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Minority Voting and Public Project Provision

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Abstract:

We propose a two-stage process called minority voting to allocate public projects in a polity. In the first period, a society decides by a simple majority decision whether to provide the public project. If the proposal in the first period is rejected, the process ends. Otherwise the process continues, but only the members of the minority keep agenda and voting rights for the second stage, in which the financing scheme is determined. In the second stage, the unanimity rule or the simple majority rule is applied. We provide a first round of relative welfare comparisons between minority voting and simple majority voting and outline our research program.

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1 Introduction

In this paper we propose a new way of allocating and financing public projects collectively in a two-stage process. In the first period, a society decides by simple majority voting whether to provide a public project. If the project is rejected, the process ends. If a majority favors the public project, the process continues, but only the members of the minority keep the agenda and voting rights for the second stage, in which the financing scheme is determined. In the second stage, the unanimity rule or the simple majority rule is applied. This scheme is called minority voting since the losing minority in the first stage obtains the exclusive right to decide on financing in the second stage. Together with a taxation protection rule, such schemes are alternative democratic procedures to simple majority voting. In this paper we compare minority voting to simple majority voting with regard to public project provision. We focus on the case where the unanimity rule is applied in the second stage under minority voting.

The following properties characterize equilibria under minority voting: When the public project is proposed in the first round, only those individuals will support the proposal who value the project highly, i.e. more than the maximum tax payment that may occur in the second stage. If the project is supported in the first stage, the supporting majority is minimal. Every supporting individual must be pivotal, since those individuals lose their voting right for the second stage.

If the project is rejected in the first stage, the collective choice process ends. If the project is adopted, an equilibrium financing scheme will involve subsidies for project losers in order to gain the support of all voting losers from the first stage. All voting winners from the first stage pay the highest admissible tax rate to finance the project and the subsidies. The agenda setter will also tax all other beneficiaries of the project in order to generate subsidies for himself.

The attractive feature of the minority voting scheme is that individuals who benefit largely from a project pay more taxes, while individuals who have little benefit, or are disadavantaged by it, will be protected from high tax payments. Moreover, minority voting with the unanimity rule in the second stage ensures that only Pareto improvements occur and that three standard inefficiencies in democratic decision-making are avoided: inefficient projects are neither proposed nor adopted; inefficient redistribution proposals are neither proposed nor adopted; when proposed, efficient projects are not rejected.

The drawback of minority voting is that efficient projects may not be proposed in the first stage. Accordingly, we compare minority voting with the standard simple-majority-rule framework, both coupled with the same tax-protection rule, and compare the relative social welfare of the schemes. In this paper, we provide a first round of relative welfare comparisons between minority voting and simple majority voting. On balance, the minority voting outperforms the simple majority voting in all circumstances except in the following constellation: A socially desirable project is adopted under the simple majority rule and redistribution costs do not outweigh the social gains while the project is not provided under minority voting.

The current paper, however, is only a first round to explore the virtues and drawbacks of minority voting in the context of public project provision. Numerous further analyses and extensions of our model should and can be performed, which are discussed in the concluding section.

Our paper is part of the recent literature on linking voting across problems. Casella (2005) introduces storable votes mechanisms, where a committee makes binary decisions repeatedly over time and where agents may store votes over time.[1] Experimental evidence has supported the efficiency gains of storable votes (Casella et al. (2006)). Jackson and Sonnenschein (2007) show that, when problems repeat themselves many times, full efficiency can be reached at the limit, and that this insight essentially applies to any collective-decision problem. In a companion paper (Fahrenberger and Gersbach (2006)) minority voting is developed for repeated project decisions where projects have a durable impact. Linkages of voting across issues can also occur through vote trading, which goes back at least to Buchanan and Tullock (1962) and Coleman (1966) and has been developed, among others, by Brams and Riker (1973), Ferejohn (1974), Philipson and Snyder (1996) or Piketty (1994).

We propose to split project and financing decisions and to introduce minority voting in such a way that, at the outset, all individuals have the same right to influence

[1] Cumulative voting is closely related to the storable votes mechanism, as individuals can cast more than one vote for one alternative under such schemes (see for example Sawyer and MacRae (1962), Brams (1975), Cox (1990), Guinier (1994) or Gerber et al. (1998)).

outcomes and minorities are protected (e.g. Guinier (1994) or Issacharoff et al. (2002)). Our proposal is aimed at resolving the tyranny of the majority problem by giving an emerging minority the exclusive right to decide about the financing scheme for a public project that a society has previously approved.

Our paper is organized as follows: In the next section, we introduce the model and the constitutional principles. In section 3, we characterize the equilibria under simple majority voting, while minority voting is discussed in section 4. In section 5 we present the relative welfare comparison, which we further extend in section 6. In section 7 we discuss an example, and section 8 concludes. All proofs can be found in the appendix.

2 Model and Constitutional Principles

2.1 Model

We consider a standard social-choice problem of public project provision and financing. Time is indexed by $\tau = 0, 1$. The first period $\tau = 0$ is the constitutional period. In the constitutional period, a society Ω of N ($N > 3, N$ uneven) risk-neutral members decides how public project provision and financing should be governed in the legislative period. Citizens are indexed by $j \in \Omega = \{1, \dots, N\}$.

In the legislative period, $\tau = 1$, each citizen is endowed with e units of a private consumption good. The community can adopt a public project with per capita costs $k > 0$. We use V_j to denote the utility of agent j from the provision of the public project. At $\tau = 0$, the utility V_j is unknown and can hence be interpreted as a random variable.

We assume that V_j is uniformly distributed on $[\underline{V}, \overline{V}]$ with $\underline{V}, \overline{V} \in \mathbb{R}$ and $\underline{V} < \overline{V}$. In the legislative period we index members of the society according to their utility levels, i.e. individual j is associated with the utility $V_j \in [\underline{V}, \overline{V}]$ with $V_1 \leq V_2 \leq V_3 \leq \ldots \leq V_N$. The vector $(V_1, \dots, V_n)$ is denoted by V.

Public projects must be financed by taxes. We assume that taxation is distortionary. Let $\lambda > 0$ denote the shadow cost of public funds. Accordingly, taxation uses $(1 + \lambda)$ of taxpayer resources in order to levy 1 unit of resources for public projects and for transfers to citizens. Hence the overall per capita costs of the public project amount to $(1+\lambda)k$. We assume that $0<\lambda<1$, as plausible values for tax distortions are considerably smaller than 100%.

We use t_j and s_j to denote a citizen j's tax payment or subsidy, respectively. We define the variable g as indicating whether the public project is proposed ($g=1$) or not ($g=0$). The utility of citizen j, denoted by U_j, in the legislative period is given by[2]

$$U_j = e + gV_j - t_j + s_j.$$

Finally, the budget constraint of the society in the legislative period is given by

$$\sum_{j\in\Omega} t_j \ = (1+\lambda)\Big[gNk + \sum_{j\in\Omega} s_j \ \Big]. \tag{1}$$

We assume throughout the paper that e is sufficiently large for agents to be able to pay taxes in all circumstances that may occur. We summarize the set of parameters that define the characteristics of the public project as $\mathcal{P}=(V,k,\lambda,N)$.

2.2 Socially Efficient Solutions

The fact that citizens are risk-neutral implies that, from an ex ante point of view or from an utilitarian perspective, it is socially efficient to provide the public project if and only if

$$\hat{V} := \frac{1}{N}\sum V_j > k(1+\lambda)$$

and taxes are raised solely to finance the public project. Any redistribution activities are detrimental from an ex ante point of view. A socially efficient tax scheme, for instance, is one where a socially desirable public project is financed by project winners and no subsidies are paid. In order to implement such a solution, a complete social contract would be necessary. We summarize our observation as follows:

Ex ante first-best allocation

Any allocation that provides the public project if and only if $\hat{V} > k(1+\lambda)$ and that raises taxes only to finance the public project is ex ante socially efficient.

[2] All tax and subsidy functions t_j and s_j are assumed to be integrable. We only discuss mechanisms where this condition is trivially fulfilled.

We follow the literature on incomplete social contracting (see Aghion and Bolton (2003) and Gersbach (2005)) and assume that society allocates public projects by democratic procedures. Given socially efficient allocations, it is important at this stage to identify the sources of inefficiencies that may arise in legislative decision-making: There are four types of inefficiencies:

(1) inefficient projects are proposed and adopted

(2) pure redistribution proposals are made and adopted

(3) efficient projects are proposed and rejected

(4) efficient projects are not proposed

The latter two inefficiencies mean that delay in undertaking public projects is costly. In the paper we assume that not adopting projects results in the status quo. In the following we examine two ways of designing the democratic process for the provision of a public project, (1) the simple majority voting scheme and (2) the minority voting scheme.

2.3 Simple Majority Voting

At the constitutional stage the society decides about the rules governing the legislative processes. The first democratic procedure is a standard majority voting scheme called SM.

Stage 1: At the start of the legislative period, citizens observe their own utility U_j and the utilities of all other individuals. Citizens decide simultaneously whether to apply for agenda setting ($\psi_j = 1$) or not ($\psi_j = 0$).

Stage 2: Among all citizens who apply, one citizen a is determined by fair randomization to set the agenda. The agenda setter proposes a project/financing package $(g, t_j, s_j)_{j\in\Omega}$. This choice is denoted by A_a.

Stage 3: Given A_a, citizens decide simultaneously whether to accept ($\delta_j(A_a) = 1$) or not ($\delta_j(A_a) = 0$). The proposal is accepted if a majority of members adopt it.

Note that if nobody applies for agenda setting, the status quo will prevail. Moreover, individuals know at the voting stage who will be taxed and who will receive subsidies if the proposal is accepted. Obviously, the status quo also prevails if a proposal to change it does not receive enough yes–votes, as required by the majority voting rule.

An equilibrium for stages 1 to 3 can be described as a set of strategies

$$\{\psi, A, \delta(\cdot)\},$$

where $\psi = (\psi_j)_{j\in\Omega}, A = (A_a)_{a\in\Omega}$, $\delta = (\delta_j)_{j\in\Omega}$ and where $\delta_j = \delta_j(A_a)$ depends on the proposed agenda A_a.

To describe the application and voting outcome in our model, we use weak dominance criteria. Elimination of weakly dominated strategies is a standard assumption for eliminating the multiplicity of equilibria based on the trembling-hand perfection of Nash equilibria.

As voting in our model is a simple binary decision and thus individuals cannot gain anything from strategic voting, this procedure implies that agents participate and vote according to their preferences, i.e. they vote for their most preferred alternative. The elimination of weakly dominated strategies with respect to voting is thus captured by the following rule:

- **(EWSV)** *Suppose an agenda setter a has been randomly drawn. Then, given an agenda A_a, the voting strategies are $\delta_j^*(A_a) = 1$ if the net benefit $u_j = gV_j + s_j - t_j$ from A_a is nonnegative and $\delta_j^*(A_a) = 0$ otherwise.*

It is obvious that (EWSV) implies unique voting equilibria, so we can also use the weak dominance criterion for the decision on whether to apply for agenda setting (stage 1).

- **(EWSA)** *Agents eliminate weakly dominated strategies in stage 1.*

Since the requirement (EWSV) ensures that the voting outcome is unique, we can use $U_j(A_a)$ to define the utility level that an agent j will achieve if agent a has proposed agenda A_a and voting has taken place. Moreover, let the set of all possible agendas be denoted by $\mathcal{A}$. In order to simplify the exposition, we assume that the following three tie-breaking rules are applied:

- If an agent j cannot strictly improve his utility by agenda setting, he will not apply for agenda setting.
- If an agenda setter knows with certainty that $g = 1$ will be rejected, he will propose $g = 0$.
- If an agenda setter is indifferent between an agenda that leads to $g = 1$ and another that yields $g = 0$, he will propose the former.

Note that $U_j(A_a)$ is based on the optimal voting strategies of all agents. For instance, $U_j(A_a) = e$ if A_a is rejected. In what follows we will assume throughout - without referring to the fact explicitly - that (EWSV), (EWSA), and the tie-breaking rules are applied.

2.4 Minority Voting

In this section we introduce an alternative democratic decision process called MV.

Stage 1: At the start of the legislative period, citizens observe their own utility V_j and the utilities of all other individuals. Citizens decide simultaneously whether to apply for agenda setting ($\psi_j = 1$) or not ($\psi_j = 0$).

Stage 2: Among all citizens who apply, one citizen a_1 is determined by fair randomization to set the agenda. The agenda setter decides whether the public project should be provided or whether a pure redistribution proposal should be considered. Denote this choice by $g_{a_1}^{MV} \in \{0, 1\}$. If nobody applies for agenda setting, the status quo prevails.

Stage 3: Citizens decide whether to accept $g_{a_1}^{MV}\big(\delta_j(g_{a_1}^{MV}) = 1\big)$ or not $\big(\delta_j(g_{a_1}^{MV}) = 0\big)$. The proposal is accepted if a majority of members adopt it. We use $\mathcal{M} = \{j \mid \delta_j(g_{a_1}^{MV}) = 0\}$ to denote the set of individuals who voted against the proposal.

Stage 4: If $g_{a_1}^{MV}$ has been adopted, i.e. if $|\mathcal{M}| < \frac{N+1}{2}$, all agents of the minority can apply to propose a financing package. Among those, a citizen a_2 is determined by fair randomization and proposes a package $(t_j, s_j)_{j\in\Omega}$. Denote this choice by F_{a_2}. If nobody applies for agenda setting, the status quo prevails.

Stage 5: Given F_{a_2}, citizens who belong to $\mathcal{M}$ decide simultaneously whether to accept $F_{a_2}\big(\delta_j(F_{a_2}) = 1\big)$ or not $\big(\delta_j(F_{a_2}) = 0\big)$. F_{a_2} is accepted if, and only if, all individuals in $\mathcal{M}$ vote $\delta_j(F_{a_2}) = 1$, i.e. the unanimity rule applies. If F_{a_2} is accepted, the plan $(g_{a_1}^{MV} = 1, F_{a_2})$ is implemented. Otherwise the status quo $(g_{a_1}^{MV} = 0,\ t_j = s_j = 0\ \ \forall\, j)$ prevails.

A number of remarks are in order here. First, there are several alternatives for resolving a situation where $g_{a_1}^{MV} = 1$ is accepted and F_{a_2} is rejected. For instance, one could allow for further rounds of financing proposals or one could design a default financing scheme to be applied together with $g_{a_1}^{MV} = 1$. Such extensions would introduce further strategic effects but they do not tend to improve the efficiency of the system.

Second, it is obvious that all individuals would like to keep their voting right in stage 3. Hence no majority can be formed for a proposal $g_{a_1}^{MV} = 0$ as supporting agents are worse off than the status quo. Therefore pure redistribution proposals will never be adopted under MV. The situation is different when $g_{a_1}^{MV} = 1$ has been proposed. Without support, the public project will not be provided. This creates incentives for individuals who benefit highly from a public project to support a proposal $g_{a_1}^{MV} = 1$.

Third, as with simple majority, to derive equilibria we use weak dominance to characterize subgame perfect equilibria. Moreover, we use the same tie-breaking rules that apply in simple majority voting for agenda setting with regard to public project provision (Stage 2). In Stage 4, we assume that all individuals apply for agenda setting and make a financing proposal as long as they are not worse off (relative to the status quo) if their proposals are adopted in Stage 5. Again, these tie-breaking rules merely simplify the exposition.

2.5 Tax Protection Rule

In the following sections we prepare the ground for the comparison of the two systems SM and MV by characterizing the equilibrium of the games. We do not impose any further rules on proposal-making, but we do assume an upper limit on taxes, denoted by $\hat{t}$. That is, a proposal that involves $t_j > \hat{t}$ for some individual j is unconstitutional, and the status quo prevails. Such tax protection rules are ubiquitous in modern democracies

(Rangel, 2005).[3] Note that the tax protection rule does not preclude an agenda setter voluntarily contributing more than $\hat{t}$ to the financing of the public project.

3 Equilibria under Simple Majority Voting

We first characterize the equilibria under SM. For this purpose we use Ω_{-j} to denote the set $\Omega\backslash\{j\}$, i.e. the society with exception of individual j. Under simple majority voting everybody stands to gain from agenda setting as this will always enable the agenda setter to propose a pure redistribution proposal that benefits him. Hence we will have $\psi_j = 1$ in any equilibrium. In Stage 2 an agenda setter a solves the following problem:

$$\begin{aligned} &\max_{(g,t_j,s_j)_{j\in\Omega}} \{U_a = e + gV_a + s_a - t_a\} \\ &\text{s.t.} \quad \sum_{j=1}^{N} t_j = (1+\lambda)\big[gNk + \sum_{j=1}^{N} s_j\big] \\ &\text{and} \quad \exists I \subset \Omega_{-a}, \text{ with } |I| = \frac{N-1}{2}, \\ &\text{such that} \quad U_j - e = gV_j + s_j - t_j \geq 0, j \in I. \end{aligned}$$

We obtain

Lemma 1

An equilibrium proposal $g = 0$ is associated with the redistribution scheme

$$\begin{aligned} t_j &= \hat{t}, j \notin I_{+a} := I \cup \{a\} \\ t_j &= 0, j \in I_{+a} \\ s_j &= 0, j \in \Omega_{-a} \\ s_a &= \frac{N-1}{2(1+\lambda)}\,\hat{t}. \end{aligned}$$

The lemma is obvious as all individuals in I_{+a} support the proposal and I_{+a} is the smallest majority the agenda setter can form. We next investigate the case $g = 1$. For this purpose we introduce the set

$$LW := \{j \in \Omega \mid V_j \geq \hat{t}\,\}$$

[3] In 1983, for instance, the German Constitutional Court declared excessive tax burdens that would fundamentally impair wealth to be unconstitutional (Reding and Müller (1999)).

Individuals belonging to LW are called large project winners. We also introduce the set

$$LW_{-a} := \begin{cases} LW\backslash\{a\} & \text{if } a \in LW \\ LW & \text{otherwise.} \end{cases}$$

We obtain

Lemma 2

An equilibrium proposal $g = 1$ is associated with

$$\begin{aligned}
s_j &= 0, t_j = \hat{t}, j \in LW_{-a} \cup \Omega\backslash I_{+a} \\
s_j &= 0, t_j = V_j, j \in I\backslash LW_{-a}, V_j \geq 0 \\
s_j &= -V_j, t_j = 0, j \in I, V_j < 0 \\
t_a &= \max\{0, -(1+\lambda)\bar{s}_a\} \\
s_a &= \max\{0, \bar{s}_a\} \\
\bar{s}_a &= \left(\sum_{j \in LW_{-a} \cup \Omega\backslash I_{+a}} \hat{t} + \sum_{j \in I\backslash LW_{-a} \text{ and } V_j \geq 0} V_j - (1+\lambda)(Nk - \sum_{j \in I \text{ and } V_j < 0} V_j) \right) (1+\lambda)^{-1}
\end{aligned}$$

with

$$I = \begin{cases} \{\frac{N+3}{2}, \ldots, N\} & \text{if } a \leq \frac{N+1}{2} \\ \{\frac{N+1}{2}, \cdots, N\}\backslash\{a\} & \text{if } a > \frac{N+1}{2}. \end{cases}$$

The proof can be found in the appendix.[4]

The crucial question is whether $g = 1$ will be chosen in equilibrium, which is equivalent to

$$(G) \;\; V_a + (1+\lambda)^{1-\text{sg}(\bar{s}_a)} \bar{s}_a(g=1) \geq s_a(g=0),$$

where

$$\text{sg}(\bar{s}_a) = \begin{cases} 1, & \bar{s}_a > 0 \\ 0, & \bar{s}_a \leq 0. \end{cases}$$

Hence by using $|LW_{-a} \cup \Omega \setminus I_{+a}| - |I| = |LW_{-a} \cap I|$, (G) can be written as

$$(G^+) \quad (1+\lambda)V_a + \sum_{j \in LW_{-a} \cap I} \hat{t} + \sum_{j \in I\backslash LW_{-a}, V_j \geq 0} V_j \geq (1+\lambda)(Nk - \sum_{j \in I, V_j < 0} V_j) \wedge \text{sg}(\bar{s}_a) = 1$$

$$(G^-) \quad V_a + \sum_{j \in LW_{-a} \cap I} \hat{t} + |I| \tfrac{\lambda}{1+\lambda} \hat{t} + \sum_{j \in I\backslash LW_{-a}, V_j \geq 0} V_j \geq (1+\lambda)(Nk - \sum_{j \in I, V_j < 0} V_j) \wedge \text{sg}(\bar{s}_a) = 0.$$

[4] Note that the tax payment of the agenda setter may be higher than $\hat{t}$ if he voluntarily decides to contribute more in order to secure the financing of the project.

In other words, if and only if the agenda setter can generate tax revenues from project winners (under $g = 1$) that are sufficiently high to finance the project and to compensate project losers, then he will propose $g = 1$.

We next state a simple observation that simplifies the characterization of the equilibria.

Lemma 3

In the simple majority voting scheme, an agenda-setter who is not one of the large project winners ($a \notin LW$) will never make a proposal that involves a tax payment for himself in order to finance the public project.

The proof can be found in the appendix. With these preliminary observations we obtain

Proposition 1

Suppose that all individuals have applied for agenda setting. Then the simple majority voting is characterized by the following equilibria:

(i) If $|LW_{-a}| \geq \frac{N-1}{2}$ and (G) holds for a proposal maker a, he offers

$$A_a^* = \begin{cases} s_j = 0 & j \in \Omega_{-a} \\ t_j = \hat{t} & j \in \Omega_{-a} \\ t_a = \max\{0, -(1+\lambda)\bar{s}_a\} & \\ s_a = \max\{0, \bar{s}_a\} & \\ \bar{s}_a = \frac{(N-1)\hat{t}}{(1+\lambda)} - Nk. & \end{cases}$$

Voting strategies are

$$\delta_j^* = \begin{cases} 1 & \text{if} \quad j \in LW \\ 1 & \text{if} \quad j = a \\ 0 & \text{otherwise.} \end{cases}$$

(ii) If $|LW_{-a}| < \frac{N-1}{2}$ and (G) holds for a proposal maker a, he offers

$$A_a^* = \begin{cases} t_j = \hat{t} & j \in LW_{-a} \cup \Omega \backslash I_{+a} \\ t_j = V_j & j \in I \backslash LW \text{ with } V_j \geq 0 \\ t_j = 0 & j \in I \backslash LW \text{ with } V_j < 0 \\ s_j = -V_j & j \in I \quad \text{with } V_j < 0 \\ t_a = \max\{0, -(1+\lambda)\bar{s}_a\} & \\ s_a = \max\{0, \bar{s}_a\} & \\ \bar{s}_a = (1+\lambda)^{-1}\{(\frac{N-1}{2} + |LW_{-a}|)\hat{t} + \sum\limits_{j \in I \backslash LW, V_j \geq 0} V_j\} - Nk + \sum\limits_{j \in I, V_j < 0} V_j. & \end{cases}$$

Voting strategies are

$$\delta_j^* = \begin{cases} 1 & j \geq \frac{N+3}{2} \\ 1 & j = \frac{N+1}{2} \text{ and } a \geq \frac{N+1}{2} \\ 1 & j = a \\ 0 & j = \frac{N+1}{2} \text{ and } a < \frac{N+1}{2} \\ 0 & \text{otherwise.} \end{cases}$$

(iii) If (G) does not hold for a proposal maker, he offers

$$A_a^* = \begin{cases} t_j = \hat{t} & \text{for an arbitrary subset } J \subset \Omega_{-a} \text{ with } |J| = \frac{N-1}{2} \\ t_j = 0 & j \in \Omega \backslash J \\ s_j = 0 & j \in \Omega \backslash J \\ s_a = \frac{N-1}{2(1+\lambda)}\,\hat{t}. \end{cases}$$

Voting strategies are

$$\delta_j^* = \begin{cases} 1 & j \in \Omega \backslash J \\ 1 & j = a \\ 0 & j \in J. \end{cases}$$

The proof of Proposition 1 is straightforward. Proposition 1 immediately implies that a proposal maker can always strictly improve his utility relative to the status quo. Hence we obtain

Corollary 1

Under the simple majority rule, every agent applies for agenda setting.

As condition (G) may hold for some proposal makers but not for others, we provide a general characterization of the equilibria in this subsection and calculate expected utilities.

3.1 General Characterization of Equilibria

For this purpose we introduce

$$\mathcal{G} := \{j \,|(G) \text{ holds } \text{ for } a = j\}$$

$\mathcal{G}$ is the set of individuals who propose $g = 1$ if they can determine the agenda. We define

$$p(G) := \frac{|\mathcal{G}|}{N}$$
$$\tilde{p}(G) := \max\{p(G) - \frac{1}{N}, 0\}.$$

The expression $p(G)$ denotes the share of individuals who will propose $g = 1$ in equilibria. Hence $p(G)$ is the probability that the public project will be provided before the agenda setter is chosen. We obtain

Proposition 2

The expected utilities are given as follows:

(i) Suppose $|LW| \geq \frac{N-1}{2}$.

α.) *If* $j \in \mathcal{G}$,

$$E[U_j] = e + \tilde{p}(G)(V_j - \hat{t}) + \frac{1}{N}(V_j + (1+\lambda)^{1-Sg(\bar{s}_a)}\bar{s}_a) - (1 - p(G))\frac{N-1}{2N}\hat{t}.$$

β.) *If* $j \notin \mathcal{G}$,

$$E[U_j] = e + p(G)(V_j - \hat{t}) + \frac{1}{N}(s_a(g=0)) - (1 - p(G) - \frac{1}{N})\,\hat{t}\,\frac{N-1}{2N}.$$

(ii) Suppose $|LW| < \frac{N-1}{2}$.

α.) *If* $j \in \mathcal{G}$,

$$E[U_j] = \begin{cases} e + \tilde{p}(G)(V_j - \hat{t}) + \frac{1}{N}(V_j + (1+\lambda)^{1-Sg(\bar{s}_a)}\bar{s}_a) - (1 - p(G))\frac{N-1}{2N}\,\hat{t}, \\ \quad j \in LW \cup \{j | j < \frac{N+1}{2}\} \\ e + \frac{1}{N}(V_j + s_a(g=1)) - (1 - p(G))\frac{N-1}{2N}\,\hat{t}, j \notin LW \text{ and } j \geq \frac{N+3}{2} \\ e + \tilde{p}(G)\frac{N-1}{2N}(V_j - \hat{t}) + \frac{1}{N}(V_j + s_a(g=1)) - (1 - p(G))\frac{N-1}{2N}\hat{t}, \\ \quad j = \frac{N+1}{2}. \end{cases}$$

β.) *If* $j \notin \mathcal{G}$,

$$E[U_j] = \begin{cases} e + p(G)(V_j - \hat{t}) + \frac{1}{N}s_a(g=0) - (1 - p(G) - \frac{1}{N})\frac{N-1}{2N}\hat{t}, \\ \quad j \in LW \cup \{j | j < \frac{N+1}{2}\} \\ e + \frac{1}{N}s_a(g=0) - (1 - p(G) - \frac{1}{N})\frac{N-1}{2N}\hat{t}, j \notin LW \text{and } j \geq \frac{N+3}{2} \\ e + p(G)\frac{N-1}{2N}(V_j - \hat{t}) + \frac{1}{N}s_a(g=0) - (1 - p(G) - \frac{1}{N})\frac{N-1}{2N}\hat{t}, \\ \quad j \notin LW, j = \frac{N+1}{2}. \end{cases}$$

Proposition 2 follows directly from the propositions in the last subsection.

4 Equilibria with Minority Voting

4.1 Financing

We next consider MV. To prepare the equilibria, it is instructive to consider voting in Stage 3 first, assuming that financing will occur with certainty in Stages 4 and 5 if $g_{a_1}^{MV} = 1$ has been adopted. We obtain

Proposition 3

Suppose individual a_1 has been chosen to set the agenda.

(i) If $|LW| \geq \frac{N+1}{2}$, the agenda setter proposes $g_{a_1}^{MV} = 1$. Exactly $\frac{N+1}{2}$ large project winners will accept the proposal.

(ii) If $|LW| < \frac{N+1}{2}$, nobody applies for agenda setting and the status quo prevails.

The proof can be found in the appendix. Recall that a proposal $g_{a_1}^{MV} = 0$ will never be supported under MV. An immediate consequence is

Corollary 2

The voting equilibria in case (i) are indeterminate with respect to which of the set of large project winners will accept the proposal if $|LW| > \frac{N+1}{2}$.

In principle, all individuals with $V_j \geq \hat{t}$ prefer the project to be accepted, but they would like to reject the proposal $g_{a_1}^{MV} = 1$ in order to keep their voting rights and the associated protection from taxation. We use the following plausible refinement of voting equilibria:

Maximal Magnanimity

Suppose $g_{a_1}^{MV} = 1$ and $|LW| \geq \frac{N+1}{2}$, then all individuals with $j \geq \frac{N+1}{2}$ cast the vote $\delta_j(g_{a_1}^{MV} = 1) = 1$, while all individuals with $j < \frac{N+1}{2}$ vote $\delta_j(g_{a_1}^{MV} = 1) = 0$.

Under Maximal Magnanimity, those individuals who benefit most exclude themselves from the financing decision. Those individuals who benefit less and are not needed to form a majority reject the proposal and can thus protect themselves against taxation.

It is in this sense that such equilibria are called Maximal Magnanimity. For future references, we note that $\mathcal{M} = \{1, \ldots, \frac{N-1}{2}\}$ if $g_{a_1}^{MV}$ has been adopted.

We next consider the financing decision under MV. For this purpose, define $LW^{>} := \{j \mid V_j > \hat{t}\}$ and suppose that $g_{a_1}^{MV}$ has been adopted. An agenda setter a_2 has to gain unanimous support among the members of $\mathcal{M}$. Moreover, in order to apply for agenda setting the individual must increase his utility. Hence, if $a_2 \in LW^{>}$ the project can be financed if

$$(F^{-})\ \ V_{a_2} + |LW_{-a_2}| \cdot \hat{t} + \sum_{j \in \Omega_{-a_2} \setminus LW} \max\{V_j, 0\} \geq (1+\lambda)[Nk - \sum_{j \in \Omega_{-a_2}} \min\{V_j, 0\}].$$

It is not necessary for the agenda-setter a_2 to be part of $LW^{>}$ for the project to be financed if

$$(F^{+})\ \ |LW| \cdot \hat{t} + \sum_{j \in \Omega \setminus LW} \max\{V_j, 0\} \geq (1+\lambda)[Nk - \sum_{j \in \Omega} \min\{V_j, 0\}].$$

holds.[5] In this way, given a certain realization $(V_j)_{j\in\Omega}$, all projects (characterized by per capita cost k) that satisfy

$$(F) = \begin{cases} (F^{-}), & a_2 \in LW^{>} \\ (F^{+}), & \text{otherwise} \end{cases}$$

can be provided. Looking at the financing problem from the other side, one can argue that a given public project can be financed if the realization of the benefit vector is such that (F) holds.

The condition (F) states that tax revenues from the set of large project winners and the aggregate benefits from small project winners are weakly larger than aggregate project costs and aggregate losses from project losers. It transpires that the left side represents the maximal tax revenues that can be generated in the political process. The right side represents the minimal aggregate expenditure needed to implement a project.

Lemma 4

(i) If $|LW| > \frac{N+1}{2}$ and (F^{+}) holds with strict inequality, then all individuals will apply for agenda setting and propose $g_{a_1}^{MV} = 1$.

[5] An agenda setter $a_2 \in LW^{>}$ may pay higher taxes than $\hat{t}$ in order to ensure the financing of the public project.

(ii) *If* $|LW| = \frac{N+1}{2}$ *and* (F^+) *holds with strict inequality, all individuals except those with* $V_j = \hat{t}$ *will apply for agenda setting and will propose* $g_{a_1}^{MV} = 1$.

(iii) *If* $|LW| \geq \frac{N+1}{2}$ *and* (F^+) *holds with equality, all individuals in* $LW^> := \{j \mid V_j > \hat{t}\}$ *will apply for agenda setting and propose* $g_{a_1}^{MV} = 1$.

(iv) *If* $|LW| > \frac{N+1}{2}$ *and* (F^-) *holds with strict inequality if* $j = a_2$ *for all* $j \in LW^> \cap \mathcal{M}$ *but* (F^+) *is not satisfied, then all individuals in* $LW^>$ *will apply for agenda setting and propose* $g_{a_1}^{MV} = 1$.

(v) *If* $|LW| > \frac{N+1}{2}$ *and* (F^-) *holds with equality if* $j = a_2$ *for at least one* $j \in LW^> \cap \mathcal{M}$, *then all individuals in* $LW^> \setminus \{j \in \mathcal{M} | (F^-)$ *does not hold or holds with equality if* $j = a_2\}$ *will apply for agenda setting and propose* $g_{a_1}^{MV} = 1$.

(vi) *In all other cases nobody will apply for agenda setting.*

The proof of Lemma 4 follows directly from the fact that the project can only be financed if (F) holds and from the tie-breaking rule that agents will not apply for agenda setting if they cannot strictly improve their utility.

4.2 Overall Equilibria

After these preliminary considerations, we can characterize the equilibria of the five-stage game. For convenience, let $\mathcal{F} = \{j \in \mathcal{M}|$ (F) holds if $a_2 = j\}$.

Proposition 4

(i) *If* $|LW| < \frac{N+1}{2}$ *or* $\mathcal{F}$ *is empty, then* $\psi_j = 0 \; \forall \; j \in \Omega$ *and the status quo prevails with* $E[U_j] = e$ *for all individuals.*

(ii) *If* $|LW| \geq \frac{N+1}{2}$ *and* $\mathcal{F} \neq \emptyset$, *we obtain the following subgame perfect equilibrium:*

Stage 1: *The individuals apply for agenda-setting as described in items (i)-(v) of Lemma 4.*

Stage 2: $g_{a_1}^{MV} = 1$

Stage 3: $\delta_j(g_{a_1}^{MV} = 1) = \begin{cases} 1, j \geq \frac{N+1}{2} \\ 0, j < \frac{N+1}{2} \end{cases}$

Stage 4: All individuals $j \in \mathcal{F}$ apply to propose a financing package and the randomly chosen agenda setter a_2 proposes

$$F^*_{a_2} = \begin{cases} t_j = \hat{t} \quad j \in LW_{-a_2} \\ t_j = V_j \quad j \in \Omega_{-a_2} \backslash LW, V_j > 0 \\ s_j = -V_j \quad j \in \Omega_{-a_2}, V_j < 0 \\ t_{a_2} = \max\{0, -(1+\lambda)\bar{s}_{a_2}\} \\ s_{a_2} = \max\{0, \bar{s}_{a_2}\} \\ \text{with } \bar{s}_{a_2} := (1+\lambda)^{-1} \sum\limits_{j \in \Omega_{-a_2}} t_j - Nk - \sum\limits_{j \in \Omega_{-a_2}} s_j. \end{cases}$$

Stage 5: $\delta_m(F^*_{a_2}) = 1, m \in \mathcal{M}$

The expected payoffs are

$$E[U_j] = \begin{cases} e + V_j - \hat{t} \quad j \in LW \setminus \mathcal{F} \\ e + (1 - \frac{1}{|\mathcal{F}|})(V_j - \hat{t}) + \frac{1}{|\mathcal{F}|}(V_j + (1+\lambda)^{1-Sg(\bar{s}_{a_2})}\bar{s}_{a_2}), \quad j \in LW \cap \mathcal{F} \\ e + \frac{1}{|\mathcal{F}|}(V_j + (1+\lambda)^{1-Sg(\bar{s}_{a_2})}\bar{s}_{a_2}), \quad j \in \mathcal{F} \setminus LW \\ e, \quad j \notin LW \cup \mathcal{F}. \end{cases}$$

The proof can be found in the appendix.

5 Welfare Comparisons

For a comparison of the two voting regimes from an ex ante perspective, three kinds of uncertainty have to be considered: the vector $(V_j)_{j\in\Omega}$ of project utilities; who the agenda setters, a, a_2, will be; and what type, j, the agent himself will be. In this way, an agent's ex ante expected utility in the simple majority voting scheme denoted by $E_0[U^{SM}]$ can be written as

$$E_0[U^{SM}] = \int_{\mathcal{V}} p(V) \sum_{m\in\Omega} p(a = m) E[U_j^{SM}|V, a] dV, \tag{2}$$

where $\mathcal{V} = [\underline{V}, \overline{V}]^N$, $p(V)$ is the density function on $\mathcal{V}$, $p(a = m)$ represents the probability that individual m will be the agenda setter, and $E[U_j^{SM}|V, a]$ denotes the expected utility of an agent given (V, a), but not knowing which j he will be.

With regard to minority voting, we have to distinguish the cases in which there is an agenda setter a_2 and those where the project will not be financed. For technical reasons it is convenient to introduce an imaginary agenda setter $a_2 = 0$ if the project will not be provided. More precisely, we make the following definition:

Definition 1

$$a_2 = \begin{cases} \textit{randomly chosen from } \mathcal{F}, & |LW| \geq \frac{N+1}{2} \wedge \mathcal{F} \neq \emptyset \\ 0, & |LW| < \frac{N+1}{2} \vee \mathcal{F} = \emptyset. \end{cases}$$

The probability that $a_2 = m$, where $m \in \mathcal{F} \cup 0$, is

$$p(a_2 = m) = \begin{cases} \frac{1}{|\mathcal{F}|}, & m \in \mathcal{F} \wedge |LW| \geq \frac{N+1}{2} \\ 0, & m = 0 \wedge \mathcal{F} \neq \emptyset \wedge |LW| \geq \frac{N+1}{2} \\ 1, & m = 0 \wedge \mathcal{F} = \emptyset \end{cases}$$

$$E[U_j^{MV}|V, 0] = e.$$

With this definition we can write the ex ante expected utility in the minority voting scheme in a similar way as for majority voting:

$$E_0[U^{MV}] = \int_{\mathcal{V}} p(V) \sum_{m \in \mathcal{F} \cup 0} p(a_2 = m) E[U_j^{MV}|V, a_2] dV. \tag{3}$$

First, it would be interesting to identify the constellations (V, a, a_2) in which an agent would prefer the minority voting scheme from an ex ante perspective, that is, if he does not know his type j. The overall comparison from an ex ante perspective then depends on how the different situations are weighted in the aggregation process. More precisely, it depends on how large the difference is in expected utilities conditional on (V, a, a_2) and what the probability weights are. In this section we take the first step. As all individuals have the same probability of being some type j, we can define social welfare as

$$W^{SM/MV} = \sum_{j \in \Omega} U_j^{SM/MV},$$

which can be interpreted as the sum of ex ante expected utilities given (V, a, a_2), though the agents do not know what j they will be. More precisely,

$$E[U_j^{MV}|V, a_2] = \frac{W^{MV}}{N}, \forall j \in \Omega$$

Similar definitions can be made for SM.

For the following comparisons, it is useful to note

Lemma 5

Whether the project is proposed and accepted depends only on the benefit vector V.[6] *This is different under SM, where it depends on the benefit level V_a of the agenda setter whether the project will be proposed or not.*

Consequently, the realization (V, a) directly determines the pair (g^{SM}, g^{MV}). It will transpire that most statements only require knowledge of (g^{SM}, g^{MV}).

Proposition 5

Suppose $|LW| < \frac{N+1}{2}$ or $\mathcal{F}$ is empty. Suppose that (G) does not hold. Then

$$E[U_j^{MV} \mid V, 0] > E[U_j^{SM} \mid V, a].$$

The proof can be found in the appendix.

The preceding proposition rests on the fact that the MV rule protects a society against inefficient redistribution proposals that will occur under SM if no project is proposed.

Proposition 6

If the project is not proposed, i.e. $g = 0$, the welfare loss due to redistribution is strictly higher under SM than under MV. If the project is provided, costs of redistribution activities are weakly higher in SM than in MV.

The proof can be found in the appendix.

For the intuition of Proposition 6, we note that $|LW| \geq \frac{N+1}{2}$ must hold if $g^{MV} = g^{SM} = 1$. As $|LW| \geq \frac{N+1}{2}$, the agenda setter in SM does not have to care about the voting behavior of all individuals $\Omega_{-a} \setminus LW$ and consequently proposes the highest tax for them. This is different for the agenda-setter a_2 in MV, as he needs the votes of the minority. In this way, total tax payments, and hence welfare losses from redistribution, must be weakly higher in SM than in MV.

Further we observe

[6] The benefit vector V determines the set of agenda setters and whether the financing condition holds.

Lemma 6

In MV only socially desirable projects will be proposed and adopted.

The proof can be found in the appendix.

We are now in a position to formulate the following result:

Proposition 7

From an ex ante social perspective, simple majority voting is strictly preferable to minority voting if and only if $(g^{SM}, g^{MV}) = (1, 0)$ *and*

$$\sum_{\Omega} V_j > (1+\lambda)Nk + \lambda \sum_{\Omega} s_j^{SM}(g^{SM} = 1).$$

In all other cases, society is at least weakly better off with the minority voting scheme.

The proof can be found in the appendix.

The previous propositions and lemmata have shown that, under the proposed minority voting scheme, the first three possible inefficiencies of legislative decision making listed in section 2.2 are avoided. For instance, Lemma 6 ensures that no inefficient projects are proposed and adopted. Proposition 5 shows that MV protects against pure redistribution proposals. However, minority voting suffers from the last inefficiency: in certain situations efficient projects are not proposed. In such cases, a simple majority scheme may be preferable from an ex ante welfare perspective. The necessary condition $(g^{SM}, g^{MV}) = (1, 0)$ for SM to be strictly preferable to MV translates into

$$\left[|LW| < \frac{N+1}{2} \vee \neg(F)\right] \wedge (G).$$

Consider the case where $|LW| \geq \frac{N+1}{2}$. Then a project would be provided in SM but not in MV if condition (G) holds and the financing condition (F) is violated ($\mathcal{F}$ is empty). In order to further characterize this case, denote by $\bar{a}_2$ the individual with the highest valuation of the project in the minority. That is, $\bar{a}_2 \in \mathcal{M} : \; V_{\bar{a}_2} \geq V_j, \; \forall j \in \mathcal{M}$. The reasoning behind this definition is that if (F) is violated when $\bar{a}_2$ is the agenda-setter a_2, it must be violated for all $j \in \mathcal{M} \setminus \bar{a}_2$. Now we can formulate

Lemma 7

Suppose $|LW| \geq \frac{N+1}{2}$, *then* $(g^{SM}, g^{MV}) = (1, 0)$ *if either* *(i)* $\frac{N-1}{1+\lambda}\hat{t} \leq Nk$ *and*

$$\begin{aligned} V_{\bar{a}_2} &+ \sum_{LW_{-\bar{a}_2}} \hat{t} + \sum_{\substack{\Omega_{-\bar{a}_2}\setminus LW \\ V_j>0}} V_j + (1+\lambda) \sum_{\substack{\Omega_{-\bar{a}_2}\setminus LW \\ V_j<0}} V_j \\ &< (1+\lambda)Nk \leq V_a + \tfrac{N-1}{2}\hat{t} + |I|\tfrac{\lambda}{1+\lambda}\hat{t}, \quad \bar{a}_2 \in LW^> \end{aligned}$$

$$\begin{aligned} \sum_{LW} \hat{t} &+ \sum_{\substack{\Omega\setminus LW \\ V_j>0}} V_j + (1+\lambda) \sum_{\substack{\Omega\setminus LW \\ V_j<0}} V_j \\ &< (1+\lambda)Nk \leq V_a + \tfrac{N-1}{2}\hat{t} + |I|\tfrac{\lambda}{1+\lambda}\hat{t}, \quad \bar{a}_2 \notin LW^> \end{aligned}$$

or

(ii) $\frac{N-1}{1+\lambda}\hat{t} > Nk$ *and*

$$\begin{aligned} V_{\bar{a}_2} &+ \sum_{LW_{-\bar{a}_2}} \hat{t} + \sum_{\substack{\Omega_{-\bar{a}_2}\setminus LW \\ V_j>0}} V_j + (1+\lambda) \sum_{\substack{\Omega_{-\bar{a}_2}\setminus LW \\ V_j<0}} V_j \\ &< (1+\lambda)Nk \leq (1+\lambda)V_a + \tfrac{N-1}{2}\hat{t}, \quad \bar{a}_2 \in LW^> \end{aligned}$$

$$\begin{aligned} \sum_{LW} \hat{t} &+ \sum_{\substack{\Omega\setminus LW \\ V_j>0}} V_j + (1+\lambda) \sum_{\substack{\Omega\setminus LW \\ V_j<0}} V_j \\ &< (1+\lambda)Nk \leq (1+\lambda)V_a + \tfrac{N-1}{2}\hat{t}, \quad \bar{a}_2 \notin LW^>. \end{aligned}$$

The proof can be found in the appendix.

According to Proposition 7, it is socially desirable for a project that would not be proposed under MV to be provided under SM if

$$\sum_{\Omega} V_j > (1+\lambda)Nk + \lambda \sum_{\Omega} s_j^{SM}(g^{SM} = 1).$$

With $|LW| \geq \frac{N+1}{2}$ and Proposition 1, this condition transforms to

$$\sum_{\Omega} V_j > Nk + \frac{\lambda}{1+\lambda}(N-1)\hat{t}.$$

Inserting this in the conditions of the previous lemma allows us to formulate

Proposition 8
If $|LW| \geq \frac{N+1}{2}$*, the situations in which simple majority voting is superior to minority voting from an ex ante social welfare point of view are characterized by either*

(i) $\frac{N-1}{1+\lambda}\hat{t} \leq Nk$ *and*

$$\begin{aligned} V_{\bar{a}_2} + \sum_{LW_{-\bar{a}_2}} \hat{t} + Nk + \tfrac{\lambda(N-1)}{1+\lambda}\hat{t} - \sum_{LW} V_j + \lambda \sum_{\substack{\Omega \setminus LW \\ V_j<0}} V_j &< (1+\lambda)Nk \\ &\leq V_a + \tfrac{N-1}{2}\hat{t} + |I|\tfrac{\lambda}{1+\lambda}\hat{t}, \\ &\quad \bar{a}_2 \in LW^{>} \end{aligned}$$

$$\begin{aligned} \sum_{LW} \hat{t} + Nk + \tfrac{\lambda(N-1)}{1+\lambda}\hat{t} - \sum_{LW} V_j + \lambda \sum_{\substack{\Omega \setminus LW \\ V_j<0}} V_j &< (1+\lambda)Nk \\ &\leq V_a + \tfrac{N-1}{2}\hat{t} + |I|\tfrac{\lambda}{1+\lambda}\hat{t}, \\ &\quad \bar{a}_2 \notin LW^{>} \end{aligned}$$

or

(ii) $\frac{N-1}{1+\lambda}\hat{t} > Nk$ *and*

$$\begin{aligned} V_{\bar{a}_2} + \sum_{LW_{-\bar{a}_2}} \hat{t} + Nk + \tfrac{\lambda(N-1)}{1+\lambda}\hat{t} - \sum_{LW} V_j + \lambda \sum_{\substack{\Omega \setminus LW \\ V_j<0}} V_j &< (1+\lambda)Nk \\ &\leq (1-\lambda)V_a + \tfrac{N-1}{2}\hat{t}, \\ &\quad \bar{a}_2 \in LW^{>} \end{aligned}$$

$$\begin{aligned} \sum_{LW} \hat{t} + Nk + \tfrac{\lambda(N-1)}{1+\lambda}\hat{t} - \sum_{LW} V_j + \lambda \sum_{\substack{\Omega \setminus LW \\ V_j<0}} V_j &< (1+\lambda)Nk \\ &\leq (1+\lambda)V_a + \tfrac{N-1}{2}\hat{t}, \\ &\quad \bar{a}_2 \notin LW^{>}. \end{aligned}$$

6 Further Aspects of Welfare

The previous welfare comparison discussed which voting scheme will result in higher expected utilities conditional on the realizations (V, a, a_2) when the individuals do not know their type. Additionally, considering uncertainty about who will be the agenda setter in SM, we can formulate the following proposition with respect to expected utilities conditional on V:

Proposition 9

If and only if $(|LW| < \frac{N+1}{2} \vee \mathcal{F} = \emptyset)$ *and*

$$\underbrace{\frac{1}{N}\sum_{a\in\mathcal{G}} E[U_j^{SM}|V,a] + (1-p(G))E[U_j^{SM}|V,a\notin\mathcal{G}]}_{E[U_j^{SM}|V]} - e > 0, \tag{4}$$

simple majority voting yields strictly higher expected levels of utility than minority voting.

The proof can be found in the appendix.

Alongside a comparison of the voting regimes with respect to ex ante expected utility, one could ask whether the outcomes under the different voting schemes would be Pareto improvements to the status quo ($U_j = e,\ \forall j \in \Omega$).

Proposition 10

Project provision under minority voting is always a Pareto improvement over the status quo. The simple majority voting scheme will result in a Pareto improvement if and only if $V_j \geq \hat{t},\ \forall j \neq a$ *and* V_a *satisfies (G).*

The proof can be found in the appendix.

7 Example

In this section we present a simple example with a homogeneous society.

Suppose that $V_j = \tilde{V} \in [\underline{V}, \overline{V}]\ \forall j \in \Omega$.

Proposition 11

If

(i) $\tilde{V} \geq \hat{t} \quad \wedge \quad \Big[\Big(\tilde{V} \geq (1+\lambda)Nk - \frac{N-1}{2}\hat{t} - |I|\frac{\lambda}{1+\lambda}\hat{t} \ \ \wedge\ \ (N-1)\hat{t} \leq (1+\lambda)Nk \Big) \ \vee \Big(\tilde{V} \geq Nk - \frac{N-1}{2(1+\lambda)}\hat{t} \ \ \wedge\ \ (N-1)\hat{t} > (1+\lambda)Nk \Big) \Big]$,

simple majority voting and the minority voting scheme yield equal levels of welfare,

(ii) $\tilde{V} \geq \hat{t} \quad \wedge \quad \neg\Big[\Big(\tilde{V} \geq (1+\lambda)Nk - \frac{N-1}{2}\hat{t} - |I|\frac{\lambda}{1+\lambda}\hat{t} \ \ \wedge\ \ (N-1)\hat{t} \leq (1+\lambda)Nk \Big) \ \vee \Big(\tilde{V} \geq Nk - \frac{N-1}{2(1+\lambda)}\hat{t} \ \ \wedge\ \ (N-1)\hat{t} > (1+\lambda)Nk \Big) \Big]$,

minority voting is strictly better than majority voting,

(iii) $\max\left\{ \frac{(1+\lambda)Nk}{1+\lambda+\frac{N-1}{2}}, \frac{Nk+\frac{\lambda}{1+\lambda}\frac{N-1}{2}\hat{t}}{N-\frac{\lambda}{1+\lambda}\frac{N-1}{2}} \right\} =: V^c < \tilde{V} < \hat{t}$ *majority voting is strictly preferable from a social perspective,*

(iv) $\tilde{V} \leq V^c$*, the minority voting scheme is superior to majority voting.*

The proof can be found in the appendix.

The example illustrates the advantages and drawbacks of minority voting. It also illustrates the importance of the tax protection level $\hat{t}$. If $\hat{t} > \tilde{V}$, a socially desirable project may not be proposed under MV.

8 Conclusion

In this paper we propose a two-stage collective process called minority voting that can avoid a variety of inefficiencies in democratic decision-making. Obviously, our proposal points up quite a number of future research avenues. First and foremost, there is a further level of design by varying the maximal tax level $\hat{t}$ in order to maximize social welfare. One might even consider a pre-voting step in which $\hat{t}$ is determined. Second, while we have focussed on unanimous decisions in the second stage under MV, it is important to compare the results with the outcome when the simple majority rule is used for the financing scheme. Third, there are a number of further variations of the model, e.g. when the benefits are not observable in the first period or when there are two public projects. These need to be examined in order to verify the merits of our proposal.

Appendix

Proof of Lemma 2

For this lemma the following observation is important: For the agenda setter it is optimal in the case of $g = 1$ to select the majority supporting his proposal by choosing set I as

$$I = \begin{cases} \{\frac{N+3}{2} \cdots, N\} & \text{if } a \leq \frac{N+1}{2} \\ \{\frac{N+1}{2}, \cdots, N\} \backslash \{a\} & \text{if } a > \frac{N+1}{2}. \end{cases}$$

Set I comprises the people with the highest values of V_j. Those individuals can be charged with higher taxes or need fewer subsidies while still supporting $g = 1$ than the other individuals. As he can charge $t_j = \hat{t}$ for the rest of the society $(\Omega \backslash I_{+a})$, he will obtain maximal tax revenues (or minimal subsidies) by choosing I.

■

Proof of Lemma 3

An agenda setter will propose $g = 1$ if (G) is satisfied. As $s_a(g = 0) = \frac{N-1}{2(1+\lambda)}\hat{t}$, (G) is written as

$$V_a + (1-\lambda)^{1-\text{sg}(\bar{s}_a)} \bar{s}_a(g = 1) \geq \frac{N-1}{2(1+\lambda)}\hat{t}. \tag{5}$$

A tax for the agenda setter means that $\bar{s}_a < 0$. Consequently, the project will only be proposed if

$$V_a > \frac{N-1}{2(1+\lambda)}\hat{t} > \hat{t}, \tag{6}$$

since $0 < \lambda < 1$ and $N \geq 5$. This contradicts $a \notin LW$.

■

Proof of Proposition 3

Suppose an individual a_1 proposes $g^{MV}_{a_1} = 1$. By the rules of MV, an individual who supports $g^{MV}_{a_1} = 1$ faces two possibilities. Either he is in a minority and $g^{MV}_{a_1} = 0$

prevails, or he is in the majority. As he will lose his voting rights, he will be taxed by $\hat{t}$ in the subsequent financing stage. Hence voting $\delta_j(g_{a_1}^{MV} = 1) = 0$ weakly dominates $\delta_j(g_{a_1}^{MV} = 1) = 1$ for all individuals with $V_j < \hat{t}$. By our tie-breaking rule, result (ii) follows.

If $|LW| \geq \frac{N+1}{2}$ and if $\frac{N+1}{2}$ large project winners accept the proposal, the best response for other large project winners is to vote $\delta_j(g_{a_1}^{MV} = 1) = 0$ as they then have a chance of becoming agenda setter in the financing stage. In turn, given the voting behavior of all other individuals, it is the best response for large project winners in the tight majority supporting $g_{a_1}^{MV} = 1$, as otherwise the status quo would prevail.

■

Proof of Proposition 4

The proof follows from backward induction. In Stage 4 the agenda setter solves the following problem:

$$\max_{(t_j,s_j)_{j\in\Omega}} \; U_{a_2} = e + V_{a_2} + s_{a_2} - t_{a_2}$$

$$\begin{aligned} \text{s.t. } \; U_m - e = V_m + s_m - t_m &\geq 0 \;\; \forall\, m \in \mathcal{M} \\ \sum_{j\in\Omega} t_j = (1+\lambda)(Nk + \sum_{j\in\Omega} s_j) & \\ t_j \leq \hat{t}, \forall j, & \end{aligned}$$

which yields the solution in the proposition. Note also that Maximal Magnanimity applies in Stage 3.

■

Proof of Proposition 5

Since the project is not proposed under MV and SM, we have

$$\begin{aligned} E[U_j^{MV} \mid V, 0] - E[U_j^{SM} \mid V, a] &= e - (e + \frac{1}{N} s_a(g=0) - \frac{N-1}{2N}\hat{t}) \\ &= \frac{N-1}{2N}\,\hat{t} - \frac{1}{N} s_a(g=0) \\ &= \hat{t}\,\frac{N-1}{2N}\Big(\frac{\lambda}{1+\lambda}\Big) > 0 \\ &> 0. \end{aligned}$$

■

Proof of Proposition 6

The first part of the proposition is obvious, as if the project is not proposed, there will be redistribution in SM but not in MV. Hence $W^{MV} = eN$ and $W^{SM} = eN + \frac{N-1}{2(1+\lambda)}\hat{t} - \frac{N-1}{2}\hat{t} < eN$

With respect to the second part, suppose the project is to be provided under both voting schemes, that is, $g^{MV} = g^{SM} = 1$. Redistribution activities cause a welfare loss of

$$\lambda \sum_{j\in\Omega} s_j.$$

Accordingly, the proposition claims that

$$\sum_{j\in\Omega} s_j^{SM} \geq \sum_{j\in\Omega} s_j^{MV}$$

Using the budget constraint of equation (1), the above condition can be written as

$$\sum_{j\in\Omega} t_j^{SM} \geq \sum_{j\in\Omega} t_j^{MV}.$$

This holds true, as in MV the tax payments are

$$\sum_{j\in\Omega} t_j^{MV} = \sum_{LW_{-a_2}} \hat{t} + \sum_{\substack{\Omega_{-a_2}\backslash LW \\ V_j>0}} V_j,$$

whereas in SM they amount to

$$\sum_{j\in\Omega} t_j^{SM} = \sum_{LW_{-a}} \hat{t} + \sum_{\Omega_{-a}\backslash LW} \hat{t}.$$

■

Proof of Lemma 6

As the agenda setter a_2 is not able to make any member of society worse off as compared to the status quo, the total taxes collected must be weakly smaller than the sum of the utilities derived from the public project by those individuals who benefit from its provision. Hence we have

$$\sum_{\Omega} \max\{V_j, 0\} \geq \sum_{\Omega} t_j = (1+\lambda)[Nk + \sum_{\Omega} s_j]. \tag{7}$$

$\sum_{\Omega} s_j$ can be split into

$$\sum_{\Omega} s_j = -\sum_{\Omega} \min\{V_j, 0\} + \sum_{\Omega} s_j^{pr}.$$

The first term reflects compensatory payments to the project losers in $\mathcal{M}$, while the second term represents purely redistributional subsidies (hence the superscript 'pr'), which in equilibrium can only be positive if individual j is the agenda setter.[7] Consequently, by using $\sum_{\Omega} V_j = \sum_{\Omega} \max\{V_j, 0\} + \sum_{\Omega} \min\{V_j, 0\}$, inequality (7) can be rewritten as

$$\sum_{\Omega} V_j \geq (1+\lambda)\big[Nk + \sum_{\Omega} s_j^{pr}\big] - \lambda \sum_{\Omega} \min\{V_j, 0\}.$$

As $(1+\lambda)\sum_{\Omega} s_j^{pr} - \lambda \sum_{\Omega} \min\{V_j, 0\} \geq 0$, we obtain

$$\sum_{\Omega} V_j \geq (1+\lambda)Nk.$$

If the above condition held with equality, then an agenda setter could not realize positive subsidies. In this case, nobody would apply for agenda-setting.

Consequently, if the project is proposed and adopted, the inequality must be strict, implying that the project is socially desirable.

■

Proof of Proposition 7

As from an ex ante point of view each individual is equally likely to assume any of the values V_j, total welfare can be measured by the sum of utilities. Since all members of the society are risk-neutral, this translates into

[7] Note that in the minority voting case $s_{a_2}^{pr} = \bar{s}_{a_2}$ if $V_{a_2} > 0$ and $s_{a_2}^{pr} = \bar{s}_{a_2} + V_{a_2}$ if $V_{a_2} < 0$. The same rule applies for simple majority voting.

$$W = \sum_{\Omega}(e + gV_j) - (1+\lambda)gNk - \lambda \sum_{\Omega} s_j$$

where we have used the budget constraint of equation (1).

From Proposition 6 we know that redistribution losses are (weakly) higher under SM than under MV if $g^{SM} = g^{MV}$. Consequently, in these cases social welfare is (weakly) higher in MV than in SM. This must also be the case if $(g^{SM}, g^{MV}) = (0, 1)$ because from Lemma 6 we know that, when the project is adopted in MV,

$$W^{MV}(g=1) = \sum_{\Omega} e + \underbrace{\sum_{\Omega} V_j - (1+\lambda)Nk - \lambda \sum_{\Omega} s_j^{MV}(g^{MV}=1)}_{\geq 0}$$

whereas in SM without project provision,

$$W^{SM}(g=0) = \sum_{\Omega} e \underbrace{-\lambda \sum_{\Omega} s_j^{SM}(g_a^{SM}=0)}_{<0}.$$

Consequently, the only possibility for SM to be strictly socially preferable is when $(g^{SM},\, g^{MV}) = (1, 0)$. A simple welfare comparison reveals that

$$W^{SM}(g^{SM}=1) = \sum_{\Omega}(e + V_j) - (1+\lambda)Nk - \lambda \sum_{\Omega} s_j^{SM}(g^{SM}=1)$$
$$> \sum_{\Omega} e = W^{MV}(g^{MV}=0)$$

if and only if

$$\sum_{\Omega} V_j > (1+\lambda)Nk + \lambda \sum_{\Omega} s_j^{SM}(g^{SM}=1).$$

■

Proof of Lemma 7

With $|LW| \geq \frac{N+1}{2}$, $\bar{s}_a \gtreqless 0$ is equivalent to $\frac{N-1}{1+\lambda}\hat{t} \gtreqless Nk$. Further, (G) can be rewritten as

$$(G) = \begin{cases} (G^-) & V_a + \frac{N-1}{2}\hat{t} + |I|\frac{\lambda}{1+\lambda}\hat{t} \geq (1+\lambda)Nk \\ (G^+) & (1+\lambda)V_a + \frac{N-1}{2}\hat{t} \geq (1+\lambda)Nk. \end{cases}$$

Consider the first of the above cases characterized by $\bar{s}_a \leq 0$. $(g^{SM}, g^{MV}) = (1,0)$ then requires $(G^-) \wedge \neg(F^-)$. As $\neg(F^-)$ can be written as

$$V_{\bar{a}_2} + |LW_{-\bar{a}_2}| \cdot \hat{t} + \sum_{j \in \Omega_{-\bar{a}_2} \setminus LW} \max\{V_j, 0\} < (1+\lambda)[Nk - \sum_{j \in \Omega_{-\bar{a}_2}} \min\{V_j, 0\}],$$

both (G^-) and $\neg(F^-)$ hold if

$$V_{\bar{a}_2} + \sum_{LW_{-\bar{a}_2}} \hat{t} + \sum_{\substack{\Omega_{-\bar{a}_2} \setminus LW \\ V_j > 0}} V_j + (1+\lambda) \sum_{\substack{\Omega_{-\bar{a}_2} \setminus LW \\ V_j < 0}} V_j \; < (1+\lambda)Nk \; \leq V_a + \frac{N-1}{2}\hat{t} + |I|\frac{\lambda}{1+\lambda}\hat{t}.$$

The other conditions of the lemma are derived analogously.

■

Proof of Proposition 9

From the discussion in the previous section we know that the majority voting scheme will only yield strictly higher expected utility compared to minority voting if $(g^{SM}, g^{MV}) = (1,0)$. According to Lemma 5, V directly determines g^{MV}. However, given V, there may be uncertainty about g^{SM}, as not every agenda setter under SM would propose the project. Hence SM would be strictly preferable to MV if the weighted expected utilities when it is socially desirable to provide the project are large enough to compensate for the situations in which adhering to the status quo would yield higher welfare. Note that, if the project is proposed, the expected utility depends on who will be the agenda setter. The reason is that different agenda setters can charge different amounts of taxes from the majority, which involves different levels of redistributional shadow costs.

■

Proof of Proposition 10

As under MV the minority must agree to the project by the unanimity rule and the majority will only approve project provision if they are members of the set LW, no individual will be worse off compared to the status quo. If no agent is strictly better off by providing the public project, no one will apply for agenda setting in the first stage. Hence project provision must involve a Pareto improvement to the status quo.

Under SM at least the members of the minority will be taxed by $\hat{t}$, as they are not necessary for proposal approval. Hence only a valuation of the project that is at least $\hat{t}$ will prevent an individual in the minority from being worse off when the project is provided compared to the status quo. V_a satisfying (G) implies that the project will be proposed and that at least the agenda setter will strictly gain in utility.[8] It is easy to see that in all other cases SM will not lead to a Pareto improvement. More precisely, if the project is not proposed, pure redistribution will leave the minority with utility lower than e. Further, if the project is proposed but there is an individual $j \neq a$ with $V_j < \hat{t}$, this person will be a member of the minority (as we know from Lemma 2) and hence will face taxes $\hat{t}$.

■

Proof of Proposition 11

Case (i)

Let $\tilde{V} > \hat{t}$. This implies that $|LW| \geq \frac{N+1}{2}$. As $a_2 \in LW^{>}$, the public project will be proposed and adopted under MV if

$$(F^-) \quad \tilde{V} + (N-1)\hat{t} \geq (1+\lambda)Nk.$$

With respect to SM, project provision implies

$$(G^-) \quad \tilde{V} + \frac{N-1}{2}\hat{t} + |I|\frac{\lambda}{1+\lambda}\hat{t} \geq (1+\lambda)Nk, \text{ if } (N-1)\hat{t} \leq (1+\lambda)Nk$$
$$(G^+) \quad (1+\lambda)\tilde{V} + \frac{N-1}{2}\hat{t} \geq (1+\lambda)Nk, \text{ if } (N-1)\hat{t} > (1+\lambda)Nk.$$

Suppose that (G^-) holds. Then (F^-) also holds. Hence the project will be provided under both regimes SM and MV. As $|LW| = N$, both agenda setter a and a_2 will

[8] The reason is that (G) implies that his utility gain is at least as high as the one he could achieve by pure redistribution.

propose $\hat{t}$ for every individual except himself. They will close the budget gap with a tax payment of their own. Both voting schemes yield equivalent tax revenues and no subsidies and thus result in equal levels of welfare.

The reasoning for (G^+) is similar. (G^+) also implies (F^-). In this case, however, the agenda setters a and a_2 receive subsidies that are the same under both voting schemes.

In the case of $\tilde{V} = \hat{t}$, the proof has to be adapted in the following way: As $a_2 \notin LW^>$, the public project will be proposed and adopted if

$$N\hat{t} > (1+\lambda)Nk$$

holds. We denote this condition by $(F^+)^>$. The assumptions involved in case (i) imply that $(F^+)^>$ holds, therefore the same reasoning applies as before.

Case (ii)
In the case of (ii), we have either $\neg(G) \wedge (F)$ or $\neg(G) \wedge \neg(F)$. Although $\neg(G) \wedge (F)$, might imply that there are higher shadow costs of public funds under MV, the sum of utilities derived from public project provision must overcompensate them, as no individual can be worse off in this voting scheme (see also the proof of Proposition 7). Further, we know from Lemma 6 that only socially desirable projects will be provided under MV. In this way, MV is superior to SM. The same holds true if $\neg(G) \wedge \neg(F)$, as verified in Proposition 6.

Case (iii)
Now consider situation (iii), where $\max\left\{\frac{(1+\lambda)Nk}{1+\lambda+\frac{N-1}{2}}, \frac{Nk+\frac{\lambda}{1+\lambda}\frac{N-1}{2}\hat{t}}{N-\frac{\lambda}{1+\lambda}\frac{N-1}{2}}\right\} =: V^c < \tilde{V} < \hat{t}$. The project will not be provided under MV, as $|LW| < \frac{N+1}{2}$. The project will be proposed under SM if (G^+) holds, which can be transformed to[9]

$$\tilde{V} \geq \frac{(1+\lambda)Nk}{1+\lambda+\frac{N-1}{2}}.$$

According to the condition in Proposition 7, it would be socially desirable to do so if

$$N\tilde{V} > Nk + \frac{\lambda}{1+\lambda}\left(\frac{N-1}{2}\tilde{V} + \frac{N-1}{2}\hat{t}\right). \tag{8}$$

[9]Note that according to Lemma 3 if $V < \hat{t}$ the agenda setter under SM would not propose $g = 1$ if he had to accept a tax for himself. Hence the project will be provided if (G^+) holds.

This inequality holds if the utilities derived from the project satisfy

$$\tilde{V} > \frac{Nk + \frac{\lambda}{1+\lambda}\frac{N-1}{2}\hat{t}}{N - \frac{\lambda}{1+\lambda}\frac{N-1}{2}}.$$

Hence, if both conditions (G^{+}) and (8) hold, a socially desirable project is provided under SM that would not be provided under MV. So in this case SM is strictly preferable to MV.

Case (iv)

Finally, for $\tilde{V} < V^{c}$, the project is not provided under either voting scheme or is only proposed under SM. However, provision under SM is not desirable from a social welfare perspective, as the redistribution losses are higher than the sum of additional utilities derived from the public good. Consequently, MV is superior to SM. ■

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