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# Climate change financial risks: pricing and portfolio allocation\*

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## Abstract

There are concerns that climate-related physical and political risks are not yet properly reflected in asset prices. To address these concerns, we develop a dynamic asset pricing framework with two sources of rare disasters: macroeconomic events and climate change. We link carbon emissions and portfolio composition with the stochastically-varying probability of climate-related events. Using theory and simulations we study the implications of the imminent threat of climate change on different market measures and on the participation of carbon-intensive assets in the market portfolio. We also obtain closed-form solutions for market prices and the Social Cost of Carbon. Our results suggest that climate change implies a positive and increasing risk premium. We also show that, with the observed trends in climate change, macroeconomic risk works as a hedge against catastrophic climate change, such that the aggregate equity premium may remain unaltered. The transition risk of climate policy substantially lowers the participation of carbon-intensive assets in the market portfolio.

*Keywords:* Climate change, Risk premia, Rare events, Policy Risk, Stranded assets

*JEL classification:* G11, G12, O44, Q51, Q54

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# 1 Introduction

In his path-breaking speech “Breaking the tragedy of the horizon” in 2015, the Governor of the Bank of England, Mark Carney, was the first to highlight the threat of climate change for the stability of the financial system and to identify the risks involved (Carney 2015). Although it is now well understood that climate-related events are increasing in frequency and intensity, the full severity of climate change has not materialized yet.<sup>1</sup> In the context of the financial system, climate-related risks could become systemic, especially if they are not properly reflected in asset pricing (Monnin 2018, European Central Bank 2019). The purpose of the present paper is to study the pricing of climate-related risks in the context of a dynamic CAPM approach with rare disasters. Our main objective is to explore how these risks can affect the different market measures, the participation in the aggregate portfolio of carbon-intensive assets, and the conditions under which these assets can become stranded.

There are two main types of climate-related risks relevant for market participants: physical risks and transition risks.<sup>2</sup> Physical risks, associated with physical damages to assets, could be event-driven (droughts, floods, storms, wildfires and crop failures) or chronic, related to long term climate shifts (e.g. sea level rise). The frequency and severity of environmental events can increase as a result of rising global temperatures (IPCC 2018, USGCRP 2018). Transition risks include, among other components,<sup>3</sup> policy risks which emerge from potential introduction of stringent carbon-pricing policies that can affect the returns of assets related to carbon-intensive technologies or processes.<sup>4</sup> Since investments in the carbon-based economy are mostly irreversible, stringent climate policies are likely to make the operation of carbon-intensive firms unprofitable, and, thereby leave assets stranded. Crucial to individual investors, appropriate pricing of climate-related risks, could lead to more informed, and thus efficient, capital allocation decisions.

Of relevance to central banks is the possibility that climate-related damages could result in financial losses that affect the stability of the financial system through the balance sheets of insurers, banks and credit flows (Carney 2015, Battiston et al. 2017, Stolbova et al. 2018). In combatting climate change, policy has so far been predominantly fiscal and

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<sup>1</sup>See Francis and Vavrus (2012), Cai et al. (2014), Hsiang et al. (2017), Francis (2017) on the frequency and severity of natural disasters.

<sup>2</sup>See (Campiglio et al. 2018) for a discussion on the climate-related risk and their relevance for financial markets and central banks.

<sup>3</sup>Transition risks also include, liability risks, technology risks, market risks and reputation risks.

<sup>4</sup>For example, Batten et al. (2016) find contrasting cumulative abnormal returns experienced by a petroleum refining company and a wind turbine manufacturer the day after the announcement of the Paris agreement in December 2015; others argue that these policy risks have not yet been fully internalized by markets, e.g. Blackrock Investment Institute (2016), Sevillano and González (2018).

little attention has been paid to the role of central banks under climate change conditions. This attitude is, however, changing both in the context of academic research but also in the context of central banks' policy. The focus of this new line of approach is to explore the role of central banks in managing financial risks stemming from climate change. Going one step further, central banks explore whether climate change goals could be a secondary objective, along with controlling inflation and promoting macroeconomic stability (Campiglio et al. 2018, Strauss Financial Times, 05/12/2018).<sup>5</sup> From their position at the heart of the global economy, central banks can – by allocating capital and risk – promote the transition to a low-carbon economy and ensure financial stability (Prudential Regulation Authority 2018).

Since climate-related risks emerge through two major transmission channels – physical disasters and transition risks – it will be natural to approach the problem of pricing climate-related risks by incorporating these type of risks into asset pricing. It should be noted here that transition risks are related to physical risks, since the increasing frequency and intensity of climate-related disasters are expected to accelerate the introduction of more, and more stringent, policies in the transition to a low-carbon economy.

The financial literature has come a long way in explaining investors' behavior towards risk, and in particular asset pricing puzzles, by introducing rare disasters like wars and economic crises – the macroeconomic disasters.<sup>6</sup> This approach suggests that a simple explanation of high risk premia is that investors are concerned about equity performance in rare events such as the Great Depression of the 1930s, or the two world wars of the 20th century. Since climate-related physical risks can also be regarded as rare disasters – not as rare as world wars, but nevertheless infrequent – it would be natural to consider an analytical framework similar to the one used to price assets under rare disasters for the pricing of these risks. The purpose, therefore, of the present paper is to study – at the same time – the pricing of macroeconomic and climate-related risks in the context of a dynamic asset pricing model. Our main objective is to explore how these risks can affect the different market measures, the participation of carbon-intensive assets in the aggregate portfolio, and the conditions under which these assets can become stranded.

With regards to our model, we consider a dynamic CAPM with rare disasters in the spirit of Barro (2009), and Wachter (2013). In these endowment economies rare disasters are captured by a Poisson process which allows for downward jumps in the return of the

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<sup>5</sup>In studying the role of Central Banks when climate changes, a more macroeconomic line of approach is to examine whether taking into account climate change in the context of dynamic stochastic general equilibrium models (DSGE) affects monetary policy when this policy is assumed to be conducted through the nominal interest rate on government bonds which follows a standard Taylor-type rule (see, e.g., Annicchiarico and Di Dio (2017), Economides and Xepapadeas (2018)).

<sup>6</sup>See Barro (2006), Barro and Ursúa (2008), Barro (2009), Wachter (2013), Pindyck and Wang (2013), Seo and Wachter (2018), Gomes et al. (2018).

underlying asset. Wachter (2013) introduces a stochastic intensity of the Poisson process, and matches the observed post-WWII equity premia and stock market volatility for the US, even for low values of relative risk aversion. To study climate-related risks, we build on this work and introduce a second Poisson process for climate-related events; these can be either natural disasters or the introduction of a stringent climate policy. We consider a portfolio of riskless bond holdings, and two types of risky capital assets: a general one, and a brown asset, the latter being responsible for CO2 emissions. Macroeconomic and climate-related disasters affect both assets, while climate-related policy events affect only the brown one.<sup>7</sup> Using our model we can set the “correct” value on market fundamentals and calculate the participation of carbon-intensive assets in the aggregate portfolio when climate change risk is taken into account.

Our proxy for climate change is the change in global average temperature relative to the mean of a given time period, i.e., the temperature anomaly. Our methodological contribution is threefold. First, we include the aforementioned stochastic intensity for the Poisson process of extreme environmental events that builds on the observed positive relationship between the frequency of climate-related events and temperature anomaly (see Figure 1).<sup>8</sup> As climate changes, the probability of extreme environmental events becomes higher in expectation, while events that once constituted the tail of its distribution become more frequent. Second, we differentiate between assets on their exposure to stringent climate policy and allow for the portfolio composition to adjust to the transition risk of climate change. Third, we also consider the effect of portfolio allocation on emissions: the higher the share of brown assets and emissions in the portfolio the higher – for a given brown-asset technology – the temperature anomaly and therefore the higher the probability of occurrence of a climate-related disaster. A reduction in the share of brown assets will have the opposite effects. In this way we establish a direct link between the composition of the aggregate portfolio and the arrival of climate related events.

Depending on investors information about climate change, this setup allows for two types of modelling. In the first case, the representative investor takes the evolution of global temperature as exogenous and not affected by his/her portfolio allocation. Exogenous temperature paths in this case could be regarded as the representative concentration pathways (RCPs) produced by the IPCC (van Vuuren et al. 2011). We then calibrate the

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<sup>7</sup>We follow the definition of Prudential Regulation Authority (2015) and consider as “brown” Tier 1 and 2 assets that are directly exposed to transition risks. Tier 1 assets include coal, oil and gas extraction companies, and conventional utilities; Tier 2 assets include firms that are energy-intensive, e.g. chemicals and mining companies. Together they account for about 30% of global equity and fixed-income investments.

<sup>8</sup>Since the link between temperature anomaly and emissions is well established (e.g. Matthews et al. (2009), Hassler et al. (2016), Brock and Xepapadeas (2017)), this establishes the link between emissions and the probability of natural disasters.

model to observed data and make projections for different market measures based on the RCPs; the portfolio weights get recalculated when we include the transition risk of climate change. We call this the *myopic* solution. In the second case, the representative investor recognizes that his/her portfolio choices will affect the temperature path and therefore the probability of climate-related events; we call this solution *optimal*.

The myopic simulation shows that climate change risk entails a positive risk premium which is increasing over time: with our calibration, the premium of climate change amounts to 0.2 % p.a. in 2010; it can increase up to 0.5 % p.a. when emissions follow the worst IPCC scenario. Most surprisingly, we find that whether this increase carries over to the overall equity premium depends on the dynamic nature of the probability of extreme environmental events, and on the severity of climate disasters. With rising emissions the increasing risk of environmental events puts a downward pressure on equity valuations, which leaves less room for prices to react to the risk of extreme events of either type. As climate changes the mean and the variance of the distribution of climate risk increase, while those of macroeconomic risk stay unaltered, thus increasing the relative importance of climate change risk even in times without disasters. This result suggests that macroeconomic risk works as a hedging strategy against the risk of climate change, and runs along the lines of Weitzman (2013).<sup>9</sup> Our calibration exercise also shows that, compared to the benchmark without policy risk, including the transition risk of environmental policy lowers the portfolio participation of brown assets substantially.

To the best of our knowledge, this is the first paper to incorporate the stochastically time-varying risk of rare disasters related to both macroeconomic events and climate change in a dynamic asset pricing framework with Poisson shocks, also with both types of risk exposures, namely physical and transition risks. Our contribution comes the closest to Bansal et al. (2016) who also include Poisson shocks due to climate change – but not due to macroeconomic events – and find that global warming carries a positive, and increasing, risk premium and reduces asset valuations. In comparison to us, they find that climate change unambiguously increases risk premia in the market. We show that this result is a limiting case in our model when we exclude macroeconomic risks. Finally, apart from optimally pricing climate change for the future, we study the effects of transition risk on portfolio composition. Also relevant to our results on portfolio composition, van der Ploeg and Rezai (2019) develop a macroeconomic model with policy shocks to understand the

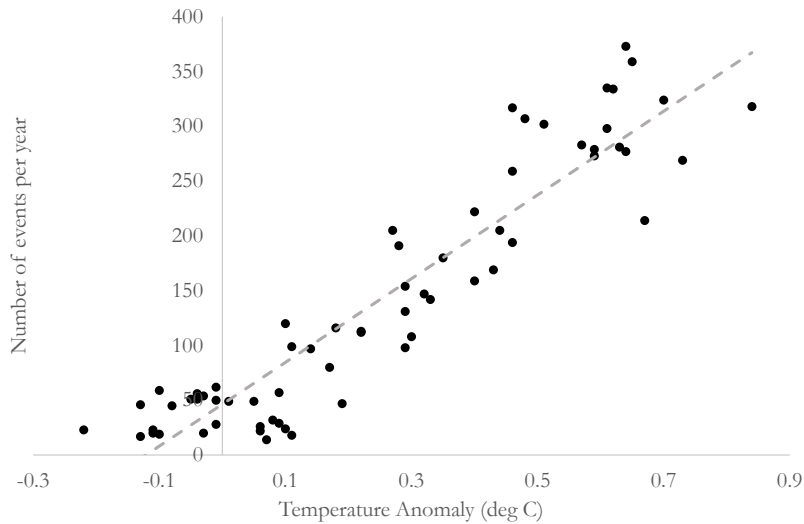
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<sup>9</sup>Considering uncertain climate change and its catastrophic effects, Weitzman (2013) suggests that emissions abatement can be used as a hedging strategy against macroeconomic risk, which implies a negative climate beta. Our result runs along the same lines but in the opposite direction. A negative climate beta is also assumed in Daniel et al. (2019), who find a declining path for the social cost of carbon as the uncertainty about the effects of carbon emissions is gradually resolved over time. The conditions for whether climate betas are positive or negative are discussed in Dietz et al. (2018).

determinants of asset stranding. As in our model, the risk of policy tipping curbs the market price of oil and gas majors and leads to stranding of carbon-intensive assets.<sup>10</sup>

The paper is organized as follows. The next section builds the theoretical framework and formally provides intuition on how climate change risks can affect risk premia. We extend the model of Wachter (2013) by adding Poisson shocks due to climate change with a stochastically time-varying probability, which we build from climate science. Section 3 deals with numerical simulations. We present our methodology, calibration, and discuss our results. Section 4 concludes.

**Figure 1: Number of climate-related disasters p.a. in the period 1955-2015 vs. temperature anomaly from the mean of the 20<sup>th</sup> century.** Source: EM-DAT The International Disasters Database [www.emdat.be](http://www.emdat.be) and [ourworldindata.org](http://ourworldindata.org)



<sup>10</sup>Additionally, Brock and Hansen (2018) study the implications of risk, ambiguity and model misspecification on climate economic modeling, while Barnett et al. (2019) incorporate these notions in an endogenous growth model with depletable energy resources and Brownian uncertainty to study asset pricing and the social implications of a changing climate. Finally, Bretschger and Vinogradova (2018) and van der Ploeg and van den Bremer (2019) develop macroeconomic models with climate-related – but not macroeconomic – disasters.

## 2 The model

### 2.1 Model setup

Time is continuous and denoted by subscript  $t$ . We consider an endowment economy with two Lucas (1978) trees. We call these assets general ( $G$ ) and brown ( $B$ ). Brown assets are carbon-intensive and subject to risk of stringent climate policy.<sup>11</sup> Assets  $\{B, G\}$  generate dividend streams  $C_G dt$  and  $C_B dt$ , that follow standard geometric jump-diffusion processes,

$$\begin{aligned}\frac{dC_{Gt}}{C_{Gt}} &= \mu_G dt + \sigma_G dW_{Gt} + \sum_{j \in \{M, E\}} (e^{Z_G^j} - 1) dQ_t^j, \\ \frac{dC_{Bt}}{C_{Bt}} &= \mu_B dt + \sigma_B dW_{Bt} + \sum_{j \in \{M, E\}} (e^{Z_B^j} - 1) dQ_t^j + (e^X - 1) dQ_t^X,\end{aligned}\tag{1}$$

with  $\mu_i, \sigma_i > 0$  being, respectively, the asset-specific drift and volatility parameters and  $i \in \{B, G\}$ . While diffusion refers to a standard Brownian motion  $W_i$ , jumps refer to infrequent adverse Poisson events that result in economic losses. Our model features two types of shocks, namely macroeconomic ( $M$ ) and environmental ( $E$ ). Macroeconomic disasters are events like wars and economic crises, while environmental disasters are severe events related to climate change like droughts, floods and wildfires. Each shock  $j \in \{M, E\}$  is expressed by an uncorrelated Poisson process  $Q^j$  with a time-varying arrival rate  $\lambda^j$ . In addition, brown assets are exposed to an infrequent policy shock  $Q^X$ . Moreover, the increasing frequency and intensity of climate-related disasters are expected to accelerate the introduction of policies towards the transition to a low-carbon economy. To capture this correlation, we assume that the Poisson intensity of policy risk is a fraction  $\pi \in [0, 1]$  of the intensity of environmental disasters, i.e.,  $\lambda^X = \pi \lambda^E$ .

Assets are denoted by subscripts and shocks by superscripts, i.e.,  $Z_i^j < 0$  denotes the drop in  $\log C_i$ ,  $i \in \{B, G\}$  when an event of type  $j \in \{M, E\}$  occurs. For ease of exposition, we assume that each asset operates in the same macroeconomic and natural environment and is subject to the same physical shock  $Z^j < 0$ ; its time-invariant distribution  $z^j$  comes from the data and is independent of all other processes. The above discussion implies that  $Z_G^M = Z_B^M = Z^M$ ,  $Z_G^E = Z_B^E = Z^E$ . When effective, stringent policy acts to further reduce the value of the brown asset by  $X < 0$ , which we assume certain for simplicity.<sup>12</sup>

With regards to macroeconomic events  $Q^M$ , we follow Wachter (2013) and assume the

<sup>11</sup>Our approach is of course simplistic. A more realistic approach would consider different shades of brown where all assets have a different degree of contribution to carbon emissions and of exposure to climate policy.

<sup>12</sup>The policy stringency  $X$  could in turn be an increasing function of the intensity of climate damages  $Z^E$ . However, this assumption would not alter the quality of the results, while it would impair the tractability of the model.



following mean-reverting process for the Poisson intensity  $\lambda^M$ :

$$d\lambda_t^M = \kappa^M(\bar{\lambda}^M - \lambda_t^M)dt + \sigma_\lambda^M \sqrt{\lambda_t^M} dW_{\lambda t}^M. \quad (2)$$

Variable  $W_\lambda^M$  is a standard Brownian motion, independent of all other processes. Parameter  $\kappa^M$  represents the adjustment speed of the process towards its mean  $\bar{\lambda}^M$ ;  $\sigma_\lambda^M$  is a volatility parameter. The solution to (2) leads to a Gamma stationary distribution for  $\lambda^M$ , provided that both  $\kappa^M$  and  $\bar{\lambda}^M$  are positive, which we will assume. This process has the attractive feature that  $\lambda^M$  can never become negative. Moreover, the square root in (2) implies that the resulting stationary distribution is highly right-skewed generating tail events, while at the same time, high realizations of  $\lambda^M$  make the process more volatile, and thus even higher realizations more likely, compared to a standard autoregressive process.<sup>13</sup>

Our main methodological contribution lies in establishing the link between climate change and the intensity  $\lambda^E$  of the Poisson process related to environmental disasters  $Q^E$ . Climate shocks are triggered by emissions caused by consumption of brown assets  $C_B$ . Let  $N > 0$  be abatement expenditure on improving the environmental efficiency of brown assets. Accordingly, effective CO2 emissions read

$$E_t = \phi(N_t)C_{Bt}, \quad (3)$$

with  $\phi(N)$  measuring emissions intensity and  $\phi'(N) < 0$ . Regarding the dynamics of climate change, it is well established that cumulative emissions are linked to the change of temperature anomaly  $T$  in a rather linear fashion.<sup>14</sup> Following the transient climate response to cumulative carbon emissions (TCRE), a simplified continuous time representation of the global temperature anomaly dynamics can be written, as in Hassler et al. (2016) and Brock and Xepapadeas (2017), as

$$\dot{T}_t = \frac{1}{H}(\Lambda E_t - \delta T_t), \quad T_0 = 0. \quad (4)$$

Parameter  $H > 0$  represents the heat capacity of climate sinks, e.g., the upper ocean layer;  $\Lambda > 0$  the TCRE; and  $\delta \in (0, 1)$  the stabilizing effect of the outgoing radiation (see also Nævdal and Oppenheimer (2007), and Heutel et al. (2016)). With  $\bar{E}$  a constant flow of emissions, equation (4) leads to a steady state for temperature anomaly  $\bar{T} = (\Lambda/\delta)\bar{E}$ . Natural disasters are increasing in frequency and intensity. Based on observations (Figure 1), we will assume that the probability of natural disasters  $\lambda^E$  depends linearly on the temperature anomaly, i.e.,  $\lambda_t^E = \tilde{\lambda}^E + \xi T_t$  with  $\{\tilde{\lambda}^E, \xi\}$  non-negative numbers.

<sup>13</sup>See Cox et al. (1985) or Wachter (2013) for more information on (2).

<sup>14</sup>See Matthews et al. (2009, 2012), Knutti (2013), Knutti and Rogelj (2015), MacDougall et al. (2017).

Differentiating the last expression with respect to time and substituting back the expression for  $E$  yields

$$\dot{\lambda}_t^E = \frac{\delta}{H}(\tilde{\lambda}^E - \lambda_t^E) + \frac{\xi}{H}\Lambda E_t.$$

Besides the expected positive relationship between the disaster probability and temperature anomaly that leads to the previous expression, the arrival rate of natural disasters is rather stochastic. We will therefore assume a process similar to (2),

$$d\lambda_t^E = \kappa^E(\bar{\lambda}_t^E - \lambda_t^E)dt + \sigma_\lambda^E \sqrt{\lambda_t^E} dW_{\lambda_t}^E, \quad (5)$$

where for ease of exposition we defined  $\kappa^E \equiv \delta/H$  and  $\bar{\lambda}_t^E \equiv \tilde{\lambda}^E + \xi(\Lambda/\delta)E_t$ , with  $E_t = \phi(N_t)C_{Bt}$ , and  $W_\lambda^E$  a standard Brownian motion, independent of all other processes. Note that, in comparison to (2), although this autoregressive process features mean reversion, the mean itself is time-varying. When emissions keep rising out of balance natural disasters are becoming more frequent in expectation but less predictable. In equilibrium, i.e., for a constant flow of net emissions (possibly zero if  $C_B = 0$ ), the solution of (5) has also a Gamma stationary distribution, where the constant mean and variance are increasing in the equilibrium flow of emissions.<sup>15</sup> Note also that, in line with observations, in our calibration  $\kappa^E < \kappa^M$  such that realizations of high probability of environmental disasters are more persistent than those of macroeconomic disasters.<sup>16</sup>

Finally, the representative agent has the continuous-time analogue of recursive Epstein-Zin preferences, as formulated by Duffie and Epstein (1992). Accordingly, we use the following recursion to define the utility function  $U$ ,

$$U_t = \mathbb{E}_t \int_t^\infty f(C_v, U_v) dv, \quad (6)$$

where

$$f(C_t, U_t) = \rho(1 - \gamma)U_t \left( \log C_t - \frac{1}{1 - \gamma} \log((1 - \gamma)U_t) \right). \quad (7)$$

Parameter  $\rho > 0$  is the subjective rate of time preference, and  $\gamma > 0$  measures relative risk

<sup>15</sup>Let  $\bar{E}$  be an equilibrium flow of emissions. Following Cox et al. (1985), we can show that the expected value and variance of  $\lambda^E$  at time  $s$ , conditional on its value at time  $t < s$ , is given by:  $\mathbb{E}[\lambda_s^E | \lambda_t^E] = \lambda_t^E e^{-(\delta/H)(s-t)} + (\tilde{\lambda}^E + \frac{\xi}{\delta}\Lambda\bar{E}) \left(1 - e^{-(\delta/H)(s-t)}\right)$  and  $\text{Var}[\lambda_s^E | \lambda_t^E] = \lambda_t^E \frac{(\sigma_\lambda^E)^2}{\delta/H} (e^{-(\delta/H)(s-t)} - e^{-2(\delta/H)(s-t)}) + (\tilde{\lambda}^E + \frac{\xi}{\delta}\Lambda\bar{E}) \frac{(\sigma_\lambda^E)^2}{2\delta/H} (1 - e^{-(\delta/H)(s-t)})^2$ . The steady-state mean and variance are  $\tilde{\lambda}^E + \frac{\xi}{\delta}\Lambda\bar{E}$  and  $(\tilde{\lambda}^E + \frac{\xi}{\delta}\Lambda\bar{E}) \frac{(\sigma_\lambda^E)^2}{2\delta/H}$ , respectively, both increasing in  $\bar{E}$ .

<sup>16</sup>For example Wachter (2013) calibrates  $\kappa^M = 0.08$  while we set  $\delta = 0.05$  PW/K and  $H = 4.58$  PW y/K to match IPCC projections. These give  $\kappa^E = 0.01$ ; see section 3.2 on calibration.

aversion. We assume for simplicity that our utility function features a unitary elasticity of intertemporal substitution (EIS). We will conventionally focus on the case of  $\gamma > 1$ , which implies a preference for early resolution of uncertainty.<sup>17</sup>

## 2.2 Equilibrium

In an endowment economy prices adjust such that aggregate consumption (the aggregate endowment) equals the sum of dividends, i.e.  $C_t = C_{Bt} + C_{Gt}$ . Let  $n_i = C_i/C$  denote the dividend share of asset  $i$ , with  $\sum_i n_i = 1$ . From Itô's Lemma, the aggregate endowment follows

$$\frac{dC_t}{C_t} = \mu_{Ct}dt + \sigma_{Ct}dW_t^T + (e^{Z_t} - 1)dQ_t^T, \quad (8)$$

with  $\mu_C = \sum_i n_i \mu_i$ ,  $\sigma_C = [n_B \sigma_B, (1 - n_B) \sigma_G]$ ,  $dW = [dW_B, dW_G]$ ,  $dQ = [dQ^M, dQ^E, dQ^X]$ , and

$$e^{Z_t} - 1 = [\underbrace{e^{Z^M} - 1}_{\text{macroeconomic shocks}}, \underbrace{e^{Z^E} - 1}_{\text{environmental shocks}}, \underbrace{n_B(e^X - 1)}_{\text{policy shocks}}]. \quad (9)$$

With unitary EIS in the utility function, the price-dividend ratio for the aggregate consumption claim is a constant (Weil 1990, Cochrane et al. 2007, Wachter 2013).<sup>18</sup> Let  $S$  denote the value of that claim, such that  $C/S = \beta$  for some constant  $\beta > 0$ . Itô's Lemma implies that  $dS/S = \mu_C dt + \sigma_C dW^T + (e^Z - 1)dQ^T$ . We consider a representative investor who maximizes lifetime utility by allocating capital income, net of consumption and abatement expenditure, between  $S$  and a perpetual zero-coupon bond  $B$ , the risk-free asset, with an instantaneous rate of return  $r$ , i.e.  $dB/B = rdt$ . Let  $n$  denote the fraction of total assets  $A$  in the risky part of the portfolio. Wealth then follows the process

$$dA_t = (A_t n_t (\mu_{Ct} - r_t + \beta) + A_t r_t - C_t - N_t) dt + A_t n_t \sigma_{Ct} dW_t^T + A_t n_t (e^{Z_t} - 1) dQ_t^T. \quad (10)$$

With  $A$ ,  $\lambda^M$  and  $\lambda^E$  as state variables, and  $V(A, \lambda^M, \lambda^E)$  the value function, using (2), (5), (10), and Itô's Lemma, optimal expenditure, and portfolio choices must satisfy the

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<sup>17</sup>A preference for the early resolution of uncertainty has become a standard notion in macroeconomic modeling, and is important to capture concerns about future growth variations, especially those that are persistent as in our model. A prime example of the adoption of such preferences in the macro-finance literature is the seminal work of Bansal and Yaron (2004).

<sup>18</sup>A constant price-dividend ratio for the price of aggregate consumption claim is the case also in Barro (2006), and Tsai and Wachter (2015) when EIS  $\rightarrow 1$ .

following Hamilton-Jacobi-Bellman equation (Duffie and Epstein 1992):

$$\begin{aligned} \sup_{C, N, n, n_B} \{ & f(C_t, V) + V_A(A_t n_t(\mu_{Ct} - r_t + \beta) + A_t r_t - C_t - N_t) + \frac{1}{2} V_{AA} A_t^2 n_t^2 \sigma_t^2 \\ & + \sum_{j=\{M, E\}} (V_{\lambda^j} \kappa^j (\bar{\lambda}_t^j - \lambda_t^j) + \frac{1}{2} V_{\lambda^j \lambda^j} (\sigma_\lambda^j)^2 \lambda_t^j + \lambda_t^j \mathbb{E}_{z^j} [\tilde{V}^j - V]) \\ & + \pi \lambda_t^E [\tilde{V}^X - V] \} = 0, \end{aligned} \quad (11)$$

with

$$\sigma_t = \sqrt{n_{Bt}^2 \sigma_B^2 + (1 - n_{Bt})^2 \sigma_G^2 + 2n_{Bt}(1 - n_{Bt}) \sigma_{GB}}, \quad (12)$$

$\sigma_{GB} \equiv \sigma_G \sigma_B \text{corr}[dW_G, dW_B]$ ;  $\tilde{V}^j \equiv V(A(1 + n(e^{Z^j} - 1)), \lambda^M, \lambda^E)$ , for  $j \in \{M, E\}$ ;  $\tilde{V}^X \equiv V(A(1 + nn_B(e^X - 1)), \lambda^M, \lambda^E)$ , the value function after the arrival of either disasters (or both together since they are uncorrelated), and policy;  $\bar{\lambda}_t^E = \tilde{\lambda}^E + \xi(\Lambda/\delta)\phi(N_t)n_{Bt}C_t$ , and  $\bar{\lambda}_t^M = \tilde{\lambda}^M$ ; and finally  $V_x, V_{xx}$  denote, respectively, first and second derivative with respect to variable  $x$ , while  $\mathbb{E}_{z^j}$  the expectations operator taken with respect to the  $z^j$ -distribution.<sup>19</sup> Assuming an interior solution we get the first order conditions w.r.t to  $C, N, n$ , and  $n_B$  :

$$f_C = V_A \left( 1 - \frac{V_{\lambda^E}}{V_A} \kappa^E \frac{\xi \Lambda}{\delta} \phi(N) n_B \right), \quad (13)$$

$$-\frac{1}{\phi'(N)} = -\frac{V_{\lambda^E}}{V_A} \kappa^E \frac{\xi \Lambda}{\delta} n_B C, \quad (14)$$

$$\begin{aligned} r = \mu_C + \beta + \frac{V_{AA} A}{V_A} n \sigma^2 + \sum_{j=\{M, E\}} \lambda^j \mathbb{E}_{z^j} \left[ \frac{\tilde{V}_A^j}{V_A} (e^{Z^j} - 1) \right] \\ + \pi \lambda^E \left[ \frac{\tilde{V}_A^X}{V_A} n_B (e^X - 1) \right], \end{aligned} \quad (15)$$

$$\begin{aligned} \mu_B + \frac{V_{AA} A}{V_A} n n_B (\sigma_B^2 - \sigma_{GB}) + \pi \lambda^E \frac{\tilde{V}_A^X}{V_A} (e^X - 1) + \frac{V_{\lambda^E}}{V_A} \kappa^E \frac{\xi \Lambda}{\delta} \phi(N) \frac{C}{An} = \\ \mu_G + \frac{V_{AA} A}{V_A} n (1 - n_B) (\sigma_G^2 - \sigma_{GB}). \end{aligned} \quad (16)$$

Equation (13) is the usual envelope condition for the price of consumption, adjusted for the externality cost of polluting emissions it creates. The polluting part of consumption,

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<sup>19</sup>From now on we will suppress the time subscript  $t$ . However, unless otherwise indicated, all variables are time-dependent.

i.e.,  $n_B C$ , increases the risk of adverse environmental events, which should then be reflected in market prices at the socially-optimal solution. Hence, equation (13) places the correct price on consumption when accounting for the externality. Using standard practices from environmental economics, from equation (11) we can calculate the social cost of carbon (SCC), i.e., the optimal carbon tax, as  $-V_E/V_A = -(V_{\lambda^E}/V_A) \times d\lambda^E/d\bar{\lambda}^E \times d\bar{\lambda}^E/dE$  with  $d\lambda^E/d\bar{\lambda}^E = \kappa^E$  and  $d\bar{\lambda}^E/dE = \xi\Lambda/\delta$ . The next paragraph further discusses its properties. This externality cost is zero when the representative investor is *myopic* and does not internalize the effects of his/her portfolio emissions on climate change. Equation (14) pins down the optimal level of abatement by creating indifference between abatement expenditure and paying the optimal pollution tax. Equation (15) is a no-arbitrage condition between the riskless and the risky part of the portfolio and pins down the riskfree rate of return. Equation (16) is a no-arbitrage condition between risky assets and gives the optimal portfolio allocation  $n_B$ ; after adjusting for their relative risk and depreciation, each asset should yield the same marginal return. We can easily verify that with equal drift and volatility parameters, and no excess policy risk for the brown asset, the myopic investor (no externality cost) would choose  $n_B = 1/2$  as an interior solution.

### 2.3 The value function and the Social Cost of Carbon

Apart from performing numerical simulations in the next section, here we aim at deriving closed-form solutions that provide intuition on risk premia under climate change risk, as well as, on the value of the social cost of carbon. For an analytical solution to exist we need to impose the following restrictions on the emissions intensity function  $\phi(N)$ : we assume that  $\phi(N) = \varphi N^{-1}$  with  $\varphi > 0$ . In equilibrium  $n = 1$  and total expenditure  $C + N$  is a constant fraction  $\beta$  of wealth  $A$ . With  $EIS = 1$  it holds that  $\beta = \rho$ .<sup>20</sup> Moreover let  $N = \nu C$ ,  $\nu > 0$ , such that  $C = (1/(1 + \nu))\rho A$ .<sup>21</sup> In Appendix A we show that the value function takes the form

$$V(A, \lambda^M, \lambda^E) = \frac{A^{1-\gamma}}{1-\gamma} e^{a + \sum_j b^j \lambda^j}, \quad (17)$$

with

$$a = \frac{1-\gamma}{\rho} \left( \mu_C - \frac{1}{2} \gamma \sigma^2 + \rho \log \left( \frac{\rho}{1+\nu} \right) \right) + \sum_{j \in \{M, E\}} b^j \frac{\kappa^j \bar{\lambda}^j}{\rho}, \quad (18)$$

<sup>20</sup>To see this combine (13) and (14) with (7), the guess (17), and  $\phi(N) = \varphi N^{-1}$ .

<sup>21</sup>With  $C = (1/(1 + \nu))\rho A$ ,  $N = (\nu/(1 + \nu))\rho A$ , and the guess of the value function (17), we can calculate the optimal abatement-consumption ratio  $\nu(n_B)$  from (14) as the solution to  $\nu^2 - \zeta(n_B)(1 + \nu) = 0$  with  $\zeta(n_B) = \frac{\varphi}{\rho} \frac{b^E \kappa^E}{\gamma - 1} \frac{\xi \Lambda}{\delta} n_B$ , and  $b^E$  defined in (19). From total differentiation at the optimal solution we get  $\nu'(n_B) > 0$ , i.e., a lower optimal abatement-consumption share for a cleaner portfolio (lower  $n_B$ ).

and

$$\begin{aligned} b^M &= \frac{\kappa^M + \rho}{(\sigma_\lambda^M)^2} - \sqrt{\left(\frac{\kappa^M + \rho}{(\sigma_\lambda^M)^2}\right)^2 - 2 \frac{\mathbb{E}_{z^M} [e^{(1-\gamma)Z^M} - 1]}{(\sigma_\lambda^M)^2}}, \\ b^E &= \frac{\kappa^E + \rho}{(\sigma_\lambda^E)^2} - \sqrt{\left(\frac{\kappa^E + \rho}{(\sigma_\lambda^E)^2}\right)^2 - 2 \frac{\mathbb{E}_{z^E} [e^{(1-\gamma)Z^E} - 1] + \pi[(1 + n_B(e^X - 1))^{1-\gamma} - 1]}{(\sigma_\lambda^E)^2}}, \end{aligned} \quad (19)$$

for  $\gamma > 1$ ,  $b_j > 0$ ,  $j \in \{M, E\}$ . Moreover, the fact that the quantities under the root of (19) have to be positive places a joint restriction on the severity of disasters, the risk aversion, the rate of time preference, the volatility of disasters and the heat capacity of climate sinks. After our conjectures about  $\phi(\cdot)$  and  $N$ , we have  $\bar{\lambda}^E = \bar{\lambda}^E + \xi(\Lambda/\delta)(\varphi/\nu)n_B$ . Having derived the expressions for  $a$ ,  $b^M$  and  $b^E$  of the value function, we can now calculate the return on the riskless asset and the market's risk premium for any  $n_B$  and  $\nu$ .

The SCC – that is, the marginal cost of an additional unit of emissions expressed in units of consumption, as defined previously with (17),  $C = (1/(1 + \nu))\rho A$ ,  $\kappa^E = \delta/H$  and  $b^E$  from above – reads

$$SCC = \frac{b^E}{\gamma - 1} \frac{\xi \Lambda}{H \rho} (1 + \nu) C. \quad (20)$$

As in many growth models with instantaneous logarithmic preferences (EIS=1) the optimal carbon tax is proportional to consumption (see among others Golosov et al. (2014), Bretschger and Karydas (2019)). Moreover, the SCC is increasing in the severity of expected damages from environmental disasters and the negative effect of stringent policies they trigger, through the sensitivity of the value function to climate-related risk  $b^E$  – the more so, the higher the share of polluting consumption  $n_B$ . Additionally, the SCC is higher, the higher are the sensitivity of risk of natural disasters to temperature increases  $\xi$ , the transient climate response parameter  $\Lambda$ , and the abatement-consumption share  $\nu$ , since at the optimum higher  $\nu$  implies that consumption is more polluting (see footnote 21); the SCC is lower, the higher are the intertemporal discount rate  $\rho$  and the heat capacity of climate sinks  $H$ .

## 2.4 Pricing climate change risk for the aggregate market

From (15) and (17), with  $n = 1$  in equilibrium, the riskfree rate reads:

$$\begin{aligned} r &= \underbrace{\rho + \mu_C - \gamma\sigma^2}_{\text{standard model}} + \sum_{j=\{M,E\}} \underbrace{\lambda^j \mathbb{E}_{z^j} [e^{-\gamma Z^j} (e^{Z^j} - 1)]}_{\text{macroec. \& environ. risk}} \\ &\quad + \underbrace{\pi \lambda^E [(1 + n_B(e^X - 1))^{-\gamma} n_B(e^X - 1)]}_{\text{policy risk}}, \end{aligned} \quad (21)$$

with  $b_E > 0$  from (19). In general, higher risk induces the representative agent to save, which reduces the riskfree rate; the greater the risk aversion  $\gamma$  the greater is this effect. Volatility aside, the risk-free rate is decreasing in the time-varying disaster probabilities  $\lambda^j$  and in the exposure to these disasters through  $Z^j$ . The effect of the general disaster risk on market fundamentals has been extensively studied in Wachter (2013). Here we concentrate on the effects of economic shocks related to climate change. Ceteris paribus, a higher share of brown assets  $n_B$  leads to greater aggregate emissions which increases the probability of natural disasters  $\lambda^E$  and the exposure to environmental policy, both reducing the risk-free rate.

We follow Abel (1999), Campbell (2003), Wachter (2013) and assume that the aggregate market pays a dividend  $D$ , being leveraged consumption, i.e.,  $D = C^\eta$ .<sup>22</sup> We show in Appendix B that the market's equity premium reads,

$$R - r = \eta\gamma\sigma^2 - \sum_{j \in \{M, E\}} \overbrace{\lambda^j \frac{1}{G} \frac{\partial G}{\partial \lambda^j}}^{\epsilon_j} b^j(\sigma_\lambda^j)^2 - \sum_{j \in \{M, E\}} \lambda^j \mathbb{E}_{z^j} \left[ (e^{-\gamma Z^j} - 1)(e^{\eta Z^j} - 1) \right] \quad (22)$$

$$- \pi \lambda^E \left[ (1 + n_B(e^X - 1))^{-\gamma} - 1 \right] \left[ (1 + n_B(e^X - 1))^\eta - 1 \right],$$

with  $G > 0$  defined below and  $\partial G / \partial \lambda^j < 0$ ,  $j \in \{M, E\}$ . Equity premia arise from the co-movement of marginal utility of the risk-averse investor with the price of the underlying asset or portfolio, in both normal times and times of disasters. The first two terms represent this correlated movement in times without disasters, while the third term represents the same thing in the event of an economic shock – triggered either from rare macroeconomic disasters such as wars and financial crises, or from natural disasters; the last term captures policy risk, i.e., the policy premium. The first term is the risk premium in the standard CAPM and is almost negligible in our calculations, while the second term arises from the time variation in disaster risk, and further increases the equity premium. The novelty of our model lies in establishing the link between emissions and the probability of extreme environmental disasters according to (5). We define whatever multiplies  $\lambda^E$  as the part of the equity premium due to climate change risk (physical and policy).  $G$  is the price-dividend ratio for the aggregate market which is calculated as:

$$G(\lambda_t^M, \lambda_t^E) = \int_0^\infty e^{a_\eta(\tau) + b_\eta^M(\tau)\lambda_t^M + b_\eta^E(\tau)\lambda_t^E} d\tau, \quad (23)$$

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<sup>22</sup>Consistent with observations (Longstaff and Piazzesi 2004) in the event of a negative shock dividends fall more than consumption when  $\eta > 1$  which we assume.

with  $a_\eta(\tau), b_\eta^j(\tau)$  solutions to the following system of differential equations:<sup>23</sup>

$$a'_\eta(\tau) = \mu_D - \mu_C - \rho + (1 - \eta)\gamma\sigma^2 + \sum_{j \in \{M, E\}} \kappa^j \bar{\lambda}^j b_\eta^j(\tau), \quad (24)$$

$$\begin{aligned} (b_\eta^M)'(\tau) &= \frac{1}{2}(\sigma_\lambda^M b_\eta^M(\tau))^2 + (b^M(\sigma_\lambda^M)^2 - \kappa^M)b_\eta^M(\tau) + \mathbb{E}_{z^M} \left[ e^{(\eta-\gamma)Z^M} - e^{(1-\gamma)Z^M} \right], \\ (b_\eta^E)'(\tau) &= \frac{1}{2}(\sigma_\lambda^E b_\eta^E(\tau))^2 + (b^E(\sigma_\lambda^E)^2 - \kappa^E)b_\eta^E(\tau) + \mathbb{E}_{z^E} \left[ e^{(\eta-\gamma)Z^E} - e^{(1-\gamma)Z^E} \right] \\ &\quad + \pi \left[ (1 + n_B(e^X - 1))^{\eta-\gamma} - (1 + n_B(e^X - 1))^{1-\gamma} \right], \end{aligned} \quad (25)$$

and  $\mu_D$  as defined in Appendix B. Since for  $\tau = 0$  an asset should pay its current dividend, boundary conditions are  $a_\eta(0) = b_\eta^M(0) = b_\eta^E(0) = 0$ . For  $\eta > 1$ , both  $a_\eta(\tau)$  and  $b_\eta^j(\tau)$  are well defined functions of  $\tau$  such that the infinite integral  $G$  converges. The solution to (25) with the previous boundary conditions yields  $b_\eta^j(\tau) < 0$ ,  $j \in \{M, E\}$ . According to (23), this implies that, ceteris paribus, draws of high disaster risk – macroeconomic and/or environmental – reduce valuations.

We established in footnote 15 that a changing environment makes climate related disasters more frequent and less predictable; an increase in carbon emissions shifts the distribution of the probability of extreme environmental events to higher draws while it flattens it at the same time (see also Figure 6). Using equation (22) we can describe how the aggregate equity market could price these changing conditions. In line with standard results in the extreme events literature, higher draws of  $\lambda^E$  unambiguously raise the equity premium through the last two term, the “static” disaster risk terms. However, the second term, due to the time-variation of risk, deserves a closer look.

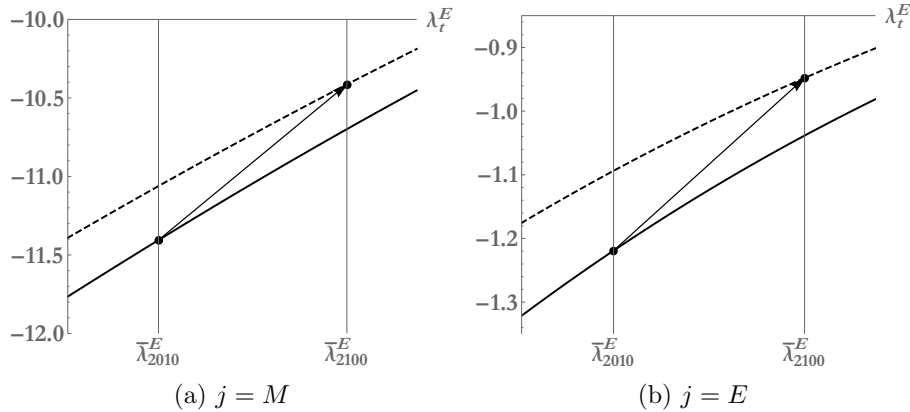
Of importance is the term  $\epsilon_j \in [-1, 0]$ , defined in (22). Loosely speaking, this term represents the risk “elasticity of valuations”, i.e., the variation in the price-dividend ratio in response to variations in macroeconomic and/or climate risk. This term can be further decomposed in two parts: the “semi-elasticity of valuations”,  $\Delta \log G / \Delta \lambda^j$ , measuring the percentage change in  $G$  from a unit increase in  $\lambda^j$ ; and the risk  $\lambda^j$  itself. On the one hand, from (23) and (24), with rising emissions the increasing expected risk of environmental events ( $\bar{\lambda}^E$ ) puts a downward pressure on equity valuations, which leaves less room for prices to react to high draws of risk of either type. This level effect reduces the magnitude of the semi-elasticity of valuations for both types of disasters. On the other hand, as climate changes, the distribution of climate risk shifts to higher draws, while the one of macroeconomic risk stays unaltered, which increases the magnitude of the elasticity of

<sup>23</sup>The proof follows Wachter (2013) and is available upon request.



valuations for climate risk through its second part, i.e. the risk itself, and the relative importance of this type of risk even in times without disasters. The equity premium of climate change is unambiguously increasing. Since, however, the risk of rare macroeconomic events makes up the largest part of the equity premium, the magnitude of the aggregate equity risk premium is only minimally affected, while – depending on calibration – it might also decline.<sup>24</sup> This result suggests that, as climate changes the risk of rare macroeconomic disasters works as a hedging strategy against the risk of climate change and should therefore be rewarded with a lower premium. The numerical results obtained in the next session confirm these effects. More specifically, the relationships between the elasticity of valuations, the aggregate equity premium, and the climate risk premium as functions of the probability of extreme environmental events are shown in Figures 2-4.<sup>25</sup>

Figure 2: “Semi-elasticity” of valuations  $\frac{1}{G} \frac{\partial G}{\partial \lambda^j}$  as climate changes. Assuming an increasing emissions path, the figures show this term for both types of risk  $j \in \{M, E\}$ , as a function of the probability of extreme environmental events  $\lambda_t^E$ . The solid line shows this term for  $\bar{\lambda}_{2010}^E$ ; the dashed line for  $\bar{\lambda}_{2100}^E$ ; the arrow shows the transition. The probability of extreme macroeconomic events  $\lambda_t^M$  is set at its equilibrium value  $\bar{\lambda}^M$ .



<sup>24</sup>This result has obviously to do with our calibration and suggests that climate change would increase the aggregate equity premium in either two cases: first, if events are more severe than what our calibration assumes; second, by setting  $\sigma_\lambda^M = 0$  or completely abstracting from macroeconomic events as usually done in the literature.

<sup>25</sup>Using stock market data Bansal et al. (2016) estimate empirically the percentage change of the P-D ratio to a unit increase in temperature, i.e.  $\Delta \log G / \Delta T$ , over one and five years, to be  $-5.5\%$  and  $-8.6\%$ , respectively. Our measure of Figure 2(b) reports  $\Delta \log G / \Delta \lambda^E$ . The linear relation between  $\lambda^E$  and  $T$  in our model implies that  $\Delta \lambda^E = \xi \Delta T$ , such that the equivalent measure becomes  $\xi \times \Delta \log G / \Delta \lambda^E$ . In the calibration part 3.2 we estimate  $\xi = 0.06$ , which yields  $\Delta \log G / \Delta T = -7.3\%$  for 2010, i.e. within the aforementioned estimates.

Figure 3: **“Elasticity of valuations”**  $\epsilon_j = \lambda^j \frac{1}{G} \frac{\partial G}{\partial \lambda^j}$  **as climate changes.** Assuming an increasing emissions path, the figures show this term for both types of risk  $j \in \{M, E\}$ , as a function of the probability of extreme environmental events  $\lambda_t^E$ . The solid line shows this term for  $\bar{\lambda}_{2010}^E$ ; the dashed line for  $\bar{\lambda}_{2100}^E$ ; the arrow shows the transition. The probability of extreme macroeconomic events  $\lambda_t^M$  is set at its equilibrium value  $\bar{\lambda}^M$ .

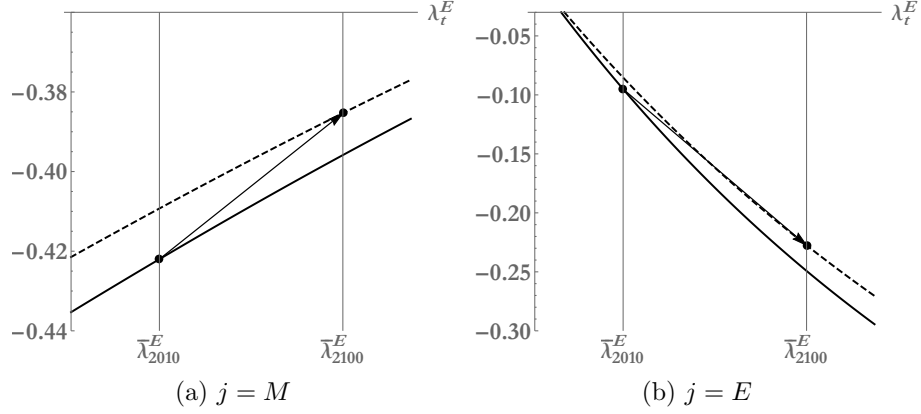
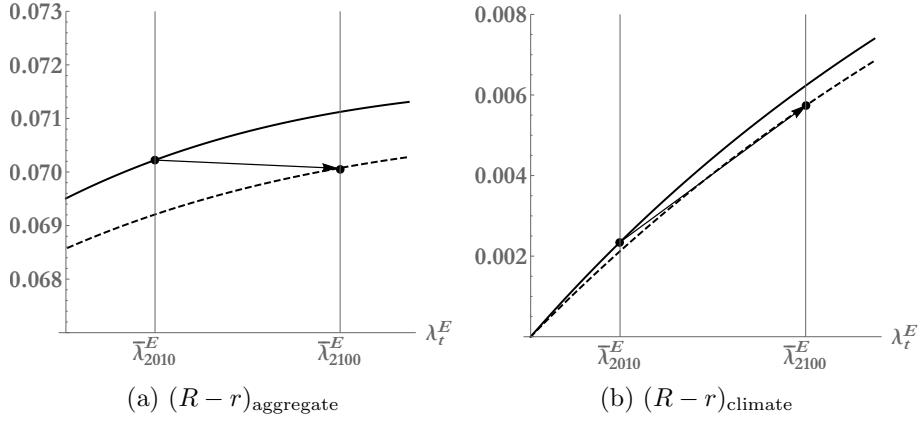


Figure 4: **The equity premium as climate changes.** Assuming an increasing emissions path, the figures show the aggregate equity premium and its part related to climate change as a function of the probability of extreme environmental events  $\lambda_t^E$ . The solid lines are for  $\bar{\lambda}_{2010}^E$ ; the dashed lines for  $\bar{\lambda}_{2100}^E$ ; the arrows show the transition. The probability of extreme macroeconomic events  $\lambda_t^M$  is set at its equilibrium value  $\bar{\lambda}^M$ .

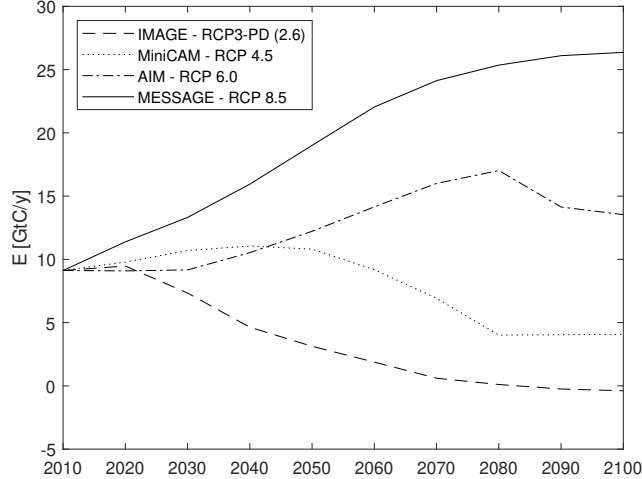


### 3 Numerical part

#### 3.1 Methodology

The current framework can be used for two types of modelling depending on investors information about climate change. In the first case, investors take emissions as given and not affected by their portfolio choices. Exogenous carbon emission / temperature paths influence the mean arrival rate of environmental shocks  $\bar{\lambda}^E$ , while the participation of dirty assets in the portfolio  $n_B$  depends only on the additional policy risk these assets are exposed to (see equation (16)); the Pigouvian tax is set to zero throughout (13)-(16) and  $\nu = N/C$  is calibrated such that model's emissions in the benchmark (year 2010) match the data, i.e.,  $E_{\text{model}} = (\varphi/\nu)n_B = E_{\text{data}}$ , from (3). For the different emission paths we consider the Representative Concentration Pathways (RCPs) produced by the IPCC for years 2010 to 2100 (van Vuuren et al. 2011); see Figure 5. We call this the *myopic* solution. As climate changes, this solution allows us to study the possible evolution of the different market measures and the portfolio participation of brown assets. In the second case, investors recognize that their portfolio choices will affect future emissions and, therefore, the probability of climate-related events. Accordingly, the Pigouvian tax in equations (13)-(16) is in place,  $\nu$  is calculated from (14), the portfolio allocation is fully optimized in each period, and the arrival rate of extreme environmental events becomes endogenous solving (5), with  $\lambda_0^E > 0$  from the data. We call this solution *optimal*.

Figure 5: IPCC projected carbon emissions



The usual methodology in asset pricing models assumes time-invariability of the system under study. This allows for a straightforward Monte-Carlo simulation using a large number of random realizations of the relevant stochastic variables. In the myopic case, however, the mean of the process for  $\lambda^E$  changes exogenously with emissions, such that our model is not time-invariant; see equation (5). We circumvent this issue by assuming that our model is time-invariant for each given level of emissions  $E_t$  and simulate the model for each  $\bar{\lambda}_t^E = \bar{\lambda}^E(E_t)$ . Figure 6 presents our methodology for the myopic case. We divide our actual time horizon (2010-2100) in decades, and run the model for 100,000 simulation years for each decade. Figure 6 presents also the resulting stationary Gamma distribution for  $\lambda^E$  in each time period, which changes as expected; see footnote 15.

In the optimal solution, our target is to identify a policy function linking our control variable  $n_B$  to the states of the economy, i.e.,  $A, \lambda^M$ , and  $\lambda^E$ . According to equation (16) and our conjectures regarding functional forms,  $n_B$  only depends on  $\lambda^E$  and on fixed parameters. For that we start with the observed  $\lambda_0^E > 0$ , solve equation (16) for  $n_B$ , and then iterate equations (5) and (16) for given simulations years. Since now (5) is time-invariant it allows for a straightforward simulation. Carbon emissions are not exogenous anymore, but depend on simulated  $n_B(\lambda^E)$ , from  $E_t = (\varphi/\nu(n_B))n_{Bt}$ . We run the optimal solution for 100,000 simulation years.<sup>26</sup>

## 3.2 Calibration

We measure time in years and calibrate the model to match observed historical market and climate data. Our initial time period is 2010. Carbon emissions are measured in GtC (1 gigaton C =  $10^9$  ton C) and temperature differences in K (Kelvin); also 1 PW =  $10^{15}$  Watts. Our benchmark is the case without policy risk, i.e.  $X = 0$  and no Pigouvian tax. Below we demonstrate our benchmark calibration; Table 1 collects the chosen parameter values.

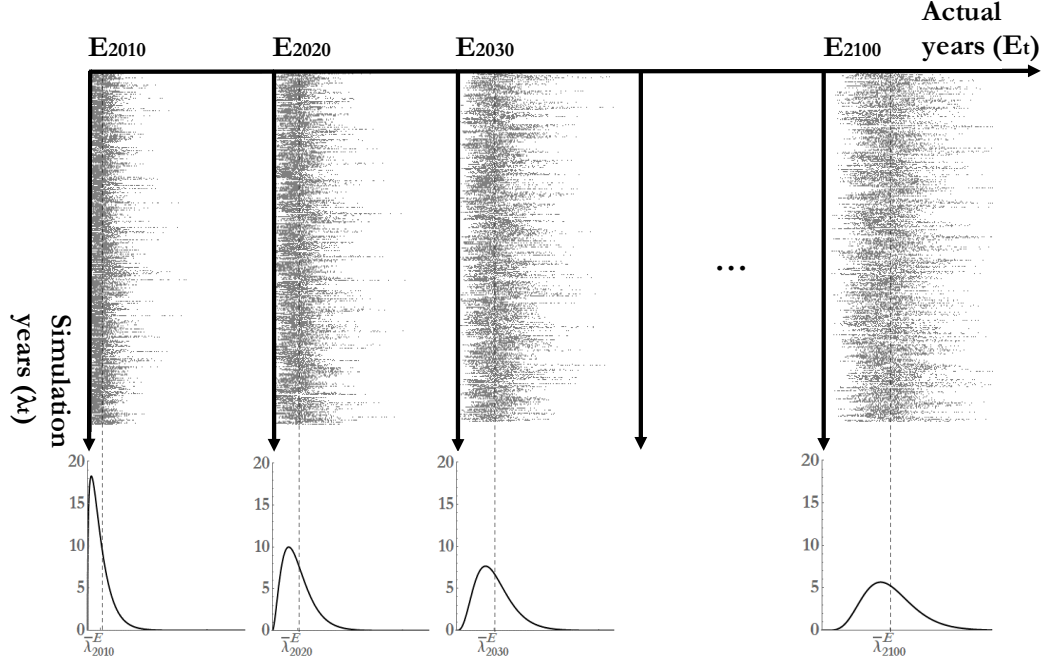
### 3.2.1 Environmental parameters

We follow Brock and Xepapadeas (2017) and set the heat capacity of climate sinks to  $H = 4.58$  PW y/K. The positive feedback of carbon emissions on increases in temperature anomaly is measured by the transient climate response with a mean value  $\bar{\Lambda} = 0.0017$  K/GtC; see also Leduc et al. (2016). Then, the relevant climate sensitivity parameter

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<sup>26</sup>Note that we do not allow for selling-short or buying-long in this model, since for example a negative  $n_B$  would result in negative emissions. Accordingly, we set emissions to a very small number for  $n_B < 0$  and to their maximum value, which occurs for  $n_B = 1$ , for  $n_B > 1$ ; the same holds for  $\nu(n_B)$ . In this way we create boundaries such that the optimal solution of  $n_B(\lambda^E)$  lies within the  $[0, 1]$  range.

Figure 6: **Schematic of the simulation methodology for the *myopic* case.** For each actual time period, we run the model for 100,000 simulation years;  $\bar{\lambda}^E$  changes with carbon emissions  $E_t$ . The figure assumes an increasing path of emissions.



in our model is  $\Lambda \equiv \bar{\Lambda}H = 0.00779$  PW y/GtC. The stabilizing parameter of outgoing radiation is set to  $\delta = 0.05$  PW/K such that the temperature anomaly for the worst RCP emissions scenario reaches an equilibrium with about 4 K in 2100, thus matching the latest IPCC projections.

### 3.2.2 Distributions of macroeconomic and environmental disasters

As shown in equation (9), the percentage decline in total wealth per capita features both environmental and macroeconomic shocks that need to be calibrated to the data. Hence, we need to construct a separate dataset for each of the two types of shocks; from these datasets we can then calculate the distribution of percentage drops  $J^j$ , as well as the average of the Poisson intensities  $\bar{\lambda}^j$ ,  $j \in \{M, E\}$ .

Table 1: **Parameters for the benchmark calibration**

All values are in annual terms

Environmental parameters	
Outgoing radiation parameter $\delta$ (PW/K)	0.05
Heat capacity of climate sinks $H$ (PW y/K)	4.58
Climate sensitivity $\Lambda$ (PW y/GtC)	0.00779
Emissions intensity $\varphi$	3.753
Parameters for the stochastic processes	
Average probability of macroeconomic disasters $\bar{\lambda}^M$	0.0369
Slope of the linear $\bar{\lambda}^E(T)$ curve $\xi$	0.06
Speed of mean reversion for macroeconomic risk $\kappa^M$	0.080
Speed of mean reversion for environmental risk $\kappa^E$	0.011
Volatility parameter for macroeconomic disasters $\sigma_\lambda^M$	0.0750
Volatility parameter for environmental disasters $\sigma_\lambda^E$	0.0839
Drift parameter for both assets $\mu_G = \mu_C = \mu_C$	0.0252
Volatility parameter for the general asset $\sigma_G$	0.0204
Volatility parameter for the brown asset $\sigma_B$	0.0221
Correlation coefficient $\text{corr}(dW_B, dW_G)$	0.8
Leverage parameter $\eta$	2.6
Probability of policy reaction to extreme climatic events $\pi$	0.6
Utility parameters	
Relative risk aversion $\gamma$	3.3
Intertemporal discount rate $\rho$	0.012

To do so we make use of different data sources as follows. As a first source we extend until 2015 the Barro-Ursúa dataset, Barro and Ursúa (2010), that collects consistent data on GDP per capita growth for 42 countries for the period 1911-2008.<sup>27</sup> For our purposes this dataset holds the real reported GDP per capita growth, i.e., after accounting of any (negative) growth effects of climate-related events. In order to calculate these growth effects of climate change we act in the following way. We first collect from the international disasters database EM-DAT (2018), all climate-related events for these 42 countries and for the 1911-2015 time period; we consider only events relevant to climate change.<sup>28</sup> We then follow the methodology of Loayza et al. (2012) and calculate the negative growth effects on GDP per capita of extreme environmental events (top 10% in each event category according to a severity index defined in that paper) for each country and each year;

<sup>27</sup>We use percentage declines in GDP per capita, instead of consumption per capita, as a proxy for damages. Both Barro (2009) and Wachter (2013) find similar results for their CAPMs with rare disasters whether they calibrate to the consumption or GDP data. Here we follow the latter.

<sup>28</sup>According to the EM-DAT categories we consider meteorological events (storms/extreme temperatures), hydrological events (floods/avalanches), and climatological events (droughts/wildfires).

from these we keep extreme events that resulted in GDP growth damages of more than 1%.<sup>29</sup> This is our first dataset including data on environmental damages. To calculate pure macroeconomic damages we add the – absolute value of – environmental damages to growth entries of the extended Barro-Ursúa dataset. This yields the real GDP per capita growth if no extreme climate-related events had occurred. To construct our second dataset containing pure macroeconomic damages we then follow the peak-to-trough methodology for cumulative fractional declines in real GDP per capita as explained in Barro and Ursúa (2008). As in the aforementioned contribution, and in Wachter (2013), we include only peak-to-trough events that resulted in GDP drops more than 10%.<sup>30</sup>

Following the methodology of Barro and Ursúa (2008) the frequency of large declines in GDP per capita in our pure macroeconomic dataset yields  $\bar{\lambda}^M = 0.0369$ . In order to construct the linear relationship  $\bar{\lambda}_t^E = \tilde{\lambda}^E + \xi(\Lambda/\delta)E_t$  for the time-varying mean of the stochastic process in (5), we divide our sample in six decades starting from the decade 1956-1965 (when the first indications of climate change became evident) and calculate  $\bar{\lambda}^E$  for each decade. The slope of the fitted line, with  $\Lambda$  and  $\delta$  from above, gives  $\xi = 0.06$ . Since the fitted linear  $\bar{\lambda}^E$ -curve crosses the x-axis for  $E > 0$ , we use instead the line defined by  $\bar{\lambda}_t^E = \max\{\bar{\lambda}_{min}^E, \tilde{\lambda}^E + \xi(\Lambda/\delta)E_t\}$ , with  $\bar{\lambda}_{min}^E = 0.0036$ , the average probability of environmental events for years 1916 to 1955, i.e., the probability that an extreme climatic event would occur irrespective of climate change, and  $\tilde{\lambda}^E = -0.00725$ . With  $E_{2010} = 9.12$  GtC we can calculate  $\bar{\lambda}_{2010}^E = 0.078$ . The frequency distributions of growth damages have a mean drop size of 22.1% for macroeconomic events and 1.58% for environmental ones.<sup>31</sup>

### 3.2.3 Other parameters

We define assets with exposure to transition risk as “brown” and follow Prudential Regulation Authority (2015) to set  $n_B = 0.3$  for 2010 in the benchmark; see footnote 7.<sup>32</sup> From

<sup>29</sup>Loayza et al. (2012) show that extreme climate-related events (top 10%) are always bad for economic growth and calculate the growth elasticities of different event types on different economic sectors: manufacturing; services; agriculture. Using World Bank data we calculate the sectoral shares of GDP for each country and then using the growth elasticities the country-specific climate-related damages in terms of GDP per capita growth for each year.

<sup>30</sup>Using the peak-to-trough methodology for macroeconomic, and not for environmental events, we implicitly make the simplifying assumption that macroeconomic events, such as wars or crises, have memory, while climate-related events are memory-less.

<sup>31</sup>The value of the mean drop size of macroeconomic events is in line with Barro and Ursúa (2008) who calculate a value of 21% for GDP disasters using a dataset of 36 countries in the time period 1870-2006; with their dataset they also calculate their  $\lambda^M = 0.0369$ . Our results in terms of drop values differ slightly due to the disentangling of GDP damages in macroeconomic and environmental.

<sup>32</sup>In addition, Oestreich and Tsiakas (2015) investigate empirically the effect of EU-ETS on German stock returns in the period 2003-2012. They divide their sample of 65 firms in clean and dirty depending on whether they received free carbon allowances or not; dirty firms occupy about 35% of that sample.

footnote 21 we get  $\nu_{2010} = \nu(n_{B,2010}, \varphi)$ . Using  $E_{2010} = \varphi n_{B,2010} / \nu_{2010}$  we get  $\varphi = 3.753$  to match  $E_{2010} = 9.12GtC$ . In order to calculate the probability  $\pi$  that climate change policy becomes effective after an extreme climate-related event, we use the Grantham-LSE (2018) database that includes all laws and legislations since the 1960s related to climate change, covering 95% of global emissions. In this database there are in total 519 laws for our 42 countries, a quarter of which refers to low carbon transition laws (Nachmany et al. 2017); with 213 severe events in our dataset we calculate  $\pi = 0.25 \times 519 / 213 \approx 0.6$ .

We set the coefficient of relative risk aversion to  $\gamma = 3.3$ .<sup>33</sup> We also follow Wachter (2013) and set the intertemporal discount rate is set to  $\rho = 0.012$ , the leverage parameter  $\eta = 2.6$ , the mean reversion parameter of the intensity of macroeconomic disasters  $\kappa^M = 0.08$  (the speed of adjustment for environmental disasters can be calculated as  $\kappa^E = \delta / H = 0.011$  per annum), the aggregate drift and the volatility parameters of the aggregate market to  $\mu_C = 0.0252$  and  $\sigma = 0.02$  per annum, respectively; see equations (10) and (12). In order for both risky assets to grow at the same rate in times without shocks, we further assume that  $\mu_B = \mu_G = \mu_C$ , while we set  $\sigma_B = 0.0221$ ,  $\sigma_G = 0.0204$ , and  $\text{corr}(dW_B, dW_G) = 0.8$ , which yield in the benchmark  $n_B = 0.3$  (equation (16)) and  $\sigma = 0.02$  (equation (12)). Finally, we need to calibrate the volatility parameters  $\sigma_\lambda^M$  and  $\sigma_\lambda^E$  for processes (2) and (5), respectively. Volatility parameters can be calculated by choosing the discriminant of (19) for both types disasters to be zero (as in Seo and Wachter (2018)). With  $\kappa^E$  from above this yields  $\sigma_\lambda^M = 0.0750$ ,  $\sigma_\lambda^E = 0.0839$  for  $X = 0$ .

### 3.3 Simulation results - myopic solution

#### 3.3.1 Benchmark simulation

As discussed above we calibrated the model to match historical data. Our benchmark calibration involves only physical risk i.e.  $X = 0$ , while emission parameters are calibrated to 2010 data. Table 2 presents the results of our benchmark simulation in contrast to historical post-WWII US data from Wachter (2013). Our model and its calibration matches observed moments of interest very well: the return on government bill matches exactly the 1.34% p.a. in the data, the equity premium generated is 7.01% p.a. in comparison to 7.05% p.a. observed in the data, while simulated equity volatility is 19.3% p.a., in comparison to observed 17.8% p.a. Next we discuss the effects of climate change on model's moments.

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<sup>33</sup>Barro (2009) sets  $\gamma = 4$ , while Wachter (2013)  $\gamma = 3$ .



Table 2: **Moments from simulated vs. historical data.**  $R^b$  is the return on government debt,  $R^e$  denotes the gross return on equity,  $AR1[P - D]$  is the first order autocorrelation of the price-dividend ratio, and SR measures the Sharpe ratio.

All values are in annual terms		
Moments	Simulation	US Data (1947-2010)
$\mathbb{E}[R^b]$	0.0134	0.0134
$\sigma(R^b)$	0.0209	0.0266
$\mathbb{E}[R^e - R^b]$	0.0701	0.0705
$\sigma(R^e)$	0.193	0.178
$AR1[P - D]$	0.932	0.920
SR	0.363	0.397

### 3.3.2 The physical risk of climate change

In this part we explore the pure effects of climate change risk on market fundamentals, with a focus on the equity premium. We simulate the model as described in Figure 6. Figure 7 presents the effect of the two extreme RCP scenarios (RCP3 and RCP8.5) on the risk premium. The risk premium on the aggregate market does not change much with emissions.

Is this because the effect of climate change is negligible in comparison to the effect of severe pure macroeconomic disasters? From (22) we can get the part of the equity premium that is solely due to climate change risk. As Figure 8 shows, in the initial period, the risk premium of climate change amounted to about 0.2% p.a.; the remaining is mainly due to the risk of rare macroeconomic disasters, and only a very small part is due to the standard CAPM's diffusion risk. With our calibration, this premium increases to 0.5% p.a. by the end of the century in the worst case scenario (RCP8.5), while it naturally ceases to exist in the RCP3 scenario where emissions fall to zero in the long run. Since climate change entails a positive premium, which is also increasing with emissions, what is the reason behind the generated constant equity premium on the aggregate market?

According to our discussion in section 2.4, from (23) and (24), higher emissions – which increase  $\bar{\lambda}^E$  – affect the way in which valuations react to the different kinds of risk. With our calibration, higher emissions reduce the magnitude of the risk elasticity of valuations for macroeconomic disasters  $\epsilon^M$  in (22), while they increase the one for environmental  $\epsilon^E$ . Therefore, increasing emissions, change the relative importance of the two sources of risk in normal times. However, since the premium due to macroeconomic disasters is greater, this holds the overall equity premium constant; see Figure (9). Finally, as expected from our theory, higher emissions unambiguously decrease the return on government debt; see Figure 10.

Figure 7: The equity premium over the gross return on government debt (annual terms)

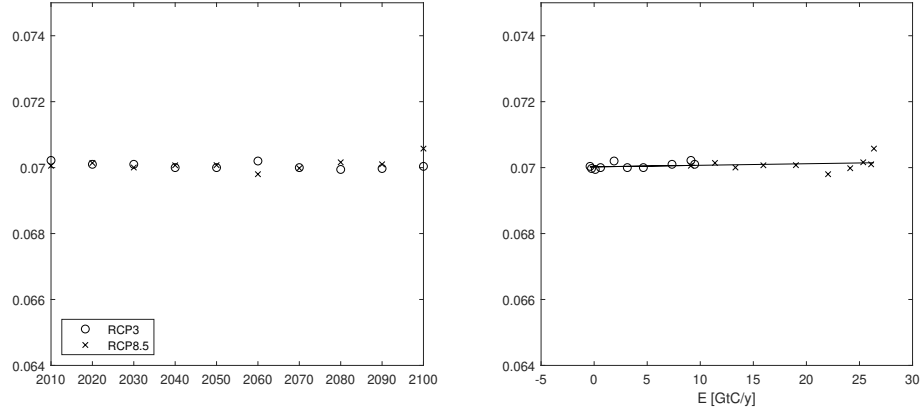


Figure 8: The premium of climate change risk (annual terms)

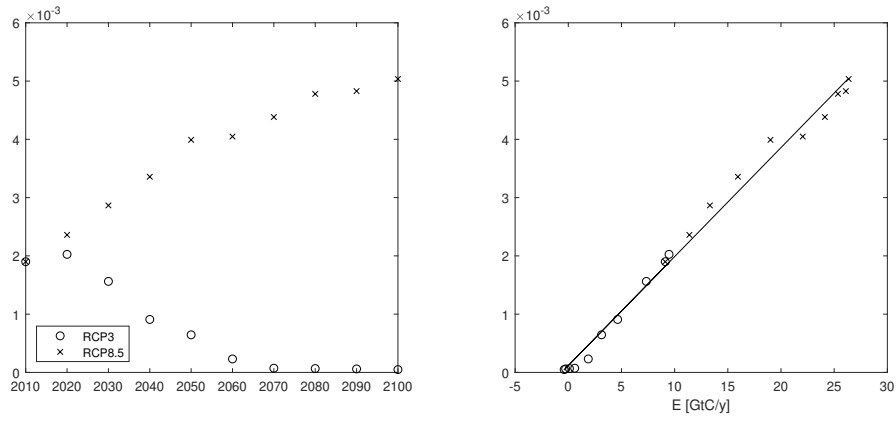


Figure 9: Elasticities of valuation for macroeconomic and environmental risks,  $\epsilon^j = \frac{\partial G}{\partial \lambda^j} \frac{\lambda^j}{G}$

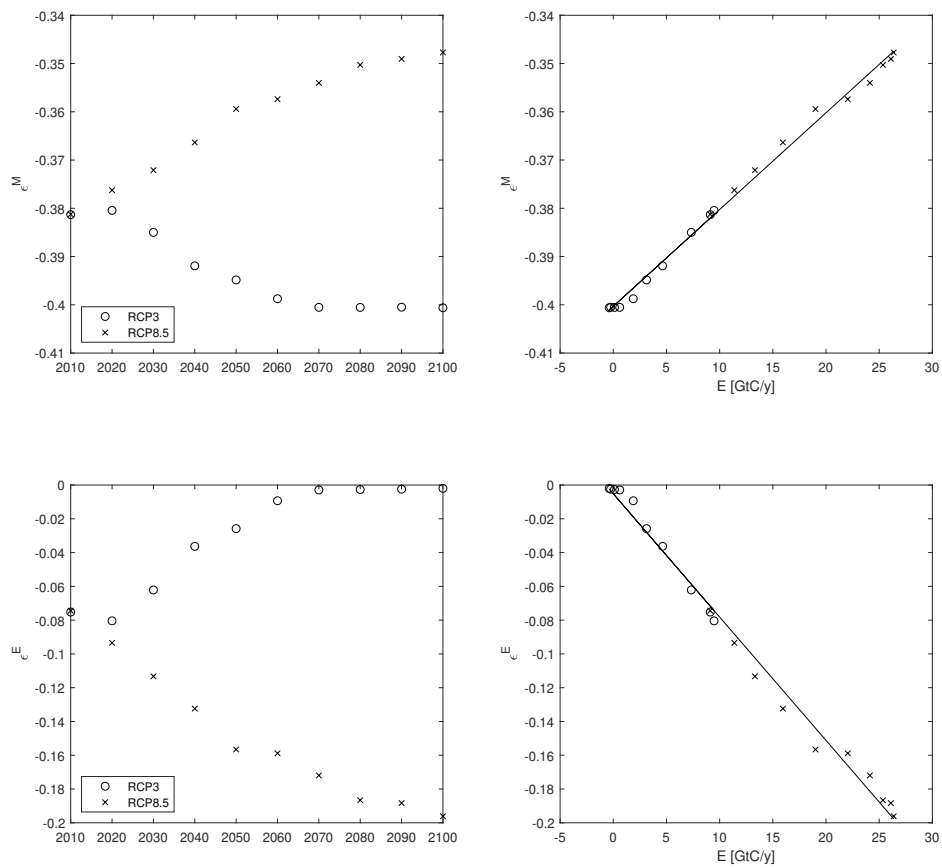
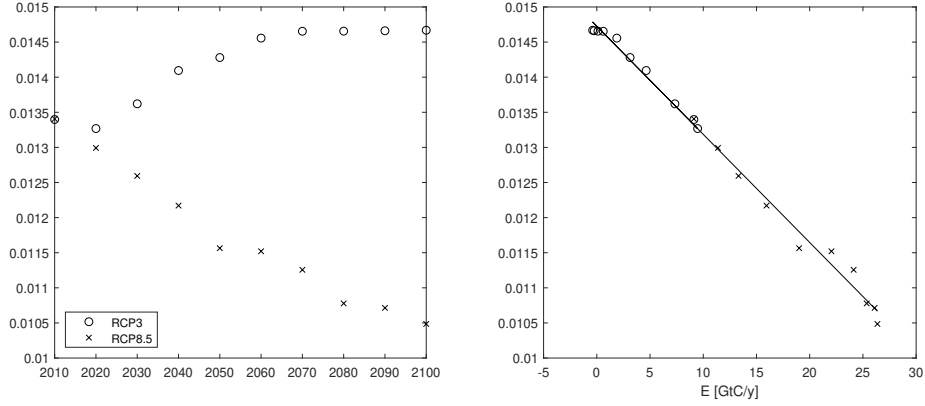


Figure 10: The effect of emissions on government bond yield (annual terms)



### 3.3.3 Policy risk and portfolio participation of brown assets

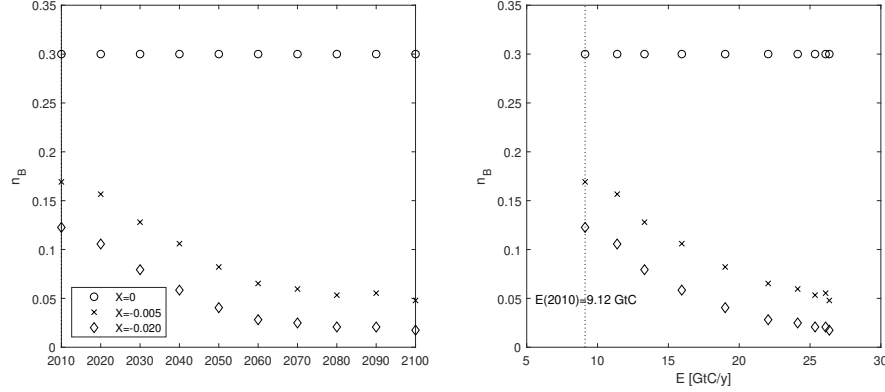
Our benchmark calibration assumes that there is no additional policy risk on brown assets, the share of which we calibrate to  $n_B = 0.3$ . In this paragraph we relax this by assuming the existence of abnormal returns following the announcement of green policies.<sup>34</sup> Ramiah et al. (2013) study the existence of such returns in Australia and document negative mean abnormal returns on the order of  $-2.8\%$  across 10 industries, including mining, oil, gas and real estate. In our model we measure abnormal returns as the mean difference in actual returns on equity at the time the policy strikes for a carbon-intensive portfolio ( $n_B = 1$ ) whose expected return is evaluated neglecting policy risk.

With the above information we calibrate  $X = -0.005$  that leads to abnormal returns of  $-2.5\%$ . In our myopic simulation, with policy risk, as the probability of extreme environmental events changes with emissions, investors optimally reallocate their portfolio by choosing  $n_B$  in each time period according to (16). Figure 11 presents the simulated portfolio participation of brown assets for worst RCP scenario for  $X = 0$ ,  $X = -0.005$  and for  $X = -0.02$ , the latter leading to abnormal returns of  $-6\%$ . This graph holds two important pieces of information. First, according to our calibration, including policy risk leads to excess portfolio participation in the benchmark period: for  $X = -0.005$  brown assets should not occupy more than 15% of the portfolio in 2020; this falls down to 10% for  $X = -0.02$ . Second, when including policy risk, there is a clear negative relationship

<sup>34</sup>For specific stocks or portfolios, abnormal returns measure the performance difference on given dates or time periods from expected returns that are calculated by an asset-pricing model.

between  $n_B$  and emissions.<sup>35</sup>

Figure 11: **Portfolio participation of brown assets for the worst IPCC emissions scenario.** The graph shows the portfolio allocation on brown assets when policy risk is taken into account, for different values of the policy parameter  $X$ .



### 3.4 Simulation results - optimal solution

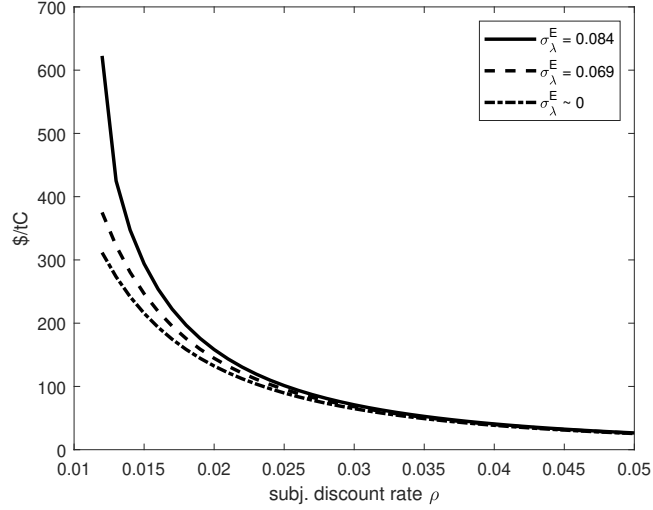
Optimal carbon taxation – as measured by the social cost of carbon – is highly influenced by the intertemporal discount rate  $\rho$ .<sup>36</sup> Figure 12 shows the optimal tax rates for different values of  $\rho$  and for different levels of stochasticity of our state variable  $\lambda^E$ , for our benchmark  $n_B = 0.3$ . From equation (20), and with our calibration of  $\rho = 0.012$  and  $\sigma_\lambda^E = 0.0839$ , the social cost of carbon amounts to about \$600 per metric ton in 2010.<sup>37</sup> Although high, this number lies close to the estimates from stochastic climate-economic models. For example the optimal carbon tax in Golosov et al. (2014) for  $\rho = 0.015$  can go up to \$500/tC in 2010. Additionally, when including a stochastic natural environment Cai et al. (2019) find, with the same value for the subjective discount rate, optimal carbon taxes ranging between \$200 – 400/tC. For  $\rho = 0.015$  equation (20) implies an optimal carbon tax of about \$300/tC, i.e. in the range of the above contributions; abstracting from uncertainty in the state variable ( $\sigma_\lambda^E \approx 0$ ), our optimal carbon tax is about \$200/tC for  $\rho = 0.015$ .

<sup>35</sup>See also van der Ploeg and Rezai (2019) on the asset stranding due to climate policy risks.

<sup>36</sup>See Golosov et al. (2014) p.70 for a discussion on the role of the intertemporal discount rate on shaping climate policy.

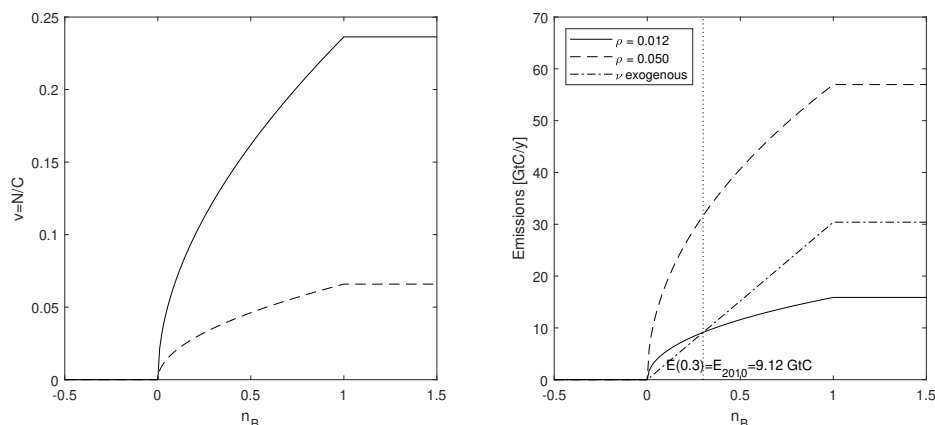
<sup>37</sup>We used the estimate of  $C_{2010} = tr.\$48$  from the World Bank for global consumption.

Figure 12: **The Social Cost of Carbon** Optimal tax rates in current dollars per ton of emitted carbon versus yearly subjective discount rate



In the optimal solution the representative investor acknowledges the fact that higher portfolio emissions have a negative feedback on the economy, raising the risk of severe climate-related disasters and the subsequent policy they trigger, i.e., equations (13)-(16) now include the externality cost of portfolio emissions. The last term in equation (16) expresses the emissions externality damage – in terms of growth – triggered by a marginal increase of brown assets in the optimal portfolio. For our calibration, the optimal solution to (16) yields  $n_B = 0$ , i.e., including the full cost of climate change does not allow for any carbon-intensive assets in the optimal portfolio; see also footnotes 21 and 26. Figure 13 shows the optimal abatement expenditure as a fraction of consumption ( $\nu$ ) and the triggered portfolio emissions for different values of  $n_B$  and  $\rho$ , as well as the resulting portfolio emissions when  $\nu$  is held constant at its myopic calibration ( $\nu = 0.123$ ). For a given share of brown assets  $n_B$ , a higher discount rate leads to a lower abatement-capital ratio  $\nu$  and higher portfolio emissions.

Figure 13: **Optimal abatement expenditure and emissions as functions of  $n_B$ .** The plots show the optimal abatement expenditure as a fraction of consumption (left) and the triggered portfolio emissions (right), both as functions of  $n_B$  and for different values of the subjective discount rate  $\rho$ . The right figure shows also the portfolio emissions when  $\nu$  is exogenous and calibrated as in the myopic solution.



## 4 Conclusion

There are concerns from market participants that the risk of climate change is not yet perfectly priced by capital markets. In order to price climate change risk we develop an asset pricing model with rare events and time-varying probabilities. Such models are shown to match observed equity premia for low values of relative risk aversion. In addition to the – already considered – risk of macroeconomic disasters such as wars and financial crises, we include the risk of extreme adverse environmental events related to climate change. Our main contribution lies in establishing the link between carbon emissions and the stochastically-varying risk of rare climatic events. Besides the physical risk, which affects the whole market, we include the transition risk of climate change, i.e., the risk of exposure to stringent environmental policies that lower the returns of carbon-intensive assets.

We confirm the result in the literature that climate change entails a positive and increasing risk premium. We show, however, that whether this ultimately carries over to the overall equity premium depends on the time variation of risk and the severity of environmental events. With rising emissions the increasing risk of environmental events puts a downward pressure on equity valuations, which leaves less room for prices to react to the risk of extreme events of either type, macroeconomic or environmental. As climate

changes, the distribution of climate risk shifts to higher draws, while that of macroeconomic risk stays unaltered, which increases the relative importance of climate change risk even in times without disasters. Since, however, the risk of rare macroeconomic events makes up the largest part of the equity premium, the magnitude of the aggregate equity risk premium is only minimally affected. We consider different emission projections and find that including the transition risk of environmental policies substantially lowers the participation of carbon-intensive assets in the market portfolio. In the planner's solution, which includes the Pigouvian tax of the environmental externality, there is no room for any carbon-intensive assets in the optimal portfolio.

## Appendix A - Deriving the value function

We substitute our conjecture (17) and  $C = (1/(1 + \nu))\rho A$  from the main text into (7) to get

$$f(A, \lambda^M, \lambda^E) = \rho A^{1-\gamma} I(\lambda^M, \lambda^E) \left( \log \left( \frac{\rho}{1 + \nu} \right) - \frac{\log I(\lambda^M, \lambda^E)}{1 - \gamma} \right), \quad (26)$$

with  $I(\lambda^M, \lambda^E) = e^{a + \sum_j b^j \lambda^j}$ . Substitute the above along with  $n = 1$  and  $C + N = \beta A$  in equilibrium in the optimized HJB equation (11) to get

$$\begin{aligned} (1 - \gamma)\rho \log \left( \frac{\rho}{1 + \nu} \right) - \rho \left( a + \sum_j b^j \lambda^j \right) + (1 - \gamma)(\mu_C - \frac{\gamma}{2}\sigma^2) \\ + \sum_j \kappa^j (\bar{\lambda}^j - \lambda^j) b^j + \frac{1}{2}(\sigma_\lambda^j)^2 (b^j)^2 \lambda^j + \lambda^j \mathbb{E}_{z^j} [e^{(1-\gamma)Z^j} - 1] \\ + \pi \lambda^E [(1 + n_B(e^X - 1))^{1-\gamma} - 1] = 0. \end{aligned} \quad (27)$$

Collecting terms in  $\lambda^j$  implies a quadratic equation for each  $b^j$  giving (19) in the main text; the solution with the negative sign in front of the square root is the one with reasonable economic properties (Wachter 2013). Collecting constant terms gives equation (18).

## Appendix B - Pricing climate change risk

Let  $m$  denote the state-price density, loosely speaking the marginal utility of the risk-averse investor. Any asset with a dividend stream  $D$  can be priced according to the usual asset pricing equation  $P_t = \mathbb{E}_t \left[ \int_t^\infty \frac{m_s}{m_t} D_s ds \right]$ . Duffie and Skiadas (1994) show that the state-price density for preferences as given by (6) and (7) in continuous time is given by  $m_t =$



$\exp \left[ \int_0^t f_U(C_s, U_s) ds \right] f_C(C_t, U_t)$ . With the value function (17) and  $C = (1/(1+\nu))\rho A$ , the Poisson jump of  $m$  reads  $\tilde{m}/m = \tilde{f}_C/f_C = (\tilde{C}/C)^{-\gamma}$ , where  $C$  follows (8). This fact and Itô's Lemma imply

$$\frac{dm}{m} = \mu_m dt + \sigma_m dW_m^T + J_m dQ^T, \quad (28)$$

with

$$\sigma_m = [-\gamma \sigma_C, b^M \sigma_\lambda^M \sqrt{\lambda^M}, b^E \sigma_\lambda^E \sqrt{\lambda^E}], \quad (29)$$

$$J_m = [e^{-\gamma Z^M} - 1, e^{-\gamma Z^E} - 1, (1 + n_B(e^X - 1))^{-\gamma} - 1], \quad (30)$$

for  $dW_m = [dW, dW_\lambda^M, dW_\lambda^E]$ , and  $dQ = [dQ^M, dQ^E, dQ^X]$ . It also follows from no-arbitrage:

$$\mu_m = -r - \sum_{j \in \{M, E\}} \lambda^j \mathbb{E}_{z^j} [e^{-\gamma Z^j} - 1] - \pi \lambda [(1 + n_B(e^X - 1))^{-\gamma} - 1]. \quad (31)$$

Since  $J_m \geq 0$ , according to (28), in the event of a macroeconomic and environmental shock marginal utility jumps upwards, increasing investor's required compensation for bearing risk. Ceteris paribus, a shift towards a greener portfolio would reduce  $m$  due to the reduction in the frequency of the catastrophic events – and the subsequently triggered policy. Having calculated the state-price density we are now in position to calculate the risk premium for the aggregate equity market. The aggregate market pays a dividend  $D$ , being leveraged consumption, i.e.  $D = C^\eta$ . From Itô's Lemma it follows directly that

$$\frac{dD}{D} = \mu_D dt + \eta \sigma_C dW^T + J_D dQ^T, \quad (32)$$

where  $\mu_D = \eta \mu_C + \frac{1}{2} \eta (\eta - 1) \sigma^2$ , and

$$J_D = [e^{\eta Z^M} - 1, e^{\eta Z^E} - 1, (1 + n_B(e^X - 1))^\eta - 1]. \quad (33)$$

We can also show (see Seo and Wachter (2018)) that the price for  $D$  reads  $P = DG(\lambda^M, \lambda^E)$  with  $G$  from (23) in the main text. Itô's Lemma on  $P = DG$  using (32) and (23) leads to the process for prices  $dP/P = \mu_P dt + \sigma_P dW_m^T + J_P dQ^T$ , with  $J_P = J_D$  and

$$\sigma_P = \left[ \eta \sigma_C, \frac{1}{G} \frac{\partial G}{\partial \lambda^M} \sigma_\lambda^M \sqrt{\lambda^M}, \frac{1}{G} \frac{\partial G}{\partial \lambda^E} \sigma_\lambda^E \sqrt{\lambda^E} \right]. \quad (34)$$

Variations in  $\lambda^j$ ,  $j \in \{M, E\}$  create variations in  $G$  and thus in stock prices, reflected by the second and third term of (34). Equity premia arise from the co-movement of marginal

utility of the risk-averse investor with the price of the underlying asset or portfolio, both in normal times and times of disasters  $R - r = -\sigma_m \sigma_P^T - [\lambda^M, \lambda^E][\mathbb{E} J_m J_P]^T$ , with  $R$  the expected return on the aggregate equity market. Using (29), (30), (33), and (34), we get (22) in the main text.

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