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CAROs: Climate Risk-Adjusted Refinancing Operations*

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Abstract

Policy makers have argued that markets are not pricing climate risk appropriately yet, which may lead to a misallocation of resources and financial instability. Climate risk-adjusted refinancing operations (CAROs) conducted by the central bank are one possible instrument to address this issue. CAROs are characterized by interest rates on reserve loans, which depend on the climate risk exposure of the assets held by the borrowing bank. If private agents and the central bank have differing beliefs about the likelihood of the transition to a low-carbon economy, the allocation emerging without CAROs is, from the central bank's perspective, suboptimal and may lead to financial instability. We find that an appropriate design of CAROs allows the central bank to influence bank lending in a way that induces the optimal allocation under its beliefs and eliminates financial instability. Moreover, we show that investment into climate risk mitigation reduces the need for central bank intervention, and that CAROs can be used to achieve specific climate-related allocation targets.

Keywords: central bank, banks, refinancing operations, interest rates, climate risk

JEL Classification: D84, E42, E43, E44, E58, G21, Q50

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1 Introduction

Climate risk is now widely recognized as a source of financial risk among academics, financial authorities, and financial market participants[1] However, financial markets do not seem to fully integrate this fact yet. There is evidence that climate risks are not adequately priced in financial markets. Central bankers are aware of this situation and often publicly highlight this market failure.[2] Such an inaccurate pricing of assets leads to distorted investment decisions as well as a potential build-up of financial risks that can even endanger financial stability, with adverse consequences for the real economy. For example, banks may suffer unexpected losses due to stranded assets and, as a consequence, may be fettered in their role as financial intermediaries.

In this context, a fiscal measure such as a carbon tax would be an effective instrument not only to internalize the climate damage associated with economic activities, but also to reduce the mispricing of assets and the potential risk of financial instability.[3] While the debate on policy measures promoting the transition to a low-carbon economy has largely focused on the fiscal dimension, the call for action by financial and monetary authorities has become stronger. Financial supervisors and central banks are both urged to adopt measures that include climate-related aspects, such as the exposure to climate risk. Regardless of the introduction of fiscal measures, mitigating the mispricing of climate risks lies within the mandate of financial supervisors and central banks to guarantee the stability of the financial system (NGFS, 2018).

Climate-related aspects can enter both financial supervision and monetary policy. Today, certain financial market participants, such as private banks, already face regulation, in the form of risk-weighted capital requirements, for instance. Accounting for climate risk in the currently used risk assessment procedures is thus a straightforward way to integrate climate considerations into a regulatory framework.[4] To the extent that climate risks endanger the financial stability and thus the effectiveness of monetary policy, central banks should also implement appropriate measures. We contribute to this discussion by outlining a potential way for central banks to account for climate-related aspects, such as climate risk, in their refinancing operations.

We study a climate-oriented monetary policy where the central bank uses differentiated interest rates in its refinancing operations, which depend on the climate risk exposure of individual bank's assets. We analyze this type of monetary policy operations in an environment characterized by private and public agents having differing beliefs about climate risk. Our analysis aims at answering the following questions: What are the implications of belief differences between private agents and the government for the real economy? From a central bank perspective, what is the optimal monetary policy in the presence of such differences? How is the optimal monetary policy affected by climate risk mitigation, concerns about financial instability and

[1]See Battiston et al. (2017), NGFS (2019), Lagarde (2020) and Fink (2020), for instance.

[2]See Rudebusch et al. (2019) and Schnabel (2020), for instance.

[3]Potential fiscal measures include, among others, carbon taxes (Nordhaus, 2013; Weitzman, 2014; Borissov et al., 2019), cap-and-trade systems for emission certificates (Gersbach and Winkler, 2011; Goulder and Schein, 2013; Greaker and Hagem, 2014), subsidies for clean investments (Acemoglu et al., 2012, 2016; Gerlagh et al., 2018; Greaker et al., 2018; Ramstein et al., 2019) and feed-in tariffs (Proença and Aubyn, 2013).

[4]Volz (2017) proposes a climate-oriented bank regulation in the form of differentiated capital requirements depending on the type of lending conducted by the individual bank. Such an approach would, for example, foresee higher risk weights and thus capital requirements for loans to emission-intensive and carbon-dependent sectors.

climate-related targets?

Various other forms of climate-oriented monetary policies have been suggested (NGFS, 2020). Campiglio (2016) discusses differentiated reserve requirements, which take the carbon footprint of the asset portfolio held by the individual financial institution into account. Such differentiated reserve requirements based on the composition of a bank's asset holdings are also discussed by Volz (2017) and Fender et al. (2019). Monnin (2018), in turn, calls for an integration of climate risk into the collateral framework used in central bank refinancing operations. Green quantitative easing, namely asset purchases by central banks that are directed towards low-carbon financial assets is another possibility (Volz, 2017). The monetary policy we consider, namely bank-specific interest rates in central bank refinancing operations, uses climate risk exposure as the conditional factor, but can be also applied more broadly: Other climate-related measures of financial assets, such as a taxonomy, could be considered as a conditional factor. The central bank policy we discuss is thus closely connected to recent proposals of green targeted long-term refinancing operations (TLTROs), see van't Klooster and van Tilburg (2020) or Batsaikhan and Jourdan (2021). Green TLTROs allow central banks to provide liquidity on a long-term basis, while inducing banks to apply more favorable financing conditions for green activities.

Our analysis is based on a static general equilibrium framework that embeds a banking sector, a government sector, comprising a central bank, and two types of loan-financed production sectors that differ in their exposure to climate risk, i.e. a riskless and a risky sector. Banks grant loans to firms which they finance through equity and deposit issuance (i.e., money creation). Moreover, banks need liquidity in the form of central bank reserves to settle interbank liabilities arising from deposit transfers among banks. The liquidity borrowed from the central bank is priced according to the individual bank's exposure to climate risk, which ultimately depends on the composition of its loan portfolio. We refer to such liquidity provisions by the central bank as "climate risk-adjusted refinancing operations", in short CAROs. Our economy either remains in business as usual or shifts to low-carbon activities, as more stringent environmental regulations are put in place. Private agents have subjective beliefs about climate risk, which lead them to attach a likelihood to the transition that may be different from the government's. We extend our baseline model by introducing investment into climate risk mitigation by firms and accounting for costly bank recapitalization, which may be necessary if banks incur sufficiently high losses in the transition. In our framework, bank recapitalization represents a proxy for financial stability.

The belief differences between private agents and the government lead to the fact that, in equilibrium, the allocation of loans is distorted from a governmental perspective. Specifically, if private agents attach a lower probability to the transition than the government, bank lending to the more climate risk-exposed production sector is excessive. As the government aims at maximizing expected welfare, taxing (subsidizing) loans to the sector which benefits (looses) from the distorted beliefs of agents, is optimal. We show that such a tax/subsidy can be implemented through CAROs conducted by the central bank. A differentiated interest rate policy on reserves allows the central bank to influence the allocation of loans in the economy, through the liquidity costs for banks. For example, if the government finds more likely that the transition occurs, compared to private agents, the central bank can counteract the belief-driven effect on the allocation of loans by setting higher marginal liquidity costs for loans allocated to the more climate risk-exposed sector. The marginal liquidity cost factors associated with loans to the two production sectors are thus at the core of the considered climate-oriented monetary policy in our setting.

As mentioned above, the central bank chooses its monetary policy to maximize the, from its point of view, expected welfare, which in our baseline model depends only on the allocation of loans (or equivalently, of capital) across production sectors. We find that the central bank can fully eliminate the belief-driven distortion of the loan allocation and induce the allocation which would emerge if private agents shared the government's beliefs and the central bank does not intervene. If agents attach a lower (higher) probability to the transition than the government, the optimal marginal liquidity cost factors set by the central bank are higher (lower) for the risky sector than for the riskless sector. We can show that the intensity of central bank intervention, as measured by the absolute difference of the marginal cost factors, increases with the belief differences between private agents and the government.

We consider several extensions to our baseline model. First, we introduce the possibility for firms to invest into climate risk mitigation technologies (CRMT). Within this setting, we can show that a higher CRMT investment reduces the intensity of the optimal central bank intervention, for any possible belief of private agents and of the government. Thus, fiscal policies in the form of a subsidy for CRMT investment can help to reduce the need for monetary policy to correct the assessment of climate risk by private agents, which is erroneous from a government perspective.

Second, we account for concerns about financial stability by modeling bank recapitalization, which is required if bank losses in the transition scenario are sufficiently large, such that the initial equity financing of banks is wiped out and shareholders must inject new equity. With costs of bank recapitalization, the central bank faces a trade-off between ruling out financial instability and correcting the belief-driven distortion of the loan allocation. This trade-off emerges from the fact that eliminating financial instability requires a shift of capital to the riskless sector that is larger than the one induced by correcting belief distortions and maximizing expected output in the economy. Accordingly, two monetary policy regimes can be identified. In the first regime, the central bank resolves concerns about financial stability by ruling out bank recapitalization. Specifically, it sets the marginal liquidity cost factor for loans to the more climate risk-exposed sector high enough to induce a sufficient shift of loans towards the less climate risk-exposed sector. In the second regime, the central bank accepts bank recapitalization in the transition but corrects the capital allocation. The choice between the two regimes is driven by a welfare comparison. We also show that if the central bank is equipped with an additional tool, in the form of quantity restrictions on reserve loans, the optimal monetary policy can at the same time rule out concerns about financial stability and correct the belief-driven distortion of the loan allocation. It turns out that, under the optimal monetary policy, the central bank may allow banks to make positive profits through the borrowing of reserves, i.e., the interest rate on reserve loans is lower than the interest rate on reserve deposits. This is the case whenever, with costly reserve borrowing at the central bank, banks would make losses that are high enough to require a recapitalization in the transition. It is then optimal for the central bank to provide an implicit subsidy to banks, by allowing them to generate profits through the borrowing of reserves, in order to prevent costly injections of new equity by shareholders. Whenever borrowing reserves is profitable, the central bank must implement quantity restrictions on reserve loans, as otherwise banks would demand an infinite amount.

Third and last, abstracting from the welfare-maximizing objective of the central bank, we also characterize the monetary policy that is needed to achieve a pre-specified target in the form of loan allocation in the economy. The less loans should be allocated to a particular sector, the higher the respective liquidity cost factor must be. Such a pre-specified target may not only be derived from climate risk considerations, but also from other sustainability objectives. For instance, the central bank may want to ensure coherence with fiscal policies and contribute to the transition to a low-carbon economy, providing support to close the green investment gap. In this particular case, the pre-specified target may represent the share of loans that banks should grant to green projects.

As a final remark, our model assumes that the loan rate on reserves varies with the climate risk exposure of the borrowing bank's asset holdings, while the deposit rate on reserves is uniform for all banks. This approach is equivalent to allowing the deposit rate on reserves to depend on the borrowing bank's climate risk exposure, while keeping the loan rate on reserves constant. The latter specification may be particularly relevant in situations where banks hold large amounts of reserves that are not matched by reserve loans from the central bank, e.g, due to large scale asset purchases by central banks (so-called "quantitative easing").

The paper is organized as follows: Section 2 relates our paper to the existing literature. Section 3 introduces the model and discusses the optimal choices of the individual agents. Section 4 studies the competitive equilibrium in our baseline model. Section 5 discusses the impact of CRMT investment by firms, while section 6 addresses concerns about financial stability. Monetary policies achieving climate-related targets are chracterized in section 7. Section 8 outlines an alternative formulation of the considered central bank policy, and discusses the application of CAROs in situations where banks hold large amount of reserves that do not originate from reserve borrowing at the central bank. Section 9 concludes.

2 Relation to Literature

Our paper relates to four strands of the literature. First, it contributes to the growing number of proposals for a green monetary policy, of which many have already been discussed in section 1. Importantly, our paper can also be seen as a formal analysis to understand the functioning of green TLTROs, as currently proposed by van't Klooster and van Tilburg (2020) and Batsaikhan and Jourdan (2021).

Second, our paper is also related to the literature on the impact of targeted long-term refinancing operations and their ability to shift resources to the desired sectors. For instance, the ECB TLTROs applied in the aftermath of the financial crisis are deemed to have significantly reduced the funding costs of banks, ultimately at the benefit of the real economy. Evidence is, for instance, provided by Andreeva and García-Posada (2021) who show that credit standards ease and loan margins narrow with a bank's uptake of TLTROs. In addition, Benetton and Fantino (2018) find that banks which used TLTROs facilities decreased their lending rates, compared to non-participating banks. They also show that market concentration and counterparty characteristics (small versus large firms, for instance) play an important role for the effect of TLTROs on the real economy. Further, as shown by Afonso and Sousa-Leite (2020), country characteristics, such as a more or less vulnerable economy, affect the pass-through of targeted long-term refinancing operations.

Third, we rely on the literature investigating the impact of climate risk on financial stability, which also plays a key role in our analysis of the optimal design of CAROs. Battiston et al. (2017), for instance, evaluate the impact of climate policies favoring (discouraging) green (brown) economic activities on the valuation of financial assets. Climate policy-induced shocks to the financial system and the pass-through to the real economy, with a specific focus on the amplification mechanisms, are also studied by Stolbova et al. (2018).

Fourth, our paper is connected to the literature on private money creation, as it accounts for the dual role of banks, providing both credit and money, in the form of bank deposits, to the real economy. Recent contributions are Gersbach and Faure (2020) and Benigno and Robatto (2019), for instance. Our monetary architecture is particularly close to the one described in Faure and Gersbach (2017) who emphasize the hierarchical structure of many modern monetary systems and analyze various stylized elements: First, the money stock available to the public mainly takes the form of deposits and is only to a minor extent in the form of cash. Second, deposits are created by commercial banks when granting loans or purchasing assets. Third, the central bank issues reserves to commercial banks that use them to settle claims between each other, which can, for example, arise from interbank deposits flows.

3 Model

3.1 Macroeconomic environment

We develop a static general equilibrium model featuring firms, households, banks and a government sector, including a central bank, as well as two goods—a capital good and a consumption good. Households are endowed with the capital good, which they sell to firms for production of the consumption good. Production of some firms is exposed to climate risk and accordingly we distinguish between riskless and risky firms. Climate risk enters our model through a positive probability of the transition to a low-carbon economy induced, for instance, by more stringent environmental regulations. The decision about the introduction of such regulations is external to our model. The economy features two macroeconomic states: The business as usual scenario without further regulations, and the transition scenario. Throughout our analysis, we allow for differences in beliefs of private agents—firms, households and banks—and beliefs of the government about the likelihood of each scenario.

We focus on a monetary economy where trades are settled instantaneously by using private money in the form of bank deposits.[5] Firms are penniless and must acquire from external

[5]We abstract from cash, which in the considered environment is without loss of generality, as the alternative money (i.e., bank deposits) is interest-bearing.

creditors the funds (i.e., deposits) needed to finance the capital good purchases from households. Due to moral hazard, repayment of firms can only be enforced by banks, so that production is fully financed with bank loans. When granting loans, banks issue deposits, which are, after the capital good sales have been settled, held by households. Parts of these deposits are used for investments into bank equity. Banks operate under unlimited liability and may experience losses, as loan repayment is risky. If bank losses are sufficiently large, banks must be recapitalized, i.e., households, as the only shareholders, must inject new equity. In our baseline model, bank recapitalization is frictionless. We also provide an extension where new equity injections lead to additional costs, which are not internalized by bank managers and shareholders in the initial equity financing decision. We use this setup to study the effect of financial stability concerns on monetary policy.

In our setup, banks must settle interbank liabilities at the central bank by using reserves. Liabilities between banks arise from interbank deposit flows following from transactions on the good markets. The needed liquidity, in the form of reserves, can be borrowed from the central bank. The interest rate on reserve loans, as set by the central bank, depends on the climate risk exposure of the loan portfolio held by the borrowing bank. By applying different liquidity cost factors on loans to riskless and risky firms, the central bank can influence the loan allocation to firms in the economy. Monetary policy is chosen by the central bank to maximize expected welfare, while the governmental budget is balanced throughout our analysis.

3.2 Timeline

As we focus on a monetary economy where trades are settled instantaneously, the timing of interactions among agents is important for our analysis. Figure 1 summarizes the events in our static framework.

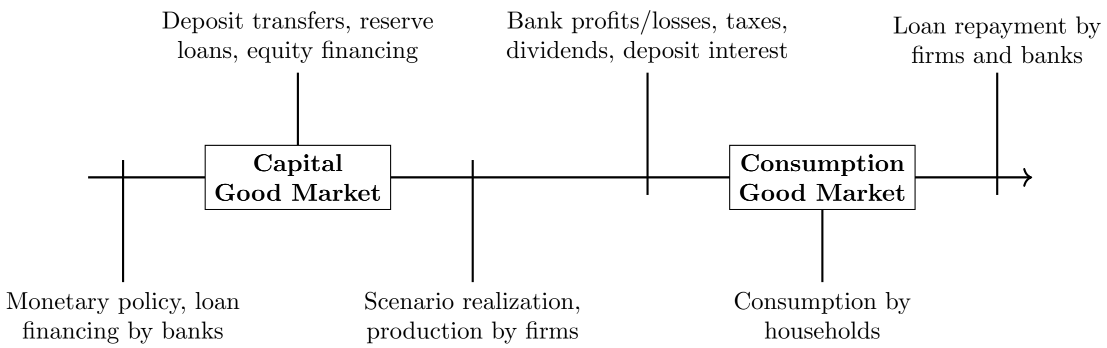

Figure 1: Timeline.

3.3 Firms

There exist two types of firms, which differ in their exposure to climate risk: Firms are either riskless (indexed by l) or risky (indexed by h). Each type of firm exists in a continuum with mass normalized to one, so that we can focus on a representative firm for each type. Firms are penniless and thus must acquire external funds in the form of deposits to finance the capital good purchases before production starts. Firms are prone to moral hazard and can only raise funds through loans from banks, as banks are the only agents in the economy that can eliminate moral hazard by monitoring. For the subsequent analysis, we assume that bank monitoring is costless and fully eliminates moral hazard.

The riskless firm purchases capital good $K_l \geq 0$ from households at a nominal price $Q > 0$.[6] It produces the consumption good with the strictly concave and deterministic technology $A_l K_l^\alpha$, where $A_l > 0$ denotes the total factor productivity and $\alpha \in (0, 1)$ represents the capital intensity. The produced consumption good is then sold to households at a nominal price $P > 0$. The revenues, in the form of deposits, are used to repay bank loans QK_l, which are subject to the interest rate $r_l^L > 0$. The firm operates with unlimited liability and maximizes profits, so that the optimization problem is in real terms given by

$$\max_{K_l \geq 0} \ A_l K_l^\alpha - (1 + r_l^L) q K_l, \tag{1}$$

where the capital good price is in terms of the consumption good, i.e., $q := Q/P$. The riskless firm demands an optimal amount K_l of the capital good if and only if the marginal return from production equals the repayment obligation per unit of the capital good, i.e., $\alpha A_l K_l^{\alpha-1} = (1 + r_l^L)q$. The following lemma outlines the resulting optimal demand of the capital good by the riskless firm.

Lemma 1 (Optimal Choice of the Riskless Firm)
The optimal demand of capital good by the riskless firm is given by

$$K_l = \left[\frac{\alpha A_l}{(1 + r_l^L) q} \right]^{\frac{1}{1-\alpha}}. \tag{2}$$

The risky firm purchases capital good $K_h \geq 0$ from households at a nominal price $Q > 0$. It produces the consumption good according to $A_{h,s} K_h^\alpha$, where $A_{h,s} > 0$ represents the stochastic total factor productivity, which depends on the scenario s, and $\alpha \in (0, 1)$ denotes the capital intensity. The scenario is given either by business as usual ($s = b$) or by the transition to a low-carbon economy ($s = t$). Private agents—firms, household and banks—believe that the transition occurs with probability $\eta_p \in (0, 1)$. In the transition scenario, more stringent environmental regulations are introduced by an official authority, whose decision making is external to our model.[7] The risky firm sells the produced consumption good $A_{h,s} K_h^\alpha$ to households at the nominal price $P > 0$. The revenues, in the form of deposits, are used to repay bank loans QK_h, which are subject to the interest rate $r_{h,s}^L > 0$ that depends on the scenario s. The risky firm operates with unlimited liability and maximizes expected profits, so

[6] Integrating fixed labor as second production input is straightforward, but does not yield additional insights. If labor is assumed to be mobile, further assumptions must be made to maintain the relevance of production by riskless *and* risky firms, such as heterogeneous consumption goods and aggregation, for instance.

[7] Even when integrating the decision about the introduction of additional regulations, the probabilistic structure for the realization of the transition can be maintained. For instance, the decision maker may not perfectly observe the support for such regulations.

that the optimization problem is in real terms given by

$$\max_{K_h \geq 0} \ \mathbb{E}_p[A_{h,s}K_h^\alpha - (1 + r_{h,s}^L)qK_h]. \tag{3}$$

The firm demands an optimal amount K_h of the capital good if and only if the expected marginal return from production equals the expected repayment obligation per unit of the capital good, i.e., $\alpha \mathbb{E}_p[A_{h,s}]K_h^{\alpha-1} = (1 + \mathbb{E}_p[r_{h,s}^L])q$. The following lemma outlines the resulting optimal demand of the capital good by the risky firm.

Lemma 2 (Optimal Choice of the Risky Firm)
The optimal demand of capital good by the risky firm is given by

$$K_h = \left[\frac{\alpha \mathbb{E}_p[A_{h,s}]}{(1 + \mathbb{E}_p[r_{h,s}^L])q}\right]^{\frac{1}{1-\alpha}}. \tag{4}$$

We impose a specific structure of loan rates, which ensures that, in each scenario, the marginal return of production equals the repayment obligation per unit of the capital good. This assumption simplifies the introduction of bank recapitalization, as outlined in section 6.

Assumption 1 (Repayment of the Risky Firm)
$(1 + r_{h,s}^L)q = \alpha A_{h,s}K_h^{\alpha-1}$ *for all* s.

With assumption 1, the aggregate firm profits in scenario s are under optimal choices of riskless and risky firms given, in real terms, by

$$\begin{aligned}\pi_s &= A_l K_l^\alpha - (1 + r_l^L)qK_l + A_{h,s}K_h^\alpha - (1 + r_{h,s}^L)qK_h \\ &= (1-\alpha)[A_l K_l^\alpha + A_{h,s}K_h^\alpha] \geq 0.\end{aligned} \tag{5}$$

3.4 Households

Households are identical and exist in a continuum with mass normalized to one, so that we can focus on a representative household. The household is endowed with capital good $K > 0$, which can be sold to firms at the nominal price $Q > 0$. The revenues from capital good sales take the form of deposits, which are credited with interest according to the rate $r^D > 0$. Deposits can be used to invest into bank equity, which yields the rate of return r_s^E in scenario s. The share of funds invested into bank equity is denoted by $\gamma \in [0, 1]$. Households own firms and thus receive profits Π_s as dividends. After accounting for governmental taxes or transfers T_s, the household uses the equity returns $\gamma(1 + r_s^E)QK$, the deposits credited with interest $(1-\gamma)(1 + r^D)QK$, and the firm profits Π_s received as dividends to purchase an amount C_s of the consumption good at the nominal price $P > 0$ from firms. The household is maximizing the expected utility, which we assume to be linear and strictly increasing in consumption. Thus, the optimization problem of the household is given, in real terms, by

$$\max_{\gamma\in[0,1]} \mathbb{E}_p[\{\gamma(1+r_s^E)+(1-\gamma)(1+r^D)\}qK+\tau_s+\pi_s], \tag{6}$$

where taxes and profits are in terms of the consumption good, i.e., $\tau_s := T_s/P$ and $\pi_s := \Pi_s/P$. The expectation operator in (6) is indexed by "p", as like all other private agents, the household has subjective beliefs about the transition, which are captured by the probability η_p. Due to the assumption of linear utility, the household invests the funds in the asset which yields the highest expected rate of return. The following lemma outlines the household's optimal choice.

Lemma 3 (Optimal Choice of the Household)
$\gamma = 1$ *(*$\gamma = 0$*) if* $\mathbb{E}_p[r_s^E] > (<) r^D$*, and* $\gamma \in [0,1]$ *otherwise.*

3.5 Government sector

The government sector comprises the central bank and the government. Via uncollateralized loans, the central bank provides liquidity to banks in the form of reserves, which the banks use to settle interbank liabilities. Reserves can be deposited at the central bank and are credited with interest according to the rate $r_{CB}^D > 0$. The repayment of reserve loans, in turn, is determined by the interest rate $r_{CB}^L(\zeta) > 0$, which depends on the share $\zeta \in [0,1]$ of loans granted to riskless firms by the borrowing bank. Specifically, we assume that the interest rates on reserves satisfy

$$1+r_{CB}^L(\zeta) = (1+r_{CB}^D)[1+\zeta\kappa_l+(1-\zeta)\kappa_h] \quad \text{subject to} \quad \zeta\kappa_l+(1-\zeta)\kappa_h \geq 0, \tag{7}$$

with $\kappa_l \in \mathbb{R}$ and $\kappa_h \in \mathbb{R}$ representing the liquidity cost factors on bank loans granted to riskless firms and risky firms, respectively. Due to the constraint $\zeta\kappa_l + (1-\zeta)\kappa_h \geq 0$, the loan rate on reserves always weakly exceeds the deposit rate on reserves, i.e., $r_{CB}^L(\zeta) \geq r_{CB}^D$, so that liquidity is costly for banks. To simplify the subsequent analysis, we reformulate equation (7) to

$$r_{CB}^L(\zeta) = r_{CB}^D[1+\zeta\tilde{\kappa}_l+(1-\zeta)\tilde{\kappa}_h] \quad \text{with} \quad \tilde{\kappa}_l = \frac{\kappa_l(1+r_{CB}^D)}{r_{CB}^D} \quad \text{and} \quad \tilde{\kappa}_h = \frac{\kappa_h(1+r_{CB}^D)}{r_{CB}^D}.$$

Given that $\tilde{\kappa}_l$ ($\tilde{\kappa}_h$) is a rescaling of κ_l (κ_h), we will also refer to $\tilde{\kappa}_l$ ($\tilde{\kappa}_h$) as the liquidity cost factor on riskless (risky) loans.

In our setting, the central bank aims at maximizing the expected welfare, not knowing which scenario realizes, by choosing the interest rate r_{CB}^D and the cost factors κ_l and κ_h. The belief of the government sector, including the central bank's, about the likelihood of the transition is given by the probability $\eta_g \in (0,1)$. Thus, the optimization problem of the central bank is given by

$$\max_{\substack{r_{CB}^D>0 \\ \kappa_l,\kappa_h\in\mathbb{R}}} \mathbb{E}_g[W_s] \qquad \text{subject to} \qquad \zeta\kappa_l+(1-\zeta)\kappa_h \geq 0, \tag{8}$$

where W_s denotes welfare in scenario s. The government has a passive role as it only distributes (finances) central bank profits (losses) Π_s^{CB} by using governmental transfers (taxes) T_s.

Throughout our analysis, we impose that the consolidated budget of the government sector is balanced, so that taxes and transfers are given by $T_s = \Pi_s^{CB}$.

Two remarks regarding the potential spread on central bank interest rates (i.e., $r_{CB}^L > r_{CB}^D$) are in order. First, we can always find an optimal monetary policy that rules out a spread on central bank rates (i.e., $\zeta\kappa_l + (1-\zeta)\kappa_h = 0$) and thus implies zero liquidity costs for banks. In fact, in the presence of financial stability concerns, any optimal monetary policy implies zero liquidity costs for banks (see section 6). Second, even if monetary policy induces a spread on central bank interest rates, this does not affect the real allocation and, importantly, not the ability of banks to repay their reserve loans to the central bank. The reason is that central bank profits, emerging from the spread on central bank rates, are distributed to households through transfers. As we abstract from cash, these transfers represent for households an increase on their deposit accounts and for banks an inflow of deposits. Deposit flows are matched by reserve flows (for a detailed description, see subsection 3.6), so that the distribution of transfers also increases the reserve holdings of banks. The latter exactly matches the missing amount of reserves needed to cover the repayment of reserve loans.

3.6 Banks

Banks are identical and exist in a continuum with mass normalized to one, so that we can focus on a representative bank. Banks are only active if they receive a positive amount of equity financing $E > 0$ from households. The bank grants loans to riskless and risky firms, which are denoted by $L_l \geq 0$ and $L_h \geq 0$, respectively. The total loan volume is then given by $L = L_l + L_h$ and the share of loans granted to riskless firms satisfies $\zeta = L_l/L$. The supply of loans and the equity financing determine the amount of deposit financing $D = L - E$, once the capital good sales have been settled and households used (parts of) their deposits to invest into bank equity.

Deposits are credited with interest according to the rate $r^D > 0$, whereas loans yield a return determined by the interest rates $r_l^L > 0$ and $r_{h,s}^L > 0$, respectively. The repayment by risky firms is uncertain, as it depends on the scenario realized, business as usual versus transition. The bank can borrow reserves L^{CB} from the central bank, which requires a repayment determined by the interest rate $r_{CB}^L(\zeta) > 0$, which depends on the portfolio allocation, as measured by the share ζ of loans granted to riskless firms. The bank can deposit reserves D^{CB} at the central bank, which yield a rate of return $r_{CB}^D > 0$. Therefore, the balance sheet identity $L + D^{CB} = D + L^{CB} + E$ applies and, taking the returns of the various assets and liabilities into account, the nominal equity returns in scenario s are given by

$$\begin{aligned}(1+r_s^E)E = (1+r_l^L)L_l + (1+r_{h,s}^L)L_h + (1+r_{CB}^D)D^{CB} \\ - (1+r^D)D - (1+r_{CB}^L(\zeta))L^{CB}. \qquad (9)\end{aligned}$$

The bank demands liquidity in the form of reserves, as transactions on the good markets lead to deposit flows among banks, which entail interbank liabilities. The latter must be settled at the central bank by using reserves, where settlement occurs on a gross basis, i.e., the liabilities from deposit outflows cannot be netted with the claims from deposit inflows. We assume that in the course of transactions on the capital good market, a share $\psi \in (0, 1]$ of deposits is temporarily outflowing.[8] Note that when the capital good market is active, deposits equal loans, and households acquire bank equity only after all capital good transactions have been settled. Accordingly, the reserve loans demanded by the bank must satisfy $L^{CB} \geq \psi L$. The pricing of reserves chosen by the central bank is such that the loan rate is weakly exceeding the deposit rate (see equation (7) in subsection 3.5), i.e., $r^L_{CB}(\zeta) \geq r^D_{CB}$ for all $\zeta \in [0,1]$. Thus, we can assume, without loss of generality, that the liquidity demand on the side of the bank is given by $L^{CB} = \psi L$. Since we focus on a representative bank, deposit outflows always match deposit inflows, such that after all capital good transactions have been settled, reserve loans must equal reserve deposits, i.e., $L^{CB} = D^{CB}$. Using the definition of the deposit financing after capital good transactions have been settled, $D = L - E$, and the definition of the share of riskless loans in the bank's loan portfolio, $\zeta = L_l/L$, the nominal equity returns (see equation (9)) can be rewritten as

$$(1 + r^E_s)E = [(1 + r^L_l)\zeta + (1 + r^L_{h,s})(1 - \zeta)]L \\ - (1 + r^D)(L - E) - [r^L_{CB}(\zeta) - r^D_{CB}]\psi L. \tag{10}$$

As reserve deposits and reserve loans satisfy $D^{CB} = L^{CB} = \psi L$, the bank's assets are given by $L + D^{CB} = (1+\psi)L$, so that the bank leverage reads $\varphi = (L + D^{CB})/E = (1+\psi)L/E$. After capital good transactions have been settled, deposit financing is given by $D = L - E$, so that the bank leverage can also be written as $\varphi = (1+\psi)(1+D/E)$. Banking operations are subject to capital requirements leading to a regulatory leverage constraint. The bank's decision about loan supply, leading to the leverage φ, must satisfy the constraint $\varphi \leq \varphi^R$, where $\varphi^R \in [1, +\infty)$ is the regulatory maximum leverage.

Using the definition of the bank leverage $\varphi = (1+\psi)L/E$, we can derive the rate of return on equity as a function of the bank leverage φ and the portfolio allocation share ζ, i.e., from equation (10), it follows that

$$r^E_s(\varphi, \zeta) := (1+\psi)^{-1}\left\{[(1 + r^L_l)\zeta + (1 + r^L_{h,s})(1 - \zeta)]\varphi \right. \\ \left. -(1 + r^D)[\varphi - (1+\psi)] - \psi[r^L_{CB}(\zeta) - r^D_{CB}]\varphi\right\} - 1,$$

which can be rewritten as

$$r^E_s(\varphi, \zeta) = (1+\psi)^{-1}[r^L_l\zeta + r^L_{h,s}(1-\zeta) - r^D - \psi(r^L_{CB}(\zeta) - r^D_{CB})]\varphi + r^D. \tag{11}$$

We also allow for an active interbank market, where the bank can borrow from, lend to and deposit with other banks. We assume that the bank cannot differentiate between deposit holdings of other banks and deposit holdings of households and firms. Thus, the interest rate on interbank deposits is given by r^D. An active interbank market, which rules out arbitrage opportunities for banks, exists if and only if the interest rate on the interbank deposits equals the interest rate on reserve deposits at the central bank.

[8] We abstract from deposit flows due to transactions on the consumption good market, as including them does not yield further insights, but complicates the analysis.

Lemma 4 (Interbank Market)
$r^D = r^D_{CB}$.

Then, using lemma 4 and the functional form of the interest rate on reserve loans, namely $r^L_{CB}(\zeta) = r^D_{CB}[1 + \zeta\tilde{\kappa}_l + (1-\zeta)\tilde{\kappa}_h]$, the rate of return on bank equity, stated in equation (11), translates into

$$r^E_s(\varphi, \zeta) = (1+\psi)^{-1}[r^L_l \zeta + r^L_{h,s}(1-\zeta) - r^D_{CB}\Psi(\zeta)]\varphi + r^D_{CB}, \tag{12}$$

where we used the notation $\Psi(\zeta) := 1 + \psi[\zeta\tilde{\kappa}_l + (1-\zeta)\tilde{\kappa}_h]$. The bank operates with unlimited liability and maximizes the shareholder value by choosing the leverage and the loan portfolio allocation. Its optimization problem is thus given by

$$\max_{\substack{\varphi \in [1,\varphi^R], \\ \zeta \in [0,1]}} \mathbb{E}_p[r^E_s(\varphi, \zeta)]. \tag{13}$$

The expectation operator in (13) is indexed by "p", as banks share the same subjective beliefs as all other private agents about the likelihood of the transition, which is captured by the probability η_p.

We now discuss the optimal choice of the bank, focusing first on the optimal leverage. As the leverage is given by $\varphi = (1+\psi)(1+D/E)$, we know that any leverage greater than $1+\psi$ implies that the bank is partly financing loans with deposits. For its decision to finance loans with deposits, and thus its decision about the leverage, the bank must evaluate the expected repayment of loans, the interest payment on deposits and the liquidity costs arising from reserve borrowing. From equation (12), which describes the rate of return on bank equity, we know that the expected rate of return from granting loans financed with deposits is given by

$$\begin{aligned} &r^L_l \zeta + \mathbb{E}_p[r^L_{h,s}](1-\zeta) - r^D_{CB}\Psi(\zeta) \\ &\qquad = [r^L_l - r^D_{CB}(1+\psi\tilde{\kappa}_l)]\zeta + [\mathbb{E}_p[r^L_{h,s}] - r^D_{CB}(1+\psi\tilde{\kappa}_h)](1-\zeta), \end{aligned}$$

where we used the definition $\Psi(\zeta) := 1 + \psi[\zeta\tilde{\kappa}_l + (1-\zeta)\tilde{\kappa}_h]$. Financing loans with deposits generates costs for the bank, due to interest payments on deposits and costly reserve borrowings. Reserves are needed, as deposits are transferred between banks in the course of transactions on the capital good market. The costs of financing one unit of loans to riskless and risky firms with deposits are therefore given by the deposit rate r^D_{CB} and the marginal liquidity costs $r^D_{CB}\psi\tilde{\kappa}_l$ and $r^D_{CB}\psi\tilde{\kappa}_h$, respectively. If the expected loan rate in one of the sectors, r^L_l and $\mathbb{E}_p[r^L_{h,s}]$ respectively, exceeds the deposit rate and the marginal costs of reserve borrowing, i.e., if it holds that

$$r^L_l > r^D_{CB}(1+\psi\tilde{\kappa}_l) \qquad \text{or} \qquad \mathbb{E}_p[r^L_{h,s}] > r^D_{CB}(1+\psi\tilde{\kappa}_h),$$

the bank can increase the expected rate of return on bank equity by extending loan financing and deposit issuance, leading to a higher leverage. Similarly, the bank makes losses by financing loans with deposits if the expected loan rates, r^L_l and $\mathbb{E}_p[r^L_{h,s}]$, are insufficient to cover the financing costs $r^D_{CB}(1+\psi\tilde{\kappa}_l)$ and $r^D_{CB}(1+\psi\tilde{\kappa}_h)$, respectively. In this case, the bank increases the expected rate of return on bank equity by reducing the loan supply and deposit issuance, leading to a lower leverage. From the previous observations, we can conclude that the bank chooses the maximum (minimum) leverage $\varphi = \varphi^R$ ($\varphi = 1$) if it holds that

$$\max\{r^L_l - r^D_{CB}\psi\tilde{\kappa}_l, \mathbb{E}_p[r^L_{h,s}] - r^D_{CB}\psi\tilde{\kappa}_h\} > (<) r^D_{CB}.$$

In all other situations, the bank makes zero profit by granting loans financed with deposits and thus is indifferent between all leverages, i.e., $\varphi \in [1, \varphi^R]$.

Next, we discuss the optimal portfolio allocation of the bank, as captured by the share ζ of loans granted to riskless firms. The portfolio allocation of the bank depends on the expected rate of return from loans to riskless and risky firms, and the associated marginal liquidity costs. Specifically, if, after accounting for the costs of deposit financing and costly reserve borrowing, the rate of return on loan financing to riskless firms is higher (lower) than the expected rate of return on loan financing to risky firms, i.e., if it holds that

$$\begin{aligned} & r^L_l - r^D_{CB}(1+\psi\tilde{\kappa}_l) > (<) \mathbb{E}_p[r^L_{h,s}] - r^D_{CB}(1+\psi\tilde{\kappa}_h) \\ \Leftrightarrow \quad & r^L_l - r^D_{CB}\psi\tilde{\kappa}_l > (<) \mathbb{E}[r^L_{h,s}] - r^D_{CB}\psi\tilde{\kappa}_h, \end{aligned}$$

the bank chooses to provide only loan financing to riskless (risky) firms, i.e., $\zeta = 1$ ($\zeta = 0$). In all other situations, the bank is indifferent between loan financing to riskless and to risky firms, i.e., $\zeta \in [0, 1]$. The optimal choice of the bank is summarized in the following lemma.

Lemma 5 (Optimal Choice of the Bank)
The bank's optimal choice of the leverage is given by $\varphi = \varphi^R$ ($\varphi = 1$) if it holds that

$$\max\{r^L_l - r^D_{CB}\psi\tilde{\kappa}_l, \mathbb{E}_p[r^L_{h,s}] - r^D_{CB}\psi\tilde{\kappa}_h\} > (<) r^D_{CB},$$

and $\varphi \in [1, \varphi^R]$ otherwise. The bank's optimal choice of the portfolio allocation is given by $\zeta = 1$ ($\zeta = 0$) if it holds that

$$r^L_l - r^D_{CB}\psi\tilde{\kappa}_l > (<) \mathbb{E}_p[r^L_{h,s}] - r^D_{CB}\psi\tilde{\kappa}_h,$$

and $\zeta \in [0, 1]$ otherwise.

4 Equilibrium Analysis

4.1 Equilibrium definition

In the subsequent analysis, we focus on competitive equilibria. For what follows, we use the notation $Y_s := A_l K^\alpha_l + A_{h,s} K^\alpha_h$ to represent the aggregate production output in scenario s.

Definition 1 (Competitive Equilibrium)
Given a monetary policy $r^D_{CB} > 0$, $\kappa_l \in \mathbb{R}$ and $\kappa_h \in \mathbb{R}$, a competitive equilibrium is a set of

prices $P > 0$ and $Q > 0$, interest rates $r^D > 0$, $r_l^L > 0$, $r_{h,s}^L > 0$ and $r_s^E > 0$, with $s \in \{b, t\}$, and choices K_l, K_h, γ, φ and ζ, so that

(i) given P, Q and r_l^L, the choice K_l maximizes the profits of the riskless firm,

(ii) given P, Q, $r_{h,s}^L$, with $s \in \{b, t\}$, the choice K_h maximizes the expected profits of the risky firm,

(iii) given P, Q, r^D and r_s^E, with $s \in \{b, t\}$, the choice γ maximizes the utility of the household,

(iv) given r_{CB}^D, κ_l, κ_h, r^D, r_l^L, $r_{h,s}^L$, with $s \in \{b, t\}$, the choices φ and ζ maximize the shareholder value of the bank,

(v) the equity, loan, capital good and consumption good markets clear, i.e., $E = \gamma QK$, $QK_l = L_l$, $QK_h = L_h$, $K_l + K_h = K$ and $C_s = Y_s$.

Note that in the definition of a competitive equilibrium, we do not account for the deposit market, as it clears by construction of the model.

4.2 Equilibrium properties

We first show that, in equilibrium, riskless and risky firms both obtain loans, and we describe the prevailing interest rates and prices. We then provide properties relating to bank leverage and welfare, and finally outline the capital allocation in the decentralized equilibrium.

Loan demand. In equilibrium, both sectors obtain a positive amount of loan financing and produce. This is due to the fact that riskless and risky firms operate with technologies that satisfy the Inada conditions, i.e., the marginal return from production is strictly increasing with lower input of capital good. As marginal productivities are directly linked to loan rates (see subsection 3.3), we can deduce that for any possible interest rates on loans, both types of firms obtain loan financing. A higher loan rate in one sector simply leads to less demand for bank loans by this respective sector, but will remain positive in any case.

Lemma 6 (Loan Demand)
In equilibrium, riskless and risky firms obtain loans, i.e., it holds that $\zeta \in (0, 1)$.

Interest rates. Using the fact that in equilibrium riskless and risky firms both demand loan financing, and by the assumption that in equilibrium, perfect competition leads to banks making zero expected profits by financing loans with deposits, we can further characterize the interest rates in our economy. Specifically, we can relate the loan rates in the two sectors to each other, and the loan rates to the interest rate on reserve deposits. First, given that in equilibrium, both types of firms demand loan financing, as shown in lemma 6, the bank must be indifferent between granting loans to riskless and to risky firms, which, using lemma 5, implies

$$r_l^L - r_{CB}^D \psi \tilde{\kappa}_l = \mathbb{E}_p[r_{h,s}^L] - r_{CB}^D \psi \tilde{\kappa}_h. \tag{14}$$

The expected loan returns adjusted for the marginal liquidity costs—hereinafter referred to as *adjusted loan rates*—must be identical across sectors. Otherwise, the bank would have no incentive to grant loans to the two types of firms. If the liquidity cost factors κ_l and κ_h equal, so that loans to both sectors are subject to the same marginal liquidity costs, the expected loan rates in both sectors equal too, i.e., it holds that $r_l^L = \mathbb{E}_p[r_{h,s}^L]$.[9] In turn, if riskless and risky loans have a differing impact on the liquidity costs, i.e., $\kappa_l \neq \kappa_h$, the expected loan rates from the two sectors will not be identical. The sector for which a lower liquidity cost factor applies will benefit from relatively better loan financing conditions, in terms of a lower interest rate on loans. For example, note that with cost factors satisfying $\kappa_l < \kappa_h$, it follows from equation (14) that loan rates in both sectors satisfy

$$r_l^L = \mathbb{E}_p[r_{h,s}^L] - r_{CB}^D \psi(\tilde{\kappa}_h - \tilde{\kappa}_l) < \mathbb{E}_p[r_{h,s}^L],$$

leaving riskless firms with better terms for bank loans than risky firms. Second, we assume perfect competition among banks, leading to zero expected profits from financing loans with deposits in equilibrium. In other words, the bank must be indifferent in equilibrium between all possible leverages, i.e., $\varphi \in [1, \varphi^R]$. Using lemma 5, this translates into the condition

$$\max\{r_l^L - r_{CB}^D \psi \tilde{\kappa}_l, \mathbb{E}_p[r_{h,s}^L] - r_{CB}^D \psi \tilde{\kappa}_h\} = r_{CB}^D,$$

which, using the equality of adjusted loan rates (see equation (14)), leads to

$$r_l^L - r_{CB}^D \psi \tilde{\kappa}_l = \mathbb{E}_p[r_{h,s}^L] - r_{CB}^D \psi \tilde{\kappa}_h = r_{CB}^D.$$

The latter two conditions relate the adjusted loan rates in the two sectors to the interest rate on reserve deposits.

Corollary 1 (Loan Rates)
In equilibrium, the loan rates satisfy $r_l^L = r_{CB}^D(1 + \psi \tilde{\kappa}_l)$ *and* $\mathbb{E}_p[r_{h,s}^L] = r_{CB}^D(1 + \psi \tilde{\kappa}_h)$.

Note that interest rates on loans are linked to firm productivity, see subsection 3.3. For loans to riskless firms, we know from the first-order condition that it holds that $(1 + r_l^L)q = \alpha A_l K_l^{\alpha-1}$. From assumption 1, we know that for loans to risky firms, it holds that $(1+r_{h,s}^L)q = \alpha A_{h,s} K_h^{\alpha-1}$.

Prices. From corollary 1, we can deduce the formation of prices P and Q in our economy, see corollary 2. Note that the price ratio P/Q is positively correlated with the interest rate r_{CB}^D on reserve deposits. Thus, an increase of r_{CB}^D leads to an increase of the consumption good price P or a decrease of the capital good price Q or both.

Corollary 2 (Prices)
In equilibrium, the prices P and Q satisfy

$$\frac{P}{Q} = \frac{(1 + r_{CB}^D)(1 + \psi \kappa_h)}{\alpha \mathbb{E}_p[A_{h,s}] K_h^{\alpha-1}}.$$

[9]Recall that $\tilde{\kappa}_l = \kappa_l(1 + r_{CB}^D)/r_{CB}^D$ and $\tilde{\kappa}_h = \kappa_h(1 + r_{CB}^D)/r_{CB}^D$. Thus, $\kappa_l = \kappa_h$ implies $\tilde{\kappa}_l = \tilde{\kappa}_h$.

Bank leverage. Using the definition of bank leverage, $\varphi = (1+\psi)L/E$, and the definition of the share of loans allocated to the riskless sector, $\zeta = L_l/L$, we can express the amount of loan financing granted to riskless firms as $L_l = \zeta(1+\psi)^{-1}\varphi E$. Similarly, the loan supply to the risky firm is given by $L_h = (1-\zeta)(1+\psi)^{-1}\varphi E$. Due to the clearing of the equity market, i.e., $E = \gamma Q K$, and the loan market, i.e., $QK_l = L_l$ and $QK_h = L_h$, we know that the amount of capital good used in production by riskless and risky firms is given by $K_l = \zeta(1+\psi)^{-1}\varphi\gamma K$ and $K_h = (1-\zeta)(1+\psi)^{-1}\varphi\gamma K$, respectively. With the clearing of the capital good market, i.e., $K_l + K_h = K$, we then obtain that the equilibrium leverage is given by $\varphi = (1+\psi)/\gamma$ and the capital good used by firms in the riskless and risky sector satisfies $K_l = \zeta K$ and $K_h = (1-\zeta)K$, respectively. As the bank is facing the regulatory leverage constraint $\varphi \leq \varphi^R$, the existence of an equilibrium is only guaranteed if $\varphi^R \geq (1+\psi)/\gamma$.

Welfare. Throughout our analysis, we focus on utilitarian welfare. Due to our assumption of linear utility for the household, welfare comprises aggregate consumption. As the scenario, business as usual versus transition, affects the productivity in the risky sector, welfare generally depends on the state s and is given by $W_s = C_s$. The following lemma provides a characterization of welfare in terms of economic fundamentals.

Lemma 7 (Welfare)
In equilibrium, welfare is given by $W_s = [A_l\zeta^\alpha + A_{h,s}(1-\zeta)^\alpha]K^\alpha$.

Capital allocation. The demand for capital good and thus the demand for loan financing in each of the sectors depends on the respective repayment obligation as determined by the loan rate (see lemma 1 and lemma 2). For both types of firms, it holds that a higher interest rate on loans reduces the demand for loan financing and, ultimately, the amount of the capital good used in production. Equation (14) relates the equilibrium loan rates in the two sectors. Specifically, the adjusted loan rates must be equal, i.e.,

$$r_l^L - r_{CB}^D\psi\tilde{\kappa}_l = \mathbb{E}_p[r_{h,s}^L] - r_{CB}^D\psi\tilde{\kappa}_h.$$

The sector for which a lower liquidity cost factor applies benefits from relatively better terms on bank loans in the form of a lower loan rate. With identical liquidity cost factors, i.e., if $\kappa_l = \kappa_h$, both sectors face identical conditions for loan financing, i.e., $r_l^L = \mathbb{E}_p[r_{h,s}^L]$, and the allocation of capital among the sectors is only driven by the relative expected productivity of riskless and risky firms, i.e., $\mathbb{E}_p[A_{h,s}]/A_l$. In turn, if, for instance, loans to risky firms are subject to higher marginal liquidity costs than loans to riskless firms, i.e., $\kappa_l < \kappa_h$, the riskless sector is facing more favorable conditions for loan financing compared to the risky sector. Compared to the case of equal liquidity cost factors, riskless firms will demand more loan financing in equilibrium and thus receive a larger share of the capital good available in the economy. The equilibrium share ζ of capital good allocated to the riskless sector, as stated in the following proposition, captures the previously described forces driving the capital allocation, namely the relative expected productivity and the impact of marginal liquidity costs on loan financing conditions.

Proposition 1 (Capital Allocation)
In equilibrium, the share of capital good allocated to the riskless sector is given by

$$\zeta = \left[1 + \left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l} \frac{1+\psi\kappa_l}{1+\psi\kappa_h} \right)^{\frac{1}{1-\alpha}} \right]^{-1}. \tag{15}$$

Note that the expected productivity of risky firms, and thus the relative expected productivity of the two sectors, is affected by the beliefs of private agents about the likelihood of the transition. Specifically, the higher the probability η_p that agents attach to the transition, the lower the expected productivity of risky firms and the higher the share ζ of capital good allocated to riskless firms.

4.3 Optimal monetary policy

We now study the optimal monetary policy that maximizes expected welfare. Without knowing the scenario realization, the central bank chooses the interest rate $r_{CB}^D > 0$ on reserve deposits, and the costs factors $\kappa_l \in \mathbb{R}$ and $\kappa_h \in \mathbb{R}$, which ultimately determine the interest rate $r_{CB}^L(\zeta)$ on reserve loans. Note that the interest rate r_{CB}^D does not affect welfare (see lemma 7), and only influences the prices in our economy (see corollary 2). Thus, the neutrality of money applies in our model and any positive interest rate $r_{CB}^D > 0$ represents an optimal choice for the central bank. The government sector, including the central bank, has its own beliefs about the introduction of more stringent environmental regulations and thus the occurrence of the transition. These beliefs translate into the probability η_g that the government associates with the transition, which may differ from the probability η_p that private agents have. Formally, the optimization problem of the central bank is given by

$$\max_{\kappa_l,\kappa_h \in \mathbb{R}} \{A_l\zeta^\alpha + \mathbb{E}_g[A_{h,s}](1-\zeta)^\alpha\}K^\alpha \qquad \text{subject to} \qquad \zeta\kappa_l + (1-\zeta)\kappa_h \geq 0,$$

where we made use of lemma 7 to represent welfare W_s.

The cost factors κ_l and κ_h implemented by the central bank influence the capital allocation ζ in the economy, as shown in proposition 1. The capital allocation is also influenced by the beliefs of private agents. For example, the less private agents believe that the transition realizes (i.e., the lower η_p), the more capital good is allocated to the risky sector (i.e., the lower ζ). The central bank uses its interest policy on reserve loans, determined by the cost factors κ_l and κ_h, to induce the capital allocation that would emerge without central bank intervention if private agents shared the beliefs of the government sector. In other words, the central bank corrects the capital allocation for the belief differences between private agents and the government sector. Note that the central bank is restricted in its choice of the cost factors κ_l and κ_h, as liquidity must be costly for banks in order to avoid arbitrage opportunities, i.e., it holds that $\zeta_g\kappa_l + (1-\zeta_g)\kappa_h \geq 0$, where ζ_g represents the optimal capital allocation. The allocation ζ_g is indexed by "g", as it crucially depends on the beliefs in the government sector (see Proposition 2).

Proposition 2 (Optimal Monetary Policy)
The central bank optimally chooses cost factors κ_l and κ_h, so that

$$\kappa_h = a\kappa_l + \frac{a-1}{\psi} \qquad \textit{and} \qquad \zeta_g\kappa_l + (1-\zeta_g)\kappa_h \geq 0,$$

where

$$a := \frac{\mathbb{E}_p[A_{h,s}]}{\mathbb{E}_g[A_{h,s}]} \qquad \textit{and} \qquad \zeta_g := \left[1 + \left(\frac{\mathbb{E}_g[A_{h,s}]}{A_l}\right)^{\frac{1}{1-\alpha}}\right]^{-1}.$$

If $\eta_g > (<)\eta_p$, it follows that $a > (<)1$ and therefore $\kappa_h > (<)\kappa_l$.

If compared to the government sector, private agents underestimate the likelihood of the transition, i.e., $\eta_g > \eta_p$, the central bank implements cost factors that satisfy $\kappa_h > \kappa_l$. Thus compared to riskless firms, risky firms face worse conditions for loan financing, as loans to risky firms increase relatively more the liquidity costs of the bank. Similarly, the central bank discourages loan financing to riskless firms by setting cost factors that satisfy $\kappa_l > \kappa_h$, whenever compared to the government sector, private agents overestimate the likelihood of the transition, $\eta_p > \eta_g$. If the beliefs of private agents match the ones of the government sector, the central bank does not have to intervene, so that it optimally sets identical cost factors, $\kappa_l = \kappa_h$, resulting in the capital allocation

$$\zeta_g = \zeta_p := \left[1 + \left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\right)^{\frac{1}{1-\alpha}}\right]^{-1}.$$

We now focus on the intensity of central bank intervention as measured by the difference between the cost factors, $|\kappa_h - \kappa_l|$.

We can show that difference of cost factors $\kappa_h - \kappa_l$ increases (decreases) with η_g (η_p), the probability associated by the government sector (private agents) to the transition. This implies that whenever beliefs satisfy $\eta_g > \eta_p$, so that $\kappa_h - \kappa_l > 0$, the intensity of central bank intervention, as measured by the absolute difference of cost factors $|\kappa_h - \kappa_l|$, increases with η_g and decreases with η_p. In turn, if beliefs satisfy $\eta_g < \eta_p$, cost factors are such that $\kappa_h - \kappa_l < 0$, and the intensity of central bank intervention decreases with η_g and increases with η_p.

Corollary 3 (Optimal Monetary Policy and Beliefs)
If the central bank chooses the monetary policy according to proposition 2, the difference between the optimal cost factors, $\kappa_h - \kappa_l$, increases with the beliefs η_g of the government sector and decreases with the beliefs η_p of private agents.

5 Climate Risk Mitigation

In this section, we extend our baseline model by accounting for the adoption of a climate risk mitigation technology (CRMT) by risky firms. Specifically, firms in the risky sector can invest parts of the acquired capital good to reduce their exposure to risk, which ultimately increases their total factor productivity in the transition scenario. For what follows, we use $i \in [0,1]$ to denote the share of capital good used for CRMT investment, so that the amount of capital good used for production is given by $(1-i)K_h$. The optimization problem of the risky firm is in real terms then given by

$$\max_{\substack{K_h \geq 0, \\ i \in [0,1]}} \mathbb{E}_p[A_{h,s}(i)((1-i)K_h)^\alpha - (1+r^L_{h,s})qK_h], \tag{16}$$

where $A_{h,s}(i)$ represents the total factor productivity in scenario s that now depends on the CRMT investment. The risky firm demands an optimal amount of capital good if the marginal productivity equals the repayment obligation per unit of capital good, i.e., if it holds that $\alpha\mathbb{E}_p[A_{h,s}(i)](1-i)^\alpha K_h^{\alpha-1} = (1+\mathbb{E}_p[r^L_{h,s}])q$. The firm chooses the share of capital good invested into CRMT optimally if the expected return from investment is maximized, i.e., if it holds that $\partial(\mathbb{E}[A_{h,s}(i)](1-i)^\alpha)/\partial i = 0$.

For the subsequent analysis, we make specific assumptions on the CRMT investment. First, CRMT investment does not affect the productivity in the business as usual scenario. Second, the marginal effect of CRMT investment on productivity in the transition scenario scales with the expected productivity.

Assumption 2 (CRMT)
$\partial A_{h,b}(i)/\partial i = 0$ and $\partial A_{h,t}(i)/\partial i = \mathbb{E}_p[A_{h,s}(i)]\beta(1-i)^{\beta-1}$, where $\beta > 0$.

The following lemma outlines the optimal choice of the risky firm, namely the demand of capital good K_h and the share i of capital good devoted to CRMT investment.

Lemma 8 (Optimal Choice of the Risky Firm with CRMT Investment)
The optimal demand of capital good and the optimal CRMT investment by the risky firm are given by

$$K_h = \left[\frac{\alpha\mathbb{E}_p[A_{h,s}(i)](1-i)^\alpha}{(1+\mathbb{E}_p[r^L_{h,s}])q}\right]^{\frac{1}{1-\alpha}} \qquad \textit{and} \qquad i = \max\left\{1-\left(\frac{\alpha}{\eta_p\beta}\right)^{\frac{1}{\beta}}, 0\right\}.$$

Note that the share i increases with the probability η_p associated by private agents to the transition and the CRMT parameter β, whereas it decreases with the capital intensity α. CRMT investment only affects the productivity in the transition scenario (see assumption 2). Thus, a higher likelihood for the transition, as given by the probability η_p, incentivizes firms to increase CRMT investment. A higher β increases the marginal return from CRMT investment, so that firms are incentivized to devote more resources to it in terms of capital good (i.e., i is increasing). In turn, a higher capital intensity α increases the marginal return from production, so that firms optimally invest less into CRMT and produce more.

We now outline the capital allocation in the decentralized economy with CRMT investment by risky firms. The share of capital good allocated to riskless firms is similar to the one in our baseline model, as it depends on the relative expected productivity in both sectors and the cost factors applied by the central bank. The only difference is represented by the impact of

CRMT investment on the total factor productivity in the risky sector. Formally, the expected productivity of risky firms $\mathbb{E}_p[A_{h,s}]$ is now replaced by the term $\mathbb{E}_p[A_{h,s}(i)](1-i)^{\alpha}$. While CRMT investment increases the expected total factor productivity by reducing the exposure to risk, it reduces the amount of capital good available for production to $(1-i)K_h$.

Proposition 3 (Capital Allocation with CRMT Investment)
The share of capital good allocated to riskless firms is given by

$$\zeta = \left[1 + \left(\frac{\mathbb{E}_p[A_{h,s}(i)](1-i)^{\alpha}}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)\right]^{-1}.$$

The optimal monetary policy chosen by the central bank is similar to the one outlined in proposition 2. In fact, the optimal cost factors and the resulting capital allocation have the same structure as before. However, the capital allocation, which from the goverment's perspective is optimal, and the central bank intervention now depend also on the CRMT investment by risky firms.

Proposition 4 (Optimal Monetary Policy with CRMT Investment)
The central bank optimally chooses cost factors κ_l and κ_h such that

$$\kappa_h = a(i)\kappa_l + \frac{a(i)-1}{\psi} \qquad \text{and} \qquad \zeta_g\kappa_l + (1-\zeta_g)\kappa_h \geq 0,$$

where

$$a(i) = \frac{\mathbb{E}_p[A_{h,s}(i)]}{\mathbb{E}_g[A_{h,s}(i)]} \qquad \text{and} \qquad \zeta_g = \left[1 + \left(\frac{\mathbb{E}_g[A_{h,s}(i)](1-i)^{\alpha}}{A_l}\right)\right]^{-1}.$$

If $\eta_g > (<)\eta_p$, it follows $a(i) > (<)1$ and therefore $\kappa_h > (<)\kappa_l$.

Finally, we are interested in the effect of CRMT investment on the intensity of central bank intervention, as measured by the absolute difference between the cost factors $|\kappa_h - \kappa_l|$. If risky firms devote a larger share of capital good to CRMT, their expected total factor productivity increases, i.e., $\mathbb{E}_p[A_{h,s}(i)]$ increases with i. We can then deduce that if the government assigns a higher (lower) probability to the transition than private agents, i.e., if $\eta_g > (<)\eta_p$, the policy parameter $a(i)$ decreases (increases) with the share i of capital good devoted to CRMT investment.

From proposition 4, we know that the policy parameter $a(i)$ is larger (smaller) than one if beliefs satisfy $\eta_g > (<)\eta_p$. We can conclude that, independent of the beliefs of private agents and the government, the policy parameter $a(i)$ is moving closer to one with increasing CRMT investment (i.e., risky firms choose a larger i). Accordingly, the intensity of central bank intervention, as measured by $|\kappa_h - \kappa_l|$, is always decreasing with CRMT investment.

Corollary 4 (Optimal Monetary Policy and CRMT Investment)
For beliefs satisfying $\eta_g > (<)\eta_p$, it holds that $\partial a(i)/\partial i < (>)0$. If the central bank chooses the monetary policy according to proposition 4, CRMT investment always reduces the intensity of

central bank intervention, as measured by the absolute difference between cost factors $|\kappa_h - \kappa_l|$.

If the government assigns a higher (lower) probability to the transition than private agents, i.e., if beliefs satisfy $\eta_g > (<)\eta_p$, the share of capital good devoted to CRMT investment in the decentralized equilibrium is lower (higher) than the government believes to be optimal, i.e.,

$$i_g = 1 - \left(\frac{\alpha}{\eta_g \beta}\right)^{\frac{1}{\beta}} > (<) i_p = 1 - \left(\frac{\alpha}{\eta_p \beta}\right)^{\frac{1}{\beta}}.$$

An appropriate subsidy (tax) on CRMT investment can incentivize risky firms to use a share i_g of capital good for CRMT investment and, ultimately, reduce the need for the central bank to intervene.

6 Bank Recapitalization

In this section, we extend our baseline model by allowing for costs arising from bank recapitalization. The latter represents a proxy for financial instability in our framework.

Banking operations are generally risky as loan repayment is uncertain but the costs arising from interest payments on deposits and reserve borrowing at the central bank are deterministic. Specifically, deposit contracts cannot be conditioned on the prevailing scenario, i.e., whether the economy remains in the business as usual or shifts to low-carbon activities. As banks operate with unlimited liability, the households, which are the only shareholders of banks in our model, may be required to inject new equity whenever the initial equity financing has been wiped out. We refer to this process as “bank recapitalization”.

Maximum leverage without bank recapitalization. Formally, the bank experiences losses if the leverage φ is sufficiently large and loan repayment of risky firms in the transition scenario, $s = t$, is not sufficient for the bank to meet the promises towards depositors and the central bank. Due to perfect competition, the bank is, in equilibrium, making zero expected profits from granting loans to firms funded with deposits. Accordingly, the equity return in the business as usual scenario can never be negative, i.e., in the business as usual scenario, bank recapitalization cannot occur. For interest rates satisfying $r_l^L \zeta + r_{h,t}^L (1-\zeta) < r_{CB}^D \Psi(\zeta)$ in the transition scenario, the maximum leverage $\varphi^S(\zeta)$ without bank recapitalization, is determined by setting the equity return to zero, i.e., $\varphi^S(\zeta)$ satisfies $1 + r_t^E(\varphi^S(\zeta), \zeta) = 0$. Using equation (12) to express the equity rate of return, the latter condition reads as

$$(1+\psi)^{-1}[r_l^L \zeta + r_{h,t}^L (1-\zeta) - r_{CB}^D \Psi(\zeta)]\varphi^S(\zeta) + 1 + r_{CB}^D = 0,$$

so that

$$\varphi^S(\zeta) = \frac{(1 + r_{CB}^D)(1+\psi)}{r_{CB}^D \Psi(\zeta) - r_l^L \zeta - r_{h,t}^L (1-\zeta)}. \tag{17}$$

Using the previous results on the equilibrium loan rates (see corollary 1), and the link between loan returns and firm productivity (see subsection 1), we can express the leverage ratio $\varphi^S(\zeta)$ using economic fundamentals, as provided in the following lemma.

Lemma 9 (Maximum Leverage without Bank Recapitalization)
The maximum leverage, ruling out bank recapitalization, is given by

$$\varphi^S(\zeta) = \frac{\mathbb{E}_p[A_{h,s}](1+\psi)}{(1+\psi\kappa_h)(1-\eta_p)(A_{h,b}-A_{h,t})(1-\zeta)}. \tag{18}$$

Costs of bank recapitalization. New equity injections have real costs, as they require negotiation and organization with shareholders. These costs are not internalized by shareholders, which are households in our model, and by banks. The costs of recapitalization scale with the amount of loans granted to the risky sector, as these ultimately cause the costly bank recapitalization. The aggregate costs in terms of the consumption good are given by

$$\lambda(1+r^L_{h,t})qK_h = \lambda\alpha A_{h,t}K_h^\alpha = \lambda\alpha A_{h,t}(1-\zeta)^\alpha K^\alpha =: \Lambda(\zeta), \tag{19}$$

where we used assumption 1, stating $(1+r^L_{h,s})q = \alpha A_{h,s}K_h^{\alpha-1}$ for all s, and the equilibrium allocation of capital as derived in subsection 4.2, leading to $K_h = (1-\zeta)K$. The parameter $\lambda \in (0, \overline{\lambda})$ is solely used for scaling purposes. We assume that the costs of bank recapitalization cannot exceed the output of the risky sector, as expected under government beliefs, i.e., $\Lambda(\zeta) < \mathbb{E}_g[A_{h,s}](1-\zeta)^\alpha K^\alpha$. Otherwise, the central bank would find it never optimal to allow for production by the risky sector in the presence of bank recapitalization. Thus, there also exists an upper bound for the parameter λ that is determined by

$$\overline{\lambda}\alpha A_{h,t}(1-\zeta)^\alpha K^\alpha = \mathbb{E}_g[A_{h,s}](1-\zeta)^\alpha K^\alpha \qquad \Leftrightarrow \qquad \overline{\lambda} = \frac{\mathbb{E}_g[A_{h,s}]}{\alpha A_{h,t}}.$$

Welfare. Due to our assumption of linear utility for the household, welfare comprises aggregate consumption and, in the case of bank recapitalization, also the costs associated with new equity injections. As the scenario business as usual versus transition affects the productivity in the risky sector and potentially leads to bank recapitalization, welfare is given by

$$W_s^\lambda = C_s - \Lambda(\zeta)\mathbb{1}\{\varphi > \varphi^S(\zeta) \wedge s = t\}.$$

The costs $\Lambda(\zeta)$ due to new equity injections arise only if the bank chooses a leverage that exposes it to a solvency risk, i.e., $\varphi > \varphi^S(\zeta)$, and if indeed more stringent environmental regulations are put in place, i.e., $s = t$. The following lemma provides a characterization of welfare in terms of model primitives.

Lemma 10 (Welfare with Bank Recapitalization)
Equilibrium welfare is $W_s^\lambda = \{A_l\zeta^\alpha + A_{h,s}(1-\zeta)^\alpha[1-\lambda\alpha\mathbb{1}\{\varphi > \varphi^S(\zeta) \wedge s = t\}]\}K^\alpha$.

We now discuss the impact of monetary policy and beliefs on bank recapitalization. First, note that the equilibrium capital allocation, as captured by the share ζ of capital good allocated to riskless firms and outlined in proposition 1, depends on the cost factors κ_l and κ_h, which are chosen by the central bank. It then follows from lemma 9 that the maximum leverage which rules out bank recapitalization also depends on these cost factors, both directly and indirectly, via the capital allocation ζ. Moreover, the costs of bank recapitalization $\Lambda(\zeta)$ also depend, through the capital allocation, on the costs factors chosen by the central bank. Accordingly, in its choice of the cost factors, the central bank must account for the effect of its policy on the capital allocation as well as on the occurrence and the associated costs of bank recapitalization.

In proposition 5, we provide the comparative statics on the maximum leverage ruling out bank recapitalization with respect to the monetary policy. We find that the leverage threshold $\varphi^S(\zeta)$ always decreases with an increasing cost factor κ_l on loans to the riskless sector. With an increasing κ_l and a fixed κ_h, the loan financing conditions for riskless firms worsen compared to the one for risky firms, leading to a larger share of loans to the risky sector within banks' portfolio in equilibrium. Banks are therefore exposed to more risk, so that the critical leverage threshold $\varphi^S(\zeta)$ ruling out bank recapitalization decreases. In addition, we find that the same leverage threshold increases with the cost factor κ_h on loans to the risky sector only if a sufficiently large share of capital is already allocated to the riskless sector, i.e., $\zeta \geq 1-\alpha$. An increasing κ_h and a fixed κ_l, lead to a worsening of loan financing conditions for risky firms, compared to riskless firms, so that in equilibrium, the latter receive even more funds from banks. This, in turn, reduces the risk exposure of banks, resulting in a higher maximum leverage $\varphi^S(\zeta)$ that rules out bank recapitalization. Finally, we show that for cost factors satisfying $\kappa_l \to -1/\psi$ or $\kappa_h \to +\infty$, banks are not facing recapitalization, i.e., the maximum leverage $\varphi^S(\zeta)$ is approaching infinity.

We also provide comparative statics on the costs of bank recapitalization with respect to the monetary policy in the form of the costs factors κ_l and κ_h. An increase in κ_l (κ_h) leads to a higher (lower) share of capital allocated to risky firms and thus to higher (lower) bank recapitalization costs $\Lambda(\zeta)$. For the extreme case, where $\kappa_l \to -1/\psi$ or $\kappa_h \to +\infty$, only riskless firms produce (i.e., $\zeta \to 1$), so that there are no costs of bank recapitalization.

Proposition 5 (Monetary Policy and Recapitalization)
The maximum leverage ruling out bank recapitalization varies with the monetary policy in the form of the cost factors κ_l and κ_h according to

$$\frac{\partial \varphi^S(\zeta)}{\partial \kappa_l} < 0, \qquad \textit{and} \qquad \frac{\partial \varphi^S(\zeta)}{\partial \kappa_h} < (\geq) 0 \qquad \textit{if and only if} \qquad \zeta < (\geq) 1-\alpha.$$

Moreover, it holds that $\lim_{\kappa_l \to -1/\psi} \varphi^S(\zeta) = \lim_{\kappa_h \to +\infty} \varphi^S(\zeta) = +\infty$.

The costs of bank recapitalization vary with the monetary policy in the form of the cost factors κ_l and κ_h according to

$$\frac{\partial \Lambda(\zeta)}{\partial \kappa_l} > 0 \qquad \textit{and} \qquad \frac{\partial \Lambda(\zeta)}{\partial \kappa_h} < 0.$$

Moreover, it holds that $\lim_{\kappa_l \to -1/\psi} \Lambda(\zeta) = \lim_{\kappa_h \to +\infty} \Lambda(\zeta) = 0$.

The occurrence of bank recapitalization and the associated costs also depend on the beliefs of private agents. Specifically, the higher the probability η_p that agents attach to the transition, the higher the maximum leverage $\varphi^S(\zeta)$ ruling out bank recapitalization and the lower the costs $\Lambda(\zeta)$ in the case of bank recapitalization. The intuition behind this result is that the more agents believe that the transition will occur, the lower the expected productivity of risky firms. In equilibrium, this leads to more production by riskless firms and thus to more loan financing to the riskless sector. Banks therefore become safer, so that bank recapitalization occurs only at a higher leverage, i.e., $\varphi^S(\zeta)$ is increasing with η_p. Since the costs of bank recapitalization $\Lambda(\zeta)$ scale with the amount of loans granted to the risky sector, the belief-driven increase in production of riskless firms also decreases bank recapitalization costs.

Proposition 6 (Beliefs and Recapitalization)
The maximum leverage $\varphi^S(\zeta)$ ruling out bank recapitalization increases with the beliefs η_p of private agents, whereas the bank recapitalization costs $\Lambda(\zeta)$ decrease with the beliefs η_p of private agents, i.e.,

$$\frac{\partial \varphi^S(\zeta)}{\partial \eta_p} > 0 \qquad \textit{and} \qquad \frac{\partial \Lambda(\zeta)}{\partial \eta_p} < 0.$$

Optimal Monetary Policy. As in section 3, the central bank aims at maximizing expected welfare by choosing the cost factors κ_l and κ_h. The neutrality of money with regard to the interest rate policy of the central bank still applies. Specifically, the interest rate r^D_{CB} on reserve deposits does not affect the real allocation and thus welfare, but only prices (see corollary 2 and lemma 10). We showed that the beliefs of private agents affect the capital allocation as well as the occurrence and costs of bank recapitalization. In its choice of the monetary policy, the central bank thus generally faces two externalities following from private agents' beliefs. First, from a central bank perspective, beliefs of private agents lead to a distortion of the capital allocation, such that one of the sectors receives, without central bank intervention, more capital good for production than it would receive under the government's beliefs. Second, private agents' beliefs can trigger bank recapitalization and reduce welfare by inducing costly equity injections, or, if bank recapitalization also exist under the government's beliefs, private agents' distorted beliefs can lead to an increase of such costs. The central bank can use the cost factors κ_l and κ_h to steer the capital allocation in the economy and thereby aim at eliminating the previously mentioned two externalities arising from agents assessing the likelihood of the transition differently from the government. However, due to the constraint on the cost factors, namely that liquidity must remain costly for banks, the monetary policy may not always be able to eliminate both externalities. In fact, the central bank faces generally a trade-off between reducing capital distortions and ruling out bank recapitalization. In subsection 6.1, we show that once the restriction on the cost factors is relaxed and once an additional central bank tool in the form of quantity restrictions for reserve loans is introduced, the monetary policy can always eliminate both externalities following from the belief difference between private agents and the government.

We can distinguish three regimes for the optimal monetary policy. In the first regime, bank recapitalization does not occur under the capital allocation induced by government beliefs and no central bank intervention, as captured by the share ζ_g. Then, the optimal monetary policy only corrects for the impact of private agents' beliefs on the capital allocation, so that the optimal central bank policy is characterized by proposition 2. However, in an economy where bank recapitalization is costly, the central bank always has incentives to choose cost factors κ_l and κ_h, not only to induce the capital allocation ζ_g, but also to maximize the leverage threshold $\varphi^S(\zeta_g)$ ruling out bank recapitalization. The highest leverage threshold $\varphi^S(\zeta_g)$ is obtained by minimizing liquidity costs for bank, as the lower the costs for borrowing reserves at the central bank, the lower the financing costs per unit of loans funded with deposits, as measured by $r^D_{CB}\Psi(\zeta_g) = r^D_{CB}[1+\zeta_g\kappa_l+(1-\zeta_g)\kappa_h]$. Accordingly, the leverage threshold $\varphi^S(\zeta_g)$ is maximized for costs factors satisfying $\zeta_g\kappa_l + (1-\zeta_g)\kappa_h = 0$.[10]

Proposition 7 (Optimal Monetary Policy without Recapitalization)
The optimal monetary policy follows proposition 2 with $\zeta_g\kappa_l+(1-\zeta_g)\kappa_h = 0$, if there is no bank recapitalization under the allocation ζ_g, i.e., $\varphi = (1+\psi)/\gamma \leq \varphi^S(\zeta_g)$.

Now suppose that under a monetary policy foreseeing cost factors κ_l and κ_h, which induce the capital allocation ζ_g and minimize liquidity costs as $\zeta_g\kappa_l + (1-\zeta_g)\kappa_h = 0$, banks are in the transition exposed to recapitalization, i.e., $\varphi = (1+\psi)/\gamma > \varphi^S(\zeta_g)$. Then, the central bank must decide between the second and third monetary policy regime. In the second regime, the central bank implements cost factors $\hat{\kappa}_l$ and $\hat{\kappa}_h$, that lead to a capital allocation $\hat{\zeta}$, which rules out recapitalization of banks, i.e., $\varphi = (1+\psi)/\gamma = \varphi^S(\hat{\zeta})$. From proposition 5, we know that there always exists such cost factors that sufficiently discourage loan financing to risky firms, compared to loan financing to riskless firms, in order to make banks safer and rule out recapitalization in the transition. Specifically, the required allocation $\hat{\zeta}$ to rule out bank recapitalization satisfies $\hat{\zeta} > \zeta_g$. Moreover, we can show that it is optimal for the central bank to also minimize liquidity costs, i.e., $\hat{\zeta}\hat{\kappa}_l + (1-\hat{\zeta})\hat{\kappa}_h = 0$, as this leads to the smallest possible distortion in the capital allocation. In other words, allowing for positive liquidity costs would require the central bank to induce, through the choice of the cost factors, a larger shift of capital towards riskless firms. Since without bank recapitalization, welfare is maximized for the capital allocation ζ_g, a greater distortion away from ζ_g cannot be optimal.

Lemma 11 (Monetary Policy Ruling Out Bank Recapitalization)
Suppose that for cost factors κ_l and κ_h inducing ζ_g (see proposition 2) and satisfying $\zeta_g\kappa_l+(1-\zeta_g)\kappa_h = 0$, it holds that $\varphi = (1+\psi)/\gamma > \varphi^S(\zeta_g)$. Then, there exist costs factors $\hat{\kappa}_l$ and $\hat{\kappa}_h$, with $\hat{\zeta}\hat{\kappa}_l + (1-\hat{\zeta})\hat{\kappa}_h = 0$, that implement the capital allocation $\hat{\zeta}$ satisfying $\varphi = (1+\psi)/\gamma = \varphi^S(\hat{\zeta})$.

In the third monetary policy regime, the central bank chooses to accept bank recapitalization in the transition scenario but corrects for the belief-driven distortion of the capital allocation. The rule for the optimal cost factors is similar to the one in proposition 2. However, the central bank must now account for the costly bank recapitalization, which only arises due to loan financing to the risky sector. From the central bank's perspective, the expected productivity of the risky sector must be adjusted for the costs associated with new equity injections in the transition. It is therefore lower than without bank recapitalization. We use the notation

[10] In section 3, bank recapitalization was frictionless, so that the maximum leverage $\varphi^S(\zeta)$ and the effect of liquidity costs on $\varphi^S(\zeta)$ were irrelevant. Thus, the optimal monetary policy, outlined in proposition 2, allowed for any positive spread between deposit rates, i.e., $\zeta_g\kappa_l + (1-\zeta_g)\kappa_h \geq 0$.

$\mathbb{E}_g^\lambda[A_{h,s}] := \mathbb{E}_g[A_{h,s}] - \eta_g \lambda \alpha A_{h,t}$ to represent the productivity in the risky sector, as expected under government beliefs and taking the costs of recapitalization of banks into account. As a result, the policy parameter $a_\lambda = \mathbb{E}_p[A_{h,s}]/\mathbb{E}_g^\lambda[A_{h,s}]$ depends on the recapitalization costs and is thus indexed by λ.

The central bank decides between the second and third regime, depending on which one yields the highest expected welfare. Formally, the central bank then prefers the second regime, ruling out bank recapitalization, over the third regime, accepting bank recapitalization and correcting the capital allocation, if it holds that

$$\begin{aligned} & \mathbb{E}_g[W_s^\lambda(\hat{\zeta})] = \mathbb{E}_g[W_s(\hat{\zeta})] \geq \mathbb{E}_g[W_s^\lambda(\zeta_g^\lambda)] \\ \Leftrightarrow \qquad & A_l[(\hat{\zeta})^\alpha - (\zeta_g^\lambda)^\alpha] \geq \mathbb{E}_g^\lambda[A_{h,s}](1-\zeta_g^\lambda)^\alpha - \mathbb{E}_g[A_{h,s}](1-\hat{\zeta})^\alpha. \end{aligned}$$

Of course, if it holds that $\zeta_g^\lambda > \hat{\zeta}$, expected welfare under the third regime—accepting bank recapitalization and correcting the belief-driven capital distortion—can never be higher than expected welfare under the second regime—ruling out bank recapitalization, i.e., if it holds that $\mathbb{E}_g[W_s^\lambda(\zeta_g^\lambda)] < \mathbb{E}_g[W_s(\hat{\zeta})]$. The details of the third monetary policy regime are provided in the following proposition.

Proposition 8 (Optimal Monetary Policy with Bank Recapitalization)
Suppose that for cost factors κ_l and κ_h inducing ζ_g (see proposition 2) and satisfying $\zeta_g \kappa_l + (1-\zeta_g)\kappa_h = 0$, it holds that $\varphi = (1+\psi)/\gamma > \varphi^S(\zeta_g)$. Then, with $\mathbb{E}_g[W_s^\lambda(\zeta_g^\lambda)] > \mathbb{E}_g[W_s(\hat{\zeta})]$, the central bank optimally chooses cost factors κ_l and κ_h such that

$$\kappa_h = a_\lambda \kappa_l + \frac{a_\lambda - 1}{\psi} \qquad \textit{and} \qquad \zeta_g^\lambda \kappa_l + (1-\zeta_g^\lambda)\kappa_h \geq 0,$$

where

$$a_\lambda = \frac{\mathbb{E}_p[A_{h,s}]}{\mathbb{E}_g^\lambda[A_{h,s}]} \qquad \textit{and} \qquad \zeta_g^\lambda = \left[1 + \left(\frac{\mathbb{E}_g^\lambda[A_{h,s}]}{A_l}\right)^{\frac{1}{1-\alpha}}\right]^{-1}.$$

Otherwise, i.e., $\mathbb{E}_g[W_s(\hat{\zeta})] \geq \mathbb{E}_g[W_s^\lambda(\zeta_g^\lambda)]$, the central bank implements cost factors $\hat{\kappa}_l$ and $\hat{\kappa}_h$ that satisfy $\hat{\zeta}\hat{\kappa}_l + (1-\hat{\zeta})\hat{\kappa}_h = 0$ and $\varphi = (1+\psi)/\gamma = \varphi^S(\hat{\zeta})$.

6.1 Quantity restrictions on reserve loans

In our previous analysis of the optimal monetary policy, we imposed that liquidity must always remain costly for banks, i.e., $\zeta\kappa_l + (1-\zeta)\kappa_h \geq 0$, in order to avoid arbitrage opportunities. For the optimal monetary policy, the latter constraint is binding, so that liquidity costs for banks are minimized. This, in turn, reduces the monetary policy instruments to essentially one cost factor, either κ_l or κ_h, as they are co-linear due to the binding constraint on liquidity costs. As a consequence, the central bank may not be able to fully eliminate both externalities following from the beliefs of private agents. If we remove the constraint on liquidity costs, the central bank has two independent instruments, which allow it to always correct for belief-driven capital distortions and avoid bank recapitalization. However, in some situations, reserve borrowing may become profitable for banks, as the optimal cost factors satisfy $\kappa_l < 0$ and $\kappa_h < 0$. To prevent arbitrage opportunities for banks, the central bank must limit the amount of reserves that the individual bank can borrow. In what follows, we denote the maximum amount of reserve loans by $\overline{L}^{CB}$. The central bank can then always fully eliminate both externalities, namely the capital distortion and the occurrence of bank recapitalization. The central bank optimally chooses cost factors, which on the one hand implement the capital allocation ζ_g—which from a central bank perspective is the optimal allocation—and, on the other hand, rules out bank recapitalization, i.e., cost factors are chosen such that $\varphi = (1+\psi)/\gamma = \varphi^S(\zeta_g)$. The following proposition outlines the optimal monetary policy without the constraint of costly liquidity and with quantity restrictions on reserve loans. It also provides the necessary and sufficient conditions under which the quantity restriction on reserve loans is indeed effective, as captured by inequality (20) in proposition 9.

Proposition 9 (Optimal Monetary Policy with Restrictions on Reserves)
The central bank optimally chooses cost factors κ_l and κ_h such that

$$\kappa_l = \frac{\mathbb{E}_g[A_{h,s}]\gamma}{\psi(1-\eta_p)(A_{h,b}-A_{h,t})(1-\zeta_g)} - \frac{1}{\psi}$$

and

$$\kappa_h = \frac{\mathbb{E}_p[A_{h,s}]\gamma}{\psi(1-\eta_p)(A_{h,b}-A_{h,t})(1-\zeta_g)} - \frac{1}{\psi}.$$

The amount of reserve loans must be restricted to the maximum $\overline{L}^{CB} = \psi QK$ if and only if liquidity is not costly, i.e., $\zeta_g\kappa_l + (1-\zeta_g)\kappa_h < 0$ or, equivalently,

$$\frac{\zeta_g}{1-\zeta_g} < \frac{(1-\eta_p)(A_{h,b}-A_{h,t})(1-\gamma) - A_{h,t}\gamma}{\mathbb{E}_g[A_{h,s}]\gamma}. \tag{20}$$

The central bank must implement quantity restrictions on reserve loans if liquidity is not priced in a way that it is costly for banks. From inequality (20) in proposition 9, it follows that this is the case if, for instance, the share ζ_g of capital good received by riskless firms or the share γ of funds used by investors for equity financing are sufficiently small. In both cases, banks are highly risky and incur large losses in the transition, requiring the central bank to provide a subsidy to banks by allowing them to generate profits through reserve borrowing, i.e., $\zeta_g\kappa_l + (1-\zeta_g)\kappa_h < 0$. If the transition realizes, these profits are sufficient for banks to compensate the losses originating from loan financing to the risky sector, in a way that bank recapitalization is ruled out. Distributing these implicit subsidies is welfare improving, as it avoids new equity injections, whose costs are not internalized by households and banks.

Note that, in the presence of quantity restriction on reserve loans, the optimal cost factors κ_l and κ_h, as chosen by the central bank, increase if agents' beliefs about the transition, captured by the probability η_p, are growing. Formally, it holds that

$$\frac{\partial \kappa_l}{\partial \eta_p} = \frac{\mathbb{E}_g[A_{h,s}]\gamma}{\psi(1-\eta_p)^2(A_{h,b}-A_{h,t})(1-\zeta_g)} > 0$$

and

$$\frac{\partial \kappa_h}{\partial \eta_p} = \frac{A_{h,t}\gamma}{\psi(1-\eta_p)^2(A_{h,b}-A_{h,t})(1-\zeta_g)} > 0.$$

However, note that, for an increasing probability η_p, the cost factor κ_l for loans to riskless firms increases more than the cost factor κ_h on loans to risky firms, i.e., it holds that

$$\frac{\partial \kappa_l}{\partial \eta_p} > \frac{\partial \kappa_h}{\partial \eta_p}.$$

Thus, if agents' beliefs about transitioning to a low-carbon economy grow, loan financing to the risky sector is discouraged less than before, under the optimal monetary policy, relative to loan financing to the riskless sector.

7 Targets

In this section, we look at the possibility for the central bank to implement a target allocation of loans and, ultimately, of production input in the form of the capital good in the economy. We denote this target allocation by the share $\zeta_t \in (0,1)$. Such a target can, for example, be derived from a policy coherence argument according to which the central bank aims at contributing to the transition to a low-carbon economy. An alternative interpretation is that the central bank aims at mitigating climate risk and the desired level of climate risk is achieved through the target allocation ζ_t.

For the subsequent analysis, we assume that the central bank deviates from its welfare-maximizing objective and solely cares about implementing the target allocation. The central bank achieves this goal by choosing the appropriate cost factors κ_l and κ_h, while keeping liquidity generally costly for banks, i.e. $\zeta_t\kappa_l + (1-\zeta_t)\kappa_h \geq 0$. Proposition 10 outlines the optimal monetary policy in the form of the cost factors κ_l and κ_h that lead to the target allocation ζ_t.

Proposition 10 (Optimal Monetary Policy with a Target)
The central bank optimally chooses the cost factors κ_l and κ_h such that

$$\kappa_h = a_t\kappa_l + \frac{a_t - 1}{\psi} \qquad \text{and} \qquad \zeta_t\kappa_l + (1-\zeta_t)\kappa_h \geq 0,$$

where it holds that

$$a_t = \frac{\mathbb{E}_p[A_{h,s}]}{A_l}\left(\frac{\zeta_t}{1-\zeta_t}\right)^{1-\alpha}.$$

If $\zeta_t > (<)\zeta_p$, it follows that $a_t > (<)1$ and therefore that $\kappa_h > (<)\kappa_l$.

First, we focus on the case where under the target allocation, more capital is shifted to riskless firms than under the beliefs of private agents and without central bank intervention ($\kappa_l = \kappa_h$), so that it holds that $\zeta_t > \zeta_p$. Then, the central bank must implement cost factors that satisfy $\kappa_h > \kappa_l$, so that risky firms face relatively worse loan financing conditions, compared to riskless firms. Second, if the central bank aims at a target allocation that foresees less capital for riskless firms than in the decentralized equilibrium without central bank intervention, i.e., $\zeta_t < \zeta_p$, the optimal cost factors satisfy $\kappa_l > \kappa_h$. Such a monetary policy penalizes riskless firms, relative to risky firms, when demanding loans from banks.

The intensity of central bank intervention, as measured by the difference of costs factors $\kappa_h - \kappa_l$, is now influenced by the target allocation ζ_t and the beliefs η_p of private agents. In contrast to section 3, the beliefs η_g of the government sector do not play a role anymore. Whenever the central bank sets a target $\zeta_t > \zeta_p$, the cost factors satisfy $\kappa_h - \kappa_l > 0$, and the intensity of central bank intervention, as measured by the absolute difference between the cost factors $|\kappa_h - \kappa_l|$, increases with ζ_t and decreases with η_p. In turn, if the target allocation satisfies $\zeta_t < \zeta_p$, the cost factors satisfy $\kappa_h - \kappa_l < 0$, and the intensity of central bank intervention decreases with ζ_t and increases with η_p.

Corollary 5 (Optimal Monetary Policy, Beliefs and Targets)
If the central bank chooses the monetary policy according to proposition 10, the difference between the optimal cost factors $\kappa_h - \kappa_l$ increases with the target ζ_t of the central bank and decreases with the belief η_p of private agents.

8 Discussion

As an alternative to the loan rate on reserves varying with the climate risk exposure of banks' asset holdings, we could also allow for a deposit rate on reserves that varies with banks' asset allocation. Both approaches yield the same result in our model. Formally, setting a constant interest rate r^D_{CB} on reserve deposits and choosing the cost factors κ_l and κ_h, such that the loan rate on reserves satisfies $r^L_{CB}(\zeta) = r^D_{CB}[1 + \zeta\tilde{\kappa}_l + (1-\zeta)\tilde{\kappa}_h]$, is equivalent to setting a constant interest rate r^L_{CB} on reserve loans and choosing the cost factors κ_l and κ_h such that the deposit rate on reserves is given by $r^D_{CB}(\zeta) = r^L_{CB}[1 - \zeta\tilde{\kappa}_l - (1-\zeta)\tilde{\kappa}_h]$.

The latter approach may be particularly relevent when banks holds large amounts of reserves without borrowing from the central bank. In such situations, banks may face no or only a small demand for reserve loans from the central bank, as liquidity in the form of central bank reserves is relatively abundant. Then, only the deposit rate on reserves, but not the loan rate on reserves, is the relevant policy instrument. In many countries banks currently hold large amounts of reserves, that are not matched with loans from the central bank. This situation is a consequence of the expansionary monetary policy central banks adopted in the aftermath of the financial crisis in 2007/08 and in the current Covid-19 crisis. Due to large scale asset purchases by central banks, so-called "quantitative easing", banks acquired tremendous amounts of reserves. Liquidity seems to be abundant and there is no or only little need to approach central banks for reserve borrowing. Accordingly, the loan rate on reserves is of minor relevance and the deposit rate on reserves emerged as key interest rate.

9 Conclusion

It has been argued that financial market participants fail to adequately account for climate risk and thereby contribute to a mispricing of assets, which leads to a misallocation of resources, a build-up of financial risks and potentially even to financial instability. There is an ongoing debate on to which extent central banks can and should intervene by adopting a climate-oriented monetary policy to correct the existing market failure. Several monetary policy instruments taking climate considerations into account have been proposed. In this paper, we study the effect of a new concept, the climate risk-adjusted refinancing operations, in short CAROs, on resource allocation and financial stability.

We developed a static general equilibrium framework that allows us to study CAROs in environments with different beliefs between private agents and the government about the likelihood of the transition. From a central bank's perspective, without intervention, the different beliefs of private agents lead to a resource allocation in the decentralized equilibrium that is suboptimal. In our baseline model, we show that by using appropriate liquidity cost factors on loans to riskless and risky firms, the central bank can induce the allocation which is optimal under its beliefs.

We extend our baseline model by introducing climate risk mitigation technologies (CRMT), by accounting for financial stability concerns and by featuring climate-related allocation targets, following, for instance, from a policy coherence argument regarding fiscal policies. We find that CRMT investment decreases the need for the central bank to intervene, no matter the beliefs of private agents and the government. Accounting for financial stability concerns, beliefs of private agents lead to a second externality next to the distorted capital allocation from the central bank's perspective, as they trigger bank recapitalization or increase its costs. This generally leads to a trade-off for the central bank between correcting the capital allocation and eliminating bank recapitalization. However, we also show that if the central bank is equipped with an additional monetary policy instrument in the form of quantity restrictions on reserve loans, it can always resolve both belief-driven externalities. Finally, we show that CAROs can be used to achieve any target allocation in the economy, which might follow from a coherence argument with fiscal policies.

Our analysis is a first attempt to formally analyze central bank refinancing operations taking climate risk into account. Similar to CAROs, the pricing of central bank reserves can be conditioned on other characteristics of bank assets. In particular, if central bank operations should take climate considerations into account, other criteria may be used, such as emission intensity or a taxonomy. Our framework can also be extended along other dimensions. First, we did not account for capital accumulation—and potentially for other dynamics—as we focused on a static environment. Second, we used a classical setup without any price rigidities and thus cannot study how CAROs and the resulting economic effects are linked to inflation. Third, we restricted firms to relying on loans from banks and did not account for other sources of financing, such as from the financial markets. The investigation of these aspects is left to future research.

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A Appendix

Proof of Lemma 1. For the optimization problem of the riskless firm, which is given by (1), the first-order condition with respect to K_l is given by

$$\alpha A_l K_l^{\alpha-1} = (1 + r_l^L)q.$$

Rearranging then yields the optimal demand of the capital good by the riskless firm

$$K_l = \left[\frac{\alpha A_l}{(1 + r_l^L)q}\right]^{\frac{1}{1-\alpha}}.$$

■

Proof of Lemma 2. For the optimization problem of the risky firm, which is given by (3), the first-order condition with respect to K_h is given by

$$\alpha \mathbb{E}_p[A_{h,s}] K_h^{\alpha-1} = (1 + \mathbb{E}_p[r_{h,s}^L])q.$$

Rearranging then yields the optimal demand of the capital good by the risky firm

$$K_h = \left[\frac{\alpha \mathbb{E}_p[A_{h,s}]}{(1 + \mathbb{E}_p[r_{h,s}^L])q}\right]^{\frac{1}{1-\alpha}}.$$

■

Proof of Lemma 3. Due to the assumption of linear utility, the household maximizes expected consumption $\mathbb{E}_p[\{\gamma(1 + r_s^E) + (1 - \gamma)(1 + r^D)\}qK + \tau_s + \pi_s]$. Thus, its optimal choice is of knife-edge type, as the household holds the asset which yields the highest expected return. Specifically, the household invests only into bank equity, i.e., $\gamma = 1$, if the expected rate of return on equity strictly exceeds the interest rate on deposits, i.e., $\mathbb{E}_p[r_s^E] > r^D$, and only holds deposits, i.e., $\gamma = 0$, if the interest rate on deposits exceeds the expected equity rate of return, i.e., $\mathbb{E}_p[r_s^E] < r^D$. If the returns on bank equity and deposits equal, i.e., $\mathbb{E}_p[r_s^E] = r^D$, the household is indifferent, i.e., $\gamma \in [0, 1]$. ■

Proof of Lemma 4. Note that reserves can be borrowed from the central bank at an interest rate $r_{CB}^L(\zeta)$ and can be deposited at the central bank at an interest rate r_{CB}^D. The interest rate for interbank loans is given by $r_{IB}^L > 0$, whereas the interest rate on interbank deposits is given by r_{IB}^D. We assume that the bank cannot differentiate between deposits held by other banks and deposit from households and firms, so that it holds $r_{IB}^D = r^D$. Interbank loans are only demanded if $r_{IB}^L \leq r_{CB}^L(\zeta)$, whereas interbank deposits are only attractive to the bank if $r^D \geq r_{CB}^D$. Otherwise, the bank would only deposit at the central bank. The liquidity provided on the interbank market through loans L^{IB} to other banks are matched by interbank deposits D^{IB} held by the borrowing banks. Thus, it holds $L^{IB} = D^{IB}$. Interbank deposits are fully withdrawn by the borrowing banks if the latter must settle deposit outflows due to transactions on the capital good market. The lending bank must settle the outflow of interbank deposits by using reserves in the amount $D^{CB} = D^{IB}$, which itself must borrow from the central bank by demanding loans L^{CB}. The revenues from interbank lending are given by $r^L_{IB} L^{IB}$, whereas the costs of interbank lending are given by $r^D D^{IB} + r^L_{CB}(\zeta) L^{CB} - r^D_{CB} D^{CB}$. Using $L^{IB} = D^{IB}$ and $L^{CB} = D^{CB} = D^{IB}$, the bank only offers interbank loans and deposits if

$$r^L_{IB} \geq r^D + r^L_{CB}(\zeta) - r^D_{CB} \qquad \Leftrightarrow \qquad r^D_{CB} - r^D \geq r^L_{CB}(\zeta) - r^L_{CB}.$$

Since the interbank market is active only if $r^D \geq r^D_{CB}$ and $r^L_{IB} \leq r^L_{CB}(\zeta)$, we can conclude that the interest rates satisfy $r^L_{IB} = r^L_{CB}(\zeta)$ and $r^D = r^D_{CB}$. ∎

Proof of Lemma 5. Note that the expected rate of return on bank equity is given by

$$\begin{aligned}\mathbb{E}_p[r^E_s(\varphi, \zeta)] &= \mathbb{E}_p[(1+\psi)^{-1}\{r^L_l \zeta + r^L_{h,s}(1-\zeta) - r^D_{CB}\Psi(\zeta)\}\varphi + r^D_{CB}] \\ &= (1+\psi)^{-1}\{(r^L_l - r^D_{CB}\psi\tilde{\kappa}_l)\zeta + (\mathbb{E}_p[r^L_{h,s}] - r^D_{CB}\psi\tilde{\kappa}_h)(1-\zeta) - r^D_{CB}\}\varphi + r^D_{CB},\end{aligned}$$

where we used the definition $\Psi(\zeta) = 1 + \psi[\zeta\tilde{\kappa}_l + (1-\zeta)\tilde{\kappa}_h]$. The equity rate of return is maximized for the maximum (minimum) possible leverage, i.e., $\varphi = \varphi^R$ ($\varphi = 1$), if the expected return per unit of loan financing, funded with deposits, is positive (negative), i.e., for some $\zeta \in [0, 1]$

$$(r^L_l - r^D_{CB}\psi\tilde{\kappa}_l)\zeta + (\mathbb{E}_p[r^L_{h,s}] - r^D_{CB}\psi\tilde{\kappa}_h)(1-\zeta) - r^D_{CB} > (<)0$$

or, equivalently,

$$\max\{r^L_l - r^D_{CB}\psi\tilde{\kappa}_l, \mathbb{E}_p[r^L_{h,s}] - r^D_{CB}\psi\tilde{\kappa}_h\} > (<) r^D_{CB}.$$

Otherwise, i.e., if $\max\{r^L_l - r^D_{CB}\psi\tilde{\kappa}_l, \mathbb{E}_p[r^L_{h,s}] - r^D_{CB}\psi\tilde{\kappa}_h\} = r^D_{CB}$, the bank is indifferent between any leverage, i.e., $\varphi \in [1, \varphi^R]$.

The bank optimally grants loan financing to the sector, which yields the highest expected return, taking the revenues from loan repayment and the costs from interest payments on deposits as well as from the borrowing of reserves at the central bank into account. That is, the bank optimally chooses to grant loans only to the riskless (risky) firms, i.e., $\zeta = 1$ ($\zeta = 0$), if

$$\begin{aligned}& r^L_l - r^D_{CB}(1+\psi\tilde{\kappa}_l) > (<)\mathbb{E}_p[r^L_{h,s}] - r^D_{CB}(1+\psi\tilde{\kappa}_h) \\ \Leftrightarrow \quad & r^L_l - r^D_{CB}\psi\tilde{\kappa}_l > (<)\mathbb{E}_p[r^L_{h,s}] - r^D_{CB}\psi\tilde{\kappa}_h.\end{aligned}$$

In all other cases, i.e., $r^L_l - r^D_{CB}\psi\tilde{\kappa}_l = \mathbb{E}_p[r^L_{h,s}] - r^D_{CB}\psi\tilde{\kappa}_h$, the bank is indifferent between loan financing to riskless and risky firms, i.e., $\zeta \in [0, 1]$. ∎

Proof of Lemma 6. According to lemma 5, it is optimal for the bank to grant loan financing to both sectors if it holds $r^L_l - r^D_{CB}(1+\psi\tilde{\kappa}_l) = \mathbb{E}_p[r^L_{h,s}] - r^D_{CB}(1+\psi\tilde{\kappa}_h)$. Using the definition of

$\tilde{\kappa}_l$ and $\tilde{\kappa}_h$, the latter inequality translates into

$$
\begin{aligned}
& r_l^L - r_{CB}^D + (1 + r_{CB}^D)\psi\kappa_l = \mathbb{E}_p[r_{h,s}^L] - r_{CB}^D + (1 + r_{CB}^D)\psi\kappa_h \\
\Leftrightarrow \quad & 1 + r_l^L - (1 + r_{CB}^D)(1 + \psi\kappa_l) = 1 + \mathbb{E}_p[r_{h,s}^L] - (1 + r_{CB}^D)(1 + \psi\kappa_h).
\end{aligned}
$$

Multiplying both sides of the inequality with the real capital good price $q = Q/P$, using the first-order condition of the riskless firm, i.e., $(1 + r_l^L)q = \alpha A_l K_l^{\alpha-1}$, and using assumption 1, i.e., $(1 + r_{h,s}^L)q = \alpha A_{h,s} K_h^{\alpha-1}$ for all s, it follows

$$
\alpha A_l K_l^{\alpha-1} - (1 + r_{CB}^D)(1 + \psi\kappa_l) = \alpha A_{h,s} K_h^{\alpha-1} - (1 + r_{CB}^D)(1 + \psi\kappa_h).
$$

Suppose the bank only grants loans $L_l = \epsilon$ to the riskless firm, so that due to the clearing of the capital good market, i.e., $K_h = K - \epsilon$, it holds that loan financing to risky firms is given by $L_h = QK_h = Q(K - \epsilon)$. As the capital allocation satisfies $K_l = \epsilon$ and $K_h = K - \epsilon$, we know that the left-hand side of the latter equation tends to infinity for ϵ approaching zero, while the right-hand side is finite. Granting only loans to the risky sector is not optimal for the bank, as, according to lemma 5, it should in such a situation only grant loans to the riskless sector. Similarly, the right-hand side converges to infinity for ϵ approaching K, while the left-hand side is finite. Granting only loans to the riskless sector is not optimal for the bank, as, according to lemma 5, it should in such a situation only grant loans to the risky sector. We can therefore conclude that in equilibrium it is never optimal for the bank to grant loan financing to only one sector. ■

Proof of Corollary 1. First, from lemma 6 we know that, in equilibrium, both riskless and risky firms demand loans, i.e., $\zeta \in (0, 1)$. Second, from lemma 5, we know that the bank is willing to grant loans to both types of firms if and only if the adjusted loan rates equal, i.e., $r_l^L - r_{CB}^D\psi\tilde{\kappa}_l = \mathbb{E}_p[r_{h,s}^L] - r_{CB}^D\psi\tilde{\kappa}_h$. Third, due to perfect competition among banks, financing loans with deposits must in equilibrium yield zero expected profits, i.e.,

$$
\begin{aligned}
& r_l^L\zeta + r_{h,s}^L(1 - \zeta) = r_{CB}^D\Psi(\zeta) \\
\Leftrightarrow \quad & (r_l^L - r_{CB}^D\psi\tilde{\kappa}_l)\zeta + (\mathbb{E}_p[r_{h,s}^L] - r_{CB}^D\psi\tilde{\kappa}_h)(1 - \zeta) = r_{CB}^D,
\end{aligned}
$$

where we used the definition $\Psi(\zeta) = 1 + \psi[\zeta\tilde{\kappa}_l + (1 - \zeta)\tilde{\kappa}_h]$. With equal adjusted loan rates, $r_l^L - r_{CB}^D\psi\tilde{\kappa}_l = \mathbb{E}_p[r_{h,s}^L] - r_{CB}^D\psi\tilde{\kappa}_h$, it follows that loan rates satisfy

$$
r_l^L - r_{CB}^D\psi\tilde{\kappa}_l = \mathbb{E}_p[r_{h,s}^L] - r_{CB}^D\psi\tilde{\kappa}_h = r_{CB}^D,
$$

ultimately leading to $r_l^L = r_{CB}^D(1 + \psi\tilde{\kappa}_l)$ and $\mathbb{E}_p[r_{h,s}^L] = r_{CB}^D(1 + \psi\tilde{\kappa}_h)$. ■

Proof of Corollary 2. Note that, from corollary 1, it follows that $\mathbb{E}_p[r_{h,s}^L] = r_{CB}^D(1 + \psi\tilde{\kappa}_h)$.

Using the definition of $\tilde{\kappa}_h$, the latter condition translates into

$$\mathbb{E}_p[r_{h,s}^L] = r_{CB}^D + (1 + r_{CB}^D)\psi\kappa_h \qquad \Leftrightarrow \qquad 1 + \mathbb{E}_p[r_{h,s}^L] = (1 + r_{CB}^D)(1 + \psi\kappa_h).$$

Multiplying both sides of the equation with the real capital good price q and using $(1+r_{h,s}^L)q = \alpha A_{h,s}K_h^{\alpha-1}$ for all s, the latter condition reads as

$$\alpha\mathbb{E}_p[A_{h,s}]K_h^{\alpha-1} = (1 + r_{CB}^D)(1 + \psi\kappa_h)q.$$

Using $q = Q/P$ it follows that prices P and Q must satisfy

$$\alpha\mathbb{E}_p[A_{h,s}]K_h^{\alpha-1} = \frac{Q}{P}(1 + r_{CB}^D)(1 + \psi\kappa_h) \qquad \Leftrightarrow \qquad \frac{P}{Q} = \frac{(1 + r_{CB}^D)(1 + \psi\kappa_h)}{\alpha\mathbb{E}_p[A_{h,s}]K_h^{\alpha-1}}.$$

■

Proof of Lemma 7. From subsection 4.2, we know that welfare is in scenario $s \in \{b, t\}$ generally given by $W_s = C_s$. Using the structure of the household's consumption, welfare reads

$$W_s = [\gamma(1 + r_s^E) + (1 - \gamma)(1 + r^D)]qK + \tau_s + \pi_s.$$

First, note that the rate of return on bank equity is, based on equation (12), given by

$$r_s^E(\varphi, \zeta) = (1 + \psi)^{-1}[r_l^L\zeta + r_{h,s}^L(1 - \zeta) - r_{CB}^D\Psi(\zeta)]\varphi + r_{CB}^D.$$

Using the equilibrium leverage $\varphi = (1 + \psi)/\gamma$ and the definition $\Psi(\zeta) = 1 + \psi[\zeta\tilde{\kappa}_l + (1 - \zeta)\tilde{\kappa}_h]$, the equity rate of return reads as

$$r_s^E(\varphi, \zeta) = [(r_l^L - r_{CB}^D\psi\tilde{\kappa}_l)\zeta + (r_{h,s}^L - r_{CB}^D\psi\tilde{\kappa}_h)(1 - \zeta) - r_{CB}^D]/\gamma + r_{CB}^D.$$

Second, based on lemma 4, the interest rates on deposits and reserves equal, i.e., $r^D = r_{CB}^D$.

Third, due to the fact that reserve loans are costly, the central bank generates profits, which in nominal terms are given by

$$\Pi_s^{CB} = r_{CB}^L(\zeta)L^{CB} - r_{CB}^D D^{CB} = [r_{CB}^L(\zeta) - r_{CB}^D]L^{CB} = [\zeta\tilde{\kappa}_l + (1 - \zeta)\tilde{\kappa}_h]\psi r_{CB}^D QK,$$

where we used the equality of reserve loans and reserve deposits, i.e., $L^{CB} = D^{CB}$, the structure of reserve loans, i.e., $L^{CB} = \psi L$, and the fact that, in equilibrium, bank loans are given by $L = QK$. Because we impose a balanced budget for the government and the central bank, central bank profits are distributed by the government through transfers, i.e.,

$$\tau_s = \pi_s^{CB} = [\zeta\tilde{\kappa}_l + (1 - \zeta)\tilde{\kappa}_h]\psi r_{CB}^D qK.$$

Fourth, from the outline in subsection 3.3, we know that the aggregate firm profits are characterized through equation (5), i.e., $\pi_s = (1 - \alpha)(A_l K_l^\alpha + A_{h,s}K_h^\alpha)$.

Thus, welfare in scenario $s \in \{b, t\}$ reads as

$$W_s = \gamma\{1 + [(r_l^L - r_{CB}^D \psi \tilde{\kappa}_l)\zeta + (r_{h,s}^L - r_{CB}^D \psi \tilde{\kappa}_h)(1-\zeta) - r_{CB}^D]/\gamma + r_{CB}^D\}qK$$

$$+ (1-\gamma)(1 + r_{CB}^D)qK + [\zeta\tilde{\kappa}_l + (1-\zeta)\tilde{\kappa}_h]\psi r_{CB}^D qK$$

$$+ (1-\alpha)(A_l K_l^\alpha + A_{h,s} K_h^\alpha)$$

and simplifies to

$$W_s = [(1 + r_l^L)\zeta + (1 + r_{h,s}^L)(1-\zeta)]qK + (1-\alpha)(A_l K_l^\alpha + A_{h,s} K_h^\alpha).$$

Using the first-order condition for the optimization problem of the riskless firm, i.e., $(1 + r_l^L)q = \alpha A_l K_l^{\alpha-1}$, assumption 1, which states that loan rates for risk firms satisfy $(1 + r_{h,s}^L)q = \alpha A_{h,s} K_h^{\alpha-1}$ for all s, and the capital allocation across riskless and risky firms, i.e., $K_l = \zeta K$ and $K_h = (1-\zeta)K$, welfare is finally given by

$$W_s = A_l K_l^\alpha + A_{h,s} K_h^\alpha = [A_l \zeta^\alpha + A_{h,s}(1-\zeta)^\alpha]K^\alpha.$$

■

Proof of Proposition 1. From corollary 1, we know that the interest rates on loans satisfy

$$r_l^L = r_{CB}^D(1 + \psi\tilde{\kappa}_l) \qquad \text{and} \qquad \mathbb{E}_p[r_{h,s}^L] = r_{CB}^D(1 + \psi\tilde{\kappa}_h),$$

so that, using the definition of $\tilde{\kappa}_l$ and $\tilde{\kappa}_h$, it follows

$$1 + r_l^L = (1 + r_{CB}^D)(1 + \psi\kappa_l) \qquad \text{and} \qquad 1 + \mathbb{E}_p[r_{h,s}^L] = (1 + r_{CB}^D)(1 + \psi\kappa_h).$$

From the latter two equations, we then obtain

$$\frac{1 + r_l^L}{1 + \psi\kappa_l} = \frac{1 + \mathbb{E}_p[r_{h,s}^L]}{1 + \psi\kappa_h} \qquad \Leftrightarrow \qquad \frac{1 + r_l^L}{1 + \mathbb{E}_p[r_{h,s}^L]} = \frac{1 + \psi\kappa_l}{1 + \psi\kappa_h}.$$

Using the first-order condition $(1+r_l^L)q = \alpha A_l K_l^{\alpha-1}$, and assumption 1, stating that $(1+r_{h,s}^L)q = \alpha A_{h,s} K_h^{\alpha-1}$ for all s, it follows

$$\frac{A_l K_l^{\alpha-1}}{\mathbb{E}_p[A_{h,s}]K_h^{\alpha-1}} = \frac{1 + \psi\kappa_l}{1 + \psi\kappa_h} \qquad \Leftrightarrow \qquad \left(\frac{K_h}{K_l}\right)^{1-\alpha} = \frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1 + \psi\kappa_l}{1 + \psi\kappa_h}.$$

Using $K_l = \zeta K$ and $K_h = (1-\zeta)K$, as derived in subsection 4.2, we obtain

$$\frac{1-\zeta}{\zeta} = \left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1 + \psi\kappa_l}{1 + \psi\kappa_h}\right)^{\frac{1}{1-\alpha}} \qquad \Leftrightarrow \qquad \zeta = \left[1 + \left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1 + \psi\kappa_l}{1 + \psi\kappa_h}\right)^{\frac{1}{1-\alpha}}\right]^{-1}.$$

■

Proof of Proposition 2. Note that the central bank faces the optimization problem

$$\max_{\kappa_l,\kappa_h\in\mathbb{R}} \{A_l\zeta^\alpha + \mathbb{E}_g[A_{h,s}](1-\zeta)^\alpha\}K^\alpha \qquad \text{subject to} \qquad \zeta\kappa_l + (1-\zeta)\kappa_h \geq 0,$$

where the share ζ of capital good allocated to riskless firms satisfies

$$\zeta = \left[1 + \left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}\right]^{-1}.$$

The optimal allocation follows by taking the derivatives of expected welfare with respect to κ_l and κ_h, and setting them to zero. With $\mu \geq 0$ denoting the Lagrange multiplier on the constraint, the optimality conditions are given by the two first-order conditions

$$\alpha A_l\zeta^{\alpha-1}\frac{\partial\zeta}{\partial\kappa_l} - \alpha\mathbb{E}_g[A_{h,s}](1-\zeta)^{\alpha-1}\frac{\partial\zeta}{\partial\kappa_l} - \mu\zeta - \mu(\kappa_l-\kappa_h)\frac{\partial\zeta}{\partial\kappa_l} = 0,$$

$$\alpha A_l\zeta^{\alpha-1}\frac{\partial\zeta}{\partial\kappa_h} - \alpha\mathbb{E}_g[A_{h,s}](1-\zeta)^{\alpha-1}\frac{\partial\zeta}{\partial\kappa_h} - \mu(1-\zeta) - \mu(\kappa_l-\kappa_h)\frac{\partial\zeta}{\partial\kappa_h} = 0,$$

and the complementary slackness condition $\mu[\zeta\kappa_l + (1-\zeta)\kappa_h] = 0$. Note that the two first-order conditions can be rewritten as

$$\alpha A_l\zeta^{\alpha-1} - \alpha\mathbb{E}_g[A_{h,s}](1-\zeta)^{\alpha-1} = \mu(\kappa_l-\kappa_h) + \mu\zeta\left(\frac{\partial\zeta}{\partial\kappa_l}\right)^{-1},$$

$$\alpha A_l\zeta^{\alpha-1} - \alpha\mathbb{E}_g[A_{h,s}](1-\zeta)^{\alpha-1} = \mu(\kappa_l-\kappa_h) + \mu(1-\zeta)\left(\frac{\partial\zeta}{\partial\kappa_h}\right)^{-1}.$$

First, we show that the Lagrange multiplier μ equals always zero. Suppose to the contrary that $\mu > 0$. Then, equating the two first-order conditions yields

$$\zeta\frac{\partial\zeta}{\partial\kappa_h} = (1-\zeta)\frac{\partial\zeta}{\partial\kappa_l}. \tag{21}$$

The derivatives of the capital allocation share ζ with respect to κ_l and κ_h are given by

$$\begin{aligned}
\frac{\partial\zeta}{\partial\kappa_l} &= -\frac{1}{1-\alpha}\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{\alpha}{1-\alpha}}\frac{\psi\mathbb{E}_p[A_{h,s}]}{A_l(1+\psi\kappa_h)} \\
&\qquad\qquad \times\left[1 + \left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}\right]^{-2} \\
&= \frac{-\zeta^2}{1-\alpha}\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}\frac{\psi}{1+\psi\kappa_l} \\
&= \frac{-\zeta(1-\zeta)}{1-\alpha}\frac{\psi}{1+\psi\kappa_l}
\end{aligned}$$

and

$$\frac{\partial \zeta}{\partial \kappa_h} = \frac{1}{1-\alpha}\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{\alpha}{1-\alpha}} \frac{\psi A_l \mathbb{E}_p[A_{h,s}](1+\psi\kappa_l)}{A_l^2(1+\psi\kappa_h)^2} \times \left[1+\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}\right]^{-2}$$

$$= \frac{\zeta^2}{1-\alpha}\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}} \frac{\psi}{1+\psi\kappa_h} = \frac{\zeta(1-\zeta)}{1-\alpha}\frac{\psi}{1+\psi\kappa_h}.$$

Using the latter two results, the condition (21) translates into

$$\frac{\zeta^2(1-\zeta)}{1-\alpha}\frac{\psi}{1+\psi\kappa_h} = \frac{-\zeta(1-\zeta)^2}{1-\alpha}\frac{\psi}{1+\psi\kappa_l} \qquad \Leftrightarrow \qquad \frac{\zeta}{1+\psi\kappa_h} = \frac{\zeta-1}{1+\psi\kappa_l}$$

and further simplifies to

$$\zeta(1+\psi\kappa_l) = (\zeta-1)(1+\psi\kappa_h) \quad \Leftrightarrow \quad \psi\{\zeta\kappa_l + (1-\zeta)\kappa_h\} = -1.$$

The latter equation contradicts the complementary slackness condition, which implies for any positive Lagrange multiplier (i.e., $\mu > 0$), $\zeta\kappa_l + (1-\zeta)\kappa_h = 0$. Thus, we can conclude that the Lagrange multiplier is always zero, i.e., $\mu = 0$. The two first-order conditions are therefore identical and given by

$$\alpha A_l \zeta^{\alpha-1} - \alpha \mathbb{E}_g[A_{h,s}](1-\zeta)^{\alpha-1} = 0.$$

This optimality condition translates into

$$A_l\zeta^{\alpha-1} = \mathbb{E}_g[A_{h,s}](1-\zeta)^{\alpha-1} \qquad \Leftrightarrow \qquad \frac{1-\zeta}{\zeta} = \left(\frac{\mathbb{E}_g[A_{h,s}]}{A_l}\right)^{\frac{1}{1-\alpha}}.$$

Further rearranging yields that the optimal capital allocation satisfies

$$\zeta = \left[1+\left(\frac{\mathbb{E}_g[A_{h,s}]}{A_l}\right)^{\frac{1}{1-\alpha}}\right]^{-1} =: \zeta_g.$$

Using the capital allocation in the decentralized equilibrium (see proposition 1), which is given by

$$\zeta = \left[1+\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}\right]^{-1},$$

we can deduce that the optimal monetary policy must satisfy

$$\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h} = \frac{\mathbb{E}_g[A_{h,s}]}{A_l} \qquad \Leftrightarrow \qquad \frac{1+\psi\kappa_h}{1+\psi\kappa_l} = \frac{\mathbb{E}_p[A_{h,s}]}{\mathbb{E}_g[A_{h,s}]} =: a.$$

Rearranging then yields

$$1 + \psi\kappa_h = (1 + \psi\kappa_l)a \quad \Leftrightarrow \quad \psi\kappa_h = \psi a\kappa_l + a - 1 \quad \Leftrightarrow \quad \kappa_h = a\kappa_l + \frac{a-1}{\psi}.$$

In addition, the cost factors must satisfy the constraint $\zeta_g\kappa_l + (1-\zeta_g)\kappa_h \geq 0$. Note that whenever $\eta_g > (<)\eta_p$ it follows $a > (<)1$ and therefore $\kappa_h > (<)\kappa_l$. ■

Proof of Corollary 3. Based on proposition 2, we know that the optimal cost factors κ_l and κ_h satisfy

$$\kappa_h = a\kappa_l + \frac{a-1}{\psi} \qquad \Leftrightarrow \qquad \frac{1+\psi\kappa_h}{1+\psi\kappa_l} = a, \qquad \text{with} \qquad a = \frac{\mathbb{E}_p[A_{h,s}]}{\mathbb{E}_g[A_{h,s}]}.$$

Note that a increases with η_g and decreases with η_p, so that we can conclude that the difference between the optimal cost factors $\kappa_h - \kappa_l$ increases with η_g and decreases with η_p. ■

Proof of Lemma 8. The first-order condition of the optimization problem of the risky firm (see equation (16)) with respect to capital good K_h is given by $\alpha\mathbb{E}_p[A_{h,s}(i)](1-i)^\alpha K_h^{\alpha-1} = (1+\mathbb{E}_p[r_{h,s}^L])q$. The latter condition can be rearranged to

$$K_h^{1-\alpha} = \frac{\alpha\mathbb{E}_p[A_{h,s}(i)](1-i)^\alpha}{(1+\mathbb{E}_p[r_{h,s}^L])q} \qquad \Leftrightarrow \qquad K_h = \left[\frac{\alpha\mathbb{E}_p[A_{h,s}(i)](1-i)^\alpha}{(1+\mathbb{E}_p[r_{h,s}^L])q}\right]^{\frac{1}{1-\alpha}},$$

which gives the optimal demand of capital good by the risky firm. The first-order condition with respect to the share i of capital good devoted to CRMT investment is for an interior solution given by

$$\mathbb{E}_p\left[\frac{\partial A_{h,s}(i)}{\partial i}\right](1-i)^\alpha = \alpha\mathbb{E}_p[A_{h,s}(i)](1-i)^{\alpha-1} \quad \Leftrightarrow \quad i = 1 - \frac{\alpha\mathbb{E}_p[A_{h,s}(i)]}{\mathbb{E}_p[\partial A_{h,s}(i)/\partial i]}.$$

Using assumption 2, we get that the share i of capital good devoted to CRMT investment simplifies to

$$i = 1 - \frac{\alpha\mathbb{E}_p[A_{h,s}(i)]}{\eta_p\mathbb{E}_p[A_{h,s}(i)]\beta(1-i)^{\beta-1}} \;\Leftrightarrow\; 1-i = \frac{\alpha}{\eta_p\beta(1-i)^{\beta-1}} \;\Leftrightarrow\; i = 1 - \left(\frac{\alpha}{\eta_p\beta}\right)^{\frac{1}{\beta}}.$$

We can conclude that $i < 1$, but we have to account for the fact that risky firms may not devote any capital good to CRMT investment if $\alpha > \eta_p\beta$. Thus, the optimal share i of capital good devoted to CRMT investment is generally given by

$$i = \max\left\{1 - \left(\frac{\alpha}{\eta_p\beta}\right)^{\frac{1}{\beta}}, 0\right\}.$$

■

Proof of Proposition 3. From corollary 1, we know that the interest rates on loans satisfy

$$r_l^L = r_{CB}^D(1+\psi\tilde{\kappa}_l) \qquad \text{and} \qquad \mathbb{E}_p[r_{h,s}^L] = r_{CB}^D(1+\psi\tilde{\kappa}_h),$$

so that, using the definition of $\tilde{\kappa}_l$ and $\tilde{\kappa}_h$, it follows

$$1+r_l^L = (1+r_{CB}^D)(1+\psi\kappa_l) \qquad \text{and} \qquad 1+\mathbb{E}_p[r_{h,s}^L] = (1+r_{CB}^D)(1+\psi\kappa_h).$$

From the latter two equations, we then obtain

$$\frac{1+r_l^L}{1+\psi\kappa_l} = \frac{1+\mathbb{E}_p[r_{h,s}^L]}{1+\psi\kappa_h} \qquad \Leftrightarrow \qquad \frac{1+r_l^L}{1+\mathbb{E}_p[r_{h,s}^L]} = \frac{1+\psi\kappa_l}{1+\psi\kappa_h}.$$

Using the first-order condition $(1+r_l^L)q = \alpha A_l K_l^{\alpha-1}$, and assumption 1 together with the fact that risky firms can invest into CRMT, both leading to $(1+r_{h,s}^L)q = \alpha A_{h,s}(i)(1-i)^\alpha K_h^{\alpha-1}$ for all s, it follows

$$\frac{A_l K_l^{\alpha-1}}{\mathbb{E}_p[A_{h,s}(i)](1-i)^\alpha K_h^{\alpha-1}} = \frac{1+\psi\kappa_l}{1+\psi\kappa_h} \quad \Leftrightarrow \quad \left(\frac{K_h}{K_l}\right)^{1-\alpha} = \frac{\mathbb{E}_p[A_{h,s}(i)](1-i)^\alpha}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}.$$

Using $K_l = \zeta K$ and $K_h = (1-\zeta)K$, as derived in subsection 4.2, we obtain

$$\frac{1-\zeta}{\zeta} = \left(\frac{\mathbb{E}_p[A_{h,s}(i)](1-i)^\alpha}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}$$

$$\Leftrightarrow \qquad \zeta = \left[1+\left(\frac{\mathbb{E}_p[A_{h,s}(i)](1-i)^\alpha}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}\right]^{-1}.$$

■

Proof of Proposition 4. Note that the central bank faces the optimization problem

$$\max_{\kappa_l,\kappa_h\in\mathbb{R}} \{A_l\zeta^\alpha + \mathbb{E}_g[A_{h,s}(i)](1-i)^\alpha(1-\zeta)^\alpha\}K^\alpha,$$

subject to $\zeta\kappa_l + (1-\zeta)\kappa_h \geq 0$, where the share ζ of capital good allocated to the riskless firm satisfies

$$\zeta = \left[1+\left(\frac{\mathbb{E}_p[A_{h,s}(i)](1-i)^\alpha}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}\right]^{-1}.$$

The optimal allocation follows by taking the derivatives of expected welfare with respect to κ_l and κ_h, and setting them to zero. With $\mu \geq 0$ denoting the Lagrange multiplier on the constraint, the optimality conditions are given by the two first-order conditions

$$\alpha A_l \zeta^{\alpha-1} \frac{\partial \zeta}{\partial \kappa_l} - \alpha \mathbb{E}_g[A_{h,s}(i)](1-i)^\alpha (1-\zeta)^{\alpha-1} \frac{\partial \zeta}{\partial \kappa_l} - \mu\zeta - \mu(\kappa_l - \kappa_h)\frac{\partial \zeta}{\partial \kappa_l} = 0,$$

$$\begin{aligned}\alpha A_l \zeta^{\alpha-1} \frac{\partial \zeta}{\partial \kappa_h} &- \alpha \mathbb{E}_g[A_{h,s}(i)](1-i)^\alpha (1-\zeta)^{\alpha-1} \frac{\partial \zeta}{\partial \kappa_h} \\ &- \mu(1-\zeta) - \mu(\kappa_l - \kappa_h)\frac{\partial \zeta}{\partial \kappa_h} = 0,\end{aligned}$$

and the complementary slackness condition $\mu[\zeta\kappa_l + (1-\zeta)\kappa_h] = 0$. Note that the two first-order conditions can be rewritten as

$$\alpha A_l \zeta^{\alpha-1} - \alpha \mathbb{E}_g[A_{h,s}(i)](1-i)^\alpha (1-\zeta)^{\alpha-1} = \mu(\kappa_l - \kappa_h) + \mu\zeta \left(\frac{\partial \zeta}{\partial \kappa_l}\right)^{-1},$$

$$\alpha A_l \zeta^{\alpha-1} - \alpha \mathbb{E}_g[A_{h,s}(i)](1-i)^\alpha (1-\zeta)^{\alpha-1} = \mu(\kappa_l - \kappa_h) + \mu(1-\zeta) \left(\frac{\partial \zeta}{\partial \kappa_h}\right)^{-1}.$$

First, we show that the Lagrange multiplier μ equals always zero. Suppose to the contrary that $\mu > 0$. Then, equating the two first-order conditions yields

$$\zeta \frac{\partial \zeta}{\partial \kappa_h} = (1-\zeta)\frac{\partial \zeta}{\partial \kappa_l}. \tag{22}$$

The derivatives of the capital allocation share ζ with respect to κ_l and κ_h are given by

$$\begin{aligned}\frac{\partial \zeta}{\partial \kappa_l} = -\frac{1}{1-\alpha} &\left(\frac{\mathbb{E}_p[A_{h,s}(i)](1-i)^\alpha}{A_l} \frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{\alpha}{1-\alpha}} \frac{\psi \mathbb{E}_p[A_{h,s}(i)](1-i)^\alpha}{A_l(1+\psi\kappa_h)} \\ &\times \left[1 + \left(\frac{\mathbb{E}_p[A_{h,s}(i)](1-i)^\alpha}{A_l} \frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}\right]^{-2} \\ = \frac{-\zeta^2}{1-\alpha} &\left(\frac{\mathbb{E}_p[A_{h,s}(i)](1-i)^\alpha}{A_l} \frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}} \frac{\psi}{1+\psi\kappa_l} \\ = \frac{-\zeta(1-\zeta)}{1-\alpha} &\frac{\psi}{1+\psi\kappa_l}\end{aligned}$$

and

$$\frac{\partial \zeta}{\partial \kappa_h} = \frac{1}{1-\alpha}\left(\frac{\mathbb{E}_p[A_{h,s}(i)](1-i)^\alpha}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{\alpha}{1-\alpha}} \frac{\psi A_l \mathbb{E}_p[A_{h,s}(i)](1-i)^\alpha(1+\psi\kappa_l)}{A_l^2(1+\psi\kappa_h)^2}$$

$$\times\left[1+\left(\frac{\mathbb{E}_p[A_{h,s}(i)](1-i)^\alpha}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}\right]^{-2}$$

$$= \frac{\zeta^2}{1-\alpha}\left(\frac{\mathbb{E}_p[A_{h,s}(i)](1-i)^\alpha}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}} \frac{\psi}{1+\psi\kappa_h}$$

$$= \frac{\zeta(1-\zeta)}{1-\alpha}\frac{\psi}{1+\psi\kappa_h}.$$

Using the latter two results, the condition (22) translates into

$$\frac{\zeta^2(1-\zeta)}{1-\alpha}\frac{\psi}{1+\psi\kappa_h} = \frac{-\zeta(1-\zeta)^2}{1-\alpha}\frac{\psi}{1+\psi\kappa_l} \qquad \Leftrightarrow \qquad \frac{\zeta}{1+\psi\kappa_h} = \frac{\zeta-1}{1+\psi\kappa_l}$$

and further simplifies to

$$\zeta(1+\psi\kappa_l) = (\zeta-1)(1+\psi\kappa_h) \quad \Leftrightarrow \quad \psi\{\zeta\kappa_l + (1-\zeta)\kappa_h\} = -1.$$

The latter equation contradicts the complementary slackness condition, which implies for any positive Lagrange multiplier (i.e., $\mu > 0$), $\zeta\kappa_l + (1-\zeta)\kappa_h = 0$. Thus, we can conclude that the Lagrange multiplier is always zero, i.e., $\mu = 0$. The two first-order conditions are therefore identical and given by

$$\alpha A_l \zeta^{\alpha-1} - \alpha\mathbb{E}_g[A_{h,s}(i)](1-i)^\alpha(1-\zeta)^{\alpha-1} = 0.$$

This optimality condition can be rearranged to

$$A_l\zeta^{\alpha-1} = \mathbb{E}_g[A_{h,s}(i)](1-i)^\alpha(1-\zeta)^{\alpha-1} \quad \Leftrightarrow \quad \frac{1-\zeta}{\zeta} = \left(\frac{\mathbb{E}_g[A_{h,s}(i)](1-i)^\alpha}{A_l}\right)^{\frac{1}{1-\alpha}}.$$

Further rearranging yields that the optimal capital allocation is given by the share

$$\zeta = \left[1+\left(\frac{\mathbb{E}_g[A_{h,s}]}{A_l}\right)^{\frac{1}{1-\alpha}}\right]^{-1} =: \zeta_g.$$

Using the capital allocation in the decentralized equilibrium (see proposition 1), which is given by

$$\zeta = \left[1+\left(\frac{\mathbb{E}_p[A_{h,s}(i)](1-i)^\alpha}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}\right]^{-1},$$

we can deduce that the optimal monetary policy must satisfy

$$\frac{\mathbb{E}_p[A_{h,s}(i)](1-i)^\alpha}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h} = \frac{\mathbb{E}_g[A_{h,s}(i)](1-i)^\alpha}{A_l}$$

$$\Leftrightarrow \qquad \frac{1+\psi\kappa_h}{1+\psi\kappa_l} = \frac{\mathbb{E}_p[A_{h,s}(i)]}{\mathbb{E}_g[A_{h,s}(i)]} =: a(i).$$

Rearranging then yields

$$1+\psi\kappa_h = (1+\psi\kappa_l)a(i) \quad \Leftrightarrow \quad \psi\kappa_h = \psi a(i)\kappa_l + a(i) - 1 \quad \Leftrightarrow \quad \kappa_h = a(i)\kappa_l + \frac{a(i)-1}{\psi}.$$

In addition, the cost factors must satisfy the constraint $\zeta_g\kappa_l + (1-\zeta_g)\kappa_h \geq 0$. Note that whenever beliefs satisfy $\eta_g > (<)\eta_p$ it follows $a(i) > (<)1$ and therefore $\kappa_h > (<)\kappa_l$. ■

Proof of Corollary 4. Note that it holds

$$a(i) = \frac{(1-\eta_p)A_{h,b} + \eta_p A_{h,t}(i)}{(1-\eta_g)A_{h,b} + \eta_g A_{h,t}(i)},$$

where we used $A_{h,b} := A_{h,b}(i)$ for all i, following from assumption 2, which states $\partial A_{h,b}(i)/\partial i = 0$. The parameter $a(i)$ varies with CRMT investment according to

$$\begin{aligned}
\frac{\partial a(i)}{\partial i} &= \frac{\mathbb{E}_g[A_{h,s}(i)]\eta_p \frac{\partial A_{h,t}(i)}{\partial i} - \mathbb{E}_p[A_{h,s}(i)]\eta_g \frac{\partial A_{h,t}(i)}{\partial i}}{(\mathbb{E}_g[A_{h,s}(i)])^2} \\
&= \frac{\partial A_{h,t}(i)}{\partial i} \frac{\eta_p[(1-\eta_g)A_{h,b} + \eta_g A_{h,t}(i)] - \eta_g[(1-\eta_p)A_{h,b} + \eta_p A_{h,t}(i)]}{(\mathbb{E}_g[A_{h,s}(i)])^2} \\
&= \frac{\partial A_{h,t}(i)}{\partial i} \frac{(\eta_p - \eta_g)A_{h,b}}{(\mathbb{E}_g[A_{h,s}(i)])^2}.
\end{aligned}$$

Based on assumption 2, we know that $\partial A_{h,t}(i)/\partial i > 0$, so that we can conclude $\partial a(i)/\partial i < (>)0$ for $\eta_g > (<)\eta_p$.

From proposition 4, we know that if beliefs satisfy $\eta_g > (<)\eta_p$, it holds $a(i) > (<)1$ and therefore $\kappa_h > (<)\kappa_l$. Accordingly, we can deduce that the difference of cost factors $\kappa_h - \kappa_l$ is positive (negative) for beliefs satisfying $\eta_g > (<)\eta_p$ and decreases (increases) with higher CRMT investment, i.e., for a larger share i. Thus, we can conclude that CRMT investment reduces, independent of the beliefs, the intensity of central bank intervention, as measured by the absolute difference between cost factors $|\kappa_h - \kappa_l|$. ■

Proof of Lemma 9. From equation (17) in subsection 3.6, we know that the maximum leverage, which rules out bank recapitalization in the transition scenario, is given by

$$\varphi^S(\zeta) = \frac{(1+r^D_{CB})(1+\psi)}{r^D_{CB}\Psi(\zeta) - r^L_l\zeta - r^L_{h,t}(1-\zeta)}.$$

To express this leverage using economic fundamentals, we first use the fact that, in equilibrium, banks must, due to perfect competition, make in expectation zero profits from granting loans funded with deposits (see subsection 4.2), i.e., $r_l^L \zeta + \mathbb{E}_p[r_{h,s}^L](1-\zeta) = r_{CB}^D \Psi(\zeta)$. Then, using $\mathbb{E}_p[r_{h,s}^L] = r_{h,t}^L + (1-\eta_p)(r_{h,b}^L - r_{h,t}^L)$, we get

$$\varphi^S(\zeta) = \frac{(1+r_{CB}^D)(1+\psi)}{(1-\eta_p)(r_{h,b}^L - r_{h,t}^L)(1-\zeta)}.$$

Moreover, from corollary 1, we know that the interest rate on loans to the risky sector satisfies

$$\mathbb{E}_p[r_{h,s}^L] = r_{CB}^D(1+\psi\tilde{\kappa}_h) \qquad \Leftrightarrow \qquad 1+\mathbb{E}_p[r_{h,s}^L] = (1+r_{CB}^D)(1+\psi\kappa_h),$$

where we used the definition $\tilde{\kappa}_h = \kappa_h(1+r_{CB}^D)/r_{CB}^D$. Accordingly, we obtain

$$\varphi^S(\zeta) = \frac{(1+\mathbb{E}_p[r_{h,s}^L])(1+\psi)}{(1+\psi\kappa_h)(1-\eta_p)(r_{h,b}^L - r_{h,t}^L)(1-\zeta)}.$$

Using assumption 1, which states $(1+r_{h,s}^L)q = \alpha A_{h,s} K_h^{\alpha-1}$ for all s, the latter expression translates into

$$\varphi^S(\zeta) = \frac{\mathbb{E}_p[A_{h,s}](1+\psi)}{(1+\psi\kappa_h)(1-\eta_p)(A_{h,b} - A_{h,t})(1-\zeta)},$$

where we used

$$\frac{1+\mathbb{E}_p[r_{h,s}^L]}{r_{h,b}^L - r_{h,t}^L} = \frac{1+\mathbb{E}_p[r_{h,s}^L]}{(1+r_{h,b}^L) - (1+r_{h,t}^L)} = \frac{(1+\mathbb{E}_p[r_{h,s}^L])q}{(1+r_{h,b}^L)q - (1+r_{h,t}^L)q}$$

that further simplifies to

$$\frac{\alpha \mathbb{E}_p[A_{h,s}]K_h^{\alpha-1}}{\alpha A_{h,b}K_h^{\alpha-1} - \alpha A_{h,t}K_h^{\alpha-1}} = \frac{\mathbb{E}_p[A_{h,s}]}{A_{h,b} - A_{h,t}}.$$

■

Proof of Lemma 10. From subsection 4.2 we know that welfare in scenario $s \in \{b,t\}$ is generally given by

$$W_s^\lambda = C_s - \Lambda(\zeta)\mathbb{1}\{\varphi > \varphi^S(\zeta) \wedge s = t\} = W_s - \Lambda\mathbb{1}\{\varphi > \varphi^S(\zeta) \wedge s = t\}.$$

Using lemma 7, which provides W_s in terms of economic fundamentals, and the costs of bank recapitalization, i.e., $\Lambda(\zeta) = \lambda\alpha A_{h,t}(1-\zeta)^\alpha K^\alpha$, we know that welfare is given by

$$W_s^\lambda = \{A_l\zeta^\alpha + A_{h,s}(1-\zeta)^\alpha[1-\lambda\alpha\mathbb{1}\{\varphi > \varphi^S(\zeta) \wedge s = t\}]\}K^\alpha.$$

■

Proof of Proposition 5. From lemma 9, we know that the maximum leverage ruling out bank recapitalization is given by

$$\varphi^S(\zeta) = \frac{\mathbb{E}_p[A_{h,s}](1+\psi)}{(1+\psi\kappa_h)(1-\eta_p)(A_{h,b}-A_{h,t})(1-\zeta)}.$$

First, taking the derivative of $\varphi^S(\zeta)$ with respect to κ_l yields

$$\begin{aligned}
\frac{\partial\varphi^S(\zeta)}{\partial\kappa_l} &= \frac{\mathbb{E}_p[A_{h,s}](1+\psi)(1+\psi\kappa_h)(1-\eta_p)(A_{h,b}-A_{h,t})}{[(1+\psi\kappa_h)(1-\eta_p)(A_{h,b}-A_{h,t})(1-\zeta)]^2}\frac{\partial\zeta}{\partial\kappa_l} \\
&= \frac{\mathbb{E}_p[A_{h,s}](1+\psi)}{(1+\psi\kappa_h)(1-\eta_p)(A_{h,b}-A_{h,t})(1-\zeta)^2}\frac{\partial\zeta}{\partial\kappa_l} \\
&= \frac{\varphi^S(\zeta)}{1-\zeta}\frac{\partial\zeta}{\partial\kappa_l}.
\end{aligned}$$

Note that it holds

$$\begin{aligned}
\frac{\partial\zeta}{\partial\kappa_l} &= \frac{-\zeta^2}{1-\alpha}\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{\alpha}{1-\alpha}}\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{\psi}{1+\psi\kappa_h} \\
&= \frac{-\psi\zeta^2}{(1-\alpha)(1+\psi\kappa_l)}\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}} \\
&= \frac{-\psi\zeta^2}{(1-\alpha)(1+\psi\kappa_l)}\frac{1-\zeta}{\zeta} \\
&= \frac{-\psi\zeta(1-\zeta)}{(1-\alpha)(1+\psi\kappa_l)} < 0.
\end{aligned}$$

Accordingly, we obtain $\partial\varphi^S(\zeta)/\partial\kappa_l < 0$. Second, taking the derivative of $\varphi^S(\zeta)$ with respect to κ_h yields

$$\frac{\partial\varphi^S(\zeta)}{\partial\kappa_h} = \frac{-\mathbb{E}_p[A_{h,s}](1+\psi)(1-\eta_p)(A_{h,b}-A_{h,t})\left[\psi(1-\zeta)-(1+\psi\kappa_h)\frac{\partial\zeta}{\partial\kappa_h}\right]}{[(1+\psi\kappa_h)(1-\eta_p)(A_{h,b}-A_{h,t})(1-\zeta)]^2},$$

which further simplifies to

$$\begin{aligned}
\frac{\partial\varphi^S(\zeta)}{\partial\kappa_h} &= \frac{\mathbb{E}_p[A_{h,s}](1+\psi)\left[(1+\psi\kappa_h)\frac{\partial\zeta}{\partial\kappa_h}-\psi(1-\zeta)\right]}{(1+\psi\kappa_h)^2(1-\eta_p)(A_{h,b}-A_{h,t})(1-\zeta)^2} \\
&= \frac{\varphi^S(\zeta)}{(1+\psi\kappa_h)(1-\zeta)}\left[(1+\psi\kappa_h)\frac{\partial\zeta}{\partial\kappa_h}-\psi(1-\zeta)\right] \\
&= \varphi^S(\zeta)\left[\frac{1}{1-\zeta}\frac{\partial\zeta}{\partial\kappa_h}-\frac{\psi}{1+\psi\kappa_h}\right].
\end{aligned}$$

Note that it holds

$$\frac{\partial\zeta}{\partial\kappa_h} = \frac{\zeta^2}{1-\alpha}\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{\alpha}{1-\alpha}}\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{(1+\psi\kappa_l)\psi}{(1+\psi\kappa_h)^2}$$

$$= \frac{\psi\zeta^2}{(1-\alpha)(1+\psi\kappa_h)}\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}$$

$$= \frac{\psi\zeta^2}{(1-\alpha)(1+\psi\kappa_h)}\frac{1-\zeta}{\zeta}$$

$$= \frac{\psi\zeta(1-\zeta)}{(1-\alpha)(1+\psi\kappa_h)} > 0.$$

Thus, we obtain

$$\frac{1}{1-\zeta}\frac{\partial\zeta}{\partial\kappa_h} \geq \frac{\psi}{1+\psi\kappa_h} \quad\Leftrightarrow\quad \frac{\zeta}{1-\alpha}\frac{\psi}{1+\psi\kappa_h} \geq \frac{\psi}{1+\psi\kappa_h} \quad\Leftrightarrow\quad \zeta \geq 1-\alpha.$$

Then, it follows

$$\frac{\partial\varphi^S(\zeta)}{\partial\kappa_h} < (\geq) 0 \qquad \text{if and only if} \qquad \zeta < (\geq) 1-\alpha.$$

Third, note that for $\kappa_l \to -1/\psi$ the share ζ of capital good allocated to riskless firms is approaching one. From the structure of $\varphi^S(\zeta)$, we can conclude that $\lim_{\kappa_l\to-1/\psi}\varphi^S(\zeta) = +\infty$.

Fourth, we consider the case where the cost factor κ_h approaches infinity. Note that it follows from the structure of $\varphi^S(\zeta)$ that we only need to evaluate the limit of $(1+\psi\kappa_h)(1-\zeta)$ to obtain the limit of $\varphi^S(\zeta)$. Moreover, it holds that

$$\lim_{\kappa_h\to+\infty}(1+\psi\kappa_h)(1-\zeta) = \lim_{\kappa_h\to+\infty}(1+\psi\kappa_h)\zeta\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}},$$

since

$$\zeta = \left[1+\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}\right]^{-1} \quad\Leftrightarrow\quad 1-\zeta = \zeta\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}.$$

Further rearranging yields

$$\lim_{\kappa_h\to+\infty}(1+\psi\kappa_h)\zeta\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}$$

$$= \lim_{\kappa_h\to+\infty}\zeta\left[\frac{\mathbb{E}_p[A_{h,s}](1+\psi\kappa_l)}{A_l}\right]^{\frac{1}{1-\alpha}}(1+\psi\kappa_h)^{-\frac{\alpha}{1-\alpha}}$$

$$= \left[\frac{\mathbb{E}_p[A_{h,s}](1+\psi\kappa_l)}{A_l}\right]^{\frac{1}{1-\alpha}}\lim_{\kappa_h\to+\infty}\frac{\zeta}{(1+\psi\kappa_h)^{\frac{\alpha}{1-\alpha}}}.$$

It follows from the structure of the equilibrium share ζ of capital good allocated to the riskless sector that in the limit all capital good is used for production by riskless firms, $\lim_{\kappa_h \to +\infty} \zeta = 1$. We therefore obtain that $\lim_{\kappa_h \to +\infty}(1 + \psi\kappa)(1 - \zeta) = 0$ and furthermore

$$\lim_{\kappa_h \to +\infty} \varphi^S(\zeta) = \lim_{\kappa_h \to +\infty} \frac{\mathbb{E}_p[A_{h,s}](1 + \psi)}{(1 + \psi\kappa_h)(1 - \eta_p)(A_{h,b} - A_{h,t})(1 - \zeta)} = +\infty.$$

Finally, let us focus on the costs of bank recapitalization $\Lambda(\zeta) = \lambda\alpha A_{h,t}(1 - \zeta)^\alpha K^\alpha$. As we showed before, it holds $\partial\zeta/\partial\kappa_l < 0$ and $\partial\zeta/\partial\kappa_h > 0$. Thus, we can conclude

$$\frac{\partial\Lambda(\zeta)}{\partial\kappa_l} > 0 \qquad \text{and} \qquad \frac{\partial\Lambda(\zeta)}{\partial\kappa_h} < 0.$$

Moreover, based on $\lim_{\kappa_l \to -1/\psi} \zeta = \lim_{\kappa_h \to +\infty} \zeta = 1$, we further know that it holds $\lim_{\kappa_l \to -1/\psi} \Lambda(\zeta) = \lim_{\kappa_h \to +\infty} \Lambda(\zeta) = 0$. ■

Proof of Proposition 6. The maximum leverage $\varphi^S(\zeta)$ ruling out bank recapitalization varies with the beliefs η_p of private agents according to

$$\begin{aligned}
\frac{\partial\varphi^S(\zeta)}{\partial\eta_p} &= \frac{-(1 + \psi\kappa_h)(1 - \eta_p)(A_{h,b} - A_{h,t})^2(1 - \zeta)(1 + \psi)}{\{(1 + \psi\kappa_h)(1 - \eta_p)(A_{h,b} - A_{h,t})(1 - \zeta)\}^2} \\
&\quad - \frac{\mathbb{E}_p[A_{h,s}](1 + \psi)(1 + \psi\kappa_h)(A_{h,b} - A_{h,t})\left[-(1 - \zeta) - (1 - \eta_p)\dfrac{\partial\zeta}{\partial\eta_p}\right]}{\{(1 + \psi\kappa_h)(1 - \eta_p)(A_{h,b} - A_{h,t})(1 - \zeta)\}^2} \\
&= (1 + \psi)\frac{-(1 - \eta_p)(A_{h,b} - A_{h,t})(1 - \zeta) + \mathbb{E}_p[A_{h,s}]\left[1 - \zeta + (1 - \eta_p)\dfrac{\partial\zeta}{\partial\eta_p}\right]}{(1 + \psi\kappa_h)(1 - \eta_p)^2(A_{h,b} - A_{h,t})(1 - \zeta)^2} \\
&= (1 + \psi)\frac{A_{h,t}(1 - \zeta) + (1 - \eta_p)\mathbb{E}_p[A_{h,s}]\dfrac{\partial\zeta}{\partial\eta_p}}{(1 + \psi\kappa_h)(1 - \eta_p)^2(A_{h,b} - A_{h,t})(1 - \zeta)^2}.
\end{aligned}$$

Since it holds

$$\begin{aligned}
\frac{\partial\zeta}{\partial\eta_p} &= -\left[1 + \left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1 + \psi\kappa_l}{1 + \psi\kappa_h}\right)^{\frac{1}{1-\alpha}}\right]^{-2} \frac{1}{1 - \alpha}\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1 + \psi\kappa_l}{1 + \psi\kappa_h}\right)^{\frac{\alpha}{1-\alpha}} \\
&\qquad \times \frac{A_{h,t} - A_{h,b}}{A_l}\frac{1 + \psi\kappa_l}{1 + \psi\kappa_h} \\
&= \frac{(A_{h,b} - A_{h,t})\zeta^2}{(1 - \alpha)\mathbb{E}_p[A_{h,s}]}\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1 + \psi\kappa_l}{1 + \psi\kappa_h}\right)^{\frac{1}{1-\alpha}} \\
&= \frac{(A_{h,b} - A_{h,t})\zeta(1 - \zeta)}{(1 - \alpha)\mathbb{E}_p[A_{h,s}]} > 0,
\end{aligned}$$

we know that $\partial\varphi^S(\zeta)/\partial\eta_p > 0$. As the bank recapitalization costs are given by $\Lambda(\zeta) = \lambda\alpha A_{h,t}(1 - \zeta)^\alpha K^\alpha$, we can conclude with $\partial\zeta/\partial\eta_p > 0$ that it holds $\partial\Lambda(\zeta)/\partial\eta_p < 0$. ■

Proof of Proposition 7. From lemma 10, we know that whenever the leverage satisfies $\varphi \leq \varphi^S(\zeta)$, no bank recapitalization oocurs and therefore it holds $W_s^\lambda = W_s$. We know that the optimal monetary policy maximizing the expected utilitarian welfare $\mathbb{E}_g[W_s]$ is characterized by proposition 2. This optimal monetary policy induces the capital allocation ζ_g by implementing cost factors that satisfy

$$\kappa_h = a\kappa_l + \frac{a-1}{\psi} \qquad \text{and} \qquad \zeta_g\kappa_l + (1-\zeta_g)\kappa_h \geq 0, \qquad \text{with} \qquad a = \frac{\mathbb{E}_p[A_{h,s}]}{\mathbb{E}_g[A_{h,s}]}.$$

From the outline in subsection 4.2, we know that the equilibrium leverage is given by $\varphi = (1+\psi)/\gamma$, so that under the optimal monetary policy bank recapitalization is only ruled out if

$$\varphi = (1+\psi)/\gamma \leq \varphi^S(\zeta_g) = \frac{\mathbb{E}_p[A_{h,s}](1+\psi)}{(1+\psi\kappa_h)(1-\eta_p)(A_{h,b} - A_{h,t})(1-\zeta_g)}.$$

The maximum leverage $\varphi^S(\zeta)$ is highest for the lowest possible cost factor κ_h on risky loans, which is obtained by imposing $\zeta_g\kappa_l + (1-\zeta_g)\kappa_h = 0$. ■

Proof of Lemma 11. By assumption, we know that with cost factors inducing the capital allocation ζ_g (see proposition 2) and satisfying $\zeta_g\kappa_l + (1-\zeta_g)\kappa_h = 0$, bank recapitalization occurs in the transition, i.e., $\varphi = (1+\psi)/\gamma > \varphi^S(\zeta_g)$. From lemma 9, we know that for the cost factors $\kappa_l \to -1/\psi$ or $\kappa_h \to \infty$, bank recapitalization does not occur, i.e., $\lim_{\kappa_l \to -1/\psi} \varphi^S(\zeta) = \lim_{\kappa_h \to +\infty} \varphi^S(\zeta) = +\infty$. As $\varphi^S(\zeta)$ is a continuous function in κ_l and κ_h, we can conclude that there exist cost factors $\hat{\kappa}_l$ and $\hat{\kappa}_h$ inducing the capital allocation $\hat{\zeta}$ with $\varphi = (1+\psi)/\gamma = \varphi^S(\hat{\zeta})$ and satisfying $\hat{\zeta}\hat{\kappa}_l + (1-\hat{\zeta})\hat{\kappa}_h = 0$. ■

Proof of Proposition 8. Note that the central bank faces the optimization problem

$$\max_{\kappa_l,\kappa_h \in \mathbb{R}} \; [A_l\zeta^\alpha + (\mathbb{E}_g[A_{h,s}] - \eta_g\lambda\alpha A_{h,t}\mathbb{1}\{\varphi > \varphi^S(\zeta)\})(1-\zeta)^\alpha]K^\alpha,$$

$$\text{subject to} \quad \zeta\kappa_l + (1-\zeta)\kappa_h \geq 0,$$

where the share ζ of capital allocated to riskless firms satisfies

$$\zeta = \left[1 + \left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}\right]^{-1}.$$

By assumption, we know that with cost factors inducing the capital allocation ζ_g (see proposition 2) and satisfying $\zeta_g\kappa_l + (1-\zeta_g)\kappa_h = 0$, bank recapitalization occurs in the transition, i.e., $\varphi = (1+\psi)/\gamma > \varphi^S(\zeta_g)$. Thus, the central bank needs to decide whether it wants to rule out bank recapitalization or accept bank recapitalization but correct the capital allocation. In the first regime, the central bank sets the cost factors $\hat{\kappa}_l$ and $\hat{\kappa}_h$ inducing the capital allocation $\hat{\zeta} > \zeta_g$ with $\varphi = (1+\psi)/\gamma = \varphi^S(\hat{\zeta})$ and satisfying $\hat{\zeta}\hat{\kappa}_l + (1-\hat{\zeta})\hat{\kappa}_h = 0$. Note that it is optimal for the central bank to minimize liquidity costs, as otherwise it would have to set costs factors inducing a capital allocation $\hat{\hat{\zeta}} > \hat{\zeta} > \zeta_g$. However, without bank recapitalization welfare only depends on the allocation of capital allocation, so that the capital allocation $\hat{\hat{\zeta}}$ yields a lower welfare then the capital allocation $\hat{\zeta}$. In the second regime, the central bank accepts bank recapitalization in the transition and corrects the belief-driven capital distortion, while accounting for the costs arising from equity injections by shareholders. Formally, the central bank then faces within this regime the optimization problem

$$\max_{\kappa_l,\kappa_h \in \mathbb{R}} [A_l \zeta^\alpha + \mathbb{E}_g^\lambda[A_{h,s}](1-\zeta)^\alpha] K^\alpha \qquad \text{subject to} \qquad \zeta\kappa_l + (1-\zeta)\kappa_h \geq 0,$$

The optimal allocation follows by taking the derivatives of welfare with respect to κ_l and κ_h, and setting them to zero. With $\mu \geq 0$ denoting the Lagrange multiplier on the constraint, the optimality conditions are given by the two first-order conditions

$$\alpha A_l \zeta^{\alpha-1} \frac{\partial \zeta}{\partial \kappa_l} - \alpha \mathbb{E}_g^\lambda[A_{h,s}](1-\zeta)^{\alpha-1} \frac{\partial \zeta}{\partial \kappa_l} - \mu\zeta - \mu(\kappa_l - \kappa_h)\frac{\partial \zeta}{\partial \kappa_l} = 0,$$

$$\alpha A_l \zeta^{\alpha-1} \frac{\partial \zeta}{\partial \kappa_h} - \alpha \mathbb{E}_g^\lambda[A_{h,s}](1-\zeta)^{\alpha-1} \frac{\partial \zeta}{\partial \kappa_h} - \mu(1-\zeta) - \mu(\kappa_l - \kappa_h)\frac{\partial \zeta}{\partial \kappa_h} = 0,$$

and the complementary slackness condition $\mu[\zeta\kappa_l + (1-\zeta)\kappa_h] = 0$. Note that the two first-order conditions can be rewritten as

$$\alpha A_l \zeta^{\alpha-1} - \alpha \mathbb{E}_g^\lambda[A_{h,s}](1-\zeta)^{\alpha-1} = \mu(\kappa_l - \kappa_h) + \mu\zeta \left(\frac{\partial \zeta}{\partial \kappa_l}\right)^{-1},$$

$$\alpha A_l \zeta^{\alpha-1} - \alpha \mathbb{E}_g^\lambda[A_{h,s}](1-\zeta)^{\alpha-1} = \mu(\kappa_l - \kappa_h) + \mu(1-\zeta) \left(\frac{\partial \zeta}{\partial \kappa_h}\right)^{-1}.$$

First, we show that the Lagrange multiplier μ equals always zero. Suppose to the contrary that $\mu > 0$. Then, equating the two first-order conditions yields

$$\zeta \frac{\partial \zeta}{\partial \kappa_h} = (1-\zeta)\frac{\partial \zeta}{\partial \kappa_l}. \tag{23}$$

The derivatives of the capital allocation share ζ with respect to κ_l and κ_h are given by

$$\begin{aligned}
\frac{\partial \zeta}{\partial \kappa_l} &= -\frac{1}{1-\alpha}\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{\alpha}{1-\alpha}} \frac{\psi \mathbb{E}_p[A_{h,s}]}{A_l(1+\psi\kappa_h)} \\
&\qquad\qquad\qquad \times \left[1 + \left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}\right]^{-2} \\
&= \frac{-\zeta^2}{1-\alpha}\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}} \frac{\psi}{1+\psi\kappa_l} \\
&= \frac{-\zeta(1-\zeta)}{1-\alpha}\frac{\psi}{1+\psi\kappa_l}
\end{aligned}$$

and

$$\frac{\partial \zeta}{\partial \kappa_h} = \frac{1}{1-\alpha}\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{\alpha}{1-\alpha}} \frac{\psi A_l \mathbb{E}_p[A_{h,s}](1+\psi\kappa_l)}{A_l^2(1+\psi\kappa_h)^2}$$

$$\times \left[1+\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}\right]^{-2}$$

$$= \frac{\zeta^2}{1-\alpha}\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}} \frac{\psi}{1+\psi\kappa_h} = \frac{\zeta(1-\zeta)}{1-\alpha}\frac{\psi}{1+\psi\kappa_h}.$$

Using the latter two results, the condition (23) translates into

$$\frac{\zeta^2(1-\zeta)}{1-\alpha}\frac{\psi}{1+\psi\kappa_h} = \frac{-\zeta(1-\zeta)^2}{1-\alpha}\frac{\psi}{1+\psi\kappa_l} \qquad \Leftrightarrow \qquad \frac{\zeta}{1+\psi\kappa_h} = \frac{\zeta-1}{1+\psi\kappa_l}$$

and further simplifies to

$$\zeta(1+\psi\kappa_l) = (\zeta-1)(1+\psi\kappa_h) \qquad \Leftrightarrow \qquad \psi\{\zeta\kappa_l + (1-\zeta)\kappa_h\} = -1.$$

The latter equation contradicts the complementary slackness condition, which implies for any positive Lagrange multiplier (i.e., $\mu > 0$), $\zeta\kappa_l + (1-\zeta)\kappa_h = 0$. Thus, we can conclude that the Lagrange multiplier is always zero, i.e., $\mu = 0$. The two first-order conditions are therefore identical and given by

$$\alpha A_l \zeta^{\alpha-1} - \alpha \mathbb{E}_g^\lambda[A_{h,s}](1-\zeta)^{\alpha-1} = 0.$$

This optimality condition can be rearranged to

$$A_l\zeta^{\alpha-1} = \mathbb{E}_g^\lambda[A_{h,s}](1-\zeta)^{\alpha-1} \qquad \Leftrightarrow \qquad \frac{1-\zeta}{\zeta} = \left(\frac{\mathbb{E}_g^\lambda[A_{h,s}]}{A_l}\right)^{\frac{1}{1-\alpha}}.$$

Further rearranging yields that the optimal capital allocation is given by the share

$$\zeta = \left[1+\left(\frac{\mathbb{E}_g^\lambda[A_{h,s}]}{A_l}\right)^{\frac{1}{1-\alpha}}\right]^{-1} =: \zeta_g^\lambda.$$

Using the capital allocation in the decentralized equilibrium (see proposition 1), which is given by

$$\zeta = \left[1+\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}\right]^{-1},$$

we can deduce that the optimal monetary policy must satisfy

$$\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h} = \frac{\mathbb{E}_g^\lambda[A_{h,s}]}{A_l} \qquad \Leftrightarrow \qquad \frac{1+\psi\kappa_h}{1+\psi\kappa_l} = \frac{\mathbb{E}_p[A_{h,s}]}{\mathbb{E}_g^\lambda[A_{h,s}]} =: a_\lambda.$$

Rearranging then yields

$$1+\psi\kappa_h=(1+\psi\kappa_l)a_\lambda \quad\Leftrightarrow\quad \psi\kappa_h=\psi a_\lambda\kappa_l+a_\lambda-1 \quad\Leftrightarrow\quad \kappa_h=a_\lambda\kappa_l+\frac{a_\lambda-1}{\psi}.$$

The central bank decides between the first and the second regime based on a welfare comparison. It implements the monetary policy inducing $\hat{\zeta}$ (ζ_g^λ) if and only if $\mathbb{E}_g[W_s(\hat{\zeta})] \geq (<) \mathbb{E}_g[W_s^\lambda(\zeta_g^\lambda)]$. ■

Proof of Proposition 9. Note that the capital allocation in the decentralized equilibrium is provided proposition 1 and given by

$$\zeta=\left[1+\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}\right]^{-1}.$$

One the hand, the central bank aims at inducing the capital allocation ζ_g, which it finds given its belief and without bank recapitalization to be optimal one, where

$$\zeta_g=\left[1+\left(\frac{\mathbb{E}_g[A_{h,s}]}{A_l}\right)^{\frac{1}{1-\alpha}}\right].$$

Equating ζ and ζ_g yields

$$\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}=\frac{\mathbb{E}_g[A_{h,s}]}{A_l} \qquad\Leftrightarrow\qquad \frac{1+\psi\kappa_h}{1+\psi\kappa_l}=\frac{\mathbb{E}_p[A_{h,s}]}{\mathbb{E}_g[A_{h,s}]}=: a.$$

Rearranging then yields

$$1+\psi\kappa_h=(1+\psi\kappa_l)a \quad\Leftrightarrow\quad \psi\kappa_h=\psi a\kappa_l+a-1 \quad\Leftrightarrow\quad \kappa_h=a\kappa_l+\frac{a-1}{\psi}.$$

On the other hand, the central bank aims at eliminating bank recapitalization. When implementing ζ_g, bank recapitalization is ruled out whenever it holds $\varphi=(1+\psi)/\gamma=\varphi^S(\zeta_g)$ or, equivalently,

$$\begin{aligned}
(1+\psi)/\gamma &= \frac{\mathbb{E}_p[A_{h,s}](1+\psi)}{(1+\psi\kappa_h)(1-\eta_p)(A_{h,b}-A_{h,t})(1-\zeta_g)}\\
\Leftrightarrow \qquad 1+\psi\kappa_h &= \frac{\mathbb{E}_p[A_{h,s}]\gamma}{(1-\eta_p)(A_{h,b}-A_{h,t})(1-\zeta_g)}\\
\Leftrightarrow \qquad \kappa_h &= \frac{\mathbb{E}_p[A_{h,s}]\gamma}{\psi(1-\eta_p)(A_{h,b}-A_{h,t})(1-\zeta_g)}-\frac{1}{\psi}.
\end{aligned}$$

Combining the two previous conditions on κ_h, we obtain

$$
\begin{aligned}
a\kappa_l + \frac{a-1}{\psi} &= \frac{\mathbb{E}_p[A_{h,s}]\gamma}{\psi(1-\eta_p)(A_{h,b}-A_{h,t})(1-\zeta_g)} - \frac{1}{\psi} \\
\Leftrightarrow \qquad \kappa_l &= \frac{\mathbb{E}_p[A_{h,s}]\gamma}{a\psi(1-\eta_p)(A_{h,b}-A_{h,t})(1-\zeta_g)} - \frac{1}{\psi} \\
\Leftrightarrow \qquad \kappa_l &= \frac{\mathbb{E}_p[A_{h,s}]\gamma}{a\psi(1-\eta_p)(A_{h,b}-A_{h,t})(1-\zeta_g)} - \frac{1}{\psi} \\
\Leftrightarrow \qquad \kappa_l &= \frac{\mathbb{E}_g[A_{h,s}]\gamma}{\psi(1-\eta_p)(A_{h,b}-A_{h,t})(1-\zeta_g)} - \frac{1}{\psi},
\end{aligned}
$$

where we used $a = \mathbb{E}_p[A_{h,s}]/\mathbb{E}_g[A_{h,s}]$. Note that banks receive an implicit subsidy by borrowing reserves, so that the central bank must implement quantity restrictions on reserve loans, if $r^L_{CB}(\zeta_g) < r^D_{CB}$ or, equivalently, $\zeta_g\kappa_l + (1-\zeta_g)\kappa_h < 0$. The latter inequality reads as

$$
\begin{aligned}
&\frac{\zeta_g\mathbb{E}_p[A_{h,s}]\gamma}{a\psi(1-\eta_p)(A_{h,b}-A_{h,t})(1-\zeta_g)} - \frac{\zeta_g}{\psi} \\
&\qquad\qquad + \frac{(1-\zeta_g)\mathbb{E}_p[A_{h,s}]\gamma}{\psi(1-\eta_p)(A_{h,b}-A_{h,t})(1-\zeta_g)} - \frac{1-\zeta_g}{\psi} < 0,
\end{aligned}
$$

which further simplifies to

$$
\begin{aligned}
&\frac{\mathbb{E}_p[A_{h,s}]\gamma[\zeta_g + a(1-\zeta_g)]}{a\psi(1-\eta_p)(A_{h,b}-A_{h,t})(1-\zeta_g)} < \frac{1}{\psi} \\
\Leftrightarrow \qquad &\frac{\zeta_g + a(1-\zeta_g)}{1-\zeta_g} < \frac{a(1-\eta_p)(A_{h,b}-A_{h,t})}{\mathbb{E}_p[A_{h,s}]\gamma} \\
\Leftrightarrow \qquad &\frac{\zeta_g}{1-\zeta_g} < a\left[\frac{(1-\eta_p)(A_{h,b}-A_{h,t})}{\mathbb{E}_p[A_{h,s}]\gamma} - 1\right]
\end{aligned}
$$

and finally reads as

$$
\begin{aligned}
&\frac{\zeta_g}{1-\zeta_g} < a\left[\frac{(1-\eta_p)(A_{h,b}-A_{h,t})}{\mathbb{E}_p[A_{h,s}]\gamma} - \frac{\mathbb{E}_p[A_{h,s}]\gamma}{\mathbb{E}_p[A_{h,s}]\gamma}\right] \\
\Leftrightarrow \qquad &\frac{\zeta_g}{1-\zeta_g} < \frac{(1-\eta_p)(A_{h,b}-A_{h,t})(1-\gamma) - A_{h,t}\gamma}{\mathbb{E}_g[A_{h,s}]\gamma}.
\end{aligned}
$$

■

Proof of Proposition 10. Suppose the central bank aims at setting cost factors such that it induces the capital allocation ζ_t. From proposition 1, we know that the capital allocation in the decentralized equilibrium is given by

$$\zeta = \left[1 + \left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}\right]^{-1}.$$

Equating ζ_t and ζ yields that the cost factors κ_l and κ_h must satisfy

$$\zeta_t = \left[1 + \left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}\right]^{-1}$$

$$\Leftrightarrow \qquad \frac{1+\psi\kappa_h}{1+\psi\kappa_l} = \frac{\mathbb{E}_p[A_{h,s}]}{A_l}\left(\frac{\zeta_t}{1-\zeta_t}\right)^{1-\alpha} =: a_t.$$

Rearranging then leads to

$$1+\psi\kappa_h = (1+\psi\kappa_l)a_t \qquad \Leftrightarrow \qquad \psi\kappa_h = \psi a_t\kappa_l + a_t - 1$$

and finally

$$\kappa_h = a_t\kappa_l + \frac{a_t-1}{\psi} \qquad \Leftrightarrow \qquad \kappa_h = a_t\kappa_l + \frac{a_t-1}{\psi}.$$

Note that the cost factors must also satisfy the constraint $\zeta_t\kappa_l + (1-\zeta_t)\kappa_h \geq 0$. Whenever it holds $\zeta_t = \zeta_p$, we know that

$$\zeta_t = \left[1 + \left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\right)^{\frac{1}{1-\alpha}}\right]^{-1} \qquad \Leftrightarrow \qquad \left(\frac{\zeta_t}{1-\zeta_t}\right)^{1-\alpha} = \frac{\mathbb{E}_p[A_{h,s}]}{A_l},$$

and thus $a_t = 1$. Accordingly, whenever $\zeta_t = \zeta_p$, there is no central bank intervention and cost factors equal, i.e., $\kappa_l = \kappa_h$. We can also conclude that for any $\zeta_t > (<)\zeta_p$, it holds $a_t > (<)1$ and therefore $\kappa_h > (<)\kappa_l$. ■

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