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Abstract

We develop a model to rationalize and examine so-called "research bubbles", i.e. research activities based on overoptimistic beliefs about the impact of this research on the economy. Research bubbles occur when researchers self-select into research activities and the government aggregates the assessment of active researchers on the way advances in research may spur innovation and growth. In an overlapping generations framework, we study the occurrence of research bubbles and show that they tend to be welfare-improving. Particular forms can even implement the socially optimal solution. However, research bubbles can collapse, and we discuss institutional devices and the role of debt financing that ensure the sustainability of such bubbles. Finally, we demonstrate that research bubbles emerge in various extensions of our baseline model.

Keywords: Endogenous Growth, Basic Research and Macro-Based Behavioral Economics

JEL Classification: E02, E71, O32, O41

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1 Introduction

Motivation

The world spends huge amounts of money on basic research and science in general. In 2014 Basic research accounted for nearly one-third (27.8 percent) of total R&D expenditures in OECD countries (OECD (2016)), with total R&D equaling 2.4% of GDP. Basic research may lead to innovations that result in technological progress and thus long-term material benefits. Historical examples are advances in biology and medicine—from X-rays to DNA sequencing—and the introduction of running water and sewer systems (see Gordon (2012) for many examples).

Nevertheless, the success of effort expended on basic research is highly uncertain, and the value of research is often difficult to assess for the generation making the investment. From 1991 to 1995, for example, only 12 percent of all university patents were ready for commercial use once they were licensed, and whether manufacturing would be feasible was known for only 8 percent (Jensen and Thursby (2001)). Estimating the value of research is also difficult. HERG-OHE-Rand (2008) find a return of 9% for medical research on physical health but state that "[...] rates of return need to be treated with extreme caution. Most aspects of the methods unavoidably involve considerable uncertainties." Due to this uncertainty, there are several examples where society has overestimated the value of newly discovered technologies. Perez (2009) documents several examples of investment in new technologies, namely canal building in England, starting in 1771, railway development in Great Britain, starting in 1829, and the establishment of internet-related companies in the US, starting in 1971. With hindsight, these instances displayed a concentration of investments, divorced from actual technology needs in the real economy. Typically, these projects were fueled by great optimism about potential real-world application. But, after an initial surge, investments often slowed down or collapsed altogether.

There are also more recent examples of such occurrences, like the Apollo Program (Gisler and Sornette (2009)) and the Human Genome Project (Gisler et al. (2011)). Also, the Google Lunar Xprize (XPRIZE Foundation (2016)) or current concentrated expenditures focused on projects such as the European Flagships seem to involve outstanding optimism."*Flagships are visionary, science-driven, large-scale research initiatives addressing grand Scientific and Technological (S&T)*

challenges. They are long-term initiatives bringing together excellent research teams across various disciplines, sharing a unifying goal and an ambitious research roadmap on how to achieve it".[1] Currently, one focus is on Graphene, a single, thin layer of graphite, that is considered the world's strongest and most conductive material, another the explanation of the human brain.

According to the literature above, these examples, while seeming very different at first glance, share three main features:

- Large basic research investments are involved which may collapse at some point in time.
- The projects are fueled by great optimism and enthusiasm about the scientific and economic benefits of the project, while a more realistic assessment would lead to more cautious calculations.
- Typically, the outcomes are disappointing compared to the initial expectations. However, over time, various types of benefits are generated. The Apollo Program, for example, led to improvements in the production of microprocessors and to greater memory capacity for computers, from which other industries have greatly benefited (Mezzucato (2014)).

We call occurrences that fulfill these criteria "research bubbles[2]". As they seem to be a pervasive feature in the discovery of knowledge, questions about the causes and the desirability of such bubbles arise. This is the focus of our paper.

One might suggest that such bubbles are the result of mere irrationality and since agents overinvest, can only be detrimental to welfare. However, we suggest that research bubbles are generated by the self-selection of researchers into research activities and result from rational decisions on the part of governments as to whether to embark on such adventures on the basis of the assessments by the researchers involved.[3] Moreover, while such bubbles may lead to disappointment and may not benefit the current generation, they tend to be desirable from a long-term

[1] For more information see http://ec.europa.eu/programmes/horizon2020/en/h2020-section/fet-flagships (accessed on 18.09.2017).

[2] Research Bubbles are, of course, quite distinct from the well observed bubbles in the financial sector. Instead, they can be understood as a subset of "social bubbles", which are defined in Gisler and Sornette (2009) and Gisler et al. (2011), occurring in the realm of public research. For a recent survey of the asset bubble literature, see Scherbina and Schlusche (2014).

[3] The optimism bias in our paper, which will be substantiated in the following sections is

perspective, taking the welfare of future generations into account. However, they may also be excessive, even from a long-run perspective.

In classic innovation-driven growth theory and its extensions, research bubbles do not figure at all (Aghion and Howitt (1992), Grossman and Helpman (1991), Romer (1990)). Cozzi (2007) is an exception, presenting a model that allows for self-fulfilling prophecies. In our model, research bubbles are not the result of multiple equilibria but arise from the government's aggregation of heterogeneous assessment by researchers who self-select into research activities.

Approach and results

More specifically, we develop a framework that rationalizes research bubbles in an overlapping generation model with endogenous growth. Here, growth stems from the accumulation of knowledge, a production factor created in the basic research sector (henceforth simply research sector). Conducting research today leads to more knowledge tomorrow. It requires labor input, and the amount of labor in the economy is a finite resource. It can be either employed in the research sector, to increase output tomorrow, or in the productive sector, to produce output today. Hence, employing labor in research means forfeiting output today for more output tomorrow, so that conducting research represents a trade-off between output today and output tomorrow. We assume that the demand for research labor is formed by a decentralized myopic government. We use the term "myopic" to indicate that the government has a shorter horizon than a social planner.

In a first simple model without bubbles, we demonstrate that the government fails to internalize the dynamic externality of research, leading to too little research activity over and against the social optimum. We find that the decentralized outcome can be improved both by lengthening the decision maker's horizon, and by an overestimation of the short-term impact of research, i.e. a research bubble.

In a second, more complex model, we introduce bubbles that derive from rational behavior of households and the government. By allowing for heterogeneous beliefs about productivity among agents in the research sector, we focus on the way agents self-select into the research sector. Those with higher beliefs, i.e. more

an aggregate phenomenon. Our definition differs from the standard explanation in psychology and behavioral economics, where individuals overestimate the likelihood of positive events and underestimate the likelihood of negative ones.

optimistic agents, will want to work in the research sector, while more pessimistic agents will choose the productive sector. The government does not know how productive research will be. It relies on the assessment of agents in the research sector for its estimate of research productivity, which, in turn, is the basis for its demand for researchers. As optimistic agents self-select into the research sector, the government overestimates productivity, and a research bubble arises.

When governments form average assessments of the technology potential from research by listening to researchers, the emerging research bubble will typically fail to reach the socially optimal level of research investments. But other aggregation methods can produce research bubbles that generate socially optimal research activities.

We further examine how research bubbles may burst and how such collapses can be avoided through institutional remedies such as establishing constitutional rules or giving optimistic researchers a big say in basic research investment. An alternative route is debt financing, where the amount of debt that the government can borrow on international capital markets depends on the amount of research conducted in the economy. Finally, we recast the occurrence of research bubbles in variant models in which research success also depends on research effort decisions. Models with alternative welfare functions of the government are also discussed.

Structure

The rest of the paper is organized as follows: In the next section we introduce the baseline model describing our research economy. Section 3 presents the same model, this time with research bubbles, and Section 4 studies the implementation of the social optimum in a decentralized economy. Section 5 discusses the potential and the drawbacks of the decentralized solution. In Section 6, we present possible extensions of the model. Section 7 concludes.

2 A research economy

Let us turn first to our baseline model without research bubbles. We use an OLG model where endogenous growth results from an increasing stock of knowledge. Knowledge is created by basic research and research is conducted in the public research sector, which competes with the production sector for skilled labor.

2.1 Households

Households live for two periods and at any point in time, two generations coexist. An agent is labeled "young" in the first period and "old" in the second period. Each generation is represented by a single household and possesses one unit of time supplied inelastically in the market for labor. Hence, total labor endowment $\overline{L}$ is normalized to 1. There is one physical commodity that can be either used for consumption or as capital for production, while capital is fully depreciated in each period. Consumption is the only source of utility. The life-time utility of a household born in period t is

$$U_t = \log(c_t^1) + \beta \log(c_{t+1}^2), \tag{1}$$

where c_t^1 denotes consumption of the physical good in the first period, c_{t+1}^2 in the second, and the parameter β stands for the discount factor, with $0 < \beta < 1$. When young, the agent makes decisions on saving and on how much time to allocate to work in the research sector, and/or the productive sector. When old, the household only consumes its savings. The variables $L_t^{S,R}$ and $L_t^{S,P}$ stand for the time the agent supplies to the research sector and the productive sector, respectively. Furthermore, s_t stands for savings, w_t^P for the wage in the productive sector, w_t^R for the wage in the research sector, and r_{t+1} for the gross interest rate. We assume that in order to finance research, wage income in the productive sector is taxed at rate τ_t . Hence, when young and old, consumption for an agent born in t are

$$c_t^1 = w_t^R L_t^{S,R} + (1 - \tau_t) w_t^P L_t^{S,P} - s_t, \tag{2}$$

$$c_{t+1}^2 = s_t r_{t+1}, \text{ and} \tag{3}$$

$$L_t^{S,R} + L_t^{S,P} = 1. \tag{4}$$

We plug in these definitions and maximize utility with respect to $L_t^{S,R}$ and s_t to obtain

$$L_t^{S,P} = \begin{cases} 0, & \text{if } w_t^R > w_t^P(1-\tau_t), \\ \text{arbitrary}, & \text{if } w_t^R = w_t^P(1-\tau_t), \\ 1, & \text{if } w_t^R < w_t^P(1-\tau_t), \end{cases}$$

and

$$s_t = \frac{\beta}{1+\beta}\left(w_t^R L_t^{S,R} + (1-\tau_t)w_t^P\left(1 - L_t^{S,R}\right)\right).$$

Throughout the paper, we focus on constellations in which both sectors are active, which in this section requires that $w_t^R = w_t^P(1-\tau_t)$. We denote this wage by w_t and obtain that savings are a constant share of income due to logarithmic utility.

The tax rate balances the budget and fulfills the following condition:

$$w_t^R L_t^R = \tau_t w_t^P (1 - L_t^R),$$

where L_t^R is the eventual market equilibrium. Hence, by the required equality of net wages, we have

$$(1-\tau_t)w_t^P L_t^R = \tau_t w_t^P (1 - L_t^R) \Rightarrow \tau_t = L_t^R, \tag{5}$$

implying that the tax rate on wage income from productive activity is equal to the share of labor in the research sector.

2.2 Productive sector

A single firm produces output using knowledge B_t, capital K_t and labor $L_t^{D,P}$, with D, P indicating demand in the productive sector. The production function takes the form

$$Y_t = (L_t^{D,P} B_t)^{1-\alpha} K_t^{\alpha}.$$

Labor is supplied by the young household, knowledge is created in the research sector, as is described below, and capital is created from the household's savings.

As capital depreciates fully within one generation, $s_t = K_{t+1}$ is the equilibrium condition. Also, this implies that r_{t+1} is the net and gross interest rate. Knowledge will be useful in production,[4] and while the output of basic research can be used free of charge, the other two production factors are rented by the firm and compensated by wage w_t^P and interest rate r_t. Hence, the profit of the firm reads

$$\Pi_t = Y_t - w_t^P L_t^{D,P} - r_t K_t.$$

The firm takes all prices as given, so that optimal behavior is described by the standard demand functions. Since both labor and capital are supplied inelastically, we obtain the standard equilibrium condition for wages and the interest rate:

$$w_t^P = (1-\alpha)\frac{Y_t}{L_t^P} \text{ and } r_t = \alpha \frac{Y_t}{K_t}.$$

2.3 Research sector

We assume that the research sector is run by a government. It employs labor $L_t^{D,R}$ to create new knowledge on the basis of the existing knowledge stock. From the government's perspective, the knowledge production function is

$$B_{t+1} = (1 + \theta \cdot L_t^{D,R})B_t.$$

The function depends on labor demand in research $L_t^{D,R}$ and on the productivity parameter θ.

Conducting research inhibits a fundamental trade-off: increasing knowledge and output tomorrow means forfeiting output today, as labor has to be reallocated from the productive sector to the research sector. Thus, it is the government's task to decide how much labor should be employed in the research sector.

The economy allows for balanced growth paths. A steady state is characterized by proposition 1.

Proposition 1. *A steady state of the economy for a given constant share of labor invested in research in each period $\hat{L}^R$ is uniquely characterized by a constant labor*

[4]Since basic research output is of no immediate commercial use, there is typically time lag between basic research and its use in production. Estimates of this time lag range between 6 and 20 years on average (see Adams (1990)).

input $\hat{L}^P$ in the productive sector and a constant return to capital $\hat{r}$. Consumption of young and old agents, c_t^1 and c_t^2, output Y_t, capital K_t and the knowledge stock B_t all grow at a constant rate $\hat{g} = \theta \hat{L}^R$.

The proof of proposition 1 can be found in the Appendix.

2.4 Decentralized solution

First, we look at a planner who is only concerned with the current generation. He has preferences over output today and discounted output tomorrow, where both depend on labor input in research. We call this the "government solution"

$$\max_{L_t^{D,R}} \log(Y_t) + \beta \log(Y_{t+1}), \quad \text{or}$$
$$\max_{L_t^{D,R}} \log\left(((1 - L_t^{D,R})B_t)^{1-\alpha} K_t^{\alpha}\right) + \beta \log\left(((1 - L_{t+1}^{D,R})B_{t+1})^{1-\alpha} K_{t+1}^{\alpha})\right)$$

where we assume that the government has the same logarithmic utility function and discount factor β as the household. Maximizing with respect to $L_t^{D,R}$ yields

$$\frac{1}{1 - L_t^{D,R}} = \frac{\beta\theta}{1 + \theta L_t^{D,R}} \quad \text{for} \quad 0 < L_t^{D,R} < 1. \tag{6}$$

The left hand side of (6) is the marginal product of labor and reflects the marginal cost of one more unit of labor in research. The right hand side is the discounted marginal product of research today on output tomorrow via an increase in the knowledge stock. A strictly positive value for $L_t^{D,R}$ exists if the expression holds with an equality.

Expression (6) contains only the contemporary value of $L_t^{D,R}$ but not $L_{t+1}^{D,R}$, so that the government's solution is static and takes the form

$$L^{D,R} = \frac{1}{1+\beta}\left(\beta - \frac{1}{\theta}\right) \quad \forall t, \tag{7}$$

where $L^{D,R}$ only depends on the parameters β and θ. A condition for research to occur in the decentralized economy is $L^{D,R} > 0 \leftrightarrow \beta > 1/\theta$. As we have assumed that β is smaller than 1, the condition can be reduced to $\theta > 1$. A more impatient government with a lower discount factor β will invest less in research.

We can be certain that we have found a utility maximum, as the second derivative of the objective function is simply

$$-(1 - L_t^{D,R})^{-2} - \beta\theta^2(1 + \theta L_t^{D,R})^{-2} < 0 \ \forall L_t^{D,R} \ \in (0, 1).$$

The static nature of the government's solution results from two structural assumptions: Logarithmic utility and a Cobb-Douglas production function. The logarithmic utility causes Y_t and Y_{t+1} to appear in the denominator of the derivative, and the production function causes them to appear in the numerator, so that they cancel out.

To ensure that the household supplies the demanded share of labor, the government sets the wage in the research sector

$$w_t^R = (1 - \tau_t)\frac{(1 - \alpha)Y_t}{1 - L_t^R} = (1 - \tau_t)w_t^P = w_t \tag{8}$$

in each period. By doing so, the government can implement its demand as an equilibrium, so that $L^{D,R}$ stands for the equilibrium value of labor in research L^R.

Definition 1. *We define an equilibrium of the economy as the paths of w_t, r_t, Y_t, K_t and B_t, given a sequence $\{L_t^R\}_{t=0}^{\infty}$ that fulfill the following conditions: The household maximizes utility, the firm maximizes profits, the government maximizes its own utility, and the market for capital clears as does the labor market, i.e. equation* (8) *holds.*

2.5 Social planner solution

Next we turn to the social optimum for the economy. The maximization problem of the social planner reads

$$\begin{aligned}\max_{\{c_t^1, c_{t+1}^2, L_t^R, B_{t+1}, K_{t+1}\}_{t=0}^{\infty}} \quad & W = \sum_{t=0}^{\infty} \beta_s^t \left(\log(c_t^1) + \beta \log(c_{t+1}^2)\right) \\ \text{s.t.} \quad & ((1 - L_t^R)B_t)^{1-\alpha} K_t^{\alpha} = c_t^1 + c_t^2 + K_{t+1}, \\ & B_{t+1} = (1 + \theta L_t^R)B_t,\end{aligned}$$

where $\beta_s \in (0, 1)$ is the social planner's discount factor and W denotes social welfare. We define λ_t as the Lagrange Multiplier on the budget constraint and μ_t

as the multiplier on the knowledge-production function, and obtain the following first order conditions:

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial c_t^1} &= \beta_s^t\left(\frac{1}{c_t^1} + \lambda_t\right) = 0, \\
\frac{\partial \mathcal{L}}{\partial c_{t+1,}^2} &= \beta_s^t\left(\frac{\beta}{c_{t+1}^2} + \beta_s\lambda_{t+1}\right) = 0, \\
\frac{\partial \mathcal{L}}{\partial K_{t+1}} &= \lambda_t - \beta_s\lambda_{t+1}\alpha\frac{Y_{t+1}}{K_{t+1}} = 0, \\
\frac{\partial \mathcal{L}}{\partial L_t^R} &= \lambda_t(1-\alpha)\frac{Y_t}{1-L_t^R} + \mu_t\theta B_t = 0, \\
\frac{\partial \mathcal{L}}{\partial B_{t+1}} &= -\mu_t + \beta_s(-\lambda_{t+1}(1-\alpha)\frac{Y_{t+1}}{B_{t+1}} + \mu_{t+1}(1+\theta L_{t+1}^R)) = 0,
\end{aligned}
$$

where $\mathcal{L}$ is the Lagrange Function. The first three conditions are common, but the last two deserve attention. To understand them, we interpret λ_t as the change in the life-time utility of an agent born in t if one more unit of output Y_t were available. Analogously, we see μ_t as the change in the life-time utility of such an agent if one more unit of knowledge B_{t+1} were available tomorrow. With this, the derivative with respect to L_t^R implies that the marginal loss from allocating one more unit of labor to research, which is λ_t times the marginal product of labor, must be equal to the benefit which results from having θB_t more units of knowledge tomorrow. In the derivative of the Lagrange Function with respect to B_{t+1}, μ_t stands for the welfare loss associated with creating one more unit of B_{t+1}. The loss is equal to the discounted sum of two different benefits. First, more knowledge tomorrow will increase production by the marginal product $(1-\alpha)Y_{t+1}/B_{t+1}$. Second, having more knowledge tomorrow will reduce the necessity to conduct research tomorrow and hence yields the benefit $\mu_{t+1}(1+\theta L_{t+1}^R)$.

From the five first-order conditions we obtain two dynamic equations. The first is the common Euler equation, and the second describes the dynamic allocation of labor in research,

$$
\frac{1}{c_t^1} = \frac{\beta_s\alpha Y_{t+1}}{K_{t+1}c_{t+1}^2}, \tag{9}
$$

$$
\frac{Y_t}{1-L_t^R} = \frac{K_{t+1}}{\alpha Y_{t+1}}\left(\frac{\theta Y_{t+1}}{1+\theta L_t^R} + \frac{Y_{t+1}}{1-L_{t+1}^R}\frac{1+\theta L_{t+1}^R}{1+\theta L_t^R}\right). \tag{10}
$$

Together with the budget-constraint and the knowledge-production function, they describe the model. If we assume $\beta_s = \beta$, we obtain the following steady state condition:

$$\frac{1}{1-L^R} = \beta\left(\frac{\theta}{1+\theta L^R} + \frac{1}{1-L^R}\right)$$
$$\Rightarrow L^R = L^{O,R} = \beta - \frac{1-\beta}{\theta}. \tag{11}$$

For the derivation of the equation, see the Appendix. We denote the steady state value of labor input in research by $L^{O,R}$ and compare it to the government's demand for research $L^{D,R} = \frac{1}{1+\beta}\left(\beta - \frac{1}{\theta}\right)$. We find three differences. First, when θ goes to infinity, the social optimum converges to β in the limit, while the government solution converges to $\beta/(1+\beta) < \beta$. Second, when θ increases, the social planner will increase his demand less than the government:

$$\frac{\partial L^{O,R}}{\partial \theta} = \frac{1-\beta}{\theta^2} < \frac{1}{(1+\beta)\theta^2} = \frac{\partial L^{D,R}}{\partial \theta},$$

which holds if and only if
$(1-\beta)(1+\beta) < 1 \Leftrightarrow \beta^2 > 0$. Third, the social planner always employs more labor in research than the government:

$$L^{O,R} = \beta - \frac{1-\beta}{\theta} > \frac{1}{1+\beta}\left(\beta - \frac{1}{\theta}\right) = L^{D,R},$$
$$\Leftrightarrow 0 < \beta^2(\theta+1),$$

which holds because we are assuming that $\beta, \theta > 0$.

To explain this result, we compare (6) and (10). We rewrite (10) as

$$\frac{1}{1-L_t^R} = \frac{K_{t+1}}{\alpha Y_{t+1}}\frac{Y_{t+1}}{Y_t}\left(\frac{\theta}{1+\theta L_t^R} + \frac{1+\theta L_{t+1}^R}{(1-L_{t+1}^R)(1+\theta L_t^R)}\right)$$

and observe that the government and the social planner discount different benefits, using different discount factors. The government only takes into consideration the immediate benefit from research that arises in the next period and uses the constant β to discount it. The social planner internalizes the additional intergenerational effects and uses $\frac{K_{t+1}}{\alpha Y_{t+1}}\frac{Y_{t+1}}{Y_t}$ to scale the benefits occurring in the future.

Note that this factor is the product of two components: first, the inverse of the marginal product of capital, which under complete depreciation is the economy's discount factor, and second, 1 plus the growth rate of output.

The social planner's awareness of the long-term benefits of research explains why the socially optimal steady state can be greater than the decentralized solution. It also explains why the social planner solution is less sensitive to changes in θ. The government only enjoys the benefits of a higher θ and increased productivity in the next period. To capitalize on the increased productivity, the government strongly raises labor input in research. The social planner, by contrast, is aware that the benefits of a higher θ extend beyond the next period. Therefore he has smaller incentives to increase labor input today

We prove that the difference between the social planner and the government solution arise only because of two differing decision horizons. For this, we show that the government solution converges to the socially optimal solution with a rising decision horizon. A government that is aware that research today has an impact on all future generations faces the following problem of maximizing the sum of all future output over investment today: $\max_{L_t^{D,R}} \sum_{s=t}^{\infty} \beta^{s-t} \log(Y_s)$, which due to logarithmic utility can simply be written as

$$\max_{L_t^{D,R}} \log(1 - L_t^{D,R}) + \sum_{s=t+1}^{\infty} \beta^{s-t} \log(1 + \theta L_t^{D,R}) \text{ or}$$
$$\max_{L_t^{D,R}} \log(1 - L_t^{D,R}) + \frac{\beta}{1-\beta} \log(1 + \theta L_t^{D,R}),$$

which yields

$$(1 + \theta L_t^{D,R})(1 - \beta) = \beta\theta(1 - L_t^{D,R})$$
$$\Rightarrow L^{D,R} = \beta - \frac{1-\beta}{\theta}.$$

This expression precisely yields the social optimum. Thus, we obtain

Proposition 2.
For the decentralized government and the social planner, the steady state levels of labor in research are given by (7) *and* (11)*. The social optimum implies more research than the decentralized solution and has a higher upper bound but is less sensitive to changes in productivity.*

2.6 Implementing the socially optimal solution via research bubbles

The preceding analysis reveals that decentralized basic research investments can yield lower social welfare. We note that a more optimistic government view, i.e. the assumption that θ is higher than it actually is, would increase welfare. However, it would do so at the expense of generation t's utility. In addition, if all governments had a more optimistic view, i.e. if their assumption, $\widetilde{\theta}$, is greater than the true value of θ, social welfare would be higher at the expense of the first few generations. In particular, if the first generations could finance part of their research expenditures by issuing debt, a combination of research bubbles and public debt could implement the socially optimal solution and make everybody better off compared to the decentralized solution. We define the following:

Definition 2. *The economy exhibits a research bubble in a particular time frame* $(0, T]$ *for some* $T \in \mathbb{R}$ *if the governments assume* $\widetilde{\theta} > \theta$ *when they decide on investment in basic research.*

In the following, we show that sufficient optimism in the decentralized economy can implement the social optimum. Assume that the government does not know the true value of θ but believes $\widetilde{\theta}$ to be the true productivity. To achieve the socially optimal outcome, this $\widetilde{\theta}$ must fulfill

$$\begin{aligned} \beta - \frac{1-\beta}{\theta} &= \frac{1}{1+\beta}\left(\beta - \frac{1}{\widetilde{\theta}}\right) \\ \Rightarrow \widetilde{\theta} &= \frac{\theta}{1-\beta^2(1+\theta)}. \end{aligned} \tag{12}$$

The implementation of the social optimum hinges on the relation between β and θ. If $1/\beta^2 - 1 > \theta$, then implementation is possible, otherwise it is not. This restriction arises, because the decentralized solution has a lower upper bound than the social planner solution, i.e. $\frac{\beta}{1+\beta} < \beta$.

Proposition 3. *If there is optimism in the decentralized economy and the government believes* $\widetilde{\theta}$, *given by* (12), *to be the true productivity, the economy will achieve the social optimum.*

In the next section we derive a microfoundation for optimistic beliefs and suggest that they are a natural outcome of decisions on basic research. Moreover, we suggest an institutional arrangement that can implement the socially optimal solution.

3 Research with heterogeneous beliefs

We next explore whether research bubbles arise naturally in scenarios where the government does not know the true parameter θ. Also, we ask whether there are institutional arrangements that support welfare-enhancing research bubbles.

We substitute the single household in each generation by a continuum of infinitely many households of measure 1. A subset of these agents holds beliefs about the parameter θ. The beliefs are heterogeneous and the government has to make an estimate for θ based on the given beliefs.

3.1 Households

The economy is populated by infinitely many agents represented by the interval $[0, 1]$ and of mass 1. All agents possess one unit of time. Hence, the overall labor endowment in the economy is 1, as before. A share $\overline{L}_B$ of all agents is able to work in the research sector and these agents hold beliefs about θ. The set of agents with the capacity to work in the research sector is $\mathcal{L}_B$. We denote agent $i \in \mathcal{L}_B$'s belief about θ as θ_i and allow it to lie in $[\theta_l, \theta_h]$, with $\theta_l < \theta < \theta_h$. Belief types are uniformly distributed in $[\theta_l, \theta_h]$, with density $\frac{1}{\theta_h - \theta_l}$, where the latter follows from the assumption that households have mass 1.
The belief determines the sector in which an agent will want to work. If an agent works in the productive sector, he earns a wage and consumes. If the agent works in the research sector, additional considerations matter, since research has a strong non-pecuniary utility component. We assume that a researcher derives utility from research achievements and thus from knowledge creation, e.g. through intrinsic means—satisfaction about achievements— or extrinsic means—such as status and prestige. More specifically, utility derived from research depends on how efficient the agent believes his research to be θ_i, so that the utility function of a scientist reads

$$U^R_{t,i} = \log(w^R_t - s^R_{t,i}) + (1+\beta)\log(1-\theta_l+\theta_i) + \beta\log(r_{t+1}s^R_{t,i}),$$
$$\text{with } \; i \in \mathcal{L}_B,$$

where we calibrate utility in such a way that the least optimistic agent, i.e. the one who holds the belief $\theta_i = \theta_l$, receives no additional utility from working in research. With this calibration we ensure that no agents receive negative utility from working in research and hence require compensation. Also, we scale this utility by $1+\beta$, as this simplifies later derivations. The utility of a worker is given by

$$U^P_{t,i} = \log((1-\tau_t)w^P_t - s^P_{t,i}) + \beta\log(r_{t+1}s^P_{t,i}).$$

Since an individual has no impact on prices and aggregate variables, an agent takes r_{t+1} as given and maximizes utility with respect to savings $s_{t,i}$. The solution of the researcher problem and the worker problem is given by the expressions

$$s^P_{t,i} = \frac{\beta(1-\tau_t)w^P_t}{1+\beta} \text{ and } s^R_{t,i} = \frac{\beta w^R_t}{1+\beta}. \tag{13}$$

Plugging these results back into the utility function, we obtain

$$U^R_{t,i} = \log\left(\frac{w^R_t}{1+\beta}\right) + (1+\beta)\log(1-\theta_l+\theta_i) + \beta\log\left(\frac{\beta r_{t+1}w^R_t}{1+\beta}\right) \text{ and}$$
$$U^P_{t,i} = \log\left(\frac{(1-\tau_t)w^P_t}{1+\beta}\right) + \beta\log\left(\frac{\beta r_{t+1}(1-\tau_t)w^P_t}{1+\beta}\right).$$

Setting both utilities equal yields proposition 4.

Proposition 4.
The critical value for researcher i's belief is

$$\theta_{crit,t} = \frac{(1-\tau_t)w^P_t}{w^R_t} - (1-\theta_l). \tag{14}$$

Hence, every household with a belief θ_i above this value $\theta_{crit,t}$ will choose to work in the research sector. Every household with a belief below $\theta_{crit,t}$ will choose the productive sector. The agent with $\theta_i = \theta_{crit,t}$ is indifferent and, by assumption, will choose the research sector.[5]

[5] For a more detailed derivation, see the Appendix.

We find that the critical belief is a linear function of the wage ratio. The greater the wage in the productive sector, the greater an agent's belief must be for him to choose the research sector. Given some wage ratio, an agent will choose the research sector if his belief θ_i lies between $\theta_{crit,t}$ and θ_h, so that labor supply is given by

$$L_t^{S,R} = \overline{L}_B \frac{\theta_h - \theta_{crit,t}}{\theta_h - \theta_l}, \tag{15}$$

i.e. the product of the share of agents able to work in the research sector $\overline{L}_B$ and those who choose to do so $\frac{\theta_h - \theta_{crit,t}}{\theta_h - \theta_l}$.

3.2 Assessment of research productivity

Unlike before, the government does *not* know the parameter θ and has to form an estimate. Hereby, the researchers' beliefs are the only available source of information, and we assume that researchers truthfully signal their belief to the government. Equipped with this set of beliefs, the government then makes the following estimate:

$$\widetilde{\theta}_t = \eta\theta_h + (1 - \eta)\theta_{crit,t}. \tag{16}$$

The parameter η $(0 < \eta < 1)$ is the weight that the government places on the most optimistic researcher belief, and $1 - \eta$ is the weight placed on the most pessimistic counterpart. At this stage we do not specify how η is eventually determined. In Section 5.3 we explore different institutional arrangements leading to particular values of η.

Two remarks are in order. First, the expressed range of beliefs $[\theta_{crit,t}, \theta_h]$ is itself more optimistic than the range of beliefs $[\theta_l, \theta_h]$ in the entire population of researchers.

Second, at this stage we assume that researchers reveal their true beliefs to the government. In section 5.2 we discuss whether researchers do indeed have incentives to reveal their true beliefs.

3.3 The government's problem

The government relies on the following estimated production function to derive research labor demand:

$$\widetilde{B}_{t+1} = (1 + \widetilde{\theta}_t \cdot L_t^{D,R})B_t.$$

This demand differs from the previous one in two ways. First, it indicates the amount of knowledge that the government believes to be available tomorrow, $\widetilde{B}_{t+1}$. Second, the parameter θ is replaced by the government's estimate $\widetilde{\theta}_t$. To ease notational complexity, $L_t^{D,R}$ again stands for the government's demand for research labor, but now for the case with *heterogeneous* beliefs. The maximization problem for the government now reads

$$\max_{L_t^{D,R}} \log(((1 - L_t^{D,R})B_t)^{1-\alpha}K_t^{\alpha}) + \beta \log(((1 - L_{t+1}^{D,R})B_t(1 + \widetilde{\theta}_t L_t^{D,R}))^{1-\alpha}K_{t+1}^{\alpha}).$$

Maximizing with respect to $L_t^{D,R}$ yields

$$\frac{1}{1 - L_t^{D,R}} = \frac{\beta\widetilde{\theta}_t}{1 + \widetilde{\theta}_t L_t^{D,R}}, \text{ with} \tag{17}$$

$\widetilde{\theta}_t$ given by (16).

Equation (17) is analogous to equation (6), but θ is now replaced by the estimate $\widetilde{\theta}_t$. However, equation (17) alone does not enable us to determine L_t^R. It depends on $\widetilde{\theta}_t$, which in turn, depends on the labor supply to the research sector. As labor is no longer supplied inelastically, it is necessary to determine labor demand and supply for research labor simultaneously. We do this by examining the labor market equilibrium in the next subsection.

3.4 Labor market equilibrium

To determine the labor market equilibrium, we solve (17) for $L_t^{D,R}$

$$L_t^{D,R} = \frac{1}{1 + \beta}\left(\beta - \frac{1}{\widetilde{\theta}_t}\right). \tag{18}$$

Using Definition (16) with $\eta = 1/2$ yields

$$L_t^{D,R} = \frac{1}{1+\beta}\left(\beta - \frac{2}{(\theta_h + \theta_{crit,t})}\right).$$

Note that unlike before, $L_t^{D,R}$ is not a fixed value but a strictly concave function of the variable $\theta_{crit,t}$. So is the labor supply from equation (15). Thus, we have two equations, labor supply and demand in two variables, research labor L_t^R, and the critical belief $\theta_{crit,t}$. This means that labor demand and the critical belief are interdependent. Labor supply depends on $\theta_{crit,t}$ because every agent with a belief higher than $\theta_{crit,t}$ supplies his labor to the research sector. Hence, the supply falls linearly with the critical value. Labor demand depends on $\theta_{crit,t}$ because the critical belief determines $\widetilde{\theta}_t$. Demand is an increasing function in $\theta_{crit,t}$: The more optimistic the statement by researchers about the productivity of research, the greater is, of course, the government's demand.

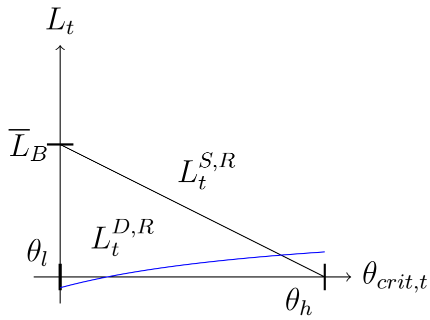

Figure 1: Labor market and optimism equilibrium.

In Figure 1 we plot supply and demand as functions of $\theta_{crit,t}$ for the purpose of illustration with the values $\beta = 0.85$, $\theta_h = 2$ and $\theta_l = 1$. Labor supply by households is shown by the linear falling function and labor demand of the government by the increasing one. Labor supply reaches its maximal value of $\overline{L}_B$ when the critical belief takes the smallest possible value. The supply decreases smoothly until $\theta_{crit,t}$ reaches θ_h. Labor demand is negative at θ_l and increases in a concave fashion, intersecting the θ-axis only once. While the first observation results from our choice of values for θ_h and θ_l, the latter results from the strict concavity of labor demand. The intersection of the curves shows the labor market equilibrium.

To obtain it analytically, we set demand equal to supply and solve for $\theta_{crit,t}$:

$$\theta_{crit,t}^2 + \theta_{crit,t}\frac{\beta(\theta_h - \theta_l)}{(1+\beta)\overline{L}_B} - \theta_h^2\left(1 - \frac{\beta}{(1+\beta)\overline{L}_B}\right) - \frac{\beta\theta_h\theta_l - 2(\theta_h - \theta_l)}{(1+\beta)\overline{L}_B} = 0. \tag{19}$$

This yields a second degree polynomial in $\theta_{crit,t}$. It depends on the boundaries of the belief interval θ_l and θ_h, the discount factor β, and the share of agents that can work in research $\overline{L}_B$. However it does not depend on the true productivity θ. As all parameters are constant, $\theta_{crit,t}$ is also constant over time.

The above polynomial has two real solutions at the most. We can show that if two solutions exist, one can be excluded. To see this, consider the demand function, which is strictly increasing for $\theta_{crit} \in \mathbb{R}$ but is not continuous everywhere, because the following holds:

$$\lim_{\theta_{crit} \xrightarrow{+} -\theta_h} L^{D,R}(\theta_{crit}) = -\infty, \quad \text{while} \quad \lim_{\theta_{crit} \xrightarrow{-} -\theta_h} L^{D,R}(\theta_{crit}) = +\infty,$$

i.e. the limits to $\theta_{crit} = -\theta_h$ from left and right are not identical. Given that demand is a strictly increasing function, we can conclude that the function becomes infinitely large on the left of $-\theta_h$, while it falls to $-\infty$ on the right of it and then increases with θ_{crit}. This explains how two intersections are possible. One of them has to lie to the left of $-\theta_h$ and is thus irrelevant.

With fixed $L_t^{D,R}$, w_t^P is given and the government needs to set w_t^R according to (14). By setting the wage in the research sector correctly, the government can implement its demand as the market equilibrium so that, as before, $L^{D,R} = L^{R,H}$, where $L^{R,H}$ is the equilibrium value in the market for research labor in the steady state. The superscript H stands for "heterogeneous beliefs".

The economy reaches the described equilibrium in the following way: First, the government hires a number of researchers and obtains the estimate $\widetilde{\theta}_t$. The expected productivity determines the government's optimal labor demand. If the optimal demand turns out to be greater, the government increases w_t^R, and hence labor supply, to lower θ_{crit}. By doing so, it hires additional, less optimistic agents and obtains, in turn, a lower average for θ. This adjustment of labor demand continues until the government hires exactly as many researchers as are justified by their aggregated belief.

In this economy, the tax rate τ_t differs from the previous one, as w_t^P and w_t^R are related by (14) and not simply $(1-\tau_t)w_t^P = w_t^R$. From labor supply, we know that any market equilibrium $L^{R,H}$ implies the following critical belief:

$$\theta_{crit} = \theta_h - \frac{(\theta_h - \theta_l)L^{R,H}}{\overline{L}_B}.$$

Substituting this expression into (14) yields

$$\theta_h - \frac{(\theta_h - \theta_l)L^{R,H}}{\overline{L}_B} = \frac{(1-\tau_t)w_t^P}{w_t^R} - (1-\theta_l),$$

or equivalently

$$w_t^R = \frac{(1-\tau_t)w_t^P}{1 + (\theta_h - \theta_l)(1 - \frac{L^{R,H}}{\overline{L}_B})}. \tag{20}$$

We find that, unlike before, researchers are now paid at a markdown. This follows, of course, from the fact that researchers have their belief as an additional source of utility and thus require less compensation for working in the research sector. Additionally, we can see that market mechanisms determine this markdown. A greater demand for research, expressed by bigger $L^{R,H}$, will lower the markdown, while a greater overall supply of researchers, expressed by a bigger $\overline{L}_B$, will increase it. Hence, the tax rate now reads

$$\tau_t = \frac{L^{R,H}}{1 + (\theta_h - \theta_l)\left(1 - \frac{L^{R,H}}{\overline{L}_B}\right)(1 - L^{R,H})},$$

which is obtained by substituting (20) into the government's budget constraint, $L^{R,H}w_t^R = \tau_t(1 - L^{R,H})w_t^P$.

3.5 Social planner solution

To derive the social planner optimum for this economy, one would have to include the following term in the previous welfare function W:

$$\sum_{t=0}^{\infty} \beta_s^t \int_{\theta_{crit,t}}^{\theta_h} \log(1 - \theta_l + \theta_i) di, \text{ with}$$
$$\theta_{crit,t} = \theta_h - \frac{(\theta_h - \theta_l) L_t^R}{\overline{L}_B},$$

which captures the additional utility for those working in research. Note that $\theta_l < \theta_{crit,t}$, so that the integral is finite. If, however, the social planner communicates the true parameter θ and it replaces the individual belief θ_i, then the integral is zero and the problem collapses to the one studied above. Next, we compare the socially optimal outcome to the decentralized outcomes in economies with and without research bubbles. Table 1 provides the parameter values that we use.

α	β	θ	$\overline{L}_B$	θ_l	θ_h
0.3	0.85	1.5	1	1	2

Table 1: Parameter values.

We derive research labor for the social planner, $L^{O,R}$ and the government, $L^{R,H}$ under a research bubble, and the equilibrium without research bubbles, L^R.

L^R	$L^{R,H}$	$L^{O,R}$
0.0991	0.1767	0.75

Table 2: Effort and labor input in research for the social planner and government.

Table 2 provides our findings, which we can summarize in one inequality: $L^R < L^{R,H} < L^{O,R}$. We find the following: First, there is a research bubble in the decentralized economy, as can be seen in the first inequality. Although the true productivity of research θ has remained the same, we find more labor dedicated to knowledge production. Second, the research bubble moves the decentralized amount of investment closer to the socially optimal one: We observe an increase of roughly 8 percentage points, when comparing the two outcomes. Third, even in the presence of a research bubble, the decentralized economy remains below the optimal outcome, as can be seen in the second inequality. We summarize our findings in proposition 5.

Proposition 5.
The steady state levels of the critical belief value θ_{crit} and of research labor $L^{R,H}$ in the decentralized economy are given by equations (15), (18), *and equation* (19). *Research labor $L^{O,R}$ in the social optimum is given by equation* (11) . *We observe a welfare-improving research bubble.*

Several remarks are in order. The government is optimistic since $\theta_{crit,t} > \theta_l$. Thus its estimate $\frac{\theta_h + \theta_{crit,t}}{2}$ is higher than the true productivity. As described above, this over-optimism and the ensuing research bubble are generated by two mechanisms, self-selection of researchers and information aggregation by the government. By self-selection we mean that agents decide themselves which sector they want to work in. More optimistic agents are willing to work in research even if w_t^R is small. Only with greater w_t^R does labor demand increase thus also attracting less optimistic researchers . Consequently, researchers are hired, beginning at the higher end of the belief distribution. The least optimistic ones are not hired, as employing all agents in research is prohibitively costly. By information aggregation we mean that the government forms an estimate about θ based only on the beliefs of the agents hired. Hence, the estimate of the government does not yield the true productivity and it demands more research than in the previous model. This is a research bubble.

If the government asked all agents about their respective beliefs, its estimate would be exactly θ, as with $\theta_l = 1$ and $\theta_h = 2$, we have $\widetilde{\theta} = 1.5 = \theta$. Yet the government receives information from a non-representative sample of the population, as only agents with $\theta_i \geq \theta_{crit}$ work in research.

One can imagine the government as an econometrician tries to measure θ. It faces random differences in the parameter because of the random distribution of beliefs. Although the government's methods are sophisticated, it overestimates the parameter, because it does not take the self-selection bias into account.

4 Implementing the socially optimal solution

In this section we explore how the socially optimal solution can be implemented by the decentralized solution in the steady state.

First we focus on whether and how the decentralized solution can implement the socially optimal solution through research bubbles. If the true productivity in the economy is θ, then equation (12) shows us the necessary size of the research bubble. The equation also provides a necessary condition for the implementation of the social optimum, given by $1/\beta^2 - 1 > \theta$. It continues to hold. However, it is not a sufficient condition, as $\widetilde{\theta} = \eta\theta_h + (1-\eta)\theta_{crit}$. Hence, we have

$$\eta\theta_h + (1-\eta)\theta_{crit} = \frac{\theta}{1-\beta^2(1+\theta)},$$

which implies that

$$\hat{\theta}_{crit} = \frac{\eta}{1-\eta}\left[\frac{1}{1-\beta^2(1+\theta)} - \theta_h\right] \text{ and thus } \theta > \frac{\theta_h(1-\beta^2)-1}{\beta^2\theta_h} \tag{21}$$

must hold for implementation. Expression (21) provides the sufficient condition for a positive value of θ_{crit}. Also, it yields the value $\hat{\theta}_{crit}$ which is the critical belief that is socially optimal. This yields the following proposition:

Proposition 6. *An optimistic view of the government* $\widetilde{\theta}_t = \eta\theta_h + (1-\eta)\theta_{crit,t}$ *can implement the social optimum if* $1/\beta^2 - 1 > \theta$ *and expression* (21) *are satisfied.*

Note that the equilibrium θ_{crit} of the decentralized economy does not have to coincide with $\hat{\theta}_{crit}$, even if the inequality from expression (21) is fulfilled. The mere possibility of implementation does not mean that the economy's research bubble will have precisely the optimal size. If $\theta_{crit} < \hat{\theta}_{crit}$ the economy's research bubble will be to large. In the opposite case it will be too small.

As a numerical example, consider the parameter values from Table 1 and the fact that the government forms an average of the researchers' beliefs, i.e. $\eta = 1/2$. In this case implementation is not possible, as $1/\beta^2 - 1 > \theta$ does not hold. However, if we consider the following set of parameter values: $\theta = 0.3$, $\theta_h = 4$, $\beta = 0.8$, and $\eta = 0.5$, we find that the necessary and sufficient conditions for implementation are met. Under these parameter values we obtain $\hat{\theta}_{crit} = 1.9524$.

However, if the government forms biased estimates of active researchers' beliefs, the socially optimal steady state can be implemented as a decentralized balanced growth path, as we show next. For this purpose we consider the steady state solution given in equation (11).

Proposition 7. *If $L^{D,R}(\eta = \frac{1}{2}) < L^{O,R} < \frac{\beta}{1+\beta}$, i.e. if the social planner solution lies between the government's demand for research labor with $\eta = \frac{1}{2}$ and the government's maximal demand, $\frac{\beta}{1+\beta}$, there exists an $\eta^* > \frac{1}{2}$ and an associated research bubble such that the decentralized solution can implement the social optimum. If $L^{O,R} < L^{D,R}(\eta = \frac{1}{2}) < \frac{\beta}{1+\beta}$, there exists an $\eta^* < \frac{1}{2}$ that implements the social optimum.*

We stress that the existence of such an η^* hinges on the aforementioned condition. If $L^{O,R} > \frac{\beta}{1+\beta}$, no amount of optimism will elevate the government's demand to the socially optimal level.

But if such an η^* exists, it can be found as follows: First, the implementation of the social optimum as a market outcome requires $L^{S,R} = L^{D,R} = L^{O,R}$. Hence, we set research labor supply equal to the socially optimal level and solve for θ_{crit}:

$$\theta_{crit} = \theta_h - \frac{\theta_h - \theta_l}{\overline{L}_B} L^{O,R}. \tag{22}$$

Next we equate demand to $L^{O,R}$ and solve for η^*:

$$\frac{1}{1+\beta}\left(\beta - \frac{2}{(\eta\theta_h + (1-\eta)\theta_{crit})}\right) = L^{O,R}, \tag{23}$$

which yields

$$\eta^* = \frac{2}{(\theta_h - \theta_{crit})(\beta - L^{O,R}(1+\beta))} - \frac{\theta_{crit}}{\theta_h - \theta_{crit}}, \tag{24}$$

where θ_{crit} is given by (22). Note the factor $(\beta - L^{O,R}(1+\beta))$ in the denominator of the expression on the right hand side of (24). Slightly rewritten, it reads $\beta\left(1 - \frac{1+\beta}{\beta}L^{O,R}\right)$, meaning that η can only be positive if and only if $L^{O,R}$ is indeed smaller than $\frac{\beta}{1+\beta}$. Note that this is a necessary but not a sufficient condition for $\eta^* > 0$.

Let us turn again to a numerical illustration under our baseline parametrization. We already know from Table 2 that $L^{O,R} = 0.75 > \frac{\beta}{1+\beta} = 0.4595$, so that implementation is not possible. This is also reflected in the negative value of $\eta^* = -0.664627$. A simple way to achieve implementation is to assume a smaller value for the true productivity θ. A decrease in θ will lower the socially optimal level of research labor but will not affect the market equilibrium, as the latter

depends on the distribution of beliefs and on other model parameters, but not on the actual research productivity. Setting $\theta = 0.3$ instead of 1.5 reduces the socially optimal solution to 0.35. It implies $\theta_{crit} = 1.65$ and $\eta^* = 25.044$, meaning that the market economy can achieve the social optimum given that the government is more than twenty times as optimistic as the most optimistic researcher.
Another way of achieving implementation is to increase the government's discount factor. There is no reason why the government's discount factor (call it β_g) should be equal to that of the social planner or the households. As the government discounts only one future period, β_g can even be greater than one. In qualitative terms this possibility produces the same results as changing θ.

5 Bursting research bubbles and prevention

5.1 Drawbacks

In this subsection we explore two possible reasons why the implementation of the socially optimal solution through research bubbles may fail.

Overstatement of beliefs
One potential drawback is that active researchers may all want to express θ_h and not $[\theta_{crit}, \theta_h]$, since overstating their belief might lead to higher research wages. This can be a drawback if the research bubble is very large, i.e. decentralized demand is equal to, or already larger than, the social optimum. If active researchers all report θ_h, then $\widetilde{\theta} = \theta_h$ instead of $\widetilde{\theta} = \eta\theta_h + (1-\eta)\theta_{crit}$ and labor demand is

$$L^{D,R} = \frac{1}{1+\beta}\left(\beta - \frac{1}{\theta_h}\right).$$

Graphically, the demand curve is shifted upwards, while supply remains unchanged, as can be seen in Figure 2. The blue curve represents the old demand for research labor while the red curve indicated the new shifting demand. Because of the upward shift, θ_{crit} is lower, and the equilibrium level of labor in research is higher.

The increase in research labor is not infinite. Even if all hired researchers over-report their belief, not all agents of the economy will be hired in the research

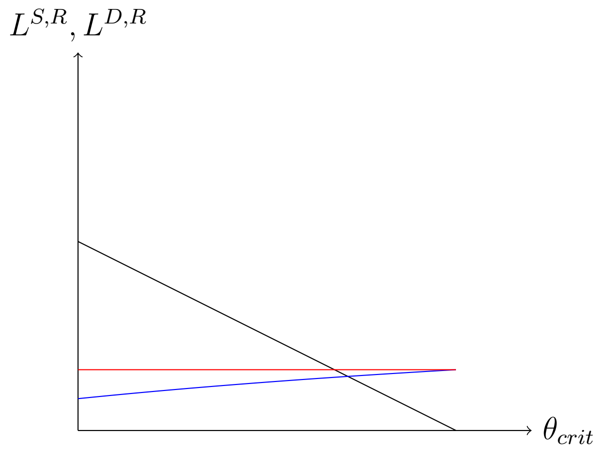

Figure 2: Labor market and optimism equilibrium with overstatement.

sector. This new equilibrium is obtained by equating labor supply with the new labor demand:

$$\theta_{crit} = \theta_h - \frac{(\theta_h - \theta_l)}{\overline{L}_B(1+\beta)}\left(\beta - \frac{1}{\theta_h}\right).$$

Collapse

Governments might learn that beliefs are too optimistic. This could happen from observing past outcomes of basic research investments by previous governments or—recognizing the selection of optimistic researchers into the research activities—by discounting the assessment of researchers. While the first source of learning might be difficult, since basic research activities are very different across time,[6] the second source of learning is more plausible. Such learning might lead to a collapse of the bubble, as we demonstrate next.

Assume a situation in which the social optimum has been achieved due to over-optimism and the government's η is equal to η^*. Now suppose a government becomes less optimistic and lowers its η to $\overline{\eta} < \eta^*$. We ask whether it will be optimal for the government to lower its demand from $L^{O,R}$ to $L^{D,R}(\overline{\eta})$, which is the labor demand associated to some $\overline{\eta}$. Once its optimism decreases, the government

[6]Moreover, if productivity is affected by macroeconomic shocks—and many varieties of such shocks are discussed in the literature—inferring the impact of basic research on GDP may be inherently difficult or impossible.

has two options. On the one hand, it can continue to demand $L^{O,R}$, while $\overline{\eta}$ and not η will enter its utility function. We assume that the government knows θ^*_{crit}, which implements the social optimum. The government can calculate its utility from maintaining the social optimum. Another thing it can do is to reassess the productivity, which will lead to a higher θ_{crit} and a lower demand for research labor. Doing this, the government does not internalize how a change in demand for labor impacts θ_{crit}. It will thus believe that, if it changes its demand, θ_{crit} will remain the same. This is a utility that the government expects to obtain. Clearly, its choice will influence θ_{crit} and influence the level of utility it actually achieves.

As the real productivity of research is always θ and not the believed value $\eta\theta_h + (1-\eta)\theta_{crit}$, we distinguish two different levels of utility. On the one hand, there is the utility that the government active in period t expects to obtain based on the anticipated productivity. On the other, there is the actually realized utility, based on the real productivity. If the government chooses to maintain the social optimum, it will expect to obtain the following utility:

$$\widetilde{u}_t^{G,O} = \log(1 - L^{O,R}) + \beta \log\left(1 + (\overline{\eta}\theta_h + (1-\overline{\eta})\theta^*_{crit})L^{O,R}\right),$$

where θ^*_{crit} is the critical value that implements the social optimum. The tilde indicates the expected value, the superscript indicates the social optimum. Actually, the government will achieve

$$u_t^{G,O} = \log(1 - L^{O,R}) + \beta \log\left(1 + \theta L^{O,R}\right).$$

If it chooses to deviate, it believes that its deviation will not influence θ_{crit}, which will remain at the level θ^*_{crit}. Hence, the government maximizes the following expression:

$$\max_{L_t^{D,R}} \widetilde{u}_t^{G}(L_t^{D,R}) = \log(1 - L_t^{D,R}) + \beta \log\left(1 + (\overline{\eta}\theta_h + (1-\overline{\eta})\theta^*_{crit})\, L_t^{D,R}\right),$$

which yields

$$\overline{L}_t^{D,R} = \overline{L}^{D,R} = \frac{1}{1+\beta}\left(\beta - \frac{1}{(\overline{\eta}\theta_h + (1-\overline{\eta})\theta^*_{crit})}\right).$$

Note that $\overline{L}^{D,R}$ is not a function but a specific value that the government believes to be the market equilibrium.

In the next step we examine whether the government will deviate from the social optimum. We have shown that the government's maximization problem is strictly concave over the domain of $L^{D,R}$, so that a maximizer of the objective function is unique and is also the global maximum. Therefore it remains to show that $L^{O,R} \neq \overline{L}^{D,R}$. To see that this is indeed the case, recall that

$$L^{O,R} = L^{D,R}(\eta^*, \theta^*_{crit}), \quad \overline{L}^{D,R} = L^{D,R}(\overline{\eta}, \theta^*_{crit}), \text{ where}$$
$$L^{D,R}(\eta, \theta^*_{crit}) = \frac{1}{1+\beta}\left(\beta - \frac{1}{(\eta\theta_h + (1-\eta)\theta^*_{crit})}\right)$$
$$\Rightarrow \frac{\partial L^{D,R}(\eta, \theta^*_{crit})}{\partial \eta} > 0,$$

so that $\overline{L}^{D,R} < L^{O,R}$, which concludes the proof. Note that we have assumed $\beta > 1/\theta$ and that $\theta_h \geq \theta$, so that $\beta > 1/\theta_h$ holds. We can be certain that the government will deviate from the social optimum if it becomes less optimistic.

The government makes its decision believing that θ_{crit} will not adjust. However, we know that θ_{crit} is likely to change. The reason is that lower optimism on the part of the government, $\overline{\eta}$, shifts the demand curve downwards and leads to a larger θ_{crit}. Therefore it is unlikely that $\overline{L}^{D,R}$ will be the new market equilibrium $L^R(\overline{\eta})$ that is associated to $\overline{\eta}$. Hence, the government will receive the following utility:

$$u^G_t(\overline{\eta}) = \log(1 - L^R(\overline{\eta})) + \beta \log\left(1 + \theta L^R(\overline{\eta})\right),$$

which we write as a function of $\overline{\eta}$ because it determines the market clearing $\theta_{crit}(\overline{\eta})$ and the labor market equilibrium $L^R(\overline{\eta})$. Thus deviation will be profitable *ex post* if $u^{G,O}_t < u^G_t(\overline{\eta})$, i.e. if the decentralized equilibrium provides higher utility than the social optimum. This is not the case, as the L^R that maximizes the expression $u^G_t = \log(1-L^R) + \beta\log\left(1 + \theta L^R\right)$ is, of course, the labor demand from the simple model. Also, remember that u^G_t is a strictly concave function and thus is decreasing on $(L^R, 1)$, so that $L^R < L^R(\overline{\eta}) < L^{O,R}$ implies $u^G_t(L^R) > u^G_t(L^R(\overline{\eta})) > u^{G,O}_t$. Hence deviation is profitable *ex post.*

5.2 Institutional remedies

We have observed that governments may resort to more realistic assessments, thus lowering basic research investments below socially optimal levels. Of course, it is not the more realistic assessments of the impact of basic research that should be prevented, but the lowering of basic research investments as a consequence. There are three possible ways of preventing such attempts. First, one could give optimistic researchers a strong say in decisions about basic research investments. Of course, these views have to be balanced to prevent excessive basic research investments. Second, one could allow the government to issue public debt the amount of which is dependent on the level of research activities. This would provide generations with more incentives to undertake the socially optimal amount of research. We will explore this case in the next subsection. Third, attempts to lower research investment could also be prevented by traditional constitutional means making deviations from the social optimum difficult for a single generation. This can be achieved, by say, committing to longer term funding plans that cannot be rapidly changed by one individual generation.

5.3 Debt financing

We have shown that the decentralized solution implies less research than the social optimum. The reason is straightforward. The decentralized government does not internalize the marginal benefits of research for future generations. To increase the the government's labor demand, one could allow to issue debt. More precisely, our goal is study whether there is some amount of debt d_t enabling the steady state social planner solution to be implemented in the decentralized case. We assume that the government has access to financial markets and can borrow at the rate r_t without any frictions. Also, we assume that debt can be fully rolled over to the next generation, i.e. if the government in period t borrows d_t, in it can always borrow at least $r_{t+1}d_t$ in $t+1$.

We propose a debt contract made up of two parts. First, the government is allowed to borrow an amount equal to the total debt level times interest in t . As described above, this allows to roll over debt. Second, the government can borrow some amount $D_t(L_t^{D,R})$, which depends on its demand for research labor. We can show that if

$$D_t(L_t^{D,R}) = \left(\frac{1}{(1-L_t^{D,R})^{\beta(1-\alpha)}} - 1\right) Y_t,$$

then the social optimum can be implemented in the market economy. To prove this, we write the modified maximization problem of the government as

$$\max_{L_t^{D,R}} \log\left(Y_t + D_t(L_t^{D,R}) + r_t d_{t-1} - r_t d_{t-1}\right) + \beta \log\left(Y_{t+1} + r_{t+1} d_t - r_{t+1} d_t\right),$$

where $d_t = D_t(L_t^{D,R}) + d_{t-1} r_t$ is the accumulated debt stock, i.e. the sum of debt taken over from the previous generation plus the additional debt from period t. We can simplify the problem to

$$\begin{aligned}
&\max_{L_t^{D,R}} \ \log\left(\frac{Y_t}{(1-L_t^{D,R})^{\beta(1-\alpha)}}\right) + \beta \log\left(Y_{t+1}\right) = \\
&\max_{L_t^{D,R}} \ \log\left(\frac{((1-L_t^{D,R})B_t)^{1-\alpha} K_t^{\alpha}}{(1-L_t^{D,R})^{\beta(1-\alpha)}}\right) + \beta \log\left(((1-L_{t+1}^{D,R})B_t(1+\theta L_t^{D,R}))^{1-\alpha} K_{t+1}^{\alpha}\right),
\end{aligned}$$

where as before, K_t and B_t are state variables in period t. In t, the government perceives L_{t+1} and K_{t+1} as independent of its choice. Due to the logarithmic utility function, the problem is a sum in which the components depending on the aforementioned variables can be omitted. This enables us to reduce the problem to

$$\max_{L_t^{D,R}} (1-\beta) \log\left(1 - L_t^{D,R}\right) + \beta \log\left(1 + \theta L_t^{D,R}\right),$$

which yields

$$\begin{aligned}
&\frac{1-\beta}{1-L_t^{D,R}} - \frac{\beta\theta}{1+\theta L_t^{D,R}} = 0 \\
&\Rightarrow L_t^{D,R} = \beta - \frac{1-\beta}{\theta} = L^{O,R}.
\end{aligned}$$

It is thus possible to implement the socially optimal steady state as labor demand in every period by allowing the government to issue debt. In the Appendix, we show that such a debt contract leads to a constant ratio of debt to output if the interest rate satisfies $r < 1 + \theta L^{O,R}$.

The socially optimal demand must be financed in order to become the new market equilibrium. For this, we assume that all debt income is used to finance wages in the research sector. At best, this would allow the government to lower income taxes τ_t to zero and pay researchers the amount $D_t(L^{O,R})$. We cannot however be certain whether $D_t(L^{O,R})$ will actually cover the government's financing need, which has increased, because the government wants to employ more researchers than before. It may even be the case that the government will have to raise the income tax rate. Therefore, when the government implements the optimal amount $L^{O,R}$ and issues debt of magnitude $D_t(L^{O,R})$, balancing the budget requires

$$L^{O,R} w_t^R = \tau_t^G (1 - L^{O,R}) w_t^P + D_t(L^{O,R}),$$

with τ_t^G being the new tax rate. The right-hand side of this equation shows the two sources of government income: Taxes and debt. Substituting w_t^R from (20) and simplifying yields

$$w_t^P \kappa \left(\frac{L^{O,R}}{1 + (1 - L^{O,R})(\theta_h - \theta_l)\left(1 - \frac{L^{O,R}}{\overline{L}_B}\right)} - \tau_t^G \right) = D_t(L^{O,R}), \qquad \text{with}$$

$$\kappa := \left(\frac{1 + (1 - L^{O,R})(\theta_h - \theta_l)\left(1 - \frac{L^{O,R}}{\overline{L}_B}\right)}{1 + (\theta_h - \theta_l)\left(1 - \frac{L^{O,R}}{\overline{L}_B}\right)} \right).$$

Note that the term

$$\frac{L^{O,R}}{1 + (1 - L^{O,R})(\theta_h - \theta_l)\left(1 - \frac{L^{O,R}}{\overline{L}_B}\right)}$$

is exactly the tax rate that would be required to finance the optimal labor demand in the absence of debt. Thus, we call it τ^S and have

$$w_t^P \kappa (\tau^S - \tau_t^G) = D_t(L^{O,R}).$$

By plugging in the definition of $D_t(L^{O,R})$, and making use of $w_t^P = (1-\alpha)Y_t/(1-L^{O,R})$, we arrive at

$$\tau^S - \tau^G = \left((1-L^{O,R})^{-\beta(1-\alpha)} - 1\right)\frac{Y_t(1-L^{O,R})}{(1-\alpha)\kappa Y_t},$$
$$\tau^G = \tau^S - \frac{1-L^{O,R}}{(1-\alpha)\kappa}\left((1-L^{O,R})^{-\beta(1-\alpha)} - 1\right),$$

and find that the government does not have to increase the tax rate to τ^S thanks to the presence of debt financing. However, it is not clear whether τ^G will be greater or smaller than the previous τ. Their relative size depends on the difference between the market and the social planner outcome and hence on parameter constellations.

6 Extensions

The model allows a number of extensions that shed further light on the role of research bubbles.

6.1 Effort in knowledge production

In this extension, the output of the research sector depends not only on the number of researchers but also on the effort they invest. The expected production function for knowledge changes to

$$\widetilde{B}_{t+1} = B_t(1 + \widetilde{\theta}_t L_t^{D,R} E_t),$$

where, as before, B_t stands for the knowledge stock in t, $\widetilde{B}_{t+1}$ for the expected knowledge stock in $t+1$, $\widetilde{\theta}_t$ for the governments' estimate of the productivity of the research sector, given by $\widetilde{\theta}_t = \eta\theta_h + (1-\eta)\theta_{crit,t}$, and $L_t^{D,R}$ for the demand for research labor. The new variable is E_t, which is the aggregate effort of researchers, i.e.

$$E_t = \int_{\theta_{crit,t}}^{\theta_h} e_{t,i} di,$$

where $e_{t,i}$ is the effort of the individual researcher i. Every θ_i corresponds to a finite value of $e_{t,i}$, making the integral finite. Researchers choose $e_{t,i}$ by maximizing utility. Unlike before, utility from research depends on the product of how efficient the agent believes his research to be, θ_i and the effort $e_{t,i}$ he invests. Effort is also associated with costs, which we capture by the cost function $C(e_{t,i}) = q\frac{e_{t,i}^2}{2}$, where $q \geq 0$ is a scaling parameter. The effort-augmented utility function for a researcher thus writes as

$$U_{t,i}^R = \log(w_t^R - s_{t,i}) + (1+\beta)\log(\theta_i e_{t,i} - q\frac{e_{t,i}^2}{2}) + \beta\log(r_{t+1}s_{t,i}).$$

Maximizing utility with respect to effort and savings yields

$$e_{t,i} = \frac{\theta_i}{q}, \;\; s_{t,i}^P = \beta\frac{(1-\tau_t)w_t^P}{1+\beta} \text{ and } s_{t,i}^R = \beta\frac{w_t^R}{1+\beta}.$$

Plugging these optimal choices into the utility of a researcher and setting it equal to the utility of a worker, we obtain the critical value for the belief θ_i:

$$\theta_{crit,t} = \sqrt{2q\frac{(1-\tau_t)w_t^P}{w_t^R}}.$$

In this extension, the government faces the following maximization problem:

$$\begin{aligned}\max_{L_t^{D,R}} &\log(((1-L_t^{D,R})B_t)^{1-\alpha}K_t^\alpha)+ \\ &\beta\log(((1-L_{t+1}^{D,R})B_t(1+\widetilde{\theta}_t E_t L_t^{D,R}))^{1-\alpha}K_{t+1}^\alpha),\end{aligned}$$

where it takes B_t, K_t and E_t as given and both K_{t+1} and $L_{t+1}^{D,R}$ as outside its sphere of influence. Hence, the problem can be written as

$$\max_{L_t^{D,R}} \log(1-L_t^{D,R}) + \beta\log(1+\widetilde{\theta}_t E_t L_t^{D,R}),$$

which gives the following demand function:

$$L_t^{D,R} = \frac{1}{1+\beta}\left(\beta - \frac{1}{E_t\widetilde{\theta}_t}\right), \text{ with} \tag{25}$$
$$E_t = \int_{\theta_{crit,t}}^{\theta_h} \frac{\theta_i}{q} di = \frac{\theta_h^2 - \theta_{crit,t}^2}{2q}.$$

Plugging in the definitions of E_t and $\widetilde{\theta}_t$, we can write demand as a function of $\theta_{crit,t}$ only:

$$L_t^{D,R} = \frac{1}{1+\beta}\left(\beta - \frac{4q}{(\theta_h^2 - \theta_{crit,t}^2)(\theta_h + \theta_{crit,t})}\right).$$

It is possible to show that this demand function is strictly concave in $\theta_{crit,t}$, which yields two possible labor market equilibria, given that labor supply is the same linear function as in the previous model [7]. Moreover, we demonstrate that the market equilibrium with greater labor in research L^R is preferred by the government.

We also explore the solution of a social planner who knows the true value of θ:

$$\max_{c_{t,i}^1, c_{t+1,i}^2, e_{t,i}, L_t^R, B_{t+1}, K_{t+1}} W(c_{t,i}^1, c_{t+1,i}^2, e_{t,i}, L_t^R, B_{t+1}, K_{t+1}),$$

with

$$W = \sum_{t=0}^{\infty} \beta_s^t \left(\int_0^1 \log(c_{t,i}^1) + \beta \log(c_{t+1,i}^2)\, di + \frac{1}{L_t^R}\int_0^{L_t^R} \log(\theta e_{t,i} - q\frac{e_{t,i}^2}{2})\, di \right),$$

subject to

$$((1-L_t^R)B_t)^{1-\alpha}K_t^{\alpha} = \int_0^1 (c_{t,i}^1 + c_{t,i}^2)\, di + K_{t+1},$$

and

$$B_{t+1} = (1 + \theta \cdot \int_0^1 e_{t,i}\, di \cdot L_t^R)B_t.$$

For a symmetric equilibrium with $e_{t,i} = E_t$ and $c_{t,i}^1 = c_t^1 = c_{t,i}^2 = c_t^2\ \forall i$, due to $\beta_s = \beta$, we find the following optimality conditions:

[7] Proofs for this claim, as well as for all others in this section, are available on request.

$$\frac{1}{c_t^1} = \frac{\alpha\beta Y_{t+1}}{K_{t+1}c_{t+1}^2}, \tag{26}$$

$$\frac{Y_t}{1-L_t^R}\frac{1}{E_t} = \frac{K_{t+1}}{\alpha Y_{t+1}}\left(\frac{\theta Y_{t+1}}{(1+\theta L_t^R E_t)} + \frac{Y_{t+1}}{(1-L_{t+1}^R)E_{t+1}}\frac{1+\theta L_{t+1}^R E_{t+1}}{(1+\theta L_t^R E_t)}\right), \tag{27}$$

$$\frac{E_t}{L_t^R}\frac{\theta - qE_t}{\theta E_t - q\frac{E_t^2}{2}} = -\frac{2(1-\alpha)}{(1-\frac{K_{t+1}}{Y_t})(1-L_t^R)}, \tag{28}$$

which in a steady state yield

$$L^R = \beta - \frac{1-\beta}{\theta E}, \quad \text{and} \tag{29}$$

$$E^2 q\left[\frac{(1-\alpha)\beta}{1-\alpha\beta} + 1 - \beta\right] - (1-\beta)\left[1 - \frac{2(1-\alpha)}{1-\alpha\beta}\right] \tag{30}$$
$$- E\left[(1-\beta)(\theta - \frac{q}{\theta}) + \frac{2(1-\alpha)}{1-\alpha\beta}\left(\beta\theta + \frac{(1-\beta)q}{2\theta}\right)\right] = 0.$$

We find that the socially optimal labor input equation, L^R, is structurally equivalent to what it was before, but that θ is replaced by θE. The same holds for the decentralized labor demand. Hence, if θ and E were the same in the decentralized economy and the social planner solution, L^R would be too low in the decentralized economy. However, if aggregate effort E was greater in the market equilibrium, this could mitigate the government's myopia. To investigate this possibility, we compare the first order condition for the individual effort of some agent i in the decentralized economy to that of the social planner:

$$\frac{\theta_i - qe_{t,i}}{\theta_i e_{t,i} - q\frac{e_{t,i}^2}{2}} + 0 = 0, \tag{31}$$

$$\frac{\theta - qe_t}{\theta e_t - q\frac{e_t^2}{2}} + \frac{2(1-\alpha)Y_t}{e_t C_t}\frac{L_t^R}{1-L_t^R} = 0, \tag{32}$$

The first equation is the first-order condition in the decentralized equilibrium. We obtain two differences. First, the individual agent bases his effort on his belief θ_i and not on θ. Second, he does not internalize the positive externality of his effort on knowledge production, which is captured by the second term in equation (32). Hence it is not clear whether an individual agent supplies more or less effort than would be socially optimal. On the one hand, the individual belief might be greater than θ, implying more effort, while on the other hand, the agent might not be aware of the externality, implying less effort. It is likely, however, that the effort externality is greater than the over-optimism, especially for more conservative agents, so that aggregation of individual effort would provide an overall level of effort that is too small compared to the social optimum.

With this finding, the implementation of the social optimum in the decentralized economy cannot be achieved by introducing government debt alone, even when the government knows θ. It is also necessary to create incentives for scientists to provide the optimum amount of effort. Hence the following wage contracts must be offered to a researcher with belief θ_i:

Proposition 8. *The social optimum can be obtained in the decentralized economy by offering researcher i with belief θ_i the wage*

$$
\begin{aligned}
w_{t,i}^R &= \widetilde{w}_t^R \left(\frac{\hat{\theta} e_{t,i} - q\frac{e_{t,i}^2}{2}}{\theta_i e_{t,i} - q\frac{e_{t,i}^2}{2}} \right)^{\frac{1}{1+\beta}} e^{\frac{\hat{G}_t e_{t,i}}{1+\beta}}, \quad \textit{with} \\
\widetilde{w}_t^R &= w_t^P \left(\hat{\theta} e_t - q\frac{e_t^2}{2} \right)^{\frac{-1}{1+\beta}} e^{-\hat{G}_t \frac{e_t}{1+\beta}}, \\
e_t &= \frac{\theta_{crit} - \theta_l}{\theta_h - \theta_l} E_t^{SOC},
\end{aligned}
$$

and allowing the government to issue debt of magnitude

$$
\frac{Y_t}{(1 - L^{D,R})^{\beta(1-\alpha)}} + r_t d_{t-1},
$$

where $\widetilde{w}_t^R$ is a fixed-wage component that is equal for all researchers. $L^{R,SOC,H}$ and E^{SOC} stand for the social planner steady state levels of research labor and aggregate effort. Furthermore, $\hat{\theta} = \frac{\theta_h - \theta_l}{\theta_h - \theta_{crit}}\theta$ and $\hat{G}_t = \frac{\theta_h - \theta_{crit}}{\theta_h - \theta_l} G_t$. θ_{crit} corresponds to the steady state level of the critical belief value that leads to the socially optimal supply of research labor. e_t is individual effort, which is the same for all researchers. τ_t is the tax rate that balances the budget, as before.

The Appendix contains the proof that this payment scheme will implement the social optimum. The intuition behind it is the following: First, we want every researcher to base his effort decision on the true parameter and not on his beliefs. Therefore $w_{t,i}^R$ depends on θ_i and θ, i.e. on the individual belief and on the scaled true parameter.[8] Second, every researcher is supposed to internalize the effect of his effort and research productivity. Therefore $\hat{G}_t$ is incorporated in the wage scheme. Third, the government is supposed to increase its labor demand, so that the debt it can issue depends on $L_t^{D,R}$. The first of our measures simplifies the task of implementing the social optimum. The second and third fulfill the task.

6.2 Linear utility

Another variant of the model is one with effort and a linear utility function for the government. Accordingly, the government's problem is

$$\max_{L_t^{D,R}} ((1 - L_t^{D,R})B_t)^{1-\alpha} K_t^{\alpha} + \beta((1 - L_{t+1}^{D,R})B_t(1 + \widetilde{\theta}_t E_t L_t^{D,R}))^{1-\alpha} K_{t+1}^{\alpha}.$$

In the following we use $\widetilde{Y}_{t+1}$ to denote the output that the government believes will be created, while Y_{t+1} is the true future output. Maximizing with respect to $L_t^{D,R}$ yields

$$\frac{Y_t}{1 - L_t^{D,R}} = \beta \frac{\widetilde{\theta}_t E_t \widetilde{Y}_{t+1}}{1 + \widetilde{\theta}_t E_t L_t^{D,R}}, \text{ with} \tag{33}$$

$$E_t = \int_{\theta_{crit,t}}^{\theta_h} \frac{\theta_i}{q} di = \frac{\theta_h^2 - \theta_{crit,t}^2}{2q} \tag{34}$$

$$\text{and } \widetilde{\theta}_t \text{ given by (16).}$$

Unlike before, the first order condition of the government is a dynamic equation in L_t^R and does not yield a time-constant value for research labor demand. However, a steady state with constant labor demand can be found

$$L^{D,R} = 1 - \frac{1}{\beta \widetilde{\theta} E} = 1 - \frac{4q}{\beta(\theta_h^2 - \theta_{crit}^2)(\theta_h + \theta_{crit})}.$$

In this case, it is possible that the government may demand more research labor than is socially optimal. Furthermore, the demand function is not strictly increasing in θ_{crit}, as can be seen from plugging in $\widetilde{\theta}$ and E. We can show that $L^{D,R}$ is a convex function in θ_{crit}, so that two market equilibria are possible. It can be

[8] Why the true parameter needs to be scaled is set out in the Appendix.

demonstrated, though, that one of them yields more utility for the government.[9]

The dynamic demand function allows for a convergence analysis. We find that the decentralized market outcome is saddle-path stable. The implementation of the social optimum steady state as a balanced growth path for the decentralized economy is possible. However, this is only the case if the initial market allocation implies a lower level of research labor. If that is so, then a combination of public debt and wage contracts for researchers allows for implementation, as discussed above.

7 Conclusion

We have developed a model that provides a rationale and a microfoundation for research bubbles. Such research bubbles emerge when researchers self-select into those activities they believe to be most promising, and when the assessments of these researchers are aggregated by the government. Furthermore, bubbles can implement socially desirable allocations. Thus specific forms of research bubbles are desirable from a long-term welfare perspective. Numerous extensions require further scrutiny, as they have the potential to shed further light on the emergence, social desirability, and downside of research bubbles, which may be a key factor of modern knowledge economies.

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[9] By plugging in $L^{D,R}$ into the government's utility function, one can analyze the governments utility as a function of θ_{crit}. It turns out to be a convex function over the interval $[\theta_l, \theta_h]$ with a local maximum. Hence, the desirability of the equilibria can be studied by their proximity to the local maximum.

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8 Appendix

8.1 Proof of proposition 1

Assume there is a balanced growth path along which $L_t^R = \hat{L}^R\ \forall t$, where $\hat{L}^R$ is constant. The saving decision implies $s_t = \frac{\beta w_t^P(1-\tau_t)}{1+\beta} = K_{t+1}$. Furthermore, we have $w_t^P = \frac{(1-\alpha)Y_t}{1-L_t^R}$. Substituting w_t^P into the savings decision yields

$$K_{t+1} = \frac{\beta(1-\tau_t)}{(1+\beta)}\frac{(1-\alpha)Y_t}{1-L_t^R}.$$

Recall that $L_t^R = \tau_t$ has to hold, so we have

$$\frac{K_{t+1}}{Y_t} = \frac{\beta(1-\alpha)}{1+\beta}. \tag{35}$$

As $\frac{K_{t+1}}{Y_t}$ is constant, K_t and Y_t grow at the same rate. We show that this rate is constant and equal to $\theta\hat{L}^R$. First, we take the logarithm of the production function

$$\log(Y_t) = (1-\alpha)\log(L_t^P) + (1-\alpha)\log(B_t) + \alpha\log(K_t).$$

Then we write the analogous equation for the next period Y_{t+1} and subtract both to obtain

$$g_{Y_t} = (1-\alpha)g_{L_t^P} + (1-\alpha)g_{B_t} + \alpha g_{K_t},$$

where $g_{(.)}$ stands for the growth rate of the variable in the index. As in the steady state, the labor share is constant in both sectors and it holds that $g_{L_t^P} = 0$. We have just found that $g_{K_t} = g_{Y_t}$, even outside the steady state, so we obtain

$$g_{Y_t} = g_{B_t} = \frac{B_{t+1}}{B_t} - 1 = \theta\hat{L}^R.$$

Hence, the interest rate $\hat{r}$ is given by

$$\hat{r} = \frac{\alpha Y_{t+1}}{K_{t+1}} = \frac{\alpha Y_{t+1}Y_t}{Y_tK_{t+1}} = \frac{\alpha(1+\theta\hat{L}^R)(1+\beta)}{\beta(1-\alpha)}, \tag{36}$$

because $Y_{t+1}/Y_t = 1 + \theta\hat{L}^R$.

8.2 Steady state of the social planner solution

For clarity we write down the Lagrange function of the social planner:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta_s^t \big[\log(c_t^1) + \beta \log(c_{t+1}^2) - \lambda_t \left(((1-L_t^R)B_t)^{1-\alpha} K_t^\alpha - c_t^1 - c_t^2 - K_{t+1} \right) \\ - \mu_t \left(B_{t+1} - (1+\theta L_t^R)B_t \right) \big],$$

where λ_t and μ_t are the Lagrange multiplier described in the main text. We derive the steady state from equations (9) and (10) in the following way: First, we note that maximizing the Lagrangian with respect to c_t^1 and c_{t+1}^2 yields $c_{t+1}^1 \frac{\beta}{\beta_s} = c_{t+1}^2$. We assume $\beta = \beta_s$, so that $c_t^1 = c_t^2$ and consequently, $c_t^1 = C_t/2$, where C_t stands for aggregate consumption in period t. We substitute this expression into (9) and obtain

$$\frac{C_{t+1}}{C_t} = \frac{\alpha\beta Y_{t+1}}{K_{t+1}}, \quad \Rightarrow \quad \frac{C_{t+1}Y_t}{C_t Y_{t+1}} = \frac{\alpha\beta Y_t}{K_{t+1}}.$$

In the steady state, aggregate consumption and output grow at the same rate, so that the left-hand side is 1. Hence we find

$$\frac{\overline{K}_{t+1}}{\alpha\overline{Y}_t} = \beta,$$

where $\overline{K}_{t+1}$ and $\overline{Y}_t$ stand for the constantly growing values of capital and output. Then we simplify (10) to

$$\frac{1}{1-L^R} = \frac{\overline{K}_{t+1}}{\alpha\overline{Y}_t} \left(\frac{\theta}{1+\theta L^R} + \frac{1}{1-L^R}\frac{1+\theta L^R}{1+\theta L^R} \right),$$

where we can substitute $\frac{\overline{K}_{t+1}}{\alpha\overline{Y}_t}$ by β,

$$1+\theta L^R = \beta(\theta - \theta L^R + 1 + \theta L^R),$$
$$\theta L^R = \beta\theta - (1-\beta).$$

8.3 Critical belief

Using our results from (13), we can write the utility of agent i, if he is a researcher, as

$$U_{t,i}^R = \log\left(\frac{w_t^R}{1+\beta}\right) + (1+\beta)\log\left(1-\theta_l+\theta_i\right) + \beta\log\left(\frac{\beta r_{t+1} w_t^R}{1+\beta}\right).$$

His utility as a worker is

$$U_{t,i}^P = \log\left(\frac{(1-\tau_t)w_t^P}{1+\beta}\right) + \beta\log\left(\frac{\beta r_{t+1}(1-\tau_t)w_t^P}{1+\beta}\right).$$

The individual decision of agent i has no impact on the interest rate r_{t+1} so that the respective interest rates are taken as equal in both cases. Hence we set $U_{t,i}^R = U_{t,i}^P$ for the agent with $\theta_i = \theta_{crit,t}$,

$$(1+\beta)\log(w_t^R) + (1+\beta)\log\left(1-\theta_l+\theta_i\right) + \beta\log\left(\beta r_{t+1}\right) - (1+\beta)\log(1+\beta)$$
$$=$$
$$(1+\beta)\log\left((1-\tau_t)w_t^P\right) + \beta\log(\beta r_{t+1}) - (1+\beta)\log(1+\beta),$$
$$\Leftrightarrow \log\left(1-\theta_l+\theta_i\right) = \log\left(\frac{(1-\tau_t)w_t^P}{w_t^R}\right),$$
$$\Leftrightarrow \theta_{crit,t} = \frac{(1-\tau_t)w_t^P}{w_t^R} - (1-\theta_l).$$

Proof of proposition 8

After substituting the optimal savings decision, the utility function of a researcher reads

$$U_{t,i}^R = \beta\log\left(\frac{\beta}{1+\beta}\right) - \log(1+\beta) + \log(w_t^R)(1+\beta) + \log\left(\theta_i e_{t,i} - q\frac{e_{t,i}^2}{2}\right) + \beta\log(r_{t+1}).$$

We want to alter the utility function of the researcher so that the derivative with respect to effort will be identical to the first order condition of the social planner. One way to do this is by setting w_t^R. Substituting w_t^R from proposition 8 changes the utility function to

$$
\begin{aligned}
U_{t,i}^{R} =& \beta\log(\frac{\beta}{1+\beta}) + (1+\beta)\log(\widetilde{w_t}^{R}) + \log(\hat{\theta}e_{t,i} - q\frac{e_{t,i}^2}{2}) + \hat{G}_t e_{t,i} + \\
& \beta\log(r_{t+1}), \quad \text{with the derivative}
\end{aligned} \tag{37}
$$

$$
\frac{\partial U_{t,i}^{R}}{\partial e_{t,i}} = \frac{\hat{\theta} - q e_{t,i}}{\hat{\theta}e_{t,i} - q\frac{e_{t,i}^2}{2}} + \hat{G}_t = 0. \tag{38}
$$

Solving for $e_{t,i}$, which is now the same for all i, yields

$$
e_t^2 - \frac{2}{q}(\hat{\theta} - \frac{q}{\hat{G}_t})e_t - \frac{2\hat{\theta}}{q\hat{G}_t} = 0, \tag{39}
$$

while the social planner FOC implies

$$
e_t^{SOC2} - \frac{2}{q}(\theta - \frac{q}{G_t})e_t^{SOC} - \frac{2\theta}{qG_t} = 0. \tag{40}
$$

Due to the definition of $\hat{\theta}$ and $\hat{G}_t$ it holds that $e_t = \frac{\theta_h - \theta_l}{\theta_h - \theta_{crit}} e_t^{SOC}$. To see this, substitute e_t in equation (39). This will yield equation (40). In the decentralized case we purposefully demand more effort from every researcher.

If e_t^{SOC} is the socially optimal individual and aggregate effort level, recall that the mass of all agents is 1. Hence in the steady state of the decentralized economy we have for effort

$$
E^D = \int_{\frac{\theta_{crit}-\theta_l}{\theta_h-\theta_l}}^{1} e\,di = e\frac{\theta_h - \theta_{crit}}{\theta_h - \theta_l} = e^{SOC}\frac{\theta_h - \theta_l}{\theta_{crit} - \theta_l}\frac{\theta_{crit} - \theta_l}{\theta_h - \theta_l} = E^{SOC}.
$$

As mentioned, substituting $e = \frac{\theta_h - \theta_l}{\theta_h - \theta_{crit}} E^D$ in the steady state version of (39) will yield the steady state version of equation (40). Hence we have successfully altered the researchers' individual decisions to replicate one of the social planner's two optimality conditions.

Next we turn to the FOC of the decentralized government when it can issue debt,

$$
\frac{1-\beta}{1-L^{D,R}} = \frac{\beta\theta E}{1+\theta E L^{D,R}}, \tag{41}
$$

which together with equation (38) replicate the two equations of the social planner problem and yield L^{SOC} and E^{SOC} as solutions.

Finally, we prove the definition of $\widetilde{w_t}^R$. Under proposition 8, we demonstrated how optimal effort and labor demand can be established. Now we turn to labor supply. As before, some agent i must be indifferent between working in research and the productive sector. We show that, if $w^R_{t,i}$ is defined as above, all agents are indifferent. To see this, recall equation (37). By construction, if the optimal $e_{t,i}$ is the same for all agents, $U^R_{t,i}$ is also the same. The following equality therefore holds for all i

$$(1+\beta)\log(\widetilde{w_t}^R) + \log(\hat{\theta}e_t - q\frac{e_t^2}{2}) + \hat{G}_t e_t = (1+\beta)\log(w_t^P),$$

which yields

$$\widetilde{w_t}^R = w_t^P \left(\hat{\theta}e_t - q\frac{e_t^2}{2}\right)^{\frac{-1}{1+\beta}} e^{-\frac{\hat{G}_t e_t}{1+\beta}},$$

as all researchers invest the same amount of effort. This concludes the proof.

Furthermore, note that in equilibrium, the following holds

$$\begin{aligned}
w^R_{t,i} &= \widetilde{w_t}^R \left(\frac{\hat{\theta}e_{t,i} - q\frac{e_{t,i}^2}{2}}{\theta_i e_{t,i} - q\frac{e_{t,i}^2}{2}}\right)^{\frac{1}{1+\beta}} e^{\frac{\hat{G}_t e_{t,i}}{1+\beta}}, \quad \text{and} \\
w^R_{t,i} &= \frac{w_t^P}{e_t(\theta_i - q\frac{e_t}{2})^{\frac{1}{1+\beta}}},
\end{aligned}$$

which means that researchers are paid the wage of the productive sector with a mark-down depending on their preferences and optimal effort. This leaves agents indifferent as to sector choice and exertion of socially optimal effort.

Finally, we prove the definition of $\widetilde{w_t}^R$. Under proposition 8 we demonstrated how the optimal effort and labor demand can be established. Now we turn to labor supply. As before, some agent i must be indifferent between working in research and the productive sector. We show that, if $w^R_{t,i}$ is defined as above, all agents are indifferent. To see this, recall equation (37). As, by construction, the optimal $e_{t,i}$ is the same for all agents, $U^R_{t,i}$ is the same as well. The following equality therefore holds for all i:

$$(1+\beta)\log(\widetilde{w}_t^{R}) + \log(\hat{\theta} e_t - q\frac{e_t^2}{2}) + \hat{G}_t e_t = (1+\beta)\log(w_t^P),$$

which yields

$$\widetilde{w}_t^{R} = w_t^P \left(\hat{\theta} e_t - q\frac{e_t^2}{2}\right)^{\frac{-1}{1+\beta}} e^{-\frac{\hat{G}_t e_t}{1+\beta}},$$

as all researchers invest the same amount of effort. This concludes the proof.

Furthermore, note that in equilibrium, the following holds:

$$w_{t,i}^R = \widetilde{w}_t^{R} \left(\frac{\hat{\theta} e_{t,i} - q\frac{e_{t,i}^2}{2}}{\theta_i e_{t,i} - q\frac{e_{t,i}^2}{2}}\right)^{\frac{1}{1+\beta}} e^{\frac{\hat{G}_t e_{t,i}}{1+\beta}},$$

$$w_{t,i}^R = \frac{w_t^P}{e_t(\theta_i - q\frac{e_t}{2})^{\frac{1}{1+\beta}}},$$

which means that researchers are paid the wage of the productive sector with a mark-down depending on their preferences and optimal effort. This leaves agents indifferent as to sector choice and exertion of socially optimal effort.

8.4 Stability analysis

In this section we analyze the stability of the steady states of the decentralized economy and the social planner solution in both models.

Decentralized solution without research bubbles

We have demonstrated that research labor is equal to the goverment's demand, given by

$$L_t^R = L^{D,R} = \frac{1}{1+\beta}\left(\beta - \frac{1}{\theta}\right).$$

It is therefore always constant and implies the following constant gross growth rate of the knowledge stock: $B_{t+1}/B_t = 1 + \theta L^R$. Note that savings are given by

$$s_t = K_{t+1} = \frac{\beta(1-\tau_t)w_t^P}{1+\beta},$$

so that substituting $w_t^P = (1-\alpha)Y_t/(1-L_t^R)$ yields

$$K_{t+1} = \frac{(1-\alpha)\beta(1-\tau_t)}{1+\beta}(1-L_t^R)^{-\alpha}B_t^{1-\alpha}K_t^{\alpha},$$

where $\tau_t = L_t^R = L^R$, as shown previously. We divide both sides by B_{t+1},

$$\frac{K_{t+1}}{B_{t+1}} = \frac{(1-\alpha)\beta}{(1+\beta)}(1-L^R)^{1-\alpha}\left(\frac{K_t}{B_t}\right)^{\alpha}\frac{B_t}{B_{t+1}}.$$

We write capital in terms of the knowledge stock and define $k_t = K_t/B_t$. Hence, given the constant growth rate of B_{t+1}/B_t, we have

$$k_{t+1} = \frac{(1-\alpha)\beta(1-\tau)}{(1+\beta)(1-L^R)^{\alpha-1}(1+\theta L^R)}k_t^{\alpha},$$

which is a simple convex policy-function in capital as the factor in front of k_t^{α} is a mere constant.

Social planner solution

To study whether the economy converges to the socially optimal steady state, we rewrite (10):

$$\begin{aligned}
\frac{1+\theta L_t^R}{1-L_t^R} &= \frac{K_{t+1}}{\alpha Y_{t+1}}\frac{Y_{t+1}}{Y_t}\left(\theta + \frac{1+\theta L_{t+1}^R}{1-L_{t+1}^R}\right), \quad \text{which implies} \\
\frac{1+\theta L_t^R}{1-L_t^R} &= \frac{K_{t+1}}{\alpha Y_{t+1}}\frac{Y_{t+1}}{Y_t}\frac{1+\theta}{1-L_{t+1}^R} \quad \text{or simply} \\
L_{t+1}^P = 1 - L_{t+1}^R &= \frac{K_{t+1}}{\alpha Y_{t+1}}\frac{Y_{t+1}}{Y_t}\frac{1-L_t^R}{1+\theta L_t^R}(1+\theta), \qquad (42)
\end{aligned}$$

so that the model dynamics are governed by (42), the knowledge production function, and the following three equations:

$$\begin{aligned}
Y_t &= K_t^\alpha (B_t L_t^P)^{1-\alpha}, \\
K_{t+1} &= Y_t - c_t^1 - c_t^2, \\
c_{t+1}^2 &= \beta\alpha \frac{Y_{t+1}}{K_{t+1}} c_t^1.
\end{aligned}$$

We can rewrite aggregate consumption $C_t = c_t^1 + c_t^2$, using the fact that the maximization of the Lagrangian with respect to c_t^1 and c_{t+1}^2 yields $c_{t+1}^1 \frac{\beta}{\beta_s} = c_{t+1}^2$. Assuming $\beta = \beta_s$, we arrive at $c_t^1 = C_t/2$. Next we express the four equations in terms of effective labor $B_t L_t^P$, defining g_{B_t} and g_{L_t} as the growth rate of knowledge and of labor input in the productive sector in period t,

$$\begin{aligned}
\frac{Y_t}{B_t L_t^P} &= y_t = k_t^\alpha, \\
k_{t+1}(1+g_{B_t})(1+g_{L_t}) &= y_t - c_t, \\
c_{t+1}(1+g_{B_t})(1+g_{L_t}) &= \beta\alpha k_{t+1}^{\alpha-1} c_t,
\end{aligned}$$

where k_t and c_t without superscript indicate capital and aggregate consumption per effective labor. Equation (10) becomes

$$\frac{L_{t+1}^P}{L_t^P} = 1 + g_{L_t} = \frac{K_{t+1}}{\alpha Y_{t+1}} \frac{Y_{t+1}}{Y_t} \frac{1}{1+\theta L_t^R}(1+\theta).$$

By using $1 + g_{B_t} = 1 + \theta L_t^R$ from the knowledge production function we obtain

$$\begin{aligned}
1 + g_{L_t} &= \frac{1}{\alpha} \frac{k_{t+1}}{y_{t+1}} \frac{y_{t+1}}{y_t} (1+g_{B_t})(1+g_{L_t}) \frac{(1+\theta)}{1+g_{B_t}}, \\
1 &= \frac{1+\theta}{\alpha} \frac{k_{t+1}}{k_t^\alpha}, \\
k_{t+1} &= \frac{\alpha}{1+\theta} k_t^\alpha. \qquad (43)
\end{aligned}$$

We arrive at a concave policy-function for capital per effective labor k_t. Next we turn to the Euler equation for consumption. By using the following relationships:

$$\begin{aligned}
1 + g_{B_t} &= 1 + \theta L_t^R, \\
1 + g_{L_t} &= \frac{K_{t+1}}{\alpha Y_t} \frac{1}{1+\theta L_t^R}(1+\theta) = \frac{Y_t - C_t}{\alpha Y_t} \frac{1}{1+\theta L_t^R}(1+\theta),
\end{aligned}$$

we obtain

$$\frac{c_{t+1}}{c_t} = \frac{1}{(1+\theta)}\beta\alpha k_{t+1}^{\alpha-1}\frac{\alpha Y_t}{Y_t - C_t},$$
$$\frac{c_{t+1}}{c_t} = \frac{\alpha^2\beta}{1+\theta}\frac{k_{t+1}^{\alpha-1}k_t^{\alpha}}{k_t^{\alpha}-c_t}. \tag{44}$$

Hence it is possible to express the system in the two equations (43) and (44) in two variables, consumption and capital per effective labor. The steady state along which g_{B_t} and all variables in terms of effective labor, and equivalently the amount of labor supplied to the productive sector are constant is given by

$$k = \left(\frac{\alpha}{1+\theta}\right)^{\frac{1}{1-\alpha}},$$
$$c = k^{\alpha}(1 - \frac{\alpha^2\beta}{1+\theta}k^{\alpha-1}) = \left(\frac{\alpha}{1+\theta}\right)^{\frac{\alpha}{1-\alpha}}(1-\alpha\beta)\,.$$

As the policy-function for capital is concave, we know that capital will converge to the steady state. The convergence of consumption is not clear. We know that for consumption to grow at a positive rate, it must hold that

$$\frac{c_{t+1}}{c_t} > 1 \Rightarrow c_t > k_t^{\alpha} - \alpha^2\beta\left(\frac{\alpha}{1+\theta}\right)^{\alpha-1}k_t^{\alpha^2}. \tag{45}$$

We use the phase diagram in Figure 3 to discuss the convergence of the model. The black vertical line is the steady state condition for capital. The curved black line shows the respective condition for consumption. The red line divides the space into two sectors: The one above the red line indicates where consumption is growing, the one below the red line shows where consumption is decreasing. As we know that capital converges to the steady state from both sides, we can draw the appropriate arrows that indicate the movement of the variables.

The diagram shows that the model is saddle-path-stable, as convergence to the steady state, which is the intersection of all three lines, does not occur from any arbitrary initial allocation of c_t and k_t. For instance, convergence from an allocation in the right upper corner, where consumption lies above the red line and capital to the right of the vertical black line, is not possible. The economy would move to $k_t = 0$. In Figure 4, the space between the green lines shows all possi-

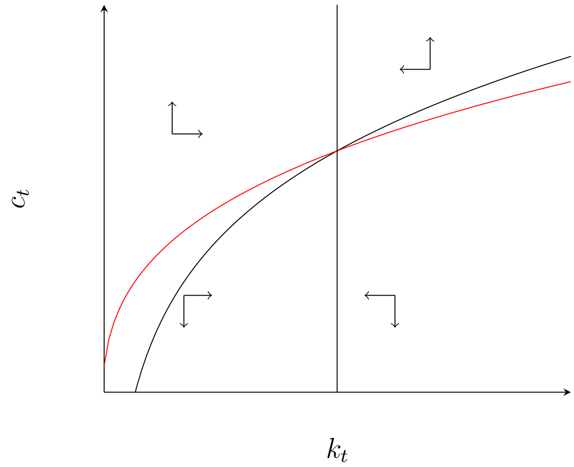

Figure 3: Phase diagram for consumption and capital per effective labor.

ble initial allocations of capital and consumption from which convergence to the unique steady state occurs.

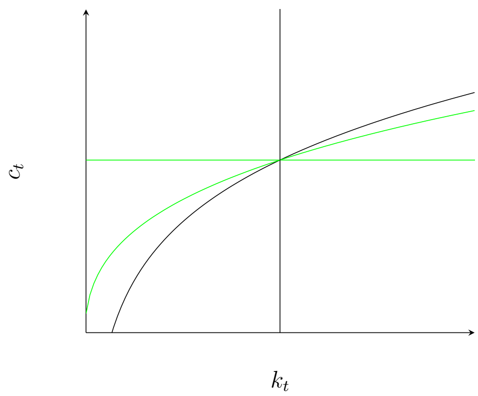

Figure 4: Set of initial allocations from which the economy converges to the steady state.

We find further support for the saddle-path-stability of the system by linearizing it around the steady state. The linearized versions of (43) and (44) are

$$\begin{pmatrix} \widehat{k}_{t+1} \\ \widehat{c}_{t+1} \end{pmatrix} = \begin{pmatrix} \alpha & 0 \\ \left(\frac{\alpha^2 c}{k} - \frac{\alpha c k^{\alpha-1}}{k^\alpha - c}\right) & \left(1 + \frac{c}{k^\alpha - c}\right) \end{pmatrix} \begin{pmatrix} \widehat{k}_t \\ \widehat{c}_t \end{pmatrix}.$$

As the second entry of the first row is 0, the Eigenvalues of the matrix are given by $\alpha = 0.3 < 1$ and $\left(1 + \frac{1}{k^\alpha - c}\right) > 1$. The first Eigenvalue is real and smaller in absolute value than 1, while the second is real and larger than 1. To see this, note that k^α is aggregate production per effective labor, while c is the aggregate consumption per effective labor. This difference is positive, so the second Eigenvalue is at least one.

Government solution with research bubbles

As we have demonstrated, the research bubbles lead to a greater input of labor in the research sector. Yet this value is constant over time, so that the dynamics of the model do not change in comparison to the model without bubbles. Thus our analysis from the section above carries over, the only difference being that L^R is greater.

8.5 Convergence under debt financing

We established above that it is theoretically possible to implement the socially optimal steady state amount of research in the decentralized economy. In this section we investigate whether the model with heterogeneous beliefs converges to its social optimum. If the optimal amount of research is implemented, i.e. if $L^R = L^{O,R}\,\forall\, t$, we have

$$K_{t+1} = \frac{\beta}{1+\beta}(w_t^R L^{O,R} + (1-\tau^G)w_t^P(1-L^{O,R})).$$

We substitute w_t^R to obtain

$$\frac{K_{t+1}}{B_{t+1}} = k_{t+1} = \frac{\beta(1-\alpha)(1-\tau^G)}{(1+\beta)(1-L^{O,R})^{\alpha-1}}\left(\frac{1+(1-L^{O,R})(\theta_h-\theta_l)\left(1-\frac{L^{O,R}}{\overline{L}_B}\right)}{1+(\theta_h-\theta_l)\left(1-\frac{L^{O,R}}{\overline{L}_B}\right)}\right)k_t^\alpha.$$

Note that this expression is a concave policy-function for k_t, as all terms in front of k_t^α are constant. Next, under conditions on the parameter values, we show that aggregate debt d_t grows at the rate of output. We have defined $D_t(L^{O,R})$ as the additional debt issued in period t. Therefore we can write aggregate debt d_t as

$$d_t = \frac{1-\gamma}{\gamma}Y_t + rd_{t-1},$$

where we write $\gamma = (1 - L^{O,R})^{\beta(1-\alpha)}$ for convenience. Hence,

$$d_t = \frac{1-\gamma}{\gamma}\left[Y_t + r_t Y_{t-1} + r_t r_{t-1} Y_{t-2} + ... + \prod_{s=1}^{t} r_s Y_0\right] \quad \text{or}$$
$$d_t = \frac{1-\gamma}{\gamma}\left[Y_t + \alpha\frac{Y_t}{K_t}Y_{t-1} + \alpha^2\frac{Y_t}{K_t}\frac{Y_{t-1}}{K_{t-1}}Y_{t-2} + ... + \alpha^t\prod_{s=1}^{t}\frac{Y_s}{K_s}Y_0\right].$$

From equation (35) we know that K_{t+1}/Y_t is constant for all t. This enables us to write

$$d_t = \frac{1-\gamma}{\gamma}Y_t\left[1 + \frac{\alpha}{\delta} + \frac{\alpha^2}{\delta^2} + ... + \frac{\alpha^t}{\delta^t}\right], \quad \text{with}$$
$$\delta = \frac{\beta(1-\alpha)}{1+\beta}.$$

For t large and if $\alpha < \delta$, i.e. $\alpha < \beta$, we can write the geometric sum as $\frac{1}{1-\frac{\alpha}{\delta}}$, so we obtain

$$\frac{d_t - d_{t-1}}{d_{t-1}} = \frac{Y_t - Y_{t-1}}{Y_{t-1}}.$$

Thus, in the long run, total debt grows with the same value as output and the ratio of public debt to GDP becomes constant.

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