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Ability Distribution and Dynamics of Wage Inequality: Unintended Consequences of Human Capital Accumulation*

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Abstract

This study investigates the dynamics of between-group and within-group wage inequality in a model with heterogeneous learning abilities putting a key emphasis on the shape of the ability distribution. In our model, intergenerational human capital externalities incentivize individuals to invest in skills, consequently reshaping the composition of the labor force by expanding the proportion of skilled workers from the lower end of ability distribution. We show that if, in the process of human capital accumulation, the skill premium increases, then wage inequality among skilled workers, as measured by the Gini coefficient, also increases. For several common distributions of abilities, the composition effect contributes to an upward shift in the between-group inequality and inequality among skilled workers. We also demonstrate that the composition effect contributes to an increase in wage inequality when ability distributions are represented by empirical distributions of students' assessment scores.
Keywords: human capital, wage inequality, skill premium, composition effect
JEL Codes: I24, J24, J31

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1 Introduction

A few years ago, in an interview, Thomas Piketty argued that "the main force pushing toward reduction in inequality has always been the diffusion of knowledge and the diffusion of education" (Saunders, 2014). This argument is difficult to refute. Undoubtedly, the expansion of education is admirable and promotes economic growth and human development.[1] However, we must not indulge in wishful thinking. Everything comes at a cost, and education is no exception. In certain circumstances, human capital accumulation may result in unintended and undesirable outcomes, such as an increase in wage inequality.

In this paper, we introduce a theoretical model of human capital accumulation, shedding light on the influence of the ability distribution's shape on the dynamics of wage inequality. Our key finding highlights that intensifying the role of human capital in economic development tends to increase wage inequality, a phenomenon observable across a broad spectrum of ability distributions.

Over the past few decades, certain nations have witnessed a notable rise in the relative number of skilled workers. During this period their corresponding wages rose faster than those of unskilled workers. Maliar et al. (2022) construct a projection for the skill premium for 2017–2037, and conclude that it will continue to grow in the US economy. Although this pattern is not uniform across all regions, it poses a conundrum that has garnered considerable attention from economists.

Wage differences are not confined solely to differences between educational strata. In several cases, a significant proportion of the overall increase in wage inequality has been attributed to the rise of residual inequality within education groups. For example, Juhn et al. (1993) and Autor et al. (2008) document this phenomenon in the United States, whereas Blundell and Etheridge (2010) and Lindley and McIntosh (2015) provide evidence of the same in the United Kingdom. Notably, graduate wage inequality has increased in several advanced economies, as highlighted by Lemieux (2006), who reveals that within-group wage inequality has grown most rapidly for graduates among all education levels in the United States.

Numerous explanations have been put forth to shed light on these interrelated phenomena, with Chusseau et al. (2008), Lemieux (2008), Nolan et al. (2019), and Hoffman et al. (2020) offering comprehensive reviews of the relevant literature.

[1] However, Wilson (2002) raises doubts regarding the value of education, stating that there exist "very serious questions about the goodness (desirability, merits, value, worth) of education that have not been faced squarely, let alone answered" (p. 327).

While the existing theoretical literature on wage inequality has primarily concentrated on wage differentials between education groups, with less emphasis on within-group wage inequality, several studies have examined both between-group and within-group wage inequality, including Galor and Moav (2000), Aghion (2002), Meckl and Zink (2004), Miyake et al. (2009), and Guvenen and Kuruscu (2012). In this paper we contribute to the literature by focusing on the role of the ability distribution. The approach adopted in our paper has some similarities with the frameworks by Galor and Moav (2000) and Meckl and Zink (2004).

In the model by Galor and Moav (2000), the efficiency units of skilled and unskilled labor are determined by the educational choices of individuals who differ in their cognitive ability and, thus, in their education and human capital. It is assumed that a rise in the rate of technological progress increases the rate of return to skills and, as a result, induces an increase in the supply of educated individuals. An increase in the return to ability and education leads to a monotonic rise in wage inequality within and between groups as the economy moves toward a steady-state equilibrium.

Meckl and Zink (2004) analyze the effect of human-capital investments of heterogeneous individuals on the dynamics of the wage structure within a neoclassical growth model. In their model, the accumulation of physical capital changes relative factor prices and thus incentives to acquire skills, thereby altering the composition of the labor force. They show that, during the transition process to the steady state, the skill premium demonstrates a U-shaped pattern, inequality among skilled workers rises and inequality among unskilled workers decreases.

In contrast to Galor and Moav (2000) and Meckl and Zink (2004), who assume a uniform distribution of ability, we study how the shape of the ability distribution impacts wage inequality. In our single-good model, the output is determined by a production function with two factors of production: skilled labor, which represents human capital, and unskilled labor. While individuals possess equal productivity as unskilled workers, they differ in their ability to acquire skills. If an individual opts for skill acquisition, her human capital depends on the amount of time devoted to education and the aggregate level of human capital within the economy. The latter factor implies that obtaining skills in an educated society is more feasible. The decision to become a skilled worker is determined by whether her total income as a skilled worker exceeds that of an unskilled worker, given her ability. At any given time, there exists a unique threshold level of ability, such that individuals with an ability level above this threshold opt to become skilled workers, while those below it choose to become unskilled workers. Intergenerational human capital externalities incentivize individuals to invest in skills, consequently reshaping the composition of the labor force by expanding the proportion of skilled workers from the lower end of the ability distribution, leading to changes in wage inequality.

In our analysis, we first demonstrate that if the elasticity of substitution between human capital and unskilled labor exceeds one, which appears empirically plausible, the fraction of skilled workers in the population rises and monotonically converges to a steady state. We then examine whether a rise in this fraction on a transition path leads to an increase in wage inequality. The answer to this question depends on the direction of the impact of the composition effect, which is contingent on the distribution of ability. We derive simple formulas for the skill premium and the Gini coefficient of wage inequality among skilled workers and then obtain the following two results: 1) if the skill premium increases with the fraction of skilled workers, then the wage inequality among skilled workers also increases; 2) while for the Pareto distribution of the ability distribution neither the skill premium nor the wage inequality among skilled workers depends on the fraction of skilled workers, for several other common distributions (uniform, log-normal, exponential, truncated normal, and logit-normal) the impact of the composition effect is such that both the skill premium and the wage inequality among skilled workers increase with the fraction of skilled workers. We also provide examples of ability distributions where the dependence of the skill premium and/or the wage inequality among skilled workers on the fraction of skilled workers is not monotonic. Based on our theoretical framework, we extend our analysis to include empirical evidence. Utilizing the results of the Trends in International Mathematics and Science Study (TIMSS) as a proxy for the ability distribution in England and the United States, we find that the composition effect contributes to an increase in wage inequality.

The remainder of this paper is organized as follows. Section 2 introduces the basic model with heterogeneous agents. In Section 3, we analyze the dynamics of the model. Section 4 describes the development of wage inequality in the basic model. Section 5 reports the results of our empirical analysis. Section 6 concludes.

2 The setting

2.1 Agents

There is a continuum $(0, 1]$ of agents. Each agent is endowed with one unit of raw labor, however they are heterogeneous in terms of learning ability. At each time, the distribution of abilities is exogenous. By $\Psi(\cdot)$ we denote the cumulative distribution function (CDF) of abilities within each generation. The function $\Psi(x)$ represents the cumulative mass of agents whose learning ability is less than or equal to x. We assume that $\Psi(\cdot)$ is continuously differentiable on (T_{min}, T_{max}), where

$$T_{min} = \sup\{x|\Psi(x) = 0\}, \ T_{max} = \begin{cases} \inf\{x|\Psi(x) = 1\}, & \{x|\Psi(x) = 1\} \neq \emptyset; \\ +\infty, & \{x|\Psi(x) = 1\} = \emptyset. \end{cases}$$

In other words, the probability density function (PDF), denoted by $\psi(\cdot)$, is continuous and strictly positive on (T_{min}, T_{max}). In the case where $T_{max} = +\infty$, we also assume that the distribution $\Psi(\cdot)$ has a finite mean.

Agents are sorted in descending order of their abilities. The ability of agent $j \in (0, 1]$ is denoted by $T(j)$. It is clear that $T(\cdot)$ is the upper quantile function of the CDF $\Psi(\cdot)$:

$$T(j) = \Psi^{-1}(1 - j), \ j \in (0, 1],$$

where $\Psi^{-1}(\cdot)$ is the inverse function of $\Psi(\cdot)$. The function $T(\cdot)$ contains all information about the distribution of abilities.[2] Its graph can be obtained from the graph of $\Psi(\cdot)$ by flipping the axes (see Figure 1). The function $\mathcal{H} : (0, 1] \to \mathbb{R}_+$ is defined by

$$\mathcal{H}(s) = \int_0^s T(j)dj$$

will be used further in the model. $\mathcal{H}(s)$ can be interpreted as the cumulative ability of the top s-th quantile. The function $\mathcal{H}(\cdot)$ is continuous, strictly increasing, and concave on $(0, 1]$. Evidently, $\mathcal{H}(0) = 0$.

Each agent makes a choice between being skilled or unskilled. If an individual decides to be unskilled, she supplies one unit of raw labor in the labor market. If an individual j living in period t decides to be skilled, she spends $e_t(j)$ of her time acquiring advanced education, while the remaining time, $1 - e_t(j)$, is spent on work. Her individual human capital $\zeta_t(j)$ depends on the total amount of effective human capital in the economy in the previous period, her ability, and time spent on education:

$$\zeta_t(j) = \chi(H_{t-1})T(j)\phi(e_t(j)),$$

where $\chi : \mathbb{R}_+ \to \mathbb{R}_+$ and $\phi : [0, 1] \to \mathbb{R}_+$ are increasing, continuous, strictly concave and bounded functions such that $\chi(0) > 0$, $\chi(H)/H \xrightarrow[H\to\infty]{} 0$ and $\phi(0) = 0$. The function $\chi(\cdot)$ captures the idea that people living in an educated society find it

[2] The expression $T(1 - j)$ is sometimes called Pen's parade.

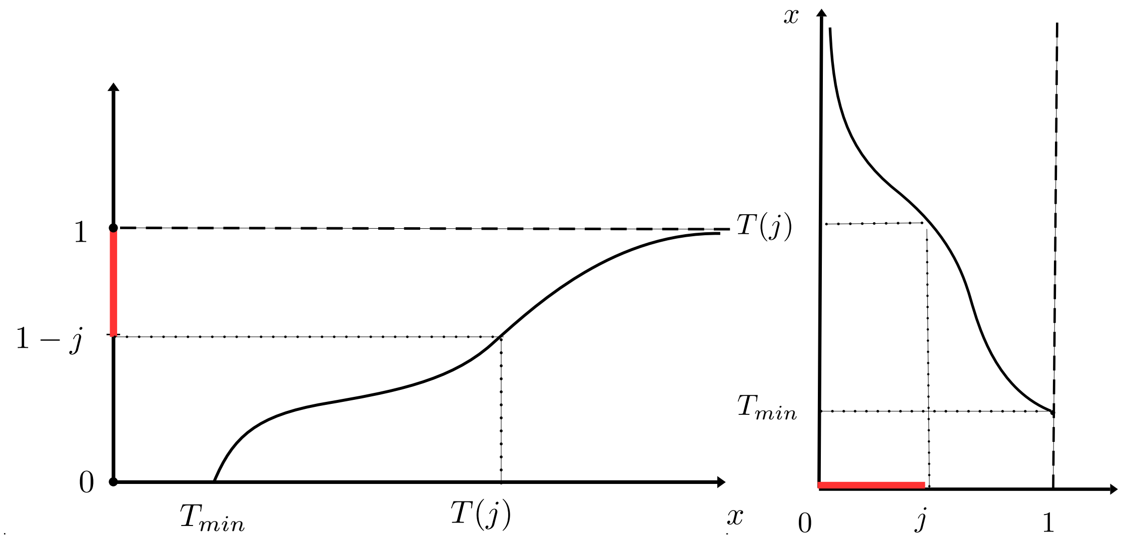


Figure 1: Graph of a CDF $\Psi(\cdot)$ and the corresponding upper quantile function $T(\cdot)$.

easier to acquire skills. The effective amount of human capital that individual j supplies in the labor market is

$$h_t(j) = (1 - e_t(j))\zeta_t(j).$$

When agent j decides to be educated, she maximizes the effective amount of her human capital by solving the following problem:

$$\max_{e \in [0,1]} \chi(H_{t-1})T(j)\phi(e_t(j))(1 - e_t(j)).$$

Since $\phi(\cdot)$ is a concave function, this problem has a unique solution, $\hat{e} = \arg\max_{e \in [0,1]} \phi(e)(1-e)$. Without loss of generality, we assume that $\phi(\hat{e})(1-\hat{e}) = 1$. Therefore, the effective amount of human capital of agent j is:

$$h_t(j) = \chi(H_{t-1})T(j).$$

To decide whether to be skilled or unskilled, an individual compares the wages she earns in the two cases. If being skilled gives her a higher wage income than being unskilled, she decides to become skilled. If the wage of an unskilled worker is higher than the wage she would make as a skilled worker, she decides to be unskilled. Formally, let w_t^H be the prevailing skilled workers' wage per unit of human capital at time t and w_t^L be the prevailing wage for unskilled workers. Individual j decides to be educated if $w_t^H h_t(j) > w_t^L$. If $w_t^H h_t(j) < w_t^L$, then she decides not to be educated and supplies one unit of unskilled labor in the labor market. If $w_t^H h_t(j) = w_t^L$, she is indifferent between being educated and uneducated. Thus, the income of agent j living in period t is equal to the maximum between $w_t^H h_t(j)$ and w_t^L.

If agent $s \in (0,1)$ is indifferent between being educated and uneducated, then all agents more capable than s decide to be educated, and the total supply of human capital is $H_t = \chi(H_{t-1})\mathcal{H}(s)$, and all agents less capable than s decide to be uneducated, and the total supply of unskilled labor is $L_t = 1 - s$.

2.2 Production

The economy produces a single consumption good. The output Y_t at time t is determined by the CES production function:

$$Y_t = F(H_t, L_t),$$

where H_t and L_t are the inputs of effective human capital and unskilled labor at time t and

$$F(H,L) = \begin{cases} \left(\alpha L^{\frac{\sigma-1}{\sigma}} + (1-\alpha)H^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, & \sigma > 0, \quad \sigma \neq 1, \\ L^{\alpha}H^{1-\alpha}, \quad \sigma = 1, \end{cases} \tag{1}$$

where σ is the elasticity of substitution between the skilled and unskilled workers. We refer to skilled and unskilled workers as gross substitutes when the elasticity of substitution $\sigma > 1$ and gross complements when $\sigma < 1$. In what follows, without any loss of generality, we assume that $\alpha = \frac{1}{2}$.

Markets for production inputs are perfectly competitive. Given the wage rates of skilled and unskilled labor at time t, w_t^H and w_t^L respectively, the representative producer chooses the level of employment of skilled labor, H_t, and unskilled labor, L_t, so as to maximize its profit:

$$(H_t, L_t) = \underset{(H,L)\in\mathbb{R}_+^2}{\arg\max}\ [F(H,L) - (w_t^H H + w_t^L L)].$$

Therefore, in equilibrium,

$$\xi(\eta_t) = \frac{w_t^L}{w_t^H}, \tag{2}$$

where $\eta_t = H_t/L_t$ is the human capital to unskilled labor ratio and $\xi(\eta)$ is the ratio of the marginal productivities of unskilled labor and human capital as depending on $\eta = H/L$:

$$\xi(\eta) := \left.\frac{F_L(H,L)}{F_H(H,L)}\right|_{H/L=\eta} = \eta^{\frac{1}{\sigma}}.$$

Note that $1/\xi(\eta) = \left.\frac{F_H(H,L)}{F_L(H,L)}\right|_{H/L=\eta}$ is the ratio of the marginal productivities of human capital and unskilled labor, which has a natural interpretation as the inverse relative demand for human capital.

3 Equilibrium dynamics

In equilibrium, the producer's profit is maximized (and equal to zero because of constant returns to scale), human capital and unskilled labor are optimally supplied, and the markets for skilled labor and unskilled labor are cleared. Formally, given the time $t-1$ total stock of human capital, $H_{t-1} \geq 0$, *a time-t equilibrium* is defined by the following conditions:

a) $\xi(H_t/L_t) = w_t^L/w_t^H$;

b) $w_t^L = w_t^H h_t(s_t)$ with $h_t(s_t) = \chi(H_{t-1})T(s_t)$;

c) $H_t = \chi(H_{t-1})\mathcal{H}(s_t), \quad L_t = 1 - s_t$.

Here s_t is the fraction of agents who decide to be educated at time t, and also, s_t is the pivotal agent that is indifferent between being educated and uneducated. Conditions (a) to (c) have standard interpretations. Condition (a) is the profit maximization condition. Condition (b) states that no agent wishes to change her decision about whether to be educated or not. Condition (c) describes the labor market equilibrium.

Therefore, in equilibrium, the fractions of both skilled and unskilled individuals in the population are positive. Situations in which all agents are skilled, or all agents are unskilled are not feasible. Indeed, if all agents decided to be unskilled, then the wage paid to skilled workers would be so high compared to the wage of unskilled workers that all unskilled agents would have incentives to become skilled. Conversely, if all agents decided to be skilled, then the wage rate of unskilled workers would be so high that each skilled agent would prefer to be unskilled.

It is easy to note that for a given H_{t-1}, the time-t equilibrium is optimal in the sense that in equilibrium, (H_t, L_t) is the solution to the following maximization problem:

$$\max F(H,L) \text{ s.t. } H \leq \chi(H_{t-1})\mathcal{H}(1-L),\ 0 \leq L \leq 1.$$

Therefore, the equilibrium output at time t is increasing in the stock of human capital at time $t-1$, H_{t-1}.

It follows from (2) and equilibrium conditions that

$$s_t = \tilde{s}(H_{t-1}),$$

where $\tilde{s}(H) : \mathbb{R}_+ \to (0,1]$ is the function defined for $H \geq 0$ as the solution to the following equation in s:

$$\xi\Big(\frac{\chi(H)\mathcal{H}(s)}{1-s}\Big) = \chi(H)T(s). \tag{3}$$

Note that for any $H \geq 0$,

$$\lim_{\eta\to 0} \xi(\eta) = 0 < \chi(H)T_{max},\ \lim_{\eta\to\infty} \xi(\eta) = \infty > \chi(H)T_{min},$$

and

$$\lim_{s\to 0} \frac{\chi(H)\mathcal{H}(s)}{1-s} = 0,\ \lim_{s\to 0} \chi(H)T(s) = \chi(H)T_{max},$$
$$\lim_{s\to 1} \frac{\chi(H)\mathcal{H}(s)}{1-s} = +\infty,\ \lim_{s\to 1} \chi(H)T(s) = \chi(H)T_{min}.$$

Hence, the LHS of Equation (3) is increasing in s, while its RHS is decreasing in s. Therefore, for any $H \geq 0$, the solution of Equation (3) exists and is unique. Graphs of both sides of Equation (3) as well as the fraction of skilled agents, s_t, effective human capital of marginal agent, h_t, and the total stocks of labor, L_t, and human capital, H_t, at the time-t equilibrium are illustrated in Figure 2.

Equation (3) can be rewritten as

$$\frac{[\mathcal{H}(s)]^{1/\sigma}}{(1-s)^{1/\sigma}T(s)} = [\chi(H)]^{1-1/\sigma}.$$

The LHS of this equation increases in s, while the RHS increases in H if $\sigma > 1$ and decreases in H if $\sigma < 1$. Therefore,

- if $\sigma > 1$, then $\tilde{s}(H)$ is monotonically increasing in H;
- if $\sigma = 1$, then $\tilde{s}(H)$ is constant in H;

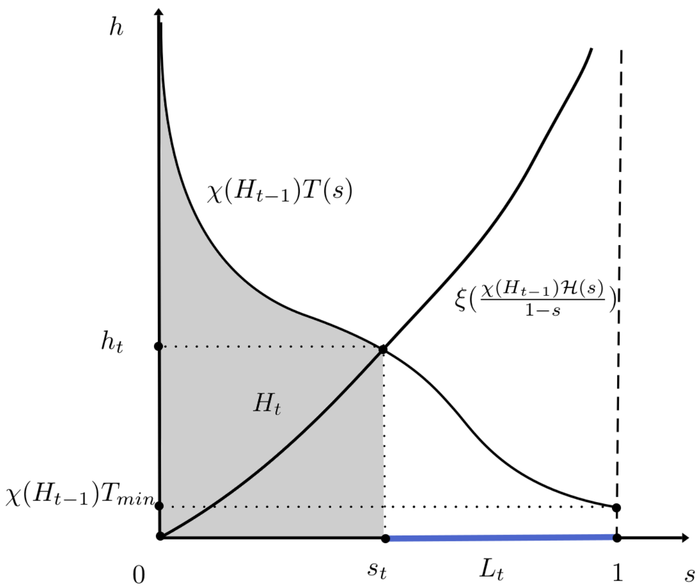


Figure 2: The time-t equilibrium.

- if $\sigma < 1$, then $\tilde{s}(H)$ is monotonically decreasing in H.

We now describe the equilibrium dynamics of our model. The dynamics of the total stock of human capital are given by:

$$H_t = \chi(H_{t-1})\mathcal{H}(s_t) = \chi(H_{t-1})\mathcal{H}(\tilde{s}(H_{t-1})). \tag{4}$$

and the steady-state equilibrium levels of human capital are determined as solutions to the following equation:

$$H = \chi(H)\mathcal{H}(\tilde{s}(H)), \tag{5}$$

which has at least one solution because $0 < \chi(0)\mathcal{H}(\tilde{s}(0))$ and $H > \chi(H)\mathcal{H}(\tilde{s}(H))$ for sufficiently large H. The qualitative picture of the equilibrium dynamics in our model is as follows.

If $\sigma > 1$, then the expression $\chi(H)\mathcal{H}(\tilde{s}(H))$ is increasing in H, and the stock of human capital (the sequence $(H_t)_{t=0}^{\infty}$ given by (4)) is either monotonically non-decreasing or non-increasing over time and hence converges to a steady-state equilibrium level. It is important to note that the stock of human capital, H_t, increases over time (or decreases over time) together with the fraction of skilled agents, s_t.[3]

[3]It should be noted that multiple steady-state equilibria may exist in this case.

In the case where $\sigma = 1$, s_t and hence $\mathcal{H}(s_t)$ are constant over time, while the stock of human capital either monotonically increases or monotonically decreases over time because $\chi(H)$ is increasing in H.

If σ is slightly smaller than 1, the stock of human capital either monotonically increases or monotonically decreases over time because the expression $\chi(H)\mathcal{H}(\tilde{s}(H))$ is still increasing in H while the fraction of skilled agents moves in the opposite direction. If σ is even smaller, the behavior of the human capital stock is not monotonic in time; moreover, in this case, the capital stock and the fraction of skilled agents change in opposite directions each time.

Among the above cases, the case where $\sigma > 1$ seems to be the most empirically relevant.[4]

4 Income inequality

Let us now examine the case in which the fraction of skilled agents in the population goes up with an accumulation of human capital and investigate the dynamics of income inequality.

The inequality between skilled and unskilled workers is measured by the skill premium defined as

$$\frac{\bar{W}^H - \bar{W}^L}{\bar{W}^L},$$

where $\bar{W}^L$ is the mean wage of unskilled workers and $\bar{W}^H$ is the mean wage of skilled workers.

Let s_t be the time-t equilibrium fraction of skilled workers in the population. Then the mean wage of skilled workers is equal to $\bar{W}_t^H = \frac{\chi(H_{t-1})\mathcal{H}(s_t)}{s_t} w_t^H$, and the mean wage of unskilled workers is equal to $\bar{W}_t^L = w_t^L$. Since $w_t^L = \chi(H_{t-1})T(s_t)w_t^H$ and $\mathcal{H}'(j) = T(j)$, we obtain that the skill premium in the time-t equilibrium is equal to $S(s_t)$, where

$$S(s) := \frac{1}{\epsilon_{\mathcal{H}}(s)} - 1,$$

with $\epsilon_{\mathcal{H}}(s) := \frac{T(s)}{\mathcal{H}(s)/s}$ being the elasticity of $\mathcal{H}(s)$.

[4]Bils et al. (2022) show that growth accounting points to a long-run elasticity of substitution across schooling groups of 4, or above.

It follows that when the stock of human capital increases and the fraction of skilled workers in the population goes up, then the skill premium either increases or decreases depending on whether the elasticity of $\mathcal{H}(s)$, $\epsilon_{\mathcal{H}}(s)$, is increasing or decreasing in s:

- if $\epsilon_{\mathcal{H}}(s)$ is increasing in s, then the skill premium $S(s)$ decreases as the fraction of skilled workers in the population goes up;
- if $\epsilon_{\mathcal{H}}(s)$ is constant in s, then the skill premium $S(s)$ does not change as the fraction of skilled workers in the population goes up;
- if $\epsilon_{\mathcal{H}}(s)$ is decreasing in s, then the skill premium $S(s)$ increases as the fraction of skilled workers in the population goes up.

As noted above, wage inequality literature has typically focused on wage differentials between educational groups, however, wages also vary within educational categories. In our model, the wages of unskilled workers are the same, but the wages of skilled workers vary.

What are the dynamics of wage inequality among skilled workers in the process of human capital accumulation? To answer this question we use the Gini coefficient to measure the wage inequality among skilled workers, which is equal to the Gini coefficient of ability inequality among skilled workers. We denote it by $G(s)$.

To calculate $G(s')$ for a given $s' > 0$, we refer to Figure 3, where it is represented as the ratio of the enclosed area between the graph of $\mathcal{H}(s)$ and the straight line segment OB to the total area of the triangle OAB (the left graph of Figure 3). Indeed, if we flip and rescale this figure, the graph of $\mathcal{H}(s)$ on the interval $[0, s']$ will become the Lorenz curve, and the straight line segment OB will become the line of perfect equality (right graph of Figure 3). Thus, it follows that

$$G(s') = \frac{\Gamma(s') - \mathcal{H}(s')s'/2}{\mathcal{H}(s')s'/2},$$

where

$$\Gamma(s) := \int_0^s \mathcal{H}(j)dj.$$

It follows that

$$G(s) = \frac{2}{\epsilon_\Gamma(s)} - 1,$$

where $\epsilon_\Gamma(s) := \frac{\mathcal{H}(s)}{\Gamma(s)/s}$ is the elasticity of $\Gamma(s)$.

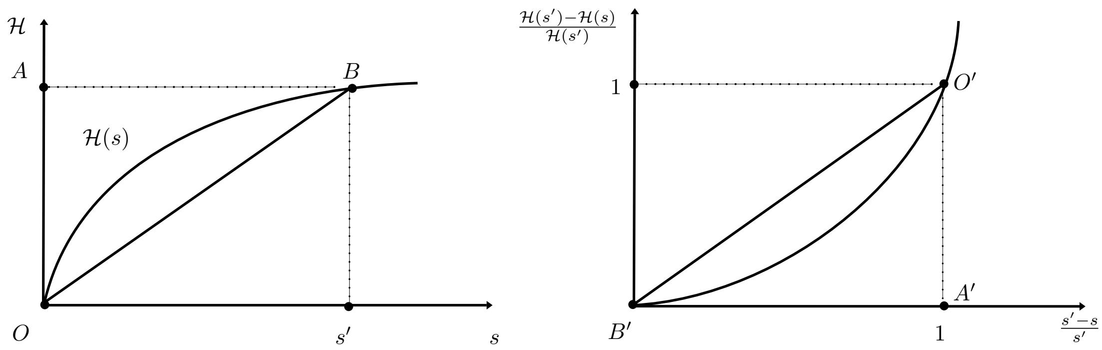


Figure 3: The graph of $\mathcal{H}(s)$ and the Lorenz curve for skilled workers.

We can see that when the stock of human capital increases and the fraction of skilled workers in the population goes up, then the Gini coefficient of wage inequality among skilled workers either increases or decreases, depending on whether the elasticity of $\Gamma(s)$, $\epsilon_\Gamma(s)$, is increasing or decreasing in s:

- if $\epsilon_\Gamma(s)$ is increasing in s, then the Gini coefficient $G(s)$ decreases as the fraction of skilled workers in the population goes up;
- if $\epsilon_\Gamma(s)$ is constant in s, then the Gini coefficient $G(s)$ does not change as the fraction of skilled workers in the population goes up;
- if $\epsilon_\Gamma(s)$ is decreasing in s, then the Gini coefficient $G(s)$ increases as the fraction of skilled workers in the population goes up.

In our model, if the skill premium increases with the fraction of skilled workers in the population, then the Gini coefficient of wage inequality among skilled workers also increases. More precisely, we formulate the following proposition.

Proposition 1. *If, for some* $0 < k < 1$, $S'(s) > 0$ *on the interval* $(0, k)$*, then* $G'(s) > 0$ *on this interval.*

Proof. Taking into account that $\epsilon_{\mathcal{H}}(s) \leq 1$ for any $s > 0$, the proposition follows directly from the following lemma, which implies that if $\epsilon'_{\mathcal{H}}(s) < 0$ on any interval $(0, k)$ with $k \leq 1$, then both $S(s)$ and $G(s)$ are increasing in s on this interval.

Lemma 1. *Let the function* $d : (0, k) \to \mathbb{R}_+$ *be continuously differentiable and*

$$D(x) = \int_0^x d(t)dt.$$

Let further $\epsilon_d(x) := \frac{d'(x)}{d(x)/x}$ *and* $\epsilon_D(x) := \frac{d(x)}{D(x)/x}$ *be the elasticities of* $d(x)$ *and* $D(x)$ *respectively.*

If $\epsilon_d(x)$ *is bounded above and* $\epsilon_d'(x) < 0$ *on* $(0, k) \subset [0, 1]$*, then* $\epsilon_D'(x) < 0$ *on* $(0, k)$*.*

The proof of the lemma is found in Appendix A. □

For some of the most common distributions of abilities we can characterize the skill premium and the Gini coefficient of wage inequality among skilled workers:

1. Pareto distribution: $\Psi(x) = 1 - (\frac{b}{x})^a, a > 1,\ x \geq b > 0$. In this case, $\epsilon_{\mathcal{H}}(s) = \frac{a-1}{a}$, $S(s) = \frac{1}{a-1}$, $G(s) = \frac{1}{2a-1}$, $S'(s) = 0$, $G'(s) = 0$.

2. Uniform distribution: $\Psi(x) = \frac{x-a}{b-a}$, $x \in [a, b]$. In this case, $\epsilon_{\mathcal{H}}(s) = 2\frac{b-s(b-a)}{2b-s(b-a)}$, $S(s) = \frac{1}{2}\frac{(b-a)s}{b-s(b-a)}$, $G(s) = \frac{1}{3}\frac{(b-a)s}{2b-s(b-a)}$, $S'(s) > 0$, $G'(s) > 0$.

3. Exponential distribution: $\Psi(x) = 1 - e^{-\lambda x}$, $\lambda > 0, x \geq 0$; $\epsilon_{\mathcal{H}}(s) = \frac{\ln s}{\ln s - 1}$; $S(s) = -1/\ln s$; $G(s) = \frac{1}{2}\frac{1}{1-\ln s}$, $S'(s) > 0$, $G'(s) > 0$

4. Log-normal distribution: $\Psi(x) = \Phi\left(\frac{\ln x - \mu}{\sigma}\right)$, $\sigma > 0, x > 0$, where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution. In this case, we have no explicit formula for the skill premium and the Gini coefficient of wage inequality among skilled workers; however, we prove in Appendix B that $S'(s) > 0$ and $G'(s) > 0$.

5. Truncated normal distribution. The truncated normal distribution is the probability distribution derived from that of a normally distributed random variable by bounding the random variable from below (by some $a \geq -\infty$) and above (by some $b \leq +\infty$ such that $b > a$). Let $\mu \in \mathbb{R}$, $\sigma^2 > 0$, and the function $y(\cdot)$ be given by $y(x) = \frac{x-\mu}{\sigma}$. Let, finally, $\alpha = y(a)$, $\beta = y(b)$.

   The cumulative distribution function of the truncated normal distribution (with parameters μ, σ, a, b), $\Psi(\cdot)$ is given by

   $$\Psi(x) = \frac{\Phi(y(x)) - \Phi(\alpha)}{\Phi(\beta) - \Phi(\alpha)}.$$

   As in the case of a log-normal distribution, we have no explicit formula for the skill premium and the Gini coefficient of wage inequality among skilled workers. However, we prove in Appendix B that $S'(s) > 0$ and $G'(s) > 0$.

6. Logit-normal distribution[5]: $\Psi(x) = \Phi\left(\frac{l(x)-\mu}{\sigma}\right)$, $\sigma > 0, x \in (0,1)$, where $l(x) = \ln(\frac{x}{1-x})$ is the logit function. Analogous to the cases of log-normal and truncated normal distributions, we develop an implicit argument to show that $S'(s) > 0$ and $G'(s) > 0$ (see Appendix B).

Thus we can formulate the following proposition.

Proposition 2. *1) If the distribution of abilities is Pareto, then both the skill premium and the Gini coefficient of wage inequality among skilled workers do not change as the fraction of skilled agents in the population increases. 2) If the distribution of abilities is uniform, exponential, log-normal, logit-normal, or truncated normal, then the skill premium and the Gini coefficient of wage inequality among skilled workers increase as the fraction of skilled agents in the population goes up.*

Our model also indicates that other measures of inequality, such as percentile ratios (P90/P50 or P90/P10), are non-decreasing. Specifically, these measures increase if the fraction of skilled agents rises between the two percentiles, and remain constant otherwise.

As shown in Appendix C, the assertion that within-group and/or between-group inequality invariably rises as the number of skilled agents increases is not universally valid. We construct several examples, showing that a high concentration of agents with similar abilities may lead to various types of dependence of inequality on the fraction of skilled agents[6]. While such distributions may be single-peaked, they are significantly different from, e.g., Gaussian distributions.

In the next section, we estimate the functions $T(\cdot)$ and $S(\cdot)$ using empirical distributions of students' assessment scores for several developed countries. Note that such scores can only be used as noisy proxies for ability. There is no way to look directly at a particular outcome of ability; therefore, we have to deal with some kind of approximation.

[5] Arthurs et al. (2019) argue that logit-normal is the best choice among low-dimensional distributions for modeling exam scores.

[6] These findings stand in contrast to the conclusions drawn by Meckl and Zink (2004). In their framework, the inequality within the skilled group exhibits an increasing trend, irrespective of the underlying distribution of abilities.

5 Some empirics

Empirical literature emphasizes the increasing role of cognitive skills in promoting economic well-being and wage determination. Hanushek and Woessmann (2008) conclude that there is strong evidence that the cognitive skills of the population, rather than mere school attainment, are powerfully related to individual earnings, the distribution of income, and economic growth. They show that countries with the largest variation in the level of ability in basic skills also have the highest degree of income inequality. On average, a one-standard-deviation increase in numeracy skills (measured by a PIAAC survey of adult skills over the full lifecycle in 23 countries) is associated with an 18 percent wage increase among prime-age workers (Hanushek et al., 2015). Ozawa et al. (2022) perform a meta-analysis of returns to wages from cognitive ability in developing countries, suggesting that a standard deviation increase in cognitive test scores was associated with a 4.5 percent increase in wages.

The data used in our study were obtained from the 2019 Wave of Trends in International Mathematics and Science Study (TIMSS). TIMSS is a series of international assessments of students' mathematics and science knowledge worldwide. TIMSS 2019 is the most recent in the TIMSS trend series, which began with the first assessments in 1995 and continued every four years. TIMSS assesses students in participating countries in their fourth year of formal schooling, provided the mean age at the time of testing is at least 9.5 years, and in their eighth year of formal schooling, provided the mean age at the time of testing is 13.5 years. The participating students come from diverse educational systems (countries or regional jurisdictions of countries) in terms of economic development, geographical location, and population size. The basic TIMSS sample design consists of at least 150 schools and one or more intact classes per grade for a student sample of approximately 4,000 students in each country. The data for each student comprise the resulting TIMSS-test score, an integer value distributed between 300 and 800. We focus on the results in mathematics of fourth grade in two advanced countries: England and the US.

Visual inspection of the data shows that the score distribution for these countries is close to bell-shaped (see histograms in Figure 4). Formal tests indicate that for each country the TIMSS score deviates[7] from a normal, truncated normal, log-

[7] We used variants of Shapiro-Wilk, Anderson-Darling, Kolmogorov-Smirnov, and Jarque-Bera tests. It turned out that with a sample size of more than $4,000$ in each country, the power of each specific test and the type of alternative hypothesis do not matter: any test rejects the null hypothesis.

normal, or logit-normal[8] distribution. Thus we evaluate the sample analog of the skill premium function $S(s)$, denoted as $\hat{S}(s)$. To do this, we estimate the empirical density using the kernel density estimation (KDE) method[9]. The red curves in Figure 4 represent kernel density estimation of cognitive test results with Silverman's rule-of-thumb bandwidth selection.

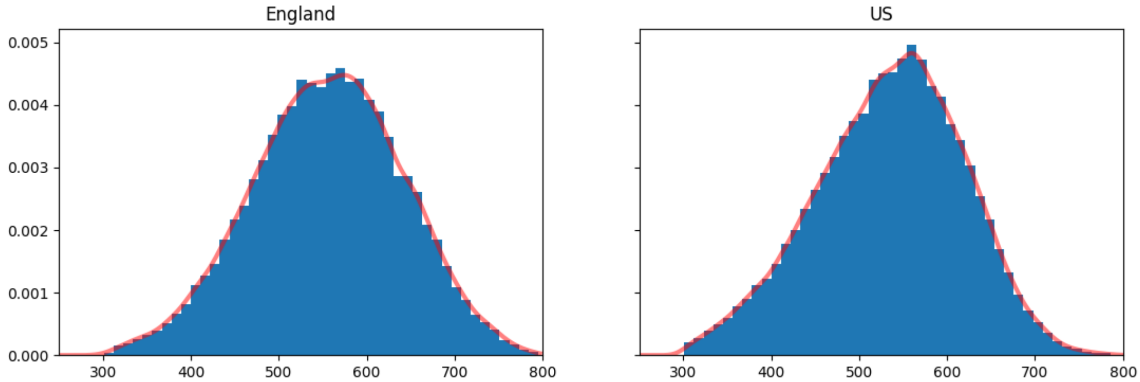


Figure 4: Score histograms and estimated densities for England and the US.

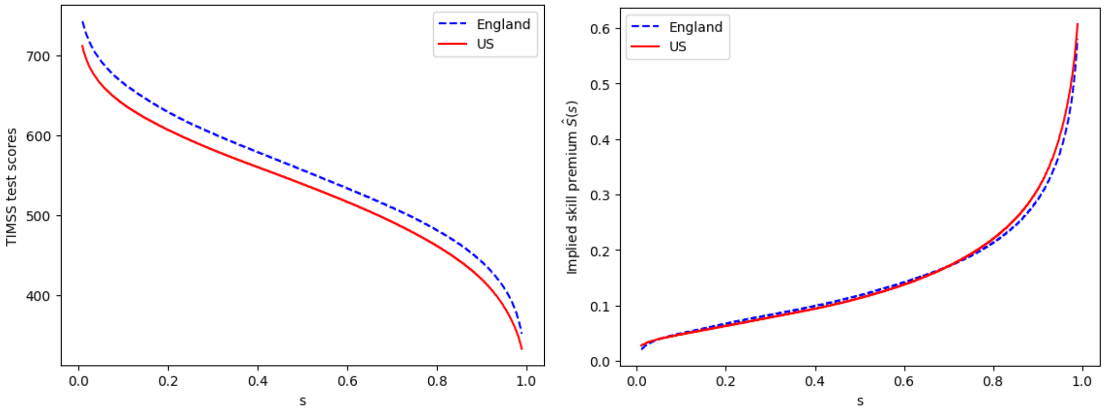
Furthermore, using estimated density, we evaluate the model implied skill premium function $\hat{S}(s)$ and explore them graphically. Figure 5a presents the estimated upper quantile function, denoted by $\hat{T}(s)$, and Figure 5b illustrates the model implied skill premium function $\hat{S}(s)$ for the selected countries. The results demonstrate that for both countries the function $\hat{S}(s)$ is increasing. Hence, it is derived from Lemma 1 and formulas that define functions $S(s)$ and $G(s)$ that, with the empirical score distribution as a proxy for the ability distribution, it holds that $S'(s) > 0$ and $G'(s) > 0$.

These distributions are typical for different cognitive ability tests in other developed countries as well (we use data from the PIAAC survey of adult skills to further support our findings). Additionaly, we incorporated data from various years to examine the temporal dynamics of the distribution. Our analysis revealed slight variations in distributions over time. Specifically, Figures 6a and 6b present the distribution curves of the TIMSS assessment results for England and the US, respectively, spanning the period from 2003 to 2019.[10] Importantly,

[8]In the case of a logit-normal distribution, the data were initially renormalized to the $(0, 1)$ interval.

[9]We used KDE realization with the Gaussian kernel function. KDE is known to be sensitive to the choice of bandwidth, whereas it is not really affected by the choice of kernel function. We use Silverman's rule of thumb for bandwidth selection, which is often used in practice. The results are robust to kernel functions and a moderate bandwidth selection.

[10]In order to ensure comparability of TIMSS scores across different years within each subject,

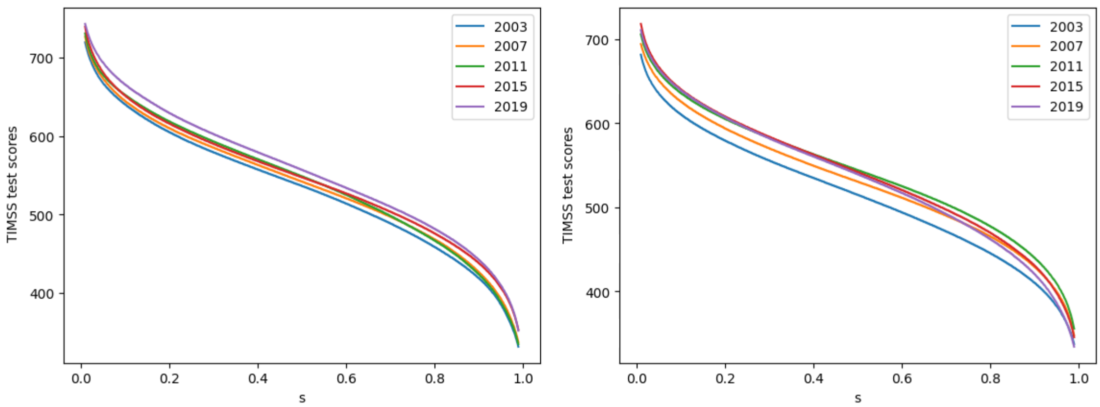


(a) graph of $\hat{T}(s)$

(b) graph of $\hat{S}(s)$

Figure 5: Estimated functions $\hat{T}(s)$ and $\hat{S}(s)$.

the observed lack of substantial variation aligns with our assumption of an exogenous and time-constant distribution of abilities. Therefore, our empirical analysis provides evidence that the composition effect can serve as a driving force that contributes to an increase in wage inequality.



(a) England

(b) the US

Figure 6: Estimated functions $\hat{T}(s)$ for the period from 2003 to 2019.

subsequent editions of the survey retain a sufficient number of items from previous waves, and the grading scale remains consistent, not being renormalized to a mean of 500 each year.

6 Conclusion

Bourguignon et al. (2005) coined the term "paradox of progress" to describe the unequalizing effect of expanding education. Their analysis of the impact of educational expansion on economic development in various developing countries found that while additional years of schooling had a generally positive impact on mean household incomes, it more often than not increased inequality measures. The "paradox of progress" has garnered considerable attention and discussion, particularly among scholars of Latin American economies. Despite the decrease in wage inequality in Latin America during the 2000s, a number of studies have found that the direct impact of educational expansion in the 1990s and 2000s on earnings inequality was actually unequalizing (see, for example, Battistón et al., 2014).

In what way can the "paradox of progress" be explained? According to Bourguignon et al. (2005), this paradox can be attributed to the convexity of the earnings functions with respect to education, which results in the same proportional increases in education leading to higher proportional increases in income for the more educated. They also highlight that their findings are not at odds with Tinbergen's (1975) view that increasing schooling levels would reduce inequality by decreasing the equilibrium rates of return to education in response to higher supply. They argue that their analysis captures only a partial effect of educational expansion while holding skill prices constant.

In some circumstances our framework suggests that the unequalizing earnings effect of expanding education can be considered as a general equilibrium phenomenon. Similar to Galor and Moav (2000), Meckl and Zink (2004), and Miyake et al. (2009), our theoretical model and empirical findings demonstrate that despite accounting for the downward pressure on skill prices resulting from a rise in supply relative to the demand for skills, the expansion of education may ultimately lead to rising wage inequality via the composition effect.

Other empirical observations regarding the impact of higher education expansion on wage inequality also provide tentative support for our conclusions. Carneiro and Lee (2011) present evidence suggesting that increases in college enrollment in the United States between 1960 and 2000 resulted in a decline in the average quality of college graduates. The primary result of Hendricks and Schoellman (2014) indicates that a widening skill disparity between high school and college-educated labor is responsible for the increase in the college wage premium from 1910 to 1960 birth cohorts in the US. Lindley and McIntosh (2015) focus on the UK and show that most of the growth in graduate wage inequality has occurred within subjects and that this growth is due to the acceptance of students from lower in the ability distribution. The case of Taiwan also illustrates that expanding access to college without maintaining its quality may result in an increase rather than a decrease in wage inequality within the economy (Keng et al. 2017).

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Appendix A. Proof of Lemma 1

We first prove that

$$\epsilon_D'(x) < 0 \ \Leftrightarrow \ \epsilon_d(x) < \epsilon_D(x) - 1, \ x > 0. \tag{A.1}$$

We have

$$\epsilon_D'(x) = \frac{\Big(d'(x)x + d(x)\Big)D(x) - xd^2(x)}{D^2(x)} = \frac{xd'(x)}{d(x)}\frac{d(x)}{D(x)} + \frac{d(x)}{D(x)} - x\frac{d(x)}{D(x)}\frac{d(x)}{D(x)}.$$

Multiplying both sides of the latter identity by x, we obtain

$$x\epsilon_D'(x) = \epsilon_D(x) - \epsilon_D(x)^2 + \epsilon_D(x)\epsilon_d(x), \ x > 0. \tag{A.2}$$

This equality proves (A.1).

Using L'Hopital's rule, we get

$$\lim_{x\to+0} \epsilon_D(x) = \lim_{x\to+0} \frac{d(x)x}{D(x)} = \lim_{x\to+0} \frac{d'(x)x + d(x)}{d(x)} = \lim_{x\to+0} \epsilon_d(x) + 1.$$

Therefore,

$$\lim_{x \to +0} \epsilon_D(x) = \lim_{x \to +0} \epsilon_d(x) + 1. \tag{A.3}$$

With this we can extend the functions $\epsilon_d(\cdot)$ and $\epsilon_D(\cdot)$ to the whole interval $[0, k)$ by putting $\epsilon_D(0) = \epsilon_d(0) + 1$.

Now suppose that while $\epsilon'_d(x) < 0$ on $[0, k)$, $\epsilon'_D(\hat{x}) \geq 0$ for some $\hat{x} \in [0, k)$.

By (A.1), $1 + \epsilon_d(\hat{x}) - \epsilon_D(\hat{x}) \geq 0$. Since $1 + \epsilon_d(0) - \epsilon_D(0) = 0$, there is $\tilde{x} \in (0, \hat{x})$ such that $\epsilon'_d(\tilde{x}) - \epsilon'_D(\tilde{x}) \geq 0$. Since, by assumption, $\epsilon'_d(\tilde{x}) < 0$, we have $\epsilon'_D(\tilde{x}) < 0$ and, taking account of (A.1), $1 + \epsilon_d(\tilde{x}) - \epsilon_D(\tilde{x}) < 0$. By continuity, there exists $\delta > 0$ such that $1 + \epsilon_d(x) - \epsilon_D(x) < 0$ for any $x \in [\tilde{x}, \tilde{x} + \delta]$.

Now let

$$\bar{x} = \inf\{x \in [\tilde{x} + \delta, k) | 1 + \epsilon_d(x) - \epsilon_D(x) \geq 0\}.$$

Clearly, $1 + \epsilon_d(\bar{x}) - \epsilon_D(\bar{x}) = 0$ and hence $\epsilon'_D(\bar{x}) = 0$. Since $\epsilon'_d(\bar{x}) < 0$, we have $(1 + \epsilon_d(\bar{x}) - \epsilon_D(\bar{x}))' < 0$. It follows that there exists $\check{x} \in (\tilde{x}, \bar{x})$ such that $1 + \epsilon_d(\check{x}) - \epsilon_D(\check{x}) \geq 0$, which is a contradiction. This contradiction proves the lemma.

Appendix B. Proof of Proposition 2.

Log-normal distribution

We want to prove that $S'(s) > 0$ and $G'(s) > 0$ in the case of a log-normal distribution. Since $\epsilon_T(j) \leq 0,\ j \in (0, 1)$, by Lemma 1, it is sufficient to show that $\epsilon'_T(j) < 0,\ j \in (0, 1)$.

First, we prove the following lemma.

Lemma 2. *Let $\Phi(\cdot)$ be the cumulative distribution and $\varphi(\cdot)$ be the probability density function of the standard normal distribution. Then*

$$(1 - \Phi(y))y < \varphi(y)\ \forall y. \tag{B.1}$$

Proof. Clearly, this inequality holds true for $y \leq 0$. In the case where $y > 0$, we have:

$$
\begin{aligned}
1-\Phi(y) &< 1-\Phi(y)+\frac{1}{\sqrt{2\pi}}\int_y^{+\infty} t^{-2}\exp\left(-t^2/2\right)dt \\
&= 1-\Phi(y)+\frac{1}{\sqrt{2\pi}}\left(-\frac{1}{t}\exp\left(-t^2/2\right)\Big|_y^{\infty}-\int_y^{+\infty}\exp\left(-t^2/2\right)dt\right)=\frac{\varphi(y)}{y},
\end{aligned}
$$

and, therefore, (B.1) also holds true for all y. $\square$

Recall that the cumulative density function of a log-normal distribution is given by

$$\Psi(x)=\Phi\left(\frac{\ln x-\mu}{\sigma}\right),\ x>0,$$

with $\sigma>0$, and the probability density function is given by

$$\psi(x)=\frac{1}{\sigma x}\varphi\left(\frac{\ln x-\mu}{\sigma}\right)=\frac{1}{x\sigma\sqrt{2\pi}}\exp\left(-\frac{(\ln x-\mu)^2}{2\sigma^2}\right).$$

It is easy to show that the elasticity $\epsilon_T(j)$ of $T(j)$ is given by

$$\epsilon_T(j)=-r(x)|_{x=T(j)}, \tag{B.2}$$

where the function $r(\cdot)$ is defined by

$$r(x)=\frac{1-\Psi(x)}{\psi(x)x},\ x>0.$$

We have $T'(j)<0,\ j\in(0,1)$. Therefore, to prove that $\epsilon_T'(j)<0,\ j\in(0,1)$, it is sufficient to verify that

$$r'(x)<0,\ x>0. \tag{B.3}$$

It is easy to note that

$$r(x)=\sigma\left(\frac{1-\Phi(\frac{\ln x-\mu}{\sigma})}{\varphi(\frac{\ln x-\mu}{\sigma})}\right). \tag{B.4}$$

Also it is not difficult to check that

$$r'(x)=-\frac{1}{x}-\left(\frac{\psi'(x)}{\psi(x)}+\frac{1}{x}\right)r(x)$$

and that

$$\frac{\psi'(x)}{\psi(x)} + \frac{1}{x} = -\frac{1}{\sigma^2 x}(\ln x - \mu).$$

Therefore, we obtain

$$r'(x) = -\frac{1}{x} + \frac{1}{\sigma^2 x}(\ln x - \mu) r(x) = \frac{1}{x}\left(-1 + \frac{\ln x - \mu}{\sigma^2} r(x)\right).$$

Taking into account (B.4), to prove (B.3), we should note that

$$\frac{\ln x - \mu}{\sigma^2} r(x) < 1 \ \forall x > 0,$$

which follows from Lemma 2.

Truncated normal distribution.

As in the case of the log-normal distribution, to prove that $S'(s) > 0$ and $G'(s) > 0$, it is sufficient to verify that the function $r(x) = \frac{1-\Psi(x)}{x\psi(x)}$ is strictly decreasing.

To do this, note that the probability density function of the truncated normal distribution (with parameters μ, σ, a, b), $\psi(\cdot)$ is given by

$$\psi(x) = \frac{\varphi(y(x))}{\sigma(\Phi(\beta) - \Phi(\alpha))},$$

where $y(x) = \frac{x-\mu}{\sigma}$, and hence $r(x) = q(y(x))$ with $q(y)$ being defined by

$$q(y) = \sigma \frac{\Phi(\beta) - \Phi(y)}{(\sigma y + \mu)\varphi(y)}.$$

Let us now prove that

$$q'(y) < 0. \tag{B.5}$$

We have

$$q'(y) = \sigma \frac{-\varphi^2(y)(\sigma y + \mu) - (\Phi(\beta) - \Phi(y))((\sigma y + \mu)\varphi'(y) + \sigma\varphi(y))}{((\sigma y + \mu)\varphi(y))^2}.$$

Therefore, to show that (B.5) is satisfied, it is sufficient to demonstrate that:

$$\frac{(\sigma y + \mu)\varphi(y)}{\sigma\Big(\Phi(\beta) - \Phi(y)\Big)} > -1 - \frac{(\sigma y + \mu)\varphi'(y)}{\sigma\varphi(y)}. \tag{B.6}$$

By Lemma 2, we have $(1 - \Phi(y))y < \varphi(y), y \in \mathbb{R}$, resulting

$$\frac{\varphi(y)}{1 - \Phi(y)} > y.$$

Thus,

$$\frac{(\sigma y + \mu)\varphi(y)}{\sigma(\Phi(\beta) - \Phi(y))} > \frac{(\sigma y + \mu)\varphi(y)}{\sigma(1 - \Phi(y))} > y^2 + \frac{\mu}{\sigma} y.$$

It is straightforward to verify that $-\frac{\varphi'(y)}{\varphi(y)} = y$ and hence $-\frac{(\sigma y+\mu)\varphi'(y)}{\sigma\varphi(y)} = y^2 + \frac{\mu}{\sigma} y$, which implies (B.6). This proves (B.5). It follows that the function $r(x) = \frac{1-\Psi(x)}{x\psi(x)}$ is strictly decreasing.

Logit-normal distribution.

The probability density function $\psi(\cdot)$ of the logit-normal distribution with parameters μ and σ is given by:

$$\psi(x) = \frac{\varphi(y(l(x)))}{\sigma x(1 - x)}, \quad x \in (0, 1),$$

where $y(x) = \frac{x-\mu}{\sigma}$ is the normalization transformation, and $l(x) = \ln(\frac{x}{1-x})$ denotes the logit-function. Again to prove that $S'(s) > 0$ and $G'(s) > 0$, we show that the function $r(x) = \frac{1-\Psi(x)}{x\psi(x)}$ is strictly decreasing. It is straightforward to show that

$$r'(x) = \frac{r(x)}{1 - x}\left(\frac{y}{\sigma x} - 1\right) - \frac{1}{x}.$$

Here y stands for $y(l(x))$. If $y > 0$ then by Lemma 2 we have $\frac{r(x)}{1-x} \equiv \sigma\frac{1-\Phi(y)}{\varphi(y)} < \frac{\sigma}{y}$. Thus we can estimate that:

$$r'(x) < \begin{cases} -\frac{r(x)}{1-x} - \frac{1}{x}, & \text{if} \quad y \leq 0, \\ -\frac{r(x)}{1-x}, & \text{if} \quad y > 0. \end{cases}$$

The latter implies $r'(x) < 0$.

Appendix C. Examples of non-increasing dependence of inequality on the fraction of skilled workers.

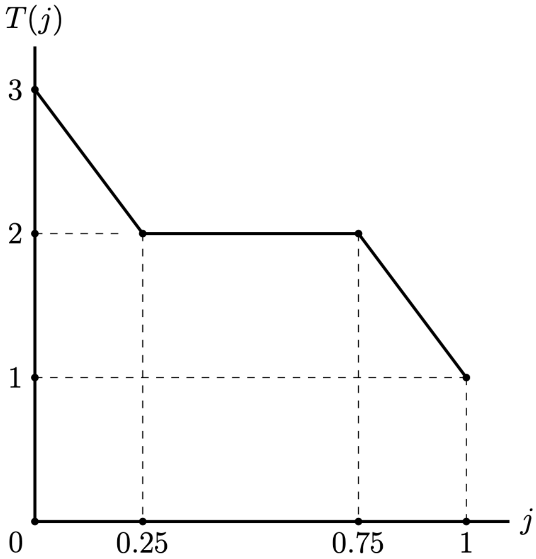
The goal of this section is to explore the boundaries of applicability of Proposition 2. We present two examples of distributions where the concentration of the mass of talented agents brings curious effects. The first example shows that there exist distributions such that there are intervals where

1. both functions $S(\cdot)$ and $G(\cdot)$ decrease,
2. $S(\cdot)$ is decreasing while $G(\cdot)$ is increasing,
3. $G(\cdot)$ is decreasing while $S(\cdot)$ is increasing.

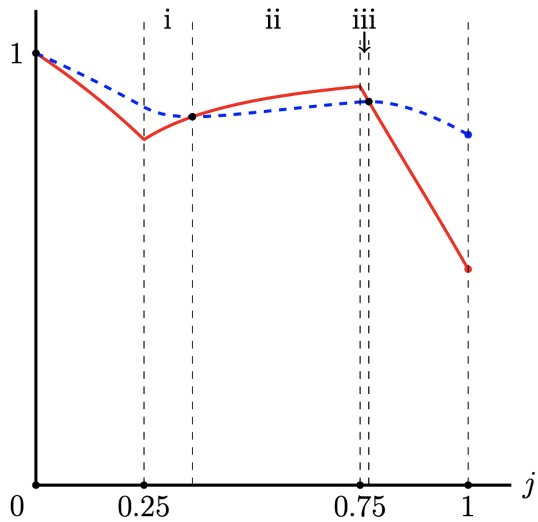
Example 1. First, we construct a distribution with finite support. It is convenient to relax the assumption that distribution is continuous. We define the distribution as a fair mixture of the uniform distribution on $[1, 3]$ and an atom at $x_0 = 2$. Therefore, the upper quantile function $T(\cdot)$ is flat in some neighborhood of $1/2$ (see Figure C.1a). The elasticities $\varepsilon_{\mathcal{H}}$ and ε_{Γ} can be then calculated directly. We do not include calculations here because they are straightforward, but rather depict the resulting graphs in Figure C.1b, which can be analyzed visually. The intervals of interest are shown in Figure C.1b. In this figure, we find i) an interval where $\varepsilon_{\mathcal{H}}$ increases while ε_{Γ} decreases, ii) an interval where both $\varepsilon_{\mathcal{H}}$ and ε_{Γ} increase, and iii) an interval where $\varepsilon_{\mathcal{H}}$ decreases while ε_{Γ} increases. On these intervals respectively i) S decreases when G increases, ii) both S and G decrease, and iii) S increases when G decreases.

Evidently, one can slightly perturb the constructed distribution so that the atom is smoothed out, but the resulting distribution still has essentially the same properties.

The second example shows that there exists a heavy-tailed distribution, such that the skill premium S may be decreasing on the fraction of skilled workers, even if the initial fraction of skilled labor is arbitrarily small. The intuition behind this result should be clear: since by Proposition 2 for a Pareto distribution, the elasticities (and thus both functions S and G) are constant, we can find by – slightly varying the density ψ in the tail of the distribution – a new distribution that leads to increasing $\varepsilon_{\mathcal{H}}$, and, thus, decreasing skill premium S.



(a) Upper quantile function $T(j)$ with an atom at $x_0 = 2$.



(b) Corresponding elasticities $\varepsilon_{\mathcal{H}}$ (solid line) and $\varepsilon_\Gamma - 1$ (dashed line).

Figure C.1: Ability distribution with an atom.

Example 2. Let the upper quantile function $T(\cdot)$ be such that $T(j) \asymp \frac{1}{j \ln^2(j)}, j \to +0$. Since $\lim\limits_{y \to +\infty} y(1 - \Psi(y)) = \lim\limits_{j \to +0} jT(j) = 0$ such a distribution has a finite mean. One can easily show, that the function $\varepsilon_{\mathcal{H}}(j) \asymp -\frac{1}{\ln(j)}, j \to +0$, and thus $\varepsilon_{\mathcal{H}}$ is increasing on some interval $(0, j^*)$.

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