fixed. Characterizing the inefficiencies in our setting, we see that the policy-maker provides a suboptimal level of public good and taxes all groups of citizens inefficiently, in particular the minority group that may be fully exploited. As a corrective measure, we first investigate the effect of constitutional limitations on taxes. This is followed by an analysis of an incentive contract that makes the policy-maker's consumption of the private good dependent on his performance. The contract is conditioned on the level of provision of the public good, which is fully observable. As a third step, we examine the effect of the combination of the incentive contract and the constitutional limitation on taxes.

While it is straightforward that constitutional limits on taxation can improve welfare in our setting, as exploitation of minorities is reduced, we show that there exists a welfare-improving incentive contract for the policy-maker that yields welfare improvements both with and without constitutional limits on taxes. The policy-maker is rewarded in units of the private good, proportionally to the level of provision of the public good. Moreover, we show that under the veil of ignorance about which of the endowment groups will provide the policy-maker, there exists an incentive contract which makes everybody better off. This provides a first result how to implement incentive contracts from an ex ante perspective.

We also explore how incentive contracts could be implemented from an interim perspective when it is already clear who has the political power. In such a setting, the incentive contract is implemented if the candidate who represents the majority is willing to introduce it. We establish conditions when welfare optimal or welfare improving incentive contracts are implementable i.e., when they make a majority better off. Finally, we show that there exists an optimal level of tax protection and a suitable incentive contract that maximizes welfare, or equivalently maximizes ex ante utility of citizens. The results are illustrated by a series of examples.

Main Results and Broader Implications

The main analysis in our paper points to broader implications. Elections are the sole device citizens have to hold their legislative and executive branches of government accountable. Of course, constitutional courts and particular oversight on the executive branch limits government's power in various ways, but surely they do not provide incentives to excel in public-good provision. Hence, there appears to be a lack of further incentive devices to motivate office-holders to provide common-interest services and public goods at the level desired by citizens. The dilemma is that with multidimensional state functions and difficult-to-measure outcomes, it is a-priori difficult to introduce high-powered incentive contracts. Still, the paper suggests and shows that incentive contracts on specific tasks with verifiable outcomes enhance welfare. This approach works better, the more constrained the office-holders are in expropriating minority groups. In other words, in societies with a balanced budget sharing to provide public goods and redistributions, introducing incentive contracts may be particularly attractive.

Organization

The rest of the paper is organized as follows: Section 2 gives an overview of relevant literature. The formal model description is provided in Section 3. Section 4 deals with the effect of constitutional limits on taxation. Section 5 analyzes the effects of an incentive contract for the policy-maker. Section 6 addresses the special case in which the policy-making elite is a finite subset of the continuum society. In that case, incentive pay for policy-makers can be considered as costless for the society. Section 7 provides an analysis of the effects of incentive pay on citizens' utilities. In particular, we ask whether incentive contracts are "implementable" in the sense that a majority of citizens would support them. In Section 8, we study the joint effect of constitutional limits on taxation and of incentive pay for policy-makers. Section 9 concludes.

2 Relation to Literature

The present paper is related to three strands of literature. First, there is a considerable literature on the characteristics of political multi-task problems. In particular, Ashworth (2005) studies political multi-tasking in legislative organizations. He categorizes tasks as constituency services and policy work, and studies the effect of reelection probability on effort allocation, given the tasks have cost-complementarity.

Hatfield and i Miquel (2006) study the effort allocation problem for a multi-task politician in the executive branch, who is responsible for the provision of multiple public goods with observable outputs. Given the multiple tasks, promising reelection to the politician yields distortionary effects on effort allocation. This article emphasizes the key role of reelection as a selection tool for citizens to choose the most competent candidate rather than an incentive tool to discipline the office-holder. Ashworth and de Mesquita (2012) examine the political multi-task problem by assuming there is no cost-complementarity for the tasks so that they can eliminate distortions. They show that there is a possible trade-off between using reelection as an incentive tool and using it as a selection tool. They approach the problem of maximizing voters' welfare from an institutional design perspective, based on how voters weigh selection compared to incentivization.

Second, since Holmström and Milgrom (1991), we know that using high-powered incentive schemes in multi-task problems in the private sector may not increase the principal's utility and may even backfire when the output of some tasks is either not verifiable or only measurable with low precision.[4] While in the private sector, it may be possible to measure the aggregate performance of a CEO for instance by a single value such as the firm's value[5], this is not possible for politicians. Hence, the use of incentive contracts—ubiquitous in the private sector—appears to be impossible in the political realm. Nevertheless, we explore the use of incentive contracts in *political* multi-task problems.[6]

There have been first attempts to explore the use of incentive contracts for politicians.[7] They have been introduced by Gersbach (2003) to incentivize politicians to invest in specific long-term projects, output of which cannot help for reelection. Making the remuneration of politicians dependent on specific policies was examined in Gersbach and Liessem (2008), which considers a politician's effort problem undertaking several tasks, when this politician can serve two terms. In this paper, we examine how to motivate office-holders when we face a multi-task problem having the characteristics outlined above: the difficulty to measure the output of some tasks, a budget determined by the office-holder, the conflicting interests of citizens with each other and with the policy-maker.

3 The Model Description

We consider a society with a continuum of citizens of measure one. There are two goods, a private good and a public good. The citizens have the same preferences over consumption pairs (x, g), where x denotes private-good consumption and g denotes public good consumption. The utility function of a representative citizen, $U : \mathbb{R}^2_+ \longrightarrow \mathbb{R}$, is given by

$$U(x, g) = u(x) + g.^{8} \tag{1}$$

[4]For a complete discussion of contract theory, see Bolton and Dewatripont (2005).

[5]On the problems of CEO salaries see e.g. Frydman and Jenter (2010).

[6]In the theory of fair allocations similar problems have been dealt with from an axiomatic point of view. One important insight is that the axioms "responsibility" and "compensation" may be in conflict (see e.g. Fleurbaey (2008)).

[7]Incentive pay is an example of a broader class of the so-called "Political Contracts", surveyed in Gersbach (2012).

[8]We choose the quasilinear utility function to rule out all substitution effects. The chosen form is more convenient for the analysis of the problem at hand. However, the other form of the quasilinear utility function, $u(g) + x$, would qualitatively lead to similar results.

The function $U(x,g)$ is additively separable with $U(0,0) = 0$. The function $u(x)$ is three-times continuously differentiable, strictly increasing, and strictly concave. Furthermore, we assume that the Inada Conditions hold for $u(x)$, that is $\lim_{x\to\infty} u'(x) = 0$, $\lim_{x\to 0} u'(x) = +$"∞." A standard example of $u(x)$ with such properties is $u(x) = x^\alpha$ with $0 < \alpha < 1$.

Citizens have heterogeneous initial endowments of the private good. With probability $0 < \theta_m < \frac{1}{2}$, a citizen is endowed with ω_m units of the private good, and with probability $\frac{1}{2} \leq \theta_M < 1$ with ω_M units, where $\theta_M + \theta_m = 1$. The exogenously-given parameters θ_M, ω_M, and ω_m are common knowledge.

Once the endowments are realized and each citizen knows his own private good endowment, by Borel's Strong Law of Large Numbers, we can think of the society as divided into a share θ_m of citizens endowed with ω_m and the complementary share endowed with ω_M. We refer to the members of the groups as the *minority endowment group* and the *majority endowment group*, respectively.

The private good serves as input in public-good production. We denote the aggregate amount of private good spent on the public good by K_g and we refer to it by public-good spending, in short. More specifically, the public-good production function is given by $g = \gamma K_g$, where a unit of K_g results in the provision level g and γ is a strictly positive parameter.

In each endowment group, each citizen is an *Elite* citizens with probability μ. Members of the Elite participate in policy-making. Once each citizen knows whether or not he is an Elite citizen, the parameter $0 \leq \mu \leq 1$ expresses the share of citizens engaged in government in each endowment group. It is exogenously-given and common knowledge. An alternative interpretation of a group of Elites is the interest group that supports the policy-maker and with whom the policy-maker shares all benefits. If a citizen does not belong to the Elites, we refer to him as a member of the *Non-elites*.

The policy-maker has the same preferences over private and public good as the other citizens.[9] The policy-maker is a member of the Elites, raises taxes, and chooses the level of public-good provision. We exclude subsidies and we will introduce a condition on the distribution of endowments to ensure that both endowment groups are taxed in the socially optimal solution. In particular, the policy-maker selects the level of private-good consumption for the majority endowment group

[9] In Section 5, we consider an incentive contract which pays the policy-maker and the Elites from his endowment group a reward in terms of private-good consumption, depending on his choice of policy. Hence, the policy-maker and the Elites' objective function differs from the other citizens.

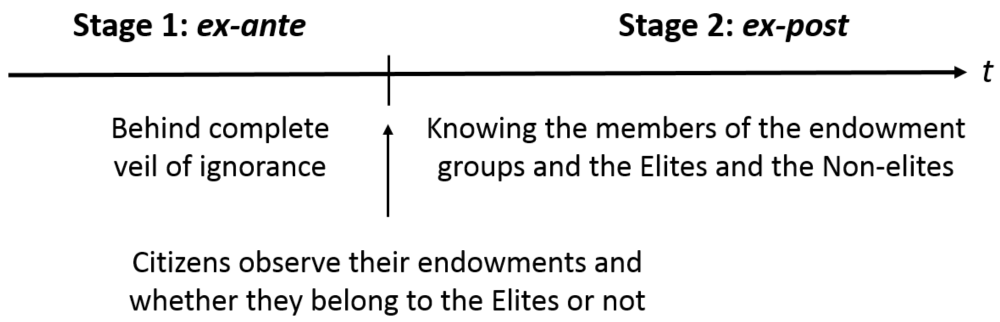

Figure 1: Timeline.

and the minority endowment group denoted by (x_M, x_m) and the public-good spending, K_g that satisfies

$$K_g = \left(\frac{1}{1+\lambda}\right)\left[\Omega - \theta_M x_M - \theta_m x_m\right], \tag{2}$$

where the society's total private-good endowment is denoted by $\Omega = \theta_M \omega_M + \theta_m \omega_m$ and the tax burden of each endowment group is given by $\theta_i(\omega_i - x_i)$, $(i = M, m)$. The parameter $(\lambda \geq 0)$ captures possible deadweight losses associated with taxation. We summarize the policy choices in the following definition.

Definition 1. *A feasible policy choice consists of a consumption plan for members of the two endowment groups* (x_M, x_m) *that satisfies* $0 \leq x_M \leq \omega_M$ *as well as* $0 \leq x_m \leq \omega_m$.
(The public-good provision with the policy choice is given by Equation (2).)

We evaluate utility and welfare at two stages. The first stage is behind a complete veil of ignorance, without information about the realization of endowments and not knowing whether an individual is a member of the Elites or the Non-elites. In the second stage, citizens observe whether they belong to the Elites or the Non-elites, and each citizen observes to which endowment group he belongs. Throughout the paper, we refer to these two stages as *ex-ante* and *ex-post*, respectively. Figure (1) shows the timeline of information revelation.

3.1 Socially Optimal Solution

As a benchmark, we consider the solution a utilitarian social planner would choose. The utilitarian social planner measures welfare by taking the sum of all citizens' utilities and maximizes the welfare function by choosing a feasible policy.

Formally, the social planner's optimization problem is given by

$$\max_{(x_M,x_m)} \qquad W(x_M,x_m) = \sum_{i=M,m} \theta_i u(x_i) + \left(\frac{\gamma}{1+\lambda}\right)\left[\Omega - \sum_{i=M,m} \theta_i x_i\right], \tag{3}$$

$$\text{subject to} \qquad 0 \le x_i \le \omega_i, \; i = M, m.$$

First, we note that if all resources in the society were spent on public-good provision, i.e. zero private-good consumption for both groups, then the marginal utility from private-good consumption would be infinite due to the Inada Conditions. Consequently, allocating zero private-good consumption is not socially optimal,

$$\lim_{x_i \to 0} \frac{\partial W}{\partial x_i} = \theta_i \underbrace{\lim_{x_i \to 0} u'(x_i)}_{=+\infty} - \frac{\gamma \theta_i}{1+\lambda} = +\infty \quad i \in \{M, m\}.$$

The following assumption guarantees that the socially optimal solution is interior:

Assumption 1. *The initial endowment of both groups satisfies*

$$u'(\omega_i) < \frac{\gamma}{1+\lambda} \quad i \in \{M, m\}.$$

With Assumption 1, if public-good spending was zero, the marginal utility from private-good consumption of both groups would be less than the constant marginal utility from public good. Thus, even a small decrease in the level of private-good consumption of both groups which yields a small increase in the level of public-good spending, is an improvement. We observe that Assumption 1 guarantees that $K_g^s = 0$ is not optimal and that at the social optimum, K_g^s is strictly positive.

Additionally, Assumption 1 states that the endowment of both groups has to be sufficiently high, such that it is socially optimal for them to participate in financing the public good. This ensures that both groups are taxed in the socially optimal solution.

We observe that at the social optimum, the private-good consumption of both groups and public-good spending are strictly positive. Thus, the socially optimal solution is interior and we find it by examining the first–order condition,

$$u'(x_i^s) = \frac{\gamma}{1+\lambda}. \tag{4}$$

Due to the strict concavity of $u(\cdot)$, there exists a unique value of x^s that satisfies Equation (4). The social optimum $(x_M, x_m) = (x^s, x^s)$ is the allocation at which the marginal welfare of the private good equals the marginal welfare of the public good. Given x^s, the implied socially optimal level of public-good spending is

$$K_g^s = \frac{1}{1+\lambda}\left[\Omega - x^s\right]. \tag{5}$$

At the socially optimal solution, social welfare cannot be improved by reshuffling resources from private consumption to the public good or by reallocating the private good between the two groups.

3.2 The Policy-maker's Optimal Solution

We next turn to the solution that is optimal from the policy-maker's point of view. The policy-maker is assumed to be a member of the majority endowment group, to reflect the majoritarian principle of democracy.

The policy-maker maximizes his utility function by choosing a feasible policy. The policy-maker's optimization problem is therefore

$$\begin{aligned} \max_{(x_M,x_m)} \quad & U(x_M,x_m) = u(x_M) + \left(\frac{\gamma}{1+\lambda}\right)\left[\Omega - \sum_{i=M,m}\theta_i x_i\right], \\ \text{subject to} \quad & 0 \le x_i \le \omega_i,\ i = M, m. \end{aligned}$$

We denote the solution to the above problem by x_M^p and x_m^p, respectively. In other words, x_M^p refers to the private-good consumption of the policy-maker's (majority) endowment group and x_m^p refers to the private-good consumption of the minority endowment group, as chosen by the policy-maker.

First, we note that the policy-maker does not derive any utility from the private-good consumption of the minority endowment group. Therefore, the policy-maker taxes this group as much as possible and sets $x_m^p = 0$.

What remains of the policy-maker's optimization problem is the trade-off between the majority endowment group's private-good consumption and public-good spending. We note that due to the Inada Conditions, at $x_M^p = 0$ the marginal utility from private-good consumption is infinite. Thus, the choice of $x_M^p = 0$ is not optimal for the policy-maker.

The following assumption ensures that the policy-maker's solution is interior and ascertains that both groups contribute to the financing of the public good:

Assumption 2. *The initial endowment of the majority endowment group satisfies*

$$u'(\omega_M) < \frac{\gamma\theta_M}{1+\lambda}. \tag{6}$$

Assumption 2 states that the majority endowment group's initial endowment is so high that makes it desirable for the policy-maker to tax his own endowment group. Thus, Assumption 2 eliminates the case in which the policy-maker subsidizes his own endowment group. Since both endowment groups are taxed by the policy-maker, public-good spending is strictly positive.

We observe that the policy-maker's choice of x_M^p is interior. To find it, we consider the first–order condition with respect to x_M^p,

$$u'\left(x_M^p\right) = \frac{\gamma\theta_M}{1+\lambda}. \tag{7}$$

And the public-good spending is given by

$$K_g^p = \frac{1}{1+\lambda}\left[\Omega - \sum_{i=M,m}\theta_i x_i^p\right]. \tag{8}$$

3.3 Sources of Non-optimality

The results in Sections 3.1 and 3.2 enable us to compare the policy choices of the utilitarian social planner and the policy-maker. The fact that the minority endowment group is excluded from power leads to two distortions in the policy-maker's choice relative to the utilitarian social planner's choice. We discuss each of these distortions in turn.

First, the minority endowment group has zero private-good consumption. The policy-maker does not care about the minority endowment group's private-good consumption. Thus, compared to the social planner, the minority's private-good consumption decreases from x^s to zero. This is strongly welfare reducing due to the Inada Conditions.

Second, the private-good consumption of majority-endowment-group citizens is higher at the policy-maker's optimum than at the social planner's optimum. From Equation (4) and Equation (7), we directly obtain

$$u'\left(x_M^p\right) = \theta_M u'(x^s). \tag{9}$$

Since $\frac{1}{2} \leq \theta_M < 1$ and $u'(\cdot)$ is strictly decreasing, we have $x_M^p > x^s$. Intuitively, we see that the policy-maker does not internalize the benefit from private-good consumption of other members of his own endowment group. Thus, compared to the social planner, he needs a higher level of private-good consumption to be indifferent between a marginal increase in his private-good consumption and a marginal increase in public-good spending.

Therefore, the policy-maker's preferred level of public-good provision differs from the socially optimal level. We note that the first distortion increases public-good spending, while the second decreases it. To determine which of the two effects dominates, we proceed with the following analysis.

First, to characterize public good under-provision and over-provision, we compare Equations (5) and (8),

$$K_g^p < K_g^s \Leftrightarrow \quad \theta_M x_M^p > x^s, \qquad \text{(under-provision)} \qquad (10)$$

$$K_g^p \geq K_g^s \Leftrightarrow \quad \theta_M x_M^p \leq x^s. \qquad \text{(over-provision)} \qquad (11)$$

If the private-good consumption of the majority endowment group is higher than the aggregate private-good consumption in the socially optimal solution, there is public good under-provision. Otherwise, there is public good over-provision.

Suppose public-good spending is at the level chosen by the social planner but only the majority endowment group consumes any private good and the minority is fully exploited. In this case, the policy-maker's private-good consumption is $\frac{x^s}{\theta_M}$. If the policy-maker's marginal utility from private-good consumption is higher than his marginal utility from the public good then he under-provides the public-good. In other words, if

$$u'\left(\frac{x^s}{\theta_M}\right) > \theta_M \frac{\gamma}{1+\lambda},$$

the policy-maker wants to increase his own private-good consumption. Thus, he deducts from public-good spending and adds to his private-good consumption. By using Equation (4), the above inequality can be rewritten as

$$\frac{1}{\theta_M} u'\left(\frac{1}{\theta_M} x^s\right) > u'\left(x^s\right). \qquad (12)$$

In the following proposition, we show that Inequality (12) holds when the third derivative of $u(\cdot)$ is positive.

Proposition 1. *If $u'''(\cdot)$ is non-negative, the public good is under-provided by the policy-maker.*

The proof of Proposition 1 is given in the Appendix. In the remainder of this paper, we focus on under-provision and make the assumption that $u'''(\cdot) \geq 0$. The following example illustrates a case of public good under-provision.

Example 1. *Let $u(x) = \sqrt{x}$. Then, $u'''(x)$ is non-negative. By Proposition 1, the* *public good is under-provided.*

We note that with $u(x) = \sqrt{x}$, Inequality (12) reduces to

$$\sqrt{\theta_M} < 1.$$

Since $\frac{1}{2} \leq \theta_M < 1$, this always holds.

To summarize our results in this section, we observe that the policy-maker does not accomplish socially optimal private-good allocation and public-good provision. We refer to the deviations from the socially optimal solution as inefficiencies in political multi-task problems and assuming $u'''(\cdot) \geq 0$, we categorize them into two main categories:

- **exploitation of the minority endowment group,**
- **public good under-provision.**

Next, we explore various corrective measures to overcome these inefficiencies.

3.4 Corrective Measures

We explore corrective measures for the observed inefficiencies in political multi-task problems outlined in the last section. First, we apply constitutional tax limits to protect the minority endowment group from exploitation. Specifically, we consider an upper limit on tax rates which prevents the policy-maker from fully taxing citizens. Second, to overcome under-provision of the public good, we introduce a political contract that involves an incentive pay for the policy-maker, and depends on the level of public good provided. Finally, we combine tax protection with this incentive contract and study the effect on social welfare.

While the first measure—constitutional tax rules—is standard and widely applied in practice,[10] the second measure is non-standard. Indeed, it is one of the purposes of this paper to explore whether such political contracts are welfare-improving on their own or in combination with tax rules.

[10] Gersbach et al. (2012) provide examples of constitutional rules that restrict taxation in the U.S. and in other countries. A famous example is from the Texas constitution (Article 8, Sec. 1(a)), which states *"Taxation shall be equal and uniform."*

4 Constitutional Limitation on Taxes

In this section, we explore the consequences of constitutional limits on taxes. We assume there is an article in the constitution that limits taxation of citizens to a maximal tax rate $b \in [0,1]$ and we investigate the optimal choice of b. The private-good consumptions chosen for both groups by the policy-maker must be non-negative and satisfy the constitutional tax limit. Accordingly, a set C of feasible policy is defined as follows:

$$C = \{(x_M, x_m) \mid (1-b)\omega_i \leq x_i \leq \omega_i, i = M, m\}. \tag{13}$$

The policy-maker solves for

$$\begin{aligned} \max_{(x_M, x_m)} \quad & U(x_M, x_m) = u(x_M) + \left(\frac{\gamma}{1+\lambda}\right)\left[\Omega - \sum_{i=M,m} \theta_i x_i\right], \\ \text{subject to} \quad & (x_M, x_m) \in C. \end{aligned}$$

Since the policy-maker does not derive any utility from the minority's private-good consumption, he taxes them as much as possible and sets

$$x_m^p = \omega_m(1-b), \tag{14}$$

Given Assumption 2, it is not optimal for the policy-maker to choose $x_M^p = \omega_M$. In fact, depending on b, the choice of x_M^p under tax limit can be interior or it can be corner solution,

$$x_M^p = \max\{x_c^p, (1-b)\omega_M\}, \tag{15}$$

where x_c^p is the level of private-good consumption that satisfies the first-order condition with respect to x_M,

$$u'\left(x_c^p\right) = \frac{\gamma \theta_M}{1+\lambda}. \tag{16}$$

Public-good spending is given by

$$K_g^p = \frac{1}{1+\lambda}\left[\Omega - \sum_{i=M,m} \theta_i x_i^p\right]. \tag{17}$$

For $b \in [0,1]$, let $x_m^p(b)$, $x_M^p(b)$ and $K_g^p(b)$ be the solution to the system of Equations (14), (15) and (17).

We denote the ex-ante utilitarian welfare under the tax limit b by $W(b)$. At the ex-ante stage, the citizens do not know yet if they belong to the Elites or to the Non-elites, nor to which endowment group they belong. However, the policy-maker's ex-post choice of policy can be anticipated. Consequently, the function $W(b)$ can be written as follows:

$$W(b) = \theta_M u(x^p_M(b)) + \theta_m u(x^p_m(b)) + \gamma K^p_g(b). \tag{18}$$

The optimal constitutional tax limit is set ex-ante in the constitution. To find the optimal choice of constitutional tax limit, we maximize the ex-ante social welfare. We first note that without tax protection ($b = 1$), the minority has zero private-good consumption. By the Inada Conditions, they have infinite marginal utility from private-good consumption. Thus, imposing a b slightly smaller than one generates a great improvement for the minority.

Additionally, we observe that if the policy-maker cannot impose any taxes ($b = 0$), there is zero public-good spending. Given Assumption 1, this cannot be optimal. In fact, allowing the policy-maker to tax at all, however little, is better than no taxation. Thus, a very small b is an improvement compared to $b = 0$.

We next establish the existence of an interior optimal tax limit and we derive the optimal value for b. The optimal b is such that no infinitesimal lump sum tax on the whole population can improve welfare.

Proposition 2.

(i) There exists a unique constitutional limit $b^* \in (0, 1)$ on tax rates, which maximizes $W(b)$.

(ii) This optimal tax limit is equal to

$$b^* = \begin{cases} 1 - \frac{x^s}{\omega_m} & \text{if } \frac{u'\left(x^p_c \frac{\omega_m}{\omega_M}\right)}{u'(x^s)} \leq 1, \\ 1 - \frac{x^p_c}{\omega_M} & \text{if } 1 < \frac{u'\left(x^p_c \frac{\omega_m}{\omega_M}\right)}{u'(x^s)} < 1 + \theta_m \left(\frac{\theta_M \omega_M}{\theta_m \omega_m}\right), \\ \tilde{b} & \text{if } \frac{u'\left(x^p_c \frac{\omega_m}{\omega_M}\right)}{u'(x^s)} \geq 1 + \theta_m \left(\frac{\theta_M \omega_M}{\theta_m \omega_m}\right), \end{cases}$$

where $\tilde{b}$ is implicitly given by

$$\frac{\gamma}{1 + \lambda} = \frac{\theta_M \omega_M}{\Omega} u'\left((1 - \tilde{b})\omega_M\right) + \frac{\theta_m \omega_m}{\Omega} u'\left((1 - \tilde{b})\omega_m\right). \tag{19}$$

The proof of Proposition 2 is given in the Appendix.

The value of b^* depends on exogenous values γ, λ, θ_m, ω_M, ω_m and on the function $u(\cdot)$. We recall that ω_m is the initial endowment of the minority endowment group. The larger ω_m, the higher the tax limit that attains the maximum social welfare. If the minority's initial endowment is large enough, such that $\omega_m \geq (\frac{x^s}{x_c^p})\omega_M$,[11] then $b^* = 1 - \frac{x^s}{\omega_m}$. However, if the minority's endowment is less than the majority's initial endowment such that $\omega_m < (\frac{x^s}{x_c^p})\omega_M$, then a smaller tax limit $b^* = 1 - \frac{x_c^p}{\omega_M}$, is the welfare maximizer. If the inequality in initial endowment of the two groups is more severe, such that $\frac{u'\left(x_c^p \frac{\omega_m}{\omega_M}\right)}{u'(x^s)} \geq 1 + \theta_m \left(\frac{\theta_M \omega_M}{\theta_m \omega_m}\right)$, then an even smaller tax limit $b^* = \tilde{b}$, maximizes welfare.

The effect of tax protection on the two inefficiencies in political multi-task problems is twofold. On the one hand, tax protection is beneficial, since it alleviates the exploitation of the minority endowment group. On the other hand, however, it exacerbates public good under-provision. Additionally, a smaller b^* yields an even lower public-good provision. We note that although the protection of the minority endowment group from full taxation yields more severe public-good under-provision[12], it improves welfare.

The following example illustrates how citizens' preferences affect the optimal tax limit and the level of public-good provision.

Example 2. *Let $\omega_m > \omega_M$. Since $x^s < x_c^p$, $\frac{x^s}{x_c^p} < 1$. Given $\frac{\omega_m}{\omega_M} > 1$, we have $\frac{x^s}{x_c^p} < \frac{\omega_m}{\omega_M}$. Thus, we have $x^s < (\frac{\omega_m}{\omega_M})x_c^p$ and given $u'(\cdot)$ is strictly decreasing,*

$$u'\left((\frac{\omega_m}{\omega_M})x_c^p\right) < u'(x^s).$$

By Proposition 2, the optimal constitutional upper bound on the tax rates, $b^* = 1 - \frac{x^s}{\omega_m}$, maximizes social welfare. Consider a utility function of the form $u(x) = x^\alpha$, where $0 < \alpha < 1$. Substituting for x^s by using Equation (4) in b^*, we obtain

$$b^* = 1 - \frac{1}{\omega_m}\left[\frac{\alpha(1+\lambda)}{\gamma}\right]^{\frac{1}{1-\alpha}}.$$

We let $x^s(\alpha)$ be the solution to Equation (4). Taking the derivative of x^s with respect to α, we note that if $\frac{1+\lambda}{\gamma} < \frac{e^{\left(\frac{1}{\alpha}\right)}}{\alpha}$, x^s is an increasing function of α. Con-

[11] We recall from Equation (4) that $u'(x^s) = \frac{\gamma}{1+\lambda}$ and from Equation (16) that $u'(x_c^p) = \frac{\gamma\theta_M}{1+\lambda}$. Given $u'(x^s) < u'(x_c^p)$ and since $u'(\cdot)$ is strictly decreasing, we have $x^s < x_c^p$.

[12] We recall from Equation (17) that the higher after-tax private-good consumption of citizens is the lower the public-good spending.

sequently, if $\frac{1+\lambda}{\gamma} < \frac{e^{\left(\frac{1}{\alpha}\right)}}{\alpha}$ *holds, the smaller* α*, the smaller* x^s*, and the larger the optimal constitutional tax limit and the level of public-good provision.*

5 Incentive Contract: General Considerations

We next introduce an incentive contract for politicians. The contract stipulates that the policy-maker receives some additional amount of the private good, depending on the level of public-good provision. This is a "Political Contract" in the sense of Gersbach (2012).[13]

For such a contract to be enforceable, it has to be conditioned on a variable connected with a verifiable performance level. We assume that the public good can be translated into a variable for which the quantifiable and verifiable dimension either exist or can be constructed. As to global warming, for instance, the quantifiable dimension might be a certain reduction of CO_2 emissions. For infrastructure projects, the number of road kilometers or of bridges built is quantifiable. The simplest example of a verifiable variable is the level of public debt.

We assume the simplest form of incentive contract, in which the policy-maker is rewarded linearly by an amount of additional private good per unit of public-good provision, and that the level of provision, g, is observable and quantifiable. In our setting, public good has a linear production function and is proportional to public-good spending ($g = \gamma K_g$). Since the technology and the production function are common knowledge, for the sake of simplicity, we make the reward conditional on the level of public-good spending. The parameter ($\beta \geq 0$) denotes the reward per unit of public-good spending and it is finite.

We recall our definition of the Elites: citizens who take part in policy-making or are members of the policy-maker's supporting interest group. We assume that the policy-maker shares the reward only with the Elites of his endowment group. Consequently, for any given consumption plan and incentive contract with parameter β, the budget constraint is given by

$$K_g + \underbrace{\mu\theta_M\beta K_g}_{\text{aggregate incentive pay}} = \frac{1}{1+\lambda}\left[\Omega - \sum_{i=M,m}\theta_i x_i\right]. \tag{20}$$

The cost of the incentive pay to the society, $C = \beta\mu\theta_M K_g$, depends on the reward

[13] By construction, the Political Contract does not interfere with the rules of liberal democracy. The rules governing the design, implementation and assessment process must be added as a new article to the constitution.

per unit of public-good spending, on the size of the majority-endowment-group Elites, and on the amount of public-good spending.

With the introduction of our incentive contract, the Elites and the Non-elites of the majority endowment group have different preferences over policies. Thus, it is useful to distinguish between the consumption level within the policy-maker's endowment group. With the introduction of incentive pay, the majority-endowment-group Elites receive a reward—in private-good consumption—which the majority-endowment-group Non-elites do not receive. For the majority-endowment-group Non-elites, we denote the level of private-good consumption by x_{MN}. For the majority-endowment-group Elites, we denote private-good consumption by x_{ME},

$$x_{ME} = x_{MN} + \beta(1+\lambda)K_g. \tag{21}$$

The first summand is the after-tax level of private-good consumption for the majority endowment group and the second summand is the reward the majority-endowment-group Elites receive due to the incentive contract. We observe from Equation (20) that the incentive pay is financed by the collected tax revenue which is reduced due to deadweight losses. This is the reason why $(1+\lambda)$ enters Equation (21).

Given this distinction between x_{ME} and x_{MN}, we modify our definition of feasible policy by replacing x_M with x_{MN}. We define the feasible policy set, C', as

$$C' = \{(x_{MN}, x_m) \mid 0 \leq x_{MN} \leq \omega_M, 0 \leq x_m \leq \omega_m\}. \tag{22}$$

Additionally, for any given level of private-good consumption (x_{MN}, x_m), the level of public-good spending—implied from the budget constraint—can be written as

$$K_g = \frac{[\Omega - \theta_M x_{MN} - \theta_m x_m]}{(1+\lambda)(1+\mu\theta_M\beta)}. \tag{23}$$

By substituting for $x_{MN} = x_{ME} - \beta(1+\lambda)K_g$ in Equation (23), public-good spending can be equivalently written as

$$K_g = \frac{[\Omega - \theta_M x_{ME} - \theta_m x_m]}{(1+\lambda)[1-\beta\theta_M(1-\mu)]}. \tag{24}$$

With incentive pay, the policy-maker's optimization problem is therefore

$$\max_{(x_{MN}, x_m)} \quad U(x_{MN}, x_m) = u\left(x_{MN} + \beta\frac{[\Omega - \theta_M x_{MN} - \theta_m x_m]}{1+\mu\theta_M\beta}\right) + \gamma\frac{[\Omega - \theta_M x_{MN} - \theta_m x_m]}{(1+\lambda)(1+\mu\theta_M\beta)},$$

subject to $\quad (x_{MN}, x_m) \in C'$.

We denote the policy-maker's optimal choice of private-good consumption for the minority endowment group, the majority-endowment-group Non-elites and the majority-endowment-group Elites, by x_m^p, x_{MN}^p and x_{ME}^p, respectively.

We immediately observe that $U(\cdot,\cdot)$ is strictly decreasing in the private-good consumption of the minority endowment group, and as a result, the policy-maker sets

$$x_m^p = 0. \tag{25}$$

The policy-maker faces a more complex trade-off between private-good consumption and public-good spending in this case, compared to the case in Section 3.2, since he receives an incentive pay in private-good consumption based on the level of public good he provides.

We observe that it is not optimal for the policy-maker to set $x_{MN}^p = \omega_M$. From Equation (24), we can see that with the incentive contract, the policy-maker's marginal utility of public-good spending is higher than his marginal utility of public-good spending without the incentive pay,

$$\frac{\gamma\theta_M}{1+\lambda} \leq \frac{\gamma\theta_M}{(1+\lambda)\left(1-\beta\theta_M(1-\mu)\right)}.$$

Additionally, with the incentive pay from Equation (21), we have $x_{ME}^p \geq x_{MN}^p$. At $x_{MN}^p = \omega_M$, given $u'(\cdot)$ is strictly decreasing, Assumption 2 yields

$$u'\left(x_{ME}^p\right) \leq \underbrace{u'\left(x_{MN}^p\right)}_{=u'(\omega_M)} < \frac{\gamma\theta_M}{1+\lambda} \leq \frac{\gamma\theta_M}{(1+\lambda)\left(1-\beta\theta_M(1-\mu)\right)}.$$

From the above inequality, we obtain $u'\left(x_{ME}^p\right) < \frac{\gamma\theta_M}{(1+\lambda)(1-\beta\theta_M(1-\mu))}$. Thus, it is not optimal for the policy-maker to set $x_{MN}^p = \omega_M$ and imposing a very small tax on his own endowment group improves his utility compared to zero taxation of the majority endowment group.

However, the choice of $x_{MN}^p = 0$ might be optimal. Although at $x_{MN}^p = 0$, the majority-endowment-group Non-elites' marginal utility of private-good consumption will be infinite due to the Inada Conditions, the policy-maker's marginal utility will not be infinite because of the incentive pay. With $x_{MN}^p = 0$, we obtain

$$K_g^p = \frac{\Omega}{(1+\lambda)(1+\mu\theta_M\beta)}, \quad \text{and} \tag{26}$$

$$x_{ME}^p = \frac{\beta\Omega}{1+\mu\theta_M\beta}, \tag{27}$$

by using Equation (23) and Equation (21), respectively.

Suppose the policy-maker fully taxes both endowment groups. If the policy-maker's marginal utility of private-good consumption is higher than his marginal utility of public good, then he prefers to tax his own endowment group less and set $x^p_{MN} > 0$. In other words, if

$$u'\left(\frac{\beta\Omega}{1+\beta\mu\theta_M}\right) \geq \frac{\gamma\theta_M}{(1+\lambda)\,(1-\beta\theta_M(1-\mu))}, \tag{28}$$

the policy-maker wants to increase his own private-good consumption. Thus, it is optimal for the policy-maker to set $x^p_{MN} = 0$, if Inequality (28) does not hold.

However, if Inequality (28) holds, the interior solution is the optimal policy for the policy-maker. We find the interior solution by examining the first-order condition with respect to x_{MN},

$$u'\left(\underbrace{x^p_{MN} + \beta\,(1+\lambda)\,K^p_g}_{=x^p_{ME}}\right) = \frac{\gamma\theta_M}{(1+\lambda)\,[1-\beta\theta_M\,(1-\mu)]}. \tag{29}$$

Substituting for $x^p_m = 0$ into Equation (24) and by using Equation (29) for x^p_{ME}, the implied level of public-good spending for the interior solution is given by

$$K^p_g = \frac{[\Omega - \theta_M x^p_{ME}]}{(1+\lambda)\,[1-\beta\theta_M\,(1-\mu)]}. \tag{30}$$

Let $x^p_{MN}(\beta)$, $x^p_{ME}(\beta)$, and $K^p_g(\beta)$ be the solution to the system of Equations (21), (29) and (30). For the intermediate result, stated in the next lemma, we require $\beta < \frac{1}{\theta_M(1-\mu)}$. This ensures $u'(\cdot) > 0$ in Equation (29) and ensures continuity of $K^p_g(\beta)$, given by Equation (30). Later, we establish that Inequality (28) requires an upper bound on the reward parameter which is strictly smaller than $\frac{1}{\theta_M(1-\mu)}$.

The following lemma states the comparative statics with respect to β for the interior solution to the policy-maker's problem:

Lemma 1. *Let $\beta < \frac{1}{\theta_M(1-\mu)}$ and $\mu \in [0,1)$. The following properties hold:*

(i) $\frac{\partial x^p_{ME}}{\partial\beta} < 0$,

(ii) $\frac{\partial K^p_g}{\partial\beta} > 0$,

(iii) $\frac{\partial x^p_{MN}}{\partial\beta} < 0$.

The proof of Lemma 1 is given in the Appendix. The results in Lemma 1 assess the incentive contract's impact on the interior policy choices. The minority endowment group's private-good consumption does not depend on the incentive pay, since the minority endowment group is fully exploited. Public-good spending is increasing with β at the interior solution. The cost of the additional public-good spending due to the incentive contract is shared by the majority-endowment-group Elites and Non-elites. Thus, the majority-endowment-group Elites' private-good consumption, $x^p_{ME}(\beta)$ is a decreasing function of β. The cost of the incentive pay has to be paid by the majority-endowment-group Non-elites. The Non-elite citizens of the policy-maker's endowment group are those citizens who are not entitled to the reward from the contract, but have to pay for its costs. The majority-endowment-group Non-elites' private-good consumption, $x^p_{MN}(\beta)$, is a decreasing function of β, due to the costs of the reward and the additional public-good spending.

The next proposition follows from Lemma 1 and establishes the admissible range for β that ensures the interior solution.

Proposition 3. *Let $\mu \in [0, 1)$.*

(i) There exists a unique $\bar{\beta} < \frac{1}{\theta_M(1-\mu)}$ that satisfies

$$u'\left(\frac{\bar{\beta}\Omega}{1+\mu\theta_M\bar{\beta}}\right) = \frac{\gamma\theta_M}{(1+\lambda)\left[1-\bar{\beta}\theta_M(1-\mu)\right]}. \tag{31}$$

(ii) The policy-maker's optimization problem has a unique optimal interior solution if and only if

$$0 \leq \beta \leq \bar{\beta}.$$

The proof of Proposition 3 is given in the Appendix.

To provide intuition about the results in Proposition 3, we compare Equation (31) with Inequality (28). At $\bar{\beta}$, the reward parameter is so high that the policy-maker's marginal utility of private-good consumption is equal to his marginal utility of public good. Thus, it is optimal for the policy-maker to set $x^p_{MN} = 0$. For all $\beta < \bar{\beta}$, the reward parameter is such that the policy-maker's marginal utility of private-good consumption is higher than his marginal utility of public good, if he sets $x^p_{MN} = 0$. Thus, the optimal solution to the policy-maker's problem is interior.

In the remainder of this section, we assume $\beta \in [0, \bar{\beta}]$ to ensure interior solution to the policy-maker's problem. Before presenting the results for the optimal incentive contract, we show the results for the case with $\mu = 1$ in the next proposition.

Proposition 4. *The incentive contract has no impact if and only if* $\mu = 1$.

The proof of Proposition 4 is given in the Appendix. The result follows from Equations (29) and (30), where the policy choice remains unchanged with the introduction of the incentive contract when $\mu = 1$.[14] To provide intuition about the above proposition, we note that at $\mu = 1$, every citizen in the majority endowment group belongs to the Elites. Since everyone in the majority endowment group is entitled to the reward and the majority-endowment-group Non-elites has no members, no one pays for the costs of the contract. In this case, the incentive pay is not effective. More precisely, the citizens' private-good consumptions do not change with the incentive pay. As a result, public-good provision is not affected by the incentive contract either. In the remainder of this paper, we focus on $\mu \in [0, 1)$.

The ex-ante welfare function as a function of the incentive parameter is given by

$$W(\beta) = \theta_M(1-\mu)u(x^p_{MN}(\beta)) + \theta_M\mu u(x^p_{ME}(\beta)) + \theta_m u(x^p_m) + \gamma K^p_g(\beta). \tag{32}$$

Due to additive separability, welfare is a weighted sum of the the citizens' utilities. The ex-ante welfare function thus depends on the policy choices, the size of the majority endowment group, and the size of the Elites. The first and second terms denote the utility of private-good consumption for the majority-endowment-group Non-elites and the majority-endowment-group Elites, respectively. The third term denotes the minority group's utility of private-good consumption and the last term denotes the society's public-good level.

The incentive parameter is set ex-ante. In Proposition 3, we have established the admissible range for β. The upper bound of β, as defined in Equation (31), solely depends on exogenous parameters and the function $u(x)$, which are common knowledge. In order to find the optimal reward, $\beta^* \in [0, \bar{\beta}]$, the ex-ante social welfare function $W(\beta)$ should be maximized.

We first note that without incentive pay ($\beta = 0$), we are back to the case in Section 3.2, where in addition to the minority's exploitation, the public good is under-provided and the majority's private-good consumption is higher than the optimal level. By Lemma 1, introducing a very small incentive pay increases the public-good spending and decreases the majority-endowment-group Elites' private-good consumption as well as the majority-endowment-group Non-elites' private-good consumption. These three effects improve social welfare. Thus, having a very

[14] At $\mu = 1$ Equation (29) is equivalent to Equation (7), and Equation (30) is equivalent to Equation (8).

small incentive pay is better than having none.

Additionally, we observe that an incentive pay with $\bar{\beta}$ cannot be optimal. Suppose the policy-maker is rewarded according to the incentive contract $\bar{\beta}$. Then, like the minority, the majority-endowment-group Non-elites have zero private-good consumption. Thus, by the Inada Conditions, they have infinite marginal utility. A small decrease of the reward parameter highly improves social welfare.

Theorem 1 assesses the existence of an optimal incentive contract.

Theorem 1. *Let $\mu \in [0,1)$. There exists $\beta^* \in (0,\bar{\beta})$ such that $W(\beta^*)$ is constrained optimal with respect to the incentive pay.*

The proof of Theorem 1 is given in the Appendix.

From the proof of Theorem 1, we obtain a formula for determining the politician's optimal reward. The reward is implicitly given by

$$\theta_M(1-\mu)u'\left(x^p_{MN}\left(\beta^*\right)\right)\left.\frac{\partial x^p_{MN}}{\partial \beta}\right|_{\beta^*}+\theta_M\mu u'\left(x^p_{ME}\left(\beta^*\right)\right)\left.\frac{\partial x^p_{ME}}{\partial \beta}\right|_{\beta^*}=$$
$$\frac{\gamma}{1+\lambda}\left[\frac{\theta_M\left.\frac{\partial x^p_{ME}}{\partial\beta}\right|_{\beta^*}}{1-\beta^*\theta_M(1-\mu)}-\frac{\theta_M^2(1-\mu)x^p_{ME}\left(\beta^*\right)}{\left(1-\beta^*\theta_M(1-\mu)\right)^2}\right].$$

To provide further insight, we next study the case $\mu=0$ as an illustrative example and we establish numerical results for β^*.

6 Incentive Contract: A Special Case

If the Elites consist of finitely many citizens ($\mu=0$), the contract is costless. The budget constraint of the society

$$K_g=\left(\frac{1}{1+\lambda}\right)\left[\Omega-\theta_M x_{MN}-\theta_m x_m\right],$$

is the same as the initial budget, without an incentive contract in Equation (2).

Consequently, the rewarded policy-maker's problem has the following form:

$$\max_{(x_{MN},x_m)}\quad U(x_{MN},x_m)=u\left(x_M+\beta\left[\Omega-\theta_M x_{MN}-\theta_m x_m\right]\right)+\gamma\frac{\left[\Omega-\theta_M x_{MN}-\theta_m x_m\right]}{(1+\lambda)},$$

subject to $\quad (x_{MN},x_m)\in C'$.

The policy-maker immediately sets $x_m^p = 0$.

By Proposition 3, the policy-maker's problem has a unique interior solution if and only if $\beta \in [0, \bar{\beta}]$. We assume $\beta \in [0, \bar{\beta}]$, where $\bar{\beta}$ is implicitly given by

$$u'\left(\bar{\beta}\Omega\right) = \frac{\gamma\theta_M}{(1+\lambda)(1-\bar{\beta}\theta_M)},$$

where we have substituted for $\mu = 0$ in Equation (31).

From the first-order condition with respect to x_{MN}, we obtain the interior solution

$$u'\left(x_{MN}^p + \beta(1+\lambda)K_g^p\right) = \left(\frac{\gamma\theta_M}{(1+\lambda)\left[1-\beta\theta_M\right]}\right).$$

The private-good consumption of the majority-endowment-group Elites differs from the majority-endowment-group Non-elites by $\beta(1+\lambda)K_g$, due to the incentive pay,

$$x_{ME}^p = x_{MN}^p + \beta(1+\lambda)K_g^p.$$

And the public-good spending is given by

$$K_g^p = \frac{\left[\Omega - \theta_M x_{ME}^p\right]}{(1+\lambda)\left[1-\beta\theta_M\right]}.$$

We note that these results are identical to those in Equations (29) and (30), with μ set equal to zero.

Intuitively, we expect the effect of incentive pay on the level of public good to be enhanced when the size of the Elites' group decreases. At $\mu = 0$, rewards can be given without social cost. This is due to the fact that only a finite number of citizens belongs to the majority-endowment-group Elites and is entitled to the reward. The rest of the majority-endowment-group citizens belongs to the Non-elites who finance the incentive pay and contribute to the additional public-good spending induced by the contract. A group of Non-elite citizens as large as the endowment group can collectively afford higher incentive pay and contribute more to the provision of the public good.[15]

By Theorem 1, there exists an incentive contract that is socially optimal at $\mu = 0$. Finding β^* for the costless contract is not feasible analytically.

Example 3. *As an example, we solve for β^* numerically, using the following set of parameters: $\mu = 0$, $\lambda = 0$, $\gamma = 1$, $\theta_M = \theta_m = \frac{1}{2}$ and $u(x) = x^{\frac{1}{2}}$. The optimal*

[15] This is also clear from Equation (30). We can see that for any given β, the public-good spending of a rewarded policy-maker is a strictly decreasing function of the size of the Elites, μ.

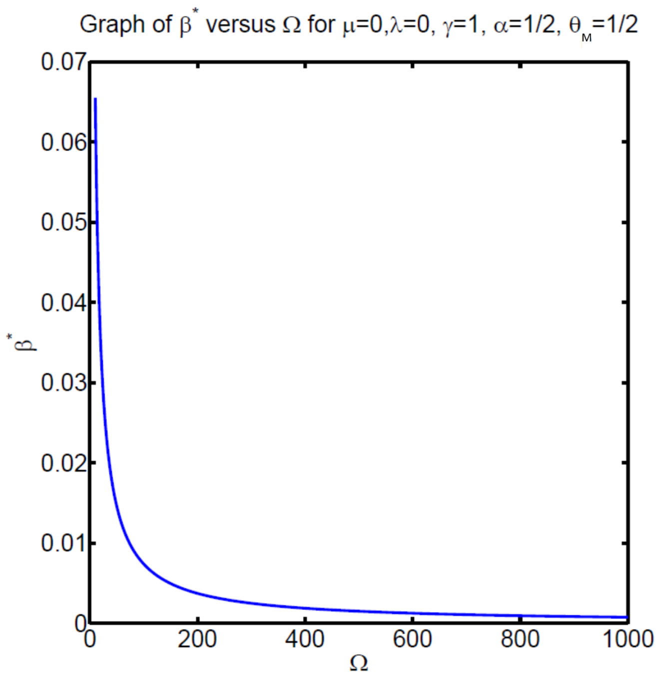

Figure 2: Numerical example.

reward parameter as a function of the total private good endowment is depicted in Figure (2). We can see that the optimal β^* value decreases, as the society's total private good endowment Ω increases.

7 Incentive Contract: Implementability

In this section, we explore the conditions for the implementability of the incentive contract. By "implementable", we mean a majority of citizens are better off with the contract than without the contract. To be more precise, for the contract to be implementable, a majority of citizens must be in favor of it.[16]

While so far, we have only focused on the ex-ante and the ex-post cases, it is now useful to consider an interim case. Suppose we are in a society where Elites are the educated citizens in the society. Citizens know whether they belong to the Elites or not at an interim stage. However, they will only observe their endowments at a later stage, when an exogenous shock to the initial endowment is realized and some citizens will have a higher endowment, while the others will have a lower one.

[16]In practice, an actual political process that can determine the implementability of the incentive contract could be a referendum or a parliamentary vote. As an extension to the main model, we discuss "Co-voting" as an alternative implementation process that promotes citizen participation.

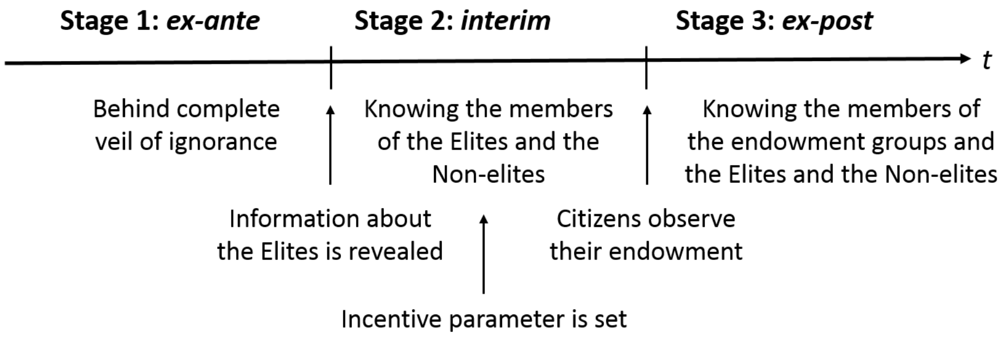

Figure 3: Timeline.

Figure (3) shows the timeline of information revelation.

At the interim stage, we consider the Elites' and the Non-elites' interest in the incentive contract separately. In other words, we evaluate the citizens' expected utilities when they do not know their endowment group yet, but know with certainty whether they belong to the Elites or to the Non-elites. The probability of belonging to the majority endowment group is the same for the Elites and the Non-elites, and equal to θ_M, and the probability of belonging to the minority endowment group for the Elites and Non-elites is equal to θ_m. The interim expected utility of an Elite and a Non-elite citizen is given by

$$\mathcal{U}_E(\beta) = \theta_M \left[u\left(x^p_{ME}(\beta)\right) + \gamma K^p_g(\beta)\right] + \theta_m \left[u\left(x^p_m\right) + \gamma K^p_g(\beta)\right] \text{ and} \tag{33}$$
$$\mathcal{U}_{NE}(\beta) = \theta_M \left[u\left(x^p_{MN}(\beta)\right) + \gamma K^p_g(\beta)\right] + \theta_m \left[u\left(x^p_m\right) + \gamma K^p_g(\beta)\right], \tag{34}$$

respectively.

Proposition 5. *(i) The Elites expect to be interim better off with the incentive contract for all $\beta \in [0, \bar{\beta}]$.*

(ii) Let $\gamma \geq (1+\lambda)^2$. The Non-elites expect to be interim better off with the incentive contract if $\mu \leq 1 - \frac{(1+\lambda)^2}{\gamma}$ and β is small enough.

The proof of Proposition 5 is given in the Appendix. We note that if $\mu \in [\frac{1}{2}, 1)$, the incentive contract is implementable. This results from the fact that the Elites have the majority and they are interim better off with the incentive contract for all β values. However, if $\mu \in \left[0, \frac{1}{2}\right)$ the implementability of the contract also depends on how the interim expected utility of the Non-elites changes with the incentive contract.

It is useful to provide intuition about Statement (ii) in Proposition 5. With probability θ_m, a Non-elite citizen belongs to the minority endowment group. The minority endowment group Non-elites are strictly better off with the incentive contract, because the public-good level increases with the incentive pay and their private-good consumption does not depend on β. However, for any citizen, the probability of belonging to the majority endowment group is higher than the probability of belonging to the minority endowment group, $\theta_M > \theta_m$. Unlike the minority endowment group Non-elites, the majority-endowment-group Non-elites are worse off with the incentive contract. Although the public-good level is higher with the incentive pay, the majority-endowment-group Non-elites' private-good consumption is decreasing in β.

For a Non-elite citizen to be interim better off with the incentive contract, it has to be that the expected positive effect of the incentive contract on the minority endowment group Non-elites' utility dominates the incentive contract's expected negative effect on the majority-endowment-group Non-elites' utility. The incentive contract's negative effect on the majority-endowment-group Non-elites' utility is a result of the costs of the incentive pay being financed by the majority-endowment-group Non-elites. With small enough β and μ, the cost of the incentive pay decreases sufficiently for the Non-elites to be interim better off with the contract. If the cost of the incentive pay is small enough, both the Elites and the Non-elites are interim better off, and the incentive contract is implementable.

The next corollary follows immediately from Proposition 5.

Corollary 1. *(i) The ex-ante optimal incentive contract β^* is implementable if and only if $\mu \in \left[\frac{1}{2}, 1\right)$.*

(ii) Any contract that makes the Non-elites interim better off is implementable and welfare-improving.

The proof of Corollary 1 is given in the Appendix.

Corollary 1 establishes that the Non-elites are interim worse off with the incentive contract at the optimal β^*. Thus, for the optimal contract to be implementable the Elites should have the majority in the society. Moreover, it shows that if the contract is such that it makes the Non-elites interim better off, it has the support of everyone in the society. Additionally, it is welfare-improving relative to the policy-maker's choice without the incentive contract.

8 Tax Protection and the Incentive Contract

8.1 The policy–maker's problem

We have argued until now that the incentive contract is a useful tool for alleviating the public-good under-provision. However, it cannot protect the minority endowment group from exploitation. A constitutional limit on tax rates can guarantee the protection of the minority endowment group from exploitation, but reduces the public-good level, which is already too low in the first place.

In this section, we combine tax protection and the incentive contract, and study their overall effect on society's welfare.

As in Section 4, we assume that there is an article in the constitution that imposes an upper bound on tax rates, say b. We note that if $b = 0$, the private good endowment of all citizens is protected by the constitution. The policy-maker cannot raise taxes and consequently, public-good spending and incentive pay equal zero. Given Assumption 1, zero public-good spending cannot be optimal from a social welfare point of view. In fact, allowing the policy-maker to impose an infintesimally small tax rate is strictly socially better than allowing no taxation whatsoever.

Moreover, we note that at $b = 1$, the problem reduces to the one without constitutional tax protection, and the results in Section 5 hold. We note that without tax protection ($b = 1$) and with incentive pay in place , at least the minority receives zero private-good consumption.[17] By the Inada Conditions, the minority citizens have infinite marginal utility from private-good consumption. Thus, choosing b slightly smaller than one leads to a large improvement for this group of citizens.

Thus, the basic argument given in Section 4 for the existence of an interior solution for the optimal tax limit is equally valid in the presence of incentive pay as it is without incentive pay.

The private-good consumptions chosen by the policy-maker should satisfy the constitutional tax limit. We define C'' as the set of feasible policies given the tax restrictions, thus

$$C'' = \left\{(x_{MN}, x_m) \mid (1-b)\omega_M \le x_{MN} \le \omega_M, (1-b)\omega_m \le x_m \le \omega_m\right\}.$$

We consider the policy-maker's optimization problem, taking both the reward and

[17]We recall from Section 5 that if β is set too high, the majority-endowment-group Non-elites' might have zero private-good consumption, additionally to the minority endowment group.

tax protection into account,

$$\max_{C''} U(x_{MN}, x_m) = u\left(x_{MN} + \beta\frac{\Omega - \theta_M x_{MN} - \theta_m x_m}{1+\mu\theta_M\beta}\right) + \gamma\frac{\Omega - \theta_M x_{MN} - \theta_m x_m}{(1+\lambda)\,(1+\mu\theta_M\beta)},$$

subject to $(x_{MN}, x_m) \in C''$.

We denote the policy-maker's choice of private-good consumption for the minority endowment group by x_m^p. We can immediately conclude that the policy-maker chooses

$$x_m^p = (1-b)\omega_m. \tag{35}$$

Therefore, the optimisation problem becomes one-dimensional and standard arguments are applicable. Since $U(x_{MN}$ is concave, there is a unique maximum. In order to find the optimal x_{MN}, we use the following first–order condition:

$$0 = \frac{1-(1-\mu)\theta_M\beta}{1+\mu\theta_M\beta} u'\left(x_{MN} + \beta\frac{\Omega - \theta_M x_{MN} - \theta_m(1-b)\omega_m}{1+\mu\theta_M\beta}\right) - \frac{\theta_M\gamma}{1+\lambda}\frac{1}{1+\mu\theta_M\beta}.$$

We observe that the behaviour of the derivative is crucially dependent on whether the inequality

$$\beta < \frac{1}{(1-\mu)\theta_M}$$

is true or not. Therefore, we need to distinguish the following three cases separately:

First, if $\theta_M > \frac{1}{(1-\mu)\theta_M}$, then we have that $\beta > \frac{1}{(1-\mu)\theta_M}$, and therefore, we find:

$$U'(x_{MN}) < 0, \forall x_{MN} \in [(1-b)\omega_M, \omega_M],$$

due to the Inada Conditions. Hence, the utility of the policy maker is strictly decreasing in x_{MN} leading to $x_{MN}^p = (1-b)\omega_M$ as the optimal choice.

Second, if $\beta = \frac{1}{(1-\mu)\theta_M}$, then one similarily obtains that the utility function for the policy maker is strictly decreasing and thus leads to the same choice for x_{MN}^p as before.

Third, we consider the case where $\beta < \frac{1}{\theta_M(1-\mu)}$. This case is the most interesting one. We emphasize that the assumption $\beta < \frac{1}{\theta_M(1-\mu)}$ guarantees the concavity of the utility function of the policy-maker. Depending on the sign of the derivative at the boundary values for x_{MN}, three different scenarios are possible a priori.

Case 1 If we have:

$$U'\big((1-b)\omega_M\big) > 0 > U'\big(\omega_M\big), \tag{36}$$

then the derivative has a unique zero due to the strict concavity of U. The optimal choice for the policy maker is denoted by x^p_{MN} and satisfies the following identity:

$$U'\big(x^p_{MN}\big) = 0.$$

By rearranging, we obtain the following implicit expression for the optimal policy x^p_{MN}:

$$u'\Big(x^p_{MN} + \beta\frac{\Omega - \theta_M x^p_{MN} - \theta_m(1-b)\omega_m}{1+\mu\theta_M\beta}\Big) = \frac{\gamma\theta_M}{1+\lambda}\frac{1}{1-(1-\mu)\theta_M\beta}. \tag{37}$$

In this case, we say that the optimal solution is an *interior solution* because the maximum lies inside the interval of possible values for x_{MN}.

Case 2 Now assume that

$$U'\big((1-b)\omega_M\big) \leq 0. \tag{38}$$

By concavity, this implies that U is decreasing in x_{MN}. Therefore, we immediately deduce that $x^p_{MN} = (1-b)\omega_M$ is the optimal choice for the policy-maker. Such a solution implementing the boundary value for x^p_{MN} is called a *corner solution*.

Case 3 A priori it might be the case that

$$U'(\omega_M\big) \geq 0. \tag{39}$$

In this case, the utility function U would be increasing in x_{MN} which implies that the optimal choice x^p_{MN} would be ω_M. However, this case is excluded by our assumptions on the function u. To see this, we notice that $U'(\omega_M) \geq 0$ is equivalent to:

$$\frac{\gamma\theta_M}{(1+\lambda)(1-(1-\mu)\theta_M\beta)} \leq u'\Big(\omega_M + \beta\frac{b\omega_m}{1+\mu\theta_M\beta}\Big).$$

Notice that due to Assumption 2, we have:

$$u'(\omega_M) < \frac{\gamma\theta_M}{1+\lambda},$$

and therefore, by the strict concavity of u:

$$u'(\omega_M) < u'\Big(\omega_M + \beta \frac{b\omega_m}{1+\mu\theta_M\beta}\Big) \leq u'(\omega_M),$$

which yields the desired contradiction.

These three cases can be verbally explained as follows: In Case 1, the policy-maker chooses some strictly positive amount of taxation, but less than the amout that is constitutionally allowed. In Case 2, the policy-maker sets the tax rate just as high as allowed by the constitution which corresponds to one of the possible boundary values for x_{MN}. We observe that Case 3 corresponds to the other boundary value in the 1-dimensional optimization where the tax rate is 0. But as we have seen, Assumption 2 ensures that the infinitesimal gain from investing in public goods is high enough to prevent this scenario from being optimal.

In order to adress the question of constitutional design, we want to further characterize the solution type depending on the parameters b and β. Therefore, we need to investigate (36). By our previous calculation in Case 3, it suffices to consider:

$$U'\big((1-b)\omega_M\big) > 0. \tag{40}$$

Inserting the definitions, we notice that this is equivalent to:

$$g(\beta) := \frac{\gamma}{1+\lambda}\frac{\theta_M}{1-(1-\mu)\theta_M\beta} < u'\Big((1-b)\omega_M + \beta\frac{b\Omega}{1+\mu\theta_M\beta}\Big) =: f(b,\beta).$$

If we calculate the derivative with respect to β for these functions, we see:

$$\begin{aligned}
\partial_\beta f(b,\beta) &= \frac{b\Omega}{\big(1+\mu\theta_M\beta\big)^2}u''\Big((1-b)\omega_M + \beta\frac{b\Omega}{1+\mu\theta_M\beta}\Big) < 0 \\
\partial_\beta g(\beta) &= \frac{\gamma\theta_M}{1+\lambda}\frac{(1-\mu)\theta_M}{\big(1-(1-\mu)\theta_M\beta\big)^2} > 0
\end{aligned}$$

Observe that we make use of $u'' < 0$. This implies that $f(b,\beta) - g(\beta)$ is strictly decreasing in β for a given b. Therefore, if we assume that b is fixed, we can obtain an explicit bound $\bar{\beta}_b$ on β such that the optimal policy is an interior solution. Namely, we define $\bar{\beta}_b \in [0, \frac{1}{(1-\mu)\theta_M}[$ by:

$$f(b,\bar{\beta}_b) = g(\bar{\beta}_b), \tag{41}$$

for any $b \in [0,1]$. If no such $\bar{\beta}_b$ exists, we define $\bar{\beta}_b = 0$. Notice that the only way that (41) could not have a solution would be if either for the given b the function

f is always strictly smaller than g or f is always strictly larger than g. The second case cannot occur due to g diverging to infinity as β approaches $\frac{1}{\theta_M(1-\mu)}$.

By our earlier observations, if $\beta < \bar{\beta}_b$, we have that (36) holds and thus the optimal choice for x^p_{MN} belongs to an interior solution. Otherwise, the optimal policy induces a corner solution. These findings are summarized in the following proposition:

Proposition 6. *Given a pair (b, β), the solution of the optimization problem faced by the policy-maker is interior if and only if $\beta < \bar{\beta}_b$. In this case, x^p_{MN} is determined by* (37)*. In all other cases, the optimal choice is given by $x^p_{MN} = (1-b)\omega_M$.*

We emphasize that the Proposition includes the case where $\beta \geq \frac{1}{(1-\mu)\theta_M}$. This is simply due to our earlier considerations.

Let us consider some concrete values of b: We notice that by definition of $b_c = 1 - \frac{x^p_c}{\omega_M}$, we have that $\bar{\beta}_b = 0$ for all $b \leq b_c$. This is simply due to

$$f(b_c, 0) = u'\big((1-b_c)\omega_M\big) = u'(x^p_c) = \frac{\gamma\theta_M}{1+\lambda} = g(0),$$

which immediately implies the desired value for $\bar{\beta}_b$ for $b \leq b_c$ due to f being monotonically increasing in b. This proves the following corollary:

Corollary 2. *If $b \leq b_c$ or $b > b_c$ but $\beta > \bar{\beta}_b$, it is optimal for the policy-maker to choose $x^p_{MN}(b, \beta) = (1-b)\omega_M$. Otherwise, $x^p_{MN}(b, \beta)$ is given by* (37) *and depends continuously differentiably on β on the interval* $(0, \bar{\beta}_b)$*.*

The differentiable dependence on β is an immediate consequence of direct investigations and the Implicit Function Theorem. Additionally, it is a direct consequence of the monotonicity and continuity properties of u' that the map $\beta \mapsto x^p_{MN}(b, \beta)$ is continuous and monotone decreasing.

Verbally, the result can be summarized as follows: First, if the constitution allows only very little taxation, then the policy-maker finds it optimal to fully exploit the scope for taxation regardless of incentive pay. Second, if the constitution allows enough scope for taxation, then the policy-maker finds it optimal to fully exploit it if and only if the incentive pay parameter is above a certain threshold $\bar{\beta}_b$ depending on the scope of taxation. If the incentive pay parameter is below this threshold, then the policy-maker's problem admits an interior solution.

Now that we have characterized the optimal solution for a given pair (b, β), we want to apply this knowledge to determine values of these parameters which maximize social welfare. The ex-ante social welfare depends on the taxation of the minority and majority groups taking into account the different private-good endowments for elites and non-elites. First, we recall that the private consumption of the elites in the majority group is given by:

$$x_{ME}^p(b,\beta) = x_{MN}^p(b,\beta) + \beta\frac{\Omega - \theta_M x_{MN}^p(b,\beta) - \theta_m(1-b)\omega_m}{1+\mu\theta_M\beta}. \tag{42}$$

Furthermore, we can explicitly calculate the public-good spending K_g^p under the policy choice of the policy-maker which is given by

$$K_g^p(b,\beta) = \frac{\Omega - \theta_M x_{MN}^p(b,\beta) - \theta_m(1-b)\omega_m}{(1+\lambda)(1+\mu\theta_M\beta)}. \tag{43}$$

As long as $\beta < \frac{1}{\theta_M(1-b)}$, we can rewrite this expression as:

$$K_g^p(b,\beta) = \frac{\Omega - \theta_M x_{ME}^p(b,\beta) - \theta_m(1-b)\omega_m}{(1+\lambda)(1-(1-\mu)\theta_M\beta)} \tag{44}$$

Combining these expressions, we can produce the following formula for the ex-ante social welfare $W(b,\beta)$:

$$W(b,\beta) := \theta_M(1-\mu)u\big(x_{MN}^p\big) + \theta_M\mu u\big(x_{ME}^p\big) + \theta_m u\big((1-b)\omega_m\big) + \gamma K_g^p. \tag{45}$$

where x_{MN}^p, x_{ME}^p and K_g^p depend on b, β as discussed in Corollary 2 and (42), (43). Our goal is to determine parameter values b and β which maximize the social welfare W. In order to do this, we first calculate the optimal β for any given b. Afterwards, we compare the optima for varying b to get a global maximum. It is clear from our previous considerations that the social welfare W depends continuously on β. This observation immediately implies the following result:

Theorem 2. *Suppose that $b \in [b_c, 1]$, then there exists a unique $\beta_b^* \in [0, \bar{\beta}_b]$ maximizing the social welfare $W(b,\beta)$ for $\beta \in [0, \bar{\beta}_b]$.*

Next, we want to discuss the optimality of corner solutions and their associated social welfare. Notice that for any corner solution, we have $x_{MN}^p = (1-b)\omega_M$ and thus the public-good spending becomes by (43):

$$K_g^p = \frac{\Omega b}{(1+\lambda)(1+\mu\theta_M\beta)} \tag{46}$$

Therefore, the social welfare has a simpler formula and only depends in a simple way on β. The following result summarizes the optimal choice of corner solutions:

Proposition 7. *Let $V(b,\beta)$ denote the social welfare under the implementation of the corner solution described in Case 2 for any pair of parameters b, β. Then for a given b, there either exists a β such that:*

$$u'\Big((1-b)\omega_M + \beta_b^* \frac{\Omega b}{1+\mu\theta_M\beta_b^*}\Big) = \frac{\gamma}{1+\lambda}, \tag{47}$$

or the left handside in the equation above is always strictly smaller than the left handside. In this case, we define $\beta_b^* = 0$. Then the social welfare V associated with corner solutions attains its maximum for a given b at β_b^*.

This result leads to the following insight: If for a given b, the β_b^* given in Proposition 7 is strictly less than $\bar{\beta}_b$, then the optimal choice of β in the maximisation of the social welfare W lies in $[0, \bar{\beta}_b[$ and therefore leads to an interior solution. If β_b^* is greater than $\bar{\beta}_b$, we need to compare the optimal value of W in $[0, \bar{\beta}_b]$ with the one of V in order to determine whether the optimal choice of the incentive pay parameter leads to an interior or corner solution. This enables us to find the optimal β for any given tax restriction b.

Lastly, let us consider the case $b \leq b_c$. Notice that $(1-b_c)\omega_M = x_c^p$ and by definition, we have:

$$u\big((1-b_c)\omega_M\big) = u(x_c^p) = \frac{\gamma\theta_M}{1+\lambda}. \tag{48}$$

Due to the proof of Proposition 7, we know that (47) can never be satisfied for $b < b_c$ due to monotonicity. Namely, we have that $\beta = 0$ solves (47) if $\theta_M = 1$ and in any other case, no solution exists. As a result, we have $\beta_b^* = 0$ for all $b \leq b_c$.

Corollary 3. *If $b \leq b_c$, then we have $\beta_b^* = 0$.*

These considerations enable a complete discussion of the optimal constitutional choice of (b, β). To conclude our discussion of Incentive Contracts, we provide a simple example where the maximal social welfare results from an interior solution to the policy maker's optimization problem:

Example 4. *Let us consider $u(x) = \sqrt{x}$ and $\lambda = 0, \gamma = 1, \theta_M = \theta_m = \frac{1}{2}$ and assume that $\omega_M < \omega_m$. We want to show that $\beta_b^* < \bar{\beta}_b$ for any $b > b_c$. This would imply that the optimal incentive pay parameter for $b > b_c$ induces an interior solution by our previous remark. Notice that in order to establish this inequality, it suffices to check:*

$$\frac{\gamma}{1+\lambda}\frac{\theta_M}{1-(1-\mu)\theta_M\bar{\beta}_b} < \frac{\gamma}{1+\lambda}, \tag{49}$$

by using (41) *and* (47) *together with the strict decreasing of* u'. *This reduces to:*

$$(1-\mu)\theta_M\bar{\beta}_b < 1-\theta_M,$$

and therefore

$$\bar{\beta}_b < \frac{1-\theta_M}{\theta_M(1-\mu)} =: \beta_0.$$

We observe that to check this inequality, it suffices by (41) *to check:*

$$\frac{\gamma}{1+\lambda} = \frac{\gamma}{1+\lambda}\frac{\theta_M}{1-(1-\mu)\theta_M\beta_0} > u'\Big((1-b)\omega_M + \beta_0\frac{b\Omega}{1+\mu\theta_M\beta_0}\Big), \tag{50}$$

for all b. *Differentiating the right handside of* (50) *with respect to* b, *we see:*

$$\begin{aligned} 0 &> \Big(-\omega_M + \frac{\Omega\beta_0}{1+\mu\theta_M\beta_0}\Big)u''\Big((1-b)\omega_M + \beta_0\frac{\Omega b}{1+\mu\theta_M\beta_0}\Big) \\ &= \frac{d}{db}\Big(u'\Big((1-b)\omega_M + \beta_0\frac{b\Omega}{1+\mu\theta_M\beta_0}\Big)\Big). \end{aligned} \tag{51}$$

The inequality above is an immediate consequence of the following:

$$\begin{aligned} -\omega_M + \frac{\Omega\beta_0}{1+\mu\theta_M\beta_0} > 0 &\Leftrightarrow \Omega\beta_0 > \omega_M(1+\mu\theta_M\beta_0) \\ &\Leftrightarrow (1-\mu)\theta_M\omega_M\beta_0 + \theta_m\omega_m > \omega_M \\ &\Leftrightarrow \frac{1}{2}\omega_M + \frac{1}{2}\omega_m > \omega_M. \end{aligned} \tag{52}$$

Notice that the last inequality holds by our choice of parameters. Therefore, we can conclude that $\bar{\beta}_b > \beta_b^*$ *by noting that the corresponding inequality holds for* $b=0$ *and using monotonicity as in* (51). *In order to conclude that the optimal pair* (b,β) *induces an interior solution, we just have to show that the optimal* b *satisfies* $b > b_c$. *But by differentiating the expression* $W(b,0)$ *with respect to* b *for* $b \le b_c$, *we see that it attains its maximum at* b_c *with positive derivative. Therefore, the social welfare attains its maximum for* (b,β) *with* $b > b_c$ *and* $\beta < \bar{\beta}_b$ *by our considerations and the optimal policy pair induces an interior solution to the policy-maker's optimization problem.*

9 Conclusion

There are good reasons to be cautious regarding incentive pay for politicians since they face political multi-task problems. Nevertheless, the paper suggests that a judicious combination of tax protection and incentive pay can improve welfare. Some further questions and extensions deserve scrutiny:

First, we can relax the assumption that the policy-maker only maximizes his personal utility. Suppose that the policy-maker is *altruistic*, i.e. he considers society's well-being together with his personal interests when choosing a policy. An altruistic motive tends to lower the welfare optimal incentive parameter, otherwise the apparatus can be applied in the same way as in the baseline model. Of course, a policy-maker who would only care about social welfare would make incentive pay superfluous.

Second, we allow more than two endowment groups and generalize the model to $n > 2$ endowment groups. The approach can be applied to such constellations. Typically, tax protection becomes more important to effectively complement incentive pay.

Finally, we can investigate what happens when candidates compete for election by announcing their desired incentive contract. In such a setting, the candidate from the majority endowment group will win the election and he will offer the incentive pay according to which he will fully tax everybody, to obtain the maximum incentive pay possible.

References

Ashworth, S. (2005). Reputational dynamics and political careers,. *Journal of Law, Economics, & Organization*, 21(2):441–466.

Ashworth, S. and de Mesquita, E. B. (2012). Multitask, accountability, and institutional design. University of Chicago Typescript.

Barro, R. J. (1973). The control of politicians: An economic model. *Public Choice*, 14(1):19–42.

Besley, T. (2004). Paying politicians: Theory and evidence. *Journal of the European Economic Association*, 2(2-3):193–215.

Bolton, P. and Dewatripont, M. (2005). *Contract Theory.* Cambridge, MA, MIT Press.

Ferejohn, J. (1986). Incumbent performance and electoral control. *Public Choice*, 50(1):5–25.

Ferraz, C. and Finan, F. (2009). Motivating politicians: The impacts of monetary incentives on quality and performance. *National Bureau of Economic Research Working Paper*, w14906.

Fleurbaey, M. (2008). *Fairness, Responsibility, and Welfare.* Oxford, Oxford University Press.

Frydman, C. and Jenter, D. (2010). CEO compensation. *Annual Review of Financial Economics*, 2:75–102.

Gersbach, H. (2003). Incentive contracts and election for politicians and the down-up problem. In Sertel, M. and Koray, S., editors, *Advances in Economic Design.* Berlin-Heidelberg, Springer.

Gersbach, H. (2012). Contractual democracy. *Review of Law and Economics*, 8(3):825–851.

Gersbach, H., Hahn, V., and Imhof, S. (2012). Tax rules. *Social Choice and Welfare*, 41(1):19–42.

Gersbach, H. and Liessem, V. (2008). Incentive contracts and elections for politicians with multi-task problems. *Journal of Economic Behavior and Organization*, 68:401–411.

Hatfield, J. W. and i Miquel, G. P. (2006). Multitasking, limited liability and political agency. Mimeo.

Holmström, B. and Milgrom, P. (1991). Multitask principle-agent analyses: Incentive contracts, asset ownership, and job design. *Journal of Law, Economics, and Organization*, 7:24–52.

Kotakorpi, K. and Poutvaara, P. (2011). Pay for politicians and candidate selection: An empirical analysis. *Journal of Public Economics*, 95(7):877–885.

Kreps, D. M. (1997). Intrinsic motivation and extrinsic incentives. *American Economic Review*, 87(2):359–364.

Maskin, E. and Tirole, J. (2004). The politician and the judge: Accountability in government. *American Economic Review*, 94(4):1034–1054.

Mattozzi, A. and Merlo, A. (2008). Political careers or career politicians? *Journal of Public Economics*, 92(3):597–608.

Messner, M. and Polborn, M. K. (2004). Paying politicians. *Journal of Public Economics*, 88(12):2423–2445.

Poutvaara, P. and Takalo, T. (2007). Candidate quality. *International Tax and Public Finance*, 14(1):7–27.

Appendix: Proofs

Proof of Proposition 1

Suppose $u'''(.)$ is non-negative. Since $u'(\cdot)$ is convex, the definition of convex function implies

$$u'(tx) \geq tu'(x) \quad \forall t \geq 1.$$

Since $\frac{1}{\theta_M} > 1$ and $u'(.)$ is convex, we obtain

$$u'\left(\frac{x^s}{\theta_M}\right) \geq \frac{1}{\theta_M} u'\left(x^s\right). \tag{53}$$

Additionally, since $\frac{1}{\theta_M} > 1$, we obtain

$$\frac{1}{\theta_M} u'\left(\frac{x^s}{\theta_M}\right) > u'\left(\frac{x^s}{\theta_M}\right), \tag{54}$$

$$\frac{1}{\theta_M} u'\left(x^s\right) > u'\left(x^s\right). \tag{55}$$

From Inequalities (53)–(55), we obtain

$$\begin{aligned} &\frac{1}{\theta_M} u'\left(\frac{x^s}{\theta_M}\right) > u'\left(\frac{x^s}{\theta_M}\right) \geq \frac{1}{\theta_M} u'\left(x^s\right) > u'\left(x^s\right), \\ &\frac{1}{\theta_M} u'\left(\frac{x^s}{\theta_M}\right) > u'\left(x^s\right). \end{aligned} \tag{56}$$

Thus, we have established that if $u'(\cdot)$ is convex, Inequality (12) holds.

Moreover, we can rewrite Inequality (56) by using Equation (9), to obtain

$$u'\left(\frac{x^s}{\theta_M}\right) > u'\left(x_M^p\right). \tag{57}$$

Since $u'(\cdot)$ is strictly decreasing, we obtain from Inequality (57)

$$\frac{x^s}{\theta_M} < x_M^p.$$

By the definition of under-provision as given in Inequality (10), the public good is under-provided if $u'(\cdot)$ is convex. $\square$

Proof of Proposition 2

We examine the maximization problem of $W(b)$ on $[0,1]$. The ex-ante social welfare as a function of b is given by Equation (18). The function $W(b)$ is defined over the compact set $[0,1]$. To prove that $W(b)$ is continuous, we first prove that $x^p_M(b)$ is continuous over $[0,1]$. For this purpose, we define

$$f(b) := (1-b)\omega_M - x^p_c,$$

over $[0,1]$.
At $b=0$, $f(0)=\omega_M - x^p_c$. By Assumption 2, we know that $\omega_M > x^p_c$. Thus, $f(0)>0$.
At $b=1$, $f(1)=-x^p_c$. Since x^p_c is the interior solution to the policy-maker's problem, it is strictly positive. Thus, $f(1)<0$.

The function $f(b)$ is continuous over the compact set $[0,1]$. By the Intermediate Value Theorem, there exists $b_c \in (0,1)$ such that $f(b_c)=0$. From $f(b_c)=0$, we obtain

$$b_c = 1 - \frac{x^p_c}{\omega_M}. \tag{58}$$

By using the critical value for b, b_c, we rewrite $x^p_M(b)$ as a piecewise function,

$$x^p_M(b) = \begin{cases} x^p_c & b_c < b \leq 1, \\ (1-b)\omega_M & 0 \leq b \leq b_c. \end{cases} \tag{59}$$

The function $x^p_M(b)$ is continuous for both $b \in [0, b_c)$ and $b \in (b_c, 1]$. To show that $x^p_M(b)$ is continuous, we now establish continuity at b_c.
For all $b > b_c$, we have $x^p_M(b) - x^p_M(b_c) = 0$. Let $\varepsilon > 0$. There exists $\delta > 0$ such that if $0 < b - b_c < \delta$, then $x^p_M(b) - x^p_M(b_c) = 0 < \epsilon$. Thus, $\lim_{b \to b_c^+} x^p_M(b)$ exists.
For all $b < b_c$, we have $\mid x^p_M(b) - x^p_M(b_c) \mid = \mid b - b_c \mid \omega_M$. Let $\varepsilon > 0$. There exists $\delta = \frac{\varepsilon}{\omega_M}$ such that if $\mid b - b_c \mid < \delta$, then $\mid x^p_M(b) - x^p_M(b_c) \mid < \delta\omega_M = \varepsilon$. Thus, $\lim_{b \to b_c^-} x^p_M(b)$ exists.
We observe that

$$\lim_{b \to {b_c}^+} x^p_M(b) = x^p_c, \text{ and}$$
$$\lim_{b \to {b_c}^-} x^p_M(b) = (1-b_c)\omega_M = x^p_c.$$

Thus, we have $\lim_{b \to b_c^+} x^p_M(b) = \lim_{b \to b_c^-} x^p_M(b) = x^p_M(b_c)$. We have established that $x^p_M(b)$ is continuous.

Since $x_m^p(b)$ and $x_M^p(b)$—as given in Equations (14) and (59), respectively—are continuous over $b \in [0,1]$ and given our assumptions on $u(\cdot)$, $W(b)$ is continuous over $b \in [0,1]$. Thus, there exists at least one maximizer of $W(b)$ on the compact set $[0,1]$.

To establish that there is a unique maximizer for $W(b)$, we examine the welfare optimization problem in detail for two separate cases: Case 1 for $b \in [b_c, 1]$, and Case 2 for $b \in [0, b_c]$.

Case 1. The optimization problem is as follows:

$$\max_{b_1 \in [b_c,1]} \quad W(b_1) = \theta_M u(x_c^p) + \theta_m u\left((1-b_1)\omega_m\right) + \frac{\gamma\left[\Omega - \theta_M x_c^p - \theta_m \omega_m (1-b_1)\right]}{1+\lambda},$$

where we have substituted for $x_M^p(b_1)$ and $x_m^p(b_1)$ from Equations (14) and (59), respectively, into Equation (18).

This is a constrained optimization problem. Thus, we construct the Lagrangian

$$L \equiv \theta_M u(x_c^p) + \theta_m u\left((1-b_1)\omega_m\right) + \frac{\gamma\left[\Omega - \theta_M x_c^p - \theta_m \omega_m (1-b_1)\right]}{1+\lambda} + r_1\left(b_c - b_1\right) + r_1'(1-b_1).$$

By the Inada Conditions, we know that $b_1 = 1$ cannot be optimal and thus it is not binding. By the complementary slackness conditions, we have $r_1' = 0$. From the first-order condition with respect to b, we obtain

$$\frac{\partial L}{\partial b_1} = -\theta_m \omega_m u'\left((1-b_1)\omega_m\right) + \theta_m \omega_m \frac{\gamma}{1+\lambda} - r_1 = 0. \tag{60}$$

Next, we establish (i) the corner solution and (ii) the interior solution by using the complementary slackness conditions.

(i) *Corner Solution:*
If $r_1 > 0$, we have $b_1^* = b_c$. Equation (60) for $r_1 > 0$ at $b_1 = b_c$ becomes

$$\underbrace{\frac{\gamma}{1+\lambda}}_{=u'(x^s)} - \frac{r_1}{\theta_m \omega_m} = u'\left(x_c^p \frac{\omega_m}{\omega_M}\right),$$

where we have substituted for $(1-b_c)\omega_m = x_c^p \frac{\omega_m}{\omega_M}$ by using Equation (58). From Equation (4), we recall that $u'(x^s) = \frac{\gamma}{1+\lambda}$. Since $r_1 > 0$, we observe that $u'(x^s) < u'\left(x_c^p \frac{\omega_m}{\omega_M}\right)$. Given $u'(\cdot)$ is strictly decreasing, we obtain $x^s > x_c^p \frac{\omega_m}{\omega_M}$. On the contrary, if $\frac{x^s}{x_c^p} \leq \frac{\omega_m}{\omega_M}$, then $r_1 \leq 0$ and the constraint is not binding. We discuss this next.

(ii) *Interior Solution:*
If $r_1 = 0$, Equation (60) becomes

$$u'\left(\left(1 - b_1^*\right)\omega_m\right) = \frac{\gamma}{1+\lambda}.$$

We recall from Equation (4) that $u'(x^s) = \frac{\gamma}{1+\lambda}$. Reordering and rewriting Equation (9), we obtain

$$b_1^* = 1 - \frac{x^s}{\omega_m}.$$

Additionally, the second-order condition is

$$\underbrace{\theta_m \omega_m^2}_{>0} \underbrace{u''\left(\left(1 - b_1^*\right)\omega_m\right)}_{<0} < 0.$$

Since the second-order condition is strictly concave. there is at most one interior maximizer of $W(b)$.

Thus, in Case 1, if $\frac{x^s}{x_c^p} > \frac{\omega_m}{\omega_M}$, the constraint is binding and $b_1^* = b_c$ where $b_c = 1 - \frac{x_c^p}{\omega_M}$ as given in Equation (58). However, if $\frac{x^s}{x_c^p} \leq \frac{\omega_m}{\omega_M}$, then the constraint is not binding and the optimization problem in Case 1 has a unique interior solution given by $b_1^* = 1 - \frac{x^s}{\omega_m}$.

Case 2. The optimization problem is as follows:

$$\max_{b_2 \in [0, b_c]} \quad W(b_2) = \theta_M u\left((1 - b_2)\omega_M\right) + \theta_m u\left((1 - b_2)\omega_m\right) + \frac{b_2 \gamma \Omega}{1+\lambda},$$

where we have substituted for $x_M^p(b_2)$ and $x_m^p(b_2)$ from Equations (14) and (59), respectively, into Equation (18).

This is a constrained optimization problem. Thus, we construct the Lagrangian

$$L \equiv \theta_M u\left((1 - b_2)\omega_M\right) + \theta_m u\left((1 - b_2)\omega_m\right) + \frac{b_2 \gamma \Omega}{1+\lambda} + r_2\left(b_c - b_2\right) - r_2' b_2.$$

By Assumption 1, we know that $b = 0$ cannot be optimal and thus it is not binding. By the complementary slackness conditions, we have $r_2' = 0$.

From the first-order condition with respect to b_2, we obtain

$$\frac{\partial L}{\partial b_2} = -\theta_M \omega_M u'\left((1 - b_2)\omega_M\right) - \theta_m \omega_m u'\left((1 - b_2)\omega_m\right) + \Omega \frac{\gamma}{1+\lambda} - r_2 = 0. \tag{61}$$

Next, we establish (i) the corner solution and (ii) the interior solution by using the complementary slackness conditions.

(i) *Corner Solution:*
If $r_2 > 0$, we have $b_2^* = b_c$. Equation (61) for $r_2 > 0$ and $b_2 = b_c$ becomes

$$-\theta_M \omega_M u'\left(x_c^p\right) - \theta_m \omega_m u'\left(x_c^p \frac{\omega_m}{\omega_M}\right) + \Omega \frac{\gamma}{1+\lambda} = r_2. \tag{62}$$

where we have substituted for $(1-b_c)\omega_m = x_c^p \frac{\omega_m}{\omega_M}$ and $(1-b_c)\omega_M = x_c^p$. There are two cases, where Equation (62) holds for $r_2 > 0$. We consider these two cases in the following:

(a.) We first establish that if $\frac{\omega_m}{\omega_M} \geq \frac{x^s}{x_c^p}$, then $b_2^* = b_c$.
We have $x_m^p(b_c) = x_c^p \frac{\omega_m}{\omega_M}$. If $\frac{\omega_m}{\omega_M} \geq \frac{x^s}{x_c^p}$, then $x_c^p \frac{\omega_m}{\omega_M} \geq x^s$. Since $u'(\cdot)$ is strictly decreasing, $u'\left(x_c^p \frac{\omega_m}{\omega_M}\right) \leq u'(x^s)$. We recall from Equation (4) that $u'(x^s) = \frac{\gamma}{1+\lambda}$, and we obtain

$$u'\left(x_c^p \frac{\omega_m}{\omega_M}\right) < \frac{\gamma}{1+\lambda}. \tag{63}$$

Additionally, we have $x_M^p(b_2) = x_c^p$. From Assumption 2, we know that

$$u'(x_c^p) < \frac{\gamma}{1+\lambda}. \tag{64}$$

If we multiply Equation (63) by $\theta_m \omega_m$ and Equation (64) by $\theta_M \omega_M$ and we take the sum, we obtain

$$\theta_m \omega_m u'\left(x_c^p \frac{\omega_m}{\omega_M}\right) + \theta_M \omega_M u'(x_c^p) < \frac{\gamma}{1+\lambda} \underbrace{\left[\theta_m \omega_m + \theta_M \omega_M\right]}_{=\Omega}. \tag{65}$$

If we reorder Inequality (65), we obtain

$$-\theta_M \omega_M u'\left(x_c^p\right) - \theta_m \omega_m u'\left(x_c^p \frac{\omega_m}{\omega_M}\right) + \Omega \frac{\gamma}{1+\lambda} > 0. \tag{66}$$

If Inequality (66) holds, the left hand side of Equation (62) is strictly positive. Thus, $r_2 > 0$. Thus, if $\frac{\omega_m}{\omega_M} > \frac{x^s}{x_c^p}$, the constraint is binding and $b_2^* = b_c$.

(b.) We now establish that if $\frac{\omega_m}{\omega_M} < \frac{x^s}{x_c^p}$ and $\frac{u'\left(x_c^p \frac{\omega_m}{\omega_m}\right)}{u'(x^s)} < 1 + \theta_m \left(\frac{\theta_M \omega_M}{\theta_m \omega_m}\right)$ then $r_2 > 0$ and $b_2^* = b_c$.
In Equation (62), we substitute for $u'(x_c^p)$ from Equation (16). Re-

ordering, we obtain

$$-\theta_m\omega_m u'\left(x_c^p\frac{\omega_m}{\omega_M}\right)+\underbrace{\frac{\gamma}{1+\lambda}}_{=u'(x^s)}\left[\theta_m\omega_m+\underbrace{(1-\theta_M)}_{=\theta_m}\theta_M\omega_M\right]=r_2. \quad (67)$$

The left hand side of Equation (67) is strictly positive if

$$u'\left(x_c^p\frac{\omega_m}{\omega_M}\right)<u'(x^s)\left[1+\theta_m\left(\frac{\theta_M\omega_M}{\theta_m\omega_m}\right)\right].$$

Thus, if $\frac{\omega_m}{\omega_M}<\frac{x^s}{x_c^p}$ and $\frac{u'\left(x_c^p\frac{\omega_m}{\omega_m}\right)}{u'(x^s)}<1+\theta_m\left(\frac{\theta_M\omega_M}{\theta_m\omega_m}\right)$, the left hand side of Equation (67) is strictly positive. Thus, $r_2>0$ and the constraint is binding, $b_2^*=b_c$.

However, if $\frac{\omega_m}{\omega_M}<\frac{x^s}{x_c^p}$ and $\frac{u'\left(x_c^p\frac{\omega_m}{\omega_m}\right)}{u'(x^s)}\geq 1+\theta_m\left(\frac{\theta_M\omega_M}{\theta_m\omega_m}\right)$, then $r_2\leq 0$ and the constraint is not binding. We discuss this next.

(ii) *Interior Solution:*
If $r_2=0$, Equation (61) becomes

$$\frac{\gamma\Omega}{1+\lambda}=\theta_M\omega_M u'\left((1-b_2^*)\omega_M\right)+\theta_m\omega_m u'\left((1-b_2^*)\omega_m\right). \quad (68)$$

The problem has an interior solution which is implicitly given by Equation (68).

Additionally, the second-order condition is

$$\underbrace{\theta_m\omega_m^2}_{>0}\underbrace{u''\left(\left(1-b_2^*\right)\omega_m\right)}_{<0}+\underbrace{\theta_M\omega_M^2}_{>0}\underbrace{u''\left(\left(1-b_2^*\right)\omega_M\right)}_{<0}<0.$$

Since the second-order condition is strictly concave. there is at most one interior maximizer of $W(b)$.

Thus, in Case 2, the constraint is binding and $b_2^*=b_c$ if (a) $\frac{\omega_m}{\omega_M}\geq\frac{x^s}{x_c^p}$ or (b) $\frac{\omega_m}{\omega_M}<\frac{x^s}{x_c^p}$ and $\frac{u'\left(x_c^p\frac{\omega_m}{\omega_m}\right)}{u'(x^s)}<1+\theta_m\left(\frac{\theta_M\omega_M}{\theta_m\omega_m}\right)$. However, if $\frac{\omega_m}{\omega_M}<\frac{x^s}{x_c^p}$ and $\frac{u'\left(x_c^p\frac{\omega_m}{\omega_m}\right)}{u'(x^s)}\geq 1+\theta_m\left(\frac{\theta_M\omega_M}{\theta_m\omega_m}\right)$, the optimization problem has a unique interior solution $b_2^*\in(0,b_c)$, implicitly given by Equation(68).

To summarize the results in Case 1 and Case 2,

- if $\frac{\omega_m}{\omega_M}\leq\frac{x^s}{x_c^p}$, the solution to Case 1 is interior $b_1^*\in(b_c,1)$. We have $W(b_1^*)\geq W(b)$ for all $b\in[b_c,1]$. In particular, we have $W(b_1^*)>W(b_c)$.

Additionally, if $\frac{\omega_m}{\omega_M} \leq \frac{x^s}{x_c^p}$, $b_2^* = b_c$ and $W(b_c) > W(b)$ for all $b \in [0, b_c)$.
We recall that $W(b)$ is continuous at b_c. Thus, if $\frac{\omega_m}{\omega_M} \leq \frac{x^s}{x_c^p}$, we have $W(b_1^*) \geq W(b)$ for all $b \in [b_c, 1]$ and $W(b_c) > W(b)$ for all $b \in [0, b_c)$. We conclude that

$$W(b_1^*) \geq W(b),\ \forall b \in [0, 1],$$

where $b_1^* = 1 - \frac{x^s}{\omega_m}$.

- if $\frac{\omega_m}{\omega_M} > \frac{x^s}{x_c^p}$, the solution to Case 1 is at the corner, $b_1^* = b_c$ and $W(b_c) > W(b)$ for all $b \in (b_c, 1]$.
  - Additionally, if $\frac{\omega_m}{\omega_M} > \frac{x^s}{x_c^p}$ and $\frac{u'\left(x_c^p \frac{\omega_m}{\omega_m}\right)}{u'(x^s)} < 1 + \theta_m \left(\frac{\theta_M \omega_M}{\theta_m \omega_m}\right)$, the solution to Case 2 is at the corner, $b_2^* = b_c$ and $W(b_c) > W(b)$ for all $b \in [0, b_c)$.
    Thus, if $\frac{\omega_m}{\omega_M} > \frac{x^s}{x_c^p}$ and $\frac{u'\left(x_c^p \frac{\omega_m}{\omega_m}\right)}{u'(x^s)} < 1 + \theta_m \left(\frac{\theta_M \omega_M}{\theta_m \omega_m}\right)$, we conclude

    $$W(b_c) \geq W(b)\ \forall b \in [0, 1],$$

    where $b_c = 1 - \frac{x_c^p}{\omega_M}$.
  - Moreover, if $\frac{\omega_m}{\omega_M} > \frac{x^s}{x_c^p}$ and $\frac{u'\left(x_c^p \frac{\omega_m}{\omega_m}\right)}{u'(x^s)} \geq 1 + \theta_m \left(\frac{\theta_M \omega_M}{\theta_m \omega_m}\right)$, the solution to Case 2 is interior $b_2^* \in (0, b_c)$. We have $W(b_2^*) \geq W(b)$ for all $b \in [0, b_c]$. In particular, we have $W(b_2^*) > W(b_c)$.
    Thus, if $\frac{\omega_m}{\omega_M} > \frac{x^s}{x_c^p}$ and $\frac{u'\left(x_c^p \frac{\omega_m}{\omega_m}\right)}{u'(x^s)} \geq 1 + \theta_m \left(\frac{\theta_M \omega_M}{\theta_m \omega_m}\right)$, we have $W(b_c) > W(b)$ for all $b \in (b_c, 1]$ and $W(b_2^*) > W(b_c)$. We conclude that

    $$W(b_2^*) \geq W(b)\ \forall b \in [0, 1],$$

    where b_2^* is implicitly given by Equation (68).

□

Proof of Lemma 1

Let $\beta < \frac{1}{\theta_M (1-\mu)}$ and $\mu \in [0, 1)$.

(i) Equation (29) gives x_{ME}^p as an implicit function of β. Given our assumptions on $u(\cdot)$, we know that $u'(\cdot)$ is differentiable. Applying the implicit function theorem to Equation (29) yields

$$\frac{\partial u'\left(x_{ME}^p(\beta)\right)}{\partial \beta} = \underbrace{\frac{\partial u'\left(x_{ME}^p\right)}{\partial x_{ME}^p}}_{<0} \cdot \frac{\partial x_{ME}^p}{\partial \beta} = \underbrace{\frac{\gamma \theta_M^2 (1-\mu)}{(1+\lambda)\left[1 - \beta \theta_M (1-\mu)\right]^2}}_{>0}.$$

We see that the marginal utility of x^p_{ME} is increasing in β. Given $u''(\cdot) < 0$, we conclude that $\frac{\partial x^p_{ME}}{\partial \beta} < 0$.

(ii) From Equation (30), we have

$$K^p_g(\beta) = \frac{\Omega - \theta_M x^p_{ME}(\beta)}{(1+\lambda)(1-\beta\theta_M(1-\mu))}.$$

For $\beta < \frac{1}{\theta_M(1-\mu)}$, the function $\frac{1}{1-\beta\theta_M(1-\mu)}$ is differentiable. Additionally, $x^p_{ME}(\beta)$ is a differentiable function of β. Thus, we conclude that $K^p_g(\beta)$ is a differentiable function.

We take the derivative of the equation above with respect to β. We obtain

$$\frac{\partial K^p_g}{\partial \beta} = \underbrace{\frac{\theta_M(1-\mu)\left[\Omega - \theta_M x^p_{ME}\right]}{(1+\lambda)\left[1-\beta\theta_M(1-\mu)\right]^2}}_{>0} - \underbrace{\frac{\theta_M\left(\frac{\partial x^p_{ME}}{\partial \beta}\right)}{(1+\lambda)\left[1-\beta\theta_M(1-\mu)\right]}}_{<0}.$$

Thus, $\frac{\partial K^p_g}{\partial \beta} > 0$.

(iii) From Equation (21), we see that $x^p_{MN}(\beta)$ is given by

$$x^p_{MN}(\beta) = x^p_{ME}(\beta) - \beta(1+\lambda)K^p_g(\beta). \tag{69}$$

By using Equation (69), we see that $x^p_{MN}(\beta)$ is a sum of two differentiable functions. Thus, it is a differentiable function of β.

Finally, we take the derivative of Equation (69) with respect to β and we obtain

$$\frac{\partial x^p_{MN}}{\partial \beta} = \underbrace{\frac{\partial x^p_{ME}}{\partial \beta}}_{<0} - \underbrace{\left[(1+\lambda)K^p_g(\beta) + \beta(1+\lambda)\frac{\partial K^p_g}{\partial \beta}\right]}_{>0}.$$

With the right hand side being negative, we conclude that $\frac{\partial x^p_{MN}}{\partial \beta} < 0$.

□

Proof of Proposition 3

Let $\mu \in [0,1)$. To prove (i), we consider the interior solution to the policy-maker's problem.

Equation (29) gives x^p_{MN} as an implicit function of β. If we substitute for $K^p_g(\beta)$ from Equation (23 in the left hand side of Equation (29) and we rewrite and reorder

Equation (29), we obtain

$$x^p_{MN}(\beta) = \underbrace{\left(\frac{1+\mu\theta_M\beta}{1-\beta\theta_M(1-\mu)}\right)}_{>0} \left[(u')^{-1}\left(\frac{\gamma\theta_M}{(1+\lambda)(1-\beta\theta_M(1-\mu))}\right) - \frac{\beta\Omega}{1+\mu\theta_M\beta}\right]. \tag{70}$$

We note that for all $\beta \in \left[0, \frac{1}{\theta_M(1-\mu)}\right)$, $x^p_{MN}(\beta)$ can only be zero if the term in the large bracket is equal to zero.

We next establish the existence of $\bar{\beta}$ such that $x^p_{MN}(\bar{\beta}) = 0$. For this purpose, we define

$$F(\beta) := (u')^{-1}\left(\frac{\gamma\theta_M}{(1+\lambda)(1-\beta\theta_M(1-\mu))}\right) - \frac{\beta\Omega}{1+\mu\theta_M\beta}. \tag{71}$$

We first show that $F(0)$ and $\lim_{\beta\to\left(\frac{1}{\theta_M(1-\mu)}\right)^-} F(\beta)$ have different signs.

To calculate $\lim_{\beta\to\left(\frac{1}{\theta_M(1-\mu)}\right)^-} F(\beta)$, we fist recall from the Inada Conditions that $\lim_{x\to 0} u'(x) = \infty$. Consequently, $\lim_{x\to\infty}(u')^{-1}(x) = 0$. Thus, we obtain

$$\begin{aligned}
\lim_{\beta\to\left(\frac{1}{\theta_M(1-\mu)}\right)^-} F(\beta) &= \lim_{\beta\to\left(\frac{1}{\theta_M(1-\mu)}\right)^-} \left[(u')^{-1}\left(\frac{\gamma\theta_M}{(1+\lambda)(1-\beta\theta_M(1-\mu))}\right) - \frac{\beta\Omega}{1+\mu\theta_M\beta}\right], \\
&= \lim_{\beta\to\left(\frac{1}{\theta_M(1-\mu)}\right)^-} (u')^{-1}\left(\frac{\gamma\theta_M}{(1+\lambda)(1-\beta\theta_M(1-\mu))}\right) - \frac{\Omega}{\theta_M}, \\
&= 0 - \frac{\Omega}{\theta_M}.
\end{aligned}$$

Thus, we have established $\lim_{\beta\to\left(\frac{1}{\theta_M(1-\mu)}\right)^-} F(\beta) < 0$.
To calculate $F(0)$, we substitute for $\beta = 0$ in Equation (71). We obtain

$$F(0) = (u')^{-1}\left(\frac{\gamma\theta_M}{1+\lambda}\right).$$

Thus, we observe that $F(0) > 0$. We have established that $F(0)$ and $\lim_{\beta\to\left(\frac{1}{\theta_M(1-\mu)}\right)^-} F(\beta)$ have different signs. Given our assumptions on $u(\cdot)$, $F(\beta)$ is a continuous function for all $\beta \in \left[0, \frac{1}{\theta_M(1-\mu)}\right)$.

Since $F(\beta)$ is continuous, there exists $c \in \left[0, \frac{1}{\theta_M(1-\mu)}\right)$ which is as close as we want it to $\frac{1}{\theta_M(1-\mu)}$ such that $F(c)$ and $\lim_{\beta\to\left(\frac{1}{\theta_M(1-\mu)}\right)^-} g(\beta)$ have the same sign. Consequently, $F(0)$ and $F(c)$ have opposite signs. Thus, by the Intermediate Value Theorem, there exists a $\bar{\beta} \in [0, c] \subset \left[0, \frac{1}{\theta_M(1-\mu)}\right)$, such that $g\left(\bar{\beta}\right) = 0$.

By Equation (70), we observe that if $F(\bar{\beta}) = 0$, then $x^p_{MN}(\bar{\beta}) = 0$.
The preceding analysis proves that there exists a $\bar{\beta}$ such that $x^p_{MN}(\bar{\beta}) = 0$ and

which satisfies $F(\bar{\beta}) = 0$, i.e.

$$(u')^{-1}\left(\frac{\gamma\theta_M}{(1+\lambda)(1-\bar{\beta}\theta_M(1-\mu))}\right) = \frac{\bar{\beta}\Omega}{1+\mu\theta_M\bar{\beta}}. \tag{72}$$

To show that $\bar{\beta}$ is unique, we recall from Lemma 1 that $x^p_{MN}(\beta)$ is strictly decreasing. Thus, there is a unique $\bar{\beta}$ such that $x^p_{MN}(\bar{\beta}) = 0$. Given Equation (70), the unique $\bar{\beta}$ that sets $x^p_{MN}(\bar{\beta}) = 0$ satisfies Equation (72). Equation (72) gives us a unique expression of $\bar{\beta}$ as an implicit function of exogenous parameters.

The preceding proves (i). Next, we prove (ii).

($\Rightarrow$) Let (x^p_{MN}, x^p_m) be optimal and let x^p_{MN} be the interior maximizer. The proof is by contradiction. Suppose $\exists\tilde{\beta} \in \left(\bar{\beta}, \frac{1}{\theta_M(1-\mu)}\right)$ such that $x^p_{MN}(\tilde{\beta})$ is the interior optimal solution to the policy-maker's problem.

By Lemma 1, we know that x^p_{MN} is a strictly decreasing function of β. Thus, given $\tilde{\beta} > \bar{\beta}$, we have $x^p_{MN}(\tilde{\beta}) < x^p_{MN}(\bar{\beta})$.

Since $x^p_{MN}(\bar{\beta}) = 0$, we conclude $x^p_{MN}(\tilde{\beta}) < 0$. Thus, $x^p_{MN}(\tilde{\beta}) \notin C'$ and consequently $x^p_{MN}(\tilde{\beta})$ is not a feasible policy and cannot be the optimal solution. This contradicts our initial assumption.

($\Leftarrow$) Let $0 \leq \beta \leq \bar{\beta}$. Equation (29) gives the interior solution to the policy-maker's problem, $x^p_{MN}(\beta)$, as an implicit function of β. We want to prove that the interior solution is the unique optimal solution. For this purpose, we first establish that $x^p_{MN}(\beta)$ is positive for all $\beta \in [0, \bar{\beta}]$.

We have established that $x^p_{MN}\left(\bar{\beta}\right) = 0$. Moreover, at $\beta = 0$, from Equation (21), we have $x^p_{MN}(0) = x^p_{ME}(0)$. By using Equation (29) to calculate $x^p_{ME}(0)$, we obtain $x^p_{ME}(0) = x^p_M$, where x^p_M is given by Equation (7).

By Lemma 1, we know that x^p_{MN} is a strictly decreasing function of β. Since x^p_{MN} is a strictly decreasing and continuous function of β, we have $x^p_{MN} \in [0, x^p_M]$ for all $\beta \in \left[0, \bar{\beta}\right]$.

Since $x^p_{MN} \geq 0$ for all $\beta \in [0, \bar{\beta}]$, given Equation (70), we obtain

$$(u')^{-1}\left(\frac{\gamma\theta_M}{(1+\lambda)(1-\beta\theta_M(1-\mu))}\right) - \frac{\beta\Omega}{1+\mu\theta_M\beta} \geq 0. \tag{73}$$

Since $(u')^{-1}(\cdot)$ is strictly decreasing, if Inequality (73) holds, we obtain

$$\frac{\gamma\theta_M}{(1+\lambda)(1-\beta\theta_M(1-\mu))} \leq u'\left(\frac{\beta\Omega}{1+\mu\theta_M\beta}\right). \tag{74}$$

Inequality (74) is the same as Inequality (28). Since Inequality (28) holds, the optimal solution to the policy-maker's problem is interior.

Additionally, we note that the second-order condition for the policy-maker's problem is given by

$$\frac{\partial^2 U}{\partial x_{MN}^2} = \frac{1}{1+\mu\theta_M\beta}\left[-\theta_M(1-\mu)\underbrace{u'\left(x_{ME}^p(\beta)\right)}_{>0}+\frac{\left(1-\beta\theta_M(1-\mu)\right)^2}{1+\mu\theta_M\beta}\underbrace{u''\left(x_{ME}^p(\beta)\right)}_{<0}\right].$$

Since the second-order condition is strictly decreasing, the policy-maker's problem has a unique interior solution. Finally, we observe that the interior solution to the policy-maker's problem uniquely maximizes the policy-maker's utility. □

Proof of Proposition 4

The proof follows immediately from the fact that the private-good consumption of all endowment groups remains unchanged with the introduction of the incentive contract when $\mu = 1$.

At $\mu = 1$, every citizen in the majority endowment group belongs to the Elites. Setting $\mu = 1$ in Equations (29), we obtain

$$u'\left(x_{ME}^p\right) = \frac{\gamma\theta_M}{1+\lambda}.$$

We note that this is equal to Equation (7). Applying the implicit function theorem yields

$$\frac{\partial u'\left(x_{ME}^p\right)}{\partial\beta} = \underbrace{\frac{\partial u'\left(x_{ME}^p\right)}{\partial x_{ME}^p}}_{<0}\cdot\frac{\partial x_{ME}^p}{\partial\beta} = 0.$$

Since $u''(\cdot) < 0$, we obtain $\frac{\partial x_{ME}^p}{\partial\beta} = 0$.

With $\mu = 1$ and every citizen in majority endowment group being an Elite citizen, we have $x_{ME}^p = x_{MN}^p$. Thus, we can conclude that $\frac{\partial x_{MN}^p}{\partial\beta} = 0$.

The minority endowment group's private-good consumption is always set to zero, $x_m^p = 0$, and does not change with β.

Given at $\mu = 1$, we have $u'\left(x_{ME}^p\right) = u'\left(x_M^p\right)$. Using Equation (30), the public-good spending at $\mu = 1$ is given by

$$K_g^p = \frac{\Omega - \theta_p x_M^p}{1+\lambda}.$$

This is equal to Equation (8). Taking the derivative with respect to β, we obtain $\frac{\partial K_g^p}{\partial \beta} = 0$.

Given $\frac{\partial x_{ME}^p}{\partial \beta} = \frac{\partial x_{MN}^p}{\partial \beta} = \frac{\partial x_m^p}{\partial \beta} = \frac{\partial K_g^p}{\partial \beta} = 0$, the incentive contract has no impact at $\mu = 1$. $\square$

Proof of Theorem 1

We examine the maximization problem of $W(\beta)$ over $[0, \bar{\beta}]$.

$$\max_{\beta \in [0,\bar{\beta}]} \quad W(\beta) = \theta_M(1-\mu)u(x_{MN}^p(\beta)) + \theta_M \mu u(x_{ME}^p(\beta)) + \theta_m u(x_m^p) + \frac{\gamma}{1+\lambda} \frac{[\Omega - \theta_M x_{ME}^p(\beta)]}{1-\beta\theta_M(1-\mu)}. \tag{75}$$

We note that for $\beta \in [0, \bar{\beta}]$, by Proposition 3, the policy-maker's problem has a unique interior optimal solution given by Equations (21), (29) and (30). We have substituted for $K_g^p(\beta)$ from Equation (30) in Equation (32). The welfare function $W(\beta)$ is a continuous function on the closed interval $[0, \bar{\beta}]$. By the Extreme Value Theorem, $W(\beta)$ has a maximum and a minimum on $[0, \bar{\beta}]$.

We first show that W is not maximized at either of the corner values for β. We take the derivative of the ex-ante welfare function with respect to β. Reordering and rewriting we obtain

$$\begin{aligned}\frac{\partial W}{\partial \beta} =& \theta_M(1-\mu)u'\left(x_{MN}^p(\beta)\right)\frac{\partial x_{MN}^p}{\partial \beta} + \theta_M \mu u'\left(x_{ME}^p(\beta)\right)\frac{\partial x_{ME}^p}{\partial \beta} \\ &+ \frac{\gamma}{1+\lambda}\left[\frac{\theta_M(1-\mu)\left[\Omega - \theta_M x_{ME}^p(\beta)\right]}{\left(1-\beta\theta_M(1-\mu)\right)^2} - \frac{\theta_M \frac{\partial x_{ME}^p}{\partial \beta}}{1-\beta\theta_M(1-\mu)}\right].\end{aligned} \tag{76}$$

At $\beta = 0$, the welfare is equal to the one under the policy-maker's policy choice without any incentive pay. To see that $\beta = 0$ is not optimal, we need to show that $\left.\frac{\partial W}{\partial \beta}\right|_{0+} > 0$. For this purpose, by using Equation (21), we first establish

$$\left.\frac{\partial x_{ME}^p}{\partial \beta}\right|_{0+} = \left.\frac{\partial x_{MN}^p}{\partial \beta}\right|_{0+} + \left[\Omega - \theta_M u'(x_M^p)\right]. \tag{77}$$

We have substituted for $x_{ME}^p(0) = x_M^p$ in Equation (77).

We now calculate Equation (76) at $\beta = 0^+$. By using Equation (77) and rewriting and reordering, we obtain

$$\left.\frac{\partial W}{\partial \beta}\right|_{0+} = \frac{\gamma\theta_M}{1+\lambda}\underbrace{(\theta_M - 1)}_{<0}\underbrace{\left.\frac{\partial x^p_{ME}}{\partial \beta}\right|_{0+}}_{<0} \tag{78}$$

$$+ \frac{\gamma\theta_M}{1+\lambda}(1-\mu)(1-\theta_M)\left[\Omega - \theta_M u'(x^p_M)\right]. \tag{79}$$

From Lemma 1, we know that $\frac{\partial x^p_{ME}}{\partial \beta} < 0$ for $\mu \in [0,1)$. Thus, Line (78) is positive. Given our assumptions on all exogenous parameters, Line (79) is also positive. We conclude that $\left.\frac{\partial W}{\partial \beta}\right|_{0+} > 0$. Thus, $\beta = 0$ cannot be optimal.

Next, we show that $W(\bar{\beta})$ is not optimal. We consider Equation (76) again. Reordering and rewriting we obtain

$$\left.\frac{\partial W}{\partial \beta}\right|_{\bar{\beta}^-} = \theta_M(1-\mu)u'\left(x^p_{MN}\left(\bar{\beta}\right)\right)\left.\frac{\partial x^p_{MN}}{\partial \beta}\right|_{\bar{\beta}^-}$$

$$+ \theta_M\mu u'\left(x^p_{ME}\left(\bar{\beta}\right)\right)\left.\frac{\partial x^p_{ME}}{\partial \beta}\right|_{\bar{\beta}^-}$$

$$+ \frac{\gamma}{1+\lambda}\left[\frac{\theta_M(1-\mu)\left[\Omega - \theta_M x^p_{ME}(\bar{\beta})\right]}{\left(1-\bar{\beta}\theta_M(1-\mu)\right)^2} - \frac{\theta_M \left.\frac{\partial x^p_{ME}}{\partial \beta}\right|_{\bar{\beta}^-}}{1-\bar{\beta}\theta_M(1-\mu)}\right].$$

From Lemma 1, we know that $\frac{\partial x^p_{MN}}{\partial \beta} < 0$ for $\mu \in [0,1)$. While the second and the third summands remain finite, the first summand goes to $(-\infty)$ when $\beta \to \bar{\beta}^-$. This is due to the fact that $x^p_{MN}(\bar{\beta}) = 0$ and that by the Inada Conditions, $u'(0) \to \infty$. Thus, at $\bar{\beta}$, the function $W(\cdot)$ is decreasing in β, and $W(\bar{\beta} - \varepsilon) > W(\bar{\beta})$, with ε having a small positive value. Consequently, $\bar{\beta}$ is not the maximizer of $W(\cdot)$.

Given that the maximum of $W(\cdot)$ is not at the corners, there exists $\beta^* \in (0, \bar{\beta})$ which is the interior maximizer of $W(.)$. □

Proof of Proposition 5

- Proof of Statement (i):
  To see if the Elites are better off with the incentive contract, we take the derivative of the Elites' interim expected utility (Equation (33)) with respect

to β and we obtain

$$\begin{aligned}\frac{\partial \mathcal{U}_E}{\partial \beta} =&\theta_M \underbrace{\frac{\partial x^p_{ME}(\beta)}{\partial \beta}}_{<0} \underbrace{u'\left(x^p_{ME}(\beta)\right)}_{>0} \\ &+ \underbrace{\frac{\gamma\theta_M}{1+\lambda}\left(\frac{1}{1-\beta\theta_M(1-\mu)}\right)\left[-\frac{\partial x^p_{ME}(\beta)}{\partial \beta} + (1-\mu)\left(\frac{\Omega - \theta_M x^p_{ME}(\beta)}{1-\beta\theta_M(1-\mu)}\right)\right]}_{>0}.\end{aligned}$$

Here, we have substituted for the public-good spending from Equation (30) and we have used our assumption about the utility of private-good consumption which is normalized to zero at $x^p_m = 0$.

From Lemma 1, we know that $\frac{\partial x^p_{ME}(\beta)}{\partial \beta} < 0$ for $\mu \in [0, 1)$. We see that the first line is negative and the second line is positive. Reordering and rewriting the equation above, we obtain

$$\begin{aligned}\frac{\partial \mathcal{U}_E}{\partial \beta} &= \frac{\gamma\theta_M(1-\mu)}{1+\lambda}\frac{[\Omega - \theta_M x^p_{ME}(\beta)]}{(1-\beta\theta_M(1-\mu))^2} \\ &\quad + \theta_M \frac{\partial x^p_{ME}(\beta)}{\partial \beta}\left(u'\left(x^p_{ME}(\beta)\right) - \frac{\gamma}{1+\lambda}\left(\frac{1}{1-\beta\theta_M(1-\mu)}\right)\right).\end{aligned}$$

While the first line is positive, the second line is the sum of a negative and a positive term. For $\frac{\partial \mathcal{U}_E}{\partial \beta}$ to be positive, ($\frac{\partial \mathcal{U}_E}{\partial \beta} > 0$), the second line has to be positive. Given $\frac{\partial x^p_{ME}(\beta)}{\partial \beta} < 0$, it is sufficient to show that

$$u'\left(x^p_{ME}(\beta)\right) - \frac{\gamma}{(1+\lambda)\left[1-\beta\theta_M(1-\mu)\right]} < 0.$$

Substituting from Equation (29), we see that the above inequality corresponds to

$$\frac{\gamma\theta_M}{(1+\lambda)\left[1-\beta\theta_M\left(1-\mu\right)\right]} - \frac{\gamma}{(1+\lambda)\left[1-\beta\theta_M(1-\mu)\right]} < 0. \tag{80}$$

Given $\frac{1}{2} \leq \theta_M < 1$, the above inequality always holds.

- Proof of Statement (ii):
  To see if the Non-elites are better off with the incentive contract, we take the derivative of the Elites' interim expected utility (Equation (34)) with respect

to β and we obtain

$$\frac{\partial \mathcal{U}_{NE}}{\partial \beta} = \theta_M \left[\underbrace{\frac{\partial x^p_{MN}(\beta)}{\partial \beta}}_{<0} \underbrace{u'\left(x^p_{MN}(\beta)\right)}_{>0} \right.$$

$$\left. + \underbrace{\frac{\gamma}{1+\lambda}\left(\frac{1}{1-\beta\theta_M(1-\mu)}\right)\left[-\frac{\partial x^p_{ME}(\beta)}{\partial \beta} + (1-\mu)\left(\frac{\Omega - \theta_M x^p_{ME}(\beta)}{1-\beta\theta_M(1-\mu)}\right)\right]}_{>0} \right].$$

In the above, we have substituted for the public-good spending from Equation (30) and we have used our assumption about the utility of private-good consumption which is normalized to zero at $x^p_m = 0$.

From Lemma 1, we know that $\frac{\partial x^p_{MN}(\beta)}{\partial \beta} < 0$ for $\mu \in [0,1)$. We see that the first line is negative and the second line is positive. For $\frac{\partial \mathcal{U}_{NE}}{\partial \beta}$ to be positive, $\frac{\partial \mathcal{U}_{NE}}{\partial \beta} \geq 0$, given $\frac{1}{2} \leq \theta_M < 1$, we have to show that the term in the large bracket above is positive,

$$\frac{\partial x^p_{MN}(\beta)}{\partial \beta} u'\left(x^p_{MN}(\beta)\right) - \frac{\partial x^p_{ME}(\beta)}{\partial \beta}\frac{\gamma}{1+\lambda}\left(\frac{1}{1-\beta\theta_M(1-\mu)}\right) + \left(\frac{\gamma(1-\mu)}{1-\beta\theta_M(1-\mu)}\right) K^p_g(\beta) \geq 0.$$

In the above inequality, we substitute for $\frac{\partial x^p_{MN}}{\partial \beta} = \frac{\partial x^p_{ME}}{\partial \beta} - (1+\lambda)K^p_g(\beta) - \beta(1+\lambda)\frac{\partial K^p_g}{\partial \beta}$. We obtain

$$u'\left(x^p_{MN}\right)\left[\frac{\partial x^p_{ME}}{\partial \beta} - (1+\lambda)K^p_g(\beta) - \beta(1+\lambda)\frac{\partial K^p_g}{\partial \beta}\right]$$

$$- \frac{\partial x^p_{ME}}{\partial \beta}\frac{\gamma}{1+\lambda}\left(\frac{1}{1-\beta\theta_M(1-\mu)}\right) + \left(\frac{\gamma(1-\mu)}{1-\beta\theta_M(1-\mu)}\right) K^p_g \geq 0.$$

We substitute for $\frac{\partial K^p_g}{\partial \beta} = \frac{\theta_M(1-\mu)}{1-\beta\theta_M(1-\mu)}K^p_g - \frac{\theta_M}{(1+\lambda)(1-\beta\theta_M(1-\mu))}\frac{\partial x^p_{ME}}{\partial \beta}$ and for $u'\left(x^p_{ME}\right)$ from Equation (29). With further reordering, we obtain

$$\frac{\partial x^p_{ME}}{\partial \beta}\underbrace{\left[\left(\frac{1+\beta\theta_M\mu}{1-\beta\theta_M(1-\mu)}\right)u'\left(x^p_{MN}\right) - \frac{1}{\theta_M}u'\left(x^p_{ME}\right)\right]}_{\leq 0}$$

$$+K^p_g \underbrace{\left[\frac{(1-\mu)}{1-\beta\theta_M(1-\mu)}\left[\frac{\gamma}{1+\lambda} - \beta\theta_M u'\left(x^p_{MN}\right)\right] - (1+\lambda)\right]}_{\geq 0} \geq 0. \tag{81}$$

Inequality (81) holds when the first bracket is negative and the second bracket is positive. The former requires $\frac{\theta_M(1+\beta\theta_M\mu)}{1-\beta\theta_M(1-\mu)} < \frac{u'\left(x^p_{ME}\right)}{u'\left(x^p_{MN}\right)}$. For small β values, this is always the case, since $\theta_M < 1$.

Similarly, for the second bracket to be positive, it is necessary for β to be small. At the limit, when β is very small, the term in the second bracket approaches $\frac{(1-\mu)\gamma}{(1+\lambda)} - (1+\lambda)$. And for $\frac{(1-\mu)\gamma}{(1+\lambda)} - (1+\lambda)$ to be positive, μ has to be $\mu \leq 1 - \frac{(1+\lambda)^2}{\gamma}$. Given our assumption that $\gamma \geq (1+\lambda)^2$, we ensure $0 \leq \mu < 1$.

□

Proof of Corollary 1

(i) To prove ($\Rightarrow$), we show that if $\mu \in \left[0, \frac{1}{2}\right)$, then β^* is not implementable.

We begin by rewriting the social welfare function in Equation (32) in terms of the sum of the Elites' and the Non-elites' interim expected utilities (as in Equations (33) and (34)).

$$\begin{aligned} W(\beta) &= \theta_M(1-\mu)u(x^p_{MN}(\beta)) + \theta_M\mu u(x^p_{ME}(\beta)) + \theta_m u(x^p_m(\beta)) + \gamma K^p_g(\beta) \\ &= \mu\left[\theta_M u\left(x^p_{ME}(\beta)\right) + \theta_m u\left(x^p_m\right) + \gamma K^p_g\left(\beta\right)\right] \\ &+ (1-\mu)\left[\theta_M u\left(x^p_{MN}(\beta)\right) + \theta_m u\left(x^p_m\right) + \gamma K^p_g\left(\beta\right)\right] \\ &= \mu\mathcal{U}_E\left(\beta\right) + (1-\mu)\mathcal{U}_{NE}\left(\beta\right). \end{aligned}$$

Taking the derivative with respect to β leads to

$$\frac{\partial W(\beta)}{\partial\beta} = \mu\frac{\partial\mathcal{U}_E}{\partial\beta} + (1-\mu)\frac{\partial\mathcal{U}_{NE}}{\partial\beta}. \tag{82}$$

Since β^* is an interior solution to the problem of maximizing $W(\beta)$, the left-hand side of Equation (82) at β^* is equal to zero. By Statement (i) in Proposition 5, $\frac{\partial\mathcal{U}_E}{\partial\beta}$ is strictly positive for all β values. Consequently, $\frac{\partial\mathcal{U}_{NE}}{\partial\beta}$ at β^* has to be strictly negative.

Given $\frac{\partial\mathcal{U}_{NE}(\beta^*)}{\partial\beta} < 0$, the Non-elites are not in favor of the contract. If the Elites do not have the majority and $\mu \in [0, \frac{1}{2})$, the contract β^* is not implementable. Equivalently, if the contract β^* is implementable, then $\mu \in [\frac{1}{2}, 1)$.

To prove ($\Leftarrow$), we recall from Proposition 5 that the Elites are better off with the contract for all $\beta \in [0, \bar{\beta}]$. We note that if $\mu \in [\frac{1}{2}, 1)$, the Elites have the majority in the society. Thus, the contract β^* has the support of the majority and it is implementable.

(ii) We first establish that any contract that makes the Non-elites interim better off is implementable.

From Statement (i) in Proposition 5 we know that the Elites are in favor of the contract for all $\beta \in [0, \bar{\beta}]$. Given Statement (ii) in Proposition 5, for $\mu \leq 1 - \frac{(1+\lambda)^3}{\gamma}$ and β small enough, the Non-elites are interim better off with the incentive contract. If the Non-elites are in favor of the contract, the contract has the support of everyone in the society and it is implementable.

We now establish that any contract that makes the Non-elites interim better off is welfare-improving. This is clear from Equation (82). On the right-hand side, the Elites' interim utility is strictly increasing for all $\beta \in [0, \bar{\beta}]$ and the Non-elites' interim utility is increasing in β for $\mu \leq 1 - \frac{(1+\lambda)^3}{\gamma}$ and β small enough. Thus, the left-hand side is strictly positive, $\frac{\partial W}{\partial \beta} > 0$.

□

Proof. We calculate the derivative of V with respect to β. This yields:

$$\begin{aligned}
\partial_\beta V(b, \beta) &= \theta_M \mu u'\Big((1-b)\omega_M + \beta \frac{\Omega b}{1+\mu\theta_M\beta}\Big) \frac{\Omega b}{\big(1+\mu\theta_M\beta\big)^2} - \frac{\gamma\mu\theta_M\Omega b}{\big(1+\mu\theta_M\beta\big)^2} \\
&= \frac{\mu\theta_M\Omega b}{\big(1+\mu\theta_M\beta\big)^2}\Big(u'\Big((1-b)\omega_M + \beta\frac{\Omega b}{1+\mu\theta_M\beta}\Big) - \frac{\gamma}{1+\lambda}\Big).
\end{aligned}$$

Clearly, the derivative is positive if and only if:

$$u'\Big((1-b)\omega_M + \beta\frac{\Omega b}{1+\mu\theta_M\beta}\Big) > \frac{\gamma}{1+\lambda},$$

and similarily it is negative if and only if the inequality with reversed order holds. Notice that if we let β tend to infinity, then:

$$\lim_{\beta\to+\infty} u'\Big((1-b)\omega_M + \beta\frac{\Omega b}{1+\mu\theta_M\beta}\Big) = u'\Big((1-b)\omega_M + \frac{\Omega b}{\mu\theta_M}\Big),$$

and by noticing that:

$$(1-b)\omega_M + \frac{\Omega b}{\mu\theta_M} \geq (1-b)\omega_M + \frac{\omega_M b}{\mu} > \omega_M,$$

we can use the strict monotone decreasing of u' to deduce:

$$u'\Big((1-b)\omega_M + \frac{\Omega b}{\mu\theta_M}\Big) \leq u'(\omega_M) < \frac{\gamma}{1+\lambda}.$$

This implies that the derivative of V will surely become negative for β large enough. Moreover, we have:

$$\frac{d}{d\beta}u'\Big((1-b)\omega_M+\beta\frac{\Omega b}{1+\mu\theta_M\beta}\Big) = -\frac{\Omega\mu\theta_M b}{\big(1+\mu\theta_M\beta\big)^2}u''\Big((1-b)\omega_M+\beta\frac{\Omega b}{1+\mu\theta_M\beta}\Big) < 0,$$

which means that the expression:

$$u'\Big((1-b)\omega_M + \beta\frac{\Omega b}{1+\mu\theta_M\beta}\Big) - \frac{\gamma}{1+\lambda},$$

is decreasing and therefore that V possesses a maximum as characterised in the Proposition. The second part of the statement follows due to the negativity of the partial derivative if it never vanishes. $\square$

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