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Monetary Policy under Subjective Beliefs of Banks: Optimal Central Bank Collateral Requirements*

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Abstract

We study how the subjective beliefs about loan repayment on the side of liquidity-constrained banks affect the central bank's choice of collateral standards in its lending facilities. Optimism on the side of banks, entailing a higher collateral value of bank loans, can lead to excessive lending and bank default. Pessimism, though, can entail insufficient lending and productivity losses. With an appropriate haircut on collateral, the central bank can perfectly neutralize the banks' belief distortions and always induce the socially optimal allocation. Under uncertainty about beliefs, the central bank's incentives to set looser collateral standards increase. This reduces the risk of deficient bank lending if sufficiently pessimistic beliefs realize. In extreme cases, monetary policy aims at mitigating productivity losses only, instead of also avoiding bank default.

Keywords: beliefs, collateral, liquidity, central bank, banks

JEL Classification: D83, D84, E51, E52, E58, G21

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1 Introduction

Central bank interest rates are considered to be the main instruments in the conduct of monetary policy. Central banks set interest rates on loans and deposits of reserves, i.e., on public money that is solely available to banks, to achieve their targets referring to price stability and economic activity. The costs of borrowing reserves at the central bank influences, for instance, the banks' decision about loan financing to the real economy or interest payments on bank deposits (Kakes and Sturm, 2002). Besides interest rates, the pricing of central bank reserves is also influenced by various other factors, which have gained considerable importance at the latest after the financial crisis of 2007/08. For example, the ECB's decision in 2008 to allow banks to pledge assets of lower quality in its lending facilities has initiated a general discussion about central banks' choice of collateral standards (Nyborg, 2017).[1] A central point of this discussion was the so-called "collateral premium", namely an increase in the market value of pledgable assets that solely originates from the fact that these assets can be used as collateral in central bank operations.[2] The collateral premium may lead to improved financing conditions for the issuer and thus influence the allocation of resources in the economy. In our work, we show that beliefs among banks, which lead to an under- or overvaluation of pledgable assets, can also have an influence on the real economic allocation through central banks' collateral framework. Our findings show that central banks may ultimately be incentivized to adjust collateral standards based on the beliefs in the economy.

Our analysis is based on the observation that private agents and the central bank repeatedly disagree about the future path of macroeconomic variables. For instance, as documented by Caballero and Simsek (2020), the beliefs of the market and the Federal Reserve regarding the appropriate future interest rate policy are constantly misaligned. The authors show that such a pattern may emerge if the market acts in an opinionated way, not considering the central bank's information as superior and, as a consequence, not willing to learn from central bank announcements. Instead, agents build their own subjective beliefs about economic fundamentals and update their beliefs solely based on the observed data. In its choice of interest rates, the central bank must thus take the continuous disagreement by private agents into account to ensure that its monetary policy achieves the desired effect. In this paper, we show that beliefs in the economy may also influence the central bank in its choice of other monetary policy instruments, namely the central bank's collateral framework. Similar to Caballero and Simsek (2020), we focus on opinionated banks in our setting, which disregard the central bank's information and stick to their own subjective beliefs about firm productivity and, ultimately, about loan repayment. We show how the central bank collateral requirements depend on the banks' beliefs and which role uncertainty on the side of the central bank about beliefs plays.

Our framework embeds a banking sector and a central bank. Banks grant loans to firms, which are financed through deposit issuance. Banks must settle interbank liabilities at the central bank by using reserves. In our model, interbank liabilities arise from deposit transfers

[1] For example, see https://www.ecb.europa.eu/press/pr/date/2008/html/pr081015.en.html (accessed on January 13, 2021) for the measures taken by the ECB in October 2008 to increase the possibility for liquidity provisions.

[2] See Mésonnier et al. (2017) and Van Bekkum et al. (2018), for instance.

among banks, which occur from transactions on the goods markets. The central bank provides reserves through secured loans, where the pledgable assets are given by the loans that banks provided to firms. Monetary policy comprises the interest rates on reserve loans and reserve deposits, and the haircut on the bank loans used as collateral in central bank lending operations. We model an economy without any price rigidities, so that the neutrality of money applies, i.e., the interest policies applied by the central bank influence prices but not the real allocation. On the contrary, the central bank's collateral framework has a real effect, as it determines banks' access to liquidity in the form of reserves, which is crucial for the banks' decision of issuing deposits and granting loans. Banks are only constrained by liquidity, so that an improved access to liquidity leads to more deposit issuance and loan financing to firms in the first place. In our setting, we distinguish between loan- and bond-financed firms. A higher haircut, leading to a lower provision of central bank reserves and less bank lending, comes at the benefit of bond-financed firms, i.e., all firms in the economy that can access financing at financial markets via bond issuance and do not rely on loan financing from banks. In turn, a smaller haircut on bank loans used as collateral for central bank loans leads to more bank lending and increases the leverage of banks, while reducing the access to finance for bond-financed firms. Banks face limited liability and constraints on equity financing. Further, the loan returns are risky, as the operations of firms are subject to productivity shocks. With a sufficiently large leverage, banks can thus be exposed to a solvency risk. Bank default is costly, as the resolution of defaulting banks requires real resources, which must be provided by the government sector.

The central bank aims at maximizing welfare, so that, when choosing the haircut on bank loans, it must consider two potential effects: First, any adjustment of the haircut may affect aggregate production, as the share of production in the two sectors changes. If the productivity of loan- and bond-financed firms differs, a change of the haircut translates directly into a change of aggregate production. Second, if the haircut set by the central bank is sufficiently small, banks can leverage enough to be exposed to a solvency risk and default on the liabilities towards depositors whenever the loan returns are low. This, in turn, reduces the production output available for consumption and, ultimately, welfare, as the resolution of bank default requires real resources. Thus, the central bank must, in its choice of collateral standards, trade off productivity losses and default costs, still accounting for the banks' beliefs about firm productivity and, ultimately, about loan repayment. While the central bank has rational beliefs and thus knows the true probability distribution of productivity shocks in the economy, banks have their own subjective beliefs about firm productivity. We allow for optimistic and pessimistic banks, which believe that, compared to the true probability distribution, firms in the loan-financed sector are more and less productive, respectively. Accordingly, there might be situations where, based on their beliefs, banks want to grant more or fewer loans to firms than socially optimal.

The optimal monetary policy in our baseline model, where agents are sufficiently optimistic about production in the loan-financed sector, is simple. Whenever default costs are small enough, the central bank aims at maximizing bank lending and allowing for bank default, which is achieved by setting a sufficiently small haircut in its lending facilities. In turn, if costs associated with bank default are large enough, the central bank aims at restricting bank lending and thereby eliminating bank default, which is achieved by setting a sufficiently large haircut on the collateral provided for reserve loans.

The optimal haircut set by the central bank varies with banks' beliefs about firm productivity. The reason is that banks' beliefs influence the collateral value of bank loans and thus shape their expectation about the access to central bank liquidity. Compared to the true probability distribution, growing optimism on the side of banks leads to an overvaluation of bank loans, causing banks to expect a greater access to liquidity from the central bank and incentivizing them to provide more deposit-financed loans in the first place. To counteract the effects of growing optimism and to restore the optimal level of bank lending, the central bank must tighten collateral requirements by applying a larger haircut. The central bank thereby brings the banks' expectation about the access to reserves back to the original level and eliminates the banks' incentives to grant more loans than before. A similar mechanism applies for growing pessimism among banks. If banks believe that the productivity of loan-financed firms is lower than before, the value of bank loans in the market decreases. Accordingly, the banks' expectation about the access to liquidity lowers, so that they initially issue less deposits and grant less loans. The central bank can steer against the banks' pessimistic beliefs by loosening collateral standards through a smaller haircut. If the central bank reduces the haircut adequately, this restores banks' incentives to grant the socially optimal level of loan financing. Given the mechanism described above, if beliefs are known, the central bank can completely neutralize belief distortions on the side of banks and always induce the socially optimal allocation through its collateral framework.

The central bank's choice of the collateral requirements becomes more challenging if it faces uncertainty about the beliefs in the economy. Without knowing the actual beliefs, the central bank chooses the haircut on bank loans to maximize *expected* welfare in our framework. The central bank faces a trade-off between loose collateral standards, leading to excessive lending and costs due to bank default for more optimistic beliefs, and tight collateral requirements, leading to deficient lending and lower aggregate output for more pessimistic beliefs. We find that compared to any situation where beliefs are known, the central bank becomes less strict in its choice of the collateral framework. Specifically, it prefers bank default—compared to restrictions on bank lending—already for a higher level of default costs. The larger the uncertainty, i.e., the further the most pessimistic and most optimistic types of beliefs are apart, the more the central bank is incentivized to prefer bank default to deficient bank lending. The reason is that the more distinct the possible types of beliefs, the more costly it is for the central bank to avoid bank default for the optimistic beliefs. Instead, the central bank allows for default in the presence of more optimistic banks, while reducing deficient bank lending for more pessimistic beliefs.

The paper is organized as follows: Section 2 relates our paper to the literature, section 3 introduces our model and outlines the individually optimal behaviour of agents, and section 4 provides the equilibrium analysis. Section 5 discusses the optimal monetary policy if the central bank knows banks' beliefs perfectly, whereas section 6 provides a study of the optimal monetary policy in the presence of belief uncertainty. Section 7 discusses the optimal monetary policy in the presence of bank regulation, of information signaling, and of mistakes made by the central bank. Section 8 concludes.

2 Relation to the Literature

Our paper relates to four strands of the literature, namely the impact of non-rational expectations on macroeconomic policies, optimal monetary policies in the presence of uncertainty, private money creation, and central bank collateral frameworks.

A vast literature discusses how non-rational beliefs among private agents may curtail or amplify the effect of macroeconomic policies. Woodford (2013) reviews various modeling approaches without the hypothesis of rational expectations. A large part of this literature assumes bounded rationality of market participants and aims at finding optimal policies adressing this friction. The bounded rationality of agents has been studied in different ways, for instance, in the form of learning (e.g., Eusepi and Preston (2011)), level k-thinking (e.g., García-Schmidt and Woodford (2019)) or cognitive discounting (e.g., Gabaix (2020)). A closely related literature assumes agents are rational but are not perfectly informed about each other's beliefs, and illustrates how the resulting coordination problems can lead to aggregate behavior that resembles some forms of bounded rationality (Woodford, 2001; Morris and Shin, 2014; Angeletos and Huo, 2021). Our paper is in line with the literature on bounded rationality and information frictions, which generally assumes that the policy maker—in our setting the central bank—is fully rational. In modeling the beliefs in the economy, our work is close to Caballero and Simsek (2020), as banks act in an opinionated way and have their own subjective beliefs, following from the fact that banks interpret data differently than the central bank, for instance. In other words, banks and the central bank agree to disagree. In this regard, our paper is also related to Simsek (2013), which studies the impact of belief disagreements among private traders on collateral constraints for credit financing.

As we study the optimal monetary policy also in settings where the central bank is uncertain about the banks' beliefs, our work is closely connected to the literature that studies robust macroeconomic polices. For example, Woodford (2010) studies the optimally robust monetary policy in the form of interest rates in the presence of so-called near-rational expectations, i.e., the agents' expectations can be generated by the true economic model and are not "too far" away from rational expectations.

Our approach also follows a recent literature that models the dual role of banks—providing loans and creating money in the form of bank deposits (e.g., Faure and Gersbach (2017), Faure and Gersbach (2018) and Benigno and Robatto (2019)). In contrast to the existing literature, we model banks as liquidity-constrained and focus on the real effects of the central bank's collateral framework. We also show how the central bank can use collateral standards in its lending facilities to neutralize belief distortions among banks.

There is a substantial literature on the central bank collateral framework and its possible impact (for an overview see Bindseil (2004), and Bindseil et al. (2017), for instance). Nyborg (2017) documents the weaking fo collateral standards in the ECB's liquidity provisions after the financial crisis 2007/08. The fact that central bank collateral requirements matter for banks' liquidity holdings has been documented by Bindseil et al. (2009), for instance. In their analysis of the ECB's weekly refinancing operations between 2000 and 2001, the authors find evidence that collateral haircuts have been set imperfectly by the ECB, leaving different collateral with different opportunity costs. Fecht et al. (2011) also study the liquidity pricing in the repo transactions with the ECB and find some indication that the collateral available to the individual bank matters for the access to liquidity. Fuhrer et al. (2016) study transactions on the Swiss Franc repo market and find that collateral scarcity matters for a banks' re-use of collateral in the acquisition of liquidity. Our work relates to this literature, as we describe optimal central bank haircut rules in the lending operations to banks and show how they vary along the banks' beliefs. We also describe the optimal haircut rule on collateral if the central bank is uncertain about the banks' beliefs.

3 Model

3.1 Macroeconomic environment

We focus on a monetary economy, where trades are settled instantaneously by using money in the form of bank deposits. There are five types of agents—firms, households, investors, banks, and a government sector, including the central bank—and two goods—a capital good and a consumption good. Households and investors are endowed with the capital good which they can sell to firms for the production of the consumption good. Firms finance capital good purchases from households and investors either by demanding loans from banks or by issuing bonds at the financial markets. The model features private and public money creation. Private money takes the form of bank deposits which are issued by banks when granting loans to firms. Public money, in turn, is represented by reserves which banks can obtain from the central bank by demanding collateralized reserve loans and which are used by banks to settle interbank liabilities.[3] Liabilities arise when deposits are transferred from one bank to another, which occurs due to trading partners at the good markets holding deposit accounts with different banks. In our model, good markets and asset markets are perfectly competitive.

Firm productivity, and thus loan repayment, is subject to idiosyncratic shocks. Banks have beliefs about the probability distribution of shocks, which may deviate from the true one. The model thus accounts for optimistic (pessimistic) banks that, compared to the true distribution, overestimate (underestimate) the probability of positive productivity shocks. The beliefs about productivity shocks determine the beliefs about repayment by borrowers and thus the expected value of bank loans. Since bank loans serve as collateral for reserve loans from the central bank, banks' beliefs about the value of bank loans translate into expectations about the access to liquidity at the central bank. The central bank sets the nominal interest rates on reserve loans and reserve deposits, as well as the haircut on bank loans when used as collateral for reserve loans. With the haircut, the central bank can directly affect the banks' access to liquidity.

We model banks as being constrained by liquidity, so that both their beliefs about loan repayment and the haircut set by the central bank matter for the banks' initial decision about loan supply and deposit issuance. Banks operate with limited equity financing and provide loan financing, which is generally risky. Thus, if the leverage becomes sufficiently large in the course of loan financing, banks are exposed to a solvency risk. Deposits issued by banks are insured by the government through guarantees. We impose an implicit deposit insurance, so that in

[3] For simplification we abstract from cash. In our framework, this is without loss of generality because holding the alternative form of money, namely bank deposits, yields a positive interest.

the case of a bank default, depositors are bailed out by the government. As the government covers bank losses in the case of default, the deposits held by households are safe. Bank default, however, has real costs, as the resolution of a defaulting bank requires efforts that must be compensated with resources in the form of the consumption good. Throughout our analysis, we assume that the consolidated budget of the government sector, including the central bank, is balanced.

3.2 Summary of events

As we focus on a monetary economy with instantaneous settlement of transactions, the timing of interactions among agents is of great importance for our analysis. Figure 1 summarizes the events in our static framework.

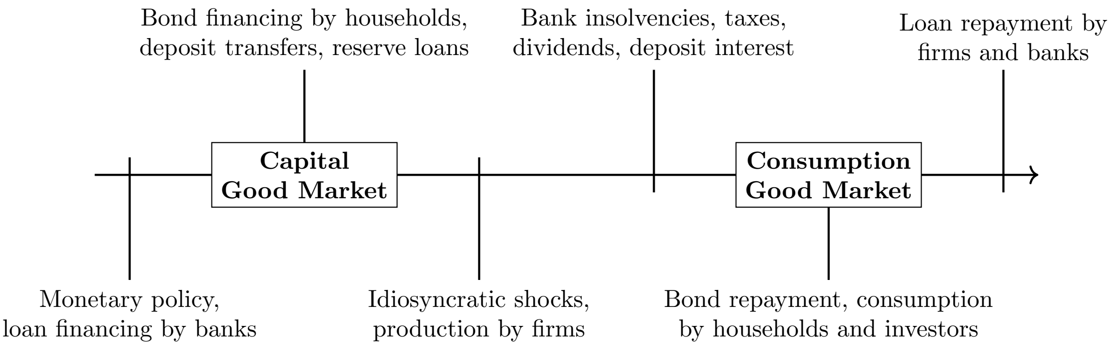

Figure 1: Timeline.

Note that all trades are settled by using bank deposits and that prices are in terms of the unit of account of the underlying currency. We use the consumption good as the numeraire of the economy. In the following subsections, we introduce the optimization problems of firms, households, investors, and banks and characterize their optimal choices. The proofs relating to the stated results can be found in appendix E.

3.3 Firms

Firms are profit-maximizing, protected by limited liability, and penniless. They purchase the capital good from households and investors to produce the consumption good. There are two types of firms, which we index by L and B. Firms of each type are ex-ante identical and exist in a continuum with mass normalized to one, so that we can focus on a representative firm for each type. Firms of type L can only obtain funds (i.e., deposits) through loans by banks, whereas firms of type B can only raise funds in a frictionless bond market.[4]

The firm of type $f \in \{L, B\}$ acquires capital good $K^f \geq 0$ on the markets from households and investors at a nominal price $Q > 0$ and transforms it into consumption good $A_s^f K^f$, where

[4] A possible justification for the assumption that L firms can only obtain loans from banks is that these firms suffer from moral hazard and banks are the only agents that can alleviate moral hazard by monitoring. The restriction that firms of type B can only access funds via the bond market serves the purpose of simplifying the subsequent analysis and can be relaxed.

$A_s^f \geq 0$ represents the available linear technology that depends on the idiosyncratic productivity shock s. The latter can be either positive ($s = \overline{s}$) or negative ($s = \underline{s}$), and thus it holds that $A_{\overline{s}}^f \geq A_{\underline{s}}^f$. The idiosyncratic productivity shocks are independent and identically distributed (i.i.d.) across firms. A positive shock occurs with probability $\eta \in (0, 1)$. Private agents—firms, households, investors and banks—have subjective beliefs which potentially deviate from the true ones. Specifically, they believe that the individual firm experiences a positive productivity shock with probability $\eta_m = \eta m$, where $m \in (0, 1/\eta)$ is the distortion factor. If the parameter m is larger (smaller) than one, we call agents and their beliefs optimistic (pessimistic).[5]

The produced consumption good $A_s^f K^f$ is sold to households and investors at a nominal price $P > 0$. The revenues, in the form of bank deposits, are then used to meet the repayment obligation towards external creditors, which, depending on the type of the firm, are banks or bond investors. The repayment is determined by the interest rate $r_s^f > 0$, which typically differs between loans and bonds. Accounting for limited liability, the expected profits of firm f are given in nominal terms by $\mathbb{E}_m[\{PA_s^f - (1 + r_s^f)Q\}^+]K^f$, where we use the notation $\{X\}^+ = \max\{X, 0\}$. Note that, due to the firms' subjective beliefs, the expectation operator is indexed by the distortion factor m. As firms are profit-maximizing, it follows that firm f's optimization problem is given in real terms by

$$\max_{K^f \geq 0} \mathbb{E}_m[\{A_s^f - (1 + r_s^f)q\}^+]K^f, \tag{1}$$

where the capital good price, denoted by a lowercase letter, is in terms of the consumption good, i.e., $q := Q/P$.

Due to limited liability, there exists no optimal, finite demand of capital good, if firm f is exposed to excess returns in at least one state. In contrast, without excess returns, firm f will be indifferent between all amounts of capital good put into production. A formal summary is provided in the following lemma.

Lemma 1 (Optimal Choice of Firms)
The optimal demand of capital good by firm $f \in \{L, B\}$ is characterized by $K^f = +\infty$ if and only if $A_s^f > (1 + r_s^f)q$ for some $s \in \{\underline{s}, \overline{s}\}$, and $K^f \in [0, +\infty)$ otherwise.

Two remarks regarding the relationship of repayment rates and firm productivity are in order. First, in any competitive equilibrium we consider, the capital good market must clear, which ultimately requires an optimal, finite demand of capital good on the side of firms. From lemma 1, we know that firms demand a finite amount of capital good if and only if the repayment obligations on external funds weakly exceed the revenues from production, i.e., if $A_s^f \leq (1+r_s^f)q$ for all f, s. Second, while agents have subjective beliefs about the probability distribution of productivity shocks, we assume that they know the economic model in all other respects perfectly. Accordingly, in equilibrium, agents' behavior cannot be subject to predictable errors. In other words, up to their beliefs, agents are fully rational, which rules out firm default in

[5] The analysis can (under additional assumptions on the trading of bank loans) also be conducted with differing beliefs among agents. To reduce complexity and highlight the relevance of banks' beliefs for the conduct of monetary policy, we focus however on the case where beliefs are shared by firms, households, investors and banks.

equilibrium. Formally, this means that it holds $A_s^f \geq (1 + r_s^f)q$ for all f, s. Based on the previous two observations, we can conclude that in any competitive equilibrium it must hold that $A_s^f = (1 + r_s^f)q$ for all f, s and firms make zero profits.

We make specific assumptions on firm productivity. First, for simplicity, we assume that bond-financed firms operate without any risk. Second, we assume that a loan-financed firm is, under the true beliefs, more productive in expectation than a bond-financed firm. This guarantees that loan-financed firms and banks—as their source of financing—are relevant for maximizing aggregate production and, ultimately, welfare. Third, when a loan-financed firm experiences a negative productivity shock, it is less productive than a bond-financed firm. As explained in subsection 4.2, the latter assumption guarantees that banks can be exposed to a solvency risk.

Assumption 1 (Firm Productivities)
$A_{\bar{s}}^B = A_{\underline{s}}^B := A^B$, $\mathbb{E}[A_s^L] \geq A^B$ *and* $A^B > A_{\underline{s}}^L$.

It follows directly from assumption 1 that a loan-financed firm is more productive than a bond-financed firm if it incurs a positive productivity shock, i.e., $A_{\bar{s}}^L > A^B$.

3.4 Households

There is a continuum of identical households with mass normalized to one, so that we can focus on a representative household. The household is endowed with capital good $K > 0$, which can be sold to firms at a nominal price $Q > 0$. The revenues are in the form of deposits and can be invested in bonds, which yield a rate of return $r^B > 0$. Deposits, in turn, are credited with interest according to the rate $r^D > 0$. The share of funds held in the form of deposits is denoted by $\gamma \in [0, 1]$. The household owns firms which distribute any available profits Π as dividends. Taking governmental taxes T^H, which are assumed to be lump-sum, and dividends Π into account, the household uses deposits credited with interest $\gamma(1 + r^D)QK$ and the revenues from bond investments $(1 - \gamma)(1 + r^B)QK$ to purchase an amount C^H of the consumption good from firms at the nominal price $P > 0$. The household maximizes utility, which we assume to be linearly increasing in consumption. Hence, the household's optimization problem is given in real terms by

$$\max_{\gamma \in [0,1]} [\gamma(1 + r^D) + (1 - \gamma)(1 + r^B)]qK + \tau^H + \pi, \tag{2}$$

where the taxes and the profits, denoted by lowercase letters, are in terms of the consumption good, i.e., $\tau^H := T^H/P$ and $\pi := \Pi/P$.

Based on the assumption of linear utility, the optimal choice of the household is of knife-edge type. The available funds are invested in the asset which yields the highest return. The following lemma provides the formal details.

Lemma 2 (Optimal Choice of the Household)
$\gamma = 1$ *(*$\gamma = 0$*) if* $r^D > (<) r^B$ *and* $\gamma \in [0, 1]$ *otherwise.*

3.5 Investors

Investors are identical and exist in a continuum of unit mass, so that we can focus on a representative investor. The investor is endowed with capital good $E > 0$, which can be sold to firms at a nominal price $Q > 0$. The revenues take the form of deposits and can be invested into equity across all active banks or bonds. The rate of return on equity for a particular bank is given by $r_s^E > 0$, which depends on the idiosyncratic shock s incurred by the respective bank. Bonds, in turn, are subject to a deterministic rate of return $r^B > 0$. The share of funds used for equity financing is denoted by $\zeta \in [0, 1]$. Taking governmental taxes T^I, which are assumed to be lump-sum, into account, the investor uses equity returns $\zeta(1 + \mathbb{E}_m[r_s^E])QE$ and the revenues from bond investments $(1 - \zeta)(1 + r^B)QE$ to purchase an amount C_m^I of the consumption good from firms at the nominal price $P > 0$. The investor maximizes the utility, which we assume to be linearly increasing in consumption. Hence, the investor's optimization problem is in real terms given by

$$\max_{\zeta \in [0,1]} [\zeta(1 + \mathbb{E}_m[r_s^E]) + (1 - \zeta)(1 + r^B)]qE + \tau^I, \tag{3}$$

where the taxes, denoted by a lowercase letter, are in terms of the consumption good, i.e., $\tau^I := T^I/P$. The expectation about the return on bank equity depends on the subjective beliefs of the investor, which may be deviate from the true one. Accordingly, the expectation operator in (3) is indexed by the distortion factor m.

Due to the assumption of linear utility, the investor's optimal choice is of knife-edge type. The available funds are used to invest into the asset which yields the highest expected return. To simplify our analysis, we assume that in the case of indifference ($\mathbb{E}_m[r_s^E] = r^B$), the investor uses all funds to invest into equity ($\zeta = 1$).

Lemma 3 (Optimal Choice of the Investor)
$\zeta = 1$ *(*$\zeta = 0$*) if and only if* $\mathbb{E}_m[r_s^E] \geq (<) r^B$.

3.6 Government sector

The government sector consists of the central bank and the government. The central bank provides banks with liquidity in the form of reserves, which banks use to settle interbank liabilities. Reserves can be borrowed from the central bank via collateralized loans. The only pledgable assets available to banks are the loans provided to firms. The value of these bank loans is reduced by a haircut $\psi \in [0, 1]$, which is determined by the central bank. The ensuing borrowing constraint on the side of banks is introduced in subsection 3.7. Reserve deposits at the central bank are credited with interest according to the rate $r_{CB}^D > 0$, while reserve loans require a repayment determined by the rate $r_{CB}^L > 0$. For simplicity, we assume that both interest rates are equal.

Assumption 2 (Reserve Rates)
$r_{CB}^D = r_{CB}^L$.

In our setting, the central bank chooses the interest rate r_{CB}^{D} and the haircut ψ in order to maximize utilitarian welfare. Details are provided in section 5, where we also characterize the optimal monetary policy.

Banks can face a solvency risk if, in the course of loan financing to firms, the leverage becomes sufficiently large; a detailed discussion is provided in subsection 3.7. In any equilibrium we consider, default by firms is ruled out (see subsection 3.3), so that banks are the only agents in our economy who can default on their liabilities. As the government insures deposits through governmental guarantees, it must balance bank losses, which in the aggregate and in nominal terms are denoted by $\Pi^{b,-}$.[6] The government finances bank losses through lump-sum taxes on households and investors. The resolution of bank default requires efforts which reduce the production output available for consumption by households and investors. Taxes are thus also required to cover the default costs, which in the aggregate and in nominal terms are given by $P\Lambda$ and are further characterized in the equilibrium analysis (see section 4). Finally, the government must use taxes to cover losses of the central bank, while it can distribute central bank profits by using transfers. We denote nominal central bank profits/ losses by Π^{CB}. Throughout our analysis, we assume that the consolidated budget of the central bank and the government is balanced, so that governmental lump-sum taxes or transfers are given in nominal terms by $T = \Pi^{b,-} - P\Lambda + \Pi^{CB}$.

3.7 Banks

There is a continuum of ex-ante identical banks with mass normalized to one, so that we can focus on a representative bank. Banks are only active if they receive equity financing $E^b > 0$ from investors. Banks and firms are matched one-to-one, which leads to banks holding non-diversified loan portfolios and being fully exposed to the idiosyncratic risk of the financed firm. When a bank is established (i.e., $E^b > 0$), the bank provides loan financing to the matched firm. The decision about loan supply L^b then determines the loans-to-equity ratio $\varphi = L^b/E^b$ and the bank's deposit financing $D^b = L^b - E^b$ once investors used (parts of) their deposits to acquire bank equity.

Transactions on the market for the capital good lead to liabilities between banks when the counterparties to a transaction hold accounts at different banks and, as a consequence, interbank deposit flows occur.[7] We assume that for each bank, a share $\alpha \in (0, 1]$ of deposits is temporarily outflowing as capital good transactions are settled. The bank will become liable for the amount of deposits going to other banks, as it adds liabilities to other banks. We assume that the bank's interbank liabilities are equally distributed across all other banks and are settled in real time, i.e., the central bank applies a gross settlement procedure. Thus, deposit outflows have to be fully settled and cannot be netted later with claims on other banks from deposit inflows. The bank must therefore borrow an amount $L^{CB} = \alpha(L^b - E^b)$ from the central bank.[8] The latter

[6] In the case of default, the bank only defaults on the deposit funding. The reserve loans can always be repaid, even in the case of default, and thus bank losses only represent the unmet liabilities towards depositors. The fact that reserves can always be repaid rests on the assumption of a representative bank (see subsection 3.7).

[7] We abstract from interbank deposit flows that are due to transactions on the market for the consumption good, as this would provide no additional insights, but would complicate our analysis.

[8] We assume that the amount of deposits used by investors to acquire bank equity does not lead to deposit outflows on the market for the capital good. Accordingly, the relevant amount of deposits, of which a share

secures its claim $(1 + r^L_{CB})L^{CB}$ by demanding collateral, which, in our setting, corresponds to the loans that banks provide to firms. The nominal value of loans, as expected by the bank before the realization of the idiosyncratic shocks, is given by $(1+\mathbb{E}_m[r^L_s])L^b$. Due to the agents' subjective beliefs, the expected value of bank loans may divert from the true one. Specifically, with a distortion factor m larger (smaller) than one, banks are optimistic (pessimistic) and, based on their expectation about repayment by borrowers, they value loans higher (lower) than under the true probability distribution. The central bank applies a haircut $\psi \in [0, 1]$ on the bank loans provided as collateral, so that $(1 - \psi)(1 + \mathbb{E}_m[r^L_s])L^b$ is the overall value of all pledgable assets, also referred to as the "collateral capacity" of the bank.

In the case of illiquidity, the bank defaults and the government seizes all assets, eliminating all potential revenues from banking. Thus, the bank's decision about loan supply and deposit issuance will always comply with the liquidity constraint

$$(1 - \psi)(1 + \mathbb{E}_m[r^L_s])L^b \geq \alpha(1 + r^L_{CB})(L^b - E^b),$$

which, using the loans-to-equity ratio $\varphi = L^b/E^b$, is equivalent to

$$(1 - \psi)(1 + \mathbb{E}_m[r^L_s])\varphi \geq \alpha(1 + r^D_{CB})(\varphi - 1),$$

where we also exploited the equality of interest rates on reserve loans and reserve deposits ($r^L_{CB} = r^D_{CB}$), following from assumption 2. The bank's beliefs about future loan repayment determine the expectation about the access to liquidity at the central bank and thus the initial decision to grant loans and finance them through deposit issuance. Using the previously outlined liquidity constraint, we can derive the maximum possible loans-to-equity ratio for which the bank is liquid. For a monetary policy (r^D_{CB} and ψ) that satisfies $\alpha(1+r^D_{CB}) > (1-\psi)(1+\mathbb{E}_m[r^L_s])$, this leverage, denoted by $\varphi^L_m(\psi)$, is determined through the condition

$$(1 - \psi)(1 + \mathbb{E}_m[r^L_s])\varphi^L_m(\psi) = \alpha(1 + r^D_{CB})[\varphi^L_m(\psi) - 1]$$

and thus leads to the maximum loans-to-equity ratio

$$\varphi^L_m(\psi) = \frac{\alpha(1 + r^D_{CB})}{\alpha(1 + r^D_{CB}) - (1 - \psi)(1 + \mathbb{E}_m[r^L_s])}.$$

Note that the ratio $\varphi^L_m(\psi)$ is indexed by the distortion factor m, as the bank's expectation about liquidity access depends on its beliefs about loan repayment. For a monetary policy satisfying $\alpha(1 + r^D_{CB}) \leq (1 - \psi)(1 + \mathbb{E}_m[r^L_s])$, the bank remains liquid for any loans-to-equity ratio, which we denote by $\varphi^L_m(\psi) = +\infty$.

As the bank can borrow reserves L^{CB} from the central bank and deposit reserves D^{CB} at the central bank, the balance sheet identity $L^b + D^{CB} = L^{CB} + D^b + E^b$ applies. We focus on a representative bank, so that deposit outflows equal deposit inflows. Accordingly, once the capital good transactions have been settled, reserve loans and reserve deposits must match,

$\alpha \in (0, 1]$ is temporarily flowing out, is given by $D^b = L^b - E^b$. Due to the equality of interest rates on reserve deposits and reserve loans (see assumption 2), we can then, without loss of generality, assume that the bank does not borrow more reserves than the required amount $\alpha(L^b - E^b)$.

i.e., formally it holds that $L^{CB} = D^{CB}$. Using $L^{CB} = \alpha(L^b - E^b)$, the bank's assets are given by $L^b + D^{CB} = (1+\alpha)L^b - \alpha E^b$ and the assets-to-equity ratio $\tilde{\varphi} = (L^b + D^{CB})/E^b$ reads $\tilde{\varphi} = (1+\alpha)L^b/E^b - \alpha = (1+\alpha)\varphi - \alpha$. For the subsequent analysis, we will mostly focus on the loans-to-equity ratio φ, as it allows for a more natural representation and analysis of the bank's optimization problem. For convenience, we will then refer to φ as the bank leverage and to $\tilde{\varphi}$ as the integrated bank leverage, which specifically accounts for the reserve holdings of the bank.

The interest rates on reserve deposits and reserve loans equal (see assumption 2). To derive the rate of return on the bank's equity financing, we can thus focus, without loss of generality, on the balance sheet identity in reduced form that is given by $L^b = D^b + E^b$, ignoring reserve deposits and reserve loans. The loans yield a return that is determined by the rate $r^L_s > 0$, which depends on the idiosyncratic shock $s \in \{\underline{s}, \overline{s}\}$ of the financed firm, whereas deposits are credited with interest according to the deterministic rate $r^D > 0$. Banking operations are protected by limited liability, so that the nominal bank equity returns satisfy $(1 + r^E_s)E^b = \{(1 + r^L_s)L^b - (1 + r^D)D^b\}^+$. These returns depend on the loan rate and therefore on the idiosyncratic shock $s \in \{\underline{s}, \overline{s}\}$ of the financed firm. We made use of the notation $\{X\}^+ = \max\{X, 0\}$ again. The rate of return per unit of bank equity then follows as $r^E_s(\varphi) := \{(r^L_s - r^D)\varphi + 1 + r^D\}^+ - 1$, where we exploited the definition of the bank leverage $\varphi = L^b/E^b$ and the fact that deposits $D^b = L^b - E^b$ are the residual funding source for loans, besides equity.

As loans are risky, the bank is exposed to a solvency risk if the leverage becomes sufficiently large in the course of loan financing to firms. For interest rates satisfying $r^D > r^L_{\underline{s}}$, the maximum leverage φ^S, which guarantees solvency of the bank in all states, is determined by

$$1 + r^E_{\underline{s}}(\varphi^S) = 0 \qquad \Leftrightarrow \qquad (r^L_{\underline{s}} - r^D)\varphi^S + 1 + r^D = 0 \qquad \Leftrightarrow \qquad \varphi^S = \frac{1 + r^D}{r^D - r^L_{\underline{s}}}.$$

For interest rates that satisfy $r^D \leq r^L_{\underline{s}}$, there is no bank leverage which exposes the bank to a solvency risk, as it holds that $1 + r^E_{\underline{s}}(\varphi) \geq 0$ for all $\varphi \geq 1$. We denote this case by $\varphi^S = +\infty$.

The bank maximizes the shareholder value, so that the optimization problem is given by

$$\max_{\varphi \in [1, \varphi^L_m(\psi)]} \mathbb{E}_m[r^E_s(\varphi)]. \tag{4}$$

The expectation operator in (4) is indexed by the distortion factor m, as the bank's beliefs about the idiosyncratic shock of the financed firm may deviate from the true ones.

In the analysis of the bank's optimal choice of leverage, we have to take into account that the bank is protected by limited liability and may face a solvency risk. First, we focus on the situation where solvency of the bank is always guaranteed, because equity financing or the haircut on bank loans used as collateral for reserve loans is sufficiently large, for instance. Formally, in any such situation, it holds that $\varphi^L_m(\psi) \leq \varphi^S$. Note that the expected rate of return on bank loans is given by $\mathbb{E}_m[r^L_s]$, whereas the interest rate on deposits is given by r^D. Thus, when granting loans funded with deposits yields profits (losses) in expectation, as it holds that $\mathbb{E}_m[r^L_s] > (<) r^D$, the bank chooses the maximum (minimum) possible leverage $\varphi = \varphi^L_m(\psi)$ ($\varphi = 1$). In other words, it supplies the maximum (minimum) possible amount of loans. If it holds that the expected interest rate on loans equals the rate of return on deposits, $\mathbb{E}_m[r_s^L] = r^D$, the bank makes zero profits by granting loans to firms which are financed through deposit issuance, and is thus indifferent between all leverages. To simplify our equilibrium analysis, we assume that in the case of indifference ($\mathbb{E}_m[r_s^L] = r^D$), the bank also chooses the maximum possible bank leverage $\varphi = \varphi_m^L(\psi)$.

Second, we consider the situation where the bank may face a solvency risk if, in the course of loan financing, the leverage grows sufficiently. Formally, such a situation is only possible if it holds that $\varphi_m^L(\psi) > \varphi^S$. The interest rate on deposits is given by r^D, whereas the expected rate of return from loans is *without* a solvency risk ($\varphi \leq \varphi^S$) given by $\mathbb{E}_m[r_s^L]$ and *with* a solvency risk ($\varphi > \varphi^S$) given by $\eta_m r_{\bar{s}}^L$. There are two types of environments in which the bank chooses the maximum possible leverage $\varphi = \varphi_m^L(\psi)$. First, even without the benefits from limited liability, financing loans with deposits is profitable ($\mathbb{E}_m[r_s^L] \geq r^D$), and thus induces the bank to grant as many loans as possible funded with deposits. Second, financing loans with deposits is not profitable without benefiting from limited liability ($\mathbb{E}_m[r_s^L] < r^D$), but the bank can leverage sufficiently, so that, with limited liability, the expected profits under the maximum leverage exceed the ones of financing loans only with equity, i.e., it holds that

$$\eta_m[(r_{\bar{s}}^L - r^D)\varphi_m^L(\psi) + 1 + r^D] > 1 + \mathbb{E}_m[r_s^L] \quad \Leftrightarrow \quad \varphi_m^L(\psi) > \frac{(1 + \mathbb{E}_m[r_s^L])/\eta_m - 1 - r^D}{r_{\bar{s}}^L - r^D}.$$

The latter condition clearly requires returns satisfying $r_{\bar{s}}^L > r^D$, namely that in the presence of a positive productivity shock of the financed firm, the interest rate on loans exceeds the interest rate on deposits. In any other environment with the possibility of a solvency risk ($\varphi_m^L(\psi) > \varphi^S$), the bank chooses to finance loans only with equity ($\varphi = 1$). The following lemma summarizes the above explanations.

Lemma 4 (Optimal Choice of the Bank)
Without the possibility of a solvency risk, i.e., if $\varphi_m^L(\psi) \leq \varphi^S$, the bank's optimal choice of leverage is given by $\varphi = \varphi_m^L(\psi)$ ($\varphi = 1$) if and only if it holds that $\mathbb{E}_m[r_s^L] \geq (<) r^D$.

With the possibility of a solvency risk, i.e., if $\varphi_m^L(\psi) > \varphi^S$, the bank's optimal choice of leverage is given by $\varphi = \varphi_m^L(\psi)$ if and only if it holds that $\mathbb{E}_m[r_s^L] \geq r^D$, or $r_{\bar{s}}^L > r^D$ and $\varphi_m^L(\psi) > [(1 + \mathbb{E}_m[r_s^L])/\eta_m - 1 - r^D]/(r_{\bar{s}}^L - r^D)$, and $\varphi = 1$ otherwise.

We also account for an interbank market, where banks are matched one-to-one and can borrow from, lend to and deposit with each other. Interbank loans are also collateralized through bank loans granted to firms, where the value of bank loans pledged for interbank loans is reduced by the haircut $\tilde{\psi} \in [0, 1]$. The collateral provided for an interbank loan can be rehypothecated when demanding a reserve loan from the central bank. The interest rates on interbank loans and interbank deposits are equal. Moreover, the bank cannot apply a different pricing on interbank deposits and deposits held by households and firms. Thus, the prevailing interest rate on the interbank market is given by $r^D > 0$.

Note that the bank granting an interbank loan must rehypothecate the pledged assets whenever the borrowing bank transfers interbank deposits to settle liabilities with other banks. The reason is that an interbank loan is completely financed with interbank deposits held by the borrowing bank. Whenever the latter must meets its liabilities with other banks, it can use the interbank deposits and transfer them to the banks it is liable to. This, however, requires the bank granting the interbank loan to hold enough liquidity to meet the liabilities arising from a transfer of interbank deposits.[9] Any bank granting an interbank loan cannot share the liquidity it obtained from the central bank or other banks. It needs this liquidity to settle its own interbank liabilities with other banks. Thus, the only way a bank can provide interbank loans and guarantee that enough liquidity is available to settle the liabilities emerging from the transfer of interbank deposits is that the bank loans pledged by the borrowing bank are completely rehypothicated at the central bank.

As stated in the following lemma, we can then establish a relationship between the terms and conditions for liquidity from the central bank, as captured by r_{CB}^D and ψ, and the standards on liquidity provision through the interbank market, namely r^D and $\tilde{\psi}$.

Lemma 5 (Interbank Market)
$(1 + r^D)(1 - \psi) = (1 + r_{CB}^D)(1 - \tilde{\psi})$.

Given a monetary policy r_{CB}^D and ψ, we can deduce that collateral requirements on the interbank market which are looser than the ones at the central bank ($\tilde{\psi} < \psi$) lead to an interest rate on the interbank market (and ultimately on bank deposits) which is higher than the interest rate on reserves ($r^D > r_{CB}^D$), and vice versa. Moreover, with identical collateral standards at the central bank and on the interbank market, the interest rates on deposits and reserves are equal.

Corollary 1 (Deposit Rate)
With $\tilde{\psi} = \psi$, it holds that $r^D = r_{CB}^D$.

For tractability of the model, we assume that the haircut on the interbank market is identical to the haircut set by the central bank ($\tilde{\psi} = \psi$). It then follows from corollary 1 that the interest rates on deposits and reserves match ($r^D = r_{CB}^D$).

Assumption 3 (Haircuts)
$\tilde{\psi} = \psi$.

4 Equilibrium Analysis

4.1 Equilibrium definition

In what follows, we focus on competitive equilibria, which are defined hereafter. We use the notation $Y := \mathbb{E}[A_s^L]K^L + A^B K^B$ to represent the aggregate production output. Note that due to the assumption that productivity shocks are i.i.d. across firms, it holds, based on the

[9] Note that we model a continuum of banks and assume that interbank liabilities of the individual bank are equally distributed across all other banks and that banks are matched one-to-one on the interbank market. It thus follows that the individual bank granting interbank loans will face a complete outflow of interbank deposits when the borrowing bank uses them to settle its liabilities with other banks.

law of large numbers, that under true beliefs, the expected production of loan-financed firms equals their aggregate production. Following the outline in subsections 3.4 and 3.5, aggregate consumption by households and investors is, under true beliefs, given by $C^H = [\gamma(1+r^D) + (1-\gamma)(1+r^B)]qK + \tau^H + \pi$ and $C^I = [\zeta(1+\mathbb{E}[r^E_s]) + (1-\zeta)(1+r^B)]qE + \tau^I$, respectively.

Definition 1 (Competitive Equilibrium)
Given a monetary policy $r^D_{CB} > 0$ and $\psi \in [0,1]$, a competitive equilibrium is a set of prices $P > 0$ and $Q > 0$, interest rates $r^L_s > 0$, $r^B > 0$, $r^D > 0$ and $r^E_s > 0$, with $s \in \{\underline{s}, \overline{s}\}$, and a set of choices K^L, K^B, γ, ζ, and φ, so that

(i) given P, Q, r^L_s, with $s \in \{\underline{s}, \overline{s}\}$, the choice K^L maximizes the expected profits of the loan-financed firm,

(ii) given P, Q, and r^B, the choice K^B maximizes the profits of the bond-financed firm,

(iii) given P, Q, r^D and r^B, the choice γ maximizes the utility of the household,

(iv) given P, Q, r^E_s, with $s \in \{\underline{s}, \overline{s}\}$, and r^B, the choice ζ maximizes the utility of the investor,

(v) given r^D_{CB}, ψ, r^L_s, with $s \in \{\underline{s}, \overline{s}\}$, and r^D, the choice φ maximizes the expected profits of the bank,

(vi) the equity, loan, capital good and consumption good markets clear, i.e., $E^b = \zeta QE$, $QK^L = \varphi E^b$, $K^L + K^B = K + E$ and $C^H + C^I = Y$.

Note that, in the definition of a competitive equilibrium, we did not account for the clearing of the deposit market, as it clears by the construction of the model.

4.2 Equilibrium properties

We first highlight some general equilibrium properties, relating to interest rates, prices, bank leverage, default costs and welfare. We then proceed by providing the necessary conditions for the existence of an equilibrium and the bank's exposure to a solvency risk.

Interest rates. In subsection 3.3, we outlined that, up to the distribution of productivity shocks, agents know the economic model perfectly. Accordingly, in equilibrium, the interest rates on loans and bonds must be directly linked to firm productivity, so that firm default is ruled out, i.e., it holds that

$$(1+r^L_s)q = A^L_s, \quad \text{with} \quad s \in \{\underline{s}, \overline{s}\}, \qquad \text{and} \qquad (1+r^B)q = A^B, \tag{5}$$

where $q = Q/P$ represents the price of the capital good in terms of the consumption good. Note that, based on assumption 1, bond-financed firms do not face productivity shocks, since it holds that $A^B_{\overline{s}} = A^B_{\underline{s}} := A^B$, and thus bonds are subject to a deterministic repayment. Moreover, from corollary 1 and assumption 3, we know that the interest rates on deposits and reserves equal ($r^D = r^D_{CB}$). From lemma 2, it follows that whenever the household invests in deposits and bonds ($0 < \gamma < 1$), the interest rates on deposits and bonds must be equal ($r^D = r^B$). Using the conditions (5), it then follows

$$(1 + r^D)q = (1 + r^D_{CB})q = A^B. \tag{6}$$

For the polar cases, where either banks issue no deposits or bond-financed firms do not operate, so that households hold no deposits or do not invest into bonds ($\gamma \in \{0, 1\}$), we impose that the interest rates on deposits and bonds are still equal ($r^D = r^B$).[10] From assumption 1, we know that it holds $A^L_{\bar{s}} > A^B > A^L_{\underline{s}}$, so that, using the conditions (5) and (6), we an conclude that the interest rate on loans is higher (lower) than the interest rate on deposits if the financed firm incurs a positive (negative) productivity shock. Formally, it holds that

$$(1 + r^L_{\bar{s}})q = A^L_{\bar{s}} > (1 + r^D)q = (1 + r^D_{CB})q = A^B > (1 + r^L_{\underline{s}})q = A^L_{\underline{s}}. \tag{7}$$

Thus, when leveraging sufficiently by financing loans with deposits, banks are exposed to a solvency risk, namely banks default, if loan repayment is low, as the financed firm incurs a negative productivity shock.

Prices. From the previously established deposit pricing condition (6), we know that the prices in our economy, namely P and Q, must satisfy

$$\frac{P}{Q} = \frac{1 + r^D_{CB}}{A^B}. \tag{8}$$

Given a capital good price Q and firm productivity A^B, the consumption good price P and the interest rate r^D_{CB} on reserves are positively correlated. An increase in the interest rate r^D_{CB} induces an increase in the consumption good price P. Similarly, given a capital good price Q and the interest rate r^D_{CB}, the consumption good price P is negatively correlated with the productivity of bond-financed firms A^B. A productivity decrease induces thus also an increase in the consumption good price.

Bank leverage. With the equilibrium conditions (5) on firms' repayment obligations, and the deposit pricing condition (6), we can express the leverage ratios φ^S and $\varphi^L_m(\psi)$, both introduced in subsection 3.7, using model primitives. Focusing on the leverage φ^S, note that it follows from condition (7) that the interest rates on deposits and loans satisfy $r^D > r^L_{\underline{s}}$. Accordingly, we know from the outline in subsection 3.7 that φ^S satisfies

$$\varphi^S = \frac{1 + r^D}{r^D - r^L_{\underline{s}}} = \frac{(1 + r^D)q}{(1 + r^D)q - (1 + r^L_{\underline{s}})q} = \frac{A^B}{A^B - A^L_{\underline{s}}}. \tag{9}$$

Given the productivity A^B of bond-financed firms, an increase of the productivity $A^L_{\underline{s}}$ of loan-financed firms for a negative productivity shock increases the leverage threshold φ^S. Similarly, given the productivity of loan-financed firms for a negativity productivity shock, an increase of

[10] In the appendix A, we outline the flow consistency of our model for the cases where households hold deposits and bonds ($0 < \gamma < 1$).

the productivity of bond-financed firms lowers the leverage threshold φ^S.

We can also express the leverage ratio $\varphi_m^L(\psi)$ in terms of economic fundamentals. Note that for a monetary policy (r_{CB}^D and ψ) satisfying $\alpha(1+r_{CB}^D) > (1-\psi)(1+\mathbb{E}_m[r_s^L])$, it follows, using conditions (5) and (6), that

$$\varphi_m^L(\psi) = \frac{\alpha(1+r_{CB}^D)q}{\alpha(1+r_{CB}^D)q - (1-\psi)(1+\mathbb{E}_m[r_s^L])q} = \frac{\alpha A^B}{\alpha A^B - (1-\psi)\mathbb{E}_m[A_s^L]}, \tag{10}$$

whereas for a monetary policy satisfying $\alpha(1+r_{CB}^D) \leq (1-\psi)(1+\mathbb{E}_m[r_s^L])$, we maintain our previous definition $\varphi_m^L(\psi) = +\infty$.

In equilibrium, the equity market and the loan market clear, so that it must formally hold that $E^b = \zeta QE$ and $QK^L = \varphi E^b$. Both market clearing conditions allow us to relate the bank leverage, the investors' equity financing decision, and real bank lending. The equilibrium leverage is given by $\varphi = K^L/(\zeta E)$ or, equivalently, real bank lending satisfies $K^L = \varphi\zeta E$. With the clearing of the capital good market, $K^L + K^B = K + E$, we know that real bond-financing is the residual of the total capital good endowment in the economy that is not used by loan-financed firms and thus is given by $K^B = K + E - K^L = K + E - \varphi\zeta E$.

Default costs. In our setting, bank default arises if the leverage of the bank is sufficiently large ($\varphi > \varphi^S$), and the financed firm incurs a negative productivity shock ($s = \underline{s}$). In terms of the consumption good, the costs of resolving bank default scale with the real amount of bank assets after repayment and are in the aggregate given by

$$\Lambda = (1-\eta)\lambda(1+r_{\underline{s}}^L)qK^L = (1-\eta)\lambda A_{\underline{s}}^L K^L, \tag{11}$$

where $\lambda \in (0,1)$ is used for scaling purposes and referred to as default cost parameter.[11] Note that in the presence of solvency risk, the default costs created by a single defaulting bank are given by $\lambda(1+r_{\underline{s}}^L)qK^L$, which, using the equilibrium conditions (5) on firms' repayment rates, read as $\lambda A_{\underline{s}}^L K^L$. As productivity shocks are i.i.d. across firms, and banks are matched one-to-one with firms, a mass $1-\eta$ of banks defaults in the presence of solvency risk. We accounted for this fact in the specification of the aggregate default costs Λ.

Welfare. Throughout our analysis, we focus on utilitarian welfare. Based on our assumption of linear utility for households and investors, welfare, which is denoted by W, is represented by aggregate consumption, so that it holds that $W = C^H + C^I$. Lemma 6 provides a characterization of welfare, using primitives of the model.

Lemma 6 (Welfare)
In equilibrium, welfare is $W = \{\mathbb{E}[A_s^L] - \mathbb{1}\{\varphi > \varphi^S\}(1-\eta)\lambda A_{\underline{s}}^L\}K^L + A^B(K+E-K^L)$.

In our framework, aggregate consumption, and thus utilitarian welfare, comprises aggregate production $\mathbb{E}[A_s^L]K^L + A^B K^B$ and costs due to bank default $(1-\eta)\lambda A_{\underline{s}}^L K^L$. Note that bank default occurs only if banks operate under a sufficiently high leverage ($\varphi > \varphi^S$). Moreover, the amount of capital good used by bond-financed firms equals the total capital good endowment in the economy less the capital good used by loan-financed firms, i.e., $K^B = K + E - K^L$.

[11]For an analysis of various other default cost specifications, see Malherbe (2020), for instance.

Existence and solvency risk. In the following, we provide necessary conditions for the existence of an equilibrium and the bank's exposure to a solvency risk. We restrict private agents to be sufficiently optimistic about the productivity of loan-financed firms. Specifically, we assume that firms, households, investors and banks always believe that a loan-financed firms is at least as productive on average as a bond-financed firm.

Assumption 4 (Beliefs)
$\mathbb{E}_m[A^L_s] \geq A^B$.

In appendix C, we provide the equilibrium analysis and a characterization of the optimal monetary policy in the case of sufficiently pessimistic agents, namely when the distortion factor m satisfies $\mathbb{E}_m[A^L_s] < A^B$.

First, note that under assumption 4, it follows that the expected loan rate weakly exceeds the deposit rate ($\mathbb{E}_m[r^L_s] \geq r^D$). This follows from the equilibrium link between productivity and the firms' repayment obligations, namely $(1 + r^L_s)q = A^L_s$ for all s and $(1 + r^D)q = (1 + r^B)q = A^B$. From lemma 4, we then know that the bank always chooses the maximum possible leverage. Moreover, due to the expected loan return exceeding the interest payments on deposits, the expected rate of return on bank equity will also be weakly larger than the interest rate on bonds ($\mathbb{E}_m[r^E_s] \geq r^B$). Using lemma 3, we can conclude that the investor uses all funds to provide equity financing for the bank.

Lemma 7 (Bank Leverage and Equity Financing)
It holds that $\varphi = \varphi^L_m(\psi)$ *and* $\zeta = 1$.

In any competitive equilibrium, the equity market must clear, so that the equity financing of banks is given by $E^b = QE$. Moreover, the clearing of the loan market, $QK^L = \varphi E^b$, allows us to express real bank lending as $K^L = \varphi^L_m(\psi)E$. The existence of an equilibrium depends crucially on the clearing of the capital good market. Specifically, we require that the loans granted by banks do not allow firms to acquire more than the entire capital good in the economy. Formally, it must hold that $QK^L = \varphi^L_m(\psi)QE \leq Q(K + E)$, which, using $\varphi^M := 1 + K/E$ to denote the maximum feasible bank leverage, is equivalent to $\varphi^L_m(\psi) \leq \varphi^M$. The latter condition leads us to a smallest feasible haircut ψ^M_m which, if implemented by the central bank, allows loan-financed firms to exactly acquire the entire capital good in the economy. Any haircut larger than ψ^M_m restricts the capital good purchases of the loan-financed sector below the maximum feasible ones and implicitly shifts capital good for production to bond-financed firms. The central bank's choice of the haircut has thus a direct influence on the capital allocation in the economy.

As outlined in subsection 3.7, the bank is exposed to a solvency risk whenever the bank's leverage is sufficiently high to surpass the threshold φ^S. Since under assumption 4, the bank always attains the maximum possible leverage, solvency risk exists whenever it holds that $\varphi^L_m(\psi) > \varphi^S$. Equation (9) provides the leverage ratio φ^S expressed using economic fundamentals. The condition $\varphi^L_m(\psi) > \varphi^S$ allows us to derive a critical haircut ψ^S_m, so that for any haircut ψ smaller than ψ^S_m the bank is exposed to solvency risk, as the leverage of the bank exceeds the threshold φ^S. The formal details are provided in the following proposition.

Proposition 1 (Existence and Solvency Risk)
A competitive equilibrium exists if it holds that

$$\psi \geq \psi_m^M := 1 - \frac{\alpha A^B}{\mathbb{E}_m[A_s^L](1 + E/K)},$$

and the bank is exposed to a solvency risk if it holds that

$$\psi < \psi_m^S := 1 - \frac{\alpha A_{\underline{s}}^L}{\mathbb{E}_m[A_s^L]}.$$

Besides the capital allocation in the economy, the choice of the haircut by the central bank also affects the bank's exposure to a solvency risk and thus the occurrence of bank default. These two channels are at the core of the subsequent analysis of the optimal monetary policy.

5 Optimal Monetary Policy

In this section, we characterize the optimal monetary policy as represented by the interest rate on reserves r_{CB}^D and the haircut ψ that applies to bank loans pledged as collateral for reserve loans. We also highlight the effect of economic fundamentals and the banks' beliefs about firm productivity and loan repayment, as captured by the distortion factor m, on the optimal monetary policy. In this section, it is assumed that the central bank perfectly knows the beliefs in the economy when deciding about the monetary policy.

While the central bank's choice of the haircut always influences the capital allocation in the economy, the central bank can only trigger or eliminate bank default whenever there is the possibility for a solvency risk. For what follows, we will focus on the case, where, at least under the maximum feasible bank leverage, the bank faces a solvency risk.

Assumption 5 (Solvency Risk)
$\varphi^M > \varphi^S$ *or , equivalently,* $\psi_m^M < \psi_m^S$.

In our setting, the central bank is maximizing utilitarian welfare. We first observe that the interest rate on reserves r_{CB}^D affects the prices in our economy (see equation (8) in subsection 4.2). From lemma 6, in turn, we know that independent of the banks' exposure to a solvency risk, welfare is not influenced by the rate of return on reserves r_{CB}^D. This is a manifestation of the neutrality of money. To the contrary, the haircut ψ set by the central bank generally influences the capital allocation in the economy as well as the banks' exposure to a solvency risk, through its impact on bank lending. With the haircut on bank loans used as collateral, the central bank can regulate the bank's access to liquidity, namely their ability to borrow reserves. As the liquidity constraint, which depends on the haircut ψ, influences the bank's initial decision to grant loans and finance them through deposit issuance, the central bank is able to affect bank lending. Taking the irrelevance of the interest rate r_{CB}^D for the real allocation into account, the optimization problem of the central bank is formally given by

$$\max_{\psi\in[0,1]} W = \max_{\psi\in[0,1]} \{\mathbb{E}[A_s^L] - \mathbb{1}\{\varphi > \varphi^S\}(1-\eta)\lambda A_{\underline{s}}^L\}K^L + A^B(K + E - K^L),$$

where we used lemma 6 to express welfare W. With the previous results on the existence of an equilibrium and the bank's exposure to a solvency risk (see proposition 1), we can rewrite the optimization problem of the central bank, as outlined in the following lemma.

Lemma 8 (The Central Bank's Optimization Problem)
The optimization problem of the central bank is

$$\max_{\psi\in[\psi_m^M,1]} \{\mathbb{E}[A_s^L] - A^B - \mathbb{1}\{\psi < \psi_m^S\}(1-\eta)\lambda A_{\underline{s}}^L\}\varphi_m^L(\psi).$$

The optimal monetary policy in the form of the haircut ψ depends on the productivity in the two production sectors, A^B and A_s^L, with $s \in \{\underline{s}, \overline{s}\}$, the default costs, as proxied by the parameter λ, and the beliefs, as captured by the distortion factor m.

In its choice of the haircut, the central bank must generally trade off productivity losses and costs due to bank default. From assumption 1, we know that under the true beliefs, a loan-financed firm is weakly more productive in expectation than a bond-financed firm ($\mathbb{E}[A_s^L] \geq A^B$). Thus, the central bank has no incentive to choose a haircut larger than ψ_m^S. Restricting the bank leverage below φ^S, by setting a haircut higher than ψ_m^S, only reduces bank lending with negative effects for aggregate production and ultimately welfare, but does not yield any benefit.

Furthermore, we know, based on assumption 5, that bank lending is not maximized under the haircut ψ_m^S, as it holds that $\varphi^M > \varphi^S$ or, equivalently, $\psi_m^M < \psi_m^S$. Any haircut lower than ψ_m^S will lead to costs due to bank default but further extend loan financing by banks. A necessary condition for the optimality of any haircut lower than ψ_m^S is that under true beliefs the expected productivity difference of loan-financed and bond-financed firms is positive, even when taking the costs of bank default into account. Formally, it must hold that

$$\mathbb{E}[A_s^L] - A^B - (1-\eta)\lambda A_{\underline{s}}^L > 0 \qquad \Leftrightarrow \qquad \lambda < \lambda^S := \frac{\mathbb{E}[A_s^L] - A^B}{(1-\eta)A_{\underline{s}}^L}.$$

With a default cost parameter satisfying $\lambda < \lambda^S$, welfare is for any situation with bank default maximized for the smallest feasible haircut ψ_m^M. Accordingly, we can conclude that the central bank optimally chooses the haircut ψ_m^S, restricting bank lending and ruling out bank default, instead of the haircut ψ_m^M, maximizing bank lending but allowing for bank default, if it holds that

$$\{\mathbb{E}[A_s^L] - A^B\}\varphi^S \geq \{\mathbb{E}[A_s^L] - A^B - (1-\eta)\lambda A_{\underline{s}}^L\}\varphi^M,$$

from which we can derive a condition on the default cost parameter λ, as represented by equation (12) in proposition 2. Note that, based on assumption 5, it holds that $\lambda^M < \lambda^S$. Accordingly, we can conclude that the central bank chooses the haircut ψ_m^S, restricting bank lending and ruling out bank default, if and only if default costs are sufficiently large, i.e., it holds that $\lambda \geq \lambda^M$.

Otherwise, the central bank optimally chooses the smallest feasible haircut ψ_m^M, maximizing bank lending and allowing for bank default. The previous explanations are summarized in the following proposition.

Proposition 2 (Optimal Monetary Policy)
The central bank optimally restricts liquidity, so that banks are not exposed to a solvency risk, by setting the haircut ψ_m^S, if and only if default costs are sufficiently large, i.e., it holds that

$$\lambda \geq \lambda^M := \left(1 - \varphi^S/\varphi^M\right) \lambda^S. \tag{12}$$

Otherwise, the central bank optimally sets the haircut ψ_m^M, maximizing bank lending and allowing for bank default.

The more optimistic agents are about productivity shocks in the loan-financed sector (i.e., m is increasing), the higher the expected value of bank loans and thus the larger the liquidity access expected by banks, which in turn leads to more loan financing in the first place. To restrict bank lending to the optimal level, the central bank must counteract the effect of agents' more optimistic beliefs by implementing tighter collateral standards in the form of a larger haircut on bank loans. Thus, the haircuts ψ_m^S and ψ_m^M, as provided in proposition 1, both increase with m. Similarly, with growing pessimism (i.e., m is decreasing), the central bank applies looser collateral requirements, adjusting the respective haircut downwards. With perfect information, the central bank can completely eliminate any belief distortions of private agents and induce the desired bank leverage (φ^S or φ^M).

Corollary 2 (Optimal Monetary Policy and Beliefs)
Suppose the central bank implements the monetary policy according to proposition 2. Then, the optimal haircut increases (decreases) with more optimistic (pessimistic) beliefs, i.e., it holds that

$$\frac{\partial \psi_m^S}{\partial m} = \frac{\alpha A_{\bar{s}}^L \eta (A_{\bar{s}}^L - A_{\underline{s}}^L)}{(\mathbb{E}_m[A_s^L])^2} > 0 \qquad \textit{and} \qquad \frac{\partial \psi_m^M}{\partial m} = \frac{\alpha A^B \eta (A_{\bar{s}}^L - A_{\underline{s}}^L)}{(\mathbb{E}_m[A_s^L])^2 (1 + E/K)} > 0.$$

Whenever the central bank optimally aims at restricting bank lending and thereby ruling out bank default, it chooses the haircut ψ_m^S which is independent of the productivity in the bond-financed sector (see proposition 1). The productivity of loan-financed firms, in turn, influences the haircut in two ways: On the one hand, an increase of the productivity for any state s leads to a higher expected value of bank loans, increasing the bank's collateral capacity. On the other hand, an increase of the productivity in the low productivity state ($s = \underline{s}$) leads to a higher leverage ratio $\varphi^S = A^B/(A^B - A_{\underline{s}}^L)$ guaranteeing the solvency of banks. If the productivity in the high productivity state increases, the value of expected bank loans increases but the leverage threshold φ^S is left unchanged. Thus, the central bank must increase the optimal haircut ψ_m^S to counteract the increase in the valuation of bank loans. If the productivity in the low productivity state increases, the value of bank loans and the critical leverage threshold φ^S both increase. The first effect incentivizes the central bank to increase the optimal haircut ψ_m^S, while the second effect incentivizes the central bank to decrease the haircut. It turns out that the second effect dominates the first one and the central bank, ultimately, lowers the optimal haircut ψ_m^S if the productivity in the low productivity state increases (see corollary 3).

If the central bank aims at implementing maximum bank lending and thereby allowing for bank default, it chooses the haircut ψ_m^M which decreases with the productivity A^B in the bond-financed sector. The haircut ψ_m^M also increases with an improved productivity of loan-financed firms in any state. A higher productivity in the loan-financed sector increases the bank's collateral capacity and allows the bank, ceteris paribus, to borrow more reserves at the central bank and to extend loan supply as well as deposit issuance in the first place. To restrict bank leverage again to the maximum feasible one φ^M, the optimal haircut ψ_m^M must increase if the productivity in the loan-financed sector increases for any state. The details are provided in the following corollary.

Corollary 3 (Optimal Monetary Policy and Productivity)
Suppose the central bank implements the monetary policy according to proposition 2. Then, the haircut ψ_m^S does not vary with the productivity of bond-financed firms, but increases (decreases) with the productivity of loan-financed firms in the high (low) productivity state, i.e.,

$$\frac{\partial \psi_m^S}{\partial A^B} = 0, \qquad \frac{\partial \psi_m^S}{\partial A_{\overline{s}}^L} = \frac{\alpha \eta_m A_{\underline{s}}^L}{(\mathbb{E}_m[A_s^L])^2} > 0, \qquad \textit{and} \qquad \frac{\partial \psi_m^S}{\partial A_{\underline{s}}^L} = -\frac{\alpha \eta_m A_{\overline{s}}^L}{(\mathbb{E}_m[A_s^L])^2} < 0.$$

In turn, the haircut ψ_m^M declines with a higher productivity of bond-financed firms, i.e.,

$$\frac{\partial \psi_m^M}{\partial A^B} = -\frac{\alpha}{\mathbb{E}_m[A_s^L](1+E/K)} < 0,$$

and increases with the productivity of loan-financed firms in both states, i.e.,

$$\frac{\partial \psi_m^M}{\partial A_{\overline{s}}^L} = \frac{\alpha A^B \eta_m}{(\mathbb{E}_m[A_s^L])^2(1+E/K)} > 0 \qquad \textit{and} \qquad \frac{\partial \psi_m^M}{\partial A_{\underline{s}}^L} = \frac{\alpha A^B (1-\eta_m)}{(\mathbb{E}_m[A_s^L])^2(1+E/K)} > 0.$$

6 Optimal Monetary Policy with Uncertainty about Beliefs

In this section, we analyze the optimal monetary policy if the central bank is uncertain about the beliefs in the economy. Formally, the central bank cannot perfectly observe the actual distortion factor $m \in (0, 1/\eta)$. We study the optimal monetary policy in a setting where there are two potential types of beliefs that realize with positive probabilities, not necessarily being uniform. We derive analytical results and study related simulations.

For the subsequent analysis, we make two assumptions on the costs of bank default. First, we assume that default costs are sufficiently large, i.e., $\lambda \geq \lambda^M$, so that, with perfect knowledge about the actual beliefs m in the economy, the optimal monetary policy would restrict bank lending in order to rule out bank default, which is achieved by implementing the haircut ψ_m^S leading to the bank leverage $\varphi_m^L(\psi_m^S) = \varphi^S$. Second, for tractability, we make the assumption that the loan-financed sector is in expectation more productive than the bond-financed sector, even when accounting for costs due to bank default, i.e., it holds that $\mathbb{E}[A_s^L] - A^B - (1-\eta)\lambda A_{\underline{s}}^L > 0$

or, equivalently, $\lambda < \lambda^S$.[12]

Assumption 6 (Default Costs)
$\lambda^S > \lambda \geq \lambda^M$.

The collateral requirements, in the form of the haircut, are set before the actual beliefs in the economy can be observed. In its choice of the haircut, the central bank aims at maximizing expected welfare and thus faces a general trade-off between restricting loan financing for some beliefs and allowing for bank default in the presence of other beliefs. A particular choice of the haircut that maximizes expected welfare will maximize actual welfare only for a specific distortion factor, say $\hat{m}$, as it leads to the bank leverage $\varphi^L_{\hat{m}}(\psi^S_{\hat{m}}) = \varphi^S$ only for beliefs that are described by the distortion factor $m = \hat{m}$. In this particular case, not only *expected* welfare but also *actual* welfare is maximized with the haircut choice $\psi^S_{\hat{m}}$. However, for more pessimistic beliefs, i.e., for the actual distortion factor m satisfying $m < \hat{m}$, the chosen haircut will induce a bank leverage $\varphi^L_m(\psi^S_{\hat{m}}) < \varphi^S$ and thereby cause a decline in aggregate production, compared to any situation where the haircut is chosen without uncertainty about beliefs. Similarly, for more optimistic beliefs, i.e., for the actual distortion factor m satisfying $m > \hat{m}$, banks attain the leverage $\varphi^L_m(\psi^S_{\hat{m}}) > \varphi^S$, so that some banks default, as the financed firm incurs a negative productivity shock. In its choice of monetary policy, the central bank must thus account for the fact that for some beliefs that may realize in the economy the chosen haircut will induce a suboptimal level of bank lending, either leading to deficient bank lending and productivity losses, or to excessive bank lending and bank default.

The beliefs in the economy can only be of two types, namely one of the two distortion factors $\underline{m} \in (0, 1/\eta)$ and $\overline{m} \in (0, 1/\eta)$, satisfying $\underline{m} < \overline{m}$, prevails. From the central bank's perspective, the distortion factors $\underline{m}$ and $\overline{m}$ realize with probability $p \in (0, 1)$ and $1 - p$, respectively. Accordingly, the expected welfare, which the central bank aims to maximize with its choice of the haircut ψ, is given by

$$\mathbb{E}[W_m(\psi)] = pW_{\underline{m}}(\psi) + (1 - p)W_{\overline{m}}(\psi).$$

Using our previous results on equilibrium welfare (see lemma 6) as well as the conditions for the existence of an equilibrium and the bank's exposure to a solvency risk (see proposition 1), we can derive a reduced form of the central bank's optimization problem, as provided in the following lemma.

Lemma 9 (The Central Bank's Optimization Problem with Uncertainty)
The optimization problem of the central bank is

$$\max_{\psi \in [\psi^M_{\overline{m}}, 1]} (\mathbb{E}[A^L_s] - A^B)\mathbb{E}[\varphi^L_m(\psi)] - (1 - \eta)\lambda A^L_{\underline{s}} \mathbb{E}[\mathbb{1}\{\psi < \psi^S_m\}\varphi^L_m(\psi)].$$

First, note that the smallest feasible haircut $\psi^S_{\underline{m}}$, which guarantees the clearing of the capital good market in the presence of the more pessimistic beliefs $\underline{m}$, satisfies $\psi^M_{\underline{m}} < \psi^M_{\overline{m}}$. Without

[12] In appendix B, we outline the optimal monetary policy when the latter assumption is not satisfied, so that default costs are sufficiently large, i.e., $\lambda \geq \lambda^S$.

knowing the actual beliefs in the economy, the central bank is unable to set the any haircut smaller than $\psi^M_{\overline{m}}$ (for instance, $\psi^M_{\underline{m}}$), as such a haircut would not allow the clearing of the capital good market if indeed the more optimistic beliefs $\overline{m}$ in the economy prevail. Thus, the smallest possible haircut the central bank can set is given by $\psi^M_{\overline{m}}$, which we already addressed in the formulation of the central bank's optimization problem (see lemma 9).

Second, note that the haircuts $\psi^S_{\underline{m}}$ and $\psi^S_{\overline{m}}$, which rule out bank default in the presence of the more pessimistic and optimistic beliefs, respectively, satisfy $\psi^S_{\underline{m}} < \psi^S_{\overline{m}}$. Accordingly, choosing the haircut $\psi^S_{\overline{m}}$ guarantees that bank default does not occur, independent of the actual beliefs in the economy. The central bank has no incentive to set any haircut larger than $\psi^S_{\overline{m}}$, since this restricts bank lending but does not yield any benefit, as ruling out bank default, for instance. Thus, the central bank chooses the optimal haircut from the closed set ranging from $\psi^M_{\overline{m}}$ to $\psi^S_{\overline{m}}$.

Third, it turns out that the optimal monetary policy depends on how close or distinct the two types of beliefs are. With sufficiently distinct beliefs $\underline{m}$ and $\overline{m}$, the smallest possible haircut $\psi^M_{\overline{m}}$ does not expose banks to a solvency risk if the more pessimistic beliefs $\underline{m}$ realize. In such a situation, it formally holds that $\varphi^L_{\underline{m}}(\psi^M_{\overline{m}}) \leq \varphi^S$. Thus, in the presence of the more pessimistic beliefs $\underline{m}$, banks never experience default, independent of the haircut set by the central bank. Instead, if beliefs $\underline{m}$ and $\overline{m}$ are sufficiently close, so that it holds $\varphi^L_{\underline{m}}(\psi^M_{\overline{m}}) > \varphi^S$, banks can be exposed to a solvency risk, at least under the smallest feasible haircut $\psi^M_{\overline{m}}$, also if the more pessimistic beliefs $\underline{m}$ prevail. From the condition $\varphi^L_{\underline{m}}(\psi^M_{\overline{m}}) = \varphi^S$, we can derive the belief threshold $\tilde{m}$ satisfying $\tilde{m} < \overline{m}$. We then classify the beliefs $\underline{m}$ and $\overline{m}$ as distinct (close) if it holds $\underline{m} \leq \tilde{m}$ ($\underline{m} > \tilde{m}$). The details are provided in the following lemma.

Lemma 10 (Belief Differences)

The beliefs $\underline{m}$ and $\overline{m}$ are distinct (close) if it holds that $\underline{m} \leq \tilde{m}$ ($\tilde{m} > \underline{m}$), where

$$\tilde{m} = \delta\overline{m} - \frac{A^L_{\underline{s}}(1-\delta)}{\eta(A^L_{\overline{s}} - A^L_{\underline{s}})}, \qquad \textit{with} \qquad \delta = \frac{(1+E/K)A^L_{\underline{s}}}{A^B}.$$

Since $\delta < 1$, it follows that $\tilde{m} < \overline{m}$.

We now characterize the optimal monetary policy in any situation where the beliefs, as represented by the distortion factors $\underline{m}$ and $\overline{m}$, are distinct (see lemma 10). With such beliefs, banks do not face a solvency risk in the presence of the more pessimistic beliefs, even if the central bank implements the smallest feasible haircut $\psi^M_{\overline{m}}$. Accordingly, in its choice of the haircut, the central bank must trade off default costs in the presence of the more optimistic beliefs and restricted loan financing in the presence of the more pessimistic beliefs. Due to our linear production technologies, the central bank in effect only chooses between accepting and eliminating bank default in case the more optimistic beliefs realize, by setting the haircuts $\psi^M_{\overline{m}}$ and $\psi^S_{\overline{m}}$, respectively. The former haircut is the smallest feasible haircut for the central bank, as it just guarantees the existence of an equilibrium in the presence of the more optimistic beliefs $\overline{m}$. The haircut $\psi^S_{\overline{m}}$, in turn, eliminates solvency risk in the presence of more optimistic beliefs. The central bank deviates from the objective of ruling out solvency risk, i.e., it implements the haircut $\psi^M_{\overline{m}}$ instead of the haircut $\psi^S_{\overline{m}}$, whenever the default costs are sufficiently low. Formally, when the default cost parameter λ satisfies $\lambda < \lambda^M_{BU}$, with λ^M_{BU} being provided in proposition

3, the central bank decides to induce the maximum bank lending and accept bank default for the more optimistic beliefs.

Proposition 3 (Optimal Monetary Policy with Uncertainty - Distinct Beliefs) *Suppose the beliefs $\underline{m}$ and $\overline{m}$ are distinct ($\underline{m} \leq \tilde{m}$). Then, the central bank optimally chooses*

(i) the haircut $\psi^M_{\overline{m}}$, accepting bank default for the more optimistic beliefs $\overline{m}$, if and only if it holds that

$$\lambda < \lambda^M_{BU} := \lambda^M + \lambda^S \frac{p}{1-p} \frac{\varphi^L_{\underline{m}}(\psi^M_{\overline{m}}) - \varphi^L_{\underline{m}}(\psi^S_{\overline{m}})}{\varphi^M},$$

(ii) the haircut $\psi^S_{\overline{m}}$, ruling out bank default for any type of beliefs, otherwise.

We can show that, compared to any situation without uncertainty about beliefs, the central bank accepts default of banks already at higher costs, as measured by the parameter λ. With certainty, in the presence of the more optimistic beliefs $\overline{m}$, the central bank accepts bank default by setting the smallest feasible haircut $\psi^M_{\overline{m}}$, whenever default costs satisfy $\lambda < \lambda^M$ (see proposition 2). In turn, with uncertainty about beliefs, the central bank must decide about the haircut on bank loans without knowing whether the beliefs in the economy are given by $\underline{m}$ or $\overline{m}$. Proposition 3 outlines that for sufficiently distinct beliefs, the central bank implements the smallest feasible haircut $\psi^M_{\overline{m}}$ whenever default costs satisfy $\lambda < \lambda^M_{BU}$. As stated in corollary 4, in the presence of belief uncertainty, the central bank chooses the smallest feasible haircut already at higher default costs, compared to any situation where it knows with certainty that beliefs are represented by the distortion factor $\overline{m}$. Formally, this means that the critical default cost parameters satisfy $\lambda^M_{BU} > \lambda^M$. This result is based on the fact that for a positive likelihood of the more pessimistic beliefs $\underline{m}$, the central bank must not only trade off restricted bank lending and default costs in the presence of the more optimistic beliefs $\overline{m}$, as in the case without uncertainty, but must also account for restricted bank lending in case the more pessimistic beliefs $\underline{m}$ realize. This incentivizes the central bank to prefer bank default over restricted bank lending already for higher default costs, compared to the situation without belief uncertainty.

Corollary 4 (Optimal Monetary Policy with Uncertainty - Distinct Beliefs) *Suppose the central bank faces uncertainty about beliefs and the beliefs are distinct ($m \leq \tilde{m}$). Then, compared to any situation where the beliefs $\overline{m}$ realize with certainty, the central bank chooses the smallest feasible haircut $\psi^M_{\overline{m}}$ already at higher default costs, i.e., $\lambda^M_{BU} > \lambda^M$.*

Next, we focus on the case where the possible beliefs are sufficiently close, i.e., $\underline{m} > \tilde{m}$, and outline the optimal monetary policy. In any such environment, even in the presence of the more pessimistic beliefs, banks are exposed to a solvency risk if the central bank chooses the smallest feasible haircut $\psi^M_{\overline{m}}$, i.e., it holds that $\varphi^L_{\underline{m}}(\psi^M_{\overline{m}}) > \varphi^S$. In its choice of the haircut, the central bank then has to take into account that banks may default independent of the actual beliefs in the economy. Due to our linear production technologies in the economy, we can state that there are three possible haircuts the central bank actually chooses. First, the central bank may allow for bank default independent of the beliefs in the economy by setting the smallest feasible haircut $\psi^M_{\overline{m}}$. Such a monetary policy is always optimal if the default costs are sufficiently small, i.e., $\lambda < \lambda^M_{BU}$, or, in other words, ruling out bank default for any type of beliefs yields a lower welfare, and the alternative of eliminating bank default for the more pessimistic beliefs does not yield a welfare gain either, which, following proposition 4, is captured by the inequality $\lambda^S_{BU} \leq \lambda^M_{BU}$. Second, the central bank may allow for default of banks in the presence of the more optimistic beliefs, but rule it out for the more pessimistic beliefs, which is achieved by setting the haircut $\psi^S_{\underline{m}}$. Such a monetary policy is optimal if the default costs are sufficiently small, i.e., $\lambda < \lambda^S_{BU}$, or, in other words, if eliminating bank default for any beliefs yields a lower welfare, and if the alternative of accepting bank default independent of beliefs does not yield a welfare gain either, i.e., $\lambda^M_{BU} < \lambda^S_{BU}$. Third, whenever it holds $\max\{\lambda^M_{BU}, \lambda^S_{BU}\} \leq \lambda$, the central bank optimally chooses to rule out bank default independent of the actual beliefs in the economy, which is achieved by setting the haircut $\psi^S_{\overline{m}}$, as it yields a higher welfare compared to the alternatives, where bank default is accepted at least for one particular type of beliefs.

Proposition 4 (Optimal Monetary Policy with Uncertainty - Close Beliefs)
Suppose the beliefs $\underline{m}$ and $\overline{m}$ are close ($\underline{m} > \tilde{m}$). Then, the central bank optimally chooses

(i) the haircut $\psi^M_{\overline{m}}$, accepting bank default for any beliefs, if and only if it holds that

$$\lambda < \lambda^M_{BU} := \lambda^M + \lambda^S \left(\frac{\varphi^S}{\varphi^M} - \frac{p\varphi^L_{\underline{m}}(\psi^S_{\overline{m}}) + (1-p)\varphi^S}{p\varphi^L_{\underline{m}}(\psi^M_{\overline{m}}) + (1-p)\varphi^M} \right),$$

and

$$\lambda^M_{BU} \geq \lambda^S_{BU} := \lambda^M + \lambda^S \left(\frac{\varphi^S}{\varphi^M} - \frac{p[\varphi^L_{\underline{m}}(\psi^S_{\overline{m}}) - \varphi^S] + (1-p)\varphi^S}{(1-p)\varphi^L_{\overline{m}}(\psi^S_{\underline{m}})} \right),$$

(ii) the haircut $\psi^S_{\underline{m}}$, ruling out bank default for the more pessimistic beliefs $\underline{m}$, if and only if it holds that $\lambda < \lambda^S_{BU}$ and $\lambda^M_{BU} < \lambda^S_{BU}$, and

(iii) the haircut $\psi^S_{\overline{m}}$, ruling out bank default for any beliefs, if and only if it holds that $\max\{\lambda^M_{BU}, \lambda^S_{BU}\} \leq \lambda$.

Next, we provide simulations to illustrate the effect of the beliefs ($\underline{m}$ and $\overline{m}$) and the probability distribution of beliefs (p) on the optimal monetary policy. The parameter specification depicted in table 1 represents our baseline, which is also in line with assumption 6.[13]

Parameter	A^B	$A^L_{\overline{s}}$	$A^L_{\underline{s}}$	η	$\underline{m}$	$\overline{m}$	E/K	α	λ
Value	1	1.6	0.8	0.5	0.5	1.5	0.1	0.5	0.3

Table 1: Parameter specification for simulations.

[13]In appendix D, we provide computational results for the case where the central bank has a uniform prior over infinitely many different types of beliefs.

For figures 2 to 5, the graphs on the left-hand side show the critical default cost parameters λ^M_{BU} (green line) and λ^S_{BU} (red line) in dependence of the considered model parameter, as well as the assumed default cost parameter λ (black line) and the critical default cost parameter λ^M (blue line) in the case without uncertainty about beliefs. The graphs on the right-hand side depict the optimal haircut ψ chosen by the central bank (black dashed line), the smallest feasible haircut $\psi^M_{\overline{m}}$ (orange solid line), the haircut $\psi^S_{\overline{m}}$ (green solid line) guaranteeing solvency of banks for any type of beliefs, and the haircut $\psi^S_{\underline{m}}$ (red solid line) ruling out bank default only for the more pessimistic beliefs $\underline{m}$.

First, we study the effect of increasing uncertainty about the beliefs in the economy, as measured by the spread between the two possible distortion factors. We assume that the distortion factors $\underline{m}$ and $\overline{m}$ are symmetrically centered around one and we only vary the spread $\overline{m} - \underline{m}$. We find that with increasing uncertainty about beliefs (i.e., the spread $\overline{m} - \underline{m}$ is growing from 0 to 0.5), the optimal monetary policy first rules out bank default independent of the actual beliefs in the economy, then allows for bank default in the presence of the more optimistic beliefs and ultimately, with sufficiently large uncertainty, allows for bank default independent of the actual beliefs in the economy. Focusing on figure 2, the graph on the left-hand side shows that for a sufficiently small spread $\overline{m}-\underline{m}$, both critical default cost parameters λ^M_{BU} (green line) and λ^S_{BU} (red line) are below the assumed default cost parameter λ, i.e., it holds that $\max\{\lambda^M_{BU}, \lambda^S_{BU}\} \leq \lambda$. From proposition 4, we know that in such a situation, the optimal monetary policy is represented by the haircut $\psi^S_{\overline{m}}$, eliminating bank default independent of the actual beliefs in the economy. Accordingly, the graph on the right-hand side of figure 2 shows that for sufficiently small belief uncertainty, as captured by the spread $\overline{m} - \underline{m}$, the optimal haircut ψ (black dashed line) coincides with the smallest possible haircut $\psi^S_{\overline{m}}$ (green solid line). This policy is optimal until the beliefs are sufficiently different, so that eliminating bank default for both types of beliefs becomes too costly in terms of welfare because in the presence of the more pessimistic beliefs, bank lending would be restricted too much. The central bank thus optimally allows for bank default in the presence of the more optimistic beliefs $\overline{m}$, but still rules it out for the more pessimistic beliefs $\underline{m}$, by setting the haircut $\psi^S_{\underline{m}}$. This policy turns out to be optimal until the two possible types of beliefs are so distinct that setting the haircut $\psi^S_{\overline{m}}$ does not trigger solvency risk for banks in case the more pessimistic beliefs prevail. This threshold in terms of the spread $\overline{m} - \underline{m}$ can be identified, using the graph on the right-hand side of figure 2, as follows: If the haircut $\psi^S_{\underline{m}}$ (red line) is below the smallest feasible haircut $\psi^S_{\overline{m}}$ (orange line), it is clear that in the presence of the more pesimistic beliefs, banks cannot face any solvency risk, even if the central bank sets the smallest feasible haircut. From this point on, choosing the haircut $\psi^S_{\underline{m}}$ is not an option for the central bank anymore. Accordingly, focusing on the graph on the left-hand side of figure 2, the red line representing the critical default cost parameter λ^S_{BU} ends at the critical spread $\overline{m} - \underline{m}$ where beliefs become sufficiently distinct, i.e., $\underline{m} \leq \tilde{m}$. For any spread $\overline{m} - \underline{m}$ larger than the critical threshold, the parameter λ^S_{BU} is not relevant for the decision of the central bank. The optimal monetary policy is now described by proposition 3. In its choice of the haircut, the central bank must assess whether maximizing bank lending but accepting bank default (i.e., setting the haircut $\psi^M_{\overline{m}}$) or restricting bank lending and ruling out bank default (i.e., setting the haircut $\psi^S_{\overline{m}}$) maximizes expected welfare.

Specifically, the central bank aims at inducing maximum bank lending by implementing the haircut $\psi^M_{\overline{m}}$ if and only if default costs are sufficiently small ($\lambda < \lambda^M_{BU}$), and aims at ruling out bank default by choosing the haircut $\psi^S_{\overline{m}}$ otherwise. As outlined in corollary 4, if beliefs are sufficiently distinct ($\underline{m} \leq \tilde{m}$), the central bank chooses, compared to the case without uncertainty and the more optimistic beliefs $\overline{m}$, the smallest feasible haircut $\psi^M_{\overline{m}}$ already at higher default costs. Formally, it holds that $\lambda^M_{BU} > \lambda^M$. Focusing on the graph on the left-hand side of figure 2, we see that for moderate spreads $\overline{m} - \underline{m}$, the central bank chooses the smallest feasible haircut $\psi^M_{\overline{m}}$ and prefers eliminating bank default over restricting bank lending. In turn, if the difference in beliefs, as measured by the spread $\overline{m} - \underline{m}$, is large enough, the central bank jumps back to the regime of avoiding bank default but accepting restrictions in bank lending. This is based on the fact that with an increasing spread $\overline{m} - \underline{m}$, λ^M_{BU} converges to λ^M as well as based on the fact that, according to our specification, it holds that $\lambda > \lambda^M$. The latter condition can be observed in the graph on the left-hand side of figure 2, as the black line is above the blue one. Thus, while for moderate differences in the beliefs, it holds that $\lambda^M_{BU} > \lambda > \lambda^M$, this turns into $\lambda \geq \lambda^M_{BU} > \lambda^M$ with sufficiently different beliefs. Overall, we can conclude that the restrained behavior of the central bank—not allowing for bank default in any case—disappears with increasing uncertainty about beliefs. Formally, this means that in its choice of the haircut, the central bank shifts from $\psi^S_{\overline{m}}$, eliminating bank default in general, to $\psi^S_{\underline{m}}$, ruling out bank default only in the presence of the more pessimistic beliefs, and ultimately shifts to $\psi^M_{\overline{m}}$, accepting bank default if the more optimistic beliefs in the economy realize. However, it has to be noted that the central bank never accepts default for both types of beliefs. In other words, it only chooses the smallest feasible haircut $\psi^M_{\overline{m}}$ when beliefs are already sufficiently distinct and banks cannot be exposed to a solvency risk in the presence of the more pessimistic beliefs anymore. The objective to avoid bank default only prevails when belief differences become extreme and the choice of haircut policy is close to the one without belief uncertainty and more optimistic beliefs. The reason for the reversal of monetary policy with growing belief differences is the limited ability of the central bank to mitigate the effects of the more pessimistic beliefs, as it cannot set a haircut that is lower than $\psi^M_{\overline{m}}$. At some point, avoiding restrictions in bank lending for the more pessimistic beliefs while allowing for bank default in the presence of the more optimistic beliefs does not yield a welfare gain, compared to the monetary policy of simply avoiding bank default for the more optimistic beliefs and "ignoring" the outcomes with the more pessimistic beliefs. In other words, setting the haircut $\psi^M_{\overline{m}}$ instead of $\psi^S_{\overline{m}}$ has only a small effect on bank lending if the more pessimistic beliefs $\underline{m}$ prevail, but generates large costs due to bank default if the more optimistic beliefs realize. Figures 3 and 4 provide the simulation results when varying the uncertainty about beliefs by changing only the more optimistic beliefs $\overline{m}$ and pessimistic beliefs $\underline{m}$, respectively. In both cases, we can deduce the same patterns in the optimal monetary policy, as outlined before.

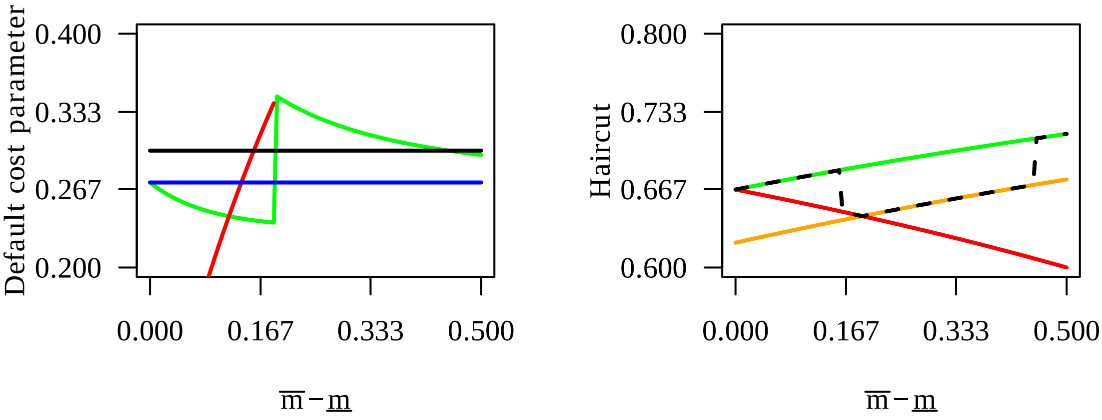

Figure 2: Varying uncertainty about beliefs with $\underline{m}$ and $\overline{m}$ symmetrically centered around one.

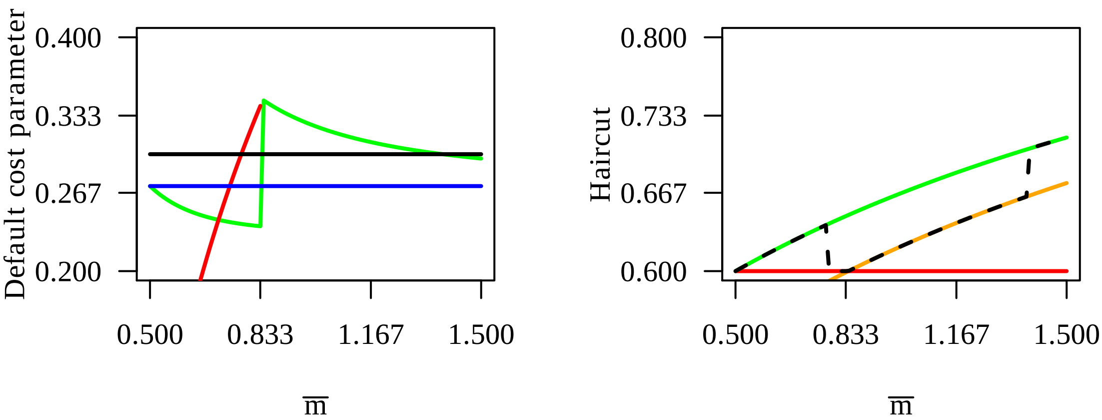

Figure 3: Varying uncertainty about beliefs with $\underline{m} = 0.5$ and increasing $\overline{m}$.

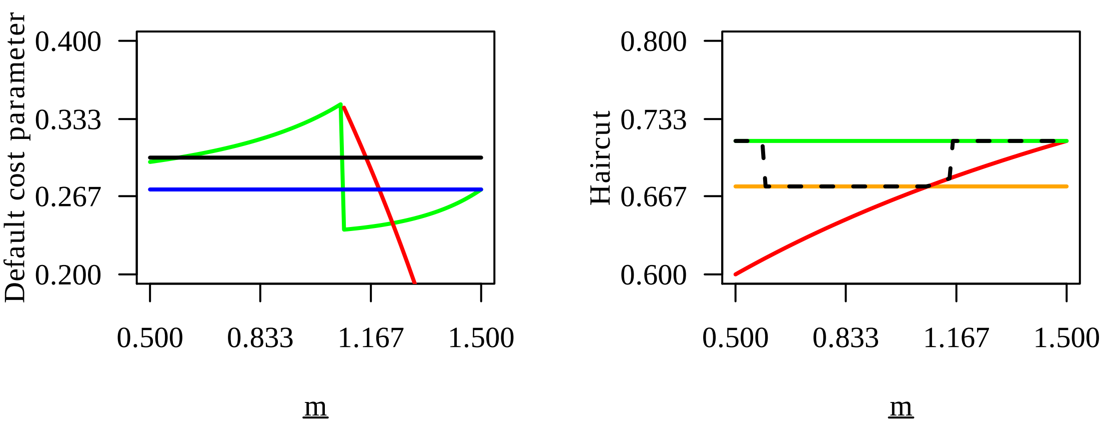

Figure 4: Varying uncertainty about beliefs with $\overline{m} = 1.5$ and increasing $\underline{m}$.

Second, we study the effect of the probability distribution of beliefs, as measured by the parameter p, on the optimal monetary policy. In its choice of the haircut, the central bank maximizes expected welfare, where the expected costs of deficient bank lending in the presence of the more pessimistic beliefs $\underline{m}$, are not only influenced by the beliefs themselves, but also by the probability p with which these beliefs emerge. When the probability that the more pessimistic beliefs realize is sufficiently small, the central bank follows the monetary policy which would be optimal without belief uncertainty. Specifically, it sets the haircut $\psi^S_{\overline{m}}$ that rules out bank default if the more optimistic beliefs $\overline{m}$ realize. In the right-hand side graph of figure 5, this pattern can be observed, as for a small probability p, the optimal haircut ψ (black dashed line) coincides with the haircut $\psi^S_{\overline{m}}$ (green line). In turn, if the probability for the more pessimistic beliefs is sufficiently large, the central bank optimally chooses to deviate from the avoidance of bank default and focus on the avoidance of bank lending restrictions, which it achieves by setting the smallest feasible haircut $\psi^M_{\overline{m}}$. Note that in this particular example, we rely on our baseline specification, so that the beliefs are sufficiently distinct, i.e., $\underline{m} \leq \tilde{m}$. Accordingly, the central bank cannot choose the haircut $\psi^S_{\underline{m}}$ and banks are not exposed to a solvency risk if the more pessimistic beliefs prevail.

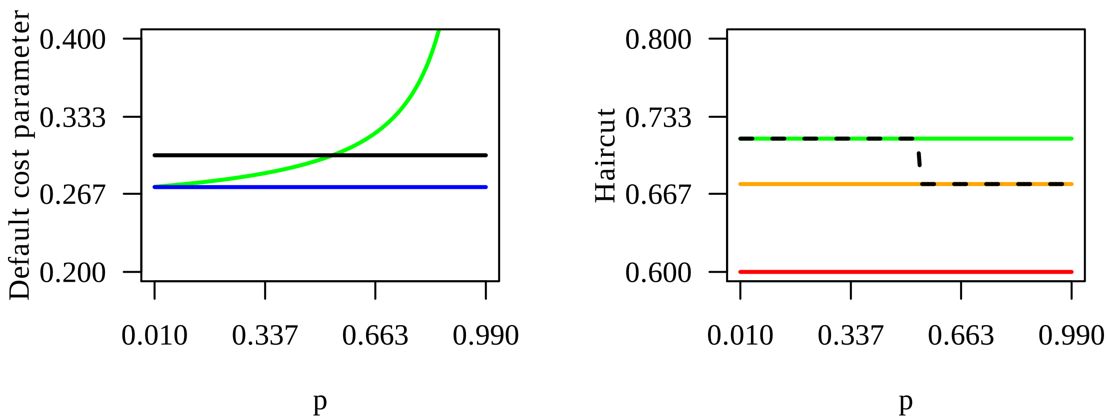

Figure 5: Varying probability p for beliefs $\underline{m}$.

7 Discussion

7.1 Bank regulation

The optimal monetary policy is independent of bank regulation in the form of capital requirements. On that account, note that capital requirements for banks lead to a regulatory leverage constraint. In other words, given the equity financing of banks, bank regulation limits loan financing to a maximum amount. Three possible scenarios may then emerge in our economy. First, capital requirements are sufficiently loose, so that with abundant liquidity, banks can grant loan financing beyond the optimal level. Then, the central bank can induce the optimal allocation by regulating bank lending adequately, which is achieved by restricting the access to liquidity through the appropriate haircut (see proposition 2). Second, the capital requirements are such that they exactly induce the optimal capital allocation in the economy. Then, the central bank should not further restrict loan financing but allow banks to indeed attain the optimal level of bank lending. This is achieved by setting the optimal haircut (see proposition 2). Third, capital requirements are sufficiently tight, so that bank lending is restricted below the optimal level. Then, it is optimal for the central bank not to restrict liquidity in a way that reduces bank lending even further. Without loss of generality, the central bank should in this case also implement the monetary policy which is optimal in the absence of bank regulation (see proposition 2).

7.2 Central bank signaling

We ruled out the possibility for the central bank to signal its information to private agents in the economy. However, in our setting, private agents have subjective beliefs and act in an opinionated way, so that even if the central bank signals its information, beliefs will not adjust in the economy. Only if we allow for private agents having distorted beliefs but not acting opinionated, such signaling by the central bank can be effective in eliminating the belief distortions in the economy.

7.3 Central bank mistakes

In our analysis, we abstracted from the possibility that the central bank makes mistakes, as we imposed that it knows the true probability distribution of productivity shocks. If the central bank is also subject to errors, monetary policy may induce a level of bank lending that is different from the optimal one. In other words, if the central bank decides about the collateral requirements in the form of the haircut without being perfectly informed about the true probability distribution of productivity shocks, it may erroneously choose a collateral framework that induces a suboptimal level of bank lending, leading to deficient loan financing to firms and productivity losses, or excessive lending and bank default.

8 Conclusion

We develop a simple framework that allows to study the optimal collateral framework in central bank lending facilities, while accounting for belief distortions in the economy leading to an over- and undervaluation of the pledged assets. Banks are liquidity-constrained, so that the central bank's choice of the haircut has a direct influence on banks' lending decisions. A larger haircut on bank loans, which can be used as collateral for reserve loans from the central bank, incentivizes banks to grant fewer loans and issue fewer deposits in the first place. Similarly, a lower haircut allows banks to borrow more reserves from the central bank and gives banks incentives to extend loan financing and deposit issuance.

When setting the haircut on bank loans in order to maximize welfare, the central bank generally trades off productivity losses due to deficient bank lending and costs due to bank default following from excessive bank lending. In our baseline model, where agents are sufficiently optimistic about the productivity of the loan-financed sector, the optimal monetary policy maximizes bank lending and allows for bank default if the default costs are sufficiently small. In turn, if the costs associated with bank default are large enough, the central bank aims at restricting bank lending, thereby eliminating a solvency risk for banks. If banks become more optimistic, the expected value of bank loans increases, causing banks to expect, ceteris paribus, an improved access to central bank liquidity. The central bank can counteract the belief changes by setting a larger haircut, thus restraining the access to reserves back to its optimal level. Similarly, growing pessimism in the economy leads to a devaluation of bank loans, ultimately requiring the central bank to loosen collateral standards in order to restore the optimal level of bank lending in the economy.

We also investigated the effect of uncertainty about beliefs on the side of the central bank. In the presence of belief uncertainty, the central bank is choosing the collateral framework in order to maximize the expected welfare in the economy. It faces the same trade-off as under certainty, namely that deficient lending leads to productivity losses, while excessive lending leads to costly bank default. However, under uncertainty about beliefs, the central bank cannot achieve the optimal capital allocation in the economy for any possible beliefs that may realize. Instead, it has to find the right balance of costs due to bank default, emerging when more optimistic beliefs realize, and productivity losses due to a suboptimal level of bank lending, emerging when more pessimistic beliefs realize. We find that with increasing uncertainty about beliefs, the restrained behavior of the central bank to avoid bank default vanishes and avoiding productivity losses due to deficient lending ultimately becomes the central bank's main objective of the central bank. Under uncertainty about beliefs, the central bank tends to become less restrictive in its choice of collateral standards.

Our simple framework allows for numerous extensions to assess the robustness of our findings. A first generalization may be represented by the introduction of strictly concave production technologies for both sectors. Second, the developed framework can be embedded into a dynamic setting, particularly accounting for an updating process of beliefs. Moreover, one may want to study various alternative formulations of default costs. These extensions are left for future research.

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A Flow consistency

In the following, we detail the flow consistency of our model for the case where banks issue deposits and bond-financed firms operate ($0 < \gamma < 1$). Note that bond investments are, as any other transaction in our economy, settled by using deposits. Thus, firms issuing bonds receive deposits from households and investors, which they can use to purchase capital good. However, deposits enter the economy only through loan financing by banks. Thus, before bonds can be purchased, loans must have been granted to firms and the respective firms must have used (some of) these deposits to purchase capital good from households and investors.

On this account, transactions on the good markets and bond financing proceed as follows. Loan-financed firms purchase capital good K^L, so that households and investors receive deposits in the amount QK^L. If households and investors decide to invest into bonds, (some of) these deposits are used to purchase bonds, so that bond-financed firms can acquire capital good. With the purchase of capital good by bond-financed firms, deposits flow back to households and investors. Clearly, if the deposits available to purchase bonds are less than the overall amount of required bond financing, QK^B, bond issuance and capital good purchase by bond-financed firms must be organized in several rounds. Assuming that firms, households and investors always use all their deposits at hand to settle transactions, the minimum number of rounds is given by

$$\sigma_1 = \left\lceil \frac{QK^B}{\xi QK^L} \right\rceil,$$

where $\lceil x \rceil$ denotes the least integer greater than or equal to x, and ξ is the share of deposits in the economy available for bond financing. If households and investors both invest into bonds, ξ takes the value of one, as all deposits in the economy can be used to purchase bonds issued by firms. If either households or banks are willing to invest into bonds and both sold already capital good to loan-financed firms, ξ takes a value less than one and essentially depends on the amount of deposits available to the respective bond investor, which, in turn, depends on how much capital good has been sold in the first place to loan-financed firms.

A similar process of transaction settlement must take place on the market for consumption good. Households and investors must use the available deposits to purchase consumption good from bond-financed firms, which then use the proceeds to meet the repayment obligations on bonds. The total amount of deposits in the economy, credited with interest, is given by $(1+r^D)QK^L$, so that the purchase of consumption good and bond repayment must be organized in at least

$$\sigma_2 = \left\lceil \frac{PA^BK^B}{(1+r^D)QK^L} \right\rceil,$$

rounds. The parameters σ_1 and σ_2 are irrelevant for our model analysis, but have been derived to illustrate that our model is flow consistent, particularly when taking the assumption that any kind of transaction is settled instantaneously by using bank deposits into account.

B Optimal monetary policy with uncertainty about beliefs and large costs of bank default

In this section, we provide additional results on the optimal monetary policy in the case where the central bank is uncertain about the beliefs in the economy. As in section 6, the beliefs can be of two types, as captured by the distortion factors $\underline{m} \in (0, 1/\eta)$ and $\overline{m} \in (0, 1/\eta)$, satisfying $\underline{m} < \overline{m}$. In section 6, we only provided the analytical results for the case where the expected productivity of loans-financed firms is, even after accounting for costs due to bank default, higher than the productivity of bond-financed firms, i.e., $\mathbb{E}[A_s^L] - A^B - (1-\eta)\lambda A_{\underline{s}}^L > 0$, or, equivalently, $\lambda < \lambda^S$ (see assumption 6). The analytical results derived under this assumption are provided in proposition 3, corollary 4 and proposition 4. In the following, we provide the results on the optimal monetary if the previous assumption does not hold and default costs are sufficiently large. Thus, when taking default costs into account, loan-financed firms are in expectation weakly less productive than bond-financed firms.

Assumption 7 (Default Costs)
$\mathbb{E}_[A_s^L] - A^B - (1-\eta)\lambda A_{\underline{s}}^L \leq 0$ *or, equivalently,* $\lambda \geq \lambda^S$.

From section 4, we know that the critical default cost parameters satisfy $\lambda^S > \lambda^M = (1 - \varphi^S/\varphi^M)\lambda^S$. Using proposition 2 and assumption 7, it follows that, with perfect knowledge about the beliefs m, the central bank would optimally set the haircut ψ_m^S, restricting bank leverage below the maximum feasible level and eliminating bank default. Under assumption 7, the central bank's optimization problem is still described by lemma 9.

We make similar observations as in section 6. First, the smallest feasible haircuts in the case of the more pessimistic and the more optimistic beliefs, respectively, satisfy $\psi_{\underline{m}}^M < \psi_{\overline{m}}^M$. Ex-ante, before the actual beliefs in the economy are revealed, the central bank cannot choose any haircut that is smaller than $\psi_{\overline{m}}^M$, as such a haircut would rule out the existence of an equilibrium if indeed, the more optimistic beliefs $\overline{m}$ realize. Accordingly, the smallest possible haircut the central bank can choose is $\psi_{\overline{m}}^M$.

Second, note that the haircuts ruling out solvency risk for the banks in the presence of the more optimistic and more pessimistic beliefs, respectively, satisfy $\psi_{\underline{m}}^S < \psi_{\overline{m}}^S$. With the haircut $\psi_{\overline{m}}^S$, bank default is eliminated independent of the beliefs. Based on assumption 1, a loan-financed firm is, under true beliefs, weakly more productive in expectation than a bond-financed firm. Accordingly, the central bank has no incentive to set a haircut that is larger than $\psi_{\overline{m}}^S$, as it only restricts bank lending but yields no benefit, such as eliminating solvency risk, for instance.

Third, the central bank will never choose a haircut that triggers bank default for both types of beliefs, as, based on our assumption on default costs (see assumption 7), such a monetary policy is clearly welfare-reducing compared to any monetary policy that simply eliminates bank default for both types of beliefs. Note that bank default occurs independent of beliefs if the chosen haircut ψ satisfies $\psi < \psi_{\underline{m}}^S$. Such a haircut choice is only feasible if it holds that $\psi_{\underline{m}}^S > \psi_{\overline{m}}^M$, where, based on our previous explanation, $\psi_{\overline{m}}^M$ is the smallest possible haircut the central bank can choose. For the analysis of the central bank's optimal haircut choice, as outlined in the following proposition, we can thus focus on the closed set $\Psi := [\max\{\psi^M_{\overline{m}}, \psi^S_{\underline{m}}\}, \psi^S_{\overline{m}}]$.

Proposition 5 (Optimal Monetary Policy with Uncertainty - Large Costs)
If it holds that $p\varphi^L_{\underline{m}}(\max\{\psi^M_{\overline{m}}, \psi^S_{\underline{m}}\}) > p\varphi^L_{\underline{m}}(\psi^S_{\overline{m}}) + (1-p)\varphi^S$, there exists a

$$\hat{\psi} \in \arg\max_{\psi \in \Psi} \lambda^M_{BU}(\psi) := \lambda^S \left\{ 1 - \frac{\varphi^S}{\varphi^L_{\overline{m}}(\psi)} + \frac{p}{1-p} \frac{\varphi^L_{\underline{m}}(\psi) - \varphi^L_{\underline{m}}(\psi^S_{\overline{m}})}{\varphi^L_{\overline{m}}(\psi)} \right\},$$

with $\lambda^M_{BU}(\hat{\psi}) > \lambda^S$, so that the central bank optimally chooses $\hat{\psi}$ whenever $\lambda < \lambda^M_{BU}(\hat{\psi})$, accepting bank default for the more optimistic beliefs $\overline{m}$. Otherwise, the central bank optimally chooses the haircut $\psi^S_{\overline{m}}$, eliminating bank default for all possible beliefs.

In the case where $\lambda = \lambda^S$, the central bank optimally chooses the haircut $\psi = \max\{\psi^M_{\overline{m}}, \psi^S_{\underline{m}}\}$ if and only if $p\varphi^L_{\underline{m}}(\max\{\psi^M_{\overline{m}}, \psi^S_{\underline{m}}\}) > p\varphi^L_{\underline{m}}(\psi^S_{\overline{m}}) + (1-p)\varphi^S$, accepting bank default for the more optimistic beliefs $\overline{m}$. Otherwise, the central bank optimally chooses the haircut $\psi^S_{\overline{m}}$, eliminating bank default for all possible beliefs.

C Pessimism

In this section, we provide the model analysis in the presence of sufficiently pessimistic beliefs. Specifically, private agents—firms, households, investors, and banks—believe that a loan-financed firm is on average weakly less productive than a bond-financed firm.

Assumption 8 (Beliefs)
$\mathbb{E}_m[A_s^L] < A^B$.

Two fundamental questions are whether banks are willing to finance loans with deposits and whether investors are willing to provide equity financing. First, note that due to the equilibrium conditions on the firms' repayment obligations—see conditions (5) in subsection 4.2 and the equality of deposit and bond rate—assumption 4 implies that the expected loan rate is lower than the deposit rate, i.e., it holds that $\mathbb{E}_m[r_s^L] < r^D$. From lemma 4, we know that in any such situation, the bank is only willing to grant loans and finance them with deposits if it makes profits if the financed firm incurs a positive productivity shock, i.e., $r_{\overline{s}}^L > r^D$, and it can leverage sufficiently, i.e., $\varphi_m^L(\psi) > [(1 + \mathbb{E}_m[r_s^L])/\eta_m - 1 - r^D]/(r_{\overline{s}}^L - r^D)$. The first condition $r_{\overline{s}}^L > r^D$ translates with the equilibrium conditions $(1 + r_s^L)q = A_s^L$, with $s \in \{\underline{s}, \overline{s}\}$, and $(1 + r^B)q = (1 + r^D)q = A^B$ (see conditions (5) and (6) in subsection 4.2) into $A_{\overline{s}}^L > A^B$, which is always satisfied, based on our assumptions on firm productivity (see assumption 1 in subsection 3.3). The second condition translates with the previous equilibrium conditions into

$$\varphi_m^L(\psi) > \frac{\mathbb{E}_m[A_s^L] - \eta_m A^B]}{\eta_m(A_{\overline{s}}^L - A^B)},$$

which, after some rearranging, yields the condition $\psi < \hat{\psi}_m$, with $\hat{\psi}_m$ being provided in lemma 11.

Second, the investor is providing equity financing for the bank ($\zeta = 1$) if and only if the expected rate of return on bank equity weakly exceeds the interest rate on bonds, i.e., $\mathbb{E}_m[r_s^E] \geq r^B$. Using the equilibrium conditions $(1+r_s^L)q = A_s^L$, with $s \in \{\underline{s}, \overline{s}\}$, and $(1+r^B)q = (1+r^D)q = A^B$, the latter inequality translates into $\psi < \tilde{\psi}_m$, with $\tilde{\psi}_m$ being provided in lemma 11.

Lemma 11 (Bank Leverage and Equity Financing)
The bank chooses the maximum (minimum) leverage $\varphi = \varphi_m^L(\psi)$ ($\varphi = 1$) if and only if it holds that

$$\psi < (\geq)\hat{\psi}_m := 1 - \frac{\alpha(A_{\underline{s}}^L - \eta_m A_{\overline{s}}^L)}{\mathbb{E}_m[A_s^L](1 - \eta_m)},$$

and the investor provides (no) equity financing $\zeta = 1$ ($\zeta = 0$) if and only if it holds that

$$\psi \leq (>)\tilde{\psi}_m := 1 - \frac{\alpha(A^B - \eta_m A_{\overline{s}}^L)}{\mathbb{E}_m[A_s^L](1 - \eta_m)}. \tag{13}$$

Furthermore, it holds that $\tilde{\psi}_m < \hat{\psi}_m$.

In section 4, we outlined that the clearing of the equity market and the capital good market leads us to real bank lending $K^L = \varphi\zeta E$. Using lemma 11, we can then deduce that real bank lending is given by $K^L = \mathbb{1}\{\psi \leq \tilde{\psi}_m\}\varphi_m^L(\psi)E$.

C.1 Optimal monetary policy

We now characterize the optimal monetary policy as represented by the interest rate on reserves r_{CB}^D and the haircut ψ that applies to bank loans pledged as collateral for reserve loans. As in section 3, the central bank perfectly knows the beliefs in the economy when deciding about the monetary policy and chooses its instruments in order to maximize utilitarian welfare. Again, the interest rate on reserves r_{CB}^D affects, in conjunction with firm productivity A^B in the bond-financed sector, the prices in our economy (see equation (8) in subsection 4.2) but not the real allocation. With the haircut on bank loans used as collateral, the central bank can regulate the banks' access to liquidity, i.e., their ability to borrow reserves. As the liquidity constraint, which depends on the haircut ψ, influences the banks' initial decision to grant loans financed through deposit issuance, the central bank is able to affect bank lending and the allocation of capital good in the economy. Taking the irrelevance of the interest rate r_{CB}^D for the real allocation into account, the optimization problem of the central bank is formally given by

$$\max_{\psi\in[0,1]} W = \max_{\psi\in[0,1]} \{\mathbb{E}[A_s^L] - \mathbb{1}\{\varphi > \varphi^S\}(1-\eta)\lambda A_{\underline{s}}^L\}K^L + A^B(K + E - K^L),$$

where real bank lending is given by $K^L = \mathbb{1}\{\psi \leq \tilde{\psi}_m\}\varphi_m^L(\psi)E$.

We can rewrite the optimization problem of the central bank, as outlined in the following lemma. First, we exploit that the fact that bank lending only occurs if the haircut satisfies $\psi \leq \tilde{\psi}_m$ (see lemma 11). Specifically, note that the condition for equity financing by the investor ($\psi \leq \tilde{\psi}_m$) is stricter than the condition for banks granting loans funded with deposits ($\psi < \hat{\psi}_m$). Second, we use a result from our analysis in section 3, stating that the bank is exposed to a solvency risk if and only if $\psi < \psi_m^S$ (see proposition 1). Third, we can show that, in the presence of sufficiently pessimistic beliefs (see assumption 8), whenever the bank is issuing deposits and the investor is providing equity financing, the bank is exposed to a solvency risk. Formally, the critical haircuts satisfy $\tilde{\psi}_m < \psi_m^S$. These three observations allow us to provide an alternative characterization of the central bank's optimization problem, as stated in the following lemma.

Lemma 12 (The Central Bank's Optimization Problem - Pessimism)
The central bank's optimization problem is

$$\max_{\psi\in[\psi_m^M,1]} \{\mathbb{E}[A_s^L] - A^B - (1-\eta)\lambda A_{\underline{s}}^L\}\mathbb{1}\{\psi \leq \tilde{\psi}_m\}\varphi_m^L(\psi).$$

With sufficiently pessimistic beliefs in the economy, the central bank faces generally two options for the implementation of its monetary policy. First, it can set loose collateral requirements in the form of a small haircut, allowing banks to leverage sufficiently and ultimately incentivizing them to grant loans funded with deposits. Second, it can set strict collateral requirements in the form of a large haircut, ruling out equity financing by investors and ultimately bank lending. The first option is preferred over the second one whenever the costs associated with bank default are sufficiently small. Formally, it must hold that $\lambda < \lambda^S$. The first option, however, is only possible if the central bank can indeed set sufficiently loose collateral requirements, while ensuring the existence of an equilibrium. Formally, the first option is feasible whenever the smallest feasible haircut satisfies $\psi_m^M < \tilde{\psi}_m$.

Proposition 6 (Optimal Monetary Policy — Pessimism)
The central bank optimally sets the haircut ψ_m^M if and only if banks can leverage enough, so that they have incentives to finance loans with deposits and investors are willing to provide equity financing, i.e., it holds that $\psi_m^M < \tilde{\psi}_m$, and default costs are sufficiently small, i.e., it holds that $\lambda < \lambda^S$. Otherwise, the central bank optimally sets the haircut $\psi = 1$, thus eliminating bank default.

Whenever the central bank optimally aims at restricting bank lending to its minimum, so that the optimal haircut is given by $\psi = 1$, monetary policy is independent of the beliefs or economic fundamentals. Instead, if the central bank aims at maixmizing bank lending and thereby accepts bank default, the optimal haircut ψ_m^M varies with beliefs and economic fundamentals, as firm productivity, for instance. For more details, see corollaries 2 and 3 in section 5.

D Simulations for a continuous belief set

In this subsection, we assume that the central bank faces uncertainty of beliefs, where the set of potential distortion factors is continuous and given by $\mathcal{M} := [\underline{m}, \overline{m}]$, with $\underline{m} > 0$ and $\overline{m} < 1/\eta$. The most pessimistic beliefs $\underline{m}$ are, however, such that they still comply with assumption 4 in section 4. Specifically, private agents must still believe that the loan-financed sector is weakly more productive than the bond-financed one under the most pessimistic beliefs $\underline{m}$.

The default costs are sufficiently large, so that with knowledge of the actual beliefs in the economy, the central bank would optimally eliminate bank default. In other words, assumption 6 in section 6 also applies for the subsequent analysis. The central bank has a uniform prior about the possible distortion factors in the set $\mathcal{M}$ and chooses the haircut ψ to maximize expected welfare

$$\int_{\underline{m}}^{\overline{m}} \frac{W_m(\psi)}{\overline{m} - \underline{m}} dm,$$

where, based on lemma 9, for a specific haircut ψ set by the central bank and a distortion factor m, welfare $W_m(\psi)$ is given by

$$W_m(\psi) = \{\mathbb{E}[A_s^L] - \mathbb{1}\{\psi < \psi_m^S\}(1-\eta)\lambda A_{\underline{s}}^L\}\varphi_m^L(\psi)E + A^B(K + E - \varphi_m^L(\psi)E).$$

We can further simplify the central bank's optimization problem by focusing only on those terms which depend on the haircut ψ, so that the optimization problem is ultimately given by

$$\max_{\psi \in [\psi_{\overline{m}}^M, 1]} \int_{\underline{m}}^{\overline{m}} \left\{\mathbb{E}[A_s^L] - A^B - \mathbb{1}\{\psi < \psi_m^S\}\lambda(1-\eta)A_{\underline{s}}^L\right\} \frac{\varphi_m^L(\psi)}{\overline{m} - \underline{m}} dm.$$

In the following, we provide simulations that illustrate the dependence of the optimal haircut ψ on the default costs (λ), the set of possible beliefs ($\underline{m}$, $\overline{m}$), and the productivity in the bond-financed sector (A^B). We provide results for small and large default costs. Specifically, we assume a default cost parameter $\lambda = 0.3$ and $\lambda = 0.5$, respectively. If not stated otherwise, the baseline for the parameter specification is the one provided in table 1 in section 6.

In the following graphs, the orange solid line illustrates the smallest feasible haircut $\psi_{\overline{m}}^M$, the dashed black line represents the optimal haircut ψ and the dotted green line depicts the smallest possible haircut $\psi_{\overline{m}}^S$ guaranteeing solvency of banks in all states if the most optimistic beliefs realize, as captured by the distortion factor $\overline{m}$. The graphs on the left hand side follow from simulations with low default costs (i.e., $\lambda = 0.3$), whereas the graphs on the right hand side follow from simulations with high default costs (i.e., $\lambda = 0.5$).

First, we study the effect of belief uncertainty on the optimal monetary policy in terms of the haircut ψ on bank loans. Figure 6 illustrates the effect of increasing uncertainty about beliefs, as represented by the spread between distortion factors $\overline{m} - \underline{m}$, with the lower bound $\underline{m}$ and the upper bound $\overline{m}$ being symmetrically centered around one. It can be observed that the central bank switches, with beliefs becoming sufficiently different, i.e., with the spread $\overline{m} - \underline{m}$ being sufficiently large, from the avoidance of bank default (achieved by setting the haircut $\psi_{\overline{m}}^S$) to the avoidance of deficient bank lending (achieved by setting the haircut $\psi_{\overline{m}}^M$). This, however, only holds if default costs are sufficiently low (i.e., $\lambda = 0.3$). With high default costs, which in our setting are represented by the default cost parameter $\lambda = 0.5$, the central bank does not deviate from its objective of ruling out bank default for any considered spread of distortion factors. Similar effects exist if the varying uncertainty about beliefs only stems from a different upper or lower bound on beliefs, see figure 7 and figure 8, respectively.

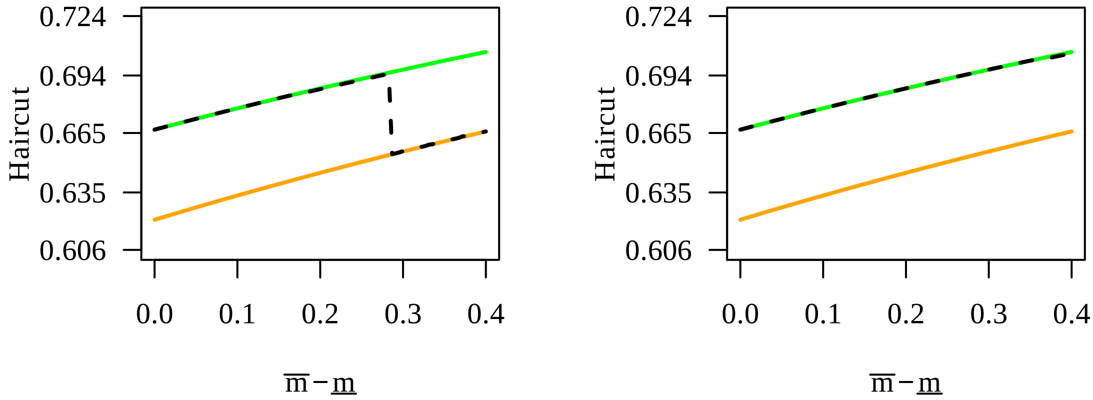

Figure 6: Varying uncertainty about a continuous belief set ranging from $\underline{m}$ to $\overline{m}$ symmetrically centered around one.

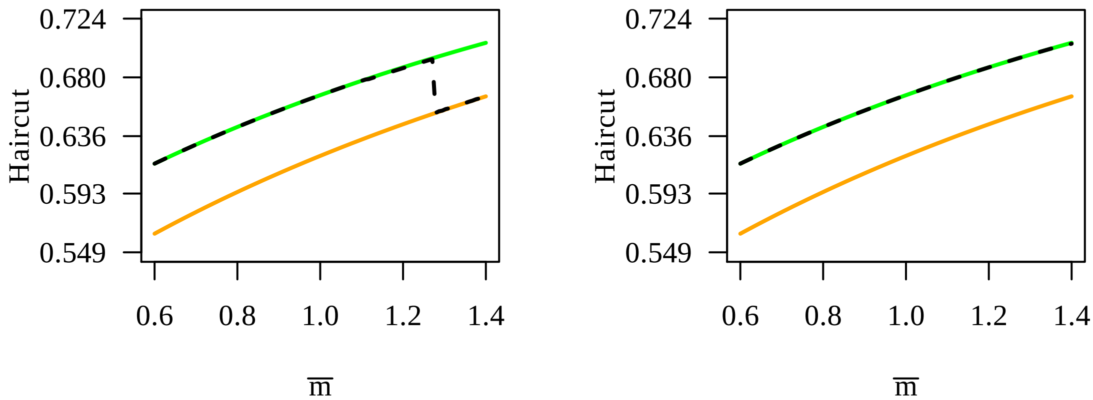

Figure 7: Varying uncertainty about a continuous belief set ranging from $\underline{m} = 0.5$ to $\overline{m}$.

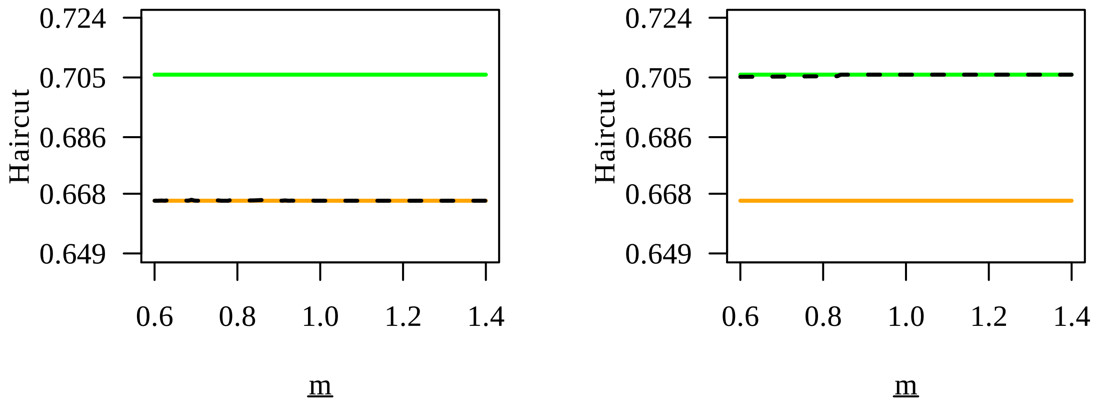

Figure 8: Varying uncertainty about a continuous belief set ranging from $\underline{m}$ to $\overline{m} = 1.5$.

Last, we study how the costs of deficient lending, as measured by the productivity difference $\mathbb{E}[A_s^L] - A^B$, influence the optimal monetary policy. Specifically, we analyze the effect of the productivity of bond-financed firms, denoted by A^B, on the central bank's choice of the haircut. It follows that with a relatively large productivity difference, i.e., if deficient lending is relatively costly compared to bank default, the central bank wants to avoid restrictions on bank lending and sets the smallest feasible haircut $\psi_{\overline{m}}^M$. In turn, if the productivity difference $\mathbb{E}[A_s^L] - A^B$ is sufficiently small, the central bank switches its objective to the avoidance of bank default and accordingly sets a haircut $\psi_{\overline{m}}^S$. This effect, however, only exists if default costs are sufficiently small (graph on the left hand side). If default costs are large, i.e., $\lambda = 0.5$, it follows that the central bank always wants to eliminate bank default for all potential distortion factors and thus sets the haircut $\psi_{\overline{m}}^S$ (see graph on the right hand side).

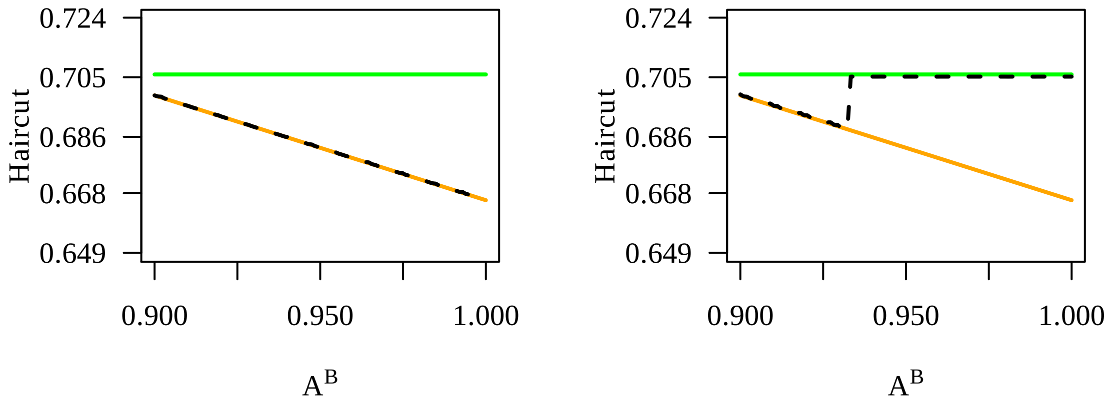

Figure 9: Varying productivity A^B of bond-financed firms.

E Proofs

Proof of Lemma 1. Firms are penniless and operate under limited liability, so that they are fully protected from losses. Accordingly, if the firm $f \in \{L, B\}$ is facing excess returns in one of the states, i.e., $A_s^f > (1 + r_s^f)q$ for some $s \in \{\underline{s}, \overline{s}\}$, the expected profits are increasing with the input K^f of capital good to production. Thus, there exists no optimal, finite demand for capital good by firm f, which we denote by $K^f = +\infty$. In contrast, without excess returns, i.e., $A_s^f \leq (1 + r_s^f)q$ for all $s \in \{\underline{s}, \overline{s}\}$, the firm f is making zero profits for any production input due to limited liability. Accordingly, the firm is indifferent between any amount of capital good put into production and the optimal demand is given by $K^f \in [0, \infty)$. ■

Proof of Lemma 2. Due to our assumption of linear utility, the household ultimately aims at maximizing consumption $C^H = [\gamma(1 + r^D) + (1 - \gamma)(1 + r^B)]qK + \tau^H + \pi$. The optimal choice of the household is thus of knife-edge type, namely the household invests the revenues from capital good sales in the asset which yields the highest return. If the deposit rate exceeds the bond rate ($r^D > r^B$), the household only holds deposits and if the bond rate exceeds the deposit rate ($r^D < r^B$), the household only invests into bonds. Otherwise ($r^D = r^B$), the household is indifferent between holding deposits and investing into bonds ($\gamma \in [0, 1]$). ■

Proof of Lemma 3. Due to our assumption of linear utility, the investor ultimately aims at maximizing consumption $C_m^I = [\zeta(1 + \mathbb{E}_m[r_s^E]) + (1 - \zeta)(1 + r^B)]qE + \tau^I$. The optimal choice of the investor is thus of knife-edge type, namely the investor uses the revenues from capital good sales to invest into the asset which yields the highest expected return. The investor's optimal choice satisfies $\zeta = 1$ if the expected rate of return on equity exceeds the one on bonds ($\mathbb{E}_m[r_s^E] > r^B$), and $\zeta = 0$, if the bond rate exceeds the expected rate of return on equity ($\mathbb{E}_m[r_s^E] < r^B$). Otherwise, the investor is indifferent between investing into bank equity and investing into bonds. In this particular case ($\mathbb{E}_m[r_s^E] = r^B$), we assume for simplicity that the investor uses all funds for investment into bank equity ($\zeta = 1$). ■

Proof of Lemma 4. First, we focus on the situation where the bank cannot face a solvency risk, as it holds that $\varphi_m^L(\psi) \leq \varphi^S$. In this case, the protection from losses through limited liability is not relevant, so that the expected rate of return on bank equity is given by

$$\mathbb{E}_m[r_s^E(\varphi)] = \mathbb{E}_m[(r_s^L - r^D)\varphi + 1 + r^D] - 1 = (\mathbb{E}_m[r_s^L] - r^D)\varphi + r^D.$$

The expected rate of return on bank equity is maximized for the leverage $\varphi = \varphi_m^L(\psi)$ if the loan and deposit rates satisfy $\mathbb{E}_m[r_s^L] > r^D$, and $\varphi = 1$ if it holds that $\mathbb{E}_m[r_s^L] < r^D$. If the expected interest rate on loans equals the interest rate on deposits ($\mathbb{E}_m[r_s^L] = r^D$), the bank is indifferent between all leverages and the optimal choice is given by $\varphi \in [1, \varphi_m^L(\psi)]$. For simplicity, we assume that in any situation where the bank is indifferent, it chooses the maximum leverage, so that it holds that $\varphi = \varphi_m^L(\psi)$. Accordingly, we can state that, without the possibility of solvency risk, the bank chooses $\varphi = \varphi_m^L(\psi)$ ($\varphi = 1$) if and only if it holds that $\mathbb{E}_m[r_s^L] \geq (<) r^D$.

Second, we focus on the situation where the bank can face a solvency risk, as it holds that

$\varphi^L_m(\psi) > \varphi^S$. A necessary condition for solvency risk is that the bank is making losses for a negative productivity shock of the financed firm. Thus, the interest rates on deposits and loans must satisfy $r^D > r^L_{\underline{s}}$. Taking the limited liability into account, with the possibility of a solvency risk, the expected rate of return on bank equity satisfies

$$\begin{aligned}\mathbb{E}_m[r^E_s(\varphi)] &= \mathbb{E}_m[\max\{(r^L_s - r^D)\varphi + 1 + r^D, 0\}] - 1 \\ &= \eta_m[(r^L_{\overline{s}} - r^D)\varphi + 1 + r^D] + \mathbb{1}\{\varphi \leq \varphi^S\}(1-\eta_m)[(r^L_{\underline{s}} - r^D)\varphi + 1 + r^D] - 1.\end{aligned}$$

If financing loans with deposits is profitable, even without benefiting from limited liability, i.e., $\mathbb{E}_m[r^L_s] \geq r^D$, the expected rate of return on bank equity is maximized for the largest possible leverage which guarantees liquidity, i.e., $\varphi = \varphi^L_m(\psi)$. This is due to the fact that if the financed firm incurs a negative productivity shock ($s = \underline{s}$), the bank makes losses (as $r^D > r^L_{\underline{s}}$) until the leverage is sufficiently high, so that the bank defaults and is protected from additional losses due to limited liability, while the bank makes always profits if the financed firm incurs a positive productivity shock, as it holds that $r^L_{\overline{s}} > r^D$.

Similarly, the expected rate of return on bank equity is maximized for $\varphi = \varphi^L_m(\psi)$ if without the benefits from limited liability financing loans with deposits is not profitable ($\mathbb{E}_m[r^L_s] < r^D$), but there are excess returns from loan financing if the financed firm incurs a positive productivity shock ($r^L_{\overline{s}} > r^D$), and the bank can leverage sufficiently, so that the expected equity return under the maximum leverage and default in the case where the financed firm incurs a negative productivity shock ($s = \underline{s}$) outweighs the expected equity return when financing loans solely with equity ($\varphi = 1$), i.e., it holds that

$$\eta_m[(r^L_{\overline{s}} - r^D)\varphi^L_m(\psi) + 1 + r^D] - 1 > \mathbb{E}_m[r^L_s] \quad \Leftrightarrow \quad \varphi^L_m(\psi) > \frac{(1 + \mathbb{E}_m[r^L_s])/\eta_m - (1 + r^D)}{r^L_{\overline{s}} - r^D}.$$

In all other cases with the possibility of solvency risk ($\varphi^L_m(\psi) > \varphi^S$), the expected rate of return on bank equity is maximized for the smallest possible leverage $\varphi = 1$. ■

Proof of Lemma 5. From lemma 4, we know that the bank is either financing loans with deposits and is liquidity-constrained as it chooses the maximum possible leverage $\varphi = \varphi^L_m(\psi)$ or finances loans solely with equity ($\varphi = 1$) and does not require any liquidity. We first focus on the situation where banks issue deposits and leverage as much as possible without risking liquidity. Note that the liquidity demand of the bank is given by $L^{CB} = \alpha(L^b - E^b)$. Thus, when borrowing liquidity from the central bank, the bank faces the liquidity constraint

$$(1 - \psi)(1 + \mathbb{E}_m[r^L_s])L^b \geq (1 + r^D_{CB})L^{CB}.$$

The repayment of the borrowed liquidity is determined by the interest rate on reserves r^D_{CB}. In turn, when borrowing liquidity on the interbank market, the bank faces the liquidity constraint

$$(1 - \tilde{\psi})(1 + \mathbb{E}_m[r^L_s])L^b \geq (1 + r^D)L^{CB},$$

where the repayment of interbank loans is determined by the interest rate r^D. As the bank is liquidity-constrained, the interbank market can only be active if the liquidity supply from other banks is weakly exceeding the liquidity supply from the central bank, i.e.,

$$\frac{(1-\psi)(1+\mathbb{E}_m[r_s^L])L^b}{1+r_{CB}^D} \leq \frac{(1-\tilde{\psi})(1+\mathbb{E}_m[r_s^L])L^b}{1+r^D}$$

$$\Leftrightarrow \quad (1+r^D)(1-\psi) \leq (1+r_{CB}^D)(1-\tilde{\psi}). \tag{14}$$

Like reserves, interbank deposits are used to settle interbank liabilities. The bank which granted an interbank loan must therefore ensure that if the interbank deposits, which have been created when the interbank loan was granted, are transferred to other banks, the liquidity (in the form of reserves) to settle the resulting interbank liability is available. If interbank deposits are transferred, the bank can use the pledged bank loans and rehypothecate them, namely use it as collateral at the central bank to borrow reserves. The maximum amount of liquidity that can be obtained by the bank, using the collateral $(1+\mathbb{E}_m[r_s^L])L^b$ associated with interbank loans, is given by $(1-\psi)(1+\mathbb{E}_m[r_s^L])L^b/(1+r_{CB}^D)$. Hence, when interbank loans are granted, it must hold that

$$\frac{(1-\tilde{\psi})(1+\mathbb{E}_m[r_s^L])L^b}{1+r^D} \geq \frac{(1-\psi)(1+\mathbb{E}_m[r_s^L])L^b}{1+r_{CB}^D}$$

$$\Leftrightarrow \quad (1+r^D)(1-\psi) \geq (1+r_{CB}^D)(1-\tilde{\psi}). \tag{15}$$

From equations (14) and (15), it follows that $(1+r^D)(1-\psi) = (1+r_{CB}^D)(1-\tilde{\psi})$. For any situation where the bank is financing loans only with equity, it issues no deposits, so that liquidity in the form of central bank reserves is irrelevant. We thus assume that when the bank chooses the smallest possible leverage $\varphi = 1$, it also holds $(1+r^D)(1-\psi) = (1+r_{CB}^D)(1-\tilde{\psi})$. ∎

Proof of Corollary 1. Lemma 5 states $(1+r^D)(1-\psi) = (1+r_{CB}^D)(1-\tilde{\psi})$. For any $\psi \in [0,1)$, imposing $\tilde{\psi} = \psi$ then yields that the interest rates on deposits and reserves are equal ($r^D = r_{CB}^D$). For $\psi = 1$, the central bank does not provide any liquidity, so that the bank will finance loans only with equity and without deposits. Accordingly, the interest rate on deposits does not play any role for the real allocation in the economy. In the case $\psi = 1$, we thus assume that it also holds that $r^D = r_{CB}^D$. ∎

Proof of Lemma 6. Due to our assumption of linear utility, utilitarian welfare represents aggregate consumption. Welfare $W = C^H + C^I$ can then be rewritten as

$$\begin{aligned} W = {} & [\gamma(1+r^D) + (1-\gamma)(1+r^B)]qK + \tau^H + \pi \\ & + [\zeta(1+\mathbb{E}[r_s^E(\varphi)]) + (1-\zeta)(1+r^B)]qE + \tau^I, \end{aligned}$$

where we used the expression for aggregate consumption of households and investors (see subsection 4.2).

First, note that, in our model, it holds that the interest rates on bonds, deposits and reserves are equal, i.e., $r^B = r^D = r^D_{CB}$. Moreover, using the conditions on the firms' repayment obligations, i.e., $(1 + r^L_s)q = A^L_s$, with $s \in \{\underline{s}, \overline{s}\}$, and $(1 + r^B)q = A^B$ (see conditions (5) in subsection 4.2), firms make zero profits ($\pi = 0$). Welfare thus reads as

$$W = (1 + r^D_{CB})qK + \tau^H + [\zeta(1 + \mathbb{E}[r^E_s(\varphi)]) + (1 - \zeta)(1 + r^D_{CB})]qE + \tau^I.$$

Second, we focus on the governmental taxes. Note that there is a representative bank, so that after the transactions on the capital good market have been settled, reserve deposits and reserve loans are equal. Moreover, based on assumption 2, the interest rates on reserve deposits and reserve loans equal. Thus, reserve loans do not bear any risk, since the bank's balance of reserve deposits always matches the repayment obligation on reserve loans, independent of the idiosyncratic productivity shock incurred by the financed firm. The central bank thus makes neither profits nor losses ($\Pi^{CB} = 0$), and the taxes imposed by the government must cover only liabilities arising from the deposit insurance and the costs due to the resolution of bank default. Specifically, the governmental taxes T in nominal terms are given by

$$T = \Pi^{b-} - P\Lambda = \{(1 - \eta)[(r^L_{\underline{s}} - r^D_{CB})\varphi + 1 + r^D_{CB}]E^b - P(1 - \eta)\lambda A^L_{\underline{s}} K^L\}\mathbb{1}\{\varphi > \varphi^S\},$$

where we used

$$\Pi^{b-} = (1 - \eta)[(r^L_{\underline{s}} - r^D_{CB})\varphi + 1 + r^D_{CB}]E^b\mathbb{1}\{\varphi > \varphi^S\}$$

to represent aggregate nominal bank losses in the case of default. If banks are exposed to a solvency risk ($\varphi > \varphi^S$), a mass $1-\eta$ of banks is defaulting, as the financed firms incur a negative productivity shock ($s = \underline{s}$). The expression for the resolution costs of bank default $P\Lambda$ follows from (11) in subsection 4.2. Using $T = T^H + T^I$, welfare is then given by

$$\begin{aligned} W = {} & (1 + r^D_{CB})qK + [\zeta(1 + \mathbb{E}[r^E_s(\varphi)]) + (1 - \zeta)(1 + r^D_{CB})]qE \\ & + \{(1 - \eta)[(r^L_{\underline{s}} - r^D_{CB})\varphi + 1 + r^D_{CB}]E^b/P - (1 - \eta)\lambda A^L_{\underline{s}} K^L\}\mathbb{1}\{\varphi > \varphi^S\}. \end{aligned}$$

Third, the expected rate of return on bank equity is given by

$$\begin{aligned} \mathbb{E}[r^E_s(\varphi)] &= \mathbb{E}[\max\{(r^L_s - r^D_{CB})\varphi + 1 + r^D_{CB}, 0\}] - 1 \\ &= \eta[(r^L_{\overline{s}} - r^D_{CB})\varphi + 1 + r^D_{CB}] + (1 - \eta)[(r^L_{\underline{s}} - r^D_{CB})\varphi + 1 + r^D_{CB}]\mathbb{1}\{\varphi \leq \varphi^S\} - 1, \end{aligned}$$

where we used that, based on corollary 1 and assumption 3, the interest rates on deposits and reserves are equal ($r^D = r^D_{CB}$). With the clearing condition for the equity market ($E^b = \zeta QE$), utilitarian welfare reads as

$$
\begin{aligned}
W &= (1+r^D_{CB})qK + \{\eta[(r^L_{\overline{s}} - r^D_{CB})\varphi + 1 + r^D_{CB}] \\
&\qquad + (1-\eta)[(r^L_{\underline{s}} - r^D_{CB})\varphi + 1 + r^D_{CB}]\}\zeta\mathbb{1}\{\varphi \le \varphi^S\}qE + (1-\zeta)(1+r^D_{CB})qE \\
&\qquad + \{(1-\eta)[(r^L_{\underline{s}} - r^D_{CB})\varphi + 1 + r^D_{CB}]\zeta qE - (1-\eta)\lambda A^L_{\underline{s}} K^L\}\mathbb{1}\{\varphi > \varphi^S\} \\
&= (1+r^D_{CB})qK + (\mathbb{E}[r^L_s] - r^D_{CB})\varphi\zeta qE + (1+r^D_{CB})qE - (1-\eta)\lambda A^L_{\underline{s}} K^L \mathbb{1}\{\varphi > \varphi^S\}.
\end{aligned}
$$

Using the equilibrium leverage $\varphi = K^L/(\zeta E)$, the conditions $(1+r^B)q = A^B$ and $(1+r^L_s)q = A^L_s$, with $s \in \{\underline{s}, \overline{s}\}$, and $(1+r^D_{CB})q = A^B$, as stated in subsection 4.2, welfare translates into

$$
\begin{aligned}
W &= A^B K + (\mathbb{E}[A^L_s] - A^B)K^L + A^B E - (1-\eta)\lambda A^L_{\underline{s}} K^L \mathbb{1}\{\varphi > \varphi^S\} \\
&= \{\mathbb{E}[A^L_s] - \mathbb{1}\{\varphi > \varphi^S\}(1-\eta)\lambda A^L_{\underline{s}}\}K^L + A^B(K + E - K^L).
\end{aligned}
$$

■

Proof of Lemma 7. In equilibrium, firm productivity and firms' repayment rates are linked. Specifically, from conditions (5) in subsection 4.2, we know that it holds $(1+r^L_s)q = A^L_s$, with $s \in \{\underline{s}, \overline{s}\}$, and $(1+r^B)q = A^B$. Moreover, deposit rate and bond rate equal (see condition (6) in subsection 4.2), so that it holds $(1+r^D)q = A^B$. With assumption 4, stating $\mathbb{E}_m[A^L_s] \ge A^B$, we can then conclude that it holds $\mathbb{E}_m[r^L_s] \ge r^D$. Using lemma 4, it then follows that the bank always chooses the maximum possible leverage $\varphi = \varphi^L_m(\psi)$.

The expected rate of return on bank equity is then given by

$$
\begin{aligned}
\mathbb{E}_m[r^E_s] &= \mathbb{E}_m[\{(r^L_s - r^D)\varphi^L_m(\psi) + 1 + r^D\}^+] - 1 \\
&= \mathbb{E}_m[\{(A^L_s - A^B)\varphi^L_m(\psi) + A^B\}^+]/q - 1,
\end{aligned}
$$

where we used $(1+r^L_s)q = A^L_s$ and $(1+r^D)q = A^B$. With $(1+r^B)q = A^B$ and the fact that, based on assumption 1, it holds $\mathbb{E}_m[A^L_s] \ge A^B$, we know that the rate of return on bank equity as expected by the investor weakly exceeds the rate of return on bonds, i.e., it holds that $\mathbb{E}_m[r^E_s] \ge r^B$. Using lemma 3, it then follows that the investor uses all available funds to invest into bank equity ($\zeta = 1$). ■

Proof of Proposition 1. Bank lending must comply with the clearing of the capital good market, so that it holds that $L^b = QK^L = \varphi^L_m(\psi)QE \le Q(K+E)$ or, equivalently, $\varphi^L_m(\psi) \le 1 + K/E$. Using (10) to express $\varphi^L_m(\psi)$ in terms of the economic fundamentals, we obtain that the inequality $\varphi^L_m(\psi) \le 1 + K/E$ reads as

$$
\frac{\alpha A^B}{\alpha A^B - (1-\psi)\mathbb{E}_m[A^L_s]} \le 1 + K/E \quad \Leftrightarrow \quad \alpha A^B \le (1 + K/E)\{\alpha A^B - (1-\psi)\mathbb{E}_m[A^L_s]\}.
$$

Rearranging yields $(1-\psi)\mathbb{E}_m[A_s^L](1+K/E) \le \alpha A^B K/E$, which finally leads to another lower bound on the haircut that is given by

$$\psi \ge \frac{\mathbb{E}_m[A_s^L](1+E/K) - \alpha A^B}{\mathbb{E}_m[A_s^L](1+E/K)} \qquad \Leftrightarrow \qquad \psi \ge \psi_m^M := 1 - \frac{\alpha A^B}{\mathbb{E}_m[A_s^L](1+E/K)}.$$

Using equations (9) and (10), which are provided in subsection 4.2, the inequality $\varphi_m^L(\psi) > \varphi^S$ translates into

$$\frac{\alpha A^B}{\alpha A^B - (1-\psi)\mathbb{E}_m[A_s^L]} > \frac{A^B}{A^B - A_{\underline{s}}^L} \qquad \Leftrightarrow \qquad \alpha(A^B - A_{\underline{s}}^L) > \alpha A^B - (1-\psi)\mathbb{E}_m[A_s^L].$$

Rearranging yields $(1-\psi)\mathbb{E}_m[A_s^L] < \alpha A_{\underline{s}}^L$, which finally provides us with a lower bound on the haircut that is given by

$$\psi < \frac{\mathbb{E}_m[A_s^L] - \alpha A_{\underline{s}}^L}{\mathbb{E}_m[A_s^L]} \qquad \Leftrightarrow \qquad \psi < \psi_m^S := 1 - \frac{\alpha A_{\underline{s}}^L}{\mathbb{E}_m[A_s^L]}.$$

■

Proof of Lemma 8. As the central bank aims at maximizing utilitarian welfare, its optimization problem is generally given by

$$\max_{\psi \in [0,1]} \{\mathbb{E}[A_s^L] - \mathbb{1}\{\varphi > \varphi^S\}(1-\eta)(1-\lambda)A_{\underline{s}}^L\}K^L + A^B(K + E - K^L),$$

where we used lemma 6 to express welfare.

First, from the outline in subsection 4.2 and lemma 7, we know that it holds that $K^L = \varphi_m^L(\psi)E$.

Second, based on proposition 1, it holds that $\varphi = \varphi_m^L(\psi) > \varphi^S$ if and only if $\psi < \psi_m^S$.

Third, we know from proposition 1 that $\varphi = \varphi_m^L(\psi) \le \varphi^M := 1 + K/E$ if and only if $\psi \ge \psi_m^M$. Accordingly, the central bank is unable to choose any haircut smaller than ψ_m^M.

Omitting all terms that do not depend on the haircut ψ, we can then conclude that the optimization problem of the central bank is

$$\max_{\psi \in [\psi_m^M, 1]} \{\mathbb{E}[A_s^L] - A^B - \mathbb{1}\{\psi < \psi_m^S\}(1-\eta)\lambda A_{\underline{s}}^L\}\varphi_m^L(\psi).$$

■

Proof of Proposition 2. First, note that without a solvency risk, welfare is maximized for the haircut ψ_m^S, as, based on assumption 1, a loan-financed firm is weakly more productive on average than a bond-financed firm ($\mathbb{E}_m[A_s^L] \ge A^B$).

Second, from lemma 8, we know that the central bank then chooses any haircut ψ lower than ψ_m^S if it holds that

$$\{\mathbb{E}[A_s^L] - A^B - (1-\eta)\lambda A_{\underline{s}}^L\}\varphi_m^L(\psi) > \{\mathbb{E}[A_s^L] - A^B\}\varphi^S,$$

where we used $\varphi^S = \varphi^L_m(\psi^S)$. Due to the linear structure of the production technologies and the default costs, we can deduce that with a solvency risk, welfare is maximized for the smallest feasible haircut ψ^M_m. Using the notation, we can thus conclude that the central bank only chooses the haircut ψ^M_m, instead of the haircut ψ^S_m, if it holds that

$$
\begin{aligned}
& \{\mathbb{E}[A^L_s] - A^B - (1-\eta)\lambda A^L_{\underline{s}}\}\varphi^M > \{\mathbb{E}[A^L_s] - A^B\}\varphi^S \\
\Leftrightarrow \quad & \{\mathbb{E}[A^L_s] - A^B\}(\varphi^M - \varphi^S) > (1-\eta)\lambda A^L_{\underline{s}}\varphi^M \\
\Leftrightarrow \quad & \frac{\mathbb{E}[A^L_s] - A^B}{(1-\eta)A^L_{\underline{s}}}\left(1 - \varphi^S/\varphi^M\right) =: \lambda^M > \lambda.
\end{aligned}
$$

A necessary condition for the central bank to optimally choose the haircut ψ^M_m is that the expected productivity difference between the loan-financed sector and the bond-financed sector is positive, even when accounting for the costs originating from bank default, i.e.,

$$
\mathbb{E}[A^L_s] - A^B - (1-\eta)\lambda A^L_{\underline{s}} > 0 \qquad \Leftrightarrow \qquad \frac{\mathbb{E}[A^L_s] - A^B}{(1-\eta)A^L_{\underline{s}}} =: \lambda^S > \lambda.
$$

We can state $\lambda^M = (1 - \varphi^S/\varphi^M)\lambda^S$. Based on assumption 5, we know that $\varphi^M > \varphi^S$ and therefore $\lambda^M < \lambda^S$. The condition $\lambda < \lambda^S$ is thus no further restriction for the central bank's choice of the haircut ψ^M_m. Hence, we know that the central bank chooses the haircut ψ^S_m if and only if $\lambda \geq \lambda^M$, and the haircut ψ^M_m otherwise. ■

Proof of Corollary 2. The haircut ψ^S_m, restricting bank lending and eliminating bank default, satisfies

$$
\psi^S_m = 1 - \frac{\alpha A^L_{\underline{s}}}{\mathbb{E}_m[A^L_s]} = 1 - \frac{\alpha A^L_{\underline{s}}}{\eta_m(A^L_{\bar{s}} - A^L_{\underline{s}}) + A^L_{\underline{s}}} = 1 - \frac{\alpha A^L_{\underline{s}}}{\eta m(A^L_{\bar{s}} - A^L_{\underline{s}}) + A^L_{\underline{s}}},
$$

where we used the definition $\eta_m = \eta m$, with $m \in (0, 1/\eta)$. The haircut depends on the productivity $\mathbb{E}_m[A^L_s]$ of loan-financed firms, as expected by the bank, and hence depends on the beliefs, as represented by the distortion factor m. Specifically,

$$
\frac{\partial \psi^S_m}{\partial m} = -\frac{-\alpha A^L_{\underline{s}}\eta(A^L_{\bar{s}} - A^L_{\underline{s}})}{[\eta m(A^L_{\bar{s}} - A^L_{\underline{s}}) + A^L_{\underline{s}}]^2} = \frac{\alpha A^L_{\underline{s}}\eta(A^L_{\bar{s}} - A^L_{\underline{s}})}{[\eta m(A^L_{\bar{s}} - A^L_{\underline{s}}) + A^L_{\underline{s}}]^2} = \frac{\alpha A^L_{\underline{s}}\eta(A^L_{\bar{s}} - A^L_{\underline{s}})}{(\mathbb{E}_m[A^L_s])^2} > 0.
$$

The haircut ψ^M_m, maximizing bank lending and allowing for bank default, satisfies

$$
\psi^M_m = 1 - \frac{\alpha A^B}{\mathbb{E}_m[A^L_s](1 + E/K)} = 1 - \frac{\alpha A^B}{[\eta m(A^L_{\bar{s}} - A^L_{\underline{s}}) + A^L_{\underline{s}}](1 + E/K)},
$$

where we again used the definition $\eta_m = \eta m$, with $m \in (0, 1/\eta)$. This haircut also depends on the productivity $\mathbb{E}_m[A^L_s]$ of loan-financed firms, as expected by the bank, and hence depends on the beliefs, as represented by the distortion factor m. Specifically,

$$\frac{\partial \psi_m^M}{\partial m} = -\frac{-\alpha A^B \eta (A^L_{\bar{s}} - A^L_{\underline{s}})(1+E/K)}{[\eta m (A^L_{\bar{s}} - A^L_{\underline{s}}) + A^L_{\underline{s}}]^2 (1+E/K)^2} = \frac{\alpha A^B \eta (A^L_{\bar{s}} - A^L_{\underline{s}})}{(\mathbb{E}_m[A^L_s])^2 (1+E/K)} > 0.$$

We can thus conclude that the optimal haircut set by the central bank, either ψ_m^S or ψ_m^M, increases if agents become more optimistic (i.e., m is increasing) and decreases if agents become more pessimistic (i.e., m is decreasing). ■

Proof of Corollary 3. The haircut ψ_m^S, restricting bank lending and eliminating bank default, is independent of the productivity of bond-financed firms, i.e.,

$$\frac{\partial \psi_m^S}{\partial A^B} = 0.$$

However, it varies with the productivity of loan-financed firms in both states. On the one hand, the derivative of the haircut with respect to $A^L_{\bar{s}}$, the productivity of loan-financed firms in the high productivity state, is given by

$$\frac{\partial \psi_m^S}{\partial A^L_{\bar{s}}} = -\frac{(-\alpha)\eta_m A^L_{\underline{s}}}{(\mathbb{E}_m[A^L_s])^2} = \frac{\alpha \eta_m A^L_{\underline{s}}}{(\mathbb{E}_m[A^L_s])^2} > 0.$$

On the other hand, the derivative with respect to $A^L_{\underline{s}}$, the productivity of loan-financed firms in the low productivity state, is given by

$$\frac{\partial \psi_m^S}{\partial A^L_{\underline{s}}} = -\frac{\mathbb{E}_m[A^L_s]\alpha - \alpha A^L_{\underline{s}}(1-\eta_m)}{(\mathbb{E}_m[A^L_s])^2} = -\frac{\alpha \eta_m A^L_{\bar{s}}}{(\mathbb{E}_m[A^L_s])^2} < 0.$$

The haircut ψ_m^M, maximizing bank lending and allowing for bank default, depends on the productivity of bond-financed firms. The derivative with respect to A^B is given by

$$\frac{\partial \psi_m^M}{\partial A^B} = -\frac{\alpha}{\mathbb{E}_m[A^L_s](1+E/K)} < 0.$$

The haircut also depends on the productivity of loan-financed firms in both states. On the one hand, the derivative with respect to $A^L_{\bar{s}}$ is given by

$$\frac{\partial \psi_m^M}{\partial A^L_{\bar{s}}} = -\frac{(-\alpha)A^B \eta_m (1+E/K)}{[\mathbb{E}_m[A^L_s](1+E/K)]^2} = \frac{\alpha A^B \eta_m}{(\mathbb{E}_m[A^L_s])^2 (1+E/K)} > 0.$$

On the other hand, the derivative with respect to $A^L_{\underline{s}}$ is given by

$$\frac{\partial \psi_m^M}{\partial A^L_{\underline{s}}} = -\frac{(-\alpha)A^B (1-\eta_m)(1+E/K)}{[\mathbb{E}_m[A^L_s](1+E/K)]^2} = \frac{\alpha A^B (1-\eta_m)}{(\mathbb{E}_m[A^L_s])^2 (1+E/K)} > 0.$$

■

Proof of Lemma 9. First, from the outline in subsection 4.2 and lemma 7, we know that for any type of beliefs $m \in \{\underline{m}, \overline{m}\}$, it holds that $K^L = \varphi^L_m(\psi)E$.

Second, based on proposition 1, it holds that $\varphi = \varphi^L_m(\psi) > \varphi^S$ if and only if $\psi < \psi^S_m$.

Third, we know from proposition 1 that $\varphi = \varphi^L_m(\psi) \leq \varphi^M := 1 + K/E$ if and only if $\psi \geq \psi^M_m$. As it holds that $\psi^M_{\underline{m}} < \psi^M_{\overline{m}}$, the central bank can, under uncertainty about beliefs, not set any haircut smaller than $\psi^M_{\overline{m}}$.

As the central bank aims at maximizing expected utilitarian welfare, its optimization problem is generally given by

$$\begin{aligned}\max_{\psi \in [\psi^M_{\overline{m}},1]} \; & p\{\mathbb{E}[A^L_s] - \mathbb{1}\{\psi > \psi^S_{\overline{m}}\}(1-\eta)(1-\lambda)A^L_{\underline{s}}\}\varphi^L_{\overline{m}}(\psi)E + pA^B(K + E - \varphi^L_{\overline{m}}(\psi)E) \\ & + (1-p)\{\mathbb{E}[A^L_s] - \mathbb{1}\{\psi > \psi^S_{\underline{m}}\}(1-\eta)(1-\lambda)A^L_{\underline{s}}\}\varphi^L_{\underline{m}}(\psi)E \\ & + (1-p)A^B(K + E - \varphi^L_{\underline{m}}(\psi)E),\end{aligned}$$

where we used lemma 6 to express welfare.

Omitting all terms that do not depend on the haircut ψ, we can then conclude that the optimization problem of the central bank is given by

$$\max_{\psi \in [\psi^M_{\overline{m}},1]} (\mathbb{E}[A^L_s] - A^B)\mathbb{E}[\varphi^L_m(\psi)] - (1-\eta)\lambda A^L_{\underline{s}}\mathbb{E}[\mathbb{1}\{\psi < \psi^S_m\}\varphi^L_m(\psi)].$$

■

Proof of Lemma 10. We say beliefs are distinct if under the smallest feasible haircut $\psi^M_{\overline{m}}$ the bank is not exposed to a solvency risk in the presence of the more pessimistic beliefs $\underline{m}$, as it holds that $\varphi^L_{\underline{m}}(\psi^M_{\overline{m}}) \leq \varphi^S$. By using the equations (9) and (10) in subsection 4.2, the latter inequality can be rewritten as

$$\frac{\alpha A^B}{\alpha A^B - (1-\psi^M_{\overline{m}})\mathbb{E}_{\underline{m}}[A^L_s]} \leq \frac{A^B}{A^B - A^L_{\underline{s}}} \quad \Leftrightarrow \quad \alpha(A^B - A^L_{\underline{s}}) \leq \alpha A^B - (1-\psi^M_{\overline{m}})\mathbb{E}_{\underline{m}}[A^L_s],$$

which is equivalent to

$$(1-\psi^M_{\overline{m}})\mathbb{E}_{\underline{m}}[A^L_s] \leq \alpha A^L_{\underline{s}} \qquad \Leftrightarrow \qquad \frac{\alpha A^B \mathbb{E}_{\underline{m}}[A^L_s]}{\mathbb{E}_{\overline{m}}[A^L_s](1+E/K)} \leq \alpha A^L_{\underline{s}},$$

where we, based on proposition 1, used

$$\psi^M_{\overline{m}} = 1 - \frac{\alpha A^B}{\mathbb{E}_{\overline{m}}[A^L_s](1+E/K)}.$$

Using the definition $\eta_m = \eta m$, further rearranging of the latter inequality yields

$$\begin{aligned} & \mathbb{E}_{\underline{m}}[A^L_s] \leq \mathbb{E}_{\overline{m}}[A^L_s](1+E/K)A^L_{\underline{s}}/A^B \\ \Leftrightarrow \quad & A^L_{\underline{s}} + \eta\underline{m}(A^L_{\overline{s}} - A^L_{\underline{s}}) \leq [A^L_{\underline{s}} + \eta\overline{m}(A^L_{\overline{s}} - A^L_{\underline{s}})](1+E/K)A^L_{\underline{s}}/A^B. \end{aligned}$$

Using the notation $\delta = (1 + E/K)A^L_{\underline{s}}/A^B$, we obtain

$$A^L_{\underline{s}}(1-\delta) \leq \eta(A^L_{\overline{s}} - A^L_{\underline{s}})(\delta\overline{m} - \underline{m}) \qquad \Leftrightarrow \qquad \frac{A^L_{\underline{s}}(1-\delta)}{\eta(A^L_{\overline{s}} - A^L_{\underline{s}})} \leq \delta\overline{m} - \underline{m},$$

or, equivalently,

$$\underline{m} \leq \tilde{m} := \delta\overline{m} - \frac{A^L_{\underline{s}}(1-\delta)}{\eta(A^L_{\overline{s}} - A^L_{\underline{s}})}.$$

Note that $\delta < 1$, as

$$(1 + E/K)\frac{A^L_{\underline{s}}}{A^B} < 1 \qquad \Leftrightarrow \qquad (1 + E/K)A^L_{\underline{s}} < A^B \qquad \Leftrightarrow \qquad A^L_{\underline{s}}E/K < A^B - A^L_{\underline{s}},$$

which then translates into

$$\frac{A^L_{\underline{s}}}{A^B - A^L_{\underline{s}}} < \frac{K}{E} \qquad \Leftrightarrow \qquad 1 + \frac{A^L_{\underline{s}}}{A^B - A^L_{\underline{s}}} < 1 + \frac{K}{E} \qquad \Leftrightarrow \qquad \frac{A^B}{A^B - A^L_{\underline{s}}} < 1 + \frac{K}{E},$$

and finally reads as $\varphi^S < \varphi^M$, which, based on assumption 5, is always satisfied. ■

Proof of Proposition 3. By assumption the beliefs $\underline{m}$ and $\overline{m}$ are distinct, i.e., it holds that $\underline{m} \leq \tilde{m}$.

First, note that there is no reason for the central bank to set a haircut larger than $\psi^S_{\overline{m}}$ which eliminates solvency risk for the more optimistic beliefs $\overline{m}$, as it only induces more restrictions on loan financing without any additional benefits, such as eliminating bank default, for instance. Using the fact that even under the smallest feasible haircut $\psi^M_{\overline{m}}$, the bank is not exposed to a solvency risk in case the more pessimistic beliefs $\underline{m}$ realize, we know that the central bank chooses a haircut $\psi \in [\psi^M_{\overline{m}}, \psi^S_{\overline{m}})$ if and only if

$$\begin{aligned}(\mathbb{E}[A^L_s] - A^B)[p\varphi^L_{\underline{m}}(\psi) + (1-p)\varphi^L_{\overline{m}}(\psi)] - (1-p)(1-\eta)\lambda A^L_{\underline{s}}\varphi^L_{\overline{m}}(\psi)& \\ > (\mathbb{E}[A^L_s] - A^B)[p\varphi^L_{\underline{m}}(\psi^S_{\overline{m}}) + (1-p)\varphi^L_{\overline{m}}(\psi^S_{\overline{m}})]&.\end{aligned}$$

With assumption 6, we know that it holds that $\mathbb{E}[A^L_s] - A^B - (1-\eta)\lambda A^L_{\underline{s}} > 0$ and thus welfare with a solvency risk for banks (i.e., the left-hand side of the latter inequality) is maximized for $\psi = \psi^M_{\overline{m}}$. Accordingly, we can state that the central bank chooses $\psi = \psi^M_{\overline{m}}$ if and only if it holds that

$$\begin{aligned}(\mathbb{E}[A^L_s] - A^B)[p\varphi^L_{\underline{m}}(\psi^M_{\overline{m}}) + (1-p)\varphi^M] - (1-p)\lambda(1-\eta)A^L_{\underline{s}}\varphi^M& \\ > (\mathbb{E}[A^L_s] - A^B)[p\varphi^L_{\underline{m}}(\psi^S_{\overline{m}}) + (1-p)\varphi^S]&,\end{aligned}$$

where we used $\varphi^M = \varphi^L_{\overline{m}}(\psi^M_{\overline{m}})$ and $\varphi^S = \varphi^L_{\overline{m}}(\psi^S_{\overline{m}})$. Rearranging of the latter inequality yields

$$(1-p)(1-\eta)\lambda A^L_{\underline{s}}\varphi^M < (\mathbb{E}[A^L_s] - A^B)\{p[\varphi^L_{\underline{m}}(\psi^M_{\overline{m}}) - \varphi^L_{\underline{m}}(\psi^S_{\overline{m}})] + (1-p)[\varphi^M - \varphi^S]\}$$

and further simplifies to

$$\lambda < \frac{\mathbb{E}[A_s^L] - A^B}{(1-\eta)A_{\underline{s}}^L} \frac{p[\varphi_{\underline{m}}^L(\psi_{\overline{m}}^M) - \varphi_{\underline{m}}^L(\psi_{\overline{m}}^S)] + (1-p)(\varphi^M - \varphi^S)}{(1-p)\varphi^M}.$$

Using the definitions $\lambda^S = (\mathbb{E}[A_s^L] - A^B)/[(1-\eta)A_{\underline{s}}^L]$ and $\lambda^M = \left(1 - \varphi^S/\varphi^M\right)\lambda^S$, the latter inequality reads

$$\lambda < \lambda^S \left(1 - \frac{\varphi^S}{\varphi^M} + \frac{p}{1-p}\frac{\varphi_{\underline{m}}^L(\psi_{\overline{m}}^M) - \varphi_{\underline{m}}^L(\psi_{\overline{m}}^S)}{\varphi^M}\right) = \lambda^M + \lambda^S \frac{p}{1-p}\frac{\varphi_{\underline{m}}^L(\psi_{\overline{m}}^M) - \varphi_{\underline{m}}^L(\psi_{\overline{m}}^S)}{\varphi^M}.$$

■

Proof of Corollary 4. Suppose the types of possible beliefs are distinct ($\underline{m} \leq \tilde{m}$). Then, it follows from proposition 3 that the central bank chooses the smallest feasible haircut $\psi_{\overline{m}}^M$ if and only if

$$\lambda < \lambda_{BU}^M = \lambda^M + \lambda^S \frac{p}{1-p}\frac{\varphi_{\underline{m}}^L(\psi_{\overline{m}}^M) - \varphi_{\underline{m}}^L(\psi_{\overline{m}}^S)}{\varphi^M}.$$

From proposition 2, we know that under perfect information and in the presence of the more optimistic beliefs $\overline{m}$, the central bank chooses the smallest feasible haircut $\psi_{\overline{m}}^M$ if and only if $\lambda < \lambda^M$. Note that it holds that

$$\frac{p}{1-p}\frac{\varphi_{\underline{m}}^L(\psi_{\overline{m}}^M) - \varphi_{\underline{m}}^L(\psi_{\overline{m}}^S)}{\varphi^M} > 0 \qquad \Leftrightarrow \qquad \varphi_{\underline{m}}^L(\psi_{\overline{m}}^M) > \varphi_{\underline{m}}^L(\psi_{\overline{m}}^S),$$

which is satisfied as, based on assumption 5, it holds that $\psi_{\overline{m}}^S > \psi_{\overline{m}}^M$. Accordingly, we can conclude that it holds that $\lambda_{BU}^M > \lambda^M$ and under belief uncertainty, the central bank chooses the smallest feasible haircut $\psi_{\overline{m}}^M$ already at a higher default cost parameter, compared to the case without uncertainty. ■

Proof of Proposition 4. By assumption the beliefs $\underline{m}$ and $\overline{m}$ are close, i.e., it holds that $\underline{m} > \tilde{m}$.

Then, adopting a haircut $\psi \in [\psi_{\overline{m}}^M, \psi_{\underline{m}}^S)$, i.e., accepting bank default for any possible type of beliefs in the economy, is welfare-improving compared to the situation without any solvency risk if and only if

$$\{\mathbb{E}[A_s^L] - A^B - (1-\eta)\lambda A_{\underline{s}}^L\}[p\varphi_{\underline{m}}^L(\psi) + (1-p)\varphi_{\overline{m}}^L(\psi)]$$
$$> (\mathbb{E}[A_s^L] - A^B)[p\varphi_{\underline{m}}^L(\psi_{\overline{m}}^S) + (1-p)\varphi_{\overline{m}}^L(\psi_{\overline{m}}^S)].$$

With assumption 6, we know that it holds that $\mathbb{E}[A_s^L] - A^B - (1-\eta)\lambda A_{\underline{s}}^L > 0$, so that welfare with a solvency risk for banks (i.e., the left-hand side of the latter inequality) is maximized for the smallest feasible haircut $\psi = \psi_{\overline{m}}^M$. Then, the central bank chooses the haircut $\psi = \psi_{\overline{m}}^M$ if and only if

$$\lambda < \frac{\mathbb{E}[A_s^L] - A^B}{(1-\eta)A_{\underline{s}}^L} \frac{\underline{\pi}[\varphi_{\underline{m}}^L(\psi_{\overline{m}}^M) - \varphi_{\underline{m}}^L(\psi_{\overline{m}}^S)] + (1-\underline{\pi})[\varphi_{\overline{m}}^L(\psi_{\overline{m}}^M) - \varphi_{\overline{m}}^L(\psi_{\overline{m}}^S)]}{\pi\varphi_{\underline{m}}^L(\psi_{\overline{m}}^M) + (1-\pi)\varphi_{\overline{m}}^L(\psi_{\overline{m}}^M)}.$$

Using $\varphi_{\overline{m}}^L(\psi_{\overline{m}}^S) = \varphi^S$ and $\varphi_{\overline{m}}^L(\psi_{\overline{m}}^M) = \varphi^M$, the latter inequality reads as

$$\lambda < \frac{\mathbb{E}[A_s^L] - A^B}{(1-\eta)A_{\underline{s}}^L} \frac{p[\varphi_{\underline{m}}^L(\psi_{\overline{m}}^M) - \varphi_{\underline{m}}^L(\psi_{\overline{m}}^S)] + (1-p)(\varphi^M - \varphi^S)}{p\varphi_{\underline{m}}^L(\psi_{\overline{m}}^M) + (1-p)\varphi^M}$$

and further simplifies to

$$\lambda < \frac{\mathbb{E}[A_s^L] - A^B}{(1-\eta)A_{\underline{s}}^L} \left(1 - \frac{p\varphi_{\underline{m}}^L(\psi_{\overline{m}}^S) + (1-p)\varphi^S}{p\varphi_{\underline{m}}^L(\psi_{\overline{m}}^M) + (1-p)\varphi^M}\right).$$

We can then state that the central bank prefers the smallest feasible haircut $\psi_{\overline{m}}^M$ over the smallest possible haircut $\psi_{\overline{m}}^S$, eliminating bank default for any beliefs, if and only if

$$\begin{aligned}
& \lambda < \frac{\mathbb{E}[A_s^L] - A^B}{(1-\eta)A_{\underline{s}}^L} \left(1 - \frac{p\varphi_{\underline{m}}^L(\psi_{\overline{m}}^S) + (1-p)\varphi^S}{p\varphi_{\underline{m}}^L(\psi_{\overline{m}}^M) + (1-p)\varphi^M}\right) \\
\Leftrightarrow \quad & \lambda < \lambda^S \left(1 - \frac{\varphi^S}{\varphi^M} + \frac{\varphi^S}{\varphi^M} - \frac{p\varphi_{\underline{m}}^L(\psi_{\overline{m}}^S) + (1-p)\varphi^S}{p\varphi_{\underline{m}}^L(\psi_{\overline{m}}^M) + (1-p)\varphi^M}\right) \\
\Leftrightarrow \quad & \lambda < \lambda^M + \lambda^S \left(\frac{\varphi^S}{\varphi^M} - \frac{p\varphi_{\underline{m}}^L(\psi_{\overline{m}}^S) + (1-p)\varphi^S}{p\varphi_{\underline{m}}^L(\psi_{\overline{m}}^M) + (1-p)\varphi^M}\right),
\end{aligned}$$

where we used the definitions $\lambda^S = (\mathbb{E}[A_s^L] - A^B)/[(1-\eta)A_{\underline{s}}^L]$ and $\lambda^M = \lambda^S\left(1 - \varphi^S/\varphi^M\right)$. The central bank may also choose a haircut $\psi \in [\psi_{\underline{m}}^S, \psi_{\overline{m}}^S)$ to only accept bank default in the presence of the more optimistic beliefs $\overline{m}$, but not in the case of the more pessimistic beliefs $\underline{m}$. Note that beliefs satisfy $\underline{m} > \tilde{m}$ or, equivalently, $\varphi_{\underline{m}}^L(\psi_{\overline{m}}^M) > \varphi^S$, so that it holds that $\psi_{\underline{m}}^S > \psi_{\overline{m}}^M$. The central bank prefers a haircut $\psi \in [\psi_{\overline{m}}^S, \psi_{\underline{m}}^S)$ over the haircut $\psi_{\overline{m}}^S$, eliminating solvency risk for any type of beliefs, if and only if

$$\begin{aligned}
& \{\mathbb{E}[A_s^L] - A^B\}p\varphi_{\underline{m}}^L(\psi) + \{\mathbb{E}[A_s^L] - A^B - (1-\eta)\lambda A_{\underline{s}}^L\}(1-p)\varphi_{\overline{m}}^L(\psi) \\
& \qquad\qquad > (\mathbb{E}[A_s^L] - A^B)[p\varphi_{\underline{m}}^L(\psi_{\overline{m}}^S) + (1-p)\varphi_{\overline{m}}^L(\psi_{\overline{m}}^S)].
\end{aligned}$$

From assumption 6, we know that it holds that $\mathbb{E}[A_s^L] - A^B - (1-\eta)\lambda A_{\underline{s}}^L > 0$, so that the left-hand side of the latter inequality is maximized for the haircut $\psi = \psi_{\underline{m}}^S$. Using $\psi = \psi_{\underline{m}}^S$, the latter inequality yields

$$\lambda < \frac{\mathbb{E}[A_s^L] - A^B}{(1-\eta)A_{\underline{s}}^L} \frac{\underline{\pi}[\varphi_{\underline{m}}^L(\psi_{\underline{m}}^S) - \varphi_{\underline{m}}^L(\psi_{\overline{m}}^S)] + (1-\underline{\pi})[\varphi_{\overline{m}}^L(\psi_{\underline{m}}^S) - \varphi_{\overline{m}}^L(\psi_{\overline{m}}^S)]}{(1-\pi)\varphi_{\overline{m}}^L(\psi_{\underline{m}}^S)}.$$

With $\varphi^L_{\overline{m}}(\psi^S_{\overline{m}}) = \varphi^S$ and $\varphi^L_{\underline{m}}(\psi^S_{\underline{m}}) = \varphi^S$, the latter inequality reads as

$$\lambda < \frac{\mathbb{E}[A^L_s] - A^B}{(1-\eta)A^L_{\underline{s}}} \frac{p[\varphi^S - \varphi^L_{\underline{m}}(\psi^S_{\overline{m}})] + (1-p)(\varphi^L_{\overline{m}}(\psi^S_{\underline{m}}) - \varphi^S)}{(1-p)\varphi^L_{\overline{m}}(\psi^S_{\underline{m}})}$$

and further simplifies to

$$\lambda < \frac{\mathbb{E}[A^L_s] - A^B}{(1-\eta)A^L_{\underline{s}}} \left(1 - \frac{p[\varphi^L_{\underline{m}}(\psi^S_{\overline{m}}) - \varphi^S] + (1-p)\varphi^S}{(1-p)\varphi^L_{\overline{m}}(\psi^S_{\underline{m}})}\right).$$

We can then state that the central bank prefers the haircut $\psi^S_{\underline{m}}$ to the smallest possible haircut $\psi^S_{\overline{m}}$, eliminating bank default for any type of beliefs, if and only if

$$\begin{aligned}
& \lambda < \frac{\mathbb{E}[A^L_s] - A^B}{(1-\eta)A^L_{\underline{s}}} \left(1 - \frac{p[\varphi^L_{\underline{m}}(\psi^S_{\overline{m}}) - \varphi^S] + (1-p)\varphi^S}{(1-p)\varphi^L_{\overline{m}}(\psi^S_{\underline{m}})}\right) \\
\Leftrightarrow \qquad & \lambda < \lambda^S \left(1 - \frac{\varphi^S}{\varphi^M} + \frac{\varphi^S}{\varphi^M} - \frac{p[\varphi^L_{\underline{m}}(\psi^S_{\overline{m}}) - \varphi^S] + (1-p)\varphi^S}{(1-p)\varphi^L_{\overline{m}}(\psi^S_{\underline{m}})}\right) \\
\Leftrightarrow \qquad & \lambda < \lambda^M + \lambda^S \left(\frac{\varphi^S}{\varphi^M} - \frac{p[\varphi^L_{\underline{m}}(\psi^S_{\overline{m}}) - \varphi^S] + (1-p)\varphi^S}{(1-p)\varphi^L_{\overline{m}}(\psi^S_{\underline{m}})}\right),
\end{aligned}$$

where we used the definitions $\lambda^S = (\mathbb{E}[A^L_s] - A^B)/[(1-\eta)A^L_{\underline{s}}]$ and $\lambda^M = \lambda^S\left(1 - \varphi^S/\varphi^M\right)$ again.

We can conclude that the central bank optimally chooses the smallest feasible haircut $\psi^M_{\overline{m}}$ if and only if $\lambda < \lambda^M_{BU}$ and $\lambda^S_{BU} \le \lambda^M_{BU}$, where

$$\lambda^S_{BU} := \lambda^M + \lambda^S \left(\frac{\varphi^S}{\varphi^M} - \frac{p[\varphi^L_{\underline{m}}(\psi^S_{\overline{m}}) - \varphi^S] + (1-p)\varphi^S}{(1-p)\varphi^L_{\overline{m}}(\psi^S_{\underline{m}})}\right).$$

Instead, if $\lambda < \lambda^S_{BU}$ and $\lambda^M_{BU} < \lambda^S_{BU}$, the central bank chooses the smallest possible haircut, eliminating bank default only in the case of the more pessimistic beliefs $\underline{m}$. Otherwise, i.e., $\lambda^M_{BU} \le \lambda$ and $\lambda^S_{BU} \le \lambda$, the central bank chooses the smallest possible haircut $\psi^S_{\overline{m}}$, ruling out bank default for any beliefs in the economy. ■

Proof of Proposition 5. First, note that independent of the actual beliefs in the economy, bank default is eliminated if the central bank sets a haircut $\psi \in [\psi^S_{\overline{m}}, 1]$. The central bank never chooses a haircut larger than $\psi^S_{\overline{m}}$, as this would simply restrict bank lending further but not yield any additional benefits, as eliminating bank default, for instance. Moreover, the central bank will never choose a haircut that triggers bank default for both types of beliefs, as, based on our assumption on default costs (see assumption 7), such a monetary policy is welfare-reducing compared to any monetary policy that simply eliminates bank default for both types of beliefs. Note that bank default occurs independent of the actual beliefs in the economy if the haircut ψ chosen by the central bank satisfies $\psi < \psi^S_{\underline{m}}$. Such a monetary policy is only feasible if it holds that $\psi^S_{\underline{m}} > \psi^M_{\overline{m}}$, where $\psi^M_{\overline{m}}$ is the smallest feasible haircut. With assumption 7, we can thus focus for the analysis of the central bank's haircut choice on the set $\Psi := [\max\{\psi^M_{\overline{m}}, \psi^S_{\underline{m}}\}, \psi^S_{\overline{m}}]$.

The central bank chooses a haircut $\psi \in [\max\{\psi^M_{\overline{m}}, \psi^S_{\underline{m}}\}, \psi^S_{\overline{m}})$, and thereby accepts default of banks in the presence of the more optimistic beliefs, if and only if

$$(\mathbb{E}[A^L_s] - A^B)[p\varphi^L_{\underline{m}}(\psi) + (1-p)\varphi^L_{\overline{m}}(\psi)] - (1-p)(1-\eta)\lambda A^L_{\underline{s}}\varphi^L_{\overline{m}}(\psi)$$
$$> (\mathbb{E}[A^L_s] - A^B)[p\varphi^L_{\underline{m}}(\psi^S_{\overline{m}}) + (1-p)\varphi^L_{\overline{m}}(\psi^S_{\overline{m}})],$$

which simplifies to

$$\frac{(\mathbb{E}[A^L_s] - A^B)}{(1-\eta)A^L_{\underline{s}}}\{p[\varphi^L_{\underline{m}}(\psi) - \varphi^L_{\underline{m}}(\psi^S_{\overline{m}})] + (1-p)[\varphi^L_{\overline{m}}(\psi) - \varphi^L_{\overline{m}}(\psi^S_{\overline{m}})]\} > \lambda(1-p)\varphi^L_{\overline{m}}(\psi).$$

Using $\varphi^L_{\overline{m}}(\psi^S_{\overline{m}}) = \varphi^S$ and $\lambda^S = (\mathbb{E}[A^L_s] - A^B)/[(1-\eta)\lambda A^L_{\underline{s}}]$, the latter inequality reads

$$\lambda < \lambda^M_{BU}(\psi) := \lambda^S\left\{1 - \frac{\varphi^S}{\varphi^L_{\overline{m}}(\psi)} + \frac{p}{1-p}\frac{\varphi^L_{\underline{m}}(\psi) - \varphi^L_{\underline{m}}(\psi^S_{\overline{m}})}{\varphi^L_{\overline{m}}(\psi)}\right\}.$$

Note that, based on assumption 7, it holds that $\lambda \geq \lambda^S$. A haircut $\psi \in [\max\{\psi^M_{\overline{m}}, \psi^S_{\underline{m}}\}, \psi^S_{\overline{m}})$ satisfies $\lambda^S < \lambda^M_{BU}(\psi)$ if and only if

$$\frac{\varphi^S}{\varphi^L_{\overline{m}}(\psi)} < \frac{p}{1-p}\frac{\varphi^L_{\underline{m}}(\psi) - \varphi^L_{\underline{m}}(\psi^S_{\overline{m}})}{\varphi^L_{\overline{m}}(\psi)} \qquad \Leftrightarrow \qquad (1-p)\varphi^S + p\varphi^L_{\underline{m}}(\psi^S_{\overline{m}}) < p\varphi^L_{\underline{m}}(\psi).$$

The right-hand side of the latter inequality is maximized for $\psi = \max\{\varphi^M_{\overline{m}}, \varphi^S_{\underline{m}}\}$. Thus, we can state that, if $p\varphi^L_{\underline{m}}(\max\{\psi^M_{\overline{m}}, \psi^S_{\underline{m}}\}) > p\varphi^L_{\underline{m}}(\psi^S_{\overline{m}}) + (1-p)\varphi^S$, there exists a

$$\hat{\psi} \in \arg\max_{\psi\in\Psi} \lambda^M_{BU}(\psi), \qquad \text{with} \qquad \lambda^M_{BU}(\hat{\psi}) > \lambda^S,$$

so that the central bank chooses $\hat{\psi}$ whenever $\lambda < \lambda^M_{BU}(\hat{\psi})$, and $\psi^S_{\overline{m}}$ otherwise. In the special case where $\lambda = \lambda^S$, we can derive a more simple monetary policy rule. The central bank chooses a haircut $\psi \in [\max\{\psi^M_{\overline{m}}, \psi^S_{\underline{m}}\}, \psi^S_{\overline{m}})$, and thereby accepts default of banks in the presence of the more optimistic beliefs $\overline{m}$, if and only if

$$(\mathbb{E}[A^L_s] - A^B)[p\varphi^L_{\underline{m}}(\psi) + (1-p)\varphi^L_{\overline{m}}(\psi)]E - (1-p)(1-\eta)\lambda A^L_{\underline{s}}\varphi^L_{\overline{m}}(\psi)E$$
$$> (\mathbb{E}[A^L_s] - A^B)[p\varphi^L_{\underline{m}}(\psi^S_{\overline{m}}) + (1-p)\varphi^L_{\overline{m}}(\psi^S_{\overline{m}})]E,$$

which simplifies to

$$\frac{(\mathbb{E}[A^L_s] - A^B)}{(1-\eta)A^L_{\underline{s}}}\{p[\varphi^L_{\underline{m}}(\psi) - \varphi^L_{\underline{m}}(\psi^S_{\overline{m}})] + (1-p)[\varphi^L_{\overline{m}}(\psi) - \varphi^L_{\overline{m}}(\psi^S_{\overline{m}})]\} > \lambda(1-p)\varphi^L_{\overline{m}}(\psi).$$

Using $\lambda = \lambda^S = (\mathbb{E}[A^L_s] - A^B)/[(1-\eta)A^L_{\underline{s}}]$, we find that the central bank chooses a haircut $\psi \in [\max\{\psi^M_{\overline{m}}, \psi^S_{\underline{m}}\}, \psi^S_{\overline{m}})$ if and only if

$$p[\varphi^L_{\underline{m}}(\psi) - \varphi^L_{\underline{m}}(\psi^S_{\overline{m}})] > (1-p)\varphi^L_{\overline{m}}(\psi^S_{\overline{m}}) \quad \Leftrightarrow \quad p\varphi^L_{\underline{m}}(\psi) > p\varphi^L_{\underline{m}}(\psi^S_{\overline{m}}) + (1-p)\varphi^L_{\overline{m}}(\psi^S_{\overline{m}}).$$

The left-hand side of the latter inequality is maximized for $\psi = \max\{\psi^M_{\underline{m}}, \psi^S_{\underline{m}}\}$. Thus, we can state that the central bank chooses the haircut $\psi = \max\{\varphi^M_{\underline{m}}, \varphi^S_{\underline{m}}\}$ if $p\varphi^L_{\underline{m}}(\max\{\varphi^M_{\underline{m}}, \varphi^S_{\underline{m}}\}) > p\varphi^L_{\underline{m}}(\psi^S_{\underline{m}}) + (1-p)\varphi^S$, and the haircut $\psi^S_{\underline{m}}$ otherwise. ■

Proof of Lemma 11. In equilibrium, firm productivity is linked to the interest rates on loans, bonds and deposits, i.e., it holds that $(1 + r^L_s)q = A^L_s$ for all $s \in \{\underline{s}, \overline{s}\}$ and $(1 + r^D)q = (1 + r^B)q = A^B$, see conditions (5) and (6) in subsection 4.2. Accordingly, assumption 8, stating $\mathbb{E}_m[A^L_s] < A^B$, implies that loan rates and the deposit rate satisfy $\mathbb{E}_m[r^L_s] < r^D$. From lemma 4, we can then deduce that the bank chooses the maximum leverage if and only if $r^L_{\overline{s}} > r^D$ and $\varphi^L_m(\psi) > [(1 + \mathbb{E}_m[r^L_s])/\eta_m - 1 - r^D]/(r^L_{\overline{s}} - r^D)$. The first condition $r^L_{\overline{s}} > r^D$ is in equilibrium equivalent to $A^L_{\overline{s}} > A^B$, which is always satisfied, based on assumption 1. The second condition translates with the equilibrium conditions $(1 + r^L_s)q = A^L_s$ for all $s \in \{\underline{s}, \overline{s}\}$ and $(1 + r^D)q = (1 + r^B)q = A^B$ into

$$\varphi^L_m(\psi) > \frac{\mathbb{E}_m[A^L_s] - \eta_m A^B}{\eta_m(A^L_{\overline{s}} - A^B)}.$$

Using the equation (10) in subsection 4.2, which expresses the leverage ratio $\varphi^L_m(\psi)$ using economic fundamentals, the latter inequality translates into

$$\frac{\alpha A^B}{\alpha A^B - (1-\psi)\mathbb{E}_m[A^L_s]} > \frac{\mathbb{E}_m[A^L_s] - \eta_m A^B}{\eta_m(A^L_{\overline{s}} - A^B)}$$

$$\Leftrightarrow \qquad \alpha A^B \eta_m(A^L_{\overline{s}} - A^B) > (\mathbb{E}_m[A^L_s] - \eta_m A^B)\{\alpha A^B - (1-\psi)\mathbb{E}_m[A^L_s]\}$$

$$\Leftrightarrow \qquad (1-\psi)(\mathbb{E}_m[A^L_s] - \eta_m A^B)\mathbb{E}_m[A^L_s] > \alpha A^B(\mathbb{E}_m[A^L_s] - \eta_m A^L_{\overline{s}})$$

$$\Leftrightarrow \qquad \psi < \hat{\psi}_m := 1 - \frac{\alpha A^B(\mathbb{E}_m[A^L_s] - \eta_m A^L_{\overline{s}})}{(\mathbb{E}_m[A^L_s] - \eta_m A^B)\mathbb{E}_m[A^L_s]}.$$

The investor provides equity financing ($\zeta = 1$) if and only if the expected rate of return on bank equity exceeds the interest rate on bonds, i.e., $\mathbb{E}_m[r^E_s] \geq r^B$, translates into

$$\eta_m[(r^L_{\overline{s}} - r^D)\varphi^L_m(\psi) + 1 + r^D] - 1 \geq r^B \qquad \Leftrightarrow \qquad \varphi^L_m(\psi) \geq \frac{(1 + r^B)/\eta_m - (1 + r^D)}{r^L_{\overline{s}} - r^D}.$$

Using the equilibrium conditions $(1+r^L_s)q = A^L_s$, with $s \in \{\underline{s}, \overline{s}\}$, and $(1+r^D)q = (1+r^B)q = A^B$, the latter inequality reads

$$\frac{\alpha A^B}{\alpha A^B - (1-\psi)\mathbb{E}_m[A^L_s]} \geq \frac{(1-\eta_m)A^B}{\eta_m(A^L_{\overline{s}} - A^B)}.$$

Further rearranging yields

$$\begin{aligned} & \alpha\eta_m(A^L_{\bar{s}} - A^B) \geq (1-\eta_m)\{\alpha A^B - (1-\psi)\mathbb{E}_m[A^L_s]\} \\ \Leftrightarrow \quad & (1-\psi)(1-\eta_m)\mathbb{E}_m[A^L_s] \geq \alpha(A^B - \eta_m A^L_{\bar{s}}) \\ \Leftrightarrow \quad & (1-\eta_m)\mathbb{E}_m[A^L_s] - \alpha(A^B - \eta_m A^L_{\bar{s}}) \geq \psi(1-\eta_m)\mathbb{E}_m[A^L_s] \end{aligned}$$

and finally leads to another upper bound on the haircut that is given by

$$\psi \leq \frac{(1-\eta_m)\mathbb{E}_m[A^L_s] - \alpha(A^B - \eta_m A^L_{\bar{s}})}{(1-\eta_m)\mathbb{E}_m[A^L_s]} \qquad \Leftrightarrow \qquad \psi \leq \tilde{\psi}_m := 1 - \frac{\alpha(A^B - \eta_m A^L_{\bar{s}})}{(1-\eta_m)\mathbb{E}_m[A^L_s]}.$$

Note that $\tilde{\psi}_m < \hat{\psi}_m$ is equivalent to

$$\begin{aligned} & \frac{\alpha A^B(\mathbb{E}_m[A^L_s] - \eta_m A^L_{\bar{s}})}{(\mathbb{E}_m[A^L_s] - \eta_m A^B)\mathbb{E}_m[A^L_s]} < \frac{\alpha(A^B - \eta_m A^L_{\bar{s}})}{(1-\eta_m)\mathbb{E}_m[A^L_s]} \\ \Leftrightarrow \quad & (1-\eta_m)A^B(\mathbb{E}_m[A^L_s] - \eta_m A^L_{\bar{s}}) < (A^B - \eta_m A^L_{\bar{s}})(\mathbb{E}_m[A^L_s] - \eta_m A^B) \\ \Leftrightarrow \quad & \eta_m A^B A^L_{\bar{s}} < \eta_m A^B \mathbb{E}_m[A^L_s] - \eta_m (A^B)^2 \\ \Leftrightarrow \quad & \eta_m \mathbb{E}_m[A^L_s](A^L_{\bar{s}} - A^B) < \eta_m A^B (A^L_{\bar{s}} - A^B). \end{aligned}$$

The latter inequality translates into $\mathbb{E}_m[A^L_s] < A^B$, which, based on assumption 8, is always satisfied. ■

Proof of Lemma 12. First, note that, based on the outline in subsection 4.2 and lemma 11, we can state that real bank lending is given by $K^L = \varphi\zeta E = \mathbb{1}\{\psi \leq \tilde{\psi}_m\}\varphi^L_m(\psi)E$.

Second, note that $\tilde{\psi}_m \leq \psi^S_m$ is equivalent to

$$\frac{\alpha A^L_{\underline{s}}}{\mathbb{E}_m[A^L_s]} \leq \frac{\alpha(A^B - \eta_m A^L_{\bar{s}})}{\mathbb{E}_m[A^L_s](1-\eta_m)} \quad \Leftrightarrow \quad (1-\eta_m)A^L_{\bar{s}} \leq A^B - \eta_m A^L_{\bar{s}} \quad \Leftrightarrow \quad \mathbb{E}_m[A^L_s] \leq A^B.$$

The latter is always satisfied, based on assumption 8. Using lemma 6, we can conclude that for a specific haircut, welfare is given by

$$W_m(\psi) = \{\mathbb{E}[A^L_s] - (1-\eta)\lambda A^L_{\underline{s}}\}\mathbb{1}\{\psi \leq \tilde{\psi}\}\varphi^L_m(\psi)E + A^B(K + E - \mathbb{1}\{\psi \leq \tilde{\psi}_m\}\varphi^L_m(\psi)E).$$

Third, from proposition 1, we know that the central bank can only set haircuts weakly higher than ψ^M_m, as otherwise the capital good market does not clear.

Omitting all terms which do not depend on the haircut ψ chosen by the central bank, we can then conclude that the central bank's optimization problem is given by

$$\max_{\psi \in [\psi^M_m, 1]} \{\mathbb{E}_m[A^L_s] - A^B - (1-\eta)\lambda A^L_{\underline{s}}\}\mathbb{1}\{\psi \leq \tilde{\psi}_m\}\varphi^L_m(\psi).$$

■

Proof of Proposition 6. Note that the central bank can only incentivize banks to grant loans funded with deposits and incentivize investors to provide equity financing for banks if the smallest feasible haircut ψ_m^M satisfies $\psi_m^M \leq \tilde{\psi}_m$. Suppose the latter condition holds. Then, it is only optimal for the central bank to provide the incentives for bank lending, i.e., setting a haircut ψ lower than $\tilde{\psi}_m$, if it holds that $\mathbb{E}[A_s^L] - A^B - (1-\eta)\lambda A_{\underline{s}}^L > 0$ or, equivalently, $\lambda < \lambda^S = (\mathbb{E}[A_s^L] - A^B)/[(1-\eta)A_{\underline{s}}^L]$. Instead, if it holds that $\psi_m^M > \tilde{\psi}_m$ or $\lambda \geq \lambda^S$, the central bank chooses to restrict bank lending and rule out bank default, by setting the haircut $\psi = 1$. Any other haircut ψ satisfying $\psi > \tilde{\psi}_m$ would also be a feasible policy for the central bank to restrict bank lending and rule out bank default. ■

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