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Time-consistent resource management with regime shifts*

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Abstract

We investigate how a resource user who is present-biased manages a renewable resource stock with variable growth that could undergo a reversible regime shift (an abrupt, persistent change in structure and function of the ecosystem supplying the resource). In a discrete-time quasi-hyperbolic discounting framework with no commitment device, and using only generic utility functions and stock transition with regime shifts, we show that there is a unique, time-consistent stationary Markov-Nash equilibrium extraction policy. Further, we find that the optimal extraction policy is increasing in the resource stock and in the degree of present bias. Overall, our results suggest that for characteristics of ecosystems commonly considered in the literature, present-biased resource users will increase extraction when faced with regime shifts.

Keywords: Renewable resources; Regime shifts; Hyperbolic Discounting; Present bias; Uncertainty; Markov Equilibrium
JEL Codes: Q20, C61, C73

1 Introduction

Many communities face dynamic renewable resource management problems. These often involve processes with some degree of natural variability in resource growth, and potential critical transitions

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that could trigger abrupt changes in the structure and function of the natural ecosystems supplying the resource. These abrupt changes – called "regime shifts(Crépin et al. (2012) and Horan et al. (2011))– occur in many ecosystems. Fisheries could undergo more or less reversible collapses (e.g. the Maine cod fisheries (Pershing et al. (2015)); forest regrowth could be adversely affected due to factors such as foragers (e.g. Moose, Crépin (2003)); an infectious disease can exist in a latent regime in which few individuals in a population are infected or in an outbreak regime in which the disease is rapidly spreading to larger parts of the population (Ludwig et al. (1978)); a coral reef can exist in a clear regime hosting a diverse multitude of species, in a bleached regime, or in an algae-dominated regime, which both host lower species diversity and provide different types of ecosystem services (Norström et al. (2009)); essential elements of the climate system itself can exist in different regimes (Lenton et al. (2008); Steffen et al. (2018)**?**). Regime shifts can be either irreversible, meaning that an ecosystem that has changed regime cannot return to its previous regime, or reversible,in which case the ecosystem can return to its previous regime when a stock variable (a pollutant or a resource stock) enters a desirable region. In any case, regime shifts can lead to adverse economic and environmental outcomes and pose unique challenges to the management of ecosystems at a wide variety of scales, from the global climate to local resources such as lakes and fisheries.

Trade-offs between the present and the future challenge the management of natural resources over long time horizons. In particular, decision makers appear to be impatient in many contexts: they are biased towards the immediate future (the 'present') over the intermediate and long term. This bias may have many explanations: low income communities are often more dependent on the resources they manage and may also exhibit greater impatience compared to less-dependent or wealthier societies (or to the same society at a distant, and wealthier, future). Alternatively, policy makers are directly responsible to current voters, and therefore more responsive to their concerns. In addition, they are also often unable to commit future policy makers to policies that appear optimal from today's perspective. Whatever the cause, present bias leads to non-constant discounting, with significant implications for dynamic resource management.

The large literature on dynamic resource management with a variety of regime shifts (e.g. Heijdra and Heijnen (2013); Peterson et al. (2003); Kang (2019); Brozović and Schlenker (2011); Polasky et al. (2011); Lemoine and Traeger (2014); Martin and Pindyck (2015); Leizarowitz and Tsur (2012)) has focused exclusively on the case of constant discounting. Few studies embed a regime shift framework in a non-constant discounting. To our knowledge, only one study, Karp and Tsur (2011), has done so, in the context of an irreversible regime shift pertaining to the climatic system.

Our objective is to address this gap, by investigating the decision process that present-biased resource users face when they manage a renewable resource stock that could undergo a regime shift that is reversible (meaning that resource growth returns to its original trajectory when the resource stock rises beyond the threshold). We focus on renewable resources with random growth and explore the response of a forward looking regulator whose decision framework is present biased (quasi-hyperbolic). Our main contribution is to situate the question of resource extraction with regime shifts in the canonical (stochastic) discrete-time quasi-hyperbolic framework without the use of very specific functional forms or very limiting assumptions. To this end, we develop a discrete-time framework of a present biased regulator who manages a renewable resource stock subject to growth shocks. The "production function" of next period stock exhibits a sharp change when entering (exiting) a –fixed–undesirable region of the state space, which constitutes the regime shift in question. Working with very general utility functions, and adopting stochastic production functions similar to those used in the current literature, we are able to establish the existence of a unique equilibrium extraction policy. As in the previous literature, we consider ecosystems both with lagged and rapid dynamics so that regime shifts and threshold crossings could depend either on pre- or post- extraction stock.

The approach of using only key properties of utility functions and transition equation (instead of special functional forms) yields many benefits: first, it will prove very helpful in identifying key drivers of resource policy more generally; and second, it enables us to place the renewable resource extraction problem with regime shifts for a present biased regulator in the framework of the familiar (stochastic) consumption-savings problems in the quasi-hyperbolic context. To our knowledge, we are the first to establish properties of the equilibrium in dynamic quasi-hyperbolic stochastic resource problems with regime shifts.

Our first main finding is that, for both cases of regime shifts, the regulator increases extraction in response to an increased threshold. This result is in contrast to the existing literature for regime shifts termed endogenous (in the terminology of Polasky et al. (2011)): Brozović and Schlenker (2011) find a non-monotonic relationship between optimal pollutant loading and the threshold level of stock while Polasky et al. (2011) report reduced extraction. A key aspect driving our result is the Edgeworth complementarity between reinvestment and the threshold, meaning that reinvestments are more beneficial at lower thresholds than at higher. Our second major result for both types of regime shifts is that equilibrium extraction is monotonically increasing in the resource stock. While this has been conjectured in a prior study considering carbon emissions by a quasi-hyperbolic regulator without regime shifts (Karp (2005)), we are unaware of any study that has established this property for resource extraction problems in general, with or without regime shifts.

When regime shifts are based upon post-extraction stock, the most common case in the literature, we establish two additional results: (i) increases in the degree of present bias lead to increased extraction; (ii) present bias and regime shifts mitigate the effects of one another upon equilibrium extraction i.e. the increase in extraction resulting from increased present bias is smaller in the presence of regime shifts (and vice-versa). Our result that increases in present bias lead to increased extraction (reduced resource reinvestment) is related to a common result in deterministic consumption-savings models of quasi-hyperbolic discounting (e.g. Krusell and Smith Jr (2003)). However, in the resource economics literature, a similar result (suggesting reduced pollution abatement) was shown only in Karp (2005) for the special linear-quadratic case, and only numerically.

In essence, our results suggest that much of the intuition from the deterministic quasi-hyperbolic consumption-saving models carries over: there is excessive extraction when decision makers are present biased, and the presence of a regime shift does not alter this intuition. Our finding that present bias and the threshold mitigate the effects of one another on equilibrium extraction, however, is novel.

The rest of the paper proceeds as follows: section 2 provides an overview of the related literature, including our interpretation of hyperbolic discounting, while section 3 details key aspects of our model set up. Section 4 provides our main results for the formulation where regime shifts depend upon pre-extraction stock, section 5 details the more common case of post-extraction stock determining growth and section 6 provides a discussion of how our findings relate to those in the broader literature. Section 7 concludes. All technical details and the proofs of all results are relegated to the Appendices.

2 Related Literature

The analysis here builds on two distinct strands of literature, the first related to management of renewable resources with regime shifts or catastrophes and the second to discounting and impatience. To aid understanding, we relate the key features of each of these aspects to those extant in the current literature.

2.1 Regime shifts and catastrophes

The most common way of modelling catastrophes is to consider catastrophic events as penalty functions with an associated hazard rate. In Clarke and Reed (1994)'s model, some random environmental process whose occurrence probability depends only on current pollution level can trigger irreversible catastrophic events. Instead, Tsur and Zemel (1996, 1998) consider events that occur as soon as the stock of pollution reaches a possibly unknown threshold but do not otherwise depend on exogenous environmental conditions. Moreover, these events are reversible, possibly with some regenerative activities. A common way of solving such models is to transform them into a deterministic control problem with the associated survival probability as the state variable. It turns out that endogenous uncertainty and reversibility are both crucial for the optimal policy outcome: they always imply more conservation.In contrast, exogenous uncertainty and/or irreversible events generate a non-monotonic relationship between uncertainty and precautionary activities: an exogenous increase in the risk of a catastrophe can increase or decrease the degree of precautionary activities undertaken by resource managers behaving optimally. A different approach, perhaps more aligned with the ecological literature, is to model a regime shift as a change in the system dynamics rather than as an abrupt and irreversible catastrophic event or sudden collapse in the resource stock. That literature often uses continuous time models with convex-concave growth function generating reinforcing dynamics (Wagener 2003; Mäler et al 2003; Brock and Starrett 2003, Crépin 2003). Even in this case the stochastic event can be modeled in a similar way, In this case, precaution is an optimal strategy as well for endogenous problems but not otherwise (Polasky et al. (2011) and see overview in Li et al. (2018)

We introduce a regime shift into the classic framework used in discrete-time stochastic renewable resource models: resource growth is subject to uncertainty, the regime shift threshold is fixed and known, and a regime shift can occur either before or after the extraction decision is made. A few prior studies, following the ecological literature, take this approach to modeling regime shifts (e.g. Peterson et al. (2003); Brozović and Schlenker (2011)), under special conditions including specific functional forms. Moving away from the focus on the transition equation, our focus is on better representing the "production function" leading to next period stock in an inherently stochastic manner. Consequently, our set-up is somewhat different from the classic time-distributed regime shift models already discussed. The focus on more appropriate representation of stochastic production implies that regime shifts now lead to the distribution function for stock exhibiting a sharp change when entering (exiting) a fixed undesirable region of the state space.

2.2 Discounting

The choice of a framework for discounting has been a long debated issue when evaluating public projects and policies with very long time horizons, including climate change. The discussion involves a wide-variety of approaches, from market-based to those based upon ethical considerations, and also includes consideration of accounting for uncertainty and shocks (see e.g. Gollier and Hammitt (2014)). Although there is no consensus regarding the "correct" approach to discounting, experimental, empirical and conceptual contributions all suggest that there is little evidence for constant discounting over long horizons, with declining discount rates the most plausible alternative suggested. Declining discount rates lead to "present bias" i.e. discounting the future more than the present, with the most common form of declining discount rates being of the (quasi-)hyperbolic form (see section 3).

Present bias over intertemporal consumption decisions may arise for many reasons. It can arise when regulators discount utility gains within their own life time (often interpreted as a generation) differently from those later on. In these cases, the regulator at generation/time t might procrastinate when faced with taking decisions that impose costs on the current generation, with benefits realized only later. This is an interpretation closer to many resource extraction settings, including climate change.[1] Alternatively, it may also arise from pure altruism for future generations, such as can arise when managing resources over the very long term, which, as pointed out in Saez-Marti and Weibull (2005), inevitably leads to changing utility-weighting over time (see also Phelps and Pollak (1968)). Another channel through which declining discount rates arise is through heterogeneity in individual time preferences and the need of aggregation in public decisions like environmental projects (Gollier and Zeckhauser (2005); Jackson and Yariv (2015)). Finally, the environmental economics literature (e.g. Sumaila and Walters (2005); Heal (2000); Karp (2005, 2007)) has focused on slightly different ways to motivate hyperbolic (or at least declining) discount rates: the Weber-Fenchel Law, the differences between inter- and intra-generational discounting or those based upon time perspective.

[1] As a practical matter, we note that many institutions of collective decision making (e.g. parliaments) are often unable to commit future decision makers, illustrated in the case of climate change by Canada's withdrawal from the Kyoto Protocol a few years after its signing (and the opting out of Japan from the second Phase of the Kyoto Protocol).

While the trade-offs involved and the analytic mechanisms are identical, our interpretation of present bias relates to the responsibility of decision makers to the current generation, rather than those of behavioural biases (such as the need for instant gratification) explored in a very large literature.[2]

Few studies in the literature address environmental problems with non-constant discounting with a focus on characterisation of the equilibrium. Prior studies on resource policy with a stock variable and (quasi-) hyperbolic discounting either restricted attention to certain specific set of policies (Karp and Tsur (2011)), on largely numerical characterisation (e.g. Karp (2005)), or on simulation frameworks (Fujii and Karp (2008)). Only one study, to our knowledge, analyses the question of equilibrium characterisation in the context of a stock variable, Gerlagh and Liski (2017), largely in the setting of special functional forms for utility. Similarly, Karp and Tsur (2011) is the only study to embed hyperbolic discounting in a model of catastrophic climate related damages (modelled as irreversible and permanent welfare loss), restricting policies to a binary set of actions for tractability (climate stabilization and business as usual). In any case, all of these studies do so in the context of deterministic state evolution,[3] and with often no clear characterisation of time-consistent policies.

2.3 The Equilibrium

A large literature studies the dynamic problem faced by a present-biased regulator who discount utility gains during their own life time/generation differently from those after their time. Furthermore, the regulator is assumed to be unable to commit to future actions and cannot dictate the decisions of future regulators. In other words, the regulator cannot commit to future actions and cannot dictate decisions to future regulators: instead, he is constrained to decisions within his 'political cycle' (in non-behavioural interpretations). The traditional way of approaching problems of hyperbolic discounting, recursive decision theory, is challenging due to time-inconsistency: what is optimal for a decision maker at time t is not so for the decision maker at time $s \neq t$, leading to many challenges to the recursive approach, including lack of continuity in preferences, and is discussed in both the theoretical and applied literature Krusell et al. (2002); Klein et al. (2008); Karp (2005).

An alternative, and common, approach is to picture the problem as a dynamic game between selves at different points in time, leading to a different set of difficulties. Recent studies, however, have attempted to unite the approaches of stochastic games to again apply recursive methods to problems with hyperbolic decision makers, and restricting attention to the more practically useful pure strategies (Balbus et al. (2014), Balbus et al. (2018)–henceforth BRW18, Nowak (2003)). Our

[2]For instance, Rabin (1998) offers a psychological basis for time variant discounting focusing among others on preference reversals and a taste for immediate gratification. Phelps and Pollak (1968) provided the first formal model of such "time varying" (and inevitably time inconsistent, see Strotz (1955)) preferences. These results were supported by other studies from both psychology (Kirby and Herrnstein (1995)) and environmental economics (Cropper et al. (2014)). Rubinstein (2003) presents experimental evidence suggesting that individuals ignore small differences and focus on large differences when comparing two alternatives while the ability to distinguish differences diminishes with time.

[3]Note that an unpublished version of Karp (2005) allowed for stochastic evolution in a framework very similar to that in Harris and Laibson (2001). Consequently, caveats similar to that for Harris and Laibson (2001) regarding the degree to which uncertainty can be accommodated are applicable (see e.g. Balbus et al. (2018)).

analysis draws upon insights from this strand of the literature. In view of its origins in stochastic dynamic (exponentially) discounted games, the appropriate notion of equilibrium is that of a Stationary Markov Nash Equilibrium (SMNE).[4] A key advantage to this approach is that, if it exists at all, the SMNE is time-consistent.[5] We note that our analysis will use this notion of equilibrium.

3 Model Details

The basic set-up is a standard regime shift model in discrete-time embedded in a stochastic quasi-hyperbolic capital accumulation setting.It can be described as follows: Let $X_t \in \mathcal{X} \subset \mathbb{R}^+$ denote a random variable for stock level at time t, with x the realized stock. Extraction, $q(X_t)$, and "reinvestment", $a(X_t)$, are the two key functions we define next. Consider a situation when after observing the stock $X_t = x$ at the beginning of period t, the decision maker chooses an extraction level, $q_t \in A(x) := [0, x)$, and leaves $a_t := x - q_t$ as the reinvestment.[6] Reinvestment and starting stock levels, together with the threshold, lead to the next period stock, X_{t+1} via the transition function (also called a 'stochastic kernel') $Q(dX_{t+1}|x, a, \underline{X})$. In our setting, realized stock levels below $\underline{X}$ trigger a shift to substantially lower stock regrowth compared to that above the threshold (constituting the 'regime shift' in question). Instead of specifying X_{t+1} as arising from the addition of an "error term" to a deterministic growth function (as for example in Brozović and Schlenker (2011)) , we directly specify the next period stock via the following stochastic specification,

$$X_{t+1} \sim Q(.|X_t = x, a_t = a). \tag{1}$$

The transition function, Q, maps the state space to itself and defines a probability distribution over the next period stock.[7]

[4]We note that what is termed a Markov Perfect equilibrium (MPE) in the literature (used e.g. in Karp and Tsur (2011)) is often also called the Markov Nash Equilibrium, according with the definitions provided in footnote 5. These equilibria thus need not be stationary, and consequently, for hyperbolically discounting agents, need not be time-consistent. This distinction is important to bear in mind, and is discussed in e.g. Balbus et al. (2015).

[5] The notion of a SMNE encompasses three distinct concepts in the stochastic (exponentially) discounted dynamic game that the decision process for a series of hyperbolic decision makers represents: stationarity; Nash equilibrium; and Markovian strategies. By stationarity of a strategy one means that a strategy is required to depend only upon the current state (resource stock), not on past history (including stock or time t). For a strategy to be a Nash equilibrium, it is required that it dominates all potential strategies, a definition familiar from deterministic, static and repeated games. Finally, a strategy is Markovian if it depends only upon the current 'stage' of the game (i.e. current state and time t). By these definitions, a stationary Markovian strategy clearly does not depend upon the stage (or time variable t) of the game, only the current state variable. In consequence, a stationary Markovian (SM) strategy that also is a Nash equilibrium is time consistent, virtually by definition (since a time-inconsistent SM strategy cannot represent a Nash equilibrium, which, recall, dominates all SM strategies). See Dutta and Sundaram (1998, §2) or §2,3.6 of Levy (2013) for precise definitions and detailed discussion of these notions.

[6]Note that in models of fisheries, what we term "reinvestment" here is termed "escapement" instead. We use the terminology of stochastic growth models in the interest of generality.

[7]More formally, it is the distribution induced by the transition probability of the Markov chain that the stock series, $\{X_t, t = 0, 1, 2, \ldots\}$, represents, under any stationary policy (extraction decision), $q(X)$. Since this is common in the literature on dynamic programming and stochastic optimal growth, we will use the simpler term "distribution of X_t" for the more formal "distribution induced by the transition probability".

We follow the literature in assuming that the effects of regime shifts are manifested directly, and exclusively, upon stocks, not upon utility. Utility U, derived only from extraction (following Karp and Tsur (2011)), is non-negative and strictly concave, possibly unbounded above. Finally, parameters $0 < \delta \leq 1$ and $0 < \beta \leq 1$ denote respectively the usual discount factor and the degree of present bias, detailed next. The objective of the decision maker can be represented by

$$U(q_t) + \beta \mathbb{E}_t \left(\sum_{t+1}^{\infty} \delta^{i-t} U(q_i) \right), \tag{2}$$

with $\mathbb{E}_t$ the expectation taken w.r.t. the time-t distribution of X_t.

The discounting aspect embodied in eq. (2) may be described briefly thus: starting at time period t, the decision maker uses the discount factor δ to compare pay-offs (consumption) between any two adjacent periods beyond $t+1$ (e.g. between $t+2$ and $t+3$) while using the factor $\beta\delta$ to compare outcomes between period t and $t+1$. This leads to the following series of discount functions: $1, \beta\delta, \beta\delta^2, \beta\delta^3, \ldots, \beta\delta^t, \ldots$. For any $\beta < 1$, this discounting framework represents declining discount rates, with larger rate of decline in the 'short-run' than in the 'long-run'. The intensity of the rate of decline increases as β decreases. Preferences exhibiting these characteristics are termed quasi-hyperbolic or present biased,[8] which is the reason for using it in our context. Note that while discount rates that decline over time may appear to be desirable from a long-run decision making perspective, this form of discounting in fact leads to strengthening the preference for immediate pay-off, relative to exponential discounting. In any case, lower values of β represent a stronger bias for the present, and any value of $\beta < 1$ captures time-inconsistent preferences.

Following the previous literature, we adopt the approach that the regulator shares the consumer's preferences,[9] and cannot commit to future actions.[10] In this case, the best a planner can do is to play a dynamic with the future planners and manipulate their decisions through his own policies for the present. In essence, the time consistent equilibrium is the only sustainable policy for the management of the resource stock, since future decision makers would deviate from any other plan, and is the notion we adopt.[11]

[8]The 'hyperbolic' aspect in the term 'quasi-hyperbolic' refers to the greater decline in the short-run compared to the long-run. This is easiest to see in the continuous time formulation (e.g. Angeletos et al. (2001)) wherein the short-run discount rate, $-\ln \beta\delta$, is larger than the long-run, $-\ln \delta$. The latter interpretation, of 'present bias', in fact follows from the observation that all future pay-offs are discontinuously discounted by the amount β, relative to the case of exponential discounting. This is easiest to see in a continuous-time formulation e.g. eq.(3) in Benhabib et al. (2010).

[9]This approach abstracts away from the question of which set of preferences the planner would like to maximise. If one believes that present bias represents a behavioural failure that an agent would like to correct, then a planner would act paternalistically and opt to maximise the "long-run" or normative preferences: the preferences an agent has without present bias (see e.g. Kang (2019)).

[10]Krusell et al. (2002),Gerlagh and Liski (2017) show that the full commitment solution would consist of one policy for today -the same as in the equilibrium- and a policy for all future periods, the optimal with exponential discounting. Instead, Karp (2005) claims that the optimal solution for today is the same as with exponential discounting, just with beta as an extra discount factor. Karp and Tsur (2011) explore the idea of "restricted" commitment where the decision maker cannot switch between policies as a more realistic alternative.

[11]For example, Hepburn et al. (2010) show how following any other policy could result to an unforeseen resource

3.1 Regime shifts

3.2 Uncertainty in resource dynamics

It will be useful to first understand how uncertainty regarding resource growth–without regime shift–is commonly modelled. Denote by $H(x, a, \epsilon)$ the possibly stochastic function that determines stock growth, which depends on "reinvestment", a, beginning-of-period stock, x, and a random growth term, ϵ. In resource economics, H is often called the "production function" of next period stock (see e.g. Ren and Polasky (2014); Mitra and Roy (2006) for the deterministic and stochastic cases respectively), and its properties are key to understanding resource problems. Consider a simplified stochastic version of the stock transition equation, defined via this production function and an additive noise term,

$$X_{t+1} = G(x, a) + \epsilon_t, \ \{\epsilon_t\} \overset{IID}{\sim} \psi \tag{3}$$

with G a deterministic function, ϵ_t taking values in $D \subset \mathbb{R}_+$ and ψ some probability distribution on (the Borel subsets of) D. The resulting stock series, $\{X_t\}_{t=1}^{\infty}$, for every stationary policy, $a(X_t)$, can be shown to lead to a Markov chain with transition function Q and associated CDF F,[12] whose properties are key to the dynamic optimisation problem.[13] Note that this is the general structure in both Leizarowitz and Tsur (2012) and Brozović and Schlenker (2011), two studies focused on regime shifts.

Clearly, for every formulation of state transition in the form in eq. (3) or the more general formulation involving H, an equivalent representation is

$$X_{t+1} \sim Q(.|x, a). \tag{4}$$

As shown in Amir (1997), direct specification of the production function in the form of eq. (4) is often more convenient than the indirect representation in the form of eq. (3) in the context of economic dynamics with a production interpretation.[14] In particular, F, the distribution function associated with Q, is the stochastic production function, and it satisfies the usual properties[15] without the necessity for additional ad hoc restrictions. Our formulation of uncertainty will follow the approach of directly specifying the relationship between X_{t+1} and x, a following the specification in eq. (4).

collapse.

[12]To see this, let $\tilde{G}(X_t, \epsilon_t) := G(X_t, a(X_t)) + \epsilon_t$. Then, by definition, the kernel Q, defined as $Q(x; B) := \mathbb{P}\left(\tilde{G}(X_t, \epsilon_t) \in B\right)$ equals $\psi\left\{k \in Z : \tilde{G}(x, z) \in B\right\}$. Thus, $\{X_t\}_{t=0}^{\infty}$ is Q-Markov, by construction.

[13]This is illustrated in the discrete-state case in Leizarowitz and Tsur (2012), where the structure of Q is derived explicitly (in §2.3).

[14]In particular, it is both more general and often more convenient to work with. To illustrate, ad hoc assumptions on ϵ (e.g. finiteness) are needed to ensure that X_{t+1} is drawn from a compact set. These assumption are often difficult to justify and also necessitate bounds upon ϵ which may depend upon X and in any case may have little to do with the structure of the problem on hand (see also the Stachurski, etc).

[15]These properties include: it is increasing in a (the greater the post-extraction stock is, the larger is the probability of obtaining higher next period stock); and yields zero next-period stock when there is zero post-extraction stock, which are the counterparts to the determinsitc production function. These aspects are discussed in greater detail in section 4.2.

3.3 Representation of Regime shifts

Regime shifts are often specified by allowing for altered system dynamics once a threshold is crossed. The threshold can be known either with certainty or only probabilistically. In these contexts, the regime shift is considered as a change in some parametric specification of the function H e.g. a change in the carrying capacity (Ren and Polasky (2014)) or in pollutant loading (BS12). Consequently, denoting by K a generic parameter that governs a change in system regime, regime shifts can be formalised as the effect of this parameter on the production function, $H(x, a, \epsilon; K)$, an approach explicitly followed in Ren and Polasky (2014). Thus, investigations can center around the effect of regime shifts upon e.g. optimal action, extraction and reinvestment, $q^*(x; K)$ and $a^*(x; K)$. In our context, the parameter K is a threshold, denoted by $\underline{X}$, that divides the state space, $\mathcal{X} \subset \mathcal{R}_+$, into two regions, a desirable one, $(\underline{X}, \infty)$, and an undesirable one, $(0, \underline{X}]$, with $\underline{X} = 0$ constituting the case of no regime shift.

Following the specification in eq. (4), the next period stock is drawn from an appropriate transition function $Q\left(.|x, a, \underline{X}\right)$, parameterised by this threshold $\underline{X}$. For this stochastic dynamic system, a (discontinuous) regime shift can be written in terms of the associated distribution function, F, as

$$F\left(X_{t+1}|X_t = x, a_t = a, \underline{X}\right) = \begin{cases} 0, & x \leq 0 \\ G_1\left(.|x, a\right), & 0 < x \leq \underline{X} \\ G_2\left(.|x, a\right), & \text{else} \end{cases}, \tag{5}$$

with G_1 and G_2 representing appropriate distribution functions. Equation (5) is the stochastic analogue of the most commonly used deterministic equation for general regime shift case, which can be written as

$$X_{t+1} = \begin{cases} H_1\left(X_t, a_t\right), & X_t \leq \underline{X} \\ H_2\left(X_t, a_t\right), & \text{else} \end{cases} \tag{6}$$

H_1 and H_2 in eq. (6) are deterministic stock growth functions, which, in view of our interpretation of the region $X_t < \underline{X}$ as being undesirable, satisfy the condition $H_2 \geq H_1$. For the stochastic production function case in eq. (5), we analogously require that probability of higher stocks are larger under G_2 than under G_1 i.e. that $\mathbb{P}\left(X_{t+1} \geq b|x \leq \underline{X}, a\right) := 1 - G_1(b|x, a) \leq \mathbb{P}\left(X_{t+1} \geq b|x > \underline{X}, a\right) := 1 - G_2(b|x, a)$, $\forall b > 0$. This requirement implies that $G_2(b|x, a) \leq G_1(b|x, a)$, $\forall b > 0$, which is equivalent to stating that the distribution of stock, X, beyond the threshold, first order stochastically dominates (FSD) that within the threshold. This requirement of FSD indeed represents the most natural extension of the deterministic notion that $H_2 \geq H_1$.

In addition, in the deterministic setting, when the threshold value increases, the desirable region of the state space shrinks. Consequently, the growth function is decreasing in the threshold, meaning naturally that X_{t+1} is decreasing in the threshold.[16] An extension of this notion to a stochastic

[16] The requirement that X_{t+1} is decreasing in the threshold is satisfied in all comparable frameworks in the literature e.g. Brozović and Schlenker (2011); Peterson et al. (2003); Ren and Polasky (2014).

setting relates to the distribution of the next period stock, and states that the distribution of stock, $F\left(y|x,a,\underline{X}\right)$, is *stochastically decreasing* in the threshold (see definition 1). This notion captures, to reiterate, the very intuitive fact that increases in the threshold, for any level of starting stock (x), makes it more likely that stock regeneration is lower.

For the case of smooth regime shift, where substantial and rapid–yet continuous–changes result around the threshold, the intuition is very similar to the case of the discontinuous shift in eq. (5). Analogous to the deterministic case , where rapid and adverse changes in H occur around the threshold leading to sizeable reduction in X_{t+1} just underneath the threshold, in the stochastic case, a rapid and substantial reduction in the probability of obtaining higher next-period stock occurs around the threshold. This can be more easily seen in a specific illustration, a modified version of eq. (5), with G_1, G_2 still representing the desirable and undesirable growth regimes respectively,

$$G_1\left(.|x,a\right)\theta\left(x,\underline{X}\right)+G_2\left(.|x,a\right)\left(1-\theta\left(x,\underline{X}\right)\right). \tag{7}$$

Here, $\theta\in[0,1]$ is a measure that changes rapidly and substantially around the threshold, and puts more weight on the undesirable distribution G_1 when $x\leq\underline{X}$, ensuring that next period stock is probabilistically lower when $x\leq\underline{X}$.[17]

For many ecosystems of interest, current extraction does not immediately determine the state of system, which may be only observed periodically or with substantial noise, possibly due to lags in the eco-system considered. In these cases, it is more plausible that rapid regime shifts are less likely. Consequently, current loading (extraction) does not directly affect threshold crossing, as is also argued in Peterson et al. (2003). This approach is encapsulated in the dependence of the regime shift upon X but not a in both the discontinuous case (in eq. (5)) and the continuous case (in eq. (7)). We begin our analysis with this case first (in section 4).[18] This is also in fact, as we shall later see, the more challenging case to study. The differences between this formulation and one that considers the post-extraction stock as the basis for regime shifts will be illustrated when we consider the case where regime shifts can occur more rapidly (in section 5).

Consider a dynamic system with stock growth as in eq. (1) subject to a continuous regime shift described just above. We discuss next a few details regarding the key features of the stochastic transition, Q, introduced in eq. (1). Since we build upon the standard framework in the literature, we provide only the necessary technical preliminaries and direct the reader to the relevant literature for fuller details, in particular Balbus et al. (2014, 2018); Nowak (2003, 2006); Amir (1997). We consider a specific, additive form of the transition structure,

$$Q\left(.|a,x,\underline{X}\right)=\left(1-P\left(\mathcal{X}|a,x,\underline{X}\right)\right)\delta_0+P\left(.|a,x,\underline{X}\right), \tag{8}$$

where $P\left(.|a,x,\underline{X}\right)$ is some probability measure satisfying certain properties to be detailed next

[17] This is easily seen to imply the following: $\theta(x\leq\underline{X})>\theta(x>\underline{X})$ and, for $X_t<\underline{X}$, $1-\theta\left(x,\underline{X}\right)<\theta\left(x,\underline{X}\right)$.

[18] More precisely, our formulation corresponds in timing to that in Peterson et al. (2003, eq.(1)), with regime shift depending upon pre-extraction stock i.e. the regime shift is independent of current extraction (loading).

and δ_0 is the Dirac measure concentrated at 0 stock. The transition function, Q, is in general a function of both the state, x, and reinvestment, a, and is fully determined by the measure P (with associated distribution function, F_P). Following the discussion in the previous section, a regime shift is encapsulated in the dependence of the measure P upon the threshold, $\underline{X}$, where F_P is stochastically decreasing in the threshold and undergoes a rapid–yet continuous–and substantial change around the threshold (as in eq. (7)).

In consequence, the distribution of X_{t+1} has an "atom" at 0 stock, meaning a strictly positive probability of reaching state 0 from any other stock value. In view of the fact that utility is assumed to be bounded below (by 0), this structure implies that reaching 0 stock leads to the 'end' of this resource (economy), and, more importantly, that the probability of this event is strictly positive, $\forall t$. The structure of the transition function in eq. (8) bears some similarity to many models in resource economics, including those of catastrophic regime shifts, resource extinction, and to models of non-convex stochastic growth with renewable resources. For instance, Leizarowitz and Tsur (2012) consider an irreversible catastrophe occurring at some random point in time; once it occurs, the stock is driven to some fixed level (possibly zero) forever. In a series of papers, Mitra and Roy (2006, 2007) consider the question of extinction of resources. In both cases, conditional on the irreversible event (catastrophe and extinction, respectively) not occurring, stock evolution is Markovian. Furthermore, this probability of occurrence is strictly positive $\forall t$ and is, moreover, a function of the extraction policy, very similar to our case above (where $1-P$, the probability of having 0 stock next period, is a function of x, a). Finally, in the framework of a stochastic growth model with non-convex production, Kamihigashi and Stachurski (2014, §4) allow for arbitrarily bad shocks to stock regeneration (which is Markovian), implying that the probability of having arbitrarily low stock is again strictly positive.[19]

3.4 Recursive Equilibrium

At time t, the forward looking regulator inherits the state variable, X_t, knows the threshold, $\underline{X}$, and makes the extraction/investment decisions, $q(x), a(x)$. Regulators at time t play an inter-generational game with all regulators at time $t+1, t+2, \ldots,$. These choices together with X_t determines X_{t+1} via the transition kernel, Q (defined in eq. (8)). We use a recursive approach to characterising the equilibrium. Consequently, our set-up parallels the more common Bellman recursion for geometric discounting, with only key differences highlighted here (see Appendix B.1 for fuller details). In this spirit, we denote by $w \in \mathcal{A} := \{w : \mathcal{X} \to A; \text{w bounded}, w(x) \in A(x)\}$ a (pure strategy) stationary Markov Nash equilibrium for a quasi-hyperbolic agent that satisfies the following functional equation

$$w(x) \in \arg\max_{q \in A(x)} \left[U(q) + \beta\delta \int_{\mathcal{X}} V_w(x') \, Q(dx' | x - q, x) \right], \tag{9}$$

[19] We note that an alternative interpretation of stock collapse is feasible. To see this, define Y to be the actual stock of a resource, with $X_t := \log(Y_t - b)$, where $b \geq 0$ is a parameter related to a certain base population level. In such a case, the use of X_t instead of Y_t implies that the collapse of X_t to 0 has the implication that stock Y_t collapses to some natural or base stock level, $1 + b$. Since the $\log$ is a bijective transformation, working with X_t instead of Y_t leads to identical optimal policies and system dynamics (on the latter point, see Stachurski (2007)).

where $V_w : \mathcal{X} \to \mathbb{R}^+$ is the continuation value function for the household of future selves. If such strategies exist (i.e. if $\mathcal{A} \neq \emptyset$), they are time-consistent. Similar to the case of more conventional recursive approaches, the continuation value function satisfies the following recursion,

$$V_w(x) = U(w(x)) + \delta \int_{\mathcal{X}} V_w(x') Q(dx'|x - q(x), x). \tag{10}$$

Now, defining the value function for the self at time period t to be

$$W_w(x) = U(w(x)) + \beta\delta \int_{\mathcal{X}} V_w(x') Q(dx'|x - q(x), x), \tag{11}$$

one obtains the relation

$$V_w(x) = \frac{1}{\beta} W_w(x) - \frac{1-\beta}{\beta} U(w(x)). \tag{12}$$

Equation (12) is the so-called generalised Bellman equation. Thus, much like the conventional Bellman equation, any fixed point, $V^\star$, of a suitably defined operator upon V_w (defined in Appendix B.1) corresponds to the value function of a time-consistent equilibrium. In general, reflecting the well-known non-uniqueness of solutions in the hyperbolic discounting case (Karp (2005); Krusell et al. (2002); Harris and Laibson (2001)), the set of fixed points needs not be a singleton (see Balbus et al. (2015, §4.1)). If, in addition, $V^\star$ is a unique fixed point, then there is a unique, pure strategy, time-consistent equilibrium extraction, $w^\star$. All of our results will use only generic properties of resource problems, inspired by the use of general regeneration functions in Polasky et al. (2011); Ren and Polasky (2014). Clearly, in keeping with the literature, our results pertain to the case of "no commitment" of future generations to current policies, the most plausible equilibrium notion explored in the literature (including in Karp and Tsur (2011).

4 Characterising the equilibrium policy

We examine the nature of the equilibrium extraction policy for the resource problem with the generalised Bellman equation in eq. (12) and the stochastic production function encapsulated in eq. (8), where we consider the case of regime shifts depending upon post-extraction stock. Proofs of all results are provided in Appendix B.2.

4.1 Uniqueness

Our main goal is to establish the uniqueness of a time-consistent eqilibrium $w^\star$, under the following set of substantive assumptions gathered under

Assumption 1. *a. $U : \mathcal{X} \to \mathbb{R}^+$ is positive, increasing and strictly concave, with $U(0) = 0$ (i.e. U is bounded below, by 0 for convenience).*

b. for $x, a \in \mathcal{X}$, the transition probability Q has the structure in eq. (8), with the measure

$P(.|a,x)$ (and associated cdf F_P) satisfying the following properties,

i. for $x \in \mathcal{X} \setminus \{0\}$, $a \in [0,x]$, $P(\mathcal{X}|a,x) < 1$, $P(\mathcal{X}|0,0) = 0$;

ii. for every bounded function $V : \mathcal{X} \to \mathbb{R}^+$, the function $I(a,x) := \int_{\mathcal{X}} V(x')\, dF_P(x'|a,x)$ is continuous in (a,X), increasing and concave in a (for every given x).[20]

Assumption 1(a) is a standard assumption on preferences, allowing for unbounded utility above, with strict concavity needed only for ensuring a unique maximum. Assumption 1(b) deals with the structure of the transition probability Q and needs a bit more explaining: (i) states that the measure P of the state space is less than one with the remaining part concentrated at 0 stock. This is in fact a technical requirement needed for uniqueness of equilibrium; (ii) ensures that the expected continuation value function is increasing and concave in investment, for any positive and bounded candidate v. In essence, this is the requirement that increases in investment lead to a larger measure P, and hence larger next period stock and this increase has a decreasing rate.[21] Both assumptions are very common in stochastic growth theory in general and models of resource extraction and regime shifts in particular: a deterministic production function $f(a;\underline{X})$ is assumed to be increasing and concave in a.[22] As for the stochastic counterpart of the deterministic case, Amir (1997) provides equivalent conditions for F_P, seen as a stochastic "production function": F_P is stochastically increasing and stochastically concave in a.
With these preliminaries concluded, we can now state our main result:

Theorem 1. *For the renewable resource extraction problem considered here, there is a unique, bounded value function, $V^\star$ and correspondingly, a unique time-consistent equilibrium extraction policy $w^\star \in \mathcal{A}$.*

4.2 Relationship between extraction, stock level and the threshold

We next turn to understanding two aspects of the equilibrium extraction policy: do increases in beginning-of-period stock and the threshold lead to unconditional increased (or reduced) reinvestment? Different papers have reached differing conclusions on these questions, as will be detailed later. In the deterministic case, properties of the state evolution structure are key to obtaining

[20] It follows from its definition in eq.(8) (and the fact that $U(0) = 0$) that integration w.r.t. the measure (induced by) Q is identical to that w.r.t the measure P, a fact that will be repeatedly used. See Lemma 2 in Balbus et al. (2018) for a formal proof.

[21] The link between properties of F_P and those of $\int_{\mathcal{X}} V(x')\, dF_P(dx'|a,x)$, the expected value of V w.r.t F, is direct (at least for increasing and bounded V): if F_P is stochastically increasing and concave in a, then so is $\int_{\mathcal{X}} V(x')\, dF_P(dx'|a,x)$ (see Appendix 1 for details).

[22] In all the state transition equations in the literature (e.g. Brozović and Schlenker (2011); Leizarowitz and Tsur (2012); Karp (2007); Peterson et al. (2003)), increases in reinvestment lead to better next-period stock. Similarly, in all these models, concavity also follows, either trivially–consequent to linearity–or explicitly, e.g. Ren and Polasky (2014). Only in cases of non-concave production functions e.g. critical depensation as in Mitra and Roy (2006), does concavity not hold.

structural results. In our case, these properties will turn out to depend upon those of the stochastic production function, $F_P(.|a, x, \underline{X})$, in particular upon substitutability between the different "inputs" in this production function. More specifically, we focus on the relationship between the action a and either the state variable X, or the parameter $\underline{X}$.[23] Instead of imposing very specific functional forms for transition, we consider minimal relationships that are required to yield properties of interest.

To understand the relationship between action and state, we briefly revisit the discrete-time dynamic setting in resource economics, considering either the case without an explicit state-based regime shift (e.g. Leizarowitz and Tsur (2012)) or with parameterised regime shifts (Ren and Polasky (2014)). In both cases,[24] X_{t+1} turns out to be separable in a and X. This means that the effect of changes in X and in a, do not interact on the margin i.e. that the "maginal returns" (in terms of increase in next period stock, X_{t+1}) to X do not vary with the a change in the level of a (and vice-versa). In any case, the notion that marginal returns to one input depend upon changes in the other is familiar as Edgeworth complementarity: two inputs to production are Edgeworth complements (substitutes) if having more of one variable increases (reduces) the marginal returns to having more of the other. Edgeworth complementarity is captured by the idea of increasing (decreasing) differences: $\tilde{H}$ (defined in footnote 24), exhibits increasing (decreasing) differences in its inputs X, a (or $\underline{X}, a$) if marginal returns to one are increasing (decreasing) in the other (see Appendix A for details).

As to the relationship between the threshold $\underline{X}$ and the reinvestment a, studies considering the effects of a threshold (e.g Brozović and Schlenker (2011), Peterson et al. (2003)) have often opted for a linear and separable relationship between $a, \underline{X}$ (by adopting a piece-wise specification for the transition equation). A notable exception is Ren and Polasky (2014) in whose parameterised transition equation ($\tilde{H}(X_t, a_t; \underline{X})$, in our notation) it is assumed that increases in the threshold (reductions in "environmental quality", in their terminology) lead to reduced stock growth. This can again be interpreted in terms of Edgeworth substitution, meaning that having "more of"$\underline{X}$ reduces the marginal returns to a, which is equivalent to stating that $\tilde{H}$ exhibits decreasing differences in $a, \underline{X}$.

Our analysis considers relationships regarding H used in these studies for the distribution of next period stock, F_P. Were F_P a deterministic function, this condition is equivalent to F_P satisfying decreasing differences in (or being submodular in) (a, x) and in $(a, \underline{X})$. However, since the production function is now stochastic, the equivalent notion is that of stochastically decreasing differences i.e. that the production function, F_P, satisfies stochastically decreasing differences in (a, x) and in $(a, \underline{X})$. In more heuristic terms (see remark 2), if $F_P(.|X, a, \underline{X})$ satisfies stochastic decreasing differences in (a, X), then the "marginal return" (the probability of drawing better stock is the next period) to

[23] We observe that the notation $.|a, x, \underline{X}$ is intended to convey dependence of the transition upon key model parameters: reinvestment, beginning-of-period stock, and threshold, with arguments not under consideration often suppressed henceforth for convenience. In addition, in common with the literature, we will refer to F_P as being e.g. stochastically increasing instead of the more cumbersome "random variable distributed as F_P" being stochastically increasing.

[24] The transition equation in both studies can be written as $X_{t+1} = H(X_t) + a_t := \tilde{H}(X_t, a_t)$, after writing extraction, h, as $X - a$.

increased stock is decreasing in a. Similarly, if $F_P(.|X, a, \underline{X})$ satisfies stochastic decreasing differences in $(a, \underline{X})$ then the marginal return to increased a is decreasing in the threshold, $\underline{X}$.

4.2.1 Is equilibrium extraction increasing in resource stock?

An important question relates to the relationship between stock levels and extraction: with exponential discounting, larger resource stocks are often considered to lead to greater extraction, but this conclusion may no longer hold in the presence of non-convexities induced by e.g. stock-dependence in the growth function, even in the absence of regime shifts (see e.g. the discussion in Knapp and Olson (1995); Krishnamurthy (2017)). In the case of non-convexity induced by regime shifts, there is (to our knowledge) no prior result regarding this aspect. Furthermore, there are competing aspects at play here: higher stocks can lead to enhanced incentives to extract but this may vary upon the actual level of the stock in relation to the threshold (since the presence of threshold can alter the incentives to extract). Consequently, even without the question of present bias, monotonic extraction with complex regeneration function is the exception rather than the norm.

Following the previous literature, we assume that higher stock levels reduce marginal benefits to reinvestment, indicating some degree of decreasing returns on marginal increases in X_{t+1}. Our view encompasses the structure of the transition function in much of the literature, which uses special functional forms with the growth function: depending only upon a or x but not both (e.g. Polasky et al. (2011)); or depending linearly upon a and x (Brozović and Schlenker (2011); Karp (2005)). What is surprising is that even under the form of non-convexity implied by a regime shift, monotonicity of extraction in stock holds. We formalise the discussion above regarding the transition and then proceed to our main result for this section.

Assumption 2. *The function $I(x, a) := \int_{\mathcal{X}} V(x')\, dF_P(dx'|a, x, \underline{X})$ is submodular (i.e. satisfies decreasing differences) in (a, x).*

We note that $\mathbb{E}[V](a, x) := \int_{\mathcal{X}} V(x')\, dF_P(dx'|a, x, \underline{X})$ is the expected value function, as a function of state, action (and threshold, argument suppressed). Thus, Assumption 2 states that, for the expected value function, increases in a (reduction in extraction) are more valuable, on the margin, when x is smaller than when it is larger. We have argued above that F_P satisfies decreasing differences in (a, x). This aspect can be shown to directly establish the required link between properties of F_P and $\mathbb{E}[V](a, x)$.[25] In consequence, Assumption 2 can be shown to hold for our problem due to the

[25] There are two parts to this argument. The first part is simple: if the value function V is increasing in stock, then it follows (from Lemma 2) that $\mathbb{E}[V](a, x)$ satisfies decreasing differences in (a, x) if $F_P(.|a, x)$ satisfies (stochastic) decreasing differences. The second part is this: intuitively, since the value function represents the value of the problem starting from stock X, and stock contributes to consumption, it is never plausible that it is everywhere (i.e. $\forall X$) decreasing in stock. However, due to many factors (including that the stock dependence of the transition kernel being rather moderate), it is likely that regions of the state space over which the (expected) value function is strictly decreasing is small; in these cases, it can be shown that if the value fuction over this region is "small enough" then arguments from Lemma 2 continue to hold.

structure of the transition function and the nature of our problem. Our main result for this section is

Theorem 2. *For the renewable resource extraction problem considered here, under assumptions 1 and 2, equilibrium extraction, w^*, is increasing in the resource stock, x.*

We are unaware of any previous study in either the literature on regime shifts or on stock pollution control with hyperbolic discounting to have established that the action variable is monotonic in state. For instance, Brozović and Schlenker (2011), by virtue of special functional forms (e.g. linearity of transition and quadratic benefit function) find pollutant loading to be stock independent; Karp (2005) hypothesises–but is unable to establish– that carbon emissions may be decreasing in existing stock of carbon. In the terminology of Karp (2005), actions and stocks in our case are "strategic complements".

It is important to note that theorem 2 does not imply that next period stock, X_{t+1} is increasing in current stock X_t. Karp (2005) provides conditions under which the trajectory of the stock is a monotonic function of time but cannot preclude the less natural case of an oscillating stock trajectory. In our case, the equivalent (to "X_{t+1} increasing in X_t") notion is that the stochastic kernel $Q(.|a,x)$ (from eq. (8)), is increasing in X. A sufficient condition for this typically involves a Lipschitz condition on extraction (implying that $\dfrac{\partial w}{\partial X} \in (0,1)$) along with the transition being stock independent, which is not fulfilled in our case. It has been noted before that in resource extraction problems with exponential discounting, stock dependence can lead to many important properties, including lipschitz continuity of extraction not necessarily holding. Our findings here suggest that stock dependence leads to essentially the same complexity in reinvestment decisions in (quasi-) hyperbolic settings too.

4.2.2 Equilibrium extraction and the threshold

Perhaps the most interesting question in our framework is how the extraction decision is affected by the threshold level of stock, $\underline{X}$, below which we enter the less desirable region, with a lower resource growth regime. One may anticipate, for instance, that increases in the threshold, leading to enlarged adverse region of the state space, will lead to reduced extraction (or increased investment).

Before proceeding with the technical analysis, it is worth articulating the intuition regarding two key properties of a transition function encapsulating a regime shift. The first is that increases in the threshold reduce the probability of reaching a larger next-period stock.This property is equivalent to stating that the transition distribution, F_P, is stochastically decreasing in the threshold, $\underline{X}$.[26] The second one pertains to the substitutability between $(a, \underline{X})$ on the margin, and we assume, following previous discussion, that the marginal reinvestment rate exhibits a degree of 'decreasing returns'. We recall that commonly used functional forms for transition (e.g. that in Polasky et al. (2011);

[26]See definition 1 for details (see also lemma 1). The underlying intuition is possibly easier to grasp via an equivalent definition: the parameter $\underline{X}$ shifts F_P in the sense of stochastic dominance, meaning that increases in $\underline{X}$ shift probability mass to the left, putting more weight on lower values of resource stock.

Brozović and Schlenker (2011)) satisfy these two properties.[27] These assumptions, we reiterate, preclude neither monotonic nor non-monotonic extraction (loading).[28]

These intuitive properties are formalised next (see Appendix B.2 for the full problem set up). We begin first with some notation. For a partially ordered set $\underline{\mathcal{X}}$, with $\underline{X} \in \underline{\mathcal{X}} \subset \mathcal{R}^+$, we define the unique equilibrium extraction policy as $w^\star_{\underline{X}}$, explicitly focusing on the dependence between extraction and the threshold (suppressing the argument x in extraction for clarity). Our subsequent analysis stands upon the following

Assumption 3. *We assume:*

a. *u does not depend on $\underline{X}$ and satisfies Assumption 1.*

b. *for any $x, a \in \mathcal{X}$ and $\underline{X} \in \underline{\mathcal{X}}$, the transition probability Q has the structure in eq. (8), with the measure $P\left(.|a, x, \underline{X}\right)$ satisfying Assumption 1.*

c. *for every bounded function $V : \mathcal{X} \to \mathbb{R}^+$, the function $I(x, a, \underline{X}) := \int_{\mathcal{X}} V\left(x'\right) dF_P\left(dx'|a, x, \underline{X}\right)$ is decreasing on $\underline{\mathcal{X}}$ and is submodular in $(a, \underline{X})$*

Assumption 3c imples that $\mathbb{E}\left[V\right](a, \underline{X}) := \int_{\mathcal{X}} V\left(x'\right) dF_P\left(dx'|a, x, \underline{X}\right)$ satisfies decreasing differences in $(a, \underline{X})$,and is decreasing in $\underline{X}$. Clearly, the latter is intuitive: increases in the threshold lead to reduction of the expected value function by increasing the size of the undesirable region of state space. The former implies that for the expected value function, increases in a (reduction in extraction) are more valuable when $\underline{X}$ is smaller than when it is larger. Put another way, the marginal benefit (in terms of the expected value function) to reinvestment is larger at lower thresholds.

We are now ready to state our main result, which is

Theorem 3. *Let Assumption 3 be satisfied. Then, equilibrium extraction, $w^*_{\underline{X}}$, is increasing in the threshold, $\underline{X}$.*

The finding of a monotonic relationship between the threshold and extraction is somewhat unique in the literature. For instance, Brozović and Schlenker (2011) find a region of non-monotonic behaviour–with an initial decrease as the threshold increases and a subsequent increase. Continuous time frameworks with a time-based regime shift (such as in Polasky et al. (2011)) do not feature this behaviour at all. Our results suggest in fact that the presence of a threshold never leads to

[27] Consider the encompassing functional form in eq. (6): a key feature is separation between $\underline{\mathrm{X}}$ and (a, x) and the assumption that the growth function is either linear (Brozović and Schlenker (2011)) or that it depends upon either x or a but not both (e.g. Polasky et al. (2011)). The first assumption directly implies in the deterministic case that X_{t+1} is decreasing in $\underline{\mathrm{X}}$ (whose extension to the stochastic case in the first property detailed above). The first and the second together imply that the marginal returns to reinvestment are decreasing in threshold levels. In fact, Ren and Polasky (2014) assume that a regime shift would not only reduce the resource production but would also lower the gross rate of return of reinvestment.

[28] To illustrate, with a very similar structure of transition–in terms of the relationship between $(a, \underline{X})$, Brozović and Schlenker (2011) and Peterson et al. (2003) report opposing findings: the former finds (under certain conditions) "precautionary" behaviour while the latter finds no such behaviour.

"precautionary behaviour " i.e. a reduction in extraction with a hope of avoiding the regime shift in question. This is also what the simulation study by Peterson et al. (2003) reports, in an investigation of emission loading.

5 Regime shifts without lags

As mentioned earlier, it is far more common in the literature to consider ecosystems with very rapid dynamics, in which case current action determines the state of the system and threshold crossing. Therefore, regime shifts are conditioned on post-extraction stock (or reinvestment) a_t and the conditions on the distributions functions described in eq.(5) are modified as follows (with obvious modifications for the continuous version of the regime shift in eq. (7))

$$F\left(X_{t+1}|X_t = x, a_t = a\right) = \begin{cases} 0, & a \leq 0 \\ G_1\left(.|x,a\right), & a \leq \underline{X} \\ G_2\left(.|x,a\right), & \text{else} \end{cases} . \tag{13}$$

Following the logic embodied in eq. (13), the transition function from eq. (8) is modified as

$$Q\left(.|a,\underline{X}\right) = \left(1 - P\left(.|a,\underline{X}\right)\right)\delta_0 + P\left(.|a,\underline{X}\right). \tag{14}$$

We note that, just as for the more general transition function in eq. (5), different shapes for the growth functions may be accommodated by specifying different shapes for the measure P (distribution F_P) in terms of the reinvestment (and the threshold).

This formulation will turn out to offer additional insights, since we are also able to establish the existence of an Euler equation, which is helpful in characterizing a few more features of the resource extraction problem and to connect it to the newer literature on hyperbolic discounting and on regime shifts. Proofs of all results in this section are collected in Appendix B.3.

5.1 Uniqueness of Equilibrium extraction

As to basic results with this frame work, under Assumption 1, Theorem 1, pertaining to the existence of a unique equilibrium extraction policy, continues to hold. Theorem 4 is an extended version of Theorem 2. We note that for this formulation of the regime shift case, assumption 3(a) is taken to hold wherever appropriate and assumption 3(b) is taken to hold throughout (with eq. (14) replacing eq. (8)).

Theorem 4. *Under Assumption 1, for the stochastic renewable resource problem for a present-biased regulator with transition following eq. (14), the optimal time-consistent policy w^* is increasing in x and Lipschitz with modulus 1.*

Theorem 4 states that: optimal extraction and reinvestment policy are increasing in the current stock, continuously everywhere differentiable and the absolute value of their derivatives is bounded above by 1. These results have two important implications: first, the latter result, that $a^{'*}(x) \in [0,1]$, will turn out to play a key role in determining the effect of β on extraction policy; second, these two together imply that F_P is stochastically increasing in the current stock (through its effect on reinvestment), meaning that future stock is increasing in current stock. We note that ours is the first study to report this finding. The only directly comparable study examining this aspect for a problem similar to ours (evaluating the effect of hyperbolic discounting on resource-related questions without regime shifts), Karp (2005), while providing a sufficient conditions for the monotonicity of future stock in current stock, is unable to definitively establish this fact: this study is also silent regarding the monotonicity of the optimal policy in carbon stock.

5.2 The Euler Equation

In much of the prior literature focused on examining a wide variety of questions with present bias, including renewable resource management and climate change (e.g. Karp (2005); Karp and Tsur (2011)), analysis has proceeded via the so-called Euler equation. This approach is often the only way to establish existence of an equilibrium policy[29] and also has the benefit of allowing an analysis of key properties of the problem, such as monotonicity (which we analysed in section 4 for a more complex case). Often, the existence of an Euler equation follows from either simply assuming that the policy function is continuously differentiable (see e.g. footnote 5 or Appendix A of Karp (2005)) or from very special functional forms and settings (e.g. Gerlagh and Liski (2017)) leading to the existence of smooth Markovian equlibria. Theorem 4 establishes the Lipschitz continuity of the equilibrium extraction policy for our case. Using this, we are able to establish that our equilibrium policy is unique, interior and, along with the value function, continuously differentiable. Subsequently, we present a version of the so-called Generalised Euler Equation (GEE, or simply 'Euler equation') for our problem, which we use subsequently to explore in some detail the nature of the relationship between present bias and resource extraction.

A technical assumption pertaining to $D_V(a) = \beta\delta \int_{\mathcal{X}} V\left(x'\right) dF_P\left(dx'|a(x), \underline{X}\right)$ is required to establish the continuous differentiability of extraction policy and the value function. This condition follows from those commonly used in the literature (e.g. Harris and Laibson (2001)) and is discussed in Appendix B.3. With this condition (Assumption 4) in place, we are ready to establish the differentiability of $w^\star$ and $V^\star$.

Theorem 5. *Under Assumptions 1 and 4, the optimal time-consistent extraction policy w^* and value function V^* are differentiable on $(0, \infty)$.*

[29]See Harris and Laibson (2001) for a comprehensive analysis of the Generalised Euler Equation for the canonical stochastic growth model, which is a setting somewhat different from ours (see also Balbus et al. (2018)). As already discussed, we approach the problem differently, and also establish the existence and uniqueness of equilibrium before we tackle the question of differentiability.

With Theorem 5 on hand, we can now write our version of the Euler equation characterising the equilibrium reinvestment. We note for future reference that all further results build upon theorem 4 and hence Assumptions 1 and 4 are taken to hold henceforth. For simplicity, we merely provide here a brief outline of the derivation of the Euler equation, and refer the reader to Appendix B.3 for details. From theorem 4, it follows that $a^*(x) := x - w^*(x)$ is a continuously differentiable equilibrium reinvestment such that $a^{\star\prime} \in [0,1]$. Then we have

$$u'(x - a^*(x)) = \beta\delta \frac{d}{da(x)} \int_{\mathcal{X}} V^*\left(x'\right) dF_P\left(dx'|a^*(x), \underline{\mathrm{X}}\right). \tag{15}$$

From the definition of the value function we obtain

$$V'^*(x) = u'(x - a^*(x))(1 - a'^*(x)) + \delta a'^*(x) \frac{d}{da(x)} \int_{\mathcal{X}} V^*\left(x'\right) dF_P\left(dx'|a^*(x), \underline{\mathrm{X}}\right). \tag{16}$$

Integrating eq. (16) wrt to x (using Amir (1997, Lemma 4.2)), we obtain, after some simplification (and abbreviating $a^*(x_t)$, $a'^*(x_t)$ to a_t^*, $a_t'^*$ to simplify notation) , the Euler equation determining the equilibrium extraction,

$$u'\left(x_t - a_t^*\right) = \mathbb{E}_t\left[u'(x_{t+1} - a_{t+1}^*)\left(\beta\delta\left(1 - a_{t+1}'^*\right) + \delta a_{t+1}'^*\right)\right], \tag{17}$$

where $\mathbb{E}_t$ denotes expectation w.r.t. $F_P\left(.|a_t^*, \underline{\mathrm{X}}\right)$. The difference with its counterpart when discounting is exponential is that the constant exponential discount factor δ of the latter is replaced with the "effective discount factor", $EDF := \beta\delta\left(1 - a_{t+1}'^*\right) + \delta a_{t+1}'^*$. This effective discount factor is endogenous and stochastic (Harris and Laibson (2001)) as it depends on the marginal propensity to reinvest (MPR). Since $\beta < 1$, the effective discount factor is positively related to the *future* MPR, $a_{t+1}'^*$. Put more plainly, the EDF increasing in $a_{t+1}'^*$ means that the future is valued more as $a_{t+1}'^*$ increases (recall that increases in the discount factor imply the future is discounted less i.e. valued more).

The intuition behind this result is as follows: Since the regulator at time t values marginal reinvestment at $t+1$ more than the $t+1$ regulator does, the regulator at time t acts strategically in an integenerational game: the lower the expected marginal propensity to reinvest at $t+1$, the less the regulator values the future and the more he will extract in period t. In summary, the fact that a present biased regulator discounts the short-term more than the long term causes him to value future reinvestment higher than the future regulator will, leading to a strategic reduction in his current reinvestment (relative to the exponentially discounting regulator). As in the literature in macroeconomics, there is excess "consumption" (extraction) of resources. We emphasise that our Euler equation was derived by establishing the existence and uniqueness of a smooth equilibrium and value function, in a framework richer than that hitherto used in the literature considering resource and environmental questions (Karp (2005); Karp and Tsur (2011); Gerlagh and Liski (2017)).

5.3 Characterising Equilibrium Extraction

A closer inspection of eq. (17) yields additional insights regarding many aspects of the resource extraction problem: the effect of β, the short term discount factor, on the time consistent equilibrium level of extraction and reinvestment; and its interaction with the threshold, $\underline{X}$. As regards the first aspect, a fundamental question is whether in fact a regulator who is present-biased would extract more than he would were he to discount exponentially. We answer this question in the affirmative in

Proposition 1. *In the renewable resource extraction problem with regime shifts described in eq. (14), equilibrium extraction w^* is increasing with present bias.*

From the discussion immediately preceding Proposition 1, it is evident that a reduction in β, which implies greater present bias, leads to higher discounting of the short-run and therefore higher extraction today. Thus, it is straight forward to see that a present-biased regulator always extracts more in an equilibrium than an exponential discounter. The strategic incentives involved in arriving at this equilibrium however are worth discussing briefly, since they also highlight the role of the MPR. As already discussed, a reduction in β has two opposing effects: on the one hand, it implies higher discounting of the future by the time-t regulator, suggesting that it is optimal to extract more at time t. On the other hand, since the time-$(t+1)$ regulator discounts the (short-run) future with $\beta\delta$, he values reinvestment even less than before. In view of the divergence between the time-t and time-$(t+1)$ regulators' valuation of the future (in particular, of time-$(t+2)$), the time-t regulator will wish to manipulate the time-$(t+1)$ regulator to reduce extraction (at time-$(t+1)$). He can do so by reducing his own time-t extraction thereby increasing time-$t+1$ stock, in response to which (with a positive future MPR), the regulator at $(t+1)$ will increase reinvestment. The fact that $0 < a'^*_{t+1} < 1$ (from Theorem 4), however means that the overall effect of a reduction in β is an increase in extraction at time t. The argument holds for any t and thus, constitutes an equilibrium.

To summarize, while present bias induces a difference between time-t and time-$t+1$ regulators' valuation of the future, the overall effect of present bias upon extraction is unambiguous only when the MPR is between 0 and 1. While Karp (2005) provides a similar analysis, he can only hypothesize that emissions would be higher with a present biased regulator. The critical difference between our study and Karp (2005) lies in Theorem 4, where we are able to show that the MPR is positive and smaller than one.

Remark 1. *We note that Theorem 4, Proposition 1, and the Euler equation (eq. (17)), are all applicable for the case of $\underline{X} = 0$. In other words, whenever a stochastic renewable resource extraction problem (without regime shifts) with present-biased agents has a transition function that can be expressed as in eq. (14) (i.e. whenever the transition only depends upon reinvestment), these three results (along of course with Theorems 1, 2) hold. Since a transition function independent of stock is the most common one used in the literature, our results here suggest that for the most common renewable resource problems with simple transition functions (e.g. linear or logistic in reinvestment), the unique time-consistent equilibrium extraction is increasing in the stock and the degree of present* *bias, and an Euler equation (as in eq. (17)) can be written down. We are unaware of results of this degree of generality (with no specific utility or transition function) regarding renewable resource problems with random growth for a present-biased decision maker.*

We turn to next to understanding what effect a regime shift has on extraction. This investigation parallels the one leading to Theorem 3, and turns out to depend upon precisely the same interaction between bio-physical considerations and economic considerations. In other words, the key consideration is whether the marginal returns to reinvestment (discussed in section 4.2.2) is increasing or decreasing with the threshold. Using the Euler equation, we provide an answer for either case (meaning that $\mathbb{E}_t\left[V\right](a, \underline{X})$ can be sub- or super-modular in $(a, \underline{X})$), in

Proposition 2. *In the renewable resource extraction problem with regime shifts described in eq. (14), equilibrium extraction, w^*, is increasing (decreasing) with the threshold, $\underline{X}$, whenever the marginal return to reinvestment is decreasing (increasing) with $\underline{X}$.*

The intuition behind this result is as follows: while the presence of a non-zero critical threshold $\underline{X}$ unambiguously reduces the expected future welfare stream, the optimal response to changes in the threshold critically depends on the nature of the regime shift, in particular on the relationship between the marginal effect of reinvestment and the threshold. In essence, when reinvestment is less (more) "productive" (on the margin) the more adverse the environmental conditions are (represented by a high $\underline{X}$), it is optimal to reduce (increase) reinvestment and increase (decrease) extraction. Notice that this effect is present even with standard exponential discounting: present bias only affects the magnitude of the effect.

While Propositions 1 and 2 established the relationship between extraction and the effect of present bias and the threshold respectively in isolation, it is of interest to understand whether their combined effects are, roughly speaking, additive. An examination of this relationship calls for some structure on the relationship of $(a, \underline{X})$ to F_P, and we assume, as in section 4.2.2, that marginal returns to reinvestment are decreasing with the threshold (i.e. that assumption 3(c) holds). Under these conditions, we summarise our findings in the following

Proposition 3. *For the renewable resource extraction problem with regime shifts described in eq. (14), the effect of present bias on extraction, on the margin, is lower at higher threshlold levels.*

Put simply, this means that a lower β leads to a smaller increase (than otherwise) in optimal extraction when regime shifts are more prominent. Similarly, more prominent regime shifts lead to a smaller increase on optimal extraction when faced with a very impatient regulator. More heuristically, then, this result may be interpreted as suggesting that the effect of increased "adverse" effects in both the threshold and present bias is not additive. [30]

[30] We note that while the dependence of optimal policy upon the discount factor has been analyzed for exponential discounting (Amir et al 1991, Amir 1996), we are unaware of any exploration of these aspects for the case of present biased discounting and definitely not for resource problems, with or without regime shifts.

Remark 2. *It is instructive to compare how bio-physical and purely economic effects interact to drive our findings, following a similar attempt in Ren and Polasky (2014) who provide conditions for what they term "aggressive management" of ecosystems. We note that our propositions 2 and 3 characterise the conditions for aggressive management. What is pertinent is that only the "bio-physical" investment effect-the marginal benefit to reinvestment,* $\frac{\partial F_P(dx'|a(x),\underline{X})}{\partial a(x)\partial \underline{X}}$ *- determines the direction of the effect of the threshold upon extraction. Unlike in Ren and Polasky (2014), "consumption smoothing" (captured by* u'*), which accounts for the varying value of extraction and the "economic investment effect" as captured by the MPR,* $a^{*\prime}$*, only scale the bio-physical effect but play no role in determining the direction of changes in extraction. From proposition 2, we find that if the bio-physical investment effect is decreasing in the threshold, the forward looking regulator reacts by increasing extraction at higher thresholds while proposition 3 suggests that the effect of an increase in present bias and the threshold ameliorate one another, in that they lead to a reduction in marginal extraction by the forward looking regulator.*

6 Discussion

Our work intersects with at least two distinct strands of literature concerned with resource extraction and pollution control, including carbon emission mitigation: the first concerned with the effect of regime shifts and the second related to examining the effects of non-exponential discounting, largely without regime shifts. It is consequently interesting to relate our findings–broadly– to relevant studies in both strands of the literature. We turn first to the strand of literature related to regime shifts with exponential discounting. Polasky et al. (2011) consider a a continuous-time renewable resource model with regime shifts modelled as changes in the system dynamics.The timing of the regime shift is stochastic, possibly depending upon the resource stock. They find that only in the case of endogenous regime shifts does the optimal policy become "precautionary", in the sense that the optimal harvest is reduced as a result of a potential regime shift. No precautionary action is taken for exogenous regime shifts while in the case of a stock collapse, it is in fact optimal to increase harvesting. Brozović and Schlenker (2011) model regime shifts with discrete-time equations of motion that differ by an additive term once (a possibly unknown) reversible threshold is crossed. They report a non-monotonic relationship between precautionary activity and uncertainty of two types: exogenous uncertainty, embedded in the system dynamics over which the policy maker has no control, and endogenous uncertainty about the threshold location. An increase in uncertainty of any type may first increase "precautionary behaviour", in terms of reduced pollutant loading, but will always eventually reduce it. Moreover, they find that optimal loading changes non-linearly as the critical threshold changes.[31] A common feature of these models is linearity in the benefits from harvesting. When allowing for a concave utility function and a more general resource growth function,

[31]For extremely low or extremely high thresholds, the probability of moving in or out of any region is very low and practically unaffected by precaution activities. Instead, for intermediate levels of threshold, there is a possibility of moving between states in either direction and additional precautionary activity has a potential economic benefit: expected pollutant stock will drop as we get closer to the threshold.

Ren and Polasky (2014) and de Zeeuw and He (2017) find that even in the case of endogenous regimes shifts with altered system dynamics, precautionary behaviour is not guaranteed.

Our results are quite different. In our discrete-time stochastic framework, regime shifts are modelled as changes in the distribution of the future stock level (with a strictly positive probability of a resource collapse), and we consider both types of regime shifts previously investigated, those with a lag between threshold crossing and shift to a new regime and those without such a lag. For both cases, we find that extraction is always increasing in the stock. For regime shifts occurring immediately after threshold crossing, we find reinvestment increasing in the stock level, a result that has not to our knowledge been report before (even with exponential discounting and regime shifts). Karp (2005), examining the case of carbon emission reduction by a quasi-hyperbolic regulator without regime shifts, characterises the conditions under which this condition may hold, without establishing that it does hold for the case considered there. Regarding extraction and the critical threshold, we show that optimal extraction is in fact increasing in the threshold: the larger the undesirable region becomes, the more the regulator would like to extract. In contrast, as already discussed, Brozović and Schlenker (2011) find a non-monotonic relationship between optimal pollutant loading and the threshold stock level. While the differences in frameworks make a direct comparisons difficult, our results overall suggest that there is no precautionary behaviour, in the sense of reduced extraction, when faced with a regime shift.

As to the second strand, non-constant discounting in long lived environmental problems with or without regime shifts has not received much attention. Karp (2005) introduces hyperbolic discounting in a deterministic discrete-time stock pollutant model for examining carbon emission policies. Unlike our study, neither uniqueness of equilibrium policy nor its monotonicity and other properties are estabilished. Gerlagh and Liski (2017) study the optimal climate and capital policy in a multi-sector deterministic growth framework, but in a setting with no regime shifts and with very specific functional forms. The only study directly comparable to ours is Karp and Tsur (2011), which examines the implications of hyperbolic discounting for climate change policy when the probability of a climate induced catastrophe depends on the stock of greenhouse gases. This model, however, is simplified for tractability, and differs from ours in many ways. They restrict attention to irreversible events where an environmental catastrophe leads to a permanent loss of income, while the level of hazard (and hence the probability of the event) cannot be reduced. Our analysis deals both with the case of abrupt events and a change in the dynamics of the economy, and the probability of either event can be controlled. They also restrict the action space to a binary choice: the planner can either abate and stabilise the hazard level or follow business as usual, letting the hazard reach its maximal level. These specifics enable them to obtain a closed form solution for the equilibrium of this game. Nonetheless, little is known regarding uniqueness or other qualitative properties of the equilibria in their framework. In contrast, using a model with unrestricted action space and for events involving abrupt changes leading changed system dynamics, we are able to uniquely characterise the equilibrium policy and establish many qualitative properties, including monotonicity in both stock and threshold. Ours appears to be the first study to establish unambiguously that increases in present bias lead to increased resource consumption (extraction).

A further insight we are able to offer relates to the interaction between economic (represented by the marginal propensity to reinvest, MPR) and biophysical (the marginal benefit to reinvestment, MBR) aspects relating to reinvestment. As the MPR is bounded in the unit interval, we find that a forward looking regulator will, on the margin, extract more "aggressively" if the MBR is decreasing in resource reinvestment (while the effect of reduced stock upon consumption merely scales this effect). Trade-offs resembling ours were reported by Ren and Polasky (2014) in a set-up with no uncertainty in growth and exponential discounting.

7 Conclusions

While there is no consensus regarding the appropriate approach to discounting over long time horizons, experimental, empirical and conceptual contributions all suggest that there is little evidence for constant discounting. However, non-constant discounting poses the challenge of time-consistency: a policy appearing optimal from one generation's perspective may not appear so from the standpoint of a subsequent generation. This is particularly relevant for issues related to resource management problems, including climate change, that involve very long time horizons. Policy making must consider this aspect since current policy makers have typically few devices to bind future policy makers' decisions. In this paper, we investigate the decision process that forward looking present biased (quasi-hyperbolic) resource users face when they manage a renewable resource stock subject to growth shocks with the added threat of a regime shift, represented by altered system dynamics. We allow for regime shifts to occur either immediately the stock falls below the threshold or with a short lag.

We are able to establish that an equilibrium not only exists but is unique and time consistent. Notably, no special functional form for utility is required to establish our key results, and assumptions made regarding stock evolution are closely related to those made in the prior literature. We find, in contrast to much of the previous literature, that there is no precautionary behaviour, in that the possibility of a regime shift does not lead to reduced extraction with a view to preventing a potential shift. To the contrary, increased thresholds in most cases lead to increased extraction, the only exception being the case when the marginal benefit of reinvestment is increasing in the threshold and where the natural system dynamics exhibit no lag. In this case, the forward looking regulator will optimally reduce extraction in response to an increase in the threshold. This response arises due to the interaction between economic and ecological features, represented respectively by the marginal propensity to reinvest (MPR) and the marginal benefit to reinvestment (MBR). When the MBR is increasing in the threshold (meaning that the benefits to reinvestment are larger at higher thresholds) and the MPR is positive and less than one, present-biased agents today reason thus: if they increase extraction today then future agents respond by reducing reinvestment, which leaves all generations thereafter worse off. The only way to prevent this is to reduce extraction this period.

Our set-up is general enough to be used to investigate our problems of regime-shifts and time inconsistencies but could be useful for general resource extraction problems, and more common time-consistent preferences as well, including recursive utility approaches. Many problems of pollution control, resource management, and drug control (see e.g. Grass et al. (2008)), to name but a few could be approached in similar ways. While our approach has accommodated a few aspects of abrupt shifts, much is still unknown regarding the effects of irreversible shifts with non-exponential discounting, cases where changes to a regime affect utility more directly, or where the threshold is known only with some uncertainty (as is likely in reality for many ecosystems). In addition, questions of welfare and policy even without abrupt shifts are challenging, with new considerations arising about many aspects including commitment mechanisms.

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Appendix A Technical Appendix

Let $F(.|\theta_1, \theta_2)$ represent a distribution function for a random variable Y defined on $D \subset \mathbb{R}_+$, parameterised by scalars θ_1, θ_2 (defined on bounded subsets of $\mathbb{R}_+$ for simplicity), and v be a bounded function defined on $\mathbb{R}_+$.

Definition 1. *(Stochastically decreasing) The random variable Y is stochastically decreasing in parameter θ_1 if, for all $b > 0$, increases in θ_1 lead to reduced probability of obtaining b. In other words, if Y is decreasing in θ_1 then $Pr(Y > b|., \theta_1) \equiv 1 - F(b|., \theta_1)$ is decreasing in θ_1.*

Remark 1. *We note that definition 1 implies that, X_{t+1}, distributed as $F(y|x, a, \underline{X})$, is stochastically decreasing in the parameter $\underline{X}$ if increases in $\underline{X}$ lead to reduced next period stock i.e. if Pr$(x_{t+1} > b|x, a) \equiv 1 - F(b|x, a, \underline{x})$ is decreasing in $\underline{X}$.*

We provide next a very general result providing an operational definition of the concepts of stochastically decreasing, convex and sub-modular. Denote by $\{F_\theta(s); \theta \in \Theta\}$, with Θ the parameter space for a vector of parameters θ, a parameterised collection of distribution functions. We note that θ is taken to be a scalar for monotonicity and convexity considerations in lemma 1.

Lemma 1. *The collection of distribution functions, $\{F_\theta(s); \theta \in \Theta\}$ is stochastically increasing (decreasing), convace (convex) or supermodular (submodular) in θ iff $1 - F_\theta$ (viewed purely as a function of θ) is increasing (decreasing), convace (convex) or supermodular (submodular) in θ.*

Definition 2. *A deterministic function G satisfies decreasing (increasing) differences in scalar arguments (a, x) if increases in one argument are less (more) valuable when more of the other is available i.e. that $G(., x') - G(., x)$ is decreasing (increasing) in a, for $x' > x$. When G is continuously differentiable w.r.t. both arguments, the condition simplifies to $\frac{\partial^2 G}{\partial a \partial x} \leq (\geq) 0$. Clearly, this is also the definition of submodularity (supermodularity), to which decreasing (increasing) differences is equivalent (for the case of scalar a and x only). See Amir (2005) for an intuitive discussion.*

Remark 2. *Lemma 1 and definition 2 make it possible to provide an intuitive interpretation of the notion of stochastic decreasing differences. A random variable distributed as F, depending upon parameters e.g. (a, x) satisfies stochastically decreasing differences (is stochastically submodular) if $\tilde{F}$ satisfies decreasing differences in (a, x). For $\tilde{F}$, decreasing differences means that $\mathbb{P}(X_{t+1} > b|x', .) - \mathbb{P}(X_{t+1} > b|x, .)$ is decreasing in a, for $x' > x$. But this simply means that the effect of a upon $\mathbb{P}(X_{t+1} > b|x', .)$ is smaller than upon $\mathbb{P}(X_{t+1} > b|x, .)$. As for the deterministic case, then, the "marginal returns" to increased stock are decreasing in reinvestment, a.*

We next introduce a lemma, which will form the cornerstone of our analysis. This result provides a set of sufficient conditions to verify many claims which relate parametric properties of a distribution function, $F(.|\theta_1, \theta_2)$, to expected value of a function, v, w.r.t to it (with θ_1 being a scalar parameter and θ_2 being a vector, two-dimensional in our case). All integrals defined hence forth will be assumed to satisfy the usual conditions (i.e. they exist, are well-defined and finite). We note that lemma 1, which is well known in the literature (Lemma 3.9.1, Topkis (2011)), provides verifiable conditions upon the distribution function $F(.|\theta_1, \theta_2)$, allowing properties defined in lemma 2 to be easily verified. Applicability to our case can be directly seen by identifying $F(.|\theta_1, \theta_2)$ with $F_P(.|a, x, \underline{X})$ and the expectation of interest with $\mathbb{E}(V) := \int_{\mathcal{X}} V(x') \, dF_P(dx'|a, x, \underline{X})$.

Lemma 2. *For $v(s)$ increasing, if $F(s|\theta_1, \theta_2)$ is*

1. *stochastically increasing (decreasing) in θ_1;*
2. *stochastically concave in θ_1;*
3. *stochastically supermodular (submodular) in θ_2;*

then it is the case that $I(\theta_1, \theta_2) := \int_{\mathcal{X}} v(s) \, dF(s|\theta_1, \theta_2)$ is

1. *increasing (decreasing) in θ_1 (for fixed θ_2);*
2. *concave in θ_1 (for fixed θ_2);*
3. *satisfies increasing (decreasing) differences in θ_2 (for fixed θ_1);*

Proof. see lemma 3.9.1, Topkis (2011). □

Remark 3. *Note that lemma 2 is most useful in applications where distribution function properties are known, but the function v cannot be explicitly written down. In particular, the only property required of the unknown function v is its non-decreasing nature.*

Appendix B Proofs

B.1 Dynamic Program and Operators

The objective of the decision maker at time t can be represented by

$$U(q_t) + \beta \mathbb{E}_t \left(\sum_{t+1}^{\infty} \delta^{i-t} U(q_i) \right), \tag{B.1}$$

where $0 < \beta \leq 1$ and $0 < \delta < 1$, u is an instantaneous utility function and $\mathbb{E}_t$ is the expectation taken w.r.t. the time-t distribution of X_t.

The dynamic program associated with the optimisation problem with quasi-hyperbolic discounting is defined analogous to the exponential discounting case. We denote by $w \in \mathcal{A} := \{w : \mathcal{X} \to A; \text{w bounded}, w(x) \in A(x)\}$ a (pure strategy) SMNE for a quasi-hyperbolic agent that satisfies the functional equation

$$w(x) \in \arg\max_{q \in A(x)} \left[U(q) + \beta\delta \int_{\mathcal{X}} V_w(x') \, Q(dx'|x - q, x) \right], \tag{B.2}$$

where $V_w : \mathcal{X} \to \mathbb{R}^+$ is the continuation value function for the household of future selves. If such strategies exist (i.e. if $.\mathcal{A} \neq \emptyset$), they are time-consistent. The continuation value function satisfies the following recursion,

$$V_w(x) = U(w(x)) + \delta \int_{\mathcal{X}} V_w(x') Q(dx'|x - q(x), x). \tag{B.3}$$

Now, defining the value function for the self at time period t to be

$$W_w(x) = U(w(x)) + \beta\delta \int_{\mathcal{X}} V_w(x') Q(dx'|x - q(x), x), \tag{B.4}$$

one obtains the relation

$$V_w(x) = \frac{1}{\beta} W_w(x) - \frac{1-\beta}{\beta} U(w(x)). \tag{B.5}$$

Equation (B.5) is the generalised Bellman equation. Any fixed point, $V^\star$, of an appropriate operator (defined below) upon the vector space comprising of function V_w corresponds to the value function of a time-consistent SMNE.

Consider a vector space, $\mathcal{V}$, of bounded (in the sup-norm), positive functions defined on the state space, $\mathcal{X}$. For $V \in \mathcal{V}$, $\beta \in (0,1]$ and $x \in \mathcal{X}$, we define an operator T as

$$TV(x) = \frac{1}{\beta} AV(x) - \frac{1-\beta}{\beta} u(BV(x)), \tag{B.6a}$$

with the operators A and B defined as

$$AV(x) = \max_{q \in A(x)} \left[u(q) + \beta\delta \int_{\mathcal{X}} V(x') Q\left(dx^{'}|x - q, x\right) \right], \tag{B.6b}$$

$$BV(x) = \arg\max_{q \in A(x)} \left[u(q) + \beta\delta \int_{\mathcal{X}} V(x') Q\left(dx^{'}|x - q, x\right) \right]. \tag{B.6c}$$

With the basic framework set up, we proceed with the proofs of our main theorems.

B.2 Regime shifts depending upon pre-extraction Stock

Monotonicity of Policies

Proof of Theorem 2

Let $w^* = BV^*$ and $V^* = TV^*$. Consider the function

$$G(q, x, V^*) = u(q) + \beta\delta \int_{\mathcal{X}} V^*\left(x^{'}\right) dF_P\left(dx^{'}|x - q, x\right). \tag{B.7}$$

Observe that G is supermodular in q on a lattice $[0, x]$, the feasible action set, which is also increasing in Veinott's strong set order. It is easy to see that submodularity of $I(x, a) := \int_{\mathcal{X}} V(x') dF_P(dx'|a, x, \underline{X})$

in (a, x) implies supermodularity in (q, x) (since $a = x - q$), which combined with $I(x, a) := \int_{\mathcal{X}} V\left(x'\right) dF_P\left(dx'|a, x, \underline{X}\right)$ being concave in a suffices for $G(q, x, V^*)$ to be supermodular in (q, x). Then, by a standard result in parametric optimisation (Theorem 2.8.1,Topkis (2011)), $w^\star(x, V^\star) := \arg\max_{q \in A(x)} G(q, x, V^\star)$ is increasing in x on $\mathcal{X}$.

Parametric Monotonicity

First, we modify the operators defined above to explicitly allow them to depend upon the parameter $\underline{X}$. $T_{\underline{X}}$ is defined, as before, on $\mathcal{V}$:

$$T_{\underline{X}}V(x) = \frac{1}{\beta} A_{\underline{X}} V(x) - \frac{1-\beta}{\beta} u(B_{\underline{X}} V(x))$$

where the pair of operators $A_{\underline{X}}$ and $B_{\underline{X}}$, also defined on $\mathcal{V}$ but with Q parameterised additionally by $\underline{X}$, are given by :

$$A_{\underline{X}} V(x) = \max_{q \in A(x)} \left[u\left(q\right) + \beta\delta \int_{\mathcal{X}} V\left(x'\right) Q\left(dx^{'}|x - q, x, \underline{X}\right) \right],$$

$$B_{\underline{X}} V(x) = \arg\max_{q \in A(x)} \left[u\left(q\right) + \beta\delta \int_{\mathcal{X}} V\left(x'\right) Q\left(dx^{'}|x - q, x, \underline{X}\right) \right].$$

We provide the following Lemma that will be useful in the proof of the next Theorem.

Lemma 1. Let $\phi : \mathcal{X} \times \underline{\mathcal{X}} \mapsto \mathbb{R}^+$ be a bounded function for each $\underline{X} \in \underline{\mathcal{X}}$ and $\phi(x, \cdot)$ is decreasing for each $x \in \mathcal{X}$. Then $\underline{X} \mapsto T_{\underline{X}}(\phi(\cdot, \underline{X}))(x)$ is a decreasing function of $\underline{X}$.

Proof. It is easy to see that for every bounded function V, Assumption 3 guarantees that a mapping $A_{\underline{X}}(V)$ is a decreasing function. Next we show that $B_{\underline{X}} V(x)$ is increasing in $\underline{X}$. For each $x \in \mathcal{X}$, let us define

$$G(q, V, \underline{X}) := u\left(q\right) + \beta\delta \int_{\mathcal{X}} V\left(x^{'}\right) P\left(dx^{'}|x - q, x, \underline{X}\right) \tag{B.8}$$

Then, for $q_1 < q_2 \leq x$, we have

$$\begin{aligned} G(q_2, V, \underline{X}) - G(q_1, V, \underline{X}) :=& u(q_2) - u(q_1) \\ &+ \beta\delta \int_{\mathcal{X}} V\left(x^{'}\right) P\left(dx^{'}|x - q_2, x, \underline{X}\right) - \beta\delta \int_{\mathcal{X}} V\left(x^{'}\right) P\left(dx^{'}|x - q_1, x, \underline{X}\right) \end{aligned}$$

From Assumption 3, we know that the function $I(x, a, \underline{X}) := \int_{\mathcal{X}} V\left(x'\right) dF_P\left(dx'|a, x, \underline{X}\right)$ has decreasing differences with $(a, \underline{X})$ so that the RHS of the above expression is increasing in $\underline{X}$ (note that $a = x - q$) which implies that $G(q, V, \underline{X})$ has increasing differences in $(q, \underline{X})$.(from the definition of increasing differences). Then by Theorem 2.8.1 in Topkis (2011), the $B_{\underline{X}} V(x) =$

$\arg\max_{q\in A(x)}[G(q,V,\underline{X})]$ is increasing in $\underline{X}$. By Lemma 2 in Balbus(2016) $V \mapsto B_{\underline{X}}(V)$ is decreasing in V. As a result, $B_{\underline{X}}(\phi(\cdot,\underline{X}))$ is increasing in $\underline{X}$ and hence $\underline{X} \mapsto T_{\underline{X}}(\phi(\cdot,\underline{X}))$ decreasing in $\underline{X}$ for any $x \in \mathcal{X}$. □

We are now ready to prove Theorem 3.

Proof of Theorem 3.

Observe that by Theorem 1, $V^*_{\underline{x}}(x) = \lim_{n\to\infty} T^n_{\underline{X}}(\mathbf{0})(x)$ (where $\mathbf{0}$ is a zero function). Applying Lemma 1, we see $T_{\underline{X}}(\mathbf{0})(x) \in \mathcal{V}$ and this expression is decreasing in $\underline{X}$. Applying Lemma 1 again, we can show that all $T^n_{\underline{X}}(\mathbf{0})(x)$ are decreasing in $\underline{X}$, hence $V^*_{\underline{X}}(x)$ is decreasing in $\underline{X}$. Finally, observe that $w^*_{\underline{X}}(x) = B_{\underline{X}}(V^*_{\underline{X}})(x)$ and using similar arguments as in the proof of Lemma 1, we find that $w^*_{\underline{X}}(x)$ is decreasing in $\underline{X}$.

An inspection of the proof clarifies that our results hold with weak submodularity of $I(x,a,\underline{X}) := \int_{\mathcal{X}} V(x')\, dF_P(dx'|a,x,\underline{X})$ in $(a,\underline{X})$. In particular, by Topkis (2011, Theorem 2.8.1), $B_{\underline{X}}V(x) = \arg\max_{q\in A(x)}[G(q,V,\underline{X})]$ is constant in $\underline{X}$ but Lemma 1 still holds.

B.3 Regime shifts depending upon post-extraction Stock

We begin by stating a set of technical conditions that will be useful for our analysis. For every bounded, positive and increasing function $V : \mathcal{X}R^+$, let $F_V(a) = \beta\delta \int_{\mathcal{X}} V\left(x'\right) dF_P\left(dx'|a(x),\underline{X}\right)$.

Assumption 4. *We assume :*

a. u is twice continuously differentiable

b. the function F_V is twice continuously differentiable on $\mathcal{X} \setminus \{0\}$

c. $\lim_{q\to 0} u'(q) = \infty$ and $\lim_{a\to 0} D'_V(a) = \infty$

d. $|u''| > 0$ and $|D''_V| > 0$ for any $x \in \mathcal{X}$

These are standard assumptions. 4(b) can be shown to follow from those commonly used in the literature e.g. Assumption F2 of Harris and Laibson (2001, §4), (along with the interchange of integral and derivative) whenever the transition function has a density, which it does in our case.[32] The first part of 4(c) is merely (one part of) the Inada condition, while the second part states that the marginal valuation of reinvestment for the expected value function is very high for low values of investment, which is a very reasonable assumption for natural resources (i.e. the last

[32]We note that F_V is defined w.r.t F_P for reasons detailed in footnote 20. Consequently, the atom at 0 stock of Q in no way affects the differentiability of F_P (even ignoring the fact that 0 stock is excluded from the domain over which the derivatives of F_V are required).

fish left unharvested has a very high value, since its absence ensures that no stock remains for the entire future). Together, they ensure that the optimal policy is interior. Finally, 4(d) is a standard condition in many dynamic models allowing continuous differentiation (via the use of an implicit function theorem).

Proof of Theorem 4 Let $w^* = BV^*$ and $V^* = TV^*$. Consider the function

$$G(q, x, V^*) = u(q) + \beta\delta \int_{\mathcal{X}} V^*(x^{'})dF_P(dx^{'}|x - q, \underline{X}). \tag{B.9}$$

Observe that G is supermodular in q on a lattice $[0, x]$, the feasible action set, which is also increasing in Veinott's strong set order. Moreover, by concavity of $I(a, \underline{X}) := \int_{\mathcal{X}} V\left(x'\right) dF_P\left(dx'|a, \underline{X}\right)$ in a, we can show that $G(q, x, V^*)$ is supermodular in (q, x). Then, by a standard result in parametric optimisation (Theorem 2.8.1,Topkis (2011)) , $w^\star(x, V^\star) := \arg\max_{q\in A(x)} G(q, x, V^\star)$ is increasing in x on $\mathcal{X}$.

Next, we rewrite the problem as function of investment a

$$H(a, x, V^*) = u(x - a) + \beta\delta \int_{\mathcal{X}} V^*(x^{'})dF_P(dx^{'}|a, \underline{X}). \tag{B.10}$$

where H is now supermodular in the choice variable a on a lattice $[0, x]$, the feasible action set, which is also increasing in Veinott's strong set order. Then, by concavity of u, it is evident that $H(a, x, V^*)$ is supermodular in (a, x), therefore by Topkis(1978) theorem, the optimal solution a^* is increasing in x on $\mathcal{X}$.

Observe that since $a^* = x - w^*$ and both a^* and w^* are increasing in x, it follows that a^* and w^* are Lipschitz with modulus 1.

Euler Equation

From theorem 4, it follows that reinvestment, $a^* = x - w^*$, is continuously differentiable. We can then write the GEE characterizing the SMNE investment $a^*(x)$:

$$u'(x - a^*(x)) = \beta\delta \frac{d}{da(x)} \int_{\mathcal{X}} V^*\left(x^{'}\right) dF_P\left(dx^{'}|a^*(x), \underline{\mathsf{X}}\right) \tag{B.11}$$

where

$$V^*(x) = u(x - a^*(x)) + \delta \int_{\mathcal{X}} V^*\left(x^{'}\right) dF_P\left(dx^{'}|a^*(x), \underline{\mathsf{X}}\right).$$

so that

$$V'^*(x) = u'(x - a^*(x))(1 - a'^*(x)) + \delta a'^*(x)\frac{d}{da(x)} \int_{\mathcal{X}} V^*\left(x^{'}\right) dF_P\left(dx^{'}|a^*(x), \underline{\mathsf{X}}\right). \tag{B.12}$$

Next we multiply (B.12) with $\frac{\partial F_P\left(dx|a(s), \underline{\mathsf{X}}\right)}{\partial a(s)}$ and integrate wrt to x. To simplify notation, we drop * from V^* and a^*.

$$\int_{\mathcal{X}} V'(x)\frac{\partial F_P\left(dx|a(s),\underline{\mathsf{X}}\right)}{\partial a(s)}dx = \int_{\mathcal{X}} u'(x-a(x))(1-a'(x))\frac{\partial F_P\left(dx|a(s),\underline{\mathsf{X}}\right)}{\partial a(s)}dx \tag{B.13}$$
$$+\,\delta\int_{\mathcal{X}} a'(x)\frac{d}{da(x)}\int_{\mathcal{X}} V\left(x^{'}\right)dF_P\left(dx^{'}|a(x),\underline{\mathsf{X}}\right)\frac{\partial F_P\left(dx|a(s),\underline{\mathsf{X}}\right)}{\partial a(s)}dx$$

Using integration by parts and Lemma 4.2 from Amir (1997) (See also proof of theorem 3.2), we can show that

$$\frac{d}{da(x)}\int_{\mathcal{X}} V\left(x^{'}\right)dF_P\left(dx^{'}|a(x),\underline{\mathsf{X}}\right) = -\int_{\mathcal{X}} V'\left(x^{'}\right)\frac{\partial F_P\left(dx^{'}|a(x),\underline{\mathsf{X}}\right)}{\partial a(x)} \tag{B.14}$$

Then using (B.11) and (B.14), we can write (B.13) as

$$\begin{aligned} u'(s-a(s)) = &-\beta\delta\int_{\mathcal{X}} u'(x-a(x))(1-a'(x))\frac{\partial F_P\left(dx|a(s),\underline{\mathsf{X}}\right)}{\partial a(s)}dx \\ &-\delta\int_{\mathcal{X}} u'(x-a(x))a'(x)\frac{\partial F_P\left(dx|a(s),\underline{\mathsf{X}}\right)}{\partial a(s)}dx \end{aligned}$$

Simplifying the above expression we get

$$u'(x-a(x)) = -\beta\delta\int_{\mathcal{X}} u'(x'-a(x'))(1+(\frac{1}{\beta}-1)a(x'))\frac{\partial F_P(dx'|a(x),\underline{\mathsf{X}})}{\partial a(x)} \tag{B.15}$$

Proof of Proposition 1.

We differentiate the right hand side of (B.15) with respect to β to get

$$-\,\delta\int_{\mathcal{X}} u'(x'-a(x'))(1+(\frac{1}{\beta}-1)a(x'))\frac{\partial F_P(dx'|a(x),\underline{\mathsf{X}})}{\partial a(x)}+ \tag{B.16}$$
$$\frac{\delta}{\beta}\int_{\mathcal{X}} u'(x'-a(x'))a(x'))\frac{\partial F_P(dx'|a(x),\underline{\mathsf{X}})}{\partial a(x)} = \delta\int_{\mathcal{X}} u'(x'-a(x'))(a(x')-1)\frac{\partial F_P(dx'|a(x),\underline{\mathsf{X}})}{\partial a(x)} > 0$$

by Assumption 1 and Theorem 4. Then, concavity of the utility function implies that a decrease in β from $\beta = 1$ to $\beta < 1$ would result in an increase in the equilibrium level of extraction. Recalling that a reduction in β implies increased present bias, it follows that an increase in present bias leads to increased extraction.

Proof Proposition 2.

We differentiate the right hand side of (B.15) with respect to $\underline{\mathsf{X}}$ to get

$$-\beta\delta \int_{\mathcal{X}} u'\left(x' - a(x')\right)\left(1 + \left(\frac{1}{\beta} - 1\right) a(x')\right) \frac{\partial F_P(dx'|a(x), \underline{\mathsf{X}})}{\partial a(x)\partial \underline{\mathsf{X}}} \tag{B.17}$$

where the sign of the expression depends on the sign of $\dfrac{\partial F_P\left(dx'|a(x), \underline{\mathsf{X}}\right)}{\partial a(x)\partial \underline{\mathsf{X}}}$.

Proof of Proposition 3.

Differentiation of eq. (B.16) with respect to $\underline{\mathsf{X}}$ yields

$$\delta \int_{\mathcal{X}} u'\left(x' - a^*(x')\right)\left(a'^*(x') - 1\right) \frac{\partial F_P(dx'|a(x), \underline{\mathsf{X}})}{\partial a(x)\partial \underline{\mathsf{X}}}. \tag{B.18}$$

Since $\frac{\partial F_P(dx'|a(x), \underline{\mathsf{X}})}{\partial a(x)\partial \underline{\mathsf{X}}} > 0$ from assumption 3, it follows that eq. (B.18) is negative.

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