Flexible Majority Rules for Central Banks*

Hans Gersbach
Department of Economics and CEPR
University of Heidelberg
Grabengasse 14
D-69117 Heidelberg, Germany
gersbach@uni-hd.de

Bernhard Pachl
Department of Economics
University of Heidelberg
Grabengasse 14
D-69117 Heidelberg, Germany
Bernhard.Pachl@urz.uni-heidelberg.de

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Abstract

We propose a flexible majority rule for central-bank councils where the size of the majority depends monotonically on the change in interest rate within a particular time frame. Small changes in the interest rate require a small share of supporting votes, possibly even less than 50%. We show that flexible majority rules are superior to simple majority rules and can implement the optimal monetary policy under a variety of circumstances.

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1 Introduction

This paper proposes a flexible majority rule for central banks. The flexible majority rule works as follows: Within a prespecified time frame, the size of the majority necessary for adopting a change in the interest rate depends on the change in the interest rate itself. For small changes in the interest rate, only a small share of the votes is required, possibly even less than 50%. For large interest rate changes, a larger majority is necessary, tending towards total unanimity.

We consider a model where N central bankers, representing countries, regions, or different constituencies within a country, decide on monetary policy. The central bank loss function is composed of the weighted loss functions of countries, regions, or constituencies. This is the typical case for the European Central Bank (ECB), but also applies to the Federal Reserve. In our example, we consider the ECB, when the monetary union is hit by a shock dividing the union into two parts. After this one part desires a change in monetary policy, while the other part wants to retain the status quo. For instance, some countries may be affected negatively by a negative supply or demand shock, and concern for their own country's welfare makes them want to ease monetary policy through interest-rate cuts. Other countries not affected by the shock will prefer no change in the interest rate. Under simple majority rule, a change in interest rate will occur, if and only if a simple majority of votes desires a change. Under flexible majority rule, small changes in the interest rate will only require a small share of supporting votes and hence a small number of countries to agree, whereas large changes in the interest rate require large majorities.

The key advantage of the flexible rule is that a number of countries hit by negative shocks can partially ease the consequences by a small interest-rate cut. Larger changes in the interest rate, however, require larger majorities, which can only be achieved if a larger number of countries are affected by the shock. The flexible majority rule aligns the severity of shocks and the socially desirable change in the interest rate. The drawbacks of simple majority rules and unanimity rules (possible exploitation by minorities, unanimity rules creating extreme veto power) can be overcome by flexible majority rules.

We distinguish two cases. First, the vote of every central banker has the same weight; second, the vote of a central banker is weighted to the same degree as his country is weighted in the central bank loss function. Our main results are as follows: First, in both cases the flexible majority rule always leads to smaller central bank losses than the simple majority rule. Second, if every vote is weighted as described above, flexible majority rules can implement the socially optimal solution. Third, it is socially optimal for small interest-rate changes within a particular time frame to be brought about by minorities - either one large country or a set of small countries. Similarly, it is socially desirable for large interest-rate changes to require large majorities. The main intuition for our results is that flexible majority rules of the kind described above can mimic aggregate social loss minimization, which calls for small interest-rate changes when shocks are small and affect only a few countries and large interest-rate changes when shocks are larger and affect many countries.

The paper is organized as follows: In section 2 we explain the flexible majority rule and relate our work to the literature. Section 3 presents our model, with the specific properties of the shock function, the assumed central bank loss function, and the constitutional process that determines how a change in interest rate is implemented. In section 4 we describe the different decision rules and their outcomes. In section 5 we discuss the results, and section 6 concludes the article. Most of the proofs can be found in appendix A, in appendix B we give a simple example involving three countries for all decision rules and appendix C is an extension to more general preferences.

2 Relation to the Literature

2.1 Regional Bias in Central Bank Decisions

A socially desirable procedure for making decisions in central bank councils has been the focus of a substantial body of recent literature, most of it centered around the ECB.

The ECB's Governing Council makes decisions about interest rates. The Council consists of the Executive Board of the ECB (president, vice-president, and four other members) and the central-bank governors of the 12 euro countries). The one person one vote principle prevails. Two main issues have been investigated. First, before the (virtual) euro was introduced in 1999, the optimal institutional design of the ECB had focused on the degree of centralization. Von Hagen and Süppel (1994), Lohmann (1997), and Bindseil (2001) have highlighted the advantages of a stronger role for the centrally nominated ECB.[1] As the current decision-making procedure relies strongly

[1]The advantages of centralization have gained renewed interest in the current process of EU enlargement (Baldwin, Berglof, Giavazzi, and Widgren (2001), and Berger, de Haan, and Inklaar (2003)).

on the national representatives, who have a political weight of about $\frac{2}{3}$ of all votes, flexible majority rules might partially act as a substitute for a lack of centralization.

Second, national and regional considerations appear to play a substantial role in ECB decision-making, as has been suggested by Heinemann and Hüfner (2002). In such circumstances, matching economic size and voting power by vote-weighting improves welfare, as we demonstrate in this paper, and as suggested for instance by Berger and de Haan (2002). Under such schemes, votes by national representatives are weighted by the member countries' share in the GDP of the euro area. We show that weighting and flexible majority rules can yield the first-best monetary policy. Our suggestion is potentially applicable to any central bank where different members of the decision-making body represent different groups or regions. Recent research has highlighted the fact that heterogeneity of preferences and even a regional bias play a significant role at the Federal Reserve. In particular, governors tend to vote against the majority when there is a significant gap between the unemployment rate in their *region* and the national rate (Meade and Sheets (2002)). Therefore flexible majority rules might also be appropriate for the Fed.

2.2 Efficient Collective Decision-Making

On a broad conceptual level, our paper addresses the optimal design of majority rules, which has a long tradition in economic and political science.

In every collective decision problem, the question arises as to which decision rule should be used in order to achieve socially desirable outcomes. One of the most widely employed decision rules is the simple majority rule, where a proposal is accepted if it obtains more than 50% of the votes. For example, in countries with a democratic constitution, most of the processes in which politicians are elected and parliamentary decisions are taken follow the simple majority rule. An early discussion of when this rule may be optimal can be found in Rae (1969) and Taylor (1969). May (1952) has shown that the simple majority rule satisfies a number of axioms that date back to the enlightment era.

Nevertheless the simple majority rule is not optimal in all cases. The classic work by Buchanan and Tullock (1962) shows that a majority other than 50% might be optimal. Other majorities are realized, for example, in the veto or the unanimity rule in the United Nations Security Council, or the $\frac{2}{3}$ majority needed for an amendment of the constitution in the Federal Republic of Germany. As shown by Caplin and Nalebuff

(1988), super majority can be designed to have desirable properties, including the elimination of cycles.

Fixed majorities can however very often lead to inefficiencies from a utilitarian perspective. Consider, for example, a collective decision problem where two groups have preferences located at two extremes. If one group is at least as big as the fixed majority needed in this decision problem,[2] it can always overrule the other group, which may lead to serious dissatisfaction on the part of the minority (see for instance Guinier (1994)) and which is not optimal from a utilitarian perspective. In this paper, we design flexible majority rules that can imitate a first-best solution in a utilitarian sense. Furthermore, in the recent past there has been a renewed interest in new decision rules. Casella (2005) suggests a system of storable votes, where the voters can choose between the possibility of voting now or storing the vote and having an additional vote in the future.[3] In this paper we design flexible majority rules for monetary policy.[4]

3 The Model

3.1 Central Bank Council

We consider a monetary union consisting of $N \in \mathbb{N}$ countries, which jointly make decisions about monetary policy in a single central bank such as the ECB. Countries are denoted by k or l. The monetary policy is decided in a central bank council where each country k delegates a central banker who represents the interest of country k. The social loss function for every single country is given by

$$L_t^k = (i_t - i_t^k)^2 \tag{1}$$

The variable i_t denotes the interest rate adopted by the central bank in period t, and i_t^k denotes the interest rate of country k ($k \in \mathcal{N} = \{1, 2, ..., N\}$) which the $k-th$ central banker wants to implement in period t. The interest rate i_t^k is the bliss point of country k and depends on a shock ϵ that occurs at the end of period $t-1$. Two

[2] In this case the majority has to be greater than 50%.

[3] Dixit and Jensen (2000) model the way in which governments could influence the central bank by offering incentive contracts.

[4] There are real-world examples of flexible majority rules in the context of public good provision, as has been pointed out by Amihai Glazer and discussed in Erlenmeier and Gersbach (2001). For instance, when a person buys property in Irvine in Southern California, he signs a contract making him a member of a homeowner association which provides local public goods and which has the right to levy annual fees. The required share of votes to implement an increase of the fees depends on the proposed fee change.

remarks about the social loss function of a country are in order. First, the social loss function can be derived from standard monetary models.[5] Second, the particular quadratic functional form considerably simplifies the representation of flexible majority rules. However, we shall see that only two assumptions about social losses are essential: firstly, social losses induce single-peaked and convex preferences in i_t which implies for aggregate social losses of the monetary union a unique optimizing interest rate in all circumstances, and secondly, this unique optimizer increases in shock size.

We arrange countries according to their weights, i.e. $g_k \leq g_l$ for all k with $k < l$. The aggregated loss function for the whole union is assumed to be given by the weighted sum of the single loss functions:

$$\mathcal{L}_t = \sum_{k=1}^{N} g_k L_t^k \tag{2}$$

with $g_k \in (0,1)$ and $g_k \leq g_l \,, \forall k < l \,, k, l \in \mathcal{N}$ and $\sum_{k=1}^{N} g_k = 1$, where g_k are the weights of the countries representing, for example, differences[6] in GDP or population.

Given the status quo in $t-1$ with an interest rate i_{t-1}, we assume that the monetary union is hit by a supply or demand shock dividing the union into two parts. One part is affected by the shock and the other part is not. We denote the countries that form the region affected by the shock by the subset $\mathcal{K}$, with $\mathcal{K} \subseteq \mathcal{N}$. Without loss of generality we will analyze positive realizations of shocks and thus possible increases in in interest rate, as negative realizations lead to corresponding declines in interest rate.

To keep things as simple as possible, we assume that if a shock occurs, every affected country is hit by the same aggregate shock. It is natural to assume that the bigger the aggregate economic weight of the affected region is, the bigger the aggregate shock will be. This results in a strictly monotonically increasing shock function $\epsilon(G_{\mathcal{K}})$ with $G_{\mathcal{K}}$ the aggregate economic weight of the affected countries, given by $G_{\mathcal{K}} = \sum_{k \in \mathcal{K}} g_k$ and $\epsilon(0) = 0$.

Furthermore, we assume that at time $t-1$ the union is in a long-term equilibrium and that i_{t-1} is the optimal interest rate in $t-1$.

Hence, the desired interest change of country k can be written as

$$i_t^k(\epsilon(G_{\mathcal{K}})) = \gamma_k \Delta i_t(\epsilon(G_{\mathcal{K}})) + i_{t-1} \tag{3}$$

[5] See e.g. Woodford (2003). Gersbach and Hahn (2001) show that this functional form of losses can be obtained if supply shocks are normally distributed.

[6] In this paper we do not focus on the determination of g_k, but take it as given.

where γ_k is a geographical indicator variable describing whether a country is affected by the shock or not and γ_k is then given by

$$\gamma_k = \begin{cases} 1 & \text{for } k \in \mathcal{K} \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

$\Delta i_t(\epsilon(G_k))$ is the desired interest rate change if the shock ϵ has occurred, with $\Delta i_t(0) = 0$. Summing up all subsets of $\mathcal{N}$, we obtain 2^N possible different shock scenarios in the union represented by $(\mathcal{K})$.[7] We assume that these shocks are distributed according to an arbitrary probability distribution. In particular, we denote by $p_{\mathcal{K}}$ the probability that all countries in $\mathcal{K}$ are affected by the shock. Note that $\sum_{n=0}^{N}\sum_{\sigma(|\mathcal{K}|=n)} p_{\mathcal{K}} = 1$ when $|\mathcal{K}|$ is the number of elements in $\mathcal{K}$ and $\sum_{\sigma(|\mathcal{K}|=n)}$ is the sum over all subsets $\mathcal{K}$ of $\mathcal{N}$ containing n elements.

If we consider that i_t can be written as

$$i_t = i_{t-1} + \Delta\hat{i}_t \tag{5}$$

where $\Delta\hat{i}_t$ is the actual change in the interest rate from period $t-1$ to period t and with (3), we can write

$$L_t^k = (\Delta\hat{i}_t - \gamma_k \Delta i_t)^2 \tag{6}$$

In the following, we drop the time index t, since we are focussing on the specific period from $t-1$ to t and do not consider permanent shocks.[8] Now we can write the social loss function of the union in any specific shock scenario, denoted by $\mathcal{L}_{\mathcal{K}}$, as[9]

$$\begin{aligned} \mathcal{L}_{\mathcal{K}} &= G_{\mathcal{K}}\left(\Delta\hat{i}\left(G_{\mathcal{K}}\right) - \Delta i\left(G_{\mathcal{K}}\right)\right)^2 + (1-G_{\mathcal{K}})\left(\Delta\hat{i}\left(G_{\mathcal{K}}\right) - 0\right)^2 & (7) \\ &= \left(\Delta\hat{i}\left(G_{\mathcal{K}}\right) - G_{\mathcal{K}}\Delta i\left(G_{\mathcal{K}}\right)\right)^2 + G_{\mathcal{K}}(1-G_{\mathcal{K}})\left(\Delta i\left(G_{\mathcal{K}}\right)\right)^2 & (8) \end{aligned}$$

For simplicity of exposition, we write (where suitable) in the following:

$$\Delta i\left(G_{\mathcal{K}}\right) = \Delta i_{\mathcal{K}} \quad \text{and} \quad \Delta\hat{i}\left(G_{\mathcal{K}}\right) = \Delta\hat{i}_{\mathcal{K}} \tag{9}$$

The expected social loss function is then given by

$$E[\mathcal{L}] = \sum_{n=0}^{N}\sum_{\sigma(|\mathcal{K}|=n)} p_{\mathcal{K}}\mathcal{L}_{\mathcal{K}} \tag{10}$$

[7] Note that degeneracies are possible if the g_i's are specified. For example, $g_1 = 0.05$, $g_2 = 0.1$, $g_3 = 0.2$, $g_4 = 0.3$, $g_5 = 0.35$. Although $\epsilon(g_1+g_5) = \epsilon(g_2+g_4)$, these are considered to be two different shocks, because the shock does not affect the same countries.

[8] We could also motivate the dropping of the time index by assuming the central bankers to be myopic.

[9] Note that we leave out ϵ and write Δi directly as a function of $G_{\mathcal{K}}$, because Δi is a strictly increasing function of ϵ, and ϵ is strictly increasing in $G_{\mathcal{K}}$.

3.2 First-Best Solution

Since $\mathcal{L}_{\mathcal{K}}$ represents the losses in every single shock scenario ($\mathcal{K}$), the expected losses $E(\mathcal{L})$ are minimized if every single $\mathcal{L}_{\mathcal{K}}(\Delta\hat{i}_{\mathcal{K}})$ is minimized. From equation (8) we see that $\mathcal{L}_{\mathcal{K}}(\Delta\hat{i}_{\mathcal{K}})$ is a parabola with the minimum at $\Delta\hat{i}^*_{\mathcal{K}} = G_{\mathcal{K}}\Delta i(G_{\mathcal{K}})$. Thus in every single shock scenario the optimal change in the interest rate is given by

$$\Delta\hat{i}^*_{\mathcal{K}} = G_{\mathcal{K}}\Delta i(G_{\mathcal{K}}) \tag{11}$$

which results in

$$\mathcal{L}^*_{\mathcal{K}} = G_{\mathcal{K}}(1 - G_{\mathcal{K}})\Delta i(G_{\mathcal{K}}) \tag{12}$$

and we end up with

$$[E(\mathcal{L})]^* = \sum_{n=0}^{N} \sum_{\sigma(|\mathcal{K}|=n)} p_{\mathcal{K}} G_{\mathcal{K}}(1 - G_{\mathcal{K}})\Delta i(G_{\mathcal{K}}) \tag{13}$$

Note that the desired interest rate change of a country affected by the shock is monotonically increasing in the size of the shock. The larger the shock, the larger is the desired interest rate of the affected countries to stabilize the shock. In the following, we will calculate expected losses for different collective decision rules determining $\Delta\hat{i}_{\mathcal{K}}$. Then we compare their expected losses among themselves and with the first-best solution.

3.3 Constitution

To examine decision rules for central banks, we consider a constitutional design problem where governments of the monetary union decide which decision rule the central bank of the union will use. The selection of the decision rule is governed by the unanimity rule and occurs under a veil of ignorance, i.e. at a time when shocks are not yet known and no conflicts of interest are present. The stages of the constitutional design process are as follows:

Stage 1: The governments of the monetary union decide unanimously on the decision rule.

Stage 2: Central bankers in the council observe whether or not their countries and other countries are affected by the shock.

Stage 3: The council decides on the change in the interest rate in accordance with the decision rule.

We will restrict rules to democratic decision processes where each central bank has one vote, which may or may not be weighted by the size of the country. The time-line of the events and decisions is illustrated in the following figure:

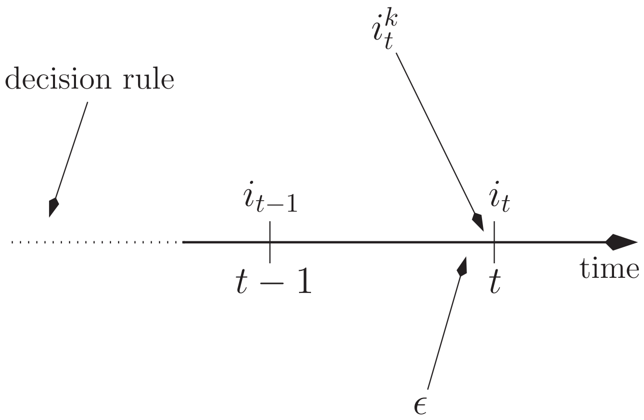

Figure 1:

Schematic figure of the decision process

4 Decision Rules

We distinguish between simple majority (*SM*) and flexible majority (*FM*) decision rules.

SM: i_{t-1} will be changed in t if and only if a change receives a majority of more than 50% of the votes. The central bank implements the maximal interest rate change that receives a majority of 50% of the votes when the interest rate is varied starting from i_{t-1}. Equivalently, the central bank implements the preferred interest rate change of the median voter.[10]

FM: i_{t-1} will be changed in t if the proposed $\Delta\hat{i}_{\mathcal{K}}$ obtains a share of $\alpha(\Delta\hat{i}_{\mathcal{K}})$ votes with $\alpha(.)$ increasing and $\alpha \in [0,1]$. The central bank implements the maximum interest rate change $\Delta\hat{i}_{\mathcal{K}}$ that receives a share of $\alpha(\Delta\hat{i}_{\mathcal{K}})$ votes when the interest rate is varied starting from i_{t-1}.

[10] When preferences are one-dimensional and single-peaked as in this paper, starting from any status quo, the median voters' most preferred outcome is the maximal change of the status quo that receives a simple majority of votes.

Practically, the FM-rule can be applied as follows. The council votes about interest changes in ascending order: $0 < \Delta i^1 < \Delta i^2 < ....$ As soon as an interest rate change does not obtain the required share of votes, the last but one interest rate change (that has been adopted) will be implemented by the central bank. The important feature of flexible majority rules is that the size of the majority α depends on the proposed interest rate change $\Delta\hat{i}_{\mathcal{K}}$. We will see that it is optimal for small interest rate changes to require a small share of votes, while large interest rate changes require a large share of supporting votes. The simple majority rule represents the standard median voter outcome. We now proceed as follows: We examine each decision rule separately and provide the comparison afterwards. The maximum interest rate change for which a supporting majority exists will be chosen. We analyze both the case where every country has only one vote, and the case where the vote of every country is weighted with its importance for overall welfare g_k. In the following we describe four different decision rules:

1. a flexible majority rule with weighted votes, indexed by FM_w.
2. a simple majority rule with weighted votes, indexed by SM_w.
3. a simple majority rule without weighted votes, indexed by SM_{nw}.
4. a flexible majority rule without weighted votes, indexed by FM_{nw}.

4.1 $\boldsymbol{FM_w}$: Flexible Majority Rule with Weighted Votes

In this case, we construct a FM_w rule that minimizes $E[\mathcal{L}]$. Here, the vote of every country is weighted by its g_k. We are looking for an optimal voting function rule $\alpha(\Delta\hat{i}_{\mathcal{K}})$. According to (7) and (9), social losses of the union for every possible shock scenario (i.e. for fixed subset $\mathcal{K}$) are given by

$$\mathcal{L}_{\mathcal{K}} = G_{\mathcal{K}}\left(\Delta\hat{i}_{\mathcal{K}} - \Delta i_{\mathcal{K}}\right)^2 + (1 - G_{\mathcal{K}})\left(\Delta\hat{i}_{\mathcal{K}}\right)^2 \tag{14}$$

From (8) and the first-best optimum we know that the aim should be to implement

$$\Delta\hat{i}_{\mathcal{K}}^{FM_w} = G_{\mathcal{K}}\Delta i_{\mathcal{K}} \tag{15}$$

via the FM_w rule. In the next proposition we establish the existence of an optimal flexible majority rule.

Proposition 1
There exists a function $\alpha^{FM_w}(\Delta\hat{i}_{\mathcal{K}})$ *which determines the share of votes in such a way that under the flexible majority rule* $\alpha^{FM_w}(\Delta\hat{i}_{\mathcal{K}})$ *the central bank council will always implement an interest rate change that minimizes the loss function* (14) *for every shock scenario.* $\alpha^{FM_w}(\Delta\hat{i}_{\mathcal{K}})$ *is given by*[11]

$$\alpha^{FM_w}(\Delta\hat{i}_{\mathcal{K}}) = \left(\Delta\hat{i}_{\mathcal{K}}^{FM_w}\right)^{-1}(\Delta\hat{i}_{\mathcal{K}}) \tag{16}$$

The proof is given in the appendix. Note that the share of votes required for support of a proposal to change the interest rate is monotonically increasing in the absolute value of the interest rate change.

Implementing the decision rule α^{FM_w} and inserting $\Delta\hat{i}_{\mathcal{K}} = [\Delta\hat{i}_{\mathcal{K}}]^{FM_w}$ in (14), we obtain

$$\mathcal{L}_{\mathcal{K}}^{FM_w} = G_{\mathcal{K}}(1 - G_{\mathcal{K}})\left(\Delta i_{\mathcal{K}}\right)^2 \tag{17}$$

and the expected loss function (10) is then given by

$$E\left(\mathcal{L}_{\mathcal{K}}^{FM_w}\right) = \sum_{n=0}^{N} \sum_{\sigma(|\mathcal{K}|=n)} p_{\mathcal{K}} G_{\mathcal{K}}(1 - G_{\mathcal{K}})\left(\Delta i_{\mathcal{K}}\right)^2 \tag{18}$$

An immediate consequence of proposition 1 is the following corollary:

Corollary 1
Applying α^{FM_w} *leads to the first-best solution.*

Corollary 1 follows from the observation that a first-best solution means implementing the interest rate change that minimizes $\mathcal{L}_{\mathcal{K}}$ for every single shock scenario. Therefore corollary 1 follows directly from (14), (15), and proposition 1.

As a very simple example, we consider a case with two countries $g_1 = 0.3$, $g_2 = 0.7$ and $\Delta i_{\{\emptyset\}} = 0$, $\Delta i_{\{1\}} = 1$, $\Delta i_{\{2\}} = 2$ and $\Delta i_{\{1,2\}} = 3$. The optimal function $\alpha(\Delta\hat{i}_{\mathcal{K}})$ is then given by

$$\alpha^{FM_w}(\Delta\hat{i}_{\mathcal{K}}) = \begin{cases} 0 & \text{if} & & & |\Delta\hat{i}_{\mathcal{K}}| & = & 0 \\ 0.3 & \text{if} & 0 & < & |\Delta\hat{i}_{\mathcal{K}}| & \leq & 0.3 \\ 0.7 & \text{if} & 0.3 & < & |\Delta\hat{i}_{\mathcal{K}}| & \leq & 1.4 \\ 1 & \text{if} & 1.4 & < & |\Delta\hat{i}_{\mathcal{K}}| & \leq & 3 \end{cases} \tag{19}$$

α^{FM_w} is calculated by determining the optimal interest rate change $\Delta\hat{i}^*_{\mathcal{K}}$ and subsequent inversion. Since $\Delta\hat{i}^*_{\mathcal{K}}$ is not continuous, this has to be done stepwise, i.e $\Delta\hat{i}^*_{\{1\}} = 0.3 \cdot 1 = 0.3$ and therefore a share of 30% can induce an interest rate change of $\Delta\hat{i}_{\{1\}} = 0.3$).

[11] $\left(\Delta\hat{i}_{\mathcal{K}}^{FM_w}\right)^{-1}$ denotes the inverse function. Recall that $\Delta\hat{i}_{\mathcal{K}}^{FM_w}$ itself is a function of $G_{\mathcal{K}}$.

Applying this flexible majority rule, we see that no change in interest rate occurs when no country is affected, whereas a change of 3 occurs when all countries are affected, because in this case every country wants an exact change of 3. If only the smaller country is affected, it will desire a change of 1. But with its share of 0.3 of the total votes, it can only implement a change up to 0.3. Since its private losses are descending in $[0, 0.3]$, the central bank will adopt a change of 0.3 in the interest rate. By the same argument, the change in interest rate will be 1.4 when only the large country is affected. For the seven different shock scenarios together, the implemented changes in the interest rate will be $\Delta\hat{i}_{\{\emptyset\}} = 0$, $\Delta\hat{i}_{\{1\}} = 0.3$, $\Delta\hat{i}_{\{2\}} = 1.4$ and $\Delta\hat{i}_{\{1,2\}} = 3$, and the social loss function of the union will also be minimized in every associated shock scenario.

4.2 SM_w: Simple Majority Rule with Weighted Votes

In the next step we examine the simple majority rule with weighted votes. In this case the change in interest rate $\Delta\hat{i}_{\mathcal{K}}^{SM_w}$ is given by

$$\Delta\hat{i}_{\mathcal{K}}^{SM_w} = \begin{cases} \Delta i_{\mathcal{K}} & \text{if} \quad G_{\mathcal{K}} > \frac{1}{2} \\ \\ 0 & \quad \text{otherwise} \end{cases} \tag{20}$$

and thus $\mathcal{L}_{\mathcal{K}}^{SM_w}$ is given by

$$\mathcal{L}_{\mathcal{K}}^{SM_w} = \begin{cases} G_{\mathcal{K}} \left(\Delta i_{\mathcal{K}}\right)^2 & \text{if} \quad G_{\mathcal{K}} \leq \frac{1}{2} \\ \\ \left(1 - G_{\mathcal{K}}\right) \left(\Delta i_{\mathcal{K}}\right)^2 & \text{if} \quad G_{\mathcal{K}} > \frac{1}{2} \end{cases} \tag{21}$$

and the expected social loss can be calculated as

$$E\left(\mathcal{L}_{\mathcal{K}}^{SM_w}\right) = \sum_{j=0}^{\bar{j}} p_{\mathcal{K}_j} G_{\mathcal{K}_j} \left(\Delta i_{\mathcal{K}_j}\right)^2 + \sum_{j=\bar{j}+1}^{2^N} p_{\mathcal{K}_j} (1 - G_{\mathcal{K}_j}) \left(\Delta i_{\mathcal{K}_j}\right)^2 \tag{22}$$

where we have arranged all 2^N $G_{\mathcal{K}}$ in the ascending series

$$0 = G_{\mathcal{K}_0} < G_{\mathcal{K}_1} \leq G_{\mathcal{K}_2} \leq ... \leq G_{\mathcal{K}_{2^N-1}} < G_{\mathcal{K}_{2^N}} = 1 \tag{23}$$

and $\bar{j}$ is determined by the fact that there exists a unique $\bar{j} \in \{0, 1..., 2^N\}$ with $G_{\mathcal{K}_{\bar{j}}} \leq \frac{1}{2}$ and $G_{\mathcal{K}_{\bar{j}+1}} > \frac{1}{2}$.[12] The expected loss under a simple majority rule includes both kinds of inefficiencies associated with collective decisions. First, interest rate changes are not

[12]The $p_{\mathcal{K}_j}$'s are the corresponding probabilities for the specific shock scenario, and $\Delta i_{\mathcal{K}_j}$ is defined similarly to $\Delta i_{\mathcal{K}}$ in (9).

implemented and are also too small, since a smaller than 50% share of the countries is affected by shocks. Second, adopted interest rate changes are too large since more than 50% of the weighted countries are affected by shocks and the minority has no impact on monetary policy.

We next turn to the simple majority rule without weighted votes.

4.3 SM_{nw}: Simple Majority Rule without Weighted Votes

In this case, the change in interest rate $\Delta i_{\mathcal{K}}^{SM_{nw}}$ is determined by

$$\Delta i_{\mathcal{K}}^{SM_{nw}} = \begin{cases} \Delta i_{\mathcal{K}} & \text{if} \quad |\mathcal{K}| > \frac{N}{2} \\ \\ 0 & \text{otherwise} \end{cases} \tag{24}$$

thus $\mathcal{L}_{\mathcal{K}}^{SM_{nw}}$ is given by

$$\mathcal{L}_{\mathcal{K}}^{SM_{nw}} = \begin{cases} G_{\mathcal{K}} \left(\Delta i_{\mathcal{K}}\right)^2 & \text{if} \quad |\mathcal{K}| \leq \frac{N}{2} \\ \\ \left(1 - G_{\mathcal{K}}\right) \left(\Delta i_{\mathcal{K}}\right)^2 & \text{if} \quad |\mathcal{K}| > \frac{N}{2} \end{cases} \tag{25}$$

and the expected social loss can be calculated as

$$E\left(\mathcal{L}_{\mathcal{K}}^{SM_{nw}}\right) = \left[\sum_{n=1}^{n \leq \frac{N}{2}} \sum_{\sigma(|\mathcal{K}|=n)} p_{\mathcal{K}} G_{\mathcal{K}} \left(\Delta i_{\mathcal{K}}\right)^2 + \sum_{n > \frac{N}{2}}^{N} \sum_{\sigma(|\mathcal{K}|=n)} p_{\mathcal{K}} (1 - G_{\mathcal{K}}) \left(\Delta i_{\mathcal{K}}\right)^2\right] \tag{26}$$

The simple majority rule without weighted votes exhibits the same sources of inefficiency as the simple majority rule with weighted votes, although inefficiencies here are more pronounced, since now more than half of the countries can change the interest rate, even if together they only have a small weight in the union.

4.4 FM_{nw}: Flexible Majority Rule without Weighted Votes

Finally, we analyze flexible majority rules without weighted votes. Since at the voting stage we can no longer distinguish between the countries on the basis of weights, we minimize over all shock possibilities where the number n of affected countries is fixed. The minimization is bounded by the fact that the change in the interest rate possible for n countries must not exceed the change in the interest rate possible for $n + 1$ countries. In order to formulate this analytically, we rearrange the order of summation in (10). First, note that for every fixed n we can arrange the $G_{\mathcal{K}}|_{|\mathcal{K}|=n}$ in a way similar to that of all $G_{\mathcal{K}}$ in (23). For n fixed, we have $\binom{N}{n}$ different $G_{\mathcal{K}}|_{|\mathcal{K}|=n}$. We order them in the ascending series

$$G_{\mathcal{K}_1^n} \leq G_{\mathcal{K}_2^n} ... \leq G_{\mathcal{K}_{j_n}^n} \leq ... \leq G_{\mathcal{K}^n_{\binom{N}{n}}} \tag{27}$$

where n is the number of affected countries and $j_n \in \{1, ...\binom{N}{n}\}$ represents the position of this shock scenario in the ordering given by (27). Now with (15) we can write for (8)[13]

$$\mathcal{L}_{\mathcal{K}_{j_n}^n} = \left(\Delta\hat{i}_{\mathcal{K}_{j_n}^n} - [\Delta\hat{i}_{\mathcal{K}_{j_n}^n}]^{FM_w}\right)^2 + (1 - G_{\mathcal{K}_{j_n}^n})\Delta i_{\mathcal{K}_{j_n}^n} \Delta\hat{i}^{FM_w}_{\mathcal{K}_{j_n}^n} \tag{28}$$

Assume now that we have an arbitrary FM_{nw} rule. Since, in this case, every central banker's vote has the same weight, this rule has to be increasing in the number n of affected countries. Thus, the rule is fully determined if, for any n, we can give a maximum possible interest rate change. We denote this interest rate change which only depends on n with $\Delta\hat{\hat{i}}(n)$, where $\Delta\hat{\hat{i}}(n)$ has to be an increasing function of n, since we cannot distinguish between large and small countries during the voting stage. $\mathcal{L}^{FM_{nw}}_{\mathcal{K}_{j_n}^n}$ is then given by

$$\mathcal{L}^{FM_{nw}}_{\mathcal{K}_{j_n}^n} = \begin{cases} \left(\Delta\hat{\hat{i}}(n) - \Delta\hat{i}^{FM_w}_{\mathcal{K}_{j_n}^n}\right)^2 + G^{-}_{\mathcal{K}_{j_n}^n} \Delta i_{\mathcal{K}_{j_n}^n} \Delta\hat{i}^{FM_w}_{\mathcal{K}_{j_n}^n} & \text{if} \quad \Delta\hat{\hat{i}}(n) \leq \Delta i_{\mathcal{K}_{j_n}^n} \\ (1 - G_{\mathcal{K}_{j_n}^n}) \left(\Delta i_{\mathcal{K}_{j_n}^n}\right)^2 & \text{if} \quad \Delta\hat{\hat{i}}(n) > \Delta i_{\mathcal{K}_{j_n}^n} \end{cases} \tag{29}$$

because if $\Delta\hat{\hat{i}}(n) > \Delta i_{\mathcal{K}_{j_n}^n}$, the n affected countries will simply minimize their own social loss function by voting for $\Delta i_{\mathcal{K}_{j_n}^n}$ but not for a higher change in the interest rate.

Now we can analytically formulate the minimization problem for the optimal FM_{nw} rule.

The optimal FM_{nw} rule must solve the following minimization problem:[14]

$$\min_{\Delta\hat{\hat{i}}(0)...\Delta\hat{\hat{i}}(N)} \sum_{n=0}^{N} \sum_{j_n=1}^{\binom{N}{n}} p_{\mathcal{K}_{j_n}^n} \mathcal{L}^{FM_{nw}}_{\mathcal{K}_{j_n}^n} \quad \text{s.t.} \quad \Delta\hat{\hat{i}}(n) \leq \Delta\hat{\hat{i}}(n+1) \tag{30}$$

Existence of the optimal FM_{nw} rule is given by

Proposition 2

The minimization problem (30) *has at least one solution.*

Since the actual calculation of a best FM_{nw} rule is very technical[15] and depends strongly on the actual values of g_k, $p_{\mathcal{K}_{j_n}^n}$ and the functional form of $\Delta i(.)$, we provide

[13] Again $\Delta i_{\mathcal{K}_{j_n}^n}$ is defined in a way similar to $\Delta i_{\mathcal{K}}$ in (9).

[14] The $p_{\mathcal{K}_{j_n}^n}$'s are again the corresponding probabilities for the specific shock scenario.

[15] This is done for a specific example in section 8.

a simple possible FM_{nw} rule which need not solve (30) but which has some appealing properties that can be exploited when we compare FM_{nw} with SM_{nw}. Consider therefore the following FM_{nw} rule:

$$\Delta\hat{i}^{FM_{nw}}(n) = \begin{cases} \Delta\hat{i}^{FM_w}_{\mathcal{K}^n_1} & \text{if} \quad n \leq \frac{N}{2} \\ \\ \Delta\hat{i}^{FM_w}_{\mathcal{K}^n_{\binom{N}{n}}} & \text{if} \quad n > \frac{N}{2} \end{cases} \tag{31}$$

This is an increasing function in n, which follows from the fact that $G_{\mathcal{K}^n_1}$ and $G_{\mathcal{K}^n_{\binom{N}{n}}}$ are both increasing in n and $G_{\mathcal{K}^n_1} \leq G_{\mathcal{K}^n_{\binom{N}{n}}}$. Applying this rule means that for fixed n, $\Delta\hat{i}^{FM_w}_{\mathcal{K}^n_1}$ for $n \leq \frac{N}{2}$ will be implemented after the voting stage, while $\Delta i_{\mathcal{K}^n_{j_n}}$ for $n > \frac{N}{2}$ will be implemented if $\Delta i_{\mathcal{K}^n_{j_n}} \leq \Delta\hat{i}^{FM_w}_{\mathcal{K}^n_{\binom{N}{n}}}$ and $\Delta\hat{i}^{FM_w}_{\mathcal{K}^n_{\binom{N}{n}}}$ if $\Delta i_{\mathcal{K}^n_{j_n}} > \Delta\hat{i}^{FM_w}_{\mathcal{K}^n_{\binom{N}{n}}}$. Altogether $\mathcal{L}^{FM_{nw}}_{\mathcal{K}^n_{j_n}}$ is then given by

$$\mathcal{L}^{FM_{nw}}_{\mathcal{K}^n_{j_n}} = \begin{cases} G_{\mathcal{K}^n_{j_n}}\left(1 - G_{\mathcal{K}^n_{j_n}}\right)\left(\Delta i_{\mathcal{K}^n_1}\right)^2 \\ \text{if} \quad \left(G_{\mathcal{K}^n_{j_n}} = G_{\mathcal{K}^n_1} \wedge n \leq \frac{N}{2}\right) \vee \left(G_{\mathcal{K}^n_{j_n}} = G_{\mathcal{K}^n_{\binom{N}{n}}} \wedge n > \frac{N}{2}\right) \\ \\ \left(\Delta\hat{i}^{FM_w}_{\mathcal{K}^n_1} - \Delta\hat{i}^{FM_w}_{\mathcal{K}^n_{j_n}}\right)^2 + G_{\mathcal{K}^n_{j_n}}\left(1 - G_{\mathcal{K}^n_{j_n}}\right)\left(\Delta i_{\mathcal{K}^n_{j_n}}\right)^2 \\ \text{if} \quad \Delta i_{\mathcal{K}^n_{j_n}} > \Delta\hat{i}^{FM_w}_{\mathcal{K}_1{}^n} \wedge n \leq \frac{N}{2} \\ \\ \left(\Delta\hat{i}_{\mathcal{K}^n_{\binom{N}{n}}} - \Delta\hat{i}^{FM_w}_{\mathcal{K}^n_{j_n}}\right)^2 + G_{\mathcal{K}^n_{j_n}}\left(1 - G_{\mathcal{K}^n_{j_n}}\right)\left(\Delta i_{\mathcal{K}^n_{j_n}}\right)^2 \\ \text{if} \quad \Delta i_{\mathcal{K}^n_{j_n}} \geq \Delta\hat{i}^{FM_w}_{\mathcal{K}^n_{\binom{N}{n}}} \wedge n > \frac{N}{2} \\ \\ \left(1 - G_{\mathcal{K}^n_{j_n}}\right)\left(\Delta i_{\mathcal{K}^n_{j_n}}\right)^2 \\ \text{otherwise} \end{cases} \tag{32}$$

Finally, we insert (32) in (10) and end up with

$$E\left(\mathcal{L}^{FM_{nw}}_{\mathcal{K}}\right) = \sum_{n=0}^{N} \sum_{j_n=1}^{\binom{N}{n}} p_{\mathcal{K}^n_{j_n}} \mathcal{L}^{FM_{nw}}_{\mathcal{K}^n_{j_n}} \tag{33}$$

The intuition for the FM_{nw} rule given in (31) is that in every shock scenario the outcome will never be worse than under the SM_{nw} rule without weighted votes. This will now be demonstrated in section 4.5.

4.5 Comparison of the Different Decision Rules

By summing up the results of the rules obtained in the former sections, we can compare the different expected losses. We obtain

Proposition 3

$$\begin{aligned} (i) \quad & E\left(\mathcal{L}_{\mathcal{K}}^{FM_w}\right) && < && E\left(\mathcal{L}_{\mathcal{K}}^{SM_w}\right) && \leq && E\left(\mathcal{L}_{\mathcal{K}}^{SM_{nw}}\right) \\ (ii) \quad & E\left(\mathcal{L}_{\mathcal{K}}^{FM_w}\right) && \leq && E\left(\mathcal{L}_{\mathcal{K}}^{FM_{nw}}\right) && < && E\left(\mathcal{L}_{\mathcal{K}}^{SM_{nw}}\right) \end{aligned} \tag{34}$$

The proof is given in the appendix, and the following corollary is deduced from this proof:

Corollary 2
In any given shock scenario we have

$$(i) \quad \mathcal{L}_{\mathcal{K}}^{FM_w} \leq \mathcal{L}_{\mathcal{K}}^{SM_w} \leq \mathcal{L}_{\mathcal{K}}^{SM_{nw}} \tag{35}$$

and there exists a FM_{nw} rule with

$$(ii) \quad \mathcal{L}_{\mathcal{K}}^{FM_w} \leq \mathcal{L}_{\mathcal{K}}^{FM_{nw}} \leq \mathcal{L}_{\mathcal{K}}^{SM_{nw}} \tag{36}$$

Note that the second part of corollary 2 is only shown for the FM_{nw} rule given in (31). This is because it need not be true for the optimal FM_{nw} rule given by the minimization problem in (30). Nevertheless, the second part of proposition 3 holds for both FM_{nw} rules without weighted votes, since in the proof we use $E\left(\mathcal{L}_{\mathcal{K}}^{FM_{nw}}\right)$ calculated by (32). The value $E(\tilde{\mathcal{L}}_{\mathcal{K}}^{FM_{nw}})$ calculated in (30) can never be greater than $E(\mathcal{L}_{\mathcal{K}}^{FM_{nw}})$ calculated in (32), where $\tilde{\mathcal{L}}_{\mathcal{K}}$ represents the social loss function derived from proposition 2.

Since the second part of corollary 2 means that there exists a FM_{nw} rule which is *ex post* never worse than the SM_{nw} rule, a plausible minimization procedure other than (30) can be considered for the flexible majority rule without weighted votes:

$$\min_{\Delta\hat{\hat{i}}(0)\ldots\Delta\hat{\hat{i}}(N)} \sum_{n=0}^{N} \sum_{j_n=1}^{\binom{N}{n}} p_{\mathcal{K}_{j_n}^n} \mathcal{L}_{\mathcal{K}_{j_n}^n}^{FM_{nw}} \qquad \begin{array}{l} \text{s.t.} \quad \Delta\hat{\hat{i}}(n) \leq \Delta\hat{\hat{i}}(n+1) \\ \text{s.t.} \quad \mathcal{L}_{\mathcal{K}}^{FM_{nw}} \leq \mathcal{L}_{\mathcal{K}}^{SM_{nw}} \end{array} \tag{37}$$

The problem given in (37) has at least one solution, which follows from the same argument as the proof of proposition 2 and from the fact that (31) is a FM_{nw} rule that satisfies $\mathcal{L}_{\mathcal{K}}^{FM_{nw}} \leq \mathcal{L}_{\mathcal{K}}^{SM_{nw}}$.

Overall, from Corollary 2 and Proposition 3 we observe that, given the weights of the votes (every vote is equally weighted or they are weighted according to the weights of the countries in the social loss function in the union), flexible majority rules are better than simple majority rules.[16] Comparing FM_w and SM_w this is obvious, since FM_w implements the first-best solution. Comparing FM_{nw} and SM_{nw} the dependence of the possible interest rate change on the share of votes helps to approximate the social optimum under FM which is not possible under SM_{nw}.

The intuition underlying the advantages of flexible majority rules runs as follows: It is socially desirable for small interest rate changes to be possible if only a small part of the union is affected by a shock. This is not possible under simple majority rules, because the 50% majority always fully determines the monetary policy. By contrast, applying flexible majority rules means that minorities can also change the interest rate to a small degree. Additionally, for the social optimum large interest rate changes should only be possible if a large part of the union is really affected by a shock. But again, simple majority rules already provide the possibility for large interest rate changes if only less than 50% of the union is affected. Under flexible majority rules, the larger the interest rate change, the larger is the required share of votes. This means that large interest rate changes can require a share of votes larger than 50%.

5 Discussion and Robustness

Our investigation suggests that flexible majority rules may be a useful tool for central banks. In appendix B we provide a detailed example of how the various decision rules can be computed.

In this section we address a variety of conceptual and practical issues which need to be dealt with.

First, allowing minorities to initiate a change in the interest rates may invite cycling in a dynamic setting, since interest rate changes might be revised immediately. Such undesirable cycling can be avoided by restricting flexible majority rules to genuine majorities or a revision rule. A revision rule stipulates that interest rate change reversals within a particular time frame, say a year, require a share of supporting votes larger than the share of opposing votes for the initial interest rate change.[17] It is still

[16]Comparing the FM_{nw} rule and the SM_w rule, we observe that without specifying $p_{\mathcal{K}}$, g_k and $\Delta i_{\mathcal{K}}(.)$ the relationship is ambiguous (as indicated by the example in section 8).

[17]Another possibility would be for the flexible majority rule not to be calculated with respect to

necessary to eliminate strategic voting under such reversal rules.[18]

Second, the construction of flexible majority rules requires only that the preferences of the countries are single-peaked and convex[19] in i_t, and that the maximization problem of the monetary union has a unique optimizer increasing in shock size. But shock scenarios can be more complicated. For instance, the sign for shocks can be different across countries. In such cases, the direction of the interest rate change must be determined by the relative size of votes supporting one direction. The increment change is determined by the flexible majority rule.

Third, we have assumed that if the members of the council have registered the overall shock and know whether they are affected, they will want to implement the same change in the interest rate. But in a given shock scenario, preferences may be different and different members may have different opinions about the appropriate change in interest rate. Such a scenario makes the optimal rule more complicated. While it is possible to calculate flexible majority rules for such cases, in practice one might want to opt for a simple step function to implement flexible majority rules, i.e. only stipulating the size of the required majority for a sequence of normalized interest rate changes 0.25%, 0.50%, 0.75%, etc.

6 Conclusion

Our discussion suggests that majority rules can be improved by making the size of the majority dependent on the proposal. This improvement will apply not only to the expected social losses but also to every single shock scenario, if the flexible majority rule is chosen properly. We have also shown that there is a first-best flexible majority rule with weighted votes.

Our investigation of flexible majority rules for central banks, however, can only be the beginning of the underlying research agenda. There are a variety of conceptual and practical issues that await further research. Nevertheless, the present paper suggests that flexible majority rules may surpass simple majority rules by a considerable margin.

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the former period, but to a long-term equilibrium interest rate. This would lower the likelihood of interest rate reversals.

[18]Details and the design of such rules are available upon request.

[19]A strict derivation of this fact for flexible majority rule with weighted votes is given in appendix C

7 Appendix A

Proof of Proposition 1:

Since $\Delta\hat{i}_{\mathcal{K}}^{FM_w} = G_{\mathcal{K}}\Delta i_{\mathcal{K}}$ is strictly increasing in $G_{\mathcal{K}}$, we can invert $\Delta\hat{i}_{\mathcal{K}}^{FM_w}$. We can then define

$$\alpha^{FM_w}(\Delta\hat{i}_{\mathcal{K}}) = \left(\Delta\hat{i}_{\mathcal{K}}^{FM_w}\right)^{-1}(\Delta\hat{i}_{\mathcal{K}}) \tag{38}$$

By constructing α^{FM_w}, in every shock scenario $\Delta\hat{i}_{\mathcal{K}}^{FM_w}$ will be implemented, as the affected countries want to incease the interest rate up to $\Delta\hat{i}_{\mathcal{K}} = \Delta i_{\mathcal{K}} \geq \Delta\hat{i}_{\mathcal{K}}^{FM_w}$. However, the FM_w rule restricts the possible change of the interest rate with a majority of $G_{\mathcal{K}}$ to the upper bound $\Delta\hat{i}_{\mathcal{K}}^{FM_w}$. We next observe that every proposal $\Delta\hat{i}_{\mathcal{K}}$ with $\Delta\hat{i}_{\mathcal{K}} < \Delta\hat{i}_{\mathcal{K}}^{FM_w}$ would lose against a proposal $\Delta\tilde{i} \in (\Delta\hat{i}_{\mathcal{K}}, \Delta\hat{i}_{\mathcal{K}}^{FM_w})$ in a pairwise decision, because all countries where the shock has occurred strictly prefer to vote for $\Delta\tilde{i}$ and, due to the construction of α^{FM_w}, $\Delta\tilde{i}$ would again be adopted, because $\Delta\tilde{i}$ needs a majority lower than or equal to $G_{\mathcal{K}}$ if it is to be accepted. Therefore, there will be an ascending pairwise ballot until $\Delta\hat{i}_{\mathcal{K}}^{FM_w}$ is reached. Altogether, α^{FM_w} minimizes $E\left[\mathcal{L}_{\mathcal{K}}^{FM_w}\right]$ because $E\left[\mathcal{L}_{\mathcal{K}}^{FM_w}\right]$ is a $2^{N+1}-1$ dimensional paraboloid in $\vec{\Delta}\hat{i} = (\Delta\hat{i}_{\mathcal{N}}^0, ..., \Delta\hat{i}_{\{\emptyset\}}^0)$.

■

Proof of Proposition 2:

This problem has at least one solution because $\tilde{\mathcal{L}}(\vec{\Delta}\hat{\hat{i}}) = \sum_{n=0}^{N}\sum_{j_n=1}^{\binom{N}{n}} p_{\mathcal{K}_{j_n}^n}\mathcal{L}_{\mathcal{K}_{j_n}^n}^{FM_{nw}}$, with $\vec{\Delta}\hat{\hat{i}} = (\Delta\hat{\hat{i}}(0), ..., \Delta\hat{\hat{i}}(N))$ is continuous in $[\mathbb{R}_0]^{N+1}$ and for $\tilde{\mathcal{L}}$ there exists a $\vec{\delta} \in [\mathbb{R}]^{N+1}$ with $\tilde{\mathcal{L}}(\vec{0}) > \tilde{\mathcal{L}}(\vec{\delta})$, $\tilde{\mathcal{L}}(\vec{\gamma})$ =const. For $\gamma_r \geq \Delta i(1)$ $\forall r \in \{0, 1..., N\}$, $\vec{\gamma} = (\gamma_0, ...\gamma_N)$ and $\delta_r < \gamma_r$.

■

Proof of Proposition 3:
Assume that $\mathcal{K}$ is fixed.

(i) If we compare (17) and (21), we see that for all $G_{\mathcal{K}} \in (0,1)$ we have $G_{\mathcal{K}}(1-G_{\mathcal{K}}) < G_{\mathcal{K}}$ and $G_{\mathcal{K}}(1-G_{\mathcal{K}}) < (1-G_{\mathcal{K}})$; since $\mathcal{L}^{FM_w} = \mathcal{L}^{SM_w}$ for $G_{\mathcal{K}} \in \{0,1\}$, we can conclude that $E\left[\mathcal{L}_{\mathcal{K}}^{FM_w}\right]$ is strictly lower than $E\left[\mathcal{L}_{\mathcal{K}}^{SM_w}\right]$.

If we compare (21) and (25), we see that $\mathcal{L}_{\mathcal{K}}^{SM_w} = \mathcal{L}_{\mathcal{K}}^{SM_{nw}}$ if $(G_{\mathcal{K}} \leq \frac{1}{2} \wedge |\mathcal{K}| \leq \frac{1}{2})$ and if $(G_{\mathcal{K}} > \frac{1}{2} \wedge |\mathcal{K}| > \frac{N}{2})$. It is also possible that for some $G_{\mathcal{K}}$ we have first $(G_{\mathcal{K}} \leq \frac{1}{2} \wedge |\mathcal{K}| > \frac{N}{2})$ and second $(G_{\mathcal{K}} > \frac{1}{2} \wedge |\mathcal{K}| \leq \frac{N}{2})$ (for example $N = 3$ and $g_1 = 0.1$, $g_2 = 0.2$, $g_3 = 0.7$). The comparison of these two cases gives

1. $G_{\mathcal{K}} \leq \frac{1}{2} \wedge |\mathcal{K}| > \frac{N}{2}$

$$\mathcal{L}_{\mathcal{K}}^{SM_w} = G_{\mathcal{K}} \left(\Delta i_{\mathcal{K}}\right)^2 \tag{39}$$
$$\mathcal{L}_{\mathcal{K}}^{SM_{nw}} = \left(1-G_{\mathcal{K}}\right) \left(\Delta i_{\mathcal{K}}\right)^2 \tag{40}$$
$$\Longleftrightarrow$$
$$\mathcal{L}_{\mathcal{K}}^{SM_w} - \mathcal{L}_{\mathcal{K}}^{SM_{nw}} = \left(2G_{\mathcal{K}}-1\right) \left(\Delta i_{\mathcal{K}}\right)^2 \leq 0 \tag{41}$$

2. $G_{\mathcal{K}} > \frac{1}{2} \wedge |\mathcal{K}| \leq \frac{N}{2}$

$$\mathcal{L}_{\mathcal{K}}^{SM_w} = \left(1-G_{\mathcal{K}}\right) \left(\Delta i_{\mathcal{K}}\right)^2 \tag{42}$$
$$\mathcal{L}_{\mathcal{K}}^{SM_{nw}} = G_{\mathcal{K}} \left(\Delta i_{\mathcal{K}}\right)^2 \tag{43}$$
$$\Longleftrightarrow$$
$$\mathcal{L}_{\mathcal{K}}^{SM_w} - \mathcal{L}_{\mathcal{K}}^{SM_{nw}} = \left(1-2G_{\mathcal{K}}\right) \left(\Delta i_{\mathcal{K}}\right)^2 < 0 \tag{44}$$

Altogether, we can conclude that $E\left[\mathcal{L}_{\mathcal{K}}^{SM_w}\right]$ is always lower than or equal to $E\left[\mathcal{L}_{\mathcal{K}}^{SM_{nw}}\right]$.

(ii) If we compare (17) and (32), we see again that for any specific shock scenario the FM_{nw} rule can never be strictly better than the FM_w rule.

In order to compare FM_{nw} and SM_{nw}, we look at the change in the interest rate implemented in these cases in a specific shock scenario. We begin with n fixed and $n \leq \frac{N}{2}$. Here in the SM_{nw} case the change in interest rate is 0, while in the FM_{nw} case the change is $\Delta\hat{i}_{\mathcal{K}_1^n}^{FM_w}$. Since 0 and $\Delta\hat{i}_{\mathcal{K}_1^n}^{FM_w}$ are both in the downward part of the parabola, the FM_{nw} rule is strictly better than SM_{nw} for every $\mathcal{K}$, with $|\mathcal{K}| = n$, $(n > 0)$. If $n > \frac{N}{2}$, SM_{nw} is as good as FM_{nw} and $\Delta i_{\mathcal{K}_{jn}^n} \leq \Delta\hat{i}_{\mathcal{K}_{\binom{N}{n}}^n}^{FM_w}$. Otherwise FM_{nw} is strictly better than SM_{nw}, because now

$\Delta i_{\mathcal{K}^n_{jn}}$ and $\Delta \hat{i}^{FM_w}_{\mathcal{K}^n_{\binom{N}{n}}}$ are in the upward part of the parabola. Accordingly, we have shown that $E\left[\mathcal{L}^{FM_{nw}}_{\mathcal{K}}\right]$ is always strictly lower than $E\left[\mathcal{L}^{SM_{nw}}_{\mathcal{K}}\right]$.

$\blacksquare$

8 Appendix B

To illustrate our results we provide a simple example, involving three countries. We assume that the weights are given by $g_1 = 0.1$, $g_2 = 0.2$ and $g_3 = 0.7$. Ordering the $G_{\mathcal{K}}$ according to (23), we obtain $G_{\mathcal{K}_0} = 0$, $G_{\mathcal{K}_1} = 0.1$, $G_{\mathcal{K}_2} = 0.2$, $G_{\mathcal{K}_3} = 0.3$, $G_{\mathcal{K}_4} = 0.7$, $G_{\mathcal{K}_5} = 0.8$, $G_{\mathcal{K}_6} = 0.9$ and $G_{\mathcal{K}_7} = 1$. Furthermore, we assume that $\Delta i\big(G_{\mathcal{K}_j}\big) = j$ and that all shocks are uniformly distributed, which implies that $p_{\mathcal{K}} = \frac{1}{7}\ \forall\ \mathcal{K}$. $\mathcal{L}^{SM_{nw}}_{\mathcal{K}}$, $\mathcal{L}^{SM_w}_{\mathcal{K}}$ and $\mathcal{L}^{FM_w}_{\mathcal{K}}$ can then be directly calculated from (25), (21) and (17). Now we calculate the FM_{nw} rules according to (30) and (37). We use the indication FM_{nw_1} for (30) and FM_{nw_2} for (37). First, we consider (29) and calculate the social loss functions for any specific shock scenario:

$$\mathcal{L}^{FM_{nw}}_{\{1\}} = \begin{cases} \left(\Delta\hat{\hat{i}}(1) - 0.1\right)^2 + 0.09 & \text{if} \quad \Delta\hat{\hat{i}}(1) < 1 \\ 0.9 & \text{otherwise} \end{cases} \tag{45}$$

$$\mathcal{L}^{FM_{nw}}_{\{2\}} = \begin{cases} \left(\Delta\hat{\hat{i}}(1) - 0.4\right)^2 + 0.64 & \text{if} \quad \Delta\hat{\hat{i}}(1) < 2 \\ 3.2 & \text{otherwise} \end{cases} \tag{46}$$

$$\mathcal{L}^{FM_{nw}}_{\{3\}} = \begin{cases} \left(\Delta\hat{\hat{i}}(1) - 2.8\right)^2 + 3.36 & \text{if} \quad \Delta\hat{\hat{i}}(1) < 4 \\ 4.8 & \text{otherwise} \end{cases} \tag{47}$$

$$\mathcal{L}^{FM_{nw}}_{\{1,2\}} = \begin{cases} \left(\Delta\hat{\hat{i}}(2) - 0.9\right)^2 + 1.89 & \text{if} \quad \Delta\hat{\hat{i}}(2) < 3 \\ 6.3 & \text{otherwise} \end{cases} \tag{48}$$

$$\mathcal{L}^{FM_{nw}}_{\{1,3\}} = \begin{cases} \left(\Delta\hat{\hat{i}}(2) - 4.0\right)^2 + 4 & \text{if} \quad \Delta\hat{\hat{i}}(2) < 5 \\ 5 & \text{otherwise} \end{cases} \tag{49}$$

$$\mathcal{L}^{FM_{nw}}_{\{2,3\}} = \begin{cases} \left(\Delta\hat{\hat{i}}(2) - 5.4\right)^2 + 3.24 & \text{if} \quad \Delta\hat{\hat{i}}(2) < 6 \\ 3.6 & \text{otherwise} \end{cases} \tag{50}$$

Obviously, in the case of $\mathcal{K} = \{\emptyset\}$, the best choice of $\Delta\hat{\hat{i}}(0)$ is $\Delta\hat{\hat{i}}(0) = 0$, while in the case of $\mathcal{K} = \{1, 2, 3\}$ $\Delta\hat{\hat{i}}(3) = 7$ is the best choice. Summing up the loss functions for $n = 1$ and $n = 2$ respectively, we obtain

$$\mathcal{L}_{|\mathcal{K}|=1} = \mathcal{L}^{FM_{nw}}_{\{1\}} + \mathcal{L}^{FM_{nw}}_{\{2\}} + \mathcal{L}^{FM_{nw}}_{\{3\}}$$
$$= \begin{cases} 3(\Delta\hat{\hat{i}}(1) - 1.1)^2 + 8.47 & \text{if} \quad \Delta\hat{\hat{i}}(1) < 1 \\ 2(\Delta\hat{\hat{i}}(1) - 1.6)^2 + 7.78 & \text{if} \quad 1 \leq \Delta\hat{\hat{i}}(1) < 2 \\ (\Delta\hat{\hat{i}}(1) - 2.8)^2 + 7.46 & \text{if} \quad 2 \leq \Delta\hat{\hat{i}}(1) < 4 \\ 8.9 & \text{if} \quad \Delta\hat{\hat{i}}(1) \geq 4 \end{cases} \tag{51}$$

and

$$\mathcal{L}_{|\mathcal{K}|=2} = \mathcal{L}^{FM_{nw}}_{\{1,2\}} + \mathcal{L}^{FM_{nw}}_{\{1,3\}} + \mathcal{L}^{FM_{nw}}_{\{2,3\}}$$
$$= \begin{cases} 3(\Delta\hat{\hat{i}}(2) - 3.4\overline{3})^2 + 19.73\overline{6} & \text{if} \quad \Delta\hat{\hat{i}}(2) < 3 \\ 2(\Delta\hat{\hat{i}}(2) - 4.7)^2 + 14.52 & \text{if} \quad 3 \leq \Delta\hat{\hat{i}}(2) < 5 \\ (\Delta\hat{\hat{i}}(2) - 5.4)^2 + 14.54 & \text{if} \quad 5 \leq \Delta\hat{\hat{i}}(2) < 6 \\ 14.9 & \text{if} \quad \Delta\hat{\hat{i}}(2) \geq 6 \end{cases} \tag{52}$$

We see at once that $\mathcal{L}_{|\mathcal{K}|=1}$ is minimized at $\Delta\hat{\hat{i}}(1) = 2.8$ and $\mathcal{L}_{|\mathcal{K}|=2}$ is minimized at $\Delta\hat{\hat{i}}(2) = 4.7$. Thus the FM_{nw_1} rule is given by

$$\Delta\hat{i}^{FM_{nw_1}}(0) = 0 \quad \Delta\hat{i}^{FM_{nw_1}}(1) = 2.8 \tag{53}$$

$$\Delta\hat{i}^{FM_{nw_1}}(2) = 4.7 \quad \Delta\hat{i}^{FM_{nw_1}}(3) = 7 \tag{54}$$

For FM_{nw_2} the solution is calculated numerically. This is feasible, as we know that the solution has to be between 0 and the beginning of the constant part of $\mathcal{L}_{|\mathcal{K}|=1}$ for $n = 1$ and between 0 and the beginning of the constant part of $\mathcal{L}_{\mathcal{K}_{|\mathcal{K}|=2}}$ for $n = 2$. We obtain $\Delta\hat{i}^{FM_{nw_2}}(1) = 0.2$ and a degenerated solution for $\Delta\hat{i}^{FM_{nw_2}}(2)$, with $\Delta\hat{i}^{FM_{nw_2}}(2) = 4.8$ or $\Delta\hat{i}^{FM_{nw_2}}(2) = 5.4$. From this we obtain two FM rules and indicate the first as $FM_{nw_{21}}$ and the second as $FM_{nw_{22}}$. They are given by

$$\Delta\hat{i}^{FM_{nw_{21}}}(0) = 0 \quad \Delta\hat{i}^{FM_{nw_{21}}}(1) = 0.2 \tag{55}$$

$$\Delta\hat{i}^{FM_{nw_{21}}}(2) = 4.8 \quad \Delta\hat{i}^{FM_{nw_{21}}}(3) = 7 \tag{56}$$

and

$$\Delta\hat{i}^{FM_{nw_{22}}}(0) = 0 \quad \Delta\hat{i}^{FM_{nw_{22}}}(1) = 0.2 \tag{57}$$

$$\Delta\hat{i}^{FM_{nw_{22}}}(2) = 5.4 \quad \Delta\hat{i}^{FM_{nw_{22}}}(3) = 7 \tag{58}$$

For completeness, we also give the shares of votes necessary for the FM_w rule:

$$\alpha^{FM_w}(\Delta\hat{i}_{\mathcal{K}}) = \begin{cases} 0 & \text{if} & & |\Delta\hat{i}_{\mathcal{K}}| & = & 0 \\ 0.1 & \text{if} & 0 < & |\Delta\hat{i}_{\mathcal{K}}| & \leq & 0.1 \\ 0.2 & \text{if} & 0.1 < & |\Delta\hat{i}_{\mathcal{K}}| & \leq & 0.4 \\ 0.3 & \text{if} & 0.4 < & |\Delta\hat{i}_{\mathcal{K}}| & \leq & 0.9 \\ 0.7 & \text{if} & 0.9 < & |\Delta\hat{i}_{\mathcal{K}}| & \leq & 2.8 \\ 0.8 & \text{if} & 2.8 < & |\Delta\hat{i}_{\mathcal{K}}| & \leq & 4.0 \\ 0.9 & \text{if} & 4.0 < & |\Delta\hat{i}_{\mathcal{K}}| & \leq & 5.4 \\ 1.0 & \text{if} & 5.4 < & |\Delta\hat{i}_{\mathcal{K}}| & \leq & 7 \end{cases} \tag{59}$$

Now we can compare all the different decision rules. The numbers are given in table 1. Comparing the columns for FM_{nw_1}, $FM_{nw_{21}}$, $FM_{nw_{22}}$ and SM_w, we see that we do indeed have cases where SM_w is better than FM_{nw} and vice versa. For example FM_{nw_1} is worse than SM_w for $G_{\mathcal{K}} = 0.1$ and better for $G_{\mathcal{K}} = 0.7$. $FM_{nw_{2j}}$ is worse than SM_w for $G_{\mathcal{K}} = 0.3$ and better for $G_{\mathcal{K}} = 0.2$ $(j = 1, 2)$. It can also be seen that in a specific shock scenario FM_{nw_1} can be worse than SM_{nw}. Take for example $G_{\mathcal{K}} = 0.2$. If we calculate the expected social losses for all decision rules, we obtain:

$$\begin{array}{llllll} E\left[\mathcal{L}_{\mathcal{K}}^{SM_{nw}}\right] & = 54 & E\left[\mathcal{L}_{\mathcal{K}}^{FM_{nw_1}}\right] & = 43.96 & E\left[\mathcal{L}_{\mathcal{K}}^{FM_{nw_{21}}}\right] & = 50.88 \\ E\left[\mathcal{L}_{\mathcal{K}}^{FM_{nw_{22}}}\right] & = 50.88 & E\left[\mathcal{L}_{\mathcal{K}}^{SM_w}\right] & = 34 & E\left[\mathcal{L}_{\mathcal{K}}^{FM_w}\right] & = 26.44 \end{array}$$

Referring only to the expected loss function, we see that, as shown generally in proposition 3, SM_{nw} is worse than FM_{nw}. Furthermore, in this specific example SM_w is better than FM_{nw}. But the opposite is also possible, as can be seen if we take another example where $g_j = g_k\ \forall j, k \in \mathcal{N}$. Here, the FM_{nw} rule coincides with FM_w, therefore $E\left[\mathcal{L}_{\mathcal{K}}^{FM_{nw}}\right]$ must be less than $E\left[\mathcal{L}_{\mathcal{K}}^{SM_w}\right]$.

$G_{\mathcal{K}_j}$	$\Delta i\left(G_{\mathcal{K}_j}\right)$	n	$\mathcal{L}_{\mathcal{K}}^{SM_{nw}}$	$\mathcal{L}_{\mathcal{K}}^{FM_{nw21}}$	$\mathcal{L}_{\mathcal{K}}^{FM_{nw22}}$	$\mathcal{L}_{\mathcal{K}}^{FM_{nw_1}}$	$\mathcal{L}_{\mathcal{K}}^{SM_w}$	$\mathcal{L}_{\mathcal{K}}^{FM_w}$
0	0	0	0	0	0	0	0	0
0.1	1	1	0.1	0.1	0.1	0.9	0.1	0.09
0.2	2	1	0.8	0.68	0.68	3.2	0.8	0.64
0.3	3	2	6.3	6.3	6.3	6.3	2.7	1.89
0.7	4	1	11.2	10.12	10.12	3.36	4.8	3.36
0.8	5	2	5	4.64	5	4.49	5	4
0.09	6	2	3.6	3.6	3.24	3.73	3.6	3.24
1	7	3	0	0	0	0	0	0

Table 1:
This table shows the different values for the different decision rules and all single shock scenarios.

9 Appendix C

In order to show that our results hold for a much broader class of preferences we write for social losses of country k

$$L^k = L(\Delta\hat{i}, \gamma_k \Delta i(G_{\mathcal{K}})) \tag{60}$$

We assume that $L(\cdot,\cdot)$ is twice continuously differentiable.

We obtain:

$$\mathcal{L}_{\mathcal{K}} = G_{\mathcal{K}} L(\Delta\hat{i}, \Delta i(G_{\mathcal{K}})) + (1 - G_{\mathcal{K}}) L(\Delta\hat{i}, 0) \tag{61}$$

Proposition 4

If social losses are single-peaked and convex in $\Delta\hat{i}$, aggregate social losses have a unique optimum.

Proof of Proposition 4:

Single-peakedness leads to $L(\Delta\hat{i}, \gamma_k \Delta i(G_{\mathcal{K}})) \gtreqless 0 \quad \text{if} \quad \Delta\hat{i} \gtreqless \gamma_k \Delta i(G_{\mathcal{K}})$

and convexity implies $\frac{\partial^2 L}{\partial(\Delta\hat{i})^2} > 0$.

The FOC for the social optimum is given by

$$\frac{\partial \mathcal{L}_{\mathcal{K}}}{\partial \Delta\hat{i}} = 0 \Longleftrightarrow \frac{\partial L(\Delta\hat{i}, \Delta i(G_{\mathcal{K}}))}{\partial \Delta\hat{i}} = \frac{G_{\mathcal{K}} - 1}{G_{\mathcal{K}}} \cdot \frac{\partial L(\Delta\hat{i}, 0)}{\partial \Delta\hat{i}} \tag{62}$$

Condition (62) has a unique solution $\Delta\hat{i}^* \in [0, \Delta i(G_{\mathcal{K}})]$, since

(i) $\frac{\partial L(\Delta\hat{i}, \Delta i(G_{\mathcal{K}}))}{\partial \Delta\hat{i}}$ and $\frac{\partial L(\Delta\hat{i}, 0)}{\partial \Delta\hat{i}}$ are increasing in $\Delta\hat{i}$.

(ii) $\frac{\partial L(\Delta\hat{i}, \Delta i(G_{\mathcal{K}}))}{\partial \Delta\hat{i}} < 0$ and $\frac{\partial L(\Delta\hat{i}, 0)}{\partial \Delta\hat{i}} > 0$ for $\Delta\hat{i} \in [0, \Delta i(G_{\mathcal{K}})]$.

(iii) $\left.\frac{\partial L(\Delta\hat{i}, \Delta i(G_{\mathcal{K}}))}{\partial \Delta\hat{i}}\right|_{\Delta\hat{i} = \Delta i(G_{\mathcal{K}})} = \left.\frac{L(\Delta\hat{i}, 0)}{\partial \Delta\hat{i}}\right|_{\Delta\hat{i} = 0} = 0$ and $\frac{G_{\mathcal{K}} - 1}{G_{\mathcal{K}}} < 0$.

Hence the solution $\Delta\hat{i}^* \in [0, \Delta i(G_{\mathcal{K}})]$ is a local optimum. The uniqueness of $\Delta\hat{i}^*$ as a global minimum is obtained by the SOC, which is given by

$$\frac{\partial^2 \mathcal{L}_{\mathcal{K}}}{\partial(\Delta\hat{i})^2} = G_{\mathcal{K}} \frac{\partial^2 L(\Delta\hat{i}, \Delta i(G_{\mathcal{K}}))}{\partial(\Delta\hat{i})^2} + (1 - G_{\mathcal{K}}) \frac{\partial^2 L(\Delta\hat{i}, 0)}{\partial(\Delta\hat{i})^2} > 0 \tag{63}$$

Condition (63) holds, because of the convexity of $L(\Delta\hat{i}, \gamma_k \Delta i(G_{\mathcal{K}}))$ in $\Delta\hat{i}$. This implies that $\mathcal{L}_{\mathcal{K}}$ is globally convex and therefore the local minimum $\Delta\hat{i}^*(G_{\mathcal{K}})$ is also a global minimum.

■

In order to construct flexible majority rules, $\Delta\hat{i}^*(G_{\mathcal{K}})$ has to be increasing in $G_{\mathcal{K}}$. The necessary conditions are derived in the following proposition:

Proposition 5
If $\frac{1}{G_{\mathcal{K}}^2} \cdot \frac{\partial L(\Delta\hat{i},0)}{\partial \Delta\hat{i}} > \frac{\partial^2 L(\Delta\hat{i},\Delta i(G_{\mathcal{K}}))}{\partial \Delta\hat{i}\, \partial \Delta i} \cdot \frac{\partial \Delta i}{\partial G_{\mathcal{K}}}$ *then* $\Delta\hat{i}^*(G_{\mathcal{K}})$ *is increasing in* $G_{\mathcal{K}}$.

Proof of Proposition 5:
If we insert the optimum $\Delta\hat{i}^*(G_{\mathcal{K}})$ into the FOC equation (62) and differentiate both sides with respect to $G_{\mathcal{K}}$, we obtain

$$\begin{aligned}
\frac{\partial^2 L(\Delta\hat{i}^*,\Delta i(G_{\mathcal{K}}))}{\partial (\Delta\hat{i}^*)^2} \cdot \frac{\partial \Delta\hat{i}^*}{\partial G_{\mathcal{K}}} + \frac{\partial^2 L(\Delta\hat{i}^*,\Delta i(G_{\mathcal{K}}))}{\partial \Delta\hat{i}^*\, \partial \Delta i} \cdot \frac{\partial \Delta i}{\partial G_{\mathcal{K}}} \quad &= \quad \frac{1}{G_{\mathcal{K}}^2} \cdot \frac{\partial L(\Delta\hat{i},0)}{\partial \Delta\hat{i}} + \frac{G_{\mathcal{K}}-1}{G_{\mathcal{K}}} \cdot \frac{\partial^2 L(\Delta\hat{i}^*,0)}{\partial (\Delta\hat{i}^*)^2} \cdot \frac{\partial \Delta\hat{i}^*}{\partial G_{\mathcal{K}}} \\
&\Longleftrightarrow \\
\frac{\partial \Delta\hat{i}^*}{\partial G_{\mathcal{K}}} \underbrace{\left[\frac{\partial^2 L(\Delta\hat{i}^*,\Delta i(G_{\mathcal{K}}))}{\partial (\Delta\hat{i}^*)^2} + \frac{G_{\mathcal{K}}-1}{G_{\mathcal{K}}} \cdot \frac{\partial^2 L(\Delta\hat{i}^*,0)}{\partial (\Delta\hat{i}^*)^2} \right]}_{>0} \quad &= \quad \frac{1}{G_{\mathcal{K}}^2} \cdot \frac{\partial L(\Delta\hat{i},0)}{\partial \Delta\hat{i}} - \frac{\partial^2 L(\Delta\hat{i}^*,\Delta i(G_{\mathcal{K}}))}{\partial \Delta\hat{i}^*\, \partial \Delta i} \cdot \frac{\partial \Delta i}{\partial G_{\mathcal{K}}}
\end{aligned} \tag{64}$$

which implies the condition given in proposition 5.

■

The proof of proposition 1 implies that the construction of the flexible majority rule with weighted votes requires that aggregate social losses have a unique optimizer and that the desired interest rate change is monotonically increasing in $G_{\mathcal{K}}$. Hence, single-peaked convex aggregate social losses and the condition $\frac{1}{G_{\mathcal{K}}^2} \cdot \frac{\partial L(\Delta\hat{i},0)}{\partial \Delta\hat{i}} > \frac{\partial^2 L(\Delta\hat{i},\Delta i(G_{\mathcal{K}}))}{\partial \Delta\hat{i}\, \partial \Delta i} \cdot \frac{\partial \Delta i}{\partial G_{\mathcal{K}}}$ ensures the existence of the flexible majority rule with weighted votes. Furthermore, all other results hold under the same conditions, as the construction of the flexible majority rule without weighted votes and the use of simple majority rules do not depend on the quadratic specification of preferences.

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