Dividing Resources by Flexible Majority Rules

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Abstract

We examine the division of resources among individuals by flexible majority rules where the majority necessary to adopt a proposal depends on the proposal itself. For instance, the size of the majority may increase with the maximal difference between the shares individuals receive. For large discount factors such rules imply an efficient even distribution of resources. For low discount factors flexible majority rules supplemented by specific agenda-setting rules such as agenda rights for the opposition guarantee envy-free distribution. Uncertainty about discount rates can make it easier to achieve efficient and envy-free allocations.

Keywords: Flexible majority rules, division of resources, unanimity rule, simple majority rule, fair division.

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1 Introduction

One of the fundamental questions in politics and social choice is which rules should be applied to distribute resources among individuals. In this paper we suggest that flexible majority rules, where the necessary majority for the adoption of a proposal depends on the proposal itself, may overcome the disadvantages of collective decision rules such as majority rules or unanimity rules.

We consider the distribution of resources by a collective choice process in a dynamic model with endogenous agenda setting in the spirit of the contributions by Baron and Ferejohn (1989), Harrington (1986, 1990), Banks and Gasmi (1987), Ferejohn, Fiorina and McKelvey (1987) and Epple and Riordan (1987). A community of N individuals decides on how to distribute resources. The group meets for a finite or an infinite number of periods. Individuals are impatient, implying that waiting is costly. In each period, an agent is chosen according to some recognition rule. The proposal is accepted or rejected by a flexible majority rule. If the proposal is accepted, resources are allocated and the process is terminated; otherwise it continues.

Under a flexible majority rule, the required majority is derived from the proposal: the larger the difference between the shares of the cake across individuals, the larger the majority needed to adopt a proposal is. As an example, consider the polar case of a threshold flexible majority rule: The simple majority rule applies as long as the maximal difference in resources across individuals is below a certain threshold; otherwise the unanimity rule applies.

We suggest that flexible majority rules can alleviate the difficulties of the unanimity rule in reaching collective decisions, while simultaneously avoiding the main drawbacks of the majority rule. This investigation is motivated by a large literature on the advantages and disadvantages of the unanimity rule.[1] Wicksell (1896) was the first to link the potential for all to benefit from collective action to the unanimity rule. The unanimity rule is the only voting rule that always leads to Pareto-efficient public good quantities and tax schemes if no delay of decisions occurs, as recognized by Wicksell (1896)[2] and in the classic contribution of Buchanan and Tullock (1962) (see Mueller

1 See Buchanan (1991) and Mueller (1996) for general surveys on constitutional political economy.
2 Also, the famous Lindahl equilibrium (Lindahl 1919) could be reached under the unanimity rule.

(1995) for a comprehensive survey). Two main criticisms have been made against the unanimity rule.[3]

First, searching for a Pareto-efficient allocation might take considerable time, particularly in large communities with diverse interests. The time required to define an issue and transaction costs to reach an agreement may become so high as to wipe out the benefits from full cooperation under the unanimity rule.

Second, the unanimity rule invites strategic behavior: an individual strategically vetoes a proposal in order to seek a greater share of the gains from collective action (see Black (1965), Buchanan and Tullock (1962), Barry (1965), Samuelson (1969)). Because of these objections, even those most favorably disposed towards the unanimity rule, such as Wicksell, Buchanan and Tullock, have argued to abandon this rule.

However, the simple or super majority rules are plagued with drawbacks as well. In particular, majority rules can lead to the exploitation of minorities. The distribution of the benefits reflects the majoritarian distribution of power in that only a minimal majority of individuals receives a positive allocation of benefits.[4] Moreover, from an ex ante or constitutional point of view, the risk of being exploited by majorities is not Pareto-efficient if voters are risk averse (see Erlenmaier and Gersbach 1999).

In this paper, we suggest that flexible majority rules might alleviate the drawbacks of the unanimity rule and the majority rule. Suppose the polity uses a flexible majority rule where the size of the majority increases continuously or even jumps with the maximal difference between the benefits individuals receive. For large discount factors, such flexible majority rules lead to an efficient and even distribution of resources although the simple majority rule governs the collective choice process in equilibrium. A proposal with a more unequal distribution of benefits would require a larger majority or even the unanimity rule, making it unattractive for an agenda setter. For low discount rates, or when individuals are uncertain about each other's discount rates, efficient and envy-free divisions of resources can still be achieved if flexible majority rules are combined with agenda-setting rules, such as opposition agenda setting or the exclusion

3 Buchanan and Tullock (1962) identify such disadvantages as external costs of decision-making.

4 However, institutional arrangements and amendment rules which govern majoritarian bodies such as legislatures can lead to more universal distributions of resources and can act as a substitute to the flexible majority rules suggested in this paper (see Shepsle 1979, Weingast 1979 and Baron and Ferejohn 1989).

of repeated agenda setting. While this paper provides a sequence of examples, a comprehensive theory about flexible majority rules is lacking. In the last section we will discuss how further research could close this gap.

The paper is organized as follows. In the next section, we relate our work to the literature. In the third section, we introduce the model. Section four deals with the division of resources within a finite number of periods. In section five we discuss how the results change under an infinite time horizon. Section 6 presents our conclusion.

2 Relation to the Literature

Our work is related to further strands of literature. First, efficiency and fairness of distribution of resources has been studied under alternative collective choice processes. Mueller (1978) and Moulin (1981) examined the veto rule. In the case of no agreement, resources are thrown away. Equilibria in such games show a strong tendency towards equal shares for each individual. We are concerned with the possibility of using majority rules and only one proposal at each voting stage to create efficient and evenly distributed benefits of collective choices.

Second, flexible majority rules, where the size of the majority depended on the share of taxed people were introduced in Erlenmaier and Gersbach (1999) to achieve socially optimal allocations of public projects from a constitutional perspective. In this paper, we introduce flexible majority rules that depend directly on the different levels of benefits individual agents receive in a collective choice process.

We can reach efficiency and envy-freeness with flexible majority rules and agenda-setting rules. However, it is important that an outside agency such as a court or a special committee in a legislature can observe and verify the benefits received by individuals which is not possible in all circumstances in actual decisions. In particular, applying flexible majority rules for policies in legislatures is not possible in all cases, since benefits might not be verifiable. Moreover, the examination of institutional structures of legislatures (Shepsle 1979, Weingast 1979, Ferejohn, Fiorina and McKelvey 1987, Gilligan and Krehbiel 1987, Baron and Ferejohn 1989) suggest that agenda amendment rules can have a great impact on legislative outcomes even within the standard majority-rule framework. How flexible majority rules could and should be embedded in an institutional framework for legislatures remains an open issue.

Finally, our discussion is related to the fairness approaches how resources should be divided. The question of fair-division procedures that allow for sharing of divisible resources has received a lot of attention in recent years as developed and summarized in important contributions by Brams and Taylor (1996), Moulin (1995), Robertson and Webb (1998) and Young (1994). Since, in our paper, individuals are identical with respect to their preferences , we use efficiency and envy-freeness as criteria for assessing allocations of collective choice processes. Our suggestion to use flexible majority rules in collective choice processes might complement this work.

3 The Model

We consider the problem of a group of N individuals distributing one unit of benefits (N odd and $N > 1$) among themselves. All individuals are assumed to be risk-neutral and have utility functions

$$U_i = \sum_{t=1}^{\infty} \delta^t \, x_i^t$$

where x_i^t is the share of the cake individual i receives in period t. Since the cake can only be distributed once, x_i^t can only be positive for one period at most.[5] δ is the discount factor which, in the first part of the paper, will be assumed to be identical across individuals. The distribution of the cake is governed by a collective choice process containing a voting rule, and by a recognition rule that determines which individual may make a proposal. Later, we supplement the collective choice process by agenda rules. The socially optimal or Pareto-efficient solutions are characterized by the following obvious proposition:

Proposition 1

A distribution is Pareto-efficient if the whole cake is distributed in $t = 1$.

As in many other instances, Pareto-efficiency is not sufficient to describe all characteristics of socially desired allocations. In particular, we will apply the no-envy test,

5 Theoretically, it would be possible to distribute the cake partially over many periods which is inefficient and does not occur in equilibria. Therefore this possibility is neglected in our analysis.

which is the dominant argument of justice in microeconomic theory.[6] Of course, the no-envy test in our simple problem requires each individual to receive the same share $\frac{1}{N}$ of the cake since utility functions of all individuals are identical.

Society uses flexible majority rules to distribute the cake. A flexible majority rule $F : [0;1] \to [0;1]$ requires the share of votes necessary to adopt a proposal to be at least $F(\Delta x)$, where:

$$\Delta x := \max_{i,j} |x_i - x_j|$$

This is the most simple formulation of flexible majority rules, depending only on the maximal difference between the benefits two individuals receive. Any measure of the extent how benefits are distributed could be used to define flexible majority rules. We give the following three examples for flexible majority rules:

- *constant majority rule with a necessary majority* $\alpha (\alpha \in [0,1])$*:*

$$F_\alpha^C(\Delta x) = \alpha$$

- *threshold flexible majority rule with parameters* $\alpha, (\alpha \in [0,1])$ *and* $\beta (\beta \geq 0)$*:*

$$F_{\alpha,\beta}^T(\Delta x) = \begin{cases} 1 & \text{if} \quad \Delta x > \beta \\ \alpha & \text{otherwise} \end{cases}$$

- *linear flexible majority rule with the slope* α*:*

$$F_\alpha^L(\Delta x) = \frac{1}{2} + \alpha \Delta x \qquad \text{with} \qquad \alpha \in [0; 1/2]$$

Note that a threshold flexible majority rule jumps from an α–majority rule to an unanimity rule as soon as individuals receive shares of the cake differing by more than β from the shares of other individuals. We consider the following sequence of events within a particular period:

Stage 1: An individual is randomly chosen to set the agenda. The agenda setter a^t proposes an allocation of the cake $x_a^t = (x_1^t, \cdots, x_N^t)$ with $\sum_{i=1}^N x_i^t \leq 1$, so that the set $\mathcal{X}$ of feasible proposals is an N–dimensional simplex.

6 The no-envy test was stated by Foley (1967) and has been further developed by Kolm (1972) and Varian (1974). Moulin (1995) provides a comprehensive discussion of the pros and cons of the no-envy test.

Stage 2: The corresponding majority decision rule for x_a^t is determined and individuals cast their votes. If the proposal receives a majority, the process is terminated. Otherwise, the collective choice process continues in the next period.

In the following we will explore the collective choice process with finite and infinite periods. Following Baron and Ferejohn (1989) we will restrict attention to subgame-perfect equilibria, in which weakly dominated strategies are eliminated.[7] Moreover, we assume the following tie-breaking rule: If an agent is indifferent between supporting and rejecting a proposal, the agent will vote yes. The tie-breaking rule is not crucial for the results.

7 This assumption is standard because of the coarseness of voting rules. The refinement significantly reduces the multiplicity of equilibria and is, therefore, a critical assumption.

4 A Finite Number of Periods

4.1 Random Recognition

To illustrate the basic structure of the model and the way flexible majority rules work, consider a simple case of a two-period collective choice process with equal probabilities of recognition for each individual, i.e. each individual will have the same likelihood of setting the agenda. We assume that if by the end of the allocation process no proposal has been accepted, each individual receives zero benefits.[8] In a finite number of periods we can work backwards to derive the subgame-perfect equilibria. We obtain:

Proposition 2

Suppose that society uses a threshold flexible majority rule with a threshold of $\beta = 1-\delta$ to distribute resources. Then, there is a unique subgame-perfect equilibrium of the two-period collective choice process:

(i) The individual recognized in the first period, i.e. the agenda setter a, proposes

$$x_a = 1 - \frac{N-1}{N}\,\delta,\; x_i = \frac{\delta}{N},\; i \neq a$$

(ii) Society uses the simple majority rule to decide on the proposal in the first period. The first proposal is thus accepted and the collective choice process is adjourned in the first period.

Proof of proposition 2:

Because the group decision is terminated after two periods, each individual of the group votes for any second-period proposal. The individual recognized at the beginning of period 2 can thus successfully propose to take the whole cake since each individual will vote for it because her expected utility is zero anyway. According to our tie-breaking rule, the proposal will be unanimously supported. Although the unanimity rule applies, the agenda setter can extract all benefits in the second period. Her benefits are therefore given by δ. Since each individual has a probability $\frac{1}{N}$ of being recognized in the second period, expected utility amounts to $\frac{\delta}{N}$ if the proposal is rejected in the

8 This is not plausible in all cases since there might exist a default distribution of benefits. As long as, however, the community does not realize some social gains at the end of the collective choice process, we can transform the problem into our framework by an appropriate normalization.

first period. As a consequence, each individual votes for any first-period proposal in which she receives at least $\frac{\delta}{N}$. Therefore, the proposal will be adopted.

It remains to be shown that the proposal is indeed optimal for the agenda setter. Note that in equilibrium

$$\Delta x := \max_{i,j} |x_i - x_j| = x_a - x_i = 1 - \delta.$$

Therefore, the simple majority rule applies. If the recognized individual tried to increase her utility by increasing x_a, Δx would necessarily be above the given threshold. Thus, the unanimity rule would apply and the proposal would be rejected since at least one individual would receive resources below $\frac{\delta}{N}$. Of course, the agenda setter does not want the proposal to be rejected since, according to her proposal, she receives a utility of $1 - \frac{N-1}{N}\delta > \frac{\delta}{N}$ if $\delta < 1$ instead of an expected utility $\frac{\delta}{N}$ when the proposal is not accepted. $\blacksquare$

It is instructive to compare our results with those of Baron and Ferejohn (1989), who have examined the same model for the simple majority rule. Under the simple majority rule, an agenda setter in the first period distributes $\frac{\delta}{N}$ to $\frac{(N-1)}{2}$ other individuals and keeps $1 - \frac{\delta(N-1)}{2N}$ for himself.[9] Although the simple majority rule governs the collective choice process in our context, every individual receives at least $\frac{\delta}{N}$.[10] More unequal distributions of resources would imply that the unanimity rule applies and such proposals would be rejected. Flexible majority rules yield a more equal distribution of benefits although the simple majority rule governs the voting process in equilibrium.

4.2 Agenda Rules and Equal Distribution

While flexible majority rules yield an efficient, relatively even, distribution of benefits if the discount factor is sufficiently large, the agenda setter in the first period can extract most of the surplus if discount factors are low. In this case, the flexible majority rule supplemented by agenda-setting rules still guarantees equal or almost equal distributions of benefits. We explore the following agenda-setting rule:

9 Under the unanimity rule the same distribution of resources as under the threshold flexible majority rule is achieved.

10 See Herrero (1985) for an analysis of the unanimity rule for an infinite number of periods.

OAS: Opposition Agenda Setting: *In the first period, each individual has the same likelihood of determining the agenda. In the second period, only individuals who have voted against the proposal can apply for agenda setting.*[11] Each individual who can apply for agenda setting has the same chance of being recognized.

Such agenda-setting features create higher incentives to reject proposals in the first period which, in turn, induces agenda setters to propose more uniform distributions. By tailoring the flexible majority rule to the agenda-setting rule, we obtain:

Proposition 3

Suppose $\delta \geq \frac{1}{N}$ and OAS. Then, there is a flexible threshold majority rule $F^T_{\alpha,0}(\Delta x)$ that yields a subgame-perfect equilibrium in which the agenda setter in the first period proposes:

$$x = \left(\frac{1}{N}, \dots, \frac{1}{N}\right)$$

The agenda setter's proposal is accepted by an α–majority rule. α is given by:

$$\alpha = \frac{N - n^*}{N} \text{ with } n^* = [\delta\, N - 1]$$[12]

The subgame-perfect equilibrium is unique with respect to outcomes.[13]

The proof of proposition 3 is given in the appendix. The important point of proposition 3 is that flexible majority rules and opposition agenda setting yield an efficient and an envy-free allocation for low discount rates. Although only an α-majority rule is applied in equilibrium, any other proposal by the agenda setter requires the unanimity rule and, because of strategic rejection, will not be adopted by the community. Strategic rejection describes the fact that under OAS individuals have a strong incentive to reject a proposal in order to determine the agenda in the future if all other individuals support the proposal under consideration.

4.3 Uncertainty about Discount Factors

Flexible majority rules can also operate differently than simple majority rules when there is uncertainty about other individual's discount factors. Suppose that the dis-

11 Obviously, the anonymity of voters is not possible for this agenda rule to be applied and voting needs to take place openly and sequentially.

12 This means the minimal natural number greater or equal than $\delta\, N - 1$.

13 Note that we assume no tie-breaking rule in this example.

count factor of agents is either $\delta_H = 1$ or $\delta_L < 1$. The probability of an individual having a discount factor δ^H (δ^L) is denoted by p_H (p_L). Each individual can only observe her own discount factor. Suppose that the society uses a threshold flexible majority rule with parameters $\alpha = \frac{1}{2}$ and $\beta = 0$.

Uncertainty about discount factors requires new considerations, since agenda setters can try to gamble. We will illustrate this point by the following example.

Assume $N = 3$. The expected utility for the individual under consideration is $\frac{\delta_H}{N} = \frac{1}{N}$ or $\frac{\delta_L}{N}$, respectively if her first-period proposal is rejected. Now assume that the individual proposes $x = (\frac{N-2\delta_L}{N}, \frac{\delta_L}{N}, \frac{\delta_L}{N})$, where the first entry of the vector defines the benefits the agenda-setter would like to donate to herself. This proposal would not be rejected by individuals with low discount factors. The recognized individual receives a share that is larger than $1/N$. With probability $(p_L)^2$, both remaining individuals have low discount factors and will accept the proposal. Expected benefits from the proposal for an agenda setter with discount rate $\delta_a \in \left\{\delta_H, \delta_L\right\}$ amount to:

$$(p_L)^2\Big(1 - \frac{(2\delta_L)}{N}\Big) + \big(1 - (p_L)^2\big)\frac{\delta_a}{N}$$

Whether the proposal $x = \left(\frac{N-2\delta_L}{N}, \frac{\delta_L}{N}, \frac{\delta_L}{N}\right)$ is profitable for an agenda setter compared, for instance, to $x = (\frac{1}{N}, \frac{1}{N}, \frac{1}{N})$ depends on the discount rate. Suppose that the agenda setter has $\delta = \delta_L$. Then, the comparison yields that the agenda setter proposes $(\frac{1}{N}, \frac{1}{N}, \frac{1}{N})$ if and only if

$$p_L \leq \frac{1}{\sqrt{3}}$$

The agenda setter could also propose $x = \left(1 - \frac{1}{N} - \frac{\delta_L}{N}, \frac{1}{N}, \frac{\delta_L}{N}\right)$ with a winning probability p_L. This proposal yields a higher expected utility than the efficient and envy-free proposal $\left(\frac{1}{N}, \frac{1}{N}, \frac{1}{N}\right)$ if $p_L > \frac{1}{2}$. Therefore, as long as $p_L \leq \frac{1}{2}$, an agenda setter with $\delta_a = \delta_L$ will not gamble by the proposal $x = \left(1 - \frac{1}{N}, \frac{\delta_L}{N}, \frac{1}{N}, \frac{\delta_L}{N}\right)$. To sum up, as long as $p_L > \frac{1}{2}$, the agenda setter will gamble which creates the risk that the distribution of resources will be delayed. Consider an individual with discount rate $\delta_H = 1$ who is recognized to set the agenda in the first period. Since

$$(p_L)^2(1 - \frac{2\,\delta_L)}{N} + (1-(p_L)^2)\frac{1}{N} > \frac{1}{N}$$

such an agenda setter will always propose $x = (1 - \frac{2\delta_L}{N}, \frac{\delta_L}{N}, \frac{\delta_L}{N})$ instead of $x = (\frac{1}{N}, \frac{1}{N}, \frac{1}{N})$. Her proposal will be accepted with probability $(p_L)^2$. However, this proposal does not only violate envy-freeness, but it is also inefficient, since there is a positive probability that resources are distributed in the second period.

In order to achieve efficient and envy-free distributions of benefits when discount rates are uncertain, we can complement the flexible majority rule and the random recognition rule by a simple agenda right rule.

ERAS: Exclusion of Repeated Agenda Setting: *The individual that was allowed to set the agenda in the first period is excluded from agenda setting in the second period.*

This assumption makes gambling much less attractive. The individual under recognition knows that its utility is zero if her proposal is rejected. Thus it is not beneficial to deviate from proposal $x = (1/N, 1/N, 1/N)$. It is straightforward to show:

Proposition 4

Suppose that the discount factor of an individual is either $\delta_H = 1$ or $\delta_L < 1$. Suppose that the community uses a threshold flexible majority rule $F^T_{\frac{1}{2},0}$ and ERAS. Then, a unique equilibrium exists where the recognized individual, independently of her discount factor, proposes the distribution:

$$x = \left(\frac{1}{N}, \dots, \frac{1}{N}\right)$$

if $p_L \le p_L^*$ for some $p_L^* > 0$. The simple majority rule applies and the proposal is accepted by all individuals in the first period.

Proposition 4 indicates that flexible majority rules can lead to efficient and uniform distribution of resources if repeated agenda setting by individuals is excluded and if the chance that individuals have a low discount factor is not too high. The exclusion of repeated agenda setting is a plausible agenda rule. With such a simple agenda setting rule, agenda setters cannot risk to propose uneven proposals because the unanimity rule would be applied and the proposal might be rejected. In this respect, uncertainty about discount rates is favorable for the community to achieve efficient and envy-free allocations.

5 Division of Resources with Infinite Periods

When the number of periods is infinite, it is well known that any distribution of benefits may be supported as a subgame-perfect equilibrium if agents are not too impatient. Following the literature (see, e.g., Baron and Ferejohn (1989)), we focus on stationary equilibria or Markov equilibria. We are interested in whether the qualitative results in the last section carry over to collective choice processes with infinite periods.

Proposition 5

Suppose that society uses threshold flexible majority rules $F^{T}_{\frac{1}{2},1-\delta}(\Delta x)$ and a random recognition rule. Then, a unique stationary equilibrium exists in which any agenda setter proposes

$$x_a = 1-(N-1)\frac{\delta}{N}$$
$$x_i = \frac{\delta}{N}, \;\; i \neq a$$

Each individual votes for any proposal in which at least $\frac{\delta}{N}$ is received. The simple majority rule applies and the proposal is accepted in the first period.

The proof of proposition 5 is given in the appendix. Proposition 5 indicates that our results carry over to infinite horizons, at least in the simple cases we have considered. Introducing agenda-setting rules apart from the random recognition rule, however, makes the derivation of stationary equilibria more complicated. Whether the simple intuitions for the combination of flexible majority rules and agenda setting rules for finite periods carry over to infinite periods remains open and will be explored in future research.

6 Discussion and Conclusions

In this paper, we have used a new type of social choice rules, namely flexible majority rules, to distribute resources in larger communities. These rules combine the advantages of majority rules of being simple and avoiding strategic delay with the advantages of unanimity rules of yielding even distributions. The functioning of the flexible majority rules is intuitively understandable. In order to avoid very uneven distributions, the majority required for adopting a proposal increases with the amount of inequality a proposal creates. This makes it much harder for proposals to win elections when they imply a very heterogeneous distribution of resources. As a consequence, the combination of flexible majority rules and agenda-setting rules can yield efficient and envy-free allocations.

Further research is necessary to investigate how and if flexible majority rules could be applied in actual political processes. Flexible majority rules must be stated rather generally to be able to be applied in a large variety of contexts. Moreover, flexible majority rules require the computation of the utility spread Δx, which must be evaluated by an independent institution. Since this may seem rather difficult in some cases, it is desirable to search for other variables which can be measured more easily and can approximate the utility spread. Finally, it remains unclear how flexible majority rules can be applied when individuals differ strongly in their preferences and more than one commodity has to be allocated among a set of individuals. While there are many open issues with respect to the application of flexible majority rules, such rules might have the potential to improve collective choice processes.

7 Appendix

Proof of proposition 3:

Similar to the proof of proposition 2, an individual that is recognized in the second period proposes to keep the whole cake. Society will accept this proposal since all other individuals will receive zero utility anyhow. Thus, utility for an agenda-setting individual in the second period amounts to δ.

We use n to denote the number of individuals refusing the first-period proposal. Hence, first-period expected utility for an individual refusing the first period proposal is $\frac{\delta}{n}$ if the first-period proposal will be rejected.

Now assume that the following flexible majority rule is used:

$$F^T_{\alpha,0}(\Delta x) = \begin{cases} 1 & \text{if} \quad \Delta x > 0 \\ \alpha & \text{if} \quad \Delta x = 0 \end{cases}$$

α is left to be determined. We use n^* to denote the number of individuals voting against the proposal in equilibrium. If no additional individual wants to vote "no" and no individual voting "yes" wants to change her position, the following inequality must hold:

$$\frac{\delta}{n^* + 1} \leq \frac{1}{N} \leq \frac{\delta}{n^*}.$$

Now n^* is uniquely determined. α, which is the fraction of individuals voting "yes" necessary to adopt the proposal, is thus given by:

$$\alpha = \frac{N - n^*}{N}$$

By construction, no individual benefits from changing her voting behavior. It remains to be shown that no proposal exists that would yield higher benefits for the recognized individual.

Clearly, the above proposal is more attractive than any proposal that would be rejected. If the recognized individual chose a different distribution of benefits with a larger share of the cake for herself, the unanimity rule would apply. But such a proposal would invite strategic rejection. If all other individuals accept the proposal, incentives to vote "no" are extremely strong for one individual, since the opposing individual could receive all of the benefits in the next period. The condition that such strategic rejection does not occur is

$$\delta \geq \frac{1}{N}$$

which is true for most plausible values of N[14] and δ and is assumed in the proposition.

■

Proof of proposition 5[15]:

It is obvious that the configuration of proposals and voting strategies is an equilibrium in any subgame. In order to establish necessity, suppose that w denotes the stationary expected continuation for an individual value when the proposal in the first period is rejected. Since all individuals are identical at the beginning of period 2, expected continuation values are equal for all individuals. Consider the proposal made by an individual in the first period. Any individual receiving $x_i \geq \delta w$ will vote for such a proposal. Because of the flexible-majority-rule requirement, suppose that the agenda setter offers $x_i = \delta w$ for all individuals in period 1. Then, expected utility for all individuals amount to

$$w = \frac{1}{N}\left[1 - (N-1)\delta w\right] + \frac{N-1}{N}\delta w$$

which implies $w = \frac{1}{N}, x_a = 1 - \frac{N-1}{N}\delta$ and $x_i = \frac{\delta}{N}$ for $i \neq a$. We have $\max /x_a - x_i/ = 1 - \delta$ and the simple majority prevails.

Note that it is impossible for the agenda setter to increase his share of the cake. An increase in x_a necessarily implies that $x_i < \frac{\delta}{N}$ for at least one individual. Therefore, since $\max /x_a - x_i/$ would be greater than $1 - \delta$ the unanimity rule applies. But since $x_i < \delta w$ for at least one individual, the proposal will be rejected and the agenda setter is worse off than with the equilibrium proposal.

■

14 It is important to note that equilibria exist where n individuals vote "no" for every n with $0 \leq n \leq n^*$. These equilibria yield the same distribution of resources since the proposal is always accepted.

15 The proof uses considerations similar to those used in Baron and Ferejohn (1989) for the simple majority rule .

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