Why one Person one Vote?

Hans Gersbach*

First version: September 1997

This version: December 1999

Abstract

We provide a justification why the core principal in liberal democracies one-person-one-vote is socially desirable. We compare two possible constitutions. Under a "fixed democracy", every person has one vote and has the same chance to propose public good provision. Under a "flexible democracy" an agenda setter can additionally propose to limit future participation in voting and agenda setting. We show that a fixed democracy induces more restrictions on attempts of majorities to tax minorities than a flexible democracy. A flexible democracy may be more suited to enable a polity to undertake public projects. This possible advantage is too small to outweigh taxation distortions and citizens unanimously favor the one-person-one-vote rule ex ante.

*Alfred-Weber-Institut, University of Heidelberg, Grabengasse 14, 69117 Heidelberg, Germany.
I am grateful to Ami Glazer, Ulrich Erlenmaier and seminar participants in Heidelberg and Konstanz for valuable suggestions and comments.

1 Introduction

The basic constitutional principles of liberal democracies is that one person should have one vote. Many ethical justifications for this principle have been developed in political philosophy and social choice. In this paper we provide a simple explanation why such a principle is socially efficient from an economic point of view.

We compare two possible constitutions. Under the “fixed democracy” constitution, every person has one vote and has the same chances to propose public good provision and financing. The policy uses the simple majority rule to decide whether proposals should be accepted or rejected. The “flexible democracy” constitution is characterized by the following set of rules: citizens who have the chance to make proposals in terms of public goods and financing can simultaneously propose that agenda setting and voting in the next period is restricted to a subset of citizens. The citizens holding the current right to participate in the voting process decide by majority rule whether to accept such proposals or to favor the status quo.

We show that a fixed democracy induces more restrictions on attempts of majorities to tax minorities and to redistribute resources than a flexible democracy. Losses due to distortionary taxation are therefore lower when the principle one-person-one vote is imposed in the constitution. A flexible democracy however, may be more suited to enable a community to implement socially beneficial public goods. In most cases this advantage is either not existent or sufficiently small and citizens unanimously favor the one-person-one-vote rule ex ante.

The paper is influenced by the seminal contribution of Buchanan and Tullock (1969) on the design of constitutions. Following a long tradition started by Rousseau (see Harsanyi (1955), Mirrless (1971) and also Wicksell (1896)), Buchanan and Tullock (1969) have examined which constitutional rules would be chosen behind a veil of ignorance (see also Rae, 1969, Taylor, 1969). In this paper, we examine which participation rules for agenda setting and voting will be chosen behind the veil of ignorance. Our model builds on Romer and Rosenthal (1983), Laffont (1995) directly on Aghion and Bolton (1998). We extend their model to a dynamic setting and endogenize the participation in democracies. Our paper is also related to a large literature on optimal collective decision rules and thus to the majoritarian logic (Buchanan, 1998). A strand of literature has singled out super majority rules (Caplin and Nalebuff, 1988) as the most preferred rules for ex post collective decisions in order to avoid Condorcet cycles (see, however, Dasgupta and Maskin, 1997). Aghion and Bolton (1998) show that simple majority rules can be optimal because it can help to overcome ex-post vested interests. In our paper, we work directly with the simple majority rule.[1] Finally, our paper complements an important and large body of literature in political philosophy [e.g. Riker 1982] on the foundations of one-person-one-vote.

2 Model and Assumptions

The basic structure of our model builds on Romer and Rosenthal (1983) and Aghion and Bolton (1998) extended to a dynamic setting. We consider the standard social choice problem of public good provision and financing. Time is infinite and indexed by $t = -1, 0, 1, \ldots$. The first period $t = -1$ is the constitutional period. In the constitutional period a society of risk-neutral members decides how public good provision and financing should be governed in all future legislative periods. We assume a continuum of citizens located uniformly on a circle with a parameter equal to one. Citizens are uniformly distributed on the circle and are indexed by $j \in [0, 1]$.

In every legislative period $t = 0, 1, 2$, each citizen is endowed with some private consumption good whose density on the unit circle is e and the community can adopt a public good g_t. Let c_j^t denote the agent $j's$ utility density from consumption of public good g_t. For simplicity of presentation, we assume that c_j^t can take two values, $c_j^t = C_h > 0$ and $c_j^t = C_l \leq 0$ so that the citizens can be divided into winners and losers. We assume that c_j^t is a private benefit that cannot be taxed.[2] The ex-ante probability that any one citizen belongs to the winner group in period t, and hence has C_h, is denoted by p_t. We shall assume that p_t is constant over time and given by p. We assume $0 < p < \frac{1}{2}$ which represents the case when a portion of the losers must be compensated in order to obtain a majority in favor of the project. $C = pC_h + (1-p)C_l$ is the expected utility from the public good. Moreover, we assume that citizen's utility types are independently distributed in each period and serially uncorrelated over time. Hence, by the law of large numbers, p equals the fraction of winners in each period. We express c_j^t in terms of the consumption good. We denote the supply of public good g_t by $g_t \in \{0, 1\}$. If we denote by τ_t^j citizen j's net subsidy payment (subsidies minus taxes), the utility of citizen j in period t, denoted by u_t^j is given by

1 Our results complement another strand of literature, started by May (1952), which has imposed the one-person-one-vote principle by requiring the anonymity property and has derived the simple majority rule satisfying this and a series of other properties [see Kelly 1987, Fishburn 1973, Bernholz and Breyer 1996.]

2 If c_j^t is a monetary return it could be taxed in addition to e. The results are unaffected.

$$u_t^j = e + c_t^j + \tau_t^j \tag{1}$$

Voters are infinitely lived and maximize expected utility U^j:

$$U^j = E\sum_{t=0}^{\infty} \delta^t\, u_t^j, \tag{2}$$

where $0 < \delta < 1$ is the discount rate and where E is the expectation operator. We often drop the index j in U^j whenever convenient.

Public goods must be financed by taxes. We assume that taxation is distortionary. Let $\lambda > 0$ denote the shadow cost of public funds. That is, taxation uses $\$(1+\lambda)$ of resources of taxpayers in order to levy \$1 for public goods or for transfers to citizens. We assume that the overall cost of a public good g_t is given by $k > 0$. The budget constraint of the society in period t is given by[3]

$$-\int_0^1 \min\left[\tau_t^j,\, 0\right] dj = k(1+\lambda)g_t + (1+\lambda)\int_0^1 \max\left[\tau_t^j,\, 0\right] dj \tag{3}$$

We assume that it is socially efficient to produce the public good. That is

$$C = p \cdot C_h + (1-p)C_l > k(1+\lambda).$$

Moreover, we assume that the maximum amount of resources that can levied through taxation is sufficient to finance the public good:

$$e > k(1+\lambda) \tag{4}$$

At the constitutional stage, the society decides about the rules governing the legislative processes. We assume that whether an individual is a winner or a loser of the project cannot be observed by other citizens since benefits are private. Moreover, it is impossible to verify, whether a particular individual is a winner or a loser in a constitutional court. However, we assume open ballots and hence individuals can be divided ex post into the majority winners and minority losers. We consider two possible constitutions

C1:
- Simple majority rule
- Equal treatment majority winners (ETW)
- Equal participation in agenda setting and voting (EPAV)

3 We assume thatthe net subsidy functions τ_t^j satisfy the integrability condition.

C2: • Simple majority rule
• ETW
• Open participation, starting with full participation

Under the second constitution a citizen who can set the agenda in period t could try to restrict future participation by proposing a set of individuals, denoted by V_{t+1}, who can determine the agenda and voting outcome in the next period. We call the constitution $C2$ a flexible democracy. The starting point is V_0 equal to the unit circle and thus full participation. The other elements of the constitutions are explained in the following. While the design of a decision rule for the legislative process is itself a matter of constitutional design, we work directly with the simple majority rule which has been proven to be optimal for such cases by Aghion and Bolton (1998).

To simplify our examination, we also work with ETW, which implies that all members of a winning majority must pay (or receive) the same amount of net transfers, denoted by τ_t^w. The equal treatment rules can be justified by the need to limit redistribution among winners. If transfers are determined completely freely, an agenda setter would increase redistribution in his favor which, however, causes more distortionary tax losses. From an ex ante point of view, ETW can therefore be justified by efficiency reasons.[4] Note how ETW works. Any proposal by the agenda setter creating a winning majority but not satisfying ETW after the votes have been casted would be unconstitutional and, hence, the status quo prevails. Since agenda setters can always benefit by making a constitutional proposal, no agenda setter would ever propose a project/financing package that violates the constitution.
For strategic considerations it is necessary to restrict equal treatment to exactly half of the total population. Hence, ETW requires that only 50 percent of the total population, who belong to the winning majority, must be treated equally. This modified treatment rule avoids any attempts of taxed individuals to join the majority and to make a proposal unconstitutional. Hence, when we use ETW we only require that half of the population belonging to the winning majority must be treated equally.[5]

For convenience, we assume a tie-breaker in case of indifferences. A new proposal wins over the status quo if the voting outcome is a tie, i.e., the proposal receives half of the votes as a measure on the unit circle.[6] The sequence of events in each

4 For detais of the argument see Gersbach 1998 together with Aghion and Bolton 1998.

5 Since we are working with a continuum of individuals and thus a countable number of individuals has measure zero, we also require that no countable number of individuals, including the agenda setter, are treated better than the 50 percent of the total population.

6 Without such a tie-breaking rule, we would face the standard problem that no equilibrium in

legislative period is summarized as follows:

Stage 1: At the start of period t, the realization of c_t^j is observed by each citizen j.

Stage 2: According to the participation rule, a citizen j is determined to set the agenda. The agenda setter proposes a project/financing package $\left(g_t, \tau_t^j\right)$ and proposes a participation rule V_{t+1} in case of C2.

Stage 3: The nation decides whether to accept or reject the proposal according to the current participation and decision rule.

It is obvious that from an ex ante point of view the socially optimal solution is given by $g_t = 1,\ t = 0, 1, 2, \ldots$. Moreover, the socially efficient tax scheme should only allow money to be raised for financing the public good, since any redistribution activities are waste from an ex ante point of view. The socially efficient tax schedule is indeterminate since citizens are risk neutral. A socially efficient scheme, for instance, is $\tau_t = \frac{k(1+\lambda)}{p}$, for all project winners and $\tau_t = 0$ for all project losers. Hence, all beneficiaries of the public good pay a tax of $\frac{k(1+\lambda)}{p}$ to cover exactly the resource costs. In order to implement such a socially optimal solution, a complete social contract would be necessary, which is infeasible since private benefits of the public project are not verifiable.

pure strategies exists. Without the tie-breaking rule, we would need to consider mixed strategy equilibria or we could discretisize the set of voters to obtain pure strategy equilibria. Since the qualitative insights of the paper are not affected by more complicated procedures to solve ties of voting outcomes, we work with the easiest solution.

3 Constitution C1: Fixed Democracy

We first examine the first constitution. Due to EPAV, every citizen has the same chance of being recognized as an agenda setter. If a citizen, denoted by a, is determined as the agenda setter, two cases can occur: either $c_t^a = C_h$ or $c_t^a = C_l$. In both cases, the agenda setter wants to create the smallest possible majority for an adoption or rejection of the public good combined with the most favorable tax/subsidy package. To simplify the description we denote by the set I_a^Δ a randomly chosen interval of individuals on the unit circle with length Δ and citizen a, i.e. we draw a random variable u, uniformly distributed on $[0, 1]$ and then choose I_j^Δ as:

$$I_a^\Delta(u) = \begin{cases} [u, u+\Delta] \cup \{a\} & \text{if } u+\Delta \leq 1, \\ [u, 1] \cup [0, u+\Delta-1] \cup \{a\} & \text{otherwise} \end{cases} \tag{5}$$

For convenience, we will drop the variable u in the following. I_a^Δ denotes a randomly chosen interval with length Δ and citizen a. We obtain

Proposition 1

(i) Suppose $c_t^a = C_h$ and $\frac{e}{(1+\lambda)} - 2k \geq -C_l$. Then, a citizen a proposes $g_t = 1$ and the tax/subsidy scheme

$$\begin{aligned} \tau_t^j &= -e \quad \text{for } j \notin I_a^{\frac{1}{2}} \\ \tau_t^j &= \frac{e}{1+\lambda} - 2k \quad \text{for } j \in I_a^{\frac{1}{2}} \end{aligned}$$

(ii) Suppose $c_t^a = C_h$ and $\frac{e}{(1+\lambda)} - 2k < -C_l$. Then, a citizen a proposes $g_t = 0$ and the tax/subsidy scheme

$$\begin{aligned} \tau_t^j &= -e \quad \text{for } j \notin I_a^{\frac{1}{2}} \\ \tau_t^j &= \frac{e}{1+\lambda} \quad \text{for } j \in I_a^{\frac{1}{2}} \end{aligned}$$

(iii) Suppose $c_t^a = C_l$. Then citizen a proposes $g_t = 0$ and the tax/subsidy scheme

$$\begin{aligned} \tau_t^j &= -e \quad \text{for } j \notin I_a^{\frac{1}{2}} \\ \tau_t^j &= \frac{e}{1+\lambda} \quad \text{for } j \in I_a^{\frac{1}{2}} \end{aligned}$$

Proof :

Point (i) follows since a project winner can subsidize project losers by taxing half of the population so that subsidized losers support the adoption of the public good. For the second point (ii), note that a project winner does not observe whether other individuals benefit or loose from the public good. If he cannot form a majority in favor of the public good by taxing half of the population, he is better off by redistributing money to the smallest possible majority which includes himself. The same considerations apply for point (iii) when the agenda setter is a loser.

■

Proposition 1 exhibits three sources of inefficiencies generated by constitution $C1$ [see also Aghion and Bolton 1998]. First, losers are always fully taxed to pay for the project and/or for redistribution. Second, a winner may not propose the project, because he cannot compensate losers sufficiently for them to adopt $g_t = 1$. Third, losers will never propose the public good. They only propose redistribution.

From proposition 1 we can calculate the expected utility of each citizen. Let us suppose that $\frac{e}{(1+\lambda)} - 2k \geq -C_l$. With probability $\frac{1}{2}$, each citizen will belong to the majority in each period. With probability p, a citizen is a beneficiary of the public good. Hence, the expected utility under $C1$ for any individual, denoted by U_{C1}, is given by:

$$U_{C1} = \sum_{t=0}^{\infty} \delta^t \left\{ \frac{1}{2} p \left\{ C + e + \frac{e}{1+\lambda} - 2k \right\} + \frac{1}{2}(1-p)\left\{ e + \frac{e}{1+\lambda} \right\} + \frac{1}{2} p \cdot C \right\} \tag{6}$$

Regrouping the terms yields:

$$U_{C1} = \sum_{t=0}^{\infty} \delta^t \left\{ \frac{e(2+\lambda)}{2(1+\lambda)} + p\big(C-k\big) \right\} = \frac{e(2+\lambda)}{2(1+\lambda)(1-\delta)} + \frac{p(C-k)}{1-\delta} \tag{7}$$

If, however, $\frac{e}{1+\lambda} - 2k < -C_l$, the public good will never be proposed. The remaining redistribution activities yield the expected utility:

$$U_{C1} = \sum_{t=0}^{\infty} \delta^t \left\{ \frac{e(2+\lambda)}{2(1+\lambda)} \right\} = \frac{e(2+\lambda)}{2(1+\lambda)(1-\delta)} \tag{8}$$

Note that for $\lambda \equiv 0$, the expected utility in this case simply amounts to:

$$U_{C1} = \frac{e}{1-\delta} \tag{9}$$

If, however, λ is very large, we obtain

$$\lim_{\lambda \to \infty} U_{C1} = \frac{e}{2(1-\delta)}$$

since individuals expect $\frac{e}{2}$ of the consumption good in every period. Everything else is lost in redistribution activities.

4 Constitution C2: Flexible Democracy

Under the constitution $C2$, a citizen a who is recognized as an agenda setter in period t proposes a participation rule V_{t+1} determining future participation. To determine V_{t+1}, we consider the decision problem when a set of voters can decide the provision of the public good in period $t+1$ and the future participation rule. Without loss of generality we can assume that V_{t+1} is a subinterval of the unit circle and given by $[a_{t+1}, b_{t+1}]$ with interval length $\Delta_{t+1} := b_{t+1} - a_{t+1}$. Obviously, $\Delta_0 = 1$.

We can simplify the determination of the participation rule. We first observe that because an individual has no influence on the voting outcome since a single vote measures zero, a nash approach would allow that any participation equal to or smaller than the winning majority in period t could occur as nash equilibrium in voting strategies of citizens. Hence, we use a standard refinement and allow a coalition of at most 50% voters to jointly deviate in their voting decision from a potential equilibrium configuration. With this refinement, we obtain:

Proposition 2

The agenda setter a in period t proposes either $V_{t+1} = \{a\}$ or V_{t+1} equal to the winning majority in period t.

Proof :

First, an agenda setter will never propose a larger participation rule V_{t+1} than the winning majority in period t. Although the likelihood that the agenda setter j belongs to the majority in future periods is not affected by the participation rule, the potential benefits from redistribution are higher for j the smaller the future majorities needed to adopt a proposal are. Second, because of ETW, the agenda setter in period t must offer the same net transfers for all members of the majority he creates. Hence, if the majority of 50% he wants to create is willing to accept the proposal without being included in future participation, the agenda setter j will propose $V_{t+1} = \{j\}$. If the majority the agenda setter wants to create is not willing to support the proposal without being in the future participation set, the agenda setter must include all members of the winning majority in V_{t+1} because of ETW. Otherwise, his proposal will not get the support of a majority.

$\blacksquare$

In the following, we assume that the second case holds, i.e. V_{t+1} equals the winning majority in period t. In section 6 we discuss under which conditions this is true.

In the case where V_{t+1} equals the winning majority in period t, we denote the measure of V_{t+1} by $|V_{t+1}|$ and obtain:

Proposition 3

Suppose that V_{t+1} equals the winning majority in period t. The measure of V_{t+1} is given by:

$$|V_{t+1}| = \frac{\Delta_t}{2} = \frac{1}{2^{t+1}} \text{ for } t = 0, 1, 2, \ldots$$

From the above proposition it is obvious that the agenda setter under the constitution $C2$ will act similarly as in the constitution $C1$ with respect to public good and to financing decisions, but now restricted by the participation rules he is facing. In period t, we obtain:

Proposition 4

(i) *Suppose $c_t^a = C_h$ and $\frac{e(2-\Delta_t)}{\Delta_t(1+\lambda)} - \frac{2k}{\Delta_t} \geq -C_l$. Then a citizen a proposes a participation rule, $g_t = 1$ and the tax/subsidy scheme*

$$\tau_t^j = -e \quad \text{for } j \notin I_a^{\frac{\Delta_t}{2}}$$
$$\tau_t^j = \frac{e(2-\Delta_t)}{\Delta_t(1+\lambda)} - \frac{2k}{\Delta_t} \quad \text{for } j \in I_a^{\frac{\Delta_t}{2}}$$

(ii) *Suppose $c_t^a = C_h$ and $\frac{e(1-\Delta_t)}{\Delta_t(1+\lambda)} - \frac{2k}{\Delta_1} < -C_l$. Then citizen a proposes a participation rule, $g_t = 0$ and the tax/subsidy scheme*

$$\tau_t^j = -e \quad \text{for } j \notin I_a^{\frac{\Delta_t}{2}}$$
$$\tau_t^j = \frac{e(2-\Delta_t)}{\Delta_t(1+\lambda)} \quad \text{for } j \in I_a^{\frac{\Delta_t}{2}}$$

(iii) *Suppose $c_t^a = C_l$. Then citizen a proposes a participation rule, $g_t = 0$ and the tax/subsidy scheme*

$$\tau_t^j = -e \quad \text{for } j \notin I_a^{\frac{\Delta_t}{2}}$$
$$\tau_t^j = \frac{e(2-\Delta_t)}{\Delta_t(1+\lambda)} \quad \text{for } j \in I_a^{\frac{\Delta_t}{2}}$$

The proof of proposition 4 follows the same observations as for proposition 1. Note that the subsidy in the first case is determined as:

$$\tau_t^j = \frac{\frac{e(1-\frac{\Delta_t}{2})}{1+\lambda} - k}{\frac{\Delta_t}{2}} = \frac{e(2-\Delta_t)}{\Delta_t(1+\lambda)} - \frac{2k}{\Delta_t} \tag{10}$$

In the case where the public good is never proposed under the constitution $C1$, the flexible democracy has the following advantage.

Proposition 5

Suppose $\frac{e}{1+\lambda} - 2k < -C_l$. *Then there exists a critical time period* $t^* > 0$ *such that under* $C2$

(i) The public good is never proposed and adopted for $t < t^*$.

(ii) The public good is proposed and adopted with probability p *for* $t \geq t^*$.

Proof :

The critical condition from proposition 4 that a project winner in period t proposes $g_t = 1$ can be rewritten as:

$$\frac{e(2-\Delta_t) - 2k(1+\lambda)}{\Delta_t(1+\lambda)} \geq -C_l \tag{11}$$

The left side is monotonically decreasing in Δ_t. Because of our financing feasibility condition $e > k(1+\lambda)$ and because of $\lim_{t\to\infty} \Delta_t = 0$, there exists a time period t^0 such that $e(2-\Delta_t) - 2k(1+\lambda)$ becomes positive for the first time. Hence, as time goes to infinity, the left side is positive and is unbounded. The mean value theorem establishes the existence and uniqueness of t^*.

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The proof of proposition 5 also provides a closed form solution for t^*. Using proposition 3, rewriting (11) and treating it as an equation yields

$$(e - k(1+\lambda)) \cdot 2^{t+1} \geq -C_l(1+\lambda) + e$$

Hence, t^* is the smallest integer, denoted by $[t]$, which satisfies

$$[t] \geq \ln_2 \frac{-C_l(1+\lambda) + e}{2(e-k)(1+\lambda)}$$

If we consider the case where $\frac{e}{1+\lambda} - 2k \geq -C_l$, we can calculate the expected utility under the constitution $C2$, denoted by U_{C2}.

$$U_{C2} = \sum_{t=0}^{\infty} \delta^t \Big[pC + \frac{\Delta_t}{2}\Big\{e + \frac{e(1-\frac{\Delta_t}{2})}{(1+\lambda)\frac{\Delta_t}{2}}\Big\} - p\frac{\Delta_t}{2}\frac{2k}{\Delta_t}\Big]$$

This expression can be simplified as:

$$\begin{aligned} U_{C2} &= \sum_{t=0}^{\infty} \delta^t \left[p(C-k) + \frac{e(2+\lambda\Delta_t)}{2(1+\lambda)}\right] \\ &= \frac{e}{2(1+\lambda)}\left(\frac{2}{1-\delta} + \frac{\lambda}{1-\frac{\delta}{2}}\right) + \frac{p(C-k)}{1-\delta} \end{aligned} \tag{12}$$

5 Fixed versus Flexible Democracy

In this section, we compare the relative efficiency of a fixed with the flexible democracy. By combining proposition (1) equations (7) and (12), we obtain:

Proposition 6

Suppose that $\frac{e}{1+\lambda} - 2k \geq -C_l$. *Then, the constitution* $C1$ *will be unanimously preferred over* $C2$.

The intuition for proposition (6) follows from the increasing redistribution volumes under $C2$ over time. All individuals have the same chance to be part of a small group that can determine the provision of the public good and can fully tax the remaining population. The losses from distortionary taxation increase, which from an ax ante point of view harm all individuals. Hence, it is in the interest of everybody to restrict future participation to one person one vote.

The situation is more complicated when the assumption in proposition (6) does not hold. Then, we obtain:

Proposition 7

Suppose that $\frac{e}{1+\lambda} - 2k < -C_l$. *Let* t^* *be the critical time period such that the public good has a chance to be adopted under* $C2$ *for periods* $t \geq t^*$. *Then* $C1$ *dominates* $C2$ *if and only if*

$$\delta^{t^*} < \frac{\lambda e}{2p(C-k)(1+\lambda)} \left(\frac{\delta}{2-\delta} \right) \tag{13}$$

The proof is given in the appendix. The efficiency condition embodied by equation 13 has intuitive interpretations. For instance, if C_h increases for given C_l and given t^*, the condition can be fulfilled less easily, since the prospect to implement public goods under $C2$ becomes more attractive. The influence of the discount rate is more ambiguous: On the one side, a higher δ increases the relative attractiveness of $C2$, because individuals value future project benefits more. On the other hand, the relatively larger future taxation losses under $C2$ also receive a higher weight in the expected utility. As condition 13 shows, the direction of the sum of these effects depends on parameter values. For a sufficiently small λ we obtain as a corollary:

Corollary 1

Suppose that $\frac{e}{1+\lambda} - 2k < C_l$. *Then, the flexible democracy* $C2$ *dominates the fixed democracy* $C1$ *for sufficiently small* λ.

Hence, if distortionary tax losses are sufficiently small, the advantage of a flexible democracy to implement socially beneficial public goods makes such a constitution more efficient.

6 Flexible Democracy and Dictatorship

In this section, we complete our discussion of the constitution $C2$ and consider the case of a flexible democracy ending immediately in dictatorship. For that purpose we consider the conditions under which $|V_{t+1}| = \frac{|V_t|}{2} = \frac{1}{2^{t+1}}$ with V_0 equal to the unit circle is a coalition proof subgame perfect equilibrium and when it is not.

We determine the conditions under which a coalition without the agenda setter credibly rejects a proposal $V_t = \{j\}$.[7] We only consider the case $t^* = 0$, i.e. an agenda setter who is a project winner, will propose the public good in all periods because compensating losers is always feasible. The case $t^* > 0$ is slightly more complicated, but the reasoning is similar. We obtain:

Proposition 8
$|V_{t+1}| = \frac{|V_t|}{2} = \frac{1}{2^{t+1}}$, $t = 0, 1, 2, \dots$, *with* V_0 *equal the unit circle is a subgame perfect coalition proof equilibrium if*

(i)

$$e(2 - 3\delta) + \delta(1 + \lambda)pk < 0$$

(ii)

$$e(3 - 2\delta) - \frac{\delta(1 - \delta)\lambda e}{2 - \delta} \leq \delta p(1 + \lambda)(C - k)$$

The preceding proposition shows that dictatorship does not occur if the expected benefits C from the project and endowments are sufficiently high. In these circumstances, individuals are better off by rejecting dictatorship proposals and by waiting for the next period. The same tends to happen for very high discount factors since future periods are sufficiently valuable for waiting to be profitable. For very low discount factors only dictatorship occurs since the majority, voting for the dictatorship, is only interested in the current benefits. In general, however, the relationship between the discount factor and non-dictatorship is nonlinear.

7 Note that $V_t = \{j\}$ is always a subgame perfect nash equilibrium since an individual vote has no influence on the voting outcome in a period. Similarly, any other participation set can be sustained as a subgame perfect nash equilibrium. Hence, we are using coalition proof equilibria as the appropriate refinement.

7 Conclusion

We have made a simple argument in favor of a fixed democracy. The present model may be useful exploring further issues in the design of constitutions. By introducing more heterogeneity with respect to benefits and numbers of project winners of public goods, one could explore rules explaining how different individuals should be treated in the legislative periods of taxes, subsidies and agenda setting. Moreover, we may be able to determine in which cases it is useful to delegate decision-making to a small subset of people as in a parliament of a representative democracy for a certain period of time.

8 Appendix

Proof of proposition 7:

Under $C1$, individuals expect

$$U_{C1} = \frac{e(2+\lambda)}{2(1+\lambda)(1-\delta)}$$

Under $C2$, the expected utility in periods $t \geq t^*$ are given by (see equation (12))

$$u_t^j = p(C-k) + \frac{e(2+\lambda\Delta_t)}{2(1+\lambda)}$$

For periods $t < t^*$, we obtain

$$u_t^j = \frac{e(2+\lambda\Delta_t)}{2(1+\lambda)}$$

Hence,

$$U_{C2} = \sum_{t=0}^{\infty} \delta^t \, \frac{e(2+\lambda\Delta_t)}{2(1+\lambda)} + \sum_{t=t^*}^{\infty} \delta^t \, p\,(C-k)$$

Using $\Delta_t = \frac{1}{2^t}$, this expression can be simplified to:

$$U_{C2} = \frac{e}{2(1+\lambda)}\left(\frac{2}{1-\delta} + \frac{2\lambda}{2-\delta}\right) + p\,(C-k)\frac{\delta^{t^*}}{1-\delta}$$

The constitution $C1$ is socially better than $C2$ if and only if $U_{C1} > U_{C2}$, and therefore

$$\frac{e}{2(1+\lambda)}\left(\frac{\lambda}{1-\delta} - \frac{2\lambda}{2-\delta}\right) - \, p\,(C-k)\frac{\delta^{t^*}}{1-\delta} > 0$$

After rearranging terms, we obtain the condition of the proposition.

■

Proof of proposition 8:

Suppose that in some period t_0 with $|V_{t_0}| = \Delta_{t_0} = \frac{1}{2^{t_0}}$, the agenda setter proposes $V_{t_0+1} = \{j\}$ together with the best possible public goods provision and financing package for $\frac{\Delta_{t_0}}{2}$ individuals. If the potentially winning majority $\frac{\Delta_{t_0}}{2}$ accepts this proposal they receive a one–time utility in this period and nothing in the future. Therefore in this case, the utility for the winning majority, denoted by U^{t_0} (yes), is given by:

$$U^{t_0}(\text{yes}) = \frac{e(1-\frac{\Delta_{t_0}}{2})}{(1+\lambda)\frac{\Delta_{t_0}}{2}} = \frac{e(2^{t_0+1}-1)}{1+\lambda}$$

To test the subgame perfect equilibrium, we can invoke the one–time deviation principle. Hence, if the potentially winning majority $\frac{\Delta_{t_0}}{2}$ rejects the proposal, they count with the equilibrium continuation in future periods. Hence, the utility for any individual except the agenda setter from a one–time deviation in period t_0, denoted by U^{t_0} (no), is

$$U^{t_0}(\text{no}) = e + \sum_{t=t_0+1}^{\infty} \delta^{t-t_0}\left[p\bar{C} + \frac{\Delta_t^*}{2\Delta_{t_0}}\left\{e + \frac{e(1-\frac{\Delta_t^*}{2})}{(1+\lambda)\frac{\Delta_t^*}{2}}\right\} - p\frac{\Delta_t^*}{2\Delta_{t_0}}\frac{2k}{\Delta_t^*}\right]$$

Since the proposal has been rejected and thus participation has not been changed in period t_0 we have $\Delta_t^* = \frac{1}{2^{t-1}}$. Therefore,

$$U^{t_0}(\text{no}) = e + \frac{\delta}{1-\delta}\left[p(C - k2^{t_0}) + 2^{t_0}\cdot\frac{e}{1+\lambda}\right] + \frac{\delta\lambda e}{(2-\delta)(1+\lambda)}$$

Comparing $U^{t_0}(\text{no})$ with $U^{t_0}(\text{yes})$ yields $U^{t_0}(\text{no}) > U^{t_0}(\text{yes})$ if and only if

$$2^{t_0}\left(\frac{e}{1+\lambda}(2-\frac{\delta}{1-\delta}) + \frac{\delta pk}{1-\delta}\right) \leq \frac{\delta}{1-\delta}pC - \frac{e}{1+\lambda} + \frac{\delta\lambda e}{(2-\delta)(1+\lambda)}$$

This inequality can only be satisfied for any value of t_0 if

$$\frac{e}{1+\lambda}\left(\frac{2-3\delta}{1-\delta}\right) + \frac{\delta pk}{1-\delta} < 0$$

This expression is equivalent to the first condition

$$e(2-3\delta) + \delta(1+\lambda)pk < 0$$

If the first condition holds the above condition is most critical for $t_0 = 0$. For $t_0 = 0$ the condition is

$$e(3-2\delta) - \frac{\delta(1-\delta)\lambda e}{2-\delta} \leq \delta p(1+\lambda)(C-k)$$

which is equivalent to the second condition.

■

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