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Abstract

We introduce a democratic procedure with voting-based proposals called "Pendular Voting". It works as follows: An agenda-setter chooses a proposal meant to replace a given status quo. In the first stage, a random sample of the population votes on the proposal. The result is made public, which may reveal information about the distribution of preferences in the electorate. Depending on the outcome, a third option (next to the proposal and the status quo) is added: This option is either closer to or more distant from the status quo than the original proposal. Then, in a second stage the entire electorate expresses pairwise social preferences over the status quo, the initial proposal, and the third option. We investigate the manipulability and exploitation of this voting procedure and its welfare effects. We show that manipulation is limited or absent and that exploitation can be avoided. Regardless of whether the agenda-setter is altruistic or selfish, Pendular Voting leads to welfare gains in expectation.

Keywords: Democracy, Manipulation, Information Sharing, Referendum
JEL Codes: C72, D70, D72

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1 Introduction

The literature has mainly studied voting games either with *heterogeneous preferences* or with *common preferences.* In the former case, each voter prefers a particular alternative, either due to his economic interests or for ideological reasons. In the latter case, all voters have the same interests: They agree to choose the "best" alternative but they receive noisy signals about which alternative is the best one. In this paper, we study situations in which voters have their own preferences but there is aggregate uncertainty about which alternative is the Condorcet winner.

In such a situation, good democratic procedures should accomplish two goals: First, they should facilitate accurate revelation and aggregation of information about the consequences of policy proposals. Second, they should adopt policies that concur with the true underlying preferences of the population. For instance, if a Condorcet winner exists, it should be discovered and then prevail against any other feasible alternative, including the status quo.

Achieving these objectives in a voting procedure is difficult for several reasons: First, there is a large number of policy proposals that can be proposed for a single issue. Second, there is uncertainty about the underlying distribution of preferences in the electorate and thus ambiguity. Third, once a proposal has been adopted, it is often quite difficult to reverse the policy. This is obvious if the policy involves physical or human capital investments such as infrastructure investments in highways, bridges, public buildings or the use of environmental resources. However, the irreversibility extends to many other policies as well. A particularly pertinent example in current events is Brexit: The delays and procedural complications in implementing Brexit have illustrated how difficult it is to join and leave a political union or even an interconnected set of multilateral treaties. Such decisions are reversible only at a high cost. Many other examples can be found for collective decisions that show a high degree of irreversibility.[1]

[1] If waiting can reveal the information, then waiting with the adoption of an irreversible alternative may be desirable (see Gersbach (1993) for the first assessment whether a majority benefits from a "wait and see" choice.)

How should we design good voting systems in such environments? In particular, how can we induce information revelation before proposals are made or how can proposals be adjusted in the light of new information before final irreversible collective decisions are taken?

In this paper, we introduce a new approach called *Pendular Voting* and examine to which extent it can resolve these issues. We aim for a democratic procedure that is robust against manipulation of information revelation by citizens but also against exploitation of information revelation by a selfish agenda-setter. In previous work (Britz and Gersbach (2020)), we have explored how manipulation and exploitation in democratic procedures can be addressed by a combination of tax incentives and transfers, which set the right incentives to reveal and share information accurately and accomplish the implementation of the Condorcet winner. In the present paper, we use a different approach: We make no use of financial incentives to change citizens' preferences. Thus, we avoid political, ethical, or constitutional problems associated with such incentive schemes. Instead, we use only a voting procedure to address the problem of information revelation and aggregation.

The voting procedure works as follows: A selfish agenda-setter chooses a proposal meant to replace a given status quo. In a first stage, only a random sample of the population votes on the proposal. The result of the first stage is made public, and may therefore reveal information about the underlying distribution of preferences in the electorate. Depending on the outcome of the first stage, a third alternative (next to the proposal and the status quo) is added: This alternative is either slightly closer to or slightly more distant from the status quo. Then, a second stage takes place: The entire electorate expresses pairwise preferences over the status quo, the initial proposal, and the newly added third alternative.

We investigate the manipulability of this voting procedure through the random sample of voters and the possibilities to exploit the procedure by an agenda setter. Finally, we study its welfare effects. We show that manipulation is limited or absent and exploitation by a selfish agenda-setter can be avoided. Our main welfare result is: Regardless of whether the agenda-setter is altruistic or selfish, Pendular Voting leads to welfare gains in expecta-

tion. In order to realize these gains, the institutional designer must choose the increments between the initial proposal and the alternatives appropriately.

The present paper aims at making the following contributions to the literature:

First, the concept of Pendular Voting allows us to further explore one of the fundamental questions in democracy research: *How can information revelation be accomplished in democratic procedures?* This question has received extensive attention in the literature, see for instance Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1997). More recent contributions are Bierbrauer and Hellwig (2016) and Britz and Gersbach (2020). When information about an underlying state of nature is dispersed among agents, there are complex incentives which may or may not lead agents to reveal, share, and aggregate their private information through a democratic procedure: In this context, Britz and Gersbach (2020) study a situation where groups of citizens coordinate their messages to strategically misrepresent information prior to a vote. Moreover, if there are turnout costs, an individual agent may free-ride on the information provided by other agents. Feddersen and Pesendorfer (1997) suggest that if some agents have information of worse quality than others, they may strategically decide to abstain and thus keep such information to themselves. Callander (2008) studies an election where citizens reap benefits from voting for the winning candidate. In the present paper, we investigate how voting processes can be organized when there is not only private information about individual preference but also about the distribution of these preferences. Ideally, a multi-stage voting process could work as follows: In an initial stage, information is collected. Then, a proposal is made in light of this information, and finally, a decisive vote takes place.

Second, our analysis of Pendular Voting complements the work of Gersbach (2015) and Gersbach et al. (2021) on so-called *assessment voting.* Their work allows for a random sample group to vote on a given proposal against the status quo earlier than the rest of the population. In a costly voting setting, the outcome of that initial vote influences turnout incentives for those outside the random sample group. However, contrary to our present work on Pendular Voting, assessment voting does not allow for modifications to

the proposal between voting stages. In this paper, we develop Pendular Voting with the aim of increasing the probability that proposals are made that are superior in terms of welfare compared to one or two voting rounds with fixed proposals.

Third, Pendular Voting allows us to suggest potential remedies for some of the problems faced by modern democracies. In the Swiss system of direct democracy, for instance, any citizen can in principle propose a change to the constitution. If the proposal is backed by a certain number of citizens in a signature collection process, then it must be put to a popular vote.[2] This rule has existed for more than a century, but one important recent development has been that the rise of social media and internet campaigns as well as population growth have made it much easier to collect the required number of signatures. At first sight, this seems to be a positive development: It can increase citizen participation. However, there are also significant drawbacks. First, an increasing number of initiatives and thus popular votes may lead to frustration among the citizenry and undermine citizens' willingness to participate in the process. Second, it becomes easier to propose extreme or unrealistic initiatives. Sometimes, such initiatives may be used by a party or interest group to strengthen their visibility and attractiveness for subgroups in society. Third, both of the above effects combined increase the likelihood that small but highly mobilized minorities with extreme views can implement their agenda, while a moderate but aloof majority stands idly by. Pendular Voting might help to address these concerns by introducing the possibility that an extreme proposal may be moderated in the process.

The aftermath of the Brexit referendum is a case in point for how Pendular Voting could improve democratic decision-making: While the referendum allowed for only two options ("Leave the European Union" vs. "Remain in the European Union"), it has since become apparent that there are many more options in reality, such as various forms of a "negotiated" Brexit, a free trade zone, or a "no-deal" Brexit. As of January 2020, it appears clear that Brexit will ultimately occur but negotiations about future relations with the EU are scheduled to take another year. Brexiteers and Remainers cite various opinion polls

[2]For a more detailed discussion of the direct democratic system in Switzerland, see Rühli and Adler (2015).

which deliver contradictory information about the British public's preferences over these options. Applying Pendular Voting could have led to a timely discovery of the underlying preferences, to a more precise and meaningful formulation of the referendum question, and ultimately to greater certainty that the outcome corresponds to the underlying preferences of the population.

Introducing the possibility of adding proposals into the model, however, leads to important information manipulation and exploitation problems, which are not present in Gersbach et al. (2021)'s study of Assessment Voting. In an ideal world, voters would reveal their preferences during the initial stage. The proposer could then adjust his proposal to fit these preferences, and the popular vote would then sanction the outcome. However, first voters may not express their true preferences in the initial stage. For instance, voters may want to feign support for an extreme policy because they hope that this will lead to a continuation of the status quo. This is similar to the problem of holding "open primaries." For instance, in the primaries to a US presidential election, supporters of the Democratic Party may want to vote for a very extreme Republican candidate, hoping that the extreme candidate wins the nomination, but then goes on to lose the election. Second, even if information is truthfully revealed in the first stage, a selfish agenda setter may exploit this information. He may propose an alternative that wins against the status quo and is implemented irreversibly. However, a majority may prefer a large set of alternatives against the proposal that is undertaken. As we will show, with Pendular Voting, we can eliminate or at least restrain these attempts and can ensure that the scheme is welfare-improving.

Our paper is also part of the broadening emerging literature on learning in dynamic collective decisions. Strulovici (2010) examines how long a committee invests in learning until a majority takes a final decision. The duration of learning in committees with heterogeneous members is characterized in Chan et al. (2018).

The paper is organized as follows: We formally introduce the model setup (Section 2) and the Pendular Voting procedure (Section 3). Then, we proceed to a backward induction analysis: That is, we first study equilibrium behavior in the second stage of the voting

procedure, and establish a sincere voting result (Section 4). In Section 5, we briefly discuss the benchmark against which we will measure the performance of the Pendular Voting procedure. In Section 6, we find results on information revelation through the Pendular Voting procedure. Intuitively, we will show that *either* the Pendular Voting procedure cannot be manipulated *or* otherwise, it leads to the same result that would also obtain under the benchmark scenario. This result is the crucial step towards the welfare results that follow: First, in Section 7, we build additional intuition by briefly addressing some special cases of the model and investigating welfare effects of Pendular Voting. Finally, in Section 8, we demonstrate that Pendular Voting leads to welfare gains in expectation, both with an altruistic and with a selfish agenda-setter.

2 Model

A society collectively decides to choose a policy $\theta \in [0, 1]$. It is convenient to define the notation:

$$\begin{aligned} \alpha^{+}(\theta) &= \min\{\theta + \mu_2,\ 1\}, \\ \alpha_{-}(\theta) &= \max\{\theta - \mu_1,\ 0\}. \end{aligned}$$

for some fixed values $\mu_1, \mu_2 > 0$. Later in this paper, we will consider μ_1 and μ_2 as parameters of institutional design. They are taken as given in the Pendular Voting procedure.

The status quo is zero. The society consists of a continuum of citizens with mass one, each of them privately informed about their type, which is some point $z \in Z$, where we normalize the type space so that $Z \in [0, 1]$. It is convenient to refer to a citizen of type z as *citizen z*.

Citizen z's utility from policy θ is given by a twice continuously differentiable utility function $u(z, \theta)$ which is symmetric around a single peak. That is, we assume $u(z, \theta) = 0$ for $z = \theta$ as well as $\partial u(z, \theta)/\partial\theta > 0$ for $\theta < z$ and $\partial u(z, \theta)/\partial\theta < 0$ for $\theta > z$. Finally, $\mid \theta' - z \mid = \mid \theta'' - z \mid$ implies $u(z, \theta') = u(z, \theta'')$.

For our results, it is only important that each citizen's preferences are single-peaked and

symmetric in the sense that equal deviations from the peak on either side lead to the same utility loss.

There is uncertainty at both the individual and aggregate levels, which we model in the same way as in Britz and Gersbach (2020): That is, we assume that there are finitely many states of nature. We denote the state space by $N = \{1, \ldots, n\}$, and use k to index the elements of N. There is a family of probability distributions on Z associated with the states of nature. We use f_k and F_k to denote the probability density function and cumulative distribution function, respectively, of the probability distribution associated to state k. In each state of nature, citizens' types are independent draws from the relevant probability distribution. Regardless of their type, citizens share a common prior belief $p = (p_1, \ldots, p_n)$ about the state of nature, where we assume that $p_k > 0$ for every $k \in N$. Due to Bayesian updating, citizen z has a posterior belief that assigns to state k the probability

$$\gamma_k(z) = \frac{f_k(z)p_k}{\sum_{j=1}^{n} f_j(z)p_j} > 0.$$

Assumption 1

1. *For any $z \in int(Z)$, we have $F_1(z) > \ldots > F_n(z)$.*
2. *For every $k \in N$ and every $z \in Z$, it holds that $\gamma_k(z) > 0$ and Bayesian updating is monotone.*

Monotone Bayesian updating means that for any $z_1, z_2 \in Z$ with $z_1 < z_2$, the posterior probability distribution $\{\gamma_k(z_2)\}_{k=1}^{n}$ stochastically dominates $\{\gamma_k(z_1)\}_{k=1}^{n}$.

In this paper, we are interested in collective decision making procedures that have "democratic" characteristics: In particular, we allow citizens to send only binary messages, and we require that an alternative needs majority approval in order to replace the status quo.

3 The Pendular Voting Procedure

In this section, we give the formal description of the *Pendular Voting procedure.*

An agenda–setter makes a proposal $\bar{\theta} \in [0, 1]$. We will allow for two different cases: The agenda–setter may be a benevolent social planner who seeks to implement the Condorcet winner.[3] Alternatively, he may also have his own interests, and be of a particular type denoted by θ_{AS}. The voting procedure consists of two stages.

1. Once the agenda–setter has made a proposal, say $\bar{\theta}$, a random sample of size λ of the population is drawn. Each member of the sample group may vote in favor of $\bar{\theta}$, or in favor of the status quo. The share of sample group members who vote in favor of $\bar{\theta}$ is denoted by δ. We define

$$\beta(\bar{\theta}, \delta) = \begin{cases} \alpha^{+}(\bar{\theta}) & \text{if } \delta \geq 1/2, \\ \alpha_{-}(\bar{\theta}) & \text{if } \delta < 1/2. \end{cases}$$

   Here, $\beta(\bar{\theta}, \delta)$ is an additional proposal determined by votes in the first round.

2. In the second stage, the entire population votes. Each voter is asked to submit his pairwise preferences over the three alternatives $\{0, \bar{\theta}, \beta(\bar{\theta}, \delta)\}$. The outcome is then determined as follows: If a majority pairwise prefers 0 to both $\bar{\theta}$ and $\beta(\bar{\theta}, \delta)$, then the outcome is the status quo 0. If a majority pairwise prefers $\bar{\theta}$ to 0, but 0 to $\beta(\bar{\theta}, \delta)$, then the outcome is $\bar{\theta}$. If a majority pairwise prefers $\beta(\bar{\theta}, \delta)$ to 0, but 0 to $\bar{\theta}$, then the outcome is $\beta(\bar{\theta}, \delta)$. If a majority pairwise prefers both $\bar{\theta}$ and $\beta(\bar{\theta}, \delta)$ to 0, then the outcome is either $\bar{\theta}$ or $\beta(\bar{\theta}, \delta)$, whichever is pairwise preferred by a majority to the other.

We assume that any ties are broken at random with equal probabilities.

We assume that citizens with the same preference ranking over the alternatives $0, \bar{\theta}$, and $\beta(\bar{\theta}, \delta)$ coordinate their votes.

The crucial feature of the Pendular Voting procedure is that the second, decisive voting round is always a three-way ballot including the initial proposal and the status quo. The third alternative on the ballot is determined by the result of the first voting round, that is,

[3] As we are looking for ways to improve democratic procedures, we do not consider utilitarian welfare but rather set as the goal to implement the Condorcet winner. That also determines the definition of benevolence of the agenda-setter. See also Britz and Gersbach (2020).

random sample group members essentially choose which additional alternative to include on the final ballot.

Throughout, we assume that citizens with the same preference ranking coordinate their votes. This is a conservative assumption: If there was no or less coordination, the scope for manipulation would be reduced, and the benefit from the Pendular Voting procedure enhanced.

4 Sincere Voting by the Population

In this section, we establish the following claim. At the second stage of the Pendular Voting procedure, all citizens find it optimal to vote sincerely, that is, in accordance with their true preferences.

For any choices of $\bar{\theta} \in \Theta = [0, 1]$ and $\delta \in [0, 1]$, we can always restate the problem of choosing from $\{0, \bar{\theta}, \beta(\bar{\theta}, \delta)\}$ as the problem of choosing from three alternatives x_0, x_1, and x_2 such that $0 = x_0 \leq x_1 \leq x_2$. Citizens cast three pairwise votes.

$$x_0 \leftrightarrow x_1,$$

$$x_0 \leftrightarrow x_2,$$

$$x_1 \leftrightarrow x_2.$$

The rules as described in the previous section can be restated as follows.

If any alternative wins two of the three pairwise votes, then it becomes the outcome of the voting procedure. If each of the three votes is won by a different alternative, then the alternative that has defeated x_0 becomes the outcome of the voting procedure.

This statement makes it clear that the Pendular Voting procedure has a certain bias against the status quo. If each of the three pairwise votes is won by a different alternative, the status quo cannot become the outcome. However, it is still true that a majority is required to place the status quo: This is an important requirement of a democratic decision making procedure. Moreover, we can justify this bias by interpreting the pairwise vote between the

two alternatives x_1 and x_2 as a "conditional" vote, similar in nature to a run-off election. It only becomes relevant when the status quo has been defeated by both alternatives.

In principle, there are six ways to rank–order the alternatives $\{x_0, x_1, x_2\}$ by some preference order $\succsim$.

$$
\begin{aligned}
x_2 &\succsim x_1 \succsim x_0, \\
x_1 &\succsim x_2 \succsim x_0, \\
x_1 &\succsim x_0 \succsim x_2, \\
x_0 &\succsim x_1 \succsim x_2, \\
x_0 &\succsim x_2 \succsim x_1, \\
x_2 &\succsim x_0 \succsim x_1.
\end{aligned}
$$

Note that the last two preference orders above are inconsistent with our assumption on single–peaked preferences. Hence, we can restrict attention to the following four preference orders:

$$
\begin{aligned}
x_2 &\succsim x_1 \succsim x_0, \\
x_1 &\succsim x_2 \succsim x_0, \\
x_1 &\succsim x_0 \succsim x_2, \\
x_0 &\succsim x_1 \succsim x_2.
\end{aligned}
$$

We assume that all citizens which share one of these four preference orders can coordinate their votes. Thus, we have to check for profitable deviations by each of the four groups, assuming that the remaining three groups vote sincerely.

As a first step, we show that citizens express their preference sincerely over the alternatives that differ from the status quo.

Proposition 1 *Citizens vote sincerely between $x_1 \leftrightarrow x_2$.*

Proof. Case 1. Suppose that x_0 wins against both x_1 and x_2. Then, the outcome of the voting procedure is x_0, regardless of the vote between $x_1 \leftrightarrow x_2$.

Case 2. Suppose that x_1 wins against x_0, but x_0 wins against x_2. Then, the outcome of the voting procedure is x_1, regardless of the vote between $x_1 \leftrightarrow x_2$.

Case 3. Suppose that x_2 wins against x_0, but x_0 wins against x_1. Then, the outcome of the voting procedure is x_2, regardless of the vote between $x_1 \leftrightarrow x_2$.

Case 4. Suppose that x_0 wins against neither x_1 nor x_2. Then, the outcome of the vote between $x_1 \leftrightarrow x_2$ selects the outcome of the whole voting procedure. In that case, it is optimal to vote sincerely.

□

Two special cases deserve attention. If the proposal is exactly equal to the status quo or to one (which is the upper bound of the policy space), then only two of the three alternatives are distinct. In that case, sincere voting obtains simply because the choice of an alternative is binary and final. Thus, there is no loss in focusing on the case where x_0, x_1, and x_2 are all distinct.

Now we are ready to show that citizens vote sincerely.

Proposition 2 *In the second stage of the Pendular Voting procedure, there is an equilibrium in which all citizens vote sincerely.*

Proof.

1. Consider the group with preference order $x_2 \succsim x_1 \succsim x_0$. If all citizens vote sincerely, x_2 is the outcome of the voting procedure if and only if a majority of the population belongs to the group at hand. If the group with preference order $x_2 \succsim x_1 \succsim x_0$ is indeed a majority, they get their most preferred alternative by sincere voting. Hence, a deviation from sincere voting can only be profitable in states of nature where the group at hand is a minority. In that case, the alternative x_2 is never implemented because a majority prefers x_1 to x_2– this follows from the premise that the three other groups vote sincerely. Now we see that the group with preference order $x_2 \succsim x_1 \succsim x_0$ can only benefit from a deviation if (i) sincere voting would lead to the outcome x_0,

and (ii) some strategic voting by the group at hand leads to the outcome x_1 instead. It follows from point (i) that, under sincere voting, x_0 wins the pairwise votes against both x_1 and x_2. But only the group with preference ranking $x_0 \succsim x_1 \succsim x_2$ sincerely prefers x_0 over x_1. It follows that a majority of the population belongs to the group with preference ranking $x_0 \succsim x_1 \succsim x_2$. Due to the premise that this group votes sincerely, the outcome of the voting procedure is x_0, regardless of any votes by the group with preference order $x_2 \succsim x_1 \succsim x_0$. Hence, point (ii) is certainly violated. We conclude that the group with preference order $x_2 \succsim x_1 \succsim x_0$ cannot have a profitable deviation from sincere voting.

2. Consider the group with preference order $x_1 \succsim x_2 \succsim x_0$. Suppose first that a majority of citizens belong to the group with preference ranking $x_2 \succsim x_1 \succsim x_0$. Since that group votes sincerely, the outcome is going to be x_2, no matter what the group with preference order $x_1 \succsim x_2 \succsim x_0$ does. The group under consideration can only profit from a deviation if a majority of the population prefers x_1 over x_2. Hence, the vote between $x_1 \leftrightarrow x_2$ is always won by x_1 and so the outcome of the vote $x_0 \leftrightarrow x_2$ is irrelevant for the outcome of the voting procedure. Indeed, x_1 is the outcome of the voting procedure if and only if it wins in the vote between $x_0 \leftrightarrow x_1$. If any strategic voting is beneficial for the group under consideration, then it must be because the vote $x_0 \leftrightarrow x_1$ is won by x_1 under strategic voting, but would be won by x_0 under sincere voting. But with sincere voting, x_1 wins against x_0. We have now shown that the group with preference order $x_1 \succsim x_2 \succsim x_0$ cannot gain by deviating from sincere voting.

3. The argument above also applies to the group with preference order $x_1 \succsim x_0 \succsim x_2$.

4. Finally, consider the group with preference order $x_0 \succsim x_1 \succsim x_2$. In a state of nature where the majority prefers x_2 to the other two alternatives, the votes of the group at hand are inconsequential. In states of nature where the group at hand is in a majority, they get their most preferred outcome from voting sincerely. Suppose that the group at hand is not in a majority, and a majority is also not in favor of x_2.

Then, the outcome under sincere voting is x_1. The only way a deviation could benefit the group at hand is if they could change the outcome from x_1 to x_0. But this is impossible, because all other citizens vote sincerely for x_1 in the vote $x_1 \leftrightarrow x_0$.

□

This result is reminiscent of the well-known *Median Voter Theorem.* It is, however, not readily implied by the Median Voter Theorem since the agenda-setter is unaware of the location of the median voter. This is a consequence of the individual and aggregate uncertainties inherent in our model.

5 Benchmark: Proposal against Status Quo

The key implication of the above result is the following. Suppose that the alternative preferred by the median voter over all alternatives qualifies for the second stage of the Pendular Voting procedure. Then, it is certain that this alternative will also be the outcome of the entire procedure. Hence, the question is how we can ensure that the alternatives present in the second round are as close as possible to preferred choice of the median voter?

The purpose of the Pendular Voting procedure is to perform better in this respect than a benchmark scenario in which the agenda-setter uses only his prior belief to choose a proposal that is then voted upon.

We now briefly consider that benchmark scenario.

Proposition 3 *In any state k with median voter $\widehat{z}_k$, if the agenda-setter chooses a proposal $\bar{\theta} < 2\widehat{z}_k$, then the proposal is accepted by a majority. Otherwise, the status quo almost always prevails.*

The reasoning behind this statement is as follows: Under single-peaked preferences, there is for each citizen $z \leq 1/2$ some proposal $\bar{\theta} > 0$ such that citizen z is indifferent between $\bar{\theta}$ and the status quo. Citizen z strictly prefers any $\bar{\theta}' \in (0, \bar{\theta})$ to either zero or $\bar{\theta}$, and he strictly prefers $\bar{\theta}$ to any $\bar{\theta}' > \bar{\theta}$. Due to our assumptions on the utility function, this critical

proposal $\bar{\theta}$ equals $2z$ for citizen z. Indeed $-(z-\bar{\theta})^2$ returns the same utility for $\bar{\theta}=0$ as for $\theta = 2z$. Finally, citizen $z > 1/2$ prefers any choice in the interval $(0,1]$ to the status quo.

6 Information Revelation through Pendular Voting

6.1 A General Result on Manipulability and Welfare

We take a proposal $\bar{\theta}$ as given, and verify under what conditions the Pendular Voting procedure reliably implements the choice from $\{0, \alpha_-(\bar{\theta}), \bar{\theta}, \alpha^+(\bar{\theta})\}$ which is closest to the median voter. This means that we assess social welfare based on the median voter's preference. This makes sense in such a type of model when discussing democratic procedures: Indeed, a democratic procedure should satisfy a requirement such as "stability to majority voting," which is similar to assessing social welfare based on the median voter's preferences.

At the first stage of the voting procedure, we take as given the proposal θ and define the following groups:

- $Z_3(\theta)$ are citizens with preference order $\alpha^+(\theta) \succsim \theta \succsim \alpha_-(\theta) \succsim 0$.
- $Z_2(\theta)$ are citizens who prefer $\theta \succsim \alpha_-(\theta) \succsim 0$ but also $\theta \succsim \alpha^+(\theta)$.
- $Z_1(\theta)$ are citizens who prefer $\alpha_-(\theta) \succsim \theta \succsim \alpha^+(\theta)$, but also $\alpha_-(\theta) \succsim 0$.
- $Z_0(\theta)$ are citizens with preference order $0 \succsim \alpha_-(\theta) \succsim \theta \succsim \alpha^+(\theta)$.

In what follows, we omit the argument θ. Note that the four groups are exhaustive, and that almost all citizens belong to exactly one group for a given θ.

We consider the following equilibrium candidate: At the first stage of the voting procedure, $Z_0 \cup Z_1$ vote No and $Z_2 \cup Z_3$ vote Yes. This equilibrium candidate is such that citizens above a threshold type vote Yes and the others vote No.

We are interested in the issue of *manipulation* in the sense that one of the four groups as defined above has an incentive to deviate from the aforementioned equilibrium candidate. Indeed, we are going to claim that Z_1, Z_2, and Z_3 have no incentive to make such a

deviation, and we examine the conditions under which Z_0 may have an incentive to deviate.

We note that our result does not rely on any assumption about the beliefs of the group members about the underlying state of nature.

We do assume that voting behavior within each of the four groups is coordinated. Note that this is a conservative assumption that biases our results in favor of manipulation, and therefore against the benefits of Pendular Voting. To see this, observe that obtaining equilibria without manipulation would be trivial if they would have to be robust only against individual citizens' deviations, and it would at least be easier if we allowed only smaller groups of individuals to coordinate their votes. Allowing groups of citizens to coordinate their deviation is an approach which concurs with recent developments in the literature. The papers by Bierbrauer and Hellwig (2016) as well as Britz and Gersbach (2020) bring together mechanism design and voting games to study public good provision problems. Bierbrauer and Hellwig show that, under certain robustness conditions, mechanisms which solve such problems must belong to a class they call "voting mechanisms." Their requirement of robust coalition proofness is predicated on the idea that citizens with concurring preferences may coordinate their strategies. Britz and Gersbach (2020) allow citizens with the same preference ranking to coordinate their votes in a democratic mechanism. In models with a continuum society, this circumvents the problem that "unilateral" deviations by a single voter are always inconsequential, and therefore a plethora of trivial equilibria arise.

The argument runs as follows: We consider the four groups defined above in turn, in descending order. We need to check for various triples of a state of nature, a proposal, and a group whether manipulations are possible. In order to appreciate the gist of the argument, note that for many such triples, the absence of manipulation is trivial: For instance, a group can never gain from manipulation if it constitutes a majority by itself. In that case, it can simply be sincere and enforce its preferred alternative by virtue of its majority. Moreover, a group can also not manipulate the process if some other group constitutes a majority – they simply do not have the power to do so. Repeating these considerations will allow us to show that the scope for any manipulation is quite restricted. Only members of Z_0 can

manipulate, and they can only do so in states in which several conditions on the relative size of the various groups are simultaneously satisfied. This will give us a set of necessary conditions for manipulation. Conversely, we will contain a set of conditions each of which is sufficient for non-manipulation.

Theorem 1 *If the Pendular Voting procedure is manipulated, then its outcome is either the status quo or the proposal θ, whichever is preferred by a majority.*

Proof. Consider the strategy profile where groups Z_0 and Z_1 vote No and groups Z_2 and Z_3 vote Yes. We check whether any one group has a profitable deviation from that strategy profile. We show first that groups Z_3, Z_2, and Z_1 have no such deviation. Then, we show that group Z_0 may have an incentive to deviate, but then the outcome is either the status quo or the proposal, whichever a majority prefers.

1. Consider Z_3's voting decision in the first round. In states where Z_3 itself is a majority, Z_3 can have their most preferred option $\alpha^+(\theta)$ if and only if it qualifies for the second round. So voting Yes at the first stage is optimal for Z_3. We will show that Z_3's choice is inconsequential in any state of nature where Z_3 is not a majority. It is straightforward that Z_3's choice is inconsequential in those states where $Z_0 \cup Z_1$ have a majority. Now consider states where neither Z_3 nor $Z_0 \cup Z_1$ are a majority. Then, $\alpha^+(\theta)$ could never win against θ in the second round. But since $Z_2 \cup Z_3$ is a majority, neither zero nor $\alpha_-(\theta)$ could win against θ in the second round either. So θ is the outcome, regardless of what happens in the first voting round, and so group Z_3 has no incentive to deviate.

2. Consider Z_2's voting decision in the first round. In states where Z_2 has a majority, they can obtain their most preferred outcome θ in the second round, regardless of the outcome of the first round. Hence, their decision at the first stage is inconsequential. In states where $Z_0 \cup Z_1$ are a majority, Z_2's choice in the first round is again inconsequential. The same is true in states where Z_3 alone has a majority. Now it remains to consider those states where neither $Z_0 \cup Z_1$, nor Z_2, nor Z_3 are a majority. In such a state, the second stage of voting cannot be won by $\alpha^+(\theta)$ (because Z_3 is a

minority), cannot be won by $\alpha_-(\theta)$ (because $Z_0 \cup Z_1$ is a minority), and cannot be won by zero because a majority $Z_2 \cup Z_3$ which prefers θ to α_- also prefers θ to zero. Hence, the outcome is θ regardless of the votes cast in the first stage. Whatever the state, Z_2 is indifferent between voting Yes or No in the first stage.

3. Consider Z_1's voting decision in the first round. It is inconsequential in all states in which either $Z_2 \cup Z_3$ or Z_0 is a majority. Indeed, consider the remaining states. Suppose that Z_1 votes No. Then $\alpha_-(\theta)$ qualifies for the second round. Since $Z_2 \cup Z_3$ is a minority, θ cannot win the second round. Since Z_0 is also a minority, zero cannot win either. So the outcome is $\alpha_-(\theta)$. Now suppose Z_1 switches from No to Yes. Then $\alpha^+(\theta)$ qualifies for the second round. Again because $Z_2 \cup Z_3$ is a minority, $\alpha^+(\theta)$ cannot win in the second round, thus the outcome is either zero or θ. But Z_1 likes $\alpha_-(\theta)$ better than zero and better than θ. Hence, whatever the state, Z_1 cannot gain from the deviation.

4. Consider Z_0's voting decision at the first stage. It is inconsequential in all states in which $Z_2 \cup Z_3$ is a majority, or in which Z_1 is a majority. In states where Z_0 by itself is a majority, the outcome of the second round is always zero, no matter what happens in the first round – again, Z_0's decision in the first round is inconsequential. Now consider the remaining states, in which neither $Z_2 \cup Z_3$, nor Z_0, nor Z_1 are a majority. Suppose first that Z_0 votes No. Then, because $Z_2 \cup Z_3$ is a minority, $\alpha_-(\theta)$ qualifies for the second round. Again because $Z_2 \cup Z_3$ is a minority, θ cannot win the second round. Since Z_0 is also a minority, zero cannot win either. So the outcome is $\alpha_-(\theta)$. Now suppose Z_0 switches from No to Yes. Then $\alpha^+(\theta)$ qualifies for the second round. Again because $Z_2 \cup Z_3$ is a minority, $\alpha^+(\theta)$ cannot win in the second round, so the outcome must be either zero or θ, whichever is preferred by a majority.

□

Theorem 1 differs from existing results in the literature on strategic voting or truthful mechanisms, in a fundamental sense: The literature typically aims at establishing results on "strategy-proofness." by demonstrating the existence of an equilibrium in which all

agents reveal their private information truthfully. Theorem 1, however, does not claim that the Pendular Voting procedure is strategy-proof. Rather, it asserts that *if* manipulation does occur, then the outcome of the Pendular Voting procedure coincides with the outcome of the benchmark procedure. Using Pendular Voting has welfare implications only if no manipulation occurs. Theorem 1 also says that only members of one particular group, called Z_0, may have incentives to vote strategically. Whether or not the members of Z_0 want to manipulate in this way depends on their probabilistic belief about the underlying state of nature. Theorem 1 itself, however, holds true regardless of these beliefs.

Recall from the previous section that we are comparing the Pendular Voting procedure to a benchmark procedure in which citizens simply choose between the proposal and the status quo without any preliminary proposal assessment. The key implication of Theorem 1 is that, whenever the Pendular Voting procedure is manipulated, its outcome coincides with the outcome of the benchmark procedure, so that welfare does not change. If the Pendular Voting procedure is not manipulated, it may still be the case that the outcome of Pendular Voting coincides with that of the benchmark procedure. Again, welfare does not change. Finally, there is a case where the Pendular Voting procedure is not manipulated, and yet leads to a different outcome than the benchmark procedure. In that case, the outcome under Pendular Voting is a welfare improvement in the sense that this outcome is preferred by a majority to that of the benchmark procedure.

Corollary 1 *Moving from the benchmark procedure to the Pendular Voting procedure, while holding the proposal constant, never leads to a welfare loss, regardless of the state.*

It is important to note that Theorem 1 and its corollary hold regardless of the prior or posterior beliefs held by any of the citizens.

6.2 A Look at Beliefs

The results in the previous subsection apply regardless of the values of the increments μ_1 and μ_2. The main message in the previous subsection is that while manipulation may be possible, we can still know the outcome of the procedure under manipulation: It is

either the status quo or the initial proposal, and the choice between these two options is in line with majority preference. Note that the entire discussion so far is independent of the beliefs held by individual groups in the population. In this subsection, we derive additional results by considering the beliefs of group Z_0. For simplicity, we assume in this subsection that $\mu_1 = \mu_2 = \mu$. The proof of Theorem 1 implies that a deviation by group Z_0 can only be profitable if the state of nature is such that all of the following conditions are simultaneously satisfied:

1. $Z_0 \cup Z_1$ constitutes a majority
2. Z_0 alone does not constitute a majority.
3. Z_1 alone does not constitute a majority.
4. A majority prefers the status quo to θ.

We note that the first condition above is implied by the fourth condition, hence we are left with three necessary conditions for manipulation.

The three necessary conditions are expressed by the three inequalities below.

$$F_k\left(\theta - \frac{1}{2}\mu\right) - F_k\left(\frac{1}{2}\theta - \frac{1}{2}\mu\right) \geq \frac{1}{2} \tag{1}$$

$$\theta \leq 2\widehat{z}_k, \tag{2}$$

$$\theta \geq 2\widehat{z}_k + \mu. \tag{3}$$

The first inequality describes the case where Z_1 has a majority, so that no manipulation is possible. If the second inequality is satisfied, then a majority prefers θ to the status quo – again, no manipulation is possible. If the third inequality is satisfied, then a majority belongs to Z_0, in which case manipulation is impossible.

Corollary 2 *Manipulation of the Pendular Voting procedure does not occur if Ineq.(1) holds. Moreover, manipulation does not occur if group Z_0 assigns sufficiently low probability to states $k \in N$ where $2\widehat{z}_k < \theta < 2\widehat{z}_k + \mu$.*

One clarifying remark is in order: While we do assume that groups can coordinate their votes, they are not able to determine their own size. Otherwise, they would know the state of nature. We stress that the assumption that groups can coordinate their votes is "conservative" in nature: If we obtain welfare results that are robust to manipulations by large groups, then they would certainly be robust to manipulations by smaller subsets of these groups.

Due to our assumptions on the probability distribution functions, every state is believed to occur with strictly positive probability.

Consider the case where $\theta+\mu \leq 2\widehat{z}_n$. This means that each of the three proposals $\alpha_-(\theta)$, θ, and $\alpha^+(\theta)$ are preferred by a majority to the status quo, and this is true in *each* state. Therefore, group Z_0 never finds it optimal to vote strategically, and hence, no manipulation is possible.

Corollary 3 *Suppose that $\theta + \mu \leq 2\widehat{z}_n$. Furthermore, suppose that there is some state $k \in N$ so that $\widehat{z}_k > \theta + \mu/2$. Then, moving from the benchmark procedure to the Pendular Voting procedure, while holding the proposal θ constant, leads to a welfare gain with strictly positive probability.*

The two necessary conditions in the corollary above boil down to a requirement that the proposal θ, which we are holding fixed here, should not be "too high." Later in the paper, we will argue that an agenda-setter, regardless of his motivation, would never have an incentive to make an excessively high proposal in the first place. Hence, the interpretation of the corollary is that switching from the benchmark procedure to the Pendular Voting procedure can indeed be expected to lead to a welfare gain.

One may wonder if the choice between the status quo, the initial proposal, and the upward and downward corrections could not more efficiently be done on a single ballot paper. There are several problems with this approach, however: First and foremost, our voting rule for the final voting round is designed for the case of a three-way ballot and does not easily generalize to a four-way ballot. Second, a four-way ballot would require the elicitation of

six, instead of three, pairwise preferences, which can be interpreted as an efficiency loss.

We stress that the arguments behind the main results in this paper do not rely on any notion of probabilistic beliefs by the citizens, except that all states occur with strictly positive probability.

7 Limit Results on Welfare Gains

The previous results reveal that no manipulation can occur if any one of the four groups alone has a majority. It is important to note that the model introduced here generalizes some of the frameworks that exist in the literature as limit cases. We discuss this in more detail in the next two subsections.

7.1 Homogeneous preferences

Suppose we modify citizens' types as follows: In state k, each citizen's type is determined by drawing some $z \in [0, 1]$ from the distribution f_k, and the type is then equal to

$$z' = qz + (1 - q)\widehat{z}_k,$$

where $\widehat{z}_k$ is defined as the median voter in state k, that is $F_k(\widehat{z}_k) = 1/2$. The choice of q does not change the location of the median voter. However, the degree to which citizens' preferences differ from each other is scaled by q. If the parameter q is close to zero, citizens have nearly homogeneous preferences. In that case, for a generic choice of the probability distributions and the concomitant median voter types, a majority of citizens will have the same ranking over the proposals and the status quo at the second stage of the Pendular Voting procedure. For small values of p, there is little conflict of interest among citizens. Hence, the problem reduces to an election in which all citizens agree to implement the best alternative but have noisy private information about which alternative is indeed "best." Voting with common preferences has been studied, among others, by Callander (2008).

One attractive feature of the Pendular Voting model is that the case of homogeneous preferences can be obtained as a special case. It helps embed the idea of Pendular Voting

in the voting literature.

7.2 Small Steps Between Proposals

Another special case that helps the intuitive understanding of the model is to consider a sufficiently small value of the parameter μ. Verbally, this means that the three alternatives that may go against the status quo are close to each other. When this is the case, then in each state, either the group Z_0 or the group Z_3 constitutes a majority. Therefore, no manipulation can occur. At the same time, in the limit as $\mu \to 0$, the outcome of the Pendular Voting procedure must lie in an ever smaller neighborhood around the outcome of the benchmark procedure. The Pendular Voting procedure always allows for some gains in social welfare compared to the benchmark scenario. How large these gains are depends on the choice of the model parameters, and, in particular, the probability distribution functions. From a practical point of view, this is a question of institutional design: Depending on the distribution of the underlying preferences, one can choose the parameter μ in such a way that welfare gains are realized in expectation. One alternative that could also be explored is that the agenda-setter himself chooses μ. Instead of assuming a single μ, the insight in this subsection easily extends to the case where $\mu_1 \neq \mu_2$, but we consider the limit as both μ_1 and μ_2 go to zero.

8 Agenda-setter's Choices

8.1 Welfare gain with same proposal

The analysis so far has led to some insights about the benefits of the Pendular Voting procedure relative to the benchmark procedure, under the premise that the proposal made under both procedures is the same. In a nutshell, in expectation, social welfare increases with the Pendular Voting procedure compared to the benchmark. The size of the welfare gain depends on μ and goes to zero as μ goes to zero.

After the analysis so far, a natural follow-up question is this: Suppose that the Pendular Voting procedure is used, and the agenda-setter anticipates the conditions for its manipula-

bility. Would the agenda-setter then want to make the same proposal as in the benchmark procedure? If the agenda-setter does so, we have shown that a welfare gain can be realized. But what happens if the agenda-setter re-optimizes even his original proposal in anticipation of proposal assessment?

8.2 Welfare change with a neighboring proposal

Suppose that an agenda-setter, whatever his preferences, would make the proposal θ_0 under the benchmark procedure. If the proposal he would make under the Pendular Voting procedure belongs to the set $\{\theta_0 - \mu_1,\ \theta_0,\ \theta_0 + \mu_2\}$, then social welfare under the Pendular Voting procedure is not less than under the benchmark procedure.

8.3 Benevolent Agenda-Setter

Proposition 4 *With a benevolent agenda–setter, social welfare under Pendular Voting is at least as high as under the benchmark.*

The agenda-setter has the option of sticking to the same proposal as under the benchmark procedure, which implies the statement. Since our assumptions do not exclude the case with just a single state of nature, it is not generally true that Pendular Voting strictly improves social welfare when the agenda-setter is benevolent. Such strict improvements do hold, however, whenever there are "enough states."

In addition, an agenda-setter could use re-optimization of the proposal in order to improve expected welfare gains even further. Which proposal is optimal, however, always depends on the underlying model parameters, and in particular on the probability distributions from which the types are drawn.

8.4 Selfish Agenda-Setter

Another important case is where the agenda-setter is selfish. One natural case to think about is the one where the agenda-setter wants to move as far as possible away from the status quo. This case can be interpreted as follows.

Consider the example in the introduction where a small but well-organized minority proposes a referendum on an extreme policy plan, which it may want to moderate in order to gain popular support. In the policy space $\theta \in [0, 1]$, we can think of 0 as "no policy change" and think of 1 as the extreme policy most preferred by the proposers. Then, the interval $(0, 1)$ is the space of all the possible compromises or moderate versions of that extreme policy. In view of this interpretation, it is natural to think of a selfish agenda-setter as having a preference for the extreme point of the policy space.

More formally, in our analysis of the *selfish agenda-setter,* we assume that this agenda-setter's utility from any policy is linearly increasing in its distance from the status quo. The question is whether moving from the benchmark procedure to the Pendular Voting procedure also leads to social welfare gains even if the agenda-setter is selfish.

Recall that θ_0 denoted the agenda-setter's proposal under the benchmark procedure. For simplicity, we consider a selfish agenda-setter who wishes to maximize the distance from the status quo, and whose utility is linear. That is, if a proposal $\theta \in [0, 1]$ is implemented, this results in a utility of θ for the agenda-setter. We use $\rho(\theta)$ to denote the probability that θ is preferred to the status quo by a majority of citizens.

Under the benchmark procedure, the agenda-setter seeks to maximize the product $\rho(\theta)\theta$, hence we know that θ_0 solves the fixed point equation given by

$$\theta_0 = -\frac{\rho(\theta_0)}{\rho'(\theta_0)}.$$

For the purpose of this analysis, we consider a family of Pendular Voting procedures parameterized by the increments μ_1 and μ_2. After the vote in the first round on a proposal θ, either the alternative $\theta - \mu_1$ or the alternative $\theta + \mu_2$ is added. This leads to the following objective function for the selfish agenda-setter:

$$\rho(\theta - \mu_1)(\theta - \mu_1) + \rho(\theta)\mu_1 + \rho(\theta + \mu_2)\mu_2,$$

This objective function captures the idea that a selfish agenda-setter wishes to maximize

θ, and we assume for simplicity that his preference over the choice of θ is linear.

The first-order condition for this optimization problem is:

$$\rho'(\theta - \mu_1)[\theta - \mu_1] + \rho(\theta - \mu_1) + \rho'(\theta)\mu_1 + \rho'(\theta + \mu_2)\mu_2 = 0.$$

Now consider the following additional assumptions on ρ:

1. In the limit as $\theta \to 0$, we have $\rho'(\theta)\theta = 0$.
2. In the limit as $\theta \to 1$, we have $\rho'(\theta) = 0$.
3. $\rho''(\theta) \geq 0$ for all θ.

For each of these assumptions, the associated claim below holds:

1. Take any $\theta \in [0, \theta_0]$ and set $\mu_1 = 0$. We can choose an appropriate μ_2 so that the selfish agenda-setter proposes θ under the Pendular Voting procedure. In order to see this, plug $\mu_1 = 0$ into the first-order condition, this gives $\rho'(\theta)\theta + \rho(\theta) + \rho'(\theta + \mu_2)\mu_2 = 0$. We establish first that it is possible to obtain $\theta = 0$ by an appropriate choice of μ_2. To this end, plug in $\theta = 0$, which yields $-\rho'(\mu_2)/\mu_2 = 1$, and thus identifies the appropriate value of μ_2, let's call it μ_2^+. We already know that setting $\mu_2 = 0$ would result in the choice of θ_0. Due to the continuity of the first-order condition and the function ρ, we can apply the intermediate value theorem to conclude that choosing $\mu_2 \in [0, \mu_2^+]$, we can obtain any $\theta \in [0, \theta_0]$.
2. Take any $\theta \in [\theta_0, 1]$ and set $\mu_2 = 0$. We can choose an appropriate μ_1 so that the selfish agenda-setter proposes θ under the Pendular Voting procedure. In order to see this, plug $\mu_2 = 0$ into the first-order condition, this gives $\rho'(\theta - \mu_1)[\theta - \mu_1] + \rho(\theta - \mu_1) + \rho'(\theta)\mu_1 = 0$. Now let $\mu_1^+ = 1 - \theta_0$. Then, the first-order condition is satisfied for $\theta = 1$. We already know that setting $\mu_1 = 0$ would result in the choice of θ_0. Invoking the continuity of the first-order condition as well as of ρ, and using the intermediate value theorem, we see that choosing $\mu_1 \in [0, \mu_1^+]$, we can obtain any $\theta \in [\theta_0, 1]$, as desired.

3. The derivatives of the first-order condition with respect to μ_1 and μ_2 have opposite signs when evaluated in the proximity of $\mu_1 = 0$ and $\mu_2 = 0$. In order to verify this claim, first consider the derivatives: $\rho''(\theta - \mu_1)[\theta - \mu_1] - 2\rho'(\theta - \mu_1) + \rho'(\theta)$ and $\rho''(\theta + \mu_2)\mu_2 + \rho'(\theta + \mu_2)$ Evaluating them at the benchmark proposal, we find $\rho''(\theta_0)\theta_0 - \rho'(\theta_0) = \rho'(\theta_0)$. Since $\rho''(\theta_0) \geq 0$ and $\rho'(\theta_0) < 0$, it follows that both partial derivatives have opposite signs.

Proposition 5 *There exists a pair $\mu = (\mu_1, \mu_2) \in \mathbb{R}^2_{++}$ such that under the associated Pendular Voting procedure, a selfish agenda-setter makes the same proposal as under the benchmark procedure.*

The most straightforward example is where $\rho(\theta) = 1 - \theta$. In this case, $\rho'(\theta) = -1$ is a constant while $\rho''(\theta) = 0$ for any θ. Thus, we can find a continuum of values for μ_1 and μ_2 such that the associated Pendular Voting procedure leads to θ_0. More specifically, the first-order condition reduces to

$$\theta = \frac{1 + \mu_1 - \mu_2}{2}.$$

Hence, any Pendular Voting procedure with $\mu_1 = \mu_2$ leads to the benchmark proposal.

Corollary 4 *If the acceptance probability function ρ is linear, then there is a continuum of pairs (μ_1, μ_2) with $\mu_1 = \mu_2$ such that the associated Pendular Voting procedure leads to the benchmark proposal.*

Manipulation by a subset of citizens can only occur if the agenda–setter makes a proposal such that a majority would rather remain at zero than accept the proposal. But if that is true, nothing higher than the proposal will ever go through. Hence, if the agenda–setter is selfish, and sticks to the same proposal as in the benchmark scenario, he can never become any worse off.

Corollary 5 *A selfish agenda–setter is better off in expectation under Pendular Voting than without it.*

Now consider social welfare under Pendular Voting if the agenda-setter is selfish. We have

shown that for an appropriate choice of the parameters μ_1 and μ_2, the agenda-setter makes the same proposal as under the benchmark procedure. Now Pendular Voting leads to a welfare gain in those states where the majority prefers a proposal in $[\theta_0 - \mu_1, \theta_0 - 0.5\mu_1]$ or a proposal in $[\theta_0 + 0.5\mu_2, \theta_0 + \mu_2]$. Hence, Pendular Voting with a selfish agenda-setter leads to expected gains in social welfare if the state space is sufficiently fine.

We observe that public information revelation is not crucial for the results: Even if citizens do not know the exact share of votes cast for either alternative in the first voting round, our results remain true. One potential concern with the Pendular Voting procedure is that random sample group members vote twice: First, they select the alternative proposal that enters the final round along with the initial proposal and the status quo. Second, they vote in the final round. This does not complicate the analysis with any strategic behavior. After all, we have shown that voting behavior in the final round is sincere.

9 Conclusion

We have provided a first analysis of voting procedures involving proposal assessment. On a qualitative level, we find that a voting procedure with proposal assessment leads to social welfare gains in expectation. A particularly attractive aspect of this result is that it holds regardless of what one assumes about the agenda-setter's self-interest. Moreover, the results are independent of any assumptions about the posterior beliefs of citizens at any point during the democratic process. On a quantitative level, the size of the expected welfare gains from proposal assessment depend on the parameters of the model, such as the underlying distribution of preferences.

The paper links with the voting literature in general, and also with an emerging strand of literature on new forms of democracy. The insights in this paper further enhance our understanding of the democratic process and, in particular, of the challenges associated with direct democracy. For instance, the present paper could help provide a theoretical foundation for the concept of a "counter-proposal" within the Swiss system of direct democracy. When an extreme policy proposal to change the constitution is put to a popular vote,

the Swiss parliament has the right to design a "counter-proposal" which may end up as the change of the constitution if it wins against the status quo and the original proposal. These counter-proposals are typically used to offer the public an opportunity for moderate changes.

In our analysis, we have considered the increment μ between the initial proposal and the alternative proposals as given. There are two potential interpretations of the given μ: First, in some applications of the model, μ may be fixed due to the natural characteristics of the decision to be taken: The actual choice may be from a discrete set, for instance. Second, we could consider the optimal choice of μ as a question of institutional design. We would then have to consider the value of μ which preserves the property that even a selfish agenda-setter cannot lead to welfare losses. One potential application could be a deliberation in a criminal trial: Instead of deciding merely between a sentence or an acquittal, there could be the option of replacing the charge with a more minor one, or adding additional charges.

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