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Abstract

We consider bilateral non–cooperative bargaining on the division of a surplus. Compared to the canonical bargaining game in the tradition of Rubinstein, we introduce additional sources of friction into the bargaining process: Implementation of an agreement and consumption of the surplus can only begin at discrete points in time, such as the first day of a month, quarter, or year. Bargaining rounds are of non–trivial length, so that counter–offers may be made without triggering costly delay. Communication between players is noisy: When players make offers, they are uncertain about the time it takes for the offer to arrive. We analyze delays and payoffs in the unique stationary equilibrium of the game. Frictions tend to make the bargaining process less efficient, but lead to a fairer surplus allocation. We establish conditions under which the equilibrium outcome converges to that in a canonical bargaining model as frictions become small.

JEL Codes: C72, C78

Keywords: Bargaining, Discount Factor, Timing, Subgame–Perfect Equilibrium, Equilibrium Delay

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1 Introduction

Many situations in economics, management, and political science can be thought of as surplus division problems: Two or more players own or are able to generate some surplus. However, they can only consume it once they have reached an agreement on its division. One classical economic example is related to labor disputes: When no labor agreement is reached, workers may go on strike, or the firm may lock them out, so that no production takes place. Workers' potential productivity can be thought of as a surplus that is available for consumption if and only if a firm and a union find agreement on its division.

Game theorists have often studied such surplus division problems using non–cooperative bargaining games in the tradition of Rubinstein (1982). This approach considers the bargaining process as a sequence of rounds. In each round, one particular player acts as the proposer and offers some division of the surplus. This offer is then accepted or rejected by the players. Acceptance of an offer ends the game. If an offer is rejected, the game moves to the next round, and the surplus shrinks due to discounting. Throughout the paper, we refer to this setup as the *canonical bargaining model.* One crucial feature of the canonical bargaining model is that offers are made, accepted, and implemented "instantaneously." These three steps are condensed into a single point in time. Thus, the canonical bargaining model assumes that it does not take the proposer any time to make and communicate an offer, nor does the responder need time to evaluate and accept the offer. A player does need time, however, in order to make a counter–offer. One concise way to put it is as follows: The canonical bargaining model assumes that time elapses only between (rather than within) rounds, and that any counter–offer ends the current round.

In the present paper, we take an alternative view. We propose a bilateral bargaining model in which negotiations are subject to frictions that lead to some delay between the points in time when an offer is made, accepted, and implemented. Moreover, each bargaining round is of exogenously fixed non–trivial length, so that a counter–offer need not end the current round.

More specifically, we make three main assumptions:

- Implementation of an agreement and consumption of the surplus can only begin at some exogenously fixed discrete points in time, such as the beginning of each new month, quarter, or year. Delay is only costly when one of these points in time goes by without agreement.

- The time between two potential dates of implementation is what we consider a "bargaining round." During any given bargaining round, one player has the right to make an initial offer at a time of his choosing. The other player may get a chance to make a counter–offer within the same bargaining round.

- When a player makes an offer (or a counter–offer), it is uncertain how long it will take for the opponent to receive it. Bargaining rounds may fail not only if an offer is rejected but also because communication is unsuccessful.

The first assumption drives a wedge between the time at which an offer is accepted and the time at which it can be implemented. Players may find agreement quickly, but have to wait for the next opportunity to implement it. The second assumption qualifies the proposer's privilege that is the driving force behind many results on the canonical bargaining model. Those findings are driven by the idea that each bargaining round is condensed into a single point in time. Under that modeling assumption, it is not meaningful to have a proposer choose the timing of his offer, or to allow a responder to make a counter–offer. In our setup, these considerations become important because each bargaining round is of non–trivial length. One question that we will address is in what sense the findings of the canonical bargaining model can be recovered in the limit as the length of each bargaining round becomes small. The third assumption introduces noise into players' communication, thus creating some friction between the time when an offer is made, and the time when it may be accepted.

In our bargaining model, a proposer faces the following trade–off: On the one hand, if the proposer makes an offer too late, he takes the risk that this offer cannot be implemented because the opponent does not receive it before the envisioned date of its implementation. On the other hand, if the proposer makes an offer way ahead of the date of implementation, the opponent may find it optimal to reject the offer and respond with a counter–offer.

One stylized example of such a bargaining process could be as follows: Suppose that a company and a prospective employee bargain under the institutional or legal constraint that working contracts can only start on the first day of a month. In order to make the hiring decision effective on February 1st, an agreement must be reached in January. Agreeing on January 20th rather than on January 10th has no immediate cost to either party. However, the closer the parties get to January 31st, the more likely it becomes that the negotiation fails due to delays in communication. If no agreement is reached by January 31st, players can continue bargaining in February. However, an agreement reached in February can only be implemented as of March 1st. This delay in implementation is costly.

Restrictions, conventions, or customs relative to the possible dates of implementation of an agreement are common: For instance, in the job market for school teachers or university lecturers, hires typically do not make sense unless they coincide with the start of a semester or academic year. Non–academic jobs typically start with the beginning of a month. Changes to the government's system of taxes or subsidies come into effect with a new fiscal year. More general legislative changes typically become effective with a new month, quarter, or year.

Our main results can be summarized as follows: In the limit as frictions between offer, acceptance, and implementation vanish, efficiency is maximized, and the ex ante expected division of the surplus corresponds to that familiar from the canonical bargaining model. If communication among players involves substantial friction, however, proposals remain lopsided even if offers are arbitrarily frequent. However, the relative payoffs of players need not depart from the canonical predictions. If potential dates of implementation are sufficiently far apart, equilibrium outcomes tend to be more fair but less efficient.

The present paper is related to two main strands of the non–cooperative bargaining literature. One strand looks at the canonical bargaining model with a variety of proposer selection protocols, and examines how exactly the protocol determines the equilibrium division of the surplus. This relationship between the distribution of proposal power and the equilibrium allocation has been explored in great detail by Hart and MasColell (1996), Laruelle and Valenciano (2008), Miyakawa (2008), Kultti and Vartiainen (2010), and Britz et al. (2010), among others.

Some related papers have studied non–cooperative bargaining in the presence of a deadline, and have suggested explanations for *deadline effects,* that is, agreements tend to be reached close to the deadline after some period of delay, see for instance Fershtman and Seidmann (1993) or Ponsati (1995). Ma and Manove (1993) consider a model in which two players negotiate in the presence of a deadline and communication is noisy.

The remainder of this paper is organized as follows: The formal model description is presented in Section 2. Then, Section 3 contains the analysis of SSPE of this model. Section 4 is devoted to some comparative statics analyses. Section 5 investigates the socially optimal length of bargaining rounds from an efficiency and fairness point of view. Section 6 concludes.

2 Bargaining game

Two players decide on the division of a surplus by non–cooperative bargaining. While bargaining takes place in continuous time $[0, \infty)$, an agreement can only be implemented at equidistant points in time $T,\ 2T,\ 3T, \ldots,$ where $T > 0$ is exogenously fixed. We refer to the time interval $[0, T]$ as the first *bargaining round,* and more generally to the time interval $[(k-1)T,\ kT]$ as the k^{th} bargaining round, for $k = 1, 2, \ldots.$

Players have a common rate of time preference $r > 0$. When it is convenient, we will work with the discount factor $\delta = e^{-rT}$ instead. As a normalization, we assume that the surplus is of size one. This implies that the discounted value of an agreement reached in the first bargaining round is δ.[1] More generally, the discounted value of an agreement in bargaining round k is δ^k.

In each bargaining round, one player is the *proposer* and the other player is the *responder.* Each bargaining round k proceeds as follows:

The proposer offers some split of the surplus, to become effective at time kT. He is free to choose at what time during bargaining round k he makes this offer. More formally, the proposer chooses a pair $(\theta, \tau) \in [0, 1] \times [0, T]$, where θ indicates the amount of surplus which the proposer offers to the responder, while $(k-1)T + \tau$ is the time at which he makes this offer. A noisy channel of communication is then used to transmit the offer to the responder. More specifically, we model communication as a Poisson process with arrival rate $\lambda > 0$. This implies that an offer made at time $(k-1)T + \tau$ reaches the responder before time kT with probability $1 - e^{-\lambda(T-\tau)}$.

If the offer fails to arrive until time kT, then we say that bargaining round k *fails,* and the game moves to round $k + 1$. Notice that this setup allows the proposer to effectively pass the opportunity to propose: He can do so by delaying his offer until time kT. We will see, however, that it is never optimal for the proposer to pass. If the responder receives the proposer's offer before time kT, the responder can either accept it or make a counter–offer. If the proposer's offer is accepted, the game ends and the proposer and responder receive utilities $\delta^k(1-\theta)$ and $\delta^k\ \theta$, respectively.[2] Now suppose that the responder does not

[1] It might seem more intuitive to assume that the physical surplus is of size $e^{rT} = 1/\delta$, so that its value is one if an agreement is implemented at the earliest possible moment. Later in the paper, however, we are going to vary T independently of r in a comparative statics analysis. Thus, we have to fix the size of the physical surplus to one.

[2] Notice that, if an offer is accepted, the proposer receives the complement of what he offered to the responder. This amounts to a tacit assumption that offers must be efficient. We could have allowed the proposer to make an offer which leaves some surplus unallocated. In equilibrium, such an offer would not

accept the offer but makes a counter–offer $\eta \in [0, 1]$. Again, an uncertain amount of time elapses until the responder's counter–offer reaches the proposer. As before, this delay in communication is modeled by a Poisson process with arrival rate λ. [3] If the counter–offer does not arrive until time kT, then round k ends in disagreement, and the game moves to round $k + 1$. If the responder's counter–offer does arrive before time kT, the proposer chooses to accept or reject it. If he rejects it, then round k ends in disagreement, and the game moves to round $k + 1$. If he accepts, then the game ends and the proposer and responder receive payoffs $\delta^k \eta$ and $\delta^k(1 - \eta)$, respectively. Whenever an agreement is not reached by time kT, we say that bargaining round k *fails.*

It remains to specify how the proposer in each round is chosen: Without loss of generality, we assume that Player 1 is the proposer in the first bargaining round. Moreover, for any $k \geq 2$, we assume that if Player $i = 1, 2$ is the proposer in bargaining round $k - 1$, then Player i is also the proposer in bargaining round k with probability m_i. With complementary probability $1 - m_i$, Player $j \neq i$ is the proposer in round k. Hence, the proposer selection follows a Markov chain with the transition matrix

$$M = \begin{pmatrix} m_1 & 1 - m_1 \\ 1 - m_2 & m_2 \end{pmatrix}.$$

We assume that $m_i < 1$ for each $i = 1, 2$, so that the Markov chain is irreducible. Its stationary distribution $\mu = (\mu_1, \mu_2)$ is given by $\mu M = \mu$, and can be written as

$$\mu_i = (1 - m_j)/(2 - m_i - m_j),$$

for each $i = 1, 2$ and $j \neq i$. One noteworthy special case is $m_1 = m_2 = 0$, which means that the role of proposer alternates from one round to the next. This is the proposer selection protocol in Rubinstein's original paper.

A *stationary strategy* for Player i consists of the following elements:

- A pair (θ_i, τ_i) such that if Player i is the proposer in round k, he proposes θ_i at time $kT + \tau_i$.

be made, so our results would not change.

[3] We assume there that if the responder makes a counter–offer she does so immediately after receiving the proposer's offer. We could have assumed instead that the responder also has the possibility to wait before making a counter–offer. This would not change the results, however: We will see that, in equilibrium, the responder has no incentive to wait.

- A map $\alpha_i : [0,1] \times [0,T] \to \{Accepts\} \cup [0,1]$ such that in any round k in which Player i is the responder, he accepts an offer θ which reaches him at time $kT + t$ if and only if $\alpha_i(\theta, t) = Accept$, and otherwise, he makes the counter–offer given by $\alpha_i(\theta, t)$.
- A map $\beta_i : [0,1] \times [0,T] \to \{Accept, Reject\}$ such that in any round k in which Player i is the proposer, he accepts a counter–offer η which reaches him at time $kT + t$ if and only if $\beta_i(\eta, t) = Accept$, and otherwise, he rejects.

A *stationary subgame–perfect equilibrium (SSPE)* is a profile of stationary strategies which is a subgame–perfect Nash equilibrium.

3 Analysis of stationary equilibrium

The purpose of this section is to derive expressions for the expected payoffs and expected delay in an SSPE. The starting point for this analysis is the stationarity property of the game just described: The subgame starting at time $(k-1)T$ for any $k \geq 2$ is equivalent to the entire game, up to the identity of the proposer. Thus, in any SSPE, there exists a quadruple (x_1, x_2, y_1, y_2) such that if Player $i = 1, 2$ is the proposer in round k, then the expected SSPE payoffs for Player i and Player $j \neq i$ in the subgame starting at time $(k-1)T$ are $\delta^k x_i$ and $\delta^k y_j$, respectively.

In what follows, we will consider a bargaining round k in which Player $i = 1, 2$ is the proposer and Player $j \neq i$ is the responder. To this end, it is useful to define the following auxiliary variables:

$$\begin{aligned}
\widetilde{x}_i &= m_i x_i + (1 - m_i) y_i, \\
\widetilde{y}_j &= m_i y_j + (1 - m_i) x_j.
\end{aligned}$$

Player i continues to act as proposer in round $k+1$ with probability m_i. Hence, $\delta \widetilde{x}_i$ and $\delta \widetilde{y}_j$ are reservation payoffs for Players i and j, respectively.

Recall that the rejection of a counter–offer by the proposer implies that bargaining can resume only in the next round. Hence, it is straightforward that in an SSPE, Player i accepts Player j's counter–offer η_j if and only if $\eta_j \geq \delta \widetilde{x}_i$. Notice that this is independent of the time at which Player i receives the counter–offer. Therefore, it would never be optimal for Player j to wait before making a counter–offer. This justifies our simplifying assumption that counter–offers are made without delay.

Now we proceed by backward induction to a history where Player j decides whether to accept Player i's offer θ_i, or to make a counter–offer. Intuitively, the later Player j receives an offer θ_i, the more risky it is for him to send a counter–offer. During each bargaining round, the responder's bargaining position gradually erodes overtime. As time approaches kT, Player j becomes more willing to make concessions to Player i. We will show that, for any given offer by Player i, there is some critical time from which onwards Player j accepts the offer. The proposition below claims that this critical point in time is given by

$$\widehat{t}_j(\theta_i) = \begin{cases} \max\left\{0,\ T - \left(\frac{1}{\lambda}\right)\ln\left(\frac{1-\delta\widetilde{x}_i-\delta\widetilde{y}_j}{1-\delta\widetilde{x}_i-\theta_i}\right)\right\} & \text{if } \theta_i < 1-\delta\widetilde{x}_i, \\ 0 & \text{if } \theta_i \geq 1-\delta\widetilde{x}_i. \end{cases}$$

The formal proof of the proposition is relegated to Appendix A.

Proposition 1. *Suppose that Player j receives Player i's offer θ_i at time $(k-1)T+t$. In an SSPE, Player j accepts if and only if $t \geq \widehat{t}_j(\theta_i)$.*

Observe that

$$\lim_{\theta_i \uparrow 1-\delta\widetilde{x}_i} T - \left(\frac{1}{\lambda}\right)\ln\left(\frac{1-\delta\widetilde{x}_i-\delta\widetilde{y}_j}{1-\delta\widetilde{x}_i-\theta_i}\right) = -\infty.$$

Hence, there is $\varepsilon > 0$ sufficiently small so that Player j accepts proposal $\theta_i = 1-\delta\widetilde{x}_i-\varepsilon$ at any time during round k. As a result, the proposer can always obtain an expected payoff strictly greater than his reservation level $\delta\widetilde{x}_i$. In particular, a proposer never finds it optimal to "pass" his opportunity to make an offer by waiting until kT. This is formally stated in the proposition below. The proof can be found in Appendix A.

Proposition 2. *In an SSPE, Player i makes an offer θ_i such that $\theta_i < 1-\delta\widetilde{x}_i$. Moreover, he makes the offer strictly earlier than at time kT.*

The next step is to show that it is optimal for Player i to make his offer exactly at the critical point in time when Player j is ready to accept it. The intuition is as follows: Player j is ready to accept a given offer from some critical point in time onwards. On the one hand, if Player i makes the offer before Player j is ready to accept it, there is a risk that the offer arrives so soon that Player j will prefer to make a counter–offer. On the other hand, if Player i makes the offer when Player j is already willing to accept it, then Player i could improve the probability of acceptance by making the same offer sightly earlier. One implication of the next proposition is that, on the path of play of an SSPE, no counter–offers will ever be made.

Proposition 3. *In an SSPE, Player i chooses the pair (θ_i, τ_i) in such a way that $\tau_i = \widehat{t}_j(\theta_i)$ and $\theta_i = (1 - e^{-\lambda(T-\tau_i)})(1 - \delta\widetilde{x}_i) + e^{-\lambda(T-\tau_i)}\delta\widetilde{y}_j$.*

The proof of Proposition 3 is relegated to Appendix A.

We have shown that no counter–offers are made on an equilibrium path of play. Hence, the only way how bargaining round k can fail is if Player i's offer does not arrive before time kT. For any $\tau_i \in [0, T]$, let us call $\pi_i(\tau_i) = e^{-\lambda(T-\tau_i)}$ the *failure probability* of an offer made at time $kT + \tau_i$. Throughout the paper, we will omit the argument τ_i if no confusion arises. One implication of Proposition 3 is that Player i's optimal choice of a pair (θ_i, τ_i) can be reduced to an optimal choice of the failure probability π_i. In case of failure, Player i expects to get $\delta\widetilde{x}_i$ in the ensuing continuation game. If bargaining round k does not fail, then Player i receives $1 - \theta_i > \delta\widetilde{x}_i$, where θ_i is implicitly determined by the choice of π_i in the way specified by Proposition 3. Therefore, Player i's expected payoff in round k equals $\delta^k\xi_i(\pi_i)$, where $\xi_i(\pi_i)$ is given by

$$\xi_i(\pi_i) = \pi_i\delta\widetilde{x}_i + (1 - \pi_i)(1 - (1 - \pi_i)(1 - \delta\widetilde{x}_i) - \pi_i\delta\widetilde{y}_j).$$

After some simplification, we obtain

$$\xi_i(\pi_i) = \delta\widetilde{x}_i + (1 - \delta\widetilde{x}_i - \delta\widetilde{y}_j)(\pi_i - \pi_i^2). \tag{1}$$

This expression for Player i's expected payoff has a straightforward interpretation: The first summand $\delta\widetilde{x}_i$ is the (discounted) continuation payoff for Player i in the next bargaining round. The expression $1 - \delta\widetilde{x}_i - \delta\widetilde{y}_j$ represents the share of the surplus which the players forgo if the current round fails. Put another way, we can interpret it as the *gain from immediate agreement*. This gain realizes with probability $1 - \pi_i$, and if it does, then our analysis so far reveals that Player i gets a share π_i of it. On the one hand, Player i can trivially ensure that his offer fails with probability one by making it only at the deadline kT. On the other hand, by making the offer immediately at time $(k-1)T$, he can reduce the failure probability in round k to $e^{-\lambda T}$. More formally, Player i's optimization problem can be written as

$$\max_{\pi_i} \ \xi_i(\pi_i) \text{ subject to } \pi_i \in [e^{-\lambda T}, 1]. \tag{2}$$

Consider the derivative

$$\partial\xi_i/\partial\pi_i = (1 - \delta\widetilde{x}_i - \delta\widetilde{y}_j)(1 - 2\pi_i).$$

We observe that $\widetilde{x}_i + \widetilde{y}_j \leq 1$ and so $1 - \delta\widetilde{x}_i - \delta\widetilde{y}_j \geq 1 - \delta > 0$. There are two cases to distinguish: Suppose first that $e^{-\lambda T} \leq 1/2$. In this case, the first–order condition

$$\partial\xi_i/\partial\pi_i = 0$$

yields $\pi_i = 1/2$. Now suppose that $e^{-\lambda T} > 1/2$. In that case, $\partial\xi_i/\partial\pi_i < 0$ for any $\pi_i \in [e^{-\lambda T}, 1]$, and hence Player i finds it optimal to choose $\pi_i = e^{-\lambda T}$. We note that the exact values of $\widetilde{x}_i$ and $\widetilde{y}_j$, as well as the identities of Players i and j, do not affect the solution to this optimization problem. Hence, we have the following proposition:

Proposition 4. *In an SSPE, τ_1 and τ_2 are chosen such that $\pi_1 = \pi_2 = \max\left\{\frac{1}{2}, e^{-\lambda T}\right\}$.*

It is noteworthy that this result is very general: It does not depend on the continuation utilities that players expect from the next round. In particular, it is independent of players' time preferences. This is somewhat surprising: One might expect that players are more willing to risk bargaining failure, and hence a delay, when they are more patient. It turns out that this is not true.

Moreover, if T is sufficiently large, the gain from immediate agreement is split fairly between the two players. There is no proposer premium. This is a consequence of our assumption that players can respond to offers by counter–offers without triggering a costly delay. Nevertheless, the complete absence of a proposer premium is not trivial: When a counter–offer is made, the potential date of implementation is closer, and thus the risk of costly delay greater than when an initial offer is made. One may therefore have intuitively expected that, even in our model, the bargaining position of the proposer is always stronger than that of the responder.

Proposition 4 has a number of additional implications that will be important in the remainder of this paper. One of them is the following corollary:

Corollary 1. *The proposer's equilibrium offer is given by*

$$\theta_i = \begin{cases} \frac{1}{2}\left(1 - \delta\widetilde{x}_i\right) + \frac{1}{2}\delta\widetilde{y}_j & \textit{if } e^{-\lambda T} \leq \frac{1}{2}, \\ (1 - e^{-\lambda T})(1 - \delta\widetilde{x}_i) + e^{-\lambda T}\delta\widetilde{y}_j & \textit{if } e^{-\lambda T} \geq \frac{1}{2}. \end{cases} \tag{3}$$

Proposition 4 says that, in an SSPE, every bargaining round which is reached fails with equal probability $\pi = \max\left\{\frac{1}{2}, e^{-\lambda T}\right\}$. This allows us to write the expected size of the surplus at the time of an agreement, discounted back to the beginning of the game, as

$$\delta v(\pi) = \delta(1 - \pi)\sum_{k=0}^{\infty}(\delta\pi)^k = \delta\left(\frac{1 - \pi}{1 - \delta\pi}\right).$$

Notice that the amount $\delta v(\pi)$ is independent of the initial proposer's identity. Therefore,

$$x_1 + y_2 = x_2 + y_1 = v(\pi). \tag{4}$$

In what follows, we will refer to the quantity $\delta v(\pi)$ as the *expected value of agreement.* In a similar manner, we can also compute the expected time at which an agreement is implemented as

$$\begin{aligned} \omega(\pi, T) = &\; T(1-\pi) + 2T\pi(1-\pi) + 3T\pi^2(1-\pi) + \dots \\ = &\; T(1-\pi)\textstyle\sum_{k=0}^{\infty}(k+1)\pi^k \\ = &\; T/(1-\pi). \end{aligned}$$

From now on, we will refer to this quantity as the *expected implementation time.*

In equilibrium, the failure probability π is chosen optimally, so that the expected value of agreement and the expected waiting time can also be thought of as the following functions of the model parameters r, λ, and T:

$$\omega(\lambda, T) = \begin{cases} 2T & \text{if } T \geq \ln(2)/\lambda, \\ \frac{T}{1-e^{-\lambda T}} & \text{if } T \leq \ln(2)/\lambda. \end{cases}$$

$$v(r, \lambda, T) = \begin{cases} \frac{1}{2e^{rT}-1} & \text{if } T \geq \ln(2)/\lambda, \\ \frac{e^{\lambda T}-1}{e^{(\lambda+r)T}-1} & \text{if } T \leq \ln(2)\lambda. \end{cases}$$

We have now introduced all preliminaries that we need to characterize SSPE payoffs and delay.

Our starting point was the existence of x_i, y_j such that if Player i is the proposer in round k, the SSPE utilities in the subgame starting in that round are given by $\delta^k x_i$ and $\delta^k y_j$. By definition of ξ_i and by Proposition 4, it follows that

$$\xi_i\left(\max\left\{\frac{1}{2}, e^{-\lambda T}\right\}\right) = x_i.$$

We can now conclude that, for any given δ, SSPE utilities x_1, x_2, y_1, and y_2 and associated continuation utilities $\widetilde{x}_1, \widetilde{x}_2, \widetilde{y}_1$, and $\widetilde{y}_2$ solve the following system of equations:

$$
\begin{aligned}
x_1 &= \delta\widetilde{x}_1 + (1-\delta v)(\pi - \pi^2), && (5)\\
x_2 &= \delta\widetilde{x}_2 + (1-\delta v)(\pi - \pi^2), && (6)\\
y_1 &= v - x_2, && (7)\\
y_2 &= v - x_1, && (8)\\
\widetilde{x}_1 &= m_1 x_1 + (1-m_1) y_1, && (9)\\
\widetilde{x}_2 &= m_2 x_2 + (1-m_2) y_2, && (10)\\
\widetilde{y}_1 &= m_2 y_1 + (1-m_2) x_1, && (11)\\
\widetilde{y}_2 &= m_1 y_2 + (1-m_1) x_2, && (12)
\end{aligned}
$$

where

$$v = (1-\pi)/(1-\delta\pi).$$

Eqns. (5)–(12) are a system of eight linearly independent equations in eight unknowns: A solution exists, and it is unique. For the remainder of our discussion, the solutions for the variables x_1 and y_2 are particularly important.

We have assumed that Player 1 is the initial proposer, so the SSPE payoffs in the entire game are δx_1 and δy_2, where the quantities x_1 and y_2 are given by

$$
\begin{aligned}
x_1 &= \left(\frac{1-\pi}{1-\delta\pi}\right)\left(\frac{\delta + (1-\delta)\pi - \delta m_1 + \delta\pi(m_1 - m_2)}{1-\delta(m_1+m_2-1)}\right), && (13)\\
y_2 &= \left(\frac{1-\pi}{1-\delta\pi}\right)\left(\frac{1 - (1-\delta)\pi - \delta m_2 - \delta\pi(m_1 - m_2)}{1-\delta(m_1+m_2-1)}\right). && (14)
\end{aligned}
$$

In Appendix B, we provide Mathematica code that can be used to verify our computation of the solution to this system of equations.
Let the tuple

$$(x_1^*, x_2^*, y_1^*, y_2^*, \widetilde{x}_1^*, \widetilde{x}_2^*, \widetilde{y}_1^*, \widetilde{y}_2^*)$$

solve the system of Eqns. (5)–(12). Define a profile of stationary strategies

$$\sigma^* = (\theta_1^*, \theta_2^*, \tau_1^*, \tau_2^*, \alpha_1^*, \alpha_2^*, \beta_1^*, \beta_2^*)$$

as follows:

- For each $i = 1, 2$, let $\theta_i^* = (1-\pi)(1-\delta\widetilde{x}_i^*) + \pi\delta\widetilde{y}_j^*$, where $\pi = \max\{\frac{1}{2}, e^{-\lambda T}\}$.
- For each $i = 1, 2$, let $\tau_i^* = \widehat{t}_j(\theta_i^*)$.

- For each $j = 1, 2$, let $\alpha_j^*(\theta, t) = \textit{Accept}$ if and only if
  $\theta \geq (1 - e^{-\lambda(T-t)})(1 - \delta\widetilde{x}_i^*) + e^{-\lambda(T-t)}\delta\widetilde{y}_j^*$.
  Otherwise, $\alpha_j^*(\theta, t) = \delta\widetilde{x}_i^*$.

- Let $\beta_i^*(\eta, t) = \textit{Accept}$ if and only if $\eta \geq \delta\widetilde{x}_i^*$, and let $\beta_i^*(\eta, t) = \textit{Reject}$ otherwise.

Theorem 1. *The profile of stationary strategies σ^* is the unique SSPE.*

Proof. Uniqueness of SSPE follows from the fact that, in an SSPE, the utilities and associated continuation utilities must satisfy Eqns. (5)–(12), and that system of equations has a unique solution. In order to show that the strategy profile σ^* is indeed an SSPE, it is straightforward that the accept/reject behavior prescribed by α_i^* and β_i^* is optimal. Moreover, the optimality of (θ_i^*, τ_i^*) follows by applying the same logic as in the proof of Proposition 3, and then observing that the solution to the optimization problem (2) is unique. □

We refer to the ratio $\rho_1 = x_1/y_2$ as Player 1's *relative payoff;* it is given by

$$\rho_1 = \frac{\delta + (1-\delta)\pi - \delta m_1 + \delta\pi(m_1 - m_2)}{1 - (1-\delta)\pi - \delta m_2 - \delta\pi(m_1 - m_2)}. \tag{15}$$

In what follows, we explore how equilibrium variables depend on the underlying model parameters. Some relevant observations can be made immediately from the above expressions:

Consider the case where $T > \ln(2)/\lambda$ and therefore the equilibrium choice of the failure probability is $\pi = 1/2$. In that case, we observe that

$$x_1 = y_2 = \left(\frac{1}{2-\delta}\right)\left(\frac{\frac{1}{2} + \frac{\delta}{2}(m_1 + m_2 - 1)}{1 - \delta(m_1 + m_2 - 1)}\right) = \left(\frac{1}{2-\delta}\right)\left(\frac{1}{2}\right).$$

Thus, the relative payoff ρ_1 equals one regardless of the value of the discount factor and the transition probabilities: For any δ and any m_1 and m_2, Players 1 and 2 receive the same expected SSPE payoff. In the canonical Rubinstein bargaining game, the discount factor and transition probabilities are the crucial determinants of bargaining power. This is not true in our setting when $T \geq \ln(2)/\lambda$: In that case, there is no advantage from being the proposer and consequently, the equilibrium surplus allocation does not depend on the proposer selection protocol. Along any equilibrium path, the proposer always offers the fair split, and the responder accepts. However, there is some degree of inefficiency which comes from the fact that each bargaining round which is reached on the equilibrium path fails with probability 1/2. Unlike in the canonical bargaining model, the expected value of agreement does not equal one but rather

$$\delta v = \delta/(2-\delta),$$

and so the expected payoff to each player is $\delta/(4-2\delta)$. This converges to 1/2 in the limit as $\delta \to 1$. In our model, when the bargaining rounds are of sufficient length, an increase in the discount factor boosts efficiency but does not change the surplus distribution. This is exactly the reverse as in the canonical bargaining model, where an increase in the discount factor makes the payoff distribution more fair, while the bargaining outcome is efficient regardless of the discount factor.

Now suppose that bargaining rounds are sufficiently short so that $e^{-\lambda T} = \pi > 1/2$. In that case, there is a proposer advantage like in the canonical bargaining model. The proposer's share does depend on the discount factor. Here, a "proposer advantage" means that the proposer gets more than half of the gain from immediate agreement in each round. In the canonical bargaining model, he gets the whole gain from immediate agreement in each round. Like in the canonical bargaining model, relative payoffs depend on the proposer protocol. In particular, we find

$$\lim_{\delta\to 1} \rho_1 = \left(\frac{1-m_1(1-\pi)-\pi m_2}{1-m_2(1-\pi)-\pi m_1}\right) = \left(\frac{1-m_1+\pi(m_1-m_2)}{1-m_2-\pi(m_1-m_2)}\right).$$

The impact of proposal power on the equilibrium allocation is weaker than in the canonical bargaining model. This is because the non–trivial length of bargaining rounds gives the responder a chance to make a counter–offer. This attenuates the extent of the proposer's strategic advantage over the responder.

Example 1. *Let us consider the numerical example with* $\lambda = 1$ *and* $T = 0.5$. *Since* $T = 0.5 < \ln(2) \approx 0.69$, *we have that* $\pi = e^{-0.5} \approx 0.607$. *Moreover, let us suppose that* $m_1 = 0.6$ *and* $m_2 = 0.4$. *This implies that the probabilities in the stationary distribution of the Markov chain are also* $\mu_1 = 0.6$ *and* $\mu_2 = 0.4$. *The limit of relative payoffs is*

$$\lim_{\delta\to 1} \rho_1 = \frac{0.4+0.2e^{-0.5}}{0.6-0.2e^{-0.5}} \approx 1.089.$$

In a canonical bargaining game in which proposer selection follows the same transition dynamics, one would expect the limit of relative payoff of Player 1 to be $\mu_1/\mu_2 = 0.6/0.4 = 1.5$.

4 The degree of noise

We are now going to examine how SSPE payoffs and delays change as we vary the arrival rate λ. First suppose that λ is arbitrarily large. In that case, Proposition 4 tells us that the proposer in bargaining round k waits almost until the deadline kT, thus ensuring that the proposal fails to arrive before kT with probability one half. The expected time for implementation of the agreement and the expected equilibrium payoffs remain constant once λ has grown beyond the point where $e^{-\lambda T} \geq 1/2$.

Proposition 5. *For any given r and T, in the limit as λ is sufficiently large, every proposer along an SSPE path of play offers the fair split, every bargaining round fails with probability $1/2$, and expected implementation time is $2T$.*

As $\lambda \to \infty$, the friction arising from noisy communication vanishes. This does not make the bargaining process more efficient, however: The proposer adjusts the timing of his offer in a way that holds the failure probability constant at one half. No matter how close to instantaneous the communication between the players is, there remains a substantial expected delay in equilibrium.

Now we turn to the case where λ is small. As a benchmark, let us briefly consider a canonical bargaining model with two players and linear utilities in which proposer selection follows a Markov chain.[4] Let $\widehat{x}_i$ and $\widehat{y}_j$ be the SSPE utilities of a proposer and a responder in any subgame of that canonical bargaining model. It is well–known that these utilities are given by the solution to the following system of equations:

$$\begin{aligned}\widehat{x}_i &= 1 - \widehat{y}_j, \\ \widehat{y}_j &= \delta m_i \widehat{y}_j + \delta(1 - m_i)\widehat{x}_j,\end{aligned}$$

for $i = 1, 2$ and $j \neq i$. The former equation follows from the fact that an SSPE of the canonical bargaining model is always efficient. In particular, agreement is reached immediately in every subgame. The latter equation captures the standard result that in an SSPE, the responder is indifferent between acceptance and rejection of the proposal. Solving this

[4]The model and resutls briefly sketched here as a benchmark are a simple special case of Britz et al. (2010): That paper studies a canonical bargaining model where proposer selection follows a Markov chain. The model in that paper is much more general, however, in the sense that it allows for an arbitrary finite number of players and a general convex set of feasible utilities.

system yields

$$\begin{aligned}\widehat{x}_i &= \frac{1-\delta m_j}{1-\delta(m_i+m_j-1)},\\ \widehat{y}_j &= \frac{\delta-\delta m_i}{1-\delta(m_i+m_j-1)},\end{aligned}$$

for $i=1,2$ and $j \neq i$. The *relative payoffs* in an SSPE of the canonical bargaining model are given by the ratio

$$\widehat{\rho}_i = \frac{1-\delta m_j}{\delta-\delta m_i}.$$

We will now use the canonical bargaining model as a benchmark to which we compare the SSPE of our bargaining model in the limit as λ goes to zero while r and T remain fixed. In that case, it is straightforward that the expected implementation time grows without bound, and so the expected value of agreement converges to zero, indeed:

$$\lim_{\lambda\to 0}\ \delta\left(\frac{1-e^{-\lambda T}}{1-\delta e^{-\lambda T}}\right)=0.$$

When λ is small enough, Player i's offer θ_i is approximately equal to Player j's continuation utility $\delta\widetilde{y}_j$. This continuation utility in turn is bounded above by the expected value of agreement δv, which again is monotone decreasing in λ and converges to zero in the limit as λ goes to zero. From inspection of Eqn. (15), we find the limit of relative payoffs

$$\lim_{\lambda\to 0}\ \rho_1 = \frac{1-\delta m_2}{\delta-\delta m_1},$$

and so we have the following proposition:

Proposition 6. *For any δ, in the limit as $\lambda \to 0$, the expected value of agreement converges to zero. Relative expected payoffs converge to the same limit as those in the canonical bargaining model. Along an equilibrium path of play, however, proposers offer almost nothing to responders.*

Intuitively, if communication is very noisy, this weakens the responder's bargaining position: Once an offer has been received, making a counter–offer would likely lead to a substantial and costly delay. Thus, a proposer only has to offer very little surplus to a responder. In that sense, when communication is noisy enough, bargaining is in a standoff: Each player finds it optimal to insist on almost the entire surplus. As a result, an equilibrium path of play looks as follows: Players keep making very lopsided offers to each other. Each of these offers is unlikely to arrive in time, so that bargaining likely keeps failing for many rounds until eventually some offer arrives in time to be accepted. A player's relative bargaining power depends on the probability with which one of his offers is eventually successful. This probability corresponds to the share of time for which this player expects to be the proposer in the long–run. This makes it intuitively clear why, with small λ, relative payoffs are driven by the distribution of proposal power in the same way as in the canonical bargaining model – even though the realized outcome is much more lopsided, and absolute expected payoffs are small.

5 Optimal length of bargaining rounds

One key assumption of the present paper is that there are equidistant points in time T, $2T$, $\ldots$ at which an agreement can be implemented, and consumption of the surplus can begin. Players make offers on how to split the surplus at the next available date. One important question is how our equilibrium predictions vary with the choice of the institutional parameter T. In particular, one might wonder which value of T is preferred by Player 1 or Player 2, and which one is preferable from an efficiency or fairness point of view. Observe that changing T has two effects: First, the length of the bargaining rounds influences the failure probability. If rounds are short enough, the proposer cannot choose his optimal failure probability of one half, but has to settle for a higher failure probability. Shorter bargaining rounds are more likely to fail. Second, when bargaining rounds are shorter, their failure becomes less costly: The next opportunity to implement agreement is less far away. So far, we have done some comparative statics analysis on the parameters r and λ, allowing us to vary the discount factor δ and the failure probability π independently of each other. As we consider changes in T, however, we are simultaneously varying both the discount factor $\delta = e^{-rT}$ and the failure probability $\pi = \max\{\frac{1}{2}, e^{-\lambda T}\}$. Both of them are non–decreasing in T. They both converge to one in the limit as $T \to 0$.

5.1 The limit case

For the purpose of this section, let us consider the expected value of agreement and the expected implementation time as a function of T, while keeping r and λ fixed, thus:

$$v(T) = \begin{cases} \frac{e^{\lambda T}-1}{e^{(\lambda+r)T}-1} & \text{if } T \leq \ln(2)/\lambda, \\ \frac{1}{2e^{rT}-1} & \text{if } T > \ln(2)/\lambda. \end{cases}$$

$$\omega(T) = \begin{cases} \left(\frac{e^{\lambda T}}{e^{\lambda T}-1}\right) T & \text{if } T \leq \ln(2)/\lambda, \\ 2T & \text{if } T > \ln(2)/\lambda. \end{cases}$$

Notice that both the expected value of agreement and the expected implementation time are continuous functions of T. In particular, they are continuous at the point $T = \ln(2)/\lambda$. It is readily apparent from the above expressions that:

Proposition 7. *The expected value of agreement in an SSPE is strictly monotonically decreasing in the length of bargaining rounds.*

The intuition is as follows: The bargaining process can be thought of as a sequence of three consecutive stages: First, bargaining may be suspended for some time because the proposer waits for the optimal moment to make an offer. Second, an offer has been made by the proposer but not yet received by the responder. Third, an agreement has been reached but the time for its implementation has not come yet. If bargaining rounds are sufficiently short, a proposer never waits before making a counter–offer which eliminates the first stage. Moreover, as bargaining rounds become shorter, an agreement is reached closer to the end of a round, and hence closer to a possible date of implementation. Thus, the third stage also vanishes. If T is small enough, a bargaining round tends to consist entirely of the time that it takes for the offer to be communicated from the proposer to the responder. Hence, it is clear that for small T, the expected implementation time must converge to the expected arrival time of the underlying Poisson process. Indeed, by applying L'Hôpital's rule, we can easily verify that

$$\lim_{T\to 0} \omega(T) = \lim_{T\to 0} \left(\frac{e^{\lambda T}}{e^{\lambda T}-1}\right) T = 1/\lambda.$$

Since the expected value of agreement is monotone decreasing in T, it is bounded above by its limit as $T \to 0$. Again applying L'Hôpital's rule, this limit can be computed as

$$\lim_{T\to 0} \delta v(T) = \lim_{T\to 0} \left(\frac{e^{\lambda T}-1}{e^{(\lambda+r)T}-1}\right) = \frac{\lambda}{\lambda + r}.$$

While delays within a round are costless if the length of the rounds is exogenously fixed, they do become an important consideration once we study the optimal length of bargaining rounds.

Finally, we consider relative payoffs of the players in the limit as $T \to 0$. From Eqn. (15) we find that

$$\begin{aligned}\lim_{T\to 0} \rho_1 &= \lim_{T\to 0}\left(\frac{e^{-rT} + (1-e^{-rT})e^{-\lambda T} - e^{-rT}m_1 + e^{-(r+\lambda)T}(m_1 - m_2)}{1 - (1-e^{-rT})e^{-\lambda T} - e^{-rT}m_2 - e^{-(r+\lambda)T}(m_1 - m_2)}\right)\\ &= \frac{1-m_2}{1-m_1}\\ &= \mu_1/\mu_2.\end{aligned}$$

Now we have established the following theorem.

Theorem 2. *In the limit as $T \to 0$, the expected value of agreement in an SSPE converges to $\lambda/(\lambda + r)$, while the ratio of Player 1's and Player 2's SSPE utilities converges to μ_1/μ_2. The SSPE proposal of Player $i = 1, 2$ converges to $\theta_i = \mu_j\left(\frac{\lambda}{\lambda+r}\right)$. The expected implementation time in an SSPE converges to $1/\lambda$.*

The literature has established some findings on the SSPE of the canonical bargaining model that hold for a wide class of proposer selection protocols: In particular, agreement is reached immediately in every subgame, and in the limit as $\delta \to 1$, the SSPE proposals of all players converge to a common limit.[5]

In our model, there is a strictly positive expected delay on the equilibrium path. Hence, it is possible for the initial proposer's advantage to vanish although the proposals themselves do not converge to the fair split. This gap between our findings and the canonical bargaining model narrows if also the friction in communication is reduced, that is, if λ is large.

Corollary 2. *Suppose λ is sufficiently large. In the limit as $T \to 0$, the expected value of agreement in an SSPE is close to one, while the ratio of Player 1's and Player 2's SSPE utilities is close to μ_1/μ_2. The SSPE proposal of Player $i = 1, 2$ to Player $j \neq i$ is close to μ_j. The expected implementation time is close to zero.*

The corollary seems intuitive: As T becomes sufficiently small, and λ sufficiently large, the frictions which distinguish our bargaining game from the canonical bargaining model become negligible, and so our results collapse into the ones familiar from the canonical bargaining model.

This intuition, however, only holds true when considering the double limit "$\lim_{\lambda\to\infty} \lim_{T\to 0}$" of equilibrium variables. By contrast, Proposition 5 has shown that the surplus is split

[5] Different versions of these results appear, among others, in Banks and Duggan (2000), Kultti and Vartiainen (2010), Laruelle and Valenciano (2008), and Britz et al. (2010).

fairly with an expected implementation time of $2T$ in the limit as $\lambda \to \infty$ for any given T. Hence, at the double limit "$\lim_{T\to 0} \lim_{\lambda\to\infty}$" equilibrium is nearly efficient but also perfectly fair.

The interpretation is as follows: Consider the case where communication is noisy to a substantial degree, while implementation of agreements is possible almost at any time. Then, the equilibrium surplus allocation is determined by the distribution of proposal power in the same way as in the canonical bargaining model. Contrary to that model, however, there is a substantial expected delay of $1/\lambda$ on the equilibrium path of play, and the equilibrium proposals of the two players do not converge to a common limit.

Now consider the case where agreements can only be implemented at few and distant points in time, while noise in the communication is negligible. In that case, the surplus is split fairly, regardless of the distribution of proposal power. The expected equilibrium delay is $2T$.

5.2 Players' preferences over the length of bargaining rounds

The main trade–off can be summarized as follows: For $T > \ln(2)/\lambda$, every proposer offers the fair split, but the bargaining outcome is inefficient. A slight decrease in T improves the payoffs of both players. From the point $T = \ln(2)/\lambda$ onwards, any further gain in equilibrium efficiency will come at a cost in terms of fairness. Recall that for any $T \geq \ln(2)/\lambda$, the expected value of agreement is given by $\delta/(2-\delta)$, or equivalently, by $(2e^{rT} - 1)^{-1}$. At the point $T = \ln(2)/\lambda$, the expected value of agreement is $(2^b - 1)^{-1}$, where we denote $b = 1 + r/\lambda$. The quantity $(2^b - 1)^{-1}$ is the greatest expected value of agreement that can be reconciled with a perfectly fair split.

In the limit as $T \to 0$, the surplus is split according to the distribution of proposal power. However, the surplus is maximized in the limit as $T \to 0$, so fairness has a price in terms of efficiency.

Suppose that the proposal power of Player 1 is greater than that of Player 2, that is, $\mu_1 > 1/2$. As T decreases from $\ln(2)/\lambda$ towards zero, we should expect to see two effects: First, the surplus increases. Second, its distribution tilts more and more in favor of Player 1. Both effects are good for Player 1, so that Player 1 prefers T to be as small as possible. For Player 2, however, the two effects work in opposite directions: Decreasing T hurts Player 2 because his share of the surplus becomes smaller, but on the other hand, Player 2 benefits because the surplus becomes bigger. Example 2 below illustrates these effects.

Example 2. *Suppose that* $r = \lambda = 1$ *so that* $e^{-rT} = e^{-\lambda T} = e^{-T}$. *Choosing* $T \in (0, \ln(2)]$ *amounts to choosing some* $e^{-T} \in [\frac{1}{2}, 1)$. *Whatever the proposer selection protocol may be, the expected value of agreement is*

$$e^{-T}v = e^{-T}/(1 + e^{-T}).$$

For the sake of this example, let us assume that $m_1 = 0.7$ *and* $m_2 = 0.3$. *By substitution into Eqn. (13), we find that Player 1's SSPE payoff is*

$$e^{-T}x_1 = \left(\frac{e^{-T}}{1 + e^{-T}}\right)\left(1.3e^{-T} - 0.6e^{-2T}\right).$$

This payoff is monotone increasing on the entire interval $e^{-T} \in [\frac{1}{2}, 1)$. *Indeed, Player 1 becomes better off the smaller* T *is. In an analogous way, by substitution into Eqn. (14), we find that Player 2's SSPE payoff is*

$$e^{-T}y_2 = \left(\frac{e^{-T}}{1 + e^{-T}}\right)\left(1 - 1.3e^{-T} + 0.6e^{-2T}\right).$$

This payoff is not monotonic in e^{-T} *on the interval* $[\frac{1}{2}, T)$. *it attains a local minimum at* $e^{-T} \approx 0.944$, *where it evaluates to* $e^{-T}y_2 \approx 0.149$. *On the interval* $[\frac{1}{2}, T)$, *the optimal choice of* e^{-T} *for Player 2 is* $e^{-T} = 1/2$, *at which point Player 2's payoff evaluates to* $e^{-T}y_2 = 1/6$.

The relation between the choice of T and the SSPE payoffs is more complex than the two effects discussed in the context of Example 2, however. In addition, we also need to take into account that the identity of the initial proposer has a non–monotonic effect on SSPE payoffs. The reason is as follows: We have assumed that Player 1 is the initial proposer in the first bargaining round. If $e^{-T} = 1/2$, all proposals are fair, so the gain from immediate agreement is split equally in each round. Hence, the identity of the initial proposer is irrelevant for the SSPE payoffs. As e^{-T} grows, however, there is a proposer advantage: The proposer in each round receives more of the gain from immediate agreement than the responder does. Due to time discounting, this effect has more bearing on the SSPE payoffs in earlier rounds than in later ones. Thus, there is an advantage to being the initial proposer. However, in the limit as e^{-T}, discounting becomes negligible, and so the premium for the initial proposer vanishes again. This is illustrated by Example 3 below.

Example 3. *Assume that* $r = \lambda = 1$ *and, moreover,* $m_1 = m_2 = 0.9$. *Also in this example, it is true that* $e^{-rT} = e^{-\lambda T} = e^{-T}$, *and that the expected value of agreement is* $e^{-T}v = e^{-T}/(1 + e^{-T})$. *By substitution into Eqns. (13) and (14), we find that Player 1's SSPE payoff is*

$$e^{-T}x_1 = \left(\frac{e^{-T}}{1 + e^{-T}}\right)\left(\frac{1.1e^{-T} - e^{-2T}}{1 - 0.8e^{-T}}\right),$$

and Player 2's SSPE payoff is

$$e^{-T}y_2 = \left(\frac{e^{-T}}{1+e^{-T}}\right)\left(\frac{1-1.9e^{-T}+e^{-2T}}{1-0.8e^{-T}}\right).$$

Player 1's SSPE payoff has a local maximum at $e^{-T} \approx 0.873$, *where it evaluates to* $e^{-T}x_1 \approx 0.306$. *Player 2's SSPE payoff has a local minimum at* $e^{-T} \approx 0.766$, *where it evaluates to* $e^{-T}y_2 \approx 0.147$. *Player 2 is best off in the limit as* $e^{-T} \to 1$, *where he receives a payoff of* $1/4$. *At the point where* $e^{-T} = 1/2$, *Player 2 would only receive* $1/6$.

In the canonical bargaining model, equilibrium offers become more fair when they can be made more frequently. In our model, the opposite may be true for some range of T: The distribution of proposal power has more and more bearing on the equilibrium allocation as $T \leq \ln(2)/\lambda$ becomes smaller.

In Example 3, the effect of the initial proposer's identity on SSPE payoffs is quite pronounced because the right to propose transitions from one player to the other only with a low probability of 0.1. Differently put, the initial proposer is likely to remain in that role for several rounds.

On the other extreme, it is also possible that the relation between T and the SSPE payoffs is determined only by the fact that smaller T implies more efficiency because other effects are quantitatively to weak to weigh in. This is illustrated by Example 4 below.

Example 4. *Assume that* $r = \lambda = 1$ *and, moreover,* $m_1 = m_2 = 0$. *Also in this example, it is true that* $e^{-rT} = e^{-\lambda T} = e^{-T}$, *and that the expected value of agreement is* $e^{-T}v = e^{-T}/(1+e^{-T})$. *By substitution into Eqns. (13) and (14), we find that Player 1's SSPE payoff is*

$$e^{-T}x_1 = \left(\frac{e^{-T}}{1+e^{-T}}\right)\left(\frac{2e^{-T}-e^{-2T}}{1+e^{-T}}\right),$$

and Player 2's SSPE payoff is

$$e^{-T}y_2 = \left(\frac{e^{-T}}{1+e^{-T}}\right)\left(\frac{1-e^{-T}+e^{-2T}}{1+e^{-T}}\right).$$

On the interval $e^{-T} \in [\frac{1}{2}, 1)$, *SSPE payoffs of both players are monotone increasing. In addition, Player 1's SSPE payoff is concave in* e^{-T} *on that interval, while Player 2's SSPE payoff is convex on that interval. Both players obtain* $1/6$ *if* $e^{-T} = 1/2$, *and both obtain* $1/4$ *in the limit as* $e^{-T} \to 1$.

The two previous examples have in common that the equilibrium split of the surplus is fair both at $T = \ln(2)/\lambda$ and in the limit as $T \to 0$. In those examples, it seems that there is no more trade–off between efficiency and fairness. So far, we have considered fairness only with regard to the ex ante *expected* split of the surplus. As discussed before, when T is small, SSPE proposals tend to be lopsided. So, the actual realized split of the surplus for small T is not fair "ex post." For instance, reconsider Example 4. In a round in which Player 1 is the proposer, his offer to Player 2 converges to 1/4 in the limit as T goes to zero. It can be checked that, when Player 2 is the proposer, he offers only 1/4 to Player 1. So the ultimate agreement will always allocate one player three times as much surplus as the other player.

5.3 Expected payoff to the final responder

In the previous subsection, we have considered which length of bargaining rounds is optimal for either of the two players, for various configurations of model parameters. Now we turn to the question how one might want to "design" an institutional environment in a way that takes both efficiency and fairness considerations into account. One simple way of doing this is to maximize the *expected payoff to the final responder.* This criterion is clearly responsive to both efficiency and fairness. Moreover, it takes into account not only the expected payoffs to the two players, but also the degree to which equilibrium proposals are lopsided.

We have shown that, in the limit as $T \to 0$, Player j is always offered $\mu_j \left(\frac{\lambda}{\lambda+r}\right)$ when he is the responder. If T is small enough, the probability that the offer which is eventually accepted is Player i's offer converges to μ_i. By symmetry, this means that the expected payoff to the final responder is $2\mu_1\mu_2 \left(\frac{\lambda}{\lambda+r}\right)$ in the limit as T goes to zero. Since $\mu_1+\mu_2 = 1$, we can also write it as $2(\mu_1 - \mu_1^2) \left(\frac{\lambda}{\lambda+r}\right)$.

If $T = \ln(2)/\lambda$, every proposer offers the fair split in an SSPE, and the expected value of agreement is $\frac{1}{2e^{rT}-1} = \frac{1}{2^b-1}$, where we recall that $b = 1+r/\lambda$. Hence, the expected payoff to the final proposer is $\frac{1/2}{2^b-1}$ if $T = \ln(2)/\lambda$.

We see that the expected payoff to the final proposer is greater at $T = \ln(2)/\lambda$ than it is for sufficiently small T if and only if the following condition holds:

$$\frac{b}{2^b - 1} \geq 4(\mu_1 - \mu_1^2). \tag{16}$$

The right–hand side equals one if $\mu_1 = 1/2$, and is strictly less than one for any other $\mu_1 \in (0, 1)$. The left–hand side is monotone decreasing on the relevant interval $[1, \infty)$, it equals one if $b = 1$, and converges to zero in the limit as b goes to infinity.

If the proposal power of both players is equal, that is, $\mu_1 = \mu_2 = 1/2$, then the expected payoff to the final responder is greater in the limit as $T \to 0$ than it is if $T = \ln(2)/\lambda$.

Notice that this is true independently of the extent to which the equilibrium proposals are lopsided.

By contrast, if players differ ever so slightly in their proposal power, one can find parameter values r and λ so that the expected payoff of the final responder is greater at $T = \ln(2)/\lambda$ than it is for sufficiently small T.

6 Conclusion

The canonical bargaining model condenses each round into a single point in time. Offers are made, accepted, and implemented instantaneously. In the present paper, we have challenged this assumption, and proposed a bilateral bargaining model including frictions that the canonical approach abstracts away from. When bargaining rounds are of non–trivial length, players may make counter–offers. When communication is noisy, bargaining may fail not only due to disagreement, but also due to unsuccessful communication. When agreements can only be implemented according to a rigid time schedule, players may want to delay offers, and inefficiencies arise.

We have established conditions under which well–known results from the canonical bargaining model can be recovered in the limit as communication becomes less noisy, and the time schedule for implementing agreements becomes more flexible. This result can be interpreted as providing the conditions under which the canonical bargaining model is robust to the presence of frictions in each bargaining round.

One important question in bargaining theory is how conflict and delay can arise, while the canonical bargaining model strongly predicts immediate agreement in all subgames. One particularly simple explanation for conflict and delay could be noisy communication. There are many different potential interpretations of what we capture by modeling noisy communication: One very literal interpretation could be that players are uncertain about the frequency with which their opponents read their e–mail. An alternative interpretation is that contracts are often very complex, and it takes an uncertain amount of time to process, understand, and evaluate the exact terms of an offer. While our model gives an explanation for equilibrium delay, its predictions about the surplus allocation are still compatible with those of the canonical approach in the limit as the time schedule for implementing agreements is flexible enough. We have demonstrated that very noisy communication cannot only lead to prolonged conflict and delay in equilibrium, but also gives players an incentive to adopt tougher bargaining positions, which leads to more lopsided proposals and less predictable surplus allocations.

Another possible interpretation of the results in this paper is that they provide a rationale for institutions that constrain the time at which agreements can be implemented: When implementation of an agreement is subject to a rigid time schedule, this may make bargaining less efficient but render the surplus allocation more fair. In that sense, a more rigid bargaining institution may be useful to protect the weaker party in the negotiation from exploitation.

Appendix A

Proof of Proposition 1.

Player i accepts a counter–offer if and only if it gives him at least $\delta\widetilde{x}_i$. If it is optimal for Player j to make a counter–offer, then that counter–offer must be $1-\delta\widetilde{x}_i$. It immediately follows that Player j accepts an offer $\theta_i \geq 1-\delta\widetilde{x}_i$ at any time. This explains why $\widehat{t}_j(\theta_i)=0$ if $\theta_i \geq 1-\delta\widetilde{x}_i$. From now on, we consider the case where $\theta_i < 1-\delta\widetilde{x}_i$. Suppose that Player j makes his best counter–offer $\eta = \delta\widetilde{x}_i$ at time $(k-1)T+t$. Then, the probability that Player i receives the counter–offer before time kT is given by $1-e^{-\lambda(T-t)}$. Hence, making the optimal counter–offer at time $(k-1)T+t$ gives Player j an expected payoff of $(1-e^{-\lambda(T-t)})(1-\delta\widetilde{x}_i)+e^{-\lambda(T-t)}\delta\widetilde{y}_j$. By a standard argument, it is optimal for Player j to accept θ_i at time $(k-1)T+t$ if and only if

$$\theta_i \geq (1-e^{-\lambda(T-t)})(1-\delta\widetilde{x}_i)+e^{-\lambda(T-t)}\delta\widetilde{y}_j.$$

Rearranging this inequality, we obtain

$$e^{-\lambda(T-t)} \geq \frac{1-\delta\widetilde{x}_i-\theta_i}{1-\delta\widetilde{x}_i-\delta\widetilde{y}_j},$$

Solving for t yields

$$t \geq T-\left(\frac{1}{\lambda}\right)\ln\left(\frac{1-\delta\widetilde{x}_i-\delta\widetilde{y}_j}{1-\delta\widetilde{x}_i-\theta_i}\right).$$

Since t is non–negative by definition, the desired expression for $\widehat{t}_j(\theta_i)$ follows.

□

Proof of Proposition 2.

Take $\varepsilon > 0$ sufficiently small, and suppose that Player i makes the offer $1-\delta\widetilde{x}_i-\varepsilon$ immediately, that is, at time $(k-1)T$. In that case, his expected payoff is

$$\begin{aligned} & (1-e^{-\lambda T})(\delta\widetilde{x}_i+\varepsilon)+e^{-\lambda T}\delta\widetilde{x}_i \\ = \quad & \delta\widetilde{x}_i+(1-e^{-\lambda T})\varepsilon > \delta\widetilde{x}_i. \end{aligned}$$

Indeed, Player i can make an offer that leads to an expected payoff strictly greater than $\delta\widetilde{x}_i$. Conversely, any choice of (θ_i,τ_i) which leads to an expected payoff of $\delta\widetilde{x}_i$ or less cannot be optimal. If Player i makes an offer θ_i such that $\theta_i \geq 1-\delta\widetilde{x}_i$, then his payoff is at most $\delta\widetilde{x}_i$. If Player i does not make an offer strictly earlier than time kT, an expected payoff of $\delta\widetilde{x}_i$ results. The proposition follows.

□

Proof of Proposition 3.

We show first that $\tau_i = \widehat{t}_j(\theta_i)$. Let us suppose by way of contradiction that Player i chooses a pair (θ_i, τ_i) such that $\tau_i > \widehat{t}_j(\theta_i)$. Then Player i's offer θ_i is accepted by Player j if and only if it arrives before time kT. Thus Player i's expected payoff is

$$(1 - e^{-\lambda(T-\tau_i)})(1-\theta_i) + e^{-\lambda(T-\tau_i)}\delta\widetilde{x}_i = 1 - \theta_i - e^{-\lambda(T-\tau_i)}(1-\theta_i-\delta\widetilde{x}_i).$$

Take $\varepsilon > 0$ sufficiently small, and consider a unilateral deviation under which Player i chooses $(\theta_i, \tau_i - \varepsilon)$ instead of (θ_i, τ_i). By the same token, Player i's expected payoff is now

$$(1 - e^{-\lambda(T-\tau_i+\varepsilon)})(1-\theta_i) + e^{-\lambda(T-\tau_i+\varepsilon)}\delta\widetilde{x}_i = 1 - \theta_i - e^{-\lambda(T-\tau_i+\varepsilon)}(1-\theta_i-\delta\widetilde{x}_i).$$

Due to Proposition 2, we have that $1 - \theta_i - \delta\widetilde{x}_i > 0$, and so the deviation is profitable.

Now suppose that Player i chooses (θ_i, τ_i) such that $\tau_i < \widehat{t}_j(\theta_i)$. There are three cases to distinguish: First, if the offer arrives between times τ_i and $\widehat{t}_j(\theta_i)$, then Player j makes the counter–offer $\eta = \delta\widetilde{x}_i$. Irrespective of whether or not this counter–offer arrives before time kT, the resulting expected payoff for Player i is $\delta\widetilde{x}_i$. Second, if the offer arrives between times $\widehat{t}_j(\theta_i)$ and kT, it is accepted, and so Player i receives $1 - \theta_i$. Third, if the offer does not arrive until time kT, then bargaining round k fails and so the expected payoff for Player i is $\delta\widetilde{x}_i$.

Thus Player i's expected payoff can be written as

$$e^{-\lambda(\widehat{t}_j(\theta)-\tau_i)}(1 - e^{-\lambda(T-\widehat{t}(\theta_i))})(1-\theta_i) + (1 - (e^{-\lambda(\widehat{t}_j(\theta_i)-\tau_i)})(1 - e^{-\lambda(T-\widehat{t}_j(\theta_i))}))\delta\widetilde{x}_i.$$

It is easy to verify that Player i has a profitable unilateral deviation by making the same offer θ_i at time $\widehat{t}_j(\theta_i)$ rather than at time τ_i, given that $1 - \theta_i > \delta\widetilde{x}_i$. We can now conclude that an optimal choice of (θ_i, τ_i) by the proposer must be such that $\tau_i = \widehat{t}_j(\theta_i)$, as desired.

Now we show that in an SSPE, Player i chooses (θ_i, τ_i) such that $\theta_i = (1 - e^{-\lambda(T-\tau_i)})(1 - \delta\widetilde{x}_i) + e^{-\lambda(T-\tau_i)}\delta\widetilde{y}_j$. The argumemt is standard: Suppose first that $\theta_i > (1 - e^{-\lambda(T-\tau_i)})(1-\delta\widetilde{x}_i) + e^{-\lambda(T-\tau_i)}\delta\widetilde{y}_j$. Then Player i has a profitable deviation in proposing $\theta_i - \varepsilon$ instead of θ_i for some $\varepsilon > 0$ sufficiently small. Now suppose that Player i offers some $\theta_i < (1 - e^{-\lambda(T-\tau_i)})(1-\delta\widetilde{x}_i) + e^{-\lambda(T-\tau_i)}\delta\widetilde{y}_j$. Then Player j makes a counter–offer in which case Player i only gets $\delta\widetilde{x}_i$. But, by Proposition 2, we have that $1 - \theta_i > \delta\widetilde{x}_i$.

□

Appendix B

In this Appendix, we provide the Mathematica code with which the solution to the system of Eqns. (5)–(12) can be verified. This code defines a total of eleven equations. The first eight of those correspond exactly to the aforementioned system of equations. The ninth equation in the code corresponds to the expected value of agreement. The tenth and eleventh equations are used to compute the equilibrium offers θ_1 and θ_2.

```
variables = {x1, x2, y1, y2, tx1, tx2, ty1, ty2, V, T1, T2};
equations = {x1 == d * tx1 + (1 - d * V) (p - p^2),
    x2 == d * tx2 + (1 - d * V) (p - p^2),
    y1 == V - x2,
    y2 == V - x1,
    tx1 == m1 * x1 + (1 - m1) y1,
    tx2 == m2 * x2 + (1 - m2) y2,
    ty1 == m2 * y1 + (1 - m2) x1,
    ty2 == m1 * y2 + (1 - m1) x2,
    V == (1 - p) / (1 - d * p),
    T1 == (1 - p) (1 - d * tx1) + p * d * ty2,
    T2 == (1 - p) (1 - d * tx2) + p * d * ty1};
eqsmatrixform = Normal[CoefficientArrays[equations, variables]];
                normal  Arrays der Koeffizienten
M = eqsmatrixform[[2]];
v = -eqsmatrixform[[1]];
sol = LinearSolve[M, v];
      löse lineare Gleichung
Print[MatrixForm[M], " ", MatrixForm[variables], " = ", MatrixForm[v]]
gib aus Matritzenform          Matritzenform               Matritzenform
Print[MatrixForm[variables], " = ", MatrixForm[Simplify[sol]]]
gib aus Matritzenform                  Matritzenform vereinfache
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 & -d & 0 & 0 & 0 & d\left(p-p^2\right) & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -d & 0 & 0 & d\left(p-p^2\right) & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ -m1 & 0 & -1+m1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -m2 & 0 & -1+m2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1+m2 & 0 & -m2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1+m1 & 0 & -m1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & d\,(1-p) & 0 & 0 & -d\,p & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & d\,(1-p) & -d\,p & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ y1 \\ y2 \\ tx1 \\ tx2 \\ ty1 \\ ty2 \\ V \\ T1 \\ T2 \end{pmatrix} = \begin{pmatrix} p-p^2 \\ p-p^2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1-p}{1-d\,p} \\ 1-p \\ 1-p \end{pmatrix}$$

$$\begin{pmatrix} x1 \\ x2 \\ y1 \\ y2 \\ tx1 \\ tx2 \\ ty1 \\ ty2 \\ V \\ T1 \\ T2 \end{pmatrix} = \begin{pmatrix} -\frac{(-1+p)\,(d+d\,m1\,(-1+p)+p-d\,(1+m2)\,p)}{(-1+d\,(-1+m1+m2))\,(-1+d\,p)} \\ -\frac{(-1+p)\,(d+d\,m2\,(-1+p)+p-d\,(1+m1)\,p)}{(-1+d\,(-1+m1+m2))\,(-1+d\,p)} \\ -\frac{(-1+p)\,(1-p+d\,(m1\,(-1+p)+p-m2\,p))}{(-1+d\,(-1+m1+m2))\,(-1+d\,p)} \\ \frac{(-1+p)\,(-1+p+d\,(m2+(-1+m1)\,p-m2\,p))}{(-1+d\,(-1+m1+m2))\,(-1+d\,p)} \\ \frac{(-1+p)\,(-1+m1+p-d\,p+(-2+d)\,m1\,p+d\,m2\,p)}{(-1+d\,(-1+m1+m2))\,(-1+d\,p)} \\ \frac{(-1+p)\,(-1+m2+p-d\,p+d\,m1\,p+(-2+d)\,m2\,p)}{(-1+d\,(-1+m1+m2))\,(-1+d\,p)} \\ -\frac{(-1+p)\,(m2+d\,(-1+m1+m2)\,(-1+p)+p-2\,m2\,p)}{(-1+d\,(-1+m1+m2))\,(-1+d\,p)} \\ -\frac{(-1+p)\,(m1+d\,(-1+m1+m2)\,(-1+p)+p-2\,m1\,p)}{(-1+d\,(-1+m1+m2))\,(-1+d\,p)} \\ \frac{-1+p}{-1+d\,p} \\ -\frac{(-1+p)\,\left(1+d^2\,(-1+m1+m2)\,p+d\,(-m2+p-2\,m1\,p)\right)}{(-1+d\,(-1+m1+m2))\,(-1+d\,p)} \\ -\frac{(-1+p)\,\left(1+d^2\,(-1+m1+m2)\,p+d\,(-m1+p-2\,m2\,p)\right)}{(-1+d\,(-1+m1+m2))\,(-1+d\,p)} \end{pmatrix}$$

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