Sustainable Development with Stock Pollution

Werner Hediger,
Institute of Agricultural Economics, ETH Zurich,
8092 Zurich, Switzerland
whediger@ethz.ch

Abstract:
The optimal control of pollution is an important challenge for sustainable development policy. On one hand, the restoration of a heavily polluted environment will require initial investment, but in general entails in a development path that is both environmentally and economically sustainable. On the other hand, the optimal trajectories in situations with an initial stock of pollution below the long-term optimum generally imply an increase in pollution and a decline of optimal consumption. Hence, the investment of the environmental rents accruing from nature’s assimilative capacity into man-made capital is required in the sense of the famous Hartwick rule to maintain a constant flow of instantaneous welfare. This would facilitate growth in consumption sufficient to compensate for the rising disutility of pollution.

Keywords: Optimal control, pollution accumulation, sustainability.

JEL classification: C61, Q52, Q56, Q58.
1 Introduction

Sustainable development is a concept of concern about the well-being of both present and future generations. It involves trade-offs between present and future use of environmental and natural resources. To address this intergenerational equity concern, various concepts of sustainability have been proposed since the publication of the “Brundtland Commission Report” (WCED, 1987). These include economic approaches that are grounded on earlier works of Solow (1974), Hartwick (1977) and Daly (1972, 1974), and the integration of ecological concepts of resilience (Common and Perrings, 1992) and critical natural capital (Pearce et al., 1994; Hediger, 1999).

The dominant concern in most economic approaches to sustainable development is about the exhaustibility of natural resources and the maintenance of some suitably defined stocks of capital. However, most models of sustainability do not explicitly consider the accumulation of pollutants in the environment, or, at least, not with the same rigor as in the environmental economics literature on optimal pollution control. The latter evolved since the publications of Keeler et al. (1972) and Plourde (1972). It covers various economic aspects of pollutant accumulation and control, such as the implication of different forms of the assimilation function (Forster, 1975; Elliott and Yarrow, 1977; Hediger, 1991a,b; Cesar and de Zeeuw, 1992; Tahvonen and Salo, 1995; Pezzey, 1996; Chevé, 2000; Toman and Withagen, 2000), economic growth and the investment in clean technologies (Asako, 1980; Luptáčik and Schubert, 1982; Van der Ploeg and Withagen, 1991; Tahvonen and Kuuluvainaen, 1993; Withagen, 1995; Chevé, 2000; Toman and Withagen, 2000; Fisher et al., 2004), and global warming (Nordhaus, 1982; Cesar and de Zeeuw, 1992; Tahvonen, 1995). However, the relationship between trajectories of optimal pollution control and sustainable development has hardly been investigated in these analyses.

Barbier and Markandya (1990) identify the minimum initial level of environmental quality required to “ensure a sustainable growth path”, which is solely characterized by an increase of the total stock of environmental assets. However, they do neither investigate the effects of the optimal trajectory upon the development of aggregate income and social welfare, nor are they explicit with regard to the accumulation and assimilation of pollution. In contrast, Chevé

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1 Investigating the role of explicit and implicit assumptions in different models of weak and strong sustainability, Hediger (2006) shows that, in order to comprehensively address the challenge of sustainable development, these models must be extended and explicitly address the accumulation and decay of pollutants.
(2000) elucidates the importance of a decline in the natural pollution decay function upon optimal pollution control and economic growth. However, her argumentation is restricted to the case of strong sustainability with a non-decreasing stock of natural capital that is simply represented by the stock of pollution. Hence, her conclusion that “a better environmental quality can be associated with higher economic growth” is only valid for the case of an initial stock of pollution above the long-term optimum. This can be referred to as problem of environmental restoration, rather than to the general problem optimal pollution control which calls for an extension of the analysis into the broader framework of weak sustainability (cf. Hediger, 2006).

Building on this background, the aim of the present article is to provide an economic analysis of optimal pollution control and sustainable development, and to build a bridge between the two bodies of literature. To this end, we start with the rather general model of Forster (1975) and gradually extend this analytical framework to gain additional insights about the relationship between optimal pollution control and the sustainability of development. The basic model is introduced in Section 2, while Section 3 provides a first discussion of the optimal pollution control trajectories from both an efficiency and sustainability point of view. It shows that the existence of constant productive capacity is not in general sufficient for sustainable development. Rather, additional investments might be required to maintain a non-declining flow of consumption. Therefore, the model is extended in Section 4 to jointly addressing optimal pollution control and capital accumulation. The extended model is also used in Section 5 to investigate the option of a Hartwick-type investment rule as a means to facilitate a constant flow of welfare along an intertemporally optimal trajectory of pollution accumulation. This is also briefly reflected from a perspective of sustainable preferences, as defined by Chichilnisky (1996, 1997), and for the case of environmental restoration. Finally, Section 6 concludes with policy recommendations.

2 The basic model

The accumulation of pollutants is a major cause of environmental degradation and adverse impacts upon human health and welfare. Examples include soil and water contamination with toxic substances or sulfur and nitrogen compounds, the eutrophication of surface waters, as well as increased concentrations of greenhouse gases and ozone depleting substances in the atmosphere. For these substances, damages to ecosystems and human interests depend on the stock of pollution, which consequently has an adverse impact on social welfare. This is
usually represented in the form of a concave utility function $U$ of instantaneous consumption $C$ and the current stock of pollution $S$, which has the following properties:

$$U(C, S)$$

$U_C > 0, \quad U_{CC} < 0, \quad U_S < 0, \quad U_{SS} < 0, \quad U_{CS} = U_{SC} = 0$ \hspace{1cm} (1)

Following Forster (1975, 1977), the last two conditions are used for reasons of tractability and to rule out the unrealistic boundary solutions of zero consumption and zero pollution. Moreover, in line with Forster’s model, we assume for the moment that the economy has a fixed productive capacity and that it produces a constant level of output $Y_0$. This is allocated to consumption $C$ and antipollution activities $Z$: 2

$$\bar{Y}_0 = C + Z$$ \hspace{1cm} (2)

For the case of constant population and technology, the sustainable use of a renewable resource (e.g., solar energy, wood, or manure) as an input to production, and in the absence of capital depreciation, equation (2) implies economic sustainability in the sense of Solow (1986). Regarding the challenge of intergenerational equity and the use of exhaustible resources, he emphasized that if one generation owes anything to its successors, it owes generalized productive capacity.

Assuming a constant and free flow of renewable resources, the dynamic problem in our analysis primarily comes from the intertemporal accumulation of pollution:

$$\dot{S} = E(C) - G(Z) - A(S)$$ \hspace{1cm} (3)

This is the excess flow of net emissions $E(C) - G(Z)$, with $E'>0, E^* > 0, G'>0$ and $G^* < 0$, above the natural capacity of pollutant assimilation, the so-called assimilative capacity $A(S)$. According to Pearce (1976), the latter refers to the environment’s capability of receiving waste, degrading it, converting it to nutrients for the occupants of the ecosystem, or to render it harmless to species even if the converted product is not required as an input to the ecosystem. Dependent on the type of pollutant this essential natural function can take different forms, such as illustrated in Figure 1. 3

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2 This assumption will be relaxed in a further step of the analysis.

Sustainable Development with Stock Pollution

Figure 1. Different forms of the assimilation function

- Case i) is the most often used functional form in the optimal pollution control literature. It is characterized by a constant proportionate rate of decay \( \alpha \), which would typically be the case for the decay of radioactive substances.

- Case ii) is rarely used in the environmental economics literature. It describes the process of saturation that is typical for chemical processes of self-purification (cf. Fiedler, 1994). It could also be an important representation of the global CO\(_2\) assimilative capacity, since the uptake of CO\(_2\) declines with the atmospheric concentration (Watson et al., 2000) and the fraction of a unit emission of CO\(_2\) remaining in the atmosphere is higher the higher the accumulated emissions are (Azar, 1995).

- Case iii) is typical for biological processes of self-purification (cf. Fiedler, 1994, 1997) that are limited by some degrader populations (e.g., microorganisms) and general environmental conditions, such as the oxygen and other nutrients available to support the degraders. This functional form has first been used in the environmental economics literature by Forster (1975), and since then gained increasing attention by other economists who are concerned about the influence of ecological processes upon the economy and their representation in economic analysis.

In this study, we concentrate on the third case which is the most interesting from an analytical and the most relevant from an ecological point of view. It combines the favorable case where the ecosystem’s assimilative capacity increases with the stock of pollution for low levels, with the consequences of saturation. The latter results in a decline of the assimilative capacity for further pollution accumulation beyond a certain turning point, say \( S_m \), and the complete
degradation of this ecosystem function at the pollution level \( S \), beyond which the assimilative capacity is zero.\(^4\) Formally, this can be represented as follows:

\[
A(S) > 0 \quad \text{for} \quad 0 < S < S_m \\
A'(S) = 0 \quad \text{for} \quad S = S_m \\
A''(S) < 0 \quad \text{for} \quad S_m < S < \overline{S} \\
A(S) = 0 \quad \text{for} \quad S = 0 \quad \text{and} \quad S \geq \overline{S}
\]

(4)

As indicated above, if the net flow of emissions from the economy exceeds this capacity, the stock of pollution accumulates in the environment and has a negative impact upon the society’s well-being. Hence, we face an intertemporal allocation problem. This is usually formulated in terms of maximizing the present value of instantaneous utility by choosing the optimal level of consumption

\[
\max_{\{C\}} \int_0^\infty e^{-\rho t} U(C, S) dt
\]

(5)

subject to \( \dot{S} = f(C) - A(S) \quad , \quad S(0) = S_0 \)

where \( \rho > 0 \) denotes the constant social discount rate and \( f(C) \) the economy’s net emissions:

\[
f(C) = E(C) - G(\overline{Y}_0 - C) \quad \text{with} \quad f' = E' + G' > 0, \quad f'' = E'' + G'' > 0
\]

(6)

Using Pontryagin’s maximum principle, this intertemporal optimization problem can be represented by in the current-value Hamiltonian

\[
H = U(C, S) + \mu[f(C) - A(S)]
\]

(7)

which is to be maximized at any point in time. The following differential equations define the optimal time path of the state and costate variables, \( S \) and \( \mu \), of this allocation problem:

\[
\dot{S} = f(C) - A(S)
\]

(8)

\[
\dot{\mu} = \mu[\rho + A'] - U_S
\]

(9)

\(^4\) In principle, this results in an irreversible pollution accumulation. However, if it is technically feasible to reduce the accumulated stock of pollution in the environment, then even the situation with a fully destroyed assimilative capacity could be reversible, but imply additional cost of “environmental restoration”. In the present article, we do not foreclose this technical option by assumption, which must be taken into consideration when discussing the results for high levels of pollution with \( A(S) = 0 \). Thus, we relax the more restrictive assumption of Chevé (2000) which implies that, as a necessary condition, the initial stock of pollution must not be above the “irreversible level” \( \overline{S} \).
Along this trajectory the allocation of the existing output to consumption and pollution abatement must continuously be adjusted, such as to satisfy the capacity constraint (2) and the subsequent first-order condition at any point in time:

$$U_c + \mu f' = 0$$

(10)

Using this optimality condition, it can be shown that the shadow price of pollution is strictly negative, which reflects that pollution generates disutility:

$$\mu = -\frac{U_c}{f'} < 0$$

(11)

Moreover, it follows from (11) and (2) that

$$\dot{\mu} = \mu \left[ \frac{U_{cc}}{U_c} - \frac{f''}{f'} \right] \dot{C} \quad \text{and} \quad \dot{C} + \dot{Z} = 0$$

(12)

And thus

$$\text{sgn}(\dot{C}) = \text{sgn}(\dot{\mu}) \quad \text{and} \quad \text{sgn}(\dot{Z}) = -\text{sgn}(\dot{C})$$

(13)

Consumption must develop with the same sign as the value of the costate variable $\mu$, while the share of output allocated to pollution abatement must go in the opposite direction.

3 Optimal pollution control with constant productive capacity

The solution of our optimal control program is characterized by a steady state where $\dot{\mu} = \dot{S} = \dot{C} = \dot{Z} = 0$ and a stable transition path from the initial state $S_0$ toward this long-term optimum. Yet, Forster (1975) already pointed out that multiple equilibria can exist for the optimal pollution control problem with a biological assimilation function. One of these long-term optima implies an intact assimilative capacity $A(S^*) > 0$. This is illustrated in Figure 2 by the intersection of the $\dot{\mu} = 0$ and $\dot{S} = 0$ demarcation curves in the point $(S^*, \mu^*)$. Here $S^*$ denotes the optimal stock of pollution in the steady state and $\mu^*$ the corresponding shadow price of pollution. The second long-term optimum is depicted in Figure 2 by the intersection of the two demarcation curves in the point $(S^x, \mu^x)$, where the assimilative capacity is completely destroyed: $A(S^x) = 0$.\(^5\)

\(^5\) This second long-term optimum does not exist for sufficiently low discount rates and high marginal damage cost of pollution.
The $\dot{\mu} = 0$ locus can be determined by setting the right hand side in equation (8) equal to zero. As shown in the Appendix, the resulting function has a pole at the pollution level $S = S_\#$, where the “pollution discount rate” $\rho + A'(S) = 0$. In contrast, the $\dot{S} = 0$ curve is the locus of the shadow price $\mu$ for the stationary optimization problem with $\dot{S} = 0$ (see in the Appendix). It can further be proven that the intersections of the $\dot{\mu} = 0$ and $\dot{S} = 0$ curves are local saddle points (cf. Cesar and de Zeeuw, 1992; Tahvonen and Withagen, 1996; Chevé, 2000). As a consequence, each of these long-term optima can only be achieved by proceeding on one of the optimal trajectories that directly and consistently flow toward the related saddle point. This is illustrated in Figure 2 by the two pairs of stable branches leading to the long-term equilibria $(S^*, \mu^*)$ and $(S^x, \mu^x)$, respectively. All other trajectories turn sooner or later away from the saddle point (Chiang, 1984). In other words, for each initial state $S_0 \neq S_\#$, exactly one shadow price $\mu$ exists on the intertemporally efficient trajectory that maximizes the net present value of instantaneous consumption benefits and pollution damage.

Figure 2. Phase diagram for the optimal pollution control problem with fixed productive capacity
For any current level of pollution $S < S^*$, the stock of pollution must grow along the optimal trajectory. As a consequence, the overall period marginal damage cost of pollution increase and the absolute value of the shadow price $\mu$ becomes larger. Since the later is negative, it follows $\mu < 0$. Given the findings in equation (13), this implies that consumption must decline over time and more resources must be allocated to pollution abatement activities, $\dot{C} < 0$ and $\dot{Z} > 0$, in order to slow down the further accumulation of pollution and stop this process in the long run at the optimal level $S^*$. Altogether, this implies a decline of social welfare $U(C,S)$ along the optimal time path, since according to equation (1)

$$
\dot{U} = U_C \dot{C} + U_S \dot{S} = \begin{cases} 
< 0 & \text{if } \dot{S} > 0 \text{ and } \dot{C} < 0 \\
= 0 & \text{if } \dot{S} > 0 \text{ and } \dot{C} < 0 \\
> 0 & \text{if } \dot{S} > 0 \text{ and } \dot{C} < 0 
\end{cases}
$$

(14)
given $U_C > 0$ and $U_S < 0$.

The same logic with the opposite signs is valid for the upper branch of the stable trajectory leading toward $S^*$; that is, for pollution levels within the interval $(S^*, S)$. In this case, the stock of pollution declines and consumption can grow along the optimal time path. As a consequence, social welfare increases over time. In contrast, all variables would remain constant in the steady-state which is characterized by $S^*$ in Table 1.

**Table 1.** Dynamic properties on the optimal pollution control trajectories

<table>
<thead>
<tr>
<th></th>
<th>$A(S) &gt; 0$</th>
<th>$A(S) = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 &lt; S &lt; S^*$</td>
<td>$\dot{S} &gt; 0$</td>
<td>$\dot{S} &gt; 0$</td>
</tr>
<tr>
<td></td>
<td>$\dot{S} = 0$</td>
<td>$\dot{S} = 0$</td>
</tr>
<tr>
<td></td>
<td>$\dot{S} &lt; 0$</td>
<td>$\dot{S} &lt; 0$</td>
</tr>
<tr>
<td>$S = S^*$</td>
<td>$\dot{S} = 0$</td>
<td>$\dot{S} = 0$</td>
</tr>
<tr>
<td></td>
<td>$\dot{S} &gt; 0$</td>
<td>$\dot{S} &gt; 0$</td>
</tr>
<tr>
<td>$S^* &lt; S &lt; S$</td>
<td>$\dot{S} &lt; 0$</td>
<td>$\dot{S} &lt; 0$</td>
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<tr>
<td></td>
<td>$\dot{S} &gt; 0$</td>
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<td></td>
<td>$\dot{S} = 0$</td>
<td>$\dot{S} = 0$</td>
</tr>
<tr>
<td>$S &lt; S^*$</td>
<td>$\dot{S} &lt; 0$</td>
<td>$\dot{S} &lt; 0$</td>
</tr>
<tr>
<td></td>
<td>$\dot{S} &gt; 0$</td>
<td>$\dot{S} &gt; 0$</td>
</tr>
</tbody>
</table>

|                | $\dot{\mu} < 0$         | $\dot{\mu} < 0$         |
|                | $\dot{\mu} = 0$         | $\dot{\mu} = 0$         |
|                | $\dot{\mu} > 0$         | $\dot{\mu} > 0$         |
| $\dot{C} < 0$  | $\dot{C} = 0$            | $\dot{C} = 0$            |
|                | $\dot{C} > 0$            | $\dot{C} > 0$            |
| $\dot{Z} > 0$  | $\dot{Z} = 0$            | $\dot{Z} = 0$            |
|                | $\dot{Z} < 0$            | $\dot{Z} < 0$            |
| $\dot{U} < 0$  | $\dot{U} = 0$            | $\dot{U} = 0$            |
|                | $\dot{U} > 0$            | $\dot{U} > 0$            |
|                | not sustainable           | sustainable              |
| $S > S^*$      | $\dot{S} > 0$            | $\dot{S} > 0$            |
|                | $\dot{S} = 0$            | $\dot{S} = 0$            |
|                | $\dot{S} < 0$            | $\dot{S} < 0$            |
| $S = S^*$      | $\dot{S} > 0$            | $\dot{S} > 0$            |
|                | $\dot{S} = 0$            | $\dot{S} = 0$            |
|                | $\dot{S} < 0$            | $\dot{S} < 0$            |
| $S < S^*$      | $\dot{S} = 0$            | $\dot{S} = 0$            |
|                | $\dot{S} > 0$            | $\dot{S} > 0$            |
| $S^* < S$      | $\dot{S} < 0$            | $\dot{S} < 0$            |
|                | $\dot{S} > 0$            | $\dot{S} > 0$            |
|                | $\dot{S} = 0$            | $\dot{S} = 0$            |
| $S < S^*$      | $\dot{S} < 0$            | $\dot{S} < 0$            |
|                | $\dot{S} > 0$            | $\dot{S} > 0$            |
| $S^* > S$      | $\dot{S} > 0$            | $\dot{S} > 0$            |
|                | $\dot{S} = 0$            | $\dot{S} = 0$            |
|                | $\dot{S} < 0$            | $\dot{S} < 0$            |
| $S > S^*$      | $\dot{S} > 0$            | $\dot{S} > 0$            |
|                | $\dot{S} = 0$            | $\dot{S} = 0$            |
|                | $\dot{S} < 0$            | $\dot{S} < 0$            |
For the two branches of the intertemporal trajectory leading toward $S^e$, we get exactly the same results (see Table 1), presuming that the steady state in $(S^e, \mu^e)$ exists. This requires that the social discount rate is not too small, and that the marginal damage cost of pollution is not too high.

Table 1 shows that the development on lower branches of the two stable trajectories in Figure 2 is not sustainable at all, neither from an environmental nor economic point of view, since both pollution increases and consumption declines over time. In contrast, development is sustainable along the optimal trajectory for an initial stock of pollution $S > S^*$ and a working assimilative capacity $A(S) > 0$. In this case, development is environmentally sustainable since the stock of pollution declines along the optimal trajectory and economically sustainable because consumption grows in line with this process. The same conclusion can be drawn for initial levels of pollution $S > S^e$, if the steady state at $S^e$ exists and if the existence of a totally destroyed assimilative capacity, $A(S) = 0$, is accepted for sustainability from a societal and ethical point of view.

For problems with a strictly increasing assimilation function, such as represented by cases i) and ii) in Figure 1, only one saddle point exists (cf. Cesar and de Zeeuw, 1992). Consequently, the above analysis reduces to the two branches of the optimal trajectory leading to the long-term optimum with $S^*$. For initial levels of pollution below this steady-state level ($S < S^*$), the development along the optimal trajectory is not sustainable. In contrast, for pollution levels above the long-term optimum ($S > S^*$), an adjustment process toward this steady state is required which is both environmentally and economically sustainable.

4 Optimal pollution control with capital accumulation

The analysis in the previous section reveals that we must distinguish two different types of the pollution control problem: (a) one with further optimal pollutant accumulation, and (b) one of environmental restoration with a net reduction of pollution. Moreover, it shows that the existence of a constant productive capacity is not in general sufficient for sustainable development. Rather, an important case exists where intertemporal efficiency requires the sacrifice of consumption and the allocation of an increasing amount of output to pollution abatement, while the stock of pollution further increases. In this case, investments are required to further increase the economy’s production capacity and to compensate for
environmental damage costs. Therefore, we relax the assumption of a fixed production capacity and replace equation (2) by

\[ Y = Y(K, \bar{L}) = C + Z + I \quad \text{with} \quad Y_K > 0, Y_{KK} < 0 \]  

(15)

The total output \( Y \) is the product of man-made capital \( K \) and a constant input of labor \( L \). It is allocated to consumption \( C \), pollution abatement \( Z \) and investment \( I \) in the stock of man-made capital \( K \). The latter consists of both physical assets and knowledge, and depreciates at a constant proportionate rate \( \delta \):

\[ \dot{K} = I - \delta K \]  

(16)

This results in an expanded optimization problem with two state variables that is represented by the following current-value Hamiltonian with the additional costate variable \( \phi \) representing the shadow price of capital:

\[ H^* = U(C, S) + \mu[E(C) - G(Z) - A(S)] + \phi[Y(K, \bar{L}) - C - Z - \delta K] \]  

(17)

Using the first-order optimality conditions, we get the subsequent set of equations that determine the long-term welfare maximizing trajectory for given initial stocks of capital and pollution, \( K_0 \) and \( S_0 \), respectively:

\[ U_C + \mu E' = \phi = -\mu G' > 0 \]  

(18)

\[ \dot{\mu} = \mu[\rho + A'] - U_s \quad , \quad \dot{S} = E(C) - G(Z) - A(S) \]  

(19)

\[ \phi = \phi[\rho + \delta - Y_K] \quad , \quad \dot{K} = Y(K, \bar{L}) - C - Z - \delta K \]  

(20)

Using equations (18) and (1), we further get

\[ \dot{C} = \frac{\phi - \mu E'}{U_{CC} + \mu E''} \begin{cases} > 0 & \text{if} \quad \phi \begin{cases} < 0 \end{cases} \mu E' \\ < 0 \end{cases} \]  

(21)

Thus, whether it would be optimal for consumption to grow depends on the relationship between \( \phi \) and \( \dot{\mu} \) times the marginal rate of emission \( E'(C) \). The first term is a function of the capital stock \( K \) and of the discount and depreciation rates \( \rho \) and \( \delta \).
\[
\phi = \phi [\rho + \delta - Y_k] \begin{cases} > 0 \quad \text{if } Y_k > \rho + \delta \\ = 0 \quad \text{if } Y_k = \rho + \delta \\ < 0 \quad \text{if } Y_k < \rho + \delta \end{cases}
\] (22)

It is positive, if the marginal productivity of capital \( Y_k \) is below the long-term optimum \( \rho + \delta \), which implies that the capital stock and thus output must shrink along the optimal trajectory \((K_0 > K^* \text{ and } \dot{K} < 0)\). In contrast, \( \phi \) is negative and the capital stock must grow if \( Y_k \) is larger than \( \rho + \delta \). In other words, for an initial stock of capital \( K_0 < K^* \) – this is below the steady-state level –, the capital stock must increase \((\dot{K} > 0)\) and the adjoint shadow price must decline \((\phi < 0)\).

As shown in the previous section, the shadow price of pollution increases according to equation (19) with the accumulation of pollution and declines if the stock of pollution decreases along the optimal trajectory. This information is combined with the above in Table 2, which indicates that along the optimal trajectory consumption should grow for low initial levels of capital and high levels of pollution, and it should decline if the initial level of capital is above the long-term optimum but pollution below. The economy is obviously in a steady state if \( \dot{C} = \phi = \dot{\mu} = 0 \), which is the case for \( \dot{K} = \dot{S} = 0 \).

**Table 2.** Optimal development of consumption for different levels of pollution and capital

<table>
<thead>
<tr>
<th>Current stock of …</th>
<th>… capital: ( K_0 &lt; K^* )</th>
<th>( K_0 = K^* )</th>
<th>( K_0 &gt; K^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>… pollution:</td>
<td>( \phi &lt; 0 )</td>
<td>( \phi = 0 )</td>
<td>( \phi &gt; 0 )</td>
</tr>
<tr>
<td>( S_0 &lt; S^* )</td>
<td>( \dot{\mu} &lt; 0, \dot{S} &gt; 0 )</td>
<td>?</td>
<td>( \phi &gt; \mu E' \Rightarrow \dot{\mu} &lt; 0 )</td>
</tr>
<tr>
<td>( S_0 = S^* )</td>
<td>( \dot{\mu} = 0, \dot{S} = 0 )</td>
<td>( \phi &lt; \mu E' \Rightarrow \dot{\mu} &lt; 0 )</td>
<td>( \phi = \mu E' \Rightarrow \dot{\mu} = 0 )</td>
</tr>
<tr>
<td>( S_0 &gt; S^* )</td>
<td>( \dot{\mu} &gt; 0, \dot{S} &lt; 0 )</td>
<td>( \phi &lt; \mu E' \Rightarrow \dot{\mu} &lt; 0 )</td>
<td>( \phi &lt; \mu E' \Rightarrow \dot{\mu} = 0 )</td>
</tr>
</tbody>
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The solution is ambiguous for the two fields in Table 2 that show a question mark. This means that, in contrast to the optimal control problem with a fixed production capacity, consumption growth can be optimal on the lower branch of the stable trajectory leading to \( S^* \) in Figure 2. This is feasible as long as more additional output is generated from the growing
capital stock than the economy must use for extended pollution abatement and for investment. Thus, a further accumulation of pollution can be sustainable

a) in the sense of “very weak sustainability” if consumption grows along the optimal trajectory, that is if \( \dot{C} \geq 0 \); and

b) in the sense of “weak sustainability” if consumption growth is sufficiently large to compensate for the pollution-induced welfare loss, that is if \( \dot{U} = U_c \dot{C} + U_s \dot{S} \geq 0 \).

Similarly, an optimal pollution control trajectory with declining stocks of pollution and capital must not necessarily go along with growth in consumption, as in the previous section where we prescribed a fixed capacity of production.

In short, the development of a growing economy which expands its productive capacity through physical and knowledge capital accumulation can be sustainable from both an economic and social welfare perspective, even in a situation with further pollution accumulation. However, to compensate the growing disutility of pollution, the rate of capital accumulation must be higher if society wants to achieve a welfare improving path of weak sustainability, rather than solely economic sustainability measured in terms of rising consumption. For a wealthy economy with a relatively high stock of man-made capital (physical assets and knowledge) which faces a problem of continued pollution accumulation, such as the enhanced greenhouse effect, the above results indicate that a further accumulation of capital according to the Ramsey rule might not be sufficient for sustainable development. Even higher rates of investment might be required to maintain a constant flow of welfare in this case.

5 Pollution, sustainability and the investment of environmental rents

The investment of resource rents has originally been proposed by Solow (1974) and Hartwick (1977) as an approach of intergenerational equity for an economy that exploits a stock on nonrenewable resources. Yet, our analysis in Section 2 shows that maintenance of an economy’s generalized productive capacity, as advocated by Solow (1986), will not in general be sufficient to maintain a constant level of consumption in the presence of pollution accumulation, to say nothing of non-declining social welfare. However, it is straightforward to think about a Hartwick-type investment rule of earmarking the ecological rents that accrue

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from using nature’s assimilative capacity to the accumulation of man-made capital. For our
intertemporal allocation problem that is represented by the Hamiltonian in equation (17), the
corresponding investment rule is given by
\[ \phi K = -\mu \dot{S} \] (23)
This results in a constant value of total capital. Consequently, the value of the Hamiltonian is
equal to the instantaneous value of social welfare:
\[ H^* = U(C, S) + 0. \]

Using the total derivation of equation (17) with respect to time and the optimality conditions
(18) to (20), it turns out that the investment of the environmental rents according to equation
(23) enables a constant flow of social welfare. It is
\[
\begin{align*}
\dot{H}^* &= U_c \dot{C} + U_s \dot{S} + \mu \dot{S} + \mu [E' \dot{C} - G' \dot{Z} - A' \dot{S}] + \phi \dot{K} + \phi [Y_k \dot{K} - \dot{C} - \dot{Z} - \delta \dot{K}] \\
&= [U_c + \mu E' - \phi] \dot{C} + [U_s + \mu - \mu A'] \dot{S} + [-\mu G' + \phi] \dot{Z} + [\phi + \phi Y_k - \phi \delta] \dot{K} \\
&= \rho \mu \dot{S} + \rho \phi \dot{K}
\end{align*}
\]
and thus
\[ \dot{U} = U_c \dot{C} + U_s \dot{S} = 0. \] (25)
As a consequence, the value of social welfare
\[ \hat{U}_0 = U(C, S) \] (26)
is the same for all points on the trajectory leading to the long-term optimum stock of pollution
\( S^* \). This is the maximum sustainable value of welfare that can be achieved, given the initial
stocks of capital and pollution \( K_0 \) and \( S_0 \).

For the important problem with \( S_0 < S^* \), this implies a lower level of consumption than the
result of the pure pollution control problem would suggest. Consumers are requested to
sacrifice some instantaneous benefits in order to maintain a constant level of welfare over
time. However, they receive a return in the long run in form of higher welfare and
consumption.

The net present value of the maximum constant welfare stream is equal to the traditional net
present value of social welfare without investment of the environmental rents:
\[ V_0 = \int_0^\infty \hat{U}_0 e^{-\rho t} dt = \int_0^\infty U(C, S) e^{-\rho t} dt \] (27)
Moreover, the optimal time path of pollution accumulation and the related shadow price are exactly the same in both cases. The difference between the two cases is only the level of instantaneous consumption and welfare. Hence, the selection of one or the other alternative is a matter of societal preferences for long-term versus short-term benefits.

In analogy to Chichilnisky’s (1996, 1997) definition of sustainable preferences, this selection problem can formally be expressed in present value terms as a weighted sum of the pure net present value of welfare maximization and the largest permanently maintainable and therefore intergenerationally equitable level of social welfare

$$V_0 = \theta \cdot \int_0^\infty U(C, S)e^{-\rho t} dt + (1 - \theta) \cdot \frac{\dot{U}_0}{\rho} \quad \text{with} \quad 0 \leq \theta \leq 1$$

(28)

In the extreme case with $\theta = 1$, priority is given to the immediate but a non-sustainable benefits of present consumption. In the other extreme with $\theta = 0$, a constant and sustainable flow of welfare and thus the long-run is prioritized. This requires the investment of the environmental rents accruing from nature’s pollution assimilation. All other cases ($0 < \theta < 1$) imply a compromise between preference given to the present and the future. However, all cases are equivalent with regard to the intertemporal efficiency, since they all imply the same trajectory of pollution accumulation for a given initial state.

Hence, the optimal trajectory with an initial stock of pollution $S_0 < S^*$ is sustainable in the conventional sense of maintaining the maximum constant flow welfare if the environmental rents are invested into any form of man-made capital that enhances the economy’s productive capacity. It is sustainable in the sense of Chichilnisky if the society finds a balance between the two extremes of pure net present value maximization and the maintenance of a constant welfare level, whereas the decision about the weighting factor $\theta$ is crucial.

For an initial stock of pollution $S_0 > S^*$ but $S_0 < S$, the investment guideline (23) should not be applied as a rule, because the optimal trajectory is generally characterized by an increase in environmental quality ($\dot{S} < 0$) and instantaneous consumption ($\dot{C} > 0$), or at least in social welfare ($\dot{U} > 0$). One exception, however, could be the restoration of water quality in a eutrophicated lake or the restoration of a contaminated soil that require costly investments in the beginning. A Hartwick-type rule could be used in such cases to make the polluters paying a fee equivalent to the environmental rent in order to amortize the related debt that is caused by the initial investment.
5 Conclusion

The control of accumulative pollution in the environment is an important task that must be included in any sustainable development policy. From a welfare economic and efficiency oriented point of view, this will not imply a strong sustainability constraint of a constant or declining stock of pollution, such as propagated by Daly (1991) and investigated by Chevé (2000). Rather, sustainable development calls for avoiding wasteful uses of scarce resource, and thus for an efficient resource allocation that theoretically goes inline with the optimal control of accumulative pollutants. As shown in this article, the latter involves three distinct types of problem that must be differently analyzed with regard to the implementation of environmental policy within a sustainable development framework.

The first one is related to the potential long-term optimum (saddle point) that can exist at a level of accumulated pollution where the ecosystem’s assimilative capacity is completely destroyed. From a purely efficiency-oriented perspective, it can be optimal to let the stock of pollution further increase if the initial level is already in the domain where the assimilative capacity is zero. From a societal perspective, however, the normative question remains whether it can be desirable to let an ecosystem die, and how the environmental quality could be restored with technical measures. This is not addressed in the present paper, and remains subject to further theoretical and applied research.

The second problem area is characterized by a relatively high initial stock of pollution that is above the long-term optimum. In this case – which has also been investigated by Chevé (2000) – optimal resource allocation results in a decline of the accumulated stock of pollution and consumption growth. Apparently, this does not pose a problem for sustainable development since the optimal trajectory satisfies both criteria of economic and environmental sustainability. Hence, a policy of optimal pollution control will in general contribute to sustainable development in situations with relatively high initial pollution and a working assimilation capacity of the environment. But, it will require a relatively high share of capacity allocated to pollution abatement, which can subsequently decline over time and allow for an increase in the rate of consumption.

The third domain of the optimal pollution control problem is the one that might gain the least attention in the public, since it refers to the situation with a relatively low initial stock of pollution below the long-term optimum. But, as shown in this article, it is the most important challenge with regard to sustainable development since the optimal pollution control
trajectory in this domain can involve a further accumulation of pollution and a decline of consumption. This is particularly the case in wealthy economies with a high initial stock of capital. In contrast, poorer economies might have the option of following a trajectory of optimal capital accumulation that generates sufficient additional output to increase antipollution efforts and, at the same time, facilitate growth in consumption. The second case can be sustainable from a weak sustainability perspective of non-declining welfare, whereas the one of the wealthy economy may be environmentally and economically unsustainable. In this case, additional investment in man-made capital will be required to achieve a sustainable development path. To this end, the economy might invest the environmental rents accruing from the use of nature’s assimilative capacity and increase its total productive capacity which is required for growth in consumption and pollution abatement activities. With the application of such a modified Hartwick-type investment rule, the economy could maintain a constant flow of welfare, even in the case of continued pollution accumulation.

The assessment of this rent is a particular research challenge. It requires adequate knowledge of the assimilative capacity and the dynamics of pollution accumulation, and it calls for the solution of the entire optimal control problem in order to provide policy information about the shadow prices of pollution and capital.

7 References


Appendix

As illustrated in Figure 2, the two long-term optima ($S^*, \mu^*$) and ($S', \mu'$) are defined by the intersection of the $\dot{\mu} = 0$ and $\dot{S} = 0$ demarcation curves. The former can be determined by setting $\dot{\mu} = 0$ in equation (8):

$$\frac{d\mu}{dt} = \frac{U}{\rho + A'} \begin{cases} < 0 & \text{for } \rho + A' > 0 \\ \rightarrow \pm \infty & \text{for } \rho + A' \rightarrow 0 \\ > 0 & \text{for } \rho + A' < 0 \end{cases}$$

(A1)

As illustrated in Figure 2, this partial equilibrium locus is zero for $S = 0$ and has a pole at the pollution level $S = S_m$, where the “pollution discount rate” $\rho + A'(S) = 0$. Moreover, it is discontinuous at the level $S = S$ where the assimilative capacity is destroyed:
\[ \mu(S)\big|_{\mu=0} = \frac{U_S(C,S)}{\rho + A'(S)} > 0 \quad \text{and} \quad \mu(S)\big|_{\mu=0} = \frac{U_S(C,S)}{\rho} < 0 \] (A2)

The gradient of the \( \mu = 0 \) curve is generally negative for \( S \leq S^* \):

\[ \frac{\partial \mu}{\partial S}\big|_{\mu=0} = \frac{U_{SS}(\rho + A') - U_S A'^*}{(\rho + A')^2} \begin{cases} < 0 & \text{for } \rho + A' > A^* U_S / U_{SS} \\ = 0 & \text{for } \rho + A' = A^* U_S / U_{SS} < 0 \\ > 0 & \text{for } \rho + A' < A^* U_S / U_{SS} \end{cases} \] (A3)

and strictly negative for \( S \geq S^* \):

\[ \frac{\partial \mu}{\partial S}\big|_{\mu=0} = \frac{U_{SS}}{\rho} < 0 \] (A4)

The \( \dot{S} = 0 \) demarcation curve is determined by including the partial stationary state condition \( f(C) = A(S) \) in equation (10). If the function \( f \) is invertible, it follows \( C = f^{-1}(A(S)) \) and thus

\[ \mu\big|_{\dot{S}=0} = -\frac{U_C(f^{-1}(A(S)),S)}{f'(f^{-1}(A(S)))} < 0 \] (A5)

From this, we get

\[ \frac{\partial \mu}{\partial S}\big|_{\dot{S}=0} = -\frac{U_{CC}(f')^{-1} A' f'' - U_C f f''(f')^{-1} A'}{(f')^2} \]

\[ = -\frac{A'}{f'} \frac{U_{CC} f' - U_C f''}{(f')^2} \begin{cases} > 0 & \text{for } 0 < S < S_m \\ = 0 & \text{for } S = S_m \text{ and } S \geq S \\ < 0 & \text{for } S_m < S \leq S \end{cases} \] (A6)

The shape of the \( \dot{S} = 0 \) curve is characterized by the \( A(S) \) curve. Starting in the point

\[ \mu^s = -\frac{U_C(f^{-1}(0),0)}{f'(f^{-1}(0))} \] (A7)

the conditional shadow price \( \mu\big|_{\dot{S}=0} \) increases for low levels of pollution, reaches the maximum at \( S = S_m \), declines afterwards, and remains constant for very high levels of pollution where the assimilative capacity is destroyed.