Abstract

Since 1980, the aggregate income of oil-exporting countries relative to that of oil-poor countries has been remarkably constant, despite structural gaps in labor productivity growth rates. We rationalize this behavior in a two-country model of asymmetric trade where resource-poor and resource-rich economies – Home and Foreign, respectively – display productivity differences in R&D, but stable income shares due to terms-of-trade dynamics. Home’s income share is positively (negatively) related to the domestic (Foreign) investment rate and to the national tax on domestic resource use. This prediction is confirmed by dynamic panel estimations for sixteen oil-poor economies. We also show that national governments have incentives to deviate from both efficient and laissez-faire allocations. In Home, increasing the tax on oil use always improves welfare through a rent-transfer mechanism. In Foreign, subsidies (taxes) on domestic oil use improve welfare only if R&D productivity is lower (higher) than in Home.

Keywords: Endogenous Growth, Exhaustible Resources, International Trade.
JEL Classification Numbers: F43, O40

1 Introduction

The functioning of modern economies crucially relies on the primary inputs obtained from exhaustible natural resources. Because of the uneven distribution of endowments of fossil fuels and minerals, many industrialized economies heavily depend on imports from resource-rich countries. The relevance of this form of trade dependence is emphasized by the statistics – fossil fuels and minerals now account for 22.5% of world merchandise trade (World Trade Organization, 2009: p.43) – and is increasingly regarded as a crucial determinant of the growth performance of resource-rich economies (Lederman and Maloney, 2007). Few studies, however, analyze in detail the implications of asymmetric trade between resource-rich and resource-poor countries from the perspective of modern growth theory. In this paper, we exploit the endogenous growth framework to analyze the determinants of national income shares, and the effects of national taxes and subsidies on resource use, in a two-country world.

Our empirical reference is the economic performance of the world’s top net exporters of oil – henceforth labelled as OEX group – relative to that of big oil-poor economies, i.e., the world’s top importers whose domestic oil production is zero or negligible – henceforth labelled as OIM group. Since 1980, the ratio between the aggregate incomes of the two groups has
been constant: as we show in section 2.1, the OIM share over the total income of the two groups is approximately 73% and its time profile over the last three decades is remarkably flat. This result is surprising in view of the productivity differentials observed during the same period: since 1980, labor productivity has stagnated, if not declined, in most OEX countries while it has substantially increased in the majority of OIM economies.

In line with these empirical facts, we build a model of asymmetric trade where productivity growth rates may differ across countries but the world equilibrium exhibits balanced growth. Specifically, we assume that a resource-poor economy (henceforth called Home) imports a final good and an exhaustible primary input from a resource-rich economy (henceforth called Foreign) and only exports its final consumption good. Both countries exhibit endogenous growth driven by R&D, and the productivity of research firms is country-specific. In this framework, several mechanisms may induce balanced growth at the world level. Abstracting from labor mobility and integrated financial markets, we emphasize the role of terms of trade: in the world equilibrium, national income shares are constant because the dynamics of the prices of traded goods compensate for productivity differences in the respective final sectors.

Our first aim is to study the determinants of national income shares and check whether the theoretical predictions are supported by empirical evidence. In the model, equilibrium shares display two characteristics. First, the income share of each country is positively (negatively) related to the domestic (other country’s) saving rate, where the respective saving rates are positively related to the domestic rates of R&D productivity. Second, because of the asymmetric structure of trade, income shares differ from market shares in final production and national policies affect income shares asymmetrically. In particular, an increase in Home’s tax on domestic resource use increases Home’s income share due to a rent-transfer mechanism; an increase in the Foreign resource tax, instead, leaves income shares unchanged.

Our panel estimations for sixteen OIM economies in relation to an aggregate group of ten OEX economies confirm the positive (negative) relation between income shares and domestic (foreign) saving rates, as well as the positive impact of domestic oil taxes on the income share of OIM economies.

Our second aim is to link the model predictions to the policy debate. The interactions between oil-rich and oil-poor economies have a prominent role in international discussions and the recent up-surge in oil prices revived the interest for the analysis of strategic tax policies in this context. One crucial question is: do economies involved in asymmetric trade have peculiar incentives to implement inefficient tax or subsidies on domestic resource use? We show that these incentives exist and are particularly strong for oil importers. If the initial state of affairs is an efficient equilibrium in which all domestic market failures are internalized, Home’s government can increase domestic welfare by raising the national resource tax above the efficient level; the Foreign government, instead, does not have any incentive to deviate from the initially efficient world equilibrium. If the initial state of affairs is a laissez-faire equilibrium, productivity differences come into play: in the empirically plausible case where productivity growth is higher in the resource-poor economy, Home’s incentive to raise the resource tax becomes stronger whereas Foreign has an incentive to subsidize domestic resource use.

As mentioned above, few studies analyze the implications of asymmetric trade between resource-rich and resource-poor countries from the perspective of modern growth theory. The trade-and-growth literature typically neglects asymmetric trade structures induced by uneven endowments of specific primary inputs. The role of terms of trade in our model is conceptually
similar to that emphasized by Acemoglu and Ventura (2002) in a world-economy model where a continuum of countries produces the same final good with heterogeneous technologies, but free trade in intermediate goods induces a stable income distribution. In the early resource economics literature, two-country models assumed that the accumulation of man-made capital inputs was either absent (Kemp and Suzuki, 1975; Brander and Djajic, 1983) or subject to diminishing returns (Chiarella, 1980; Van Geldrop and Withagen, 1993). The parallel literature on endogenous growth with natural resources as inputs, pioneered by Barbier (1999) and Scholz and Ziemes (1999), generally refers to closed or small open economies: to our knowledge, the two-country setting is only considered in two recent papers by Daubanes and Grimaud (2006) and Peretto and Valente (2010) that differ from the present analysis in both aims and means.\(^1\) A common element linking the early contributions of Brander and Djajic (1983) to the present analysis and to Daubanes and Grimaud (2006) is the existence of a rent-transfer mechanism induced by national resource taxes. The argument that oil-importing countries may use domestic taxation to extract part of the rents accruing to oil producers dates back to Bergstrom (1982), and plays an important role in our results regarding the welfare effects of discretionary national policies.

2 Growth, Trade and Resource Dependence

2.1 Empirical Facts

Our theoretical analysis focuses on exhaustible resources and can be applied to several types of minerals and fossil fuels. The main empirical reference, however, is the relative economic performance of oil-rich and oil-poor economies. The first column of Table 1 lists a group of countries, labeled as $OEX$, which comprises the seventeen top oil exporters at the world level over the period 1980-2008.\(^2\) These economies are ranked by average yearly net exports of oil in physical terms, and satisfy two requirements: during the relevant period, each country (i) has never been a net oil importer and (ii) steadily appeared in the top exporters list in each single year.\(^3\) In Table 1, the ratio between oil consumption and production in physical terms highlights different degrees of dependence and/or specialization. Angola, Oman, Norway and Qatar consumed on average less than 10% of total oil production, whereas Canada exceeded 70%. Due to data availability, real output growth rates for $OEX$ countries are calculated for

\(^1\) Daubanes and Grimaud (2006) use a North-South model where growth is exclusively driven by the technology of the oil-poor economy: there are no productivity gaps and terms of trade are excluded by the homogeneity of the final consumption good. They study national policies in the presence of perfectly integrated financial markets and characterize social optimality at the world level under the assumption that oil use generates pollution externalities. Peretto and Valente (2009) assume identical R&D technologies between countries in a non-scale model of endogenous growth featuring both vertical and horizontal innovations. They analyze the effects of resource booms, i.e. unexpected discoveries of new resource stocks on innovation rates and relative welfare.

\(^2\) We consider seventeen countries and draw the line below Kazakhstan because the subsequent positions are occupied by countries exporting much less oil in absolute terms. Over the 1980-1992 period, the Russian Federation would be replaced by USSR, and the 17th top exporter would be Egypt, whose average yearly net exports have been nearly one half of the preceding country, Canada. Over the 1992-2008 period, the 18th top exporter is Colombia, with similar figures in proportion to Kazakhstan. Indonesia and United Kingdom are excluded from the computations since they both turned from net exporters in the 1980s to net importers nowadays.

\(^3\) The position of USSR between 1980-1990, not displayed in Table 1, has been taken over by the new countries emerged from the disaggregation – especially by the Russian Federation and Kazakhstan.
two subsets: the first, labeled as $OEX_{15}$, excludes Iraq and Lybia and covers the 1992-2006 period; the second, labeled as $OEX_{13}$, also excludes Russia and Kazakhstan and covers the 1980-2006 period.\(^4\) In the most recent period, Middle-East countries are those growing faster.

<table>
<thead>
<tr>
<th>$OEX$ Countries</th>
<th>Oil Net Exports*</th>
<th>Oil Cons./Prod.**</th>
<th>Real GDP Growth***</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saudi Arabia</td>
<td>7042</td>
<td>8022</td>
<td>16.9%</td>
</tr>
<tr>
<td>Russian Federation</td>
<td>-</td>
<td>4751</td>
<td>37.9%</td>
</tr>
<tr>
<td>Norway</td>
<td>1957</td>
<td>2752</td>
<td>7.2%</td>
</tr>
<tr>
<td>Iran</td>
<td>2089</td>
<td>2483</td>
<td>34.9%</td>
</tr>
<tr>
<td>Venezuela</td>
<td>2103</td>
<td>2432</td>
<td>17.8%</td>
</tr>
<tr>
<td>United Arab Emirates</td>
<td>1926</td>
<td>2232</td>
<td>13.6%</td>
</tr>
<tr>
<td>Kuwait</td>
<td>1632</td>
<td>1964</td>
<td>11.2%</td>
</tr>
<tr>
<td>Nigeria</td>
<td>1666</td>
<td>1901</td>
<td>12.8%</td>
</tr>
<tr>
<td>Mexico</td>
<td>1420</td>
<td>1482</td>
<td>57.0%</td>
</tr>
<tr>
<td>Algeria</td>
<td>1244</td>
<td>1422</td>
<td>13.6%</td>
</tr>
<tr>
<td>Libya</td>
<td>1233</td>
<td>1319</td>
<td>14.0%</td>
</tr>
<tr>
<td>Iraq</td>
<td>1236</td>
<td>1124</td>
<td>30.0%</td>
</tr>
<tr>
<td>Angola</td>
<td>639</td>
<td>899</td>
<td>4.1%</td>
</tr>
<tr>
<td>Oman</td>
<td>653</td>
<td>778</td>
<td>6.8%</td>
</tr>
<tr>
<td>Canada</td>
<td>536</td>
<td>752</td>
<td>73.1%</td>
</tr>
<tr>
<td>Qatar</td>
<td>592</td>
<td>751</td>
<td>7.5%</td>
</tr>
<tr>
<td>Kazakhstan</td>
<td>-</td>
<td>594</td>
<td>29.1%</td>
</tr>
</tbody>
</table>

Table 1. Selected top oil exporters. * Average yearly net exports of oil, thousands barrels per day; ** Average yearly oil consumption-to-production ratio; *** Average yearly growth rate of real GDP; Sources - * and ** EIA (2009); *** World Bank (2009) and IMF (2009) for Angola and Qatar.

We contrast the $OEX$ country sample by seventeen economies that (i) appear in the list of top oil-importers and (ii) rely on imported oil for domestic use. Specifically, since the list of top importers includes top producers like the US and Brazil, we have excluded all countries producing more than 10% of the oil they consume domestically.\(^5\) The resulting list of seventeen oil-poor, oil-importing countries is labelled as $OIM$ and is reported in Table 2, ranked by average yearly net imports of oil in physical terms (1992-2008). The third column shows the ratio between net imports and domestic oil consumption, which does not fall short of 90% (Netherlands, with 89%, is the only limit case and we have decided to include it). Data on real GDP growth are available for the whole set of countries, which we label as $OIM_{-17}$, over the 1990-2006 period. The subset $OIM_{-15}$, which excludes Germany and Poland, covers the whole 1980-2006 period. Not surprisingly, the fastest-growing economies in the sample are the so-called Asian Tigers, i.e., South Korea, Singapore and Taiwan.

\(^4\)Average yearly growth rates are calculated as $(y_{t+\Delta}/y_t)^{1/\Delta} - 1$, where $y_t$ is current price gross domestic product in year $t$.

\(^5\)Excluded oil-importing countries (and their yearly average Net Import/Consumption ratio over 1992-2008) are: United States (53%), China (31%), India (62%), Thailand (74%), Brazil (26%), Ukraine (77%), South Africa (57%), Pakistan (81%), Australia (25%).
<table>
<thead>
<tr>
<th>OIM Countries</th>
<th>Oil Net Imports*</th>
<th>Oil Net Imp./Prod.**</th>
<th>Real GDP Growth***</th>
</tr>
</thead>
<tbody>
<tr>
<td>Japan</td>
<td>5076</td>
<td>3113</td>
<td>98.0%</td>
</tr>
<tr>
<td>Germany</td>
<td>-</td>
<td>1542</td>
<td>94.9%</td>
</tr>
<tr>
<td>South Korea</td>
<td>1495</td>
<td>1202</td>
<td>99.8%</td>
</tr>
<tr>
<td>France</td>
<td>1838</td>
<td>1105</td>
<td>95.4%</td>
</tr>
<tr>
<td>Italy</td>
<td>1714</td>
<td>999</td>
<td>92.5%</td>
</tr>
<tr>
<td>Spain</td>
<td>1178</td>
<td>797</td>
<td>97.9%</td>
</tr>
<tr>
<td>Taiwan</td>
<td>660</td>
<td>486</td>
<td>99.8%</td>
</tr>
<tr>
<td>Netherlands</td>
<td>702</td>
<td>452</td>
<td>89.0%</td>
</tr>
<tr>
<td>Singapore</td>
<td>499</td>
<td>387</td>
<td>99.1%</td>
</tr>
<tr>
<td>Belgium</td>
<td>534</td>
<td>350</td>
<td>98.2%</td>
</tr>
<tr>
<td>Turkey</td>
<td>474</td>
<td>335</td>
<td>91.0%</td>
</tr>
<tr>
<td>Poland</td>
<td>359</td>
<td>227</td>
<td>94.0%</td>
</tr>
<tr>
<td>Greece</td>
<td>326</td>
<td>225</td>
<td>97.7%</td>
</tr>
<tr>
<td>Sweden</td>
<td>376</td>
<td>218</td>
<td>99.4%</td>
</tr>
<tr>
<td>Philippines</td>
<td>269</td>
<td>189</td>
<td>96.9%</td>
</tr>
<tr>
<td>Portugal</td>
<td>261</td>
<td>179</td>
<td>98.9%</td>
</tr>
<tr>
<td>Switzerland</td>
<td>267</td>
<td>158</td>
<td>99.2%</td>
</tr>
</tbody>
</table>

Table 2. Selected oil-poor net importers. * Average yearly net imports of oil, thousands barrels per day; ** Average yearly oil imports-to-consumption ratio; *** Average yearly growth rate of real GDP. Sources - * and ** EIA (2009); *** World Bank (2009) and IMF (2009) for Taiwan.

The first empirical fact concerning the two country groups refers to the behavior of gross domestic product (GDP) and gross national income (GNI) calculated in purchasing power parity (PPP). Computing the income shares of each group over the total income of both groups in each year, the resulting time paths of income shares are remarkably flat. This result is robust to alternative PPP-adjusted measures of GNI and GDP, both in constant and in current prices. Figure 1, upper graphs, shows that the GDP share of OEX-13 versus OIM-15 countries in 2006 are almost identical to the respective values observed in 1980, and do not exhibit notable variations over time. The same result is obtained by comparing GDP shares of OEX-15 versus OIM-17 countries between 1990 and 2006, as well as in nominal GNI shares despite slight changes in the countries' sample due to data availability - see Figure 1, lower graphs.

The second empirical fact is related to labor productivity growth. The upper graphs in Figure 2 show the time paths of labor productivity levels for OEX and OIM countries, reported by the International Labor Organization (2009) and based on the estimates of the Total Economy Database of the Conference Board. Notice that labor productivity levels are not directly comparable for a significant fraction of OEX economies: due to the lack of basic data, labor productivity for African and Middle-East countries is calculated as ratio

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5We are here referring to the "LP person GK" time series reported in the Total Economy Database. The underlying calculations of the Conference Board define labor productivity as the ratio between PPP-adjusted (Geary-Kamis) real GDP in 1990 international dollars and number of persons engaged in work.
between real GDP and labor force instead of employed persons, which results in highly overestimated productivity levels. We circumvent this problem by normalizing labor productivity in 1980 equal to unity for each country and by focusing on labor productivity growth. The resulting time paths are reported in the lower-left graph of Figure 2, and suggest a substantial productivity growth differential in favor of OIM economies. Apart from Canada and Norway, oil exporters have been falling behind oil-importing countries in terms of aggregate labor productivity.

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Figure 1: Upper graphs: current price (dotted line) and constant price (bold line) GDP (adjusted for PPP) shares of oil-exporters and oil-importers. Source: authors calculations on World Bank data (IMF data for Angola 1980-1984 and Taiwan 1980-2006). Lower graphs: current price GNI shares (adjusted for PPP) of oil-exporters (excluding Angola) and oil-importers (excluding Taiwan). Source: authors calculations on World Bank data.

If employment rates are assumed to be substantially stable over time in the Middle-East and the African countries of the OEX sample, the time paths of normalized labor productivity reported in Figure 2 (lower graphs) coincide with those that would be obtained from ‘exact’ data on labor productivity levels (i.e., levels of the ratio between real GDP and employed persons, instead of labor force).
The existence of substantial gaps in productivity growth rates suggests that stationary income shares may originate, at least in part, in the compensating effect of the prices of traded goods. In the next subsections, we present a general equilibrium model of endogenous growth in which income shares are constant because structural gaps in productivity growth are compensated by terms of trade dynamics.

2.2 The Model: General Assumptions

The world comprises two countries - or economic areas - called Home \((h)\) and Foreign \((f)\) and indexed by \(i = f, h\). Each economy produces a final good that the residents of both countries consume in the same proportions due to identical preferences. In both economies, producing the final good requires man-made intermediate inputs and a natural resource extracted from an exhaustible stock, e.g., oil, which is entirely owned by Foreign. As a consequence, the structure of international trade is asymmetric: Home only exports its final good whereas Foreign exports its final good and the natural resource. The engine of growth is represented by R&D activity that expands the number of varieties of man-made intermediate products – e.g., as in Rivera-Batiz and Romer (1991). Intermediates’ producers earn monopoly rents.
and the productivity of R&D firms is enhanced by knowledge spillovers from past research whereby scale effects are eliminated.

Since the competitive equilibrium is inefficient under laissez-faire, we assume that national governments have access to three fiscal instruments that allow them to correct for domestic market failures: a subsidy to R&D investment, a tax on final producers, and a tax on domestic resource use. Our primary interest, however, is not the characterization of efficiency-oriented policies but rather the role of national resource taxes as potential strategic instruments. In reality, the governments of several resource-poor countries impose taxes on imported resources in order to capture part of the rents accruing to the primary sectors of resource-rich economies. We address this issue by studying in detail the general equilibrium effects of resource taxes and the welfare-tax relationship for each country.

2.3 Final Producers, Intermediate Sectors and R&D

Final Producers. In economy \(i\), final output consists of \(Y_i\) units of a country-specific consumption good. Assuming constant returns to scale, the final sector behaves like a single competitive firm producing good \(i\) by means of \(M_i\) varieties of differentiated man-made intermediate products (henceforth called intermediates), labor, and a natural resource extracted from an exhaustible resource stock (henceforth called resource). The production function is

\[
Y_i = \int_0^{M_i} (X_i(m_i))^\alpha dm_i \cdot (v_i L_i)^\beta R_i^\gamma,
\]

where \(t \in [0, \infty)\) is the time index, \(X_i(m_i)\) is the quantity of the \(m_i\)-th variety, \(m_i \in [0, M_i]\), of intermediate input employed in production, \(v_i L_i\) represents labor in efficiency units with \(v_i\) denoting the productive efficiency of each worker and \(L_i\) the number of workers, and \(R_i\) is the flow of natural resource used in the final sector of country \(i\). Elasticity parameters satisfy \(0 < \alpha, \beta, \gamma < 1\) and \(\alpha + \beta + \gamma = 1\). Since the engine of growth is represented by increases in the number of intermediate products \(M_i\), we assume for simplicity that workers efficiency \(v_i\) grows at the exogenous constant rate \(\eta_i > 0\) and that labor is supplied inelastically: \(L_h\) and \(L_f\) are constant over time and coincide with population size in Home and Foreign, respectively. The law of one price holds for all traded goods: the quantities \((Y_h, Y_f)\) are sold at the respective world prices \((P_h, P_f)\) and the exhaustible resource is sold to all final producers at the same world price \(P_R\). Since labor and intermediates are not traded internationally, the nominal wage and the price of each \(m_i\)-th intermediate, respectively denoted \(P^i_L\) and \(P^i_{X(m_i)}\), are country-specific. Production costs are affected by fiscal policy: we denote by \(b_i\) the ad valorem tax on the purchases of intermediates and by \(\tau_i\) the ad valorem tax on domestic resource use. Both \(b_i\) and \(\tau_i\) are assumed to be constant in order to preserve the balanced-growth properties of the world equilibrium.\(^8\) The profit-maximizing conditions in the final sector read

\[
P^i_L L_i = \beta P^i_Y Y_i, \quad (2)
\]

\[
P_R R_i (1 + \tau_i) = \gamma P^i_Y Y_i, \quad (3)
\]

\[
P^i_{X(m_i)} (1 + b_i) = \alpha P^i_Y (X_i(m_i))^{\alpha-1} (v_i L_i)^\beta R_i^\gamma, \quad (4)
\]

\(^8\)This assumption does not affect the generality of our results. As shown in section 4, both efficient allocations and laissez-faire equilibria exhibit balanced growth in each instant. Decentralizing efficient allocations thus requires implementing constant taxes.
where (4) is valid for each $m_i \in [0, M_i]$.

**Intermediate sector.** Each variety is produced by a monopolist who holds the relevant patent and maximizes instantaneous profits taking the demand schedule (4) as given. Production requires $\zeta (X_i (m_i))$ units of final good, where $\zeta (\cdot)$ is the cost function, and profits read $\Pi_i (m_i) = P^i_{X(m_i)} X_i (m_i) - P^i Y \zeta (X_i (m_i))$. Assuming a linear cost function $\zeta (X_i (m_i)) = \zeta X_i (m_i)$ with $\zeta > 0$, profit maximization requires
\[
P^i_{X(m_i)} = (1/\alpha) \zeta P^i_Y
\]
for each $m_i \in [0, M_i]$, and therefore symmetric quantities and profits across varieties. This implies that output is proportional to the number of varieties, according to the reduced Cobb-Douglas form (see Appendix)
\[
Y_i = \left( \frac{\alpha^2 / \zeta}{1 + b_i} \right) \cdot M_i (v_i L_i)^{\frac{\beta}{1-\alpha}} (R_i)^{\frac{\gamma}{1-\alpha}}
\]
where $1 - \alpha = \beta + \gamma$ guarantees decreasing marginal returns to labor and to the natural resource.

**R&D Sector.** The number of intermediates’ varieties $M_i$ grows over time by virtue of R&D activity pursued by competitive firms that develop new blueprints and sell the relevant patents to incumbent monopolists. Symmetry across intermediates allows us to represent R&D firms as a consolidated sector earning zero profits due to free-entry. Developing blueprints requires investing units of the domestic final good purchased by the R&D sector. Assuming a linear technology, each unit purchased has a constant marginal productivity – taken as given by R&D firms – equal to $\phi_i > 0$ in country $i$. For each unit invested the R&D firm receives from the national government a subsidy at constant rate $a_i > 0$. Denoting by $Z_i$ the total amount invested by R&D firms, total investment in country $i$ is $Z_i (1 + a_i)$ and the increase in the number of varieties equals
\[
\dot{M}_i = \phi_i \cdot Z_i (1 + a_i) .
\]
Denoting by $V_i$ the value of each patent sold in country $i$, the zero-profit condition is\(^9\)
\[
V_i = P^i_X (1 + a_i)
\]
The value of each patent sold to an incumbent producer equals the present value of future monopoly profits, which implies the standard no-arbitrage condition
\[
r_i (t) = \frac{\Pi_i (t)}{V_i (t)} + \frac{\dot{V}_i (t)}{V_i (t)}
\]
where $r_i (t)$ is the equilibrium rate of return to private investment in country $i$. We assume that the productivity of the R&D sector is affected by externalities whereby the current marginal productivity of investment, $\phi_i$, is positively influenced by past research effort. These

\(^9\)Aggregate profits of the R&D sector equal $V_i \dot{M}_i - P^i_Y Z_i = V_i \phi_i Z_i (1 + a_i) - P^i_Y Z_i$, so that condition (8) maximizes R&D profits for a given marginal productivity $\phi_i$ and implies zero profits for each firm. The same condition is equivalently obtained assuming free entry in the R&D business for an indefinite number of firms. Indeed, condition (8) says that the value of each innovation must equal the private net cost paid by R&D firms.
externalities take the form of knowledge spillovers, exactly as in models à la Lucas (1988) where the diffusion of public knowledge implies an un-compensated transmission of human capital across generations. In the present context, the productivity of each R&D firm is higher the better the "current state of technology attained by virtue of previous research". This concept of state-of-the-art in research is conveniently measured by the ratio between the number of existing varieties and current output levels, \( M_i / Y_i \). We thus posit a linear function

\[
\phi_i (t) \equiv \varphi_i \cdot (M_i (t) / Y_i (t))
\]

(10)

where \( \varphi_i > 0 \) is a constant proportionality factor determining the social productivity of R&D. Equation (10) implies that the growth rate of intermediates' varieties increases linearly with the economy-wide rate of R&D investment: from (7), we have

\[
\frac{\dot{M}_i}{M_i} = \varphi_i \frac{Z_i (1 + a_i)}{Y_i}.
\]

(11)

As mentioned in Barro and Sala-i-Martin (2004: p.300-302), linear laws like (11) generally exhibit two desirable properties. First, they eliminate scale effects by making the equilibrium growth rate of output independent of the size of endowments. Second, they are empirically plausible since, in most industrialized countries, the growth rate of productivity appears to be positively related to the ratio between R&D expenditures and output, with a proportionality coefficient that is relatively stable over time.

2.4 Resource Extraction in Foreign

The total flow of resource extracted in Foreign equals the sum of resource use in both countries and is denoted by \( R(t) \equiv R_h(t) + R_f(t) \) in each instant. The resource stock \( Q(t) \) is non-renewable and is given at \( t = 0 \). Extracting firms are competitive and take the world price of the resource \( P_R \) as given. For simplicity, extraction costs are zero. The owners of extracting firms are households in Foreign, each of whom earns the same fraction \( 1/L_f \) of rents. Normalizing the mass of firms to unity, the representative firm maximizes the present-value stream of profits discounted at the domestic equilibrium interest rate

\[
\int_0^\infty P_R(t) R(t) e^{-\int_t^\infty r_f(v) dv} dt,
\]

subject to the dynamic resource constraint \( \dot{Q}(t) = -R(t) \), with \( Q(0) = Q_0 > 0 \). This solution to this dynamic problem is characterized by the optimality conditions

\[
\dot{P}_R(t) = P_R(t) r_f(t),
\]

(13)

\[
Q_0 = \int_0^\infty R(t) dt.
\]

(14)

Equation (13) is the standard Hotelling rule asserting that, along an optimal extraction path, the resource rent must grow over time at a rate equal to the rate of return to alternative investments in the economy. Equation (14) is the intertemporal resource constraint requiring asymptotic exhaustion of the resource stock: leaving unexploited resources in the ground is not optimal. Note that we would obtain identical results if we assumed that the resource stock is incorporated in private wealth: in this case, each household is endowed with a fraction \( 1/L_f \) of the initial stock and directly extracts the resource in accordance with Hotelling's rule (13).
2.5 Governments, Households and Trade Balance

Governments. The public sector in country $i = h, f$ finances public R&D expenditures by means of the ad valorem taxes on intermediates' purchases and resource use. Ruling out public debt, balanced budget is achieved in each instant by compensating possible imbalances with a lump-sum transfer $F_i$ imposed on each household:

$$a_i P_i^h Z_i = F_i L_i + b_i M_i P_i^f X_i + \tau_i P_R R_i,$$

where $F_i$ is a lump sum tax if positive, a subsidy if negative.

Households. Each economy $i$ is populated by $L_i$ homogeneous households. Following a standard procedure, the consumer problem is solved in two steps. First, each consumer decides how to allocate expenditures between imported and domestically-produced final goods. Second, households decide the intertemporal profile of consumption expenditures by maximizing present-value utility subject to the dynamic wealth constraint. Denoting by $c^j_i$ the quantity of the good produced in country $j$ and individually consumed in country $i$, the instantaneous utility function of each resident in country $i$ reads

$$u_i(c^h_i, c^f_i) = \ln \left( \frac{E^h_i}{E^f_i} \right),$$

where the weighting parameters $\epsilon$ and $1 - \epsilon$ indicate the preference taste for Home and Foreign goods, respectively. The first-step problem consists of choosing consumed quantities in each instant by maximizing (16) subject to the individual expenditure constraint

$$E^c_i / L_i = P_i^h c^h_i + P_i^f c^f_i,$$

where $E^c_i$ is aggregate consumption expenditure in country $i$. The solution to this problem yields the indirect utility function

$$u_i = \ln \left[ \frac{E^c_i}{L_i} \right]$$

The second-step problem then consists of choosing the path of consumption expenditures that maximizes the present discounted value of the stream of utilities enjoyed over time,

$$U_i \equiv \int_0^\infty e^{-\rho t} \ln \left[ \frac{\omega(t) \cdot (E^c_i(t) / L_i)}{\omega} \right] dt,$$

where $\rho > 0$ is the pure time-preference rate and the path of $\omega(t)$ is taken as given by the household. Since the two economies are populated by identical households, individual private wealth consists of a fraction $(1/L_i)$ of the total value of the assets in the economy $(V_M)$ representing ownership of firms. Denoting the value of individual assets by $n_i \equiv (V_M) / L_i$, the dynamic constraints read

$$\dot{n}_h = r_h n_h + P_i^h E^h_i / L_h - F_h,$$

$$\dot{n}_f = r_f n_f + P_i^f E^f_i / L_f - P_R (R / L_f) - F_f,$$

where $r_i n_i + P_L$ is income from assets and labor, and $P_R (R / L_f)$ is resource income for each Foreign resident. Each consumer maximizes (18) subject to the relevant dynamic constraint using the path of $E^c_i / L_i$ as control variable. As shown in the Appendix, the resulting optimality conditions imply

$$\dot{E}^c_i(t) / E^c_i(t) = r_i(t) - \rho.$$
which is the standard Keynes-Ramsey rule asserting that the growth rate of consumption expenditures equals the difference between the rate of return to assets and the time-preference rate.

**Trade.** Ruling out international trade in assets, we have balanced trade in each instant

\[
P_R R_h + P_Y^f L_h c_h^f = P_Y^h L_f c_f^h. \tag{22}
\]

Condition (22) asserts that the value of total exports of the Foreign country, resources plus exported consumption goods, equals the value of consumption goods imported from the Home country. This asymmetric structure of international trade is relevant for the results of the model: country \( f \), the resource-rich economy, exhibits a structural deficit in the exchange of final goods and this implies that the allocation of domestic output between consumption and its competing uses generally induces feedback effects on the trade equilibrium.

**Aggregate Constraints.** To simplify the notation, we denote aggregate R&D expenditures of country \( i \) as \( E_i^d \equiv P_i^j Z_i (1 + a_i) \), aggregate spending in intermediates’ production as \( E_i^p \equiv P_i^j \times M_i X_i \). Adding consumption expenditures, the total expenditure index of country \( i \) is defined as \( E_i \equiv E_i^c + E_i^d + E_i^f \). Aggregating the individual wealth constraints across households and substituting the profit-maximizing conditions of the various sectors, we obtain (see Appendix)

\[
E_h \equiv E_h^c + E_h^d = P_Y^h Y_h - P_R R_h, \tag{23}
\]

\[
E_f \equiv E_f^c + E_f^d + E_f^f = P_Y^f Y_f + P_R R_h.
\]

These aggregate constraints imply equality between domestic expenditures and aggregate incomes of residents. From (23), total incomes in Home equal the value of final output less the value of resource rents accruing to Foreign. In (24), resource rents are added to the value of final production in Foreign to obtain total Foreign incomes. In the analysis, we will denote the aggregate income share of Home as

\[
s_h(t) = \frac{E_h(t)}{E_h(t) + E_f(t)}
\]

and, accordingly, the income share of Foreign as \( s_f(t) = 1 - s_h(t) \).

### 2.6 Competitive World Equilibrium

The competitive world equilibrium can be characterized as follows. First note that, in each country, the equilibrium rates of return read (see Appendix)

\[
r_i = \frac{\dot{P}_i}{P_Y} + \left[ \frac{\alpha (1 - \alpha) (1 + a_i)}{1 + b_i} \dot{\varphi}_i + \frac{1}{1 - \alpha} \eta_i + \frac{\gamma}{1 - \alpha} \dot{R}_i \right] = \frac{\dot{P}_i}{P_Y} + \Omega_i, \tag{25}
\]

where \( \Omega_i \) equals the central term in square brackets and is a measure of physical productivity growth in the final sector of country \( i \). Productivity growth incorporates the effect of three country-specific characteristics: the social productivity of R&D (which is crucially determined by \( \varphi_i \)), the growth rate of labor efficiency (\( \eta_i \)), and the rate of increase in domestic resource
use \( \dot{R}_i/R_i \). From (25), we can decompose the interest rate differential between Home and Foreign into a price component and a productivity term:

\[
\dot{r}_h - \dot{r}_f = \left( \frac{\dot{P}_h}{P_h} - \frac{\dot{P}_f}{P_f} \right) + \Omega_h - \Omega_f. \tag{26}
\]

The price component in round brackets is a standard terms-of-trade effect whereby price dynamics interact with possible gaps in productivity growth – henceforth structural gaps – represented by differences between \( \Omega_h \) and \( \Omega_f \). In this regard, the current assumptions on technology and preferences imply that terms of trade effects exactly compensate for structural gaps in each point in time, which yields the equalization of equilibrium interest rates:

**Proposition 1** *In the competitive equilibrium, interest rates are equalized, \( r_h(t) = r_f(t) \) in each \( t \in [0, \infty) \), and*

\[
\frac{\dot{P}_h}{P_h} - \frac{\dot{P}_f}{P_f} = \Omega_f - \Omega_h. \tag{27}
\]

Interest rate equalization implies that consumption expenditures grow at the same rate in the two countries. We now show that this in turn implies balanced growth at the world level in each instant, and therefore constant income shares.

The key variable determining national income shares is the ratio between resource-use flows, which we denote as \( \theta(t) \equiv R_h(t)/R_f(t) \) and henceforth call relative resource use. Substituting the respective country indices in (3) we obtain the basic relation

\[
\theta(t) = \frac{R_h(t)}{R_f(t)} = \frac{\tilde{\gamma}_h}{\tilde{\gamma}_f} \cdot \frac{P_h(t)Y_h(t)}{P_f(t)Y_f(t)} \quad \text{in each} \quad t \in [0, \infty), \tag{28}
\]

where \( \tilde{\gamma}_i \equiv \gamma (1 + \tau_i)^{-1} \) is the tax-adjusted resource elasticity in final production. Result (28) follows from no-arbitrage over the resource price at the international level, and shows that relative resource use equals the ratio between the values of final output weighted by the respective tax-adjusted resource elasticities. Combining (28) with the balanced trade condition, we obtain a complete characterization of the competitive world equilibrium:

**Proposition 2** *The world competitive equilibrium exhibits a constant relative resource use \( \theta(t) = \theta \) in each \( t \in [0, \infty) \). Output, resource use, expenditures and the mass of varieties grow at rates*

\[
\begin{align*}
\dot{Y}_h/Y_h &= \Omega_h - \rho \quad \text{and} \quad \dot{Y}_f/Y_f = \Omega_f - \rho, \tag{29} \\
\dot{R}_h/R_h &= \dot{R}_f/R_f = -\rho, \tag{30} \\
\dot{E}_h/E_h &= \dot{E}_f/E_f = r_i - \rho \quad \text{with} \quad r_h = r_f, \tag{31} \\
\dot{M}_i/M_i &= \varphi_i \alpha (1 - \alpha) (1 + a_i) (1 + b_i)^{-1} - \rho, \tag{32}
\end{align*}
\]

and income shares are constant:

\[
\begin{align*}
\sigma_h &= \frac{\hat{\theta}(\tilde{\gamma}_f/\tilde{\gamma}_h)}{1 + \hat{\theta}(\tilde{\gamma}_f/\tilde{\gamma}_h)} (1 - \tilde{\gamma}_h) \quad \text{and} \quad \sigma_f = \frac{1 + \tilde{\gamma}_f \hat{\theta}}{1 + (\tilde{\gamma}_f/\tilde{\gamma}_h) \hat{\theta}}. \tag{33}
\end{align*}
\]
The balanced growth path followed by the world economy embodies some general characteristics of two-country models of trade. Result (29) shows that differences between the growth rates of physical final output in the two countries are due to structural gaps between the respective productivity indices ($\Omega_h \neq \Omega_f$), which reflect possible differences in R&D productivity ($\varphi_h \neq \varphi_f$), in taxes on intermediates’ purchases ($b_h \neq b_f$), or in labor efficiency growth rates ($\eta_h \neq \eta_f$).

Income shares are constant by virtue of the terms-of-trade mechanism already outlined in Proposition 1: price dynamics compensate for possible differences in physical productivity growth rates, which implies balanced growth at the world level. This mechanism is conceptually similar to that emphasized by Acemoglu and Ventura (2002) in a model where countries exhibit heterogeneous AK technologies: despite the presence of productivity differences, the world income distribution is stable over time because terms of trade effects in the international market for intermediates compensate for productivity differences. In our model, national income shares are constant because structural gaps in physical productivity growth rates are offset by the dynamics of final goods prices. The fundamental difference between our framework and standard two-country models of growth follows from the asymmetric structure of trade and this is reflected in equations (33). Since income shares are affected by the degree of Home’s dependence on the exhaustible resources supplied by Foreign, $\tilde{\gamma}_h$, the world competitive equilibrium exhibits two peculiar features. First, each country’s income share differs from its share of world final output. Second, national taxes on resource use have asymmetric effects in the two countries. We address these points in the next section. In passing, it is worth noting that the present model exhibits closed-form solutions: the explicit time paths of resource use, physical output levels and relative prices are derived in Appendix, and will be exploited in deriving our main results.

3 National Income Shares: Theory and Evidence

3.1 Income Shares and Asymmetric Trade

The asymmetric structure of international trade implies that each country’s income share differs from its share of world final output: substituting (28) in (33), we obtain

$$s_h = \frac{(P^h Y_h)/(P^f Y_f)}{1 + (P^h Y_h)/(P^f Y_f)} \cdot \left[ 1 - \tilde{\gamma}_h \right],$$  \hspace{1cm} (34)

which clarifies that $s_h$ is the product of two factors. The first is Home’s final output share while the second term in square brackets represents the effect of trade dependence in primary inputs — i.e., the fact that Home must use a fraction $\tilde{\gamma}_h$ of revenues from final-good sales to finance resource imports. The output share term in (34) is determined by investment rates and, hence, by productivity parameters. As shown in Appendix, the output ratio between Home and Foreign reduces to

$$\frac{P^h Y_h}{P^f Y_f} = \frac{\epsilon}{1 - \epsilon } \cdot \frac{1 - I_f}{1 - I_h},$$  \hspace{1cm} (35)

and

$$I_i \equiv \frac{\varphi_i (1 - \alpha) (1 + a_i) - \rho (1 + b_i)}{\varphi_i (1 + b_i)} + \frac{\alpha^2}{1 + b_i},$$  \hspace{1cm} (36)
where $I_i$ is one possible definition of domestic investment rate: the right hand side of (36) equals the fraction of final output invested in R&D activity (first term) plus the fraction of output spent in producing intermediates (second term) in country $i$. Given (34), results (35)-(36) imply that each country’s income share is positively (negatively) related to the domestic (other country’s) investment rate and, through the investment rate, positively (negatively) related to the domestic (other country’s) level of R&D productivity. There is, however, another relevant variable influencing the income shares, $s_h$ and $s_f$, namely the tax rate on resource use in Home. The effects of resource taxes on income shares and on the other variables of interest are briefly described below.

### 3.2 Income Shares and Resource Taxes

Due to asymmetric trade, national taxes on resource use have asymmetric effects in the two countries: an increase in $\tau_h$ generally raises Home’s income relative to Foreign whereas variations in $\tau_f$ leave income shares unchanged. Before proving these results, we recall that national resource taxes have counteracting effects on the equilibrium level of relative resource use: from (28) and (35), we have

$$\bar{\theta} = \frac{1 + \tau_f}{1 + \tau_h} \cdot \frac{\epsilon}{1 - \epsilon} \cdot \frac{1 - I_f}{1 - I_h}. \quad (37)$$

Equation (37) shows that relative resource use also depends on the respective investment rates, but coincides with the output ratio $(P_Y^h Y_h)/(P_Y^f Y_f)$ if only if resource taxes are equal in the two countries.

The effects of ceteris paribus variations in resource taxes can be analyzed for each country on the basis of the closed form solution of the model (see Appendix). Considering variations in Home’s resource tax, we obtain the following

**Proposition 3** An increase in Home’s tax rate on domestic resource use reduces relative resource use, reduces physical output in Home and increases physical output in Foreign,

$$\frac{\partial \bar{\theta}}{\partial \tau_h} < 0, \quad \frac{\partial Y_h}{\partial \tau_h} < 0, \quad \frac{\partial Y_f}{\partial \tau_h} > 0, \quad \frac{\partial (Y_h/Y_f)}{\partial \tau_h} < 0,$$

the output ratio is unchanged but Home’s income share increases:

$$\frac{\partial (P_Y^h Y_h)/(P_Y^f Y_f)}{\partial \tau_h} = 0, \quad \frac{\partial s_h}{\partial \tau_h} > 0, \quad \frac{\partial s_f}{\partial \tau_h} < 0.$$

The intuition behind Proposition 3 is as follows. An increase in $\tau_h$ raises the marginal cost of resource use in Home’s final sector: the contraction in resource demand lowers Home’s relative resource use as well as Home’s physical output, both in absolute and in relative terms. Despite the decline in resource use, the output ratio $(P_Y^h Y_h)/(P_Y^f Y_f)$ is unchanged because prices adjust in order to compensate for the decline in the physical output ratio:

$$\frac{\partial (P_Y^h / P_Y^f)}{\partial \tau_h} = -\frac{\partial (Y_h/Y_f)}{\partial \tau_h}. \quad (38)$$

The fact that Home’s income share increases is exclusively due to trade dependence in primary inputs: by taxing domestic resource use, the Home economy retains part of the resource rents.
accruing to Foreign and redistributes it among Home residents. This rent transfer mechanism is evident in equation (34): since nominal output shares are independent of \( \tau_h \), the only effect of an increase in Home’s resource tax is the reduction in \( \tilde{\gamma}_h \), i.e. the fraction of Home’s output used to pay resource rents to Foreign, which increases the trade-dependence term.

The effects of asymmetric trade become furthermore clear when considering Foreign tax policy. The following Proposition establishes that an increase in \( \tau_f \) does not affect income shares:

**Proposition 4** An increase in Foreign’ tax rate on resource use increases \( \tilde{\theta} \) and reduces Foreign physical output in both absolute and relative terms:

\[
\frac{\partial \tilde{\theta}}{\partial \tau_f} > 0, \quad \frac{\partial Y_h}{\partial \tau_f} > 0, \quad \frac{\partial Y_f}{\partial \tau_f} < 0, \quad \frac{\partial (Y_h/Y_f)}{\partial \tau_f} > 0.
\]

Output and income shares are not affected:

\[
\partial \left( \frac{P^h Y_h}{P^f Y_f} \right)/\partial \tau_f = \partial s_h/\partial \tau_f = \partial s_f/\partial \tau_f = 0.
\]

Proposition 4 hinges on the fact that resource taxes in Foreign do not induce rent transfer between countries. An increase in \( \tau_f \) raises the marginal cost of resource use in the Foreign final sector, and the contraction in resource demand increases Home’s relative resource use while reducing Foreign physical output in both absolute and relative terms. However, tax revenues are redistributed within the Foreign economy and this implies that world income shares are unaffected by the variation in \( \tau_f \).

### 3.3 Determinants of Income Shares: An Empirical Test

Our results on the determination of income shares can be summarized as follows: The income share of resource-poor economy is (i) positively related to the national tax on domestic resource use, (ii) positively related to the domestic investment rate, and (iii) negatively related to the investment rate of the resource-rich economy; the respective investment rates are, in turn, (iv) positively related to the respective levels of R&D productivity.\(^{10}\)

We now test these predictions empirically. Using a dynamic panel-estimation technique, we check whether the income share of each oil-importing country depends positively on its own investment rate, negatively on the average investment rate of oil-exporting countries, and positively on domestic oil taxes. The time period is 1980-2008, and the countries for which we have data are sixteen \( OIL \) countries – namely Belgium, France, Germany, Greece, Italy, Japan, South Korea, Netherlands, Philippines, Poland, Portugal, Singapore, Spain, Sweden, Switzerland, Turkey – and the ten \( OEX \) countries – i.e., Algeria, Canada, Iran, Kuwait, Mexico, Norway, Oman, Saudi Arabia, United Arab Emirates, and Venezuela. This is the country sample which best represents the framework of our theoretical model and for which the relevant data are nearly completely available, except for taxes in the Philippines and Singapore.

\(^{10}\)Prediction (i) follows from Proposition 3, whereas predictions (ii)-(iv) follows from substituting equations (35)-(36) in (34).
Table 3: Estimation results for income shares of oil-importing countries

Arellano-Bond dynamic panel-data estimation

Endogenous variable: shareoim

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*** p<0.01, ** p<0.05, * p<0.1

Standard errors in parentheses

In order to focus on long-run effects and to avoid the impact of business cycles, we build five-year averages; the considered periods are: 1980-84, 1985-89, 1990-94, 1995-99, 2000-04, and 2005-08. To capture the dynamic development we include lags of the dependent variable. By construction, the emerging unobserved panel-level effects are then correlated with the lagged dependent variables, making standard estimators inconsistent. That is why the Arellano-Bond dynamic panel-data estimation is used; it provides a consistent generalized method-of-moments (GMM) estimator for the parameters of this model.

We use online data from the World Bank (2009) for the macroeconomic variables and from the International Energy Agency (IEA, 2009) for resource taxes. Specifically, incomes shares are calculated for each oil-importing country as the ratio between its GNI level and the sum of the GNIs of all oil-exporting countries, which we label by $shareoim$. For the investment rates, we take gross capital formation as a percentage of GDP for both oil-importing and -exporting countries. In the case of oil importers, the variable is denoted by $investoim$; for oil exporters we calculate the average investment rate - with population size used as the weighting factor.
- to get the parameter investoex. Resource taxes are measured by taxes on light fuel oil and labelled with oiltax. Further control variables are education expenditures as a percentage of GDP (eduexp, the investment rate for human capital), research expenditures as a percentage of GDP (rdexp, the investment rate for knowledge capital), population size pop, and central government debt as a percentage of GDP cgovdebt.

The results are presented in Table 3, which includes six representative equations [1]-[6]. In all equations we include the (first) lag of the endogenous variable which is significant at the 1%-level in all specifications; this confirms that the estimation method is appropriate. In [1] we start by testing the impact of the investment shares in both types of countries. As can be seen from the results, the theoretical model is confirmed by the estimations as domestic investment affects the oil-importers' income share positively while the opposite holds true for the impact of foreign investment rates. The next equation [2] exhibits that also the domestic investment rate in human capital eduexp is positive for the income share, which also holds for all the other specifications.

In [3], oil taxes are included. It appears as very favorable for the theoretical model that taxation has the predicted positive sign; the significance is 5% or 10% according to the specification. Thus according to the empirical results, oil-importing countries can indeed increase their share of total income by raising domestic oil taxes, which is a remarkable finding.

Population size pop, i.e. the scale of the economy, has no significant effect in any specification, mainly because the endogenous lagged variable already captures this effect. Similarly, research expenditures rdexp as well as central government debt cgovdebt have no significant impact and do not change our major findings. In view of these results, the model predictions regarding the determination of income shares appear to be consistent with the available empirical evidence on oil-importing and oil-exporting countries.

4 Efficiency and Policy

All the results derived in section 2.6 hold in any competitive equilibrium defined at given policy instruments. In the following subsections, we describe the characteristics of laissez-faire equilibria and efficient allocations — i.e., allocations in which domestic market failures induced by R&D externalities and monopolistic competition are neutralized by fiscal authorities through taxes and subsidies. We then analyze the welfare consequences of discretionary variations of national resource taxes.

4.1 Laissez-Faire Equilibria

The laissez-faire equilibrium is represented by the case \( \tau_i = b_i = a_i = 0 \) in each country. The resulting allocation is easily derived from the previous equations. In particular, from (37), relative resource use under laissez-faire equals

\[
\hat{\theta}_{\text{laissez-faire}} = \frac{\epsilon}{1 - \epsilon} \cdot \frac{1 - \alpha + \left( \rho/\varphi_f \right)}{1 - \alpha + \left( \rho/\varphi_h \right)}.
\]  

Equation (39) shows that relative resource use is positively (negatively) related to R&D productivity in Home (Foreign) through the terms \( \varphi_h \) and \( \varphi_f \). This result is peculiar to the laissez-faire equilibrium, which is however inefficient by construction. In each country, monopolistic competition in the intermediate sector and knowledge spillovers in the R&D sector
imply that the economy misallocates domestic final output between R&D investment, consumption and production of intermediates. The following subsection describes the set of taxes and subsidies through which fiscal authorities can restore efficiency at the country level.

4.2 Conditional Efficiency

We define conditionally-efficient allocation for a given country an allocation in which domestic market failures generated by monopoly pricing and R&D spillovers are fully internalized. Formally, an allocation is conditionally efficient for country \(i\) if domestic output is allocated in such a way that it maximizes present-value utility \(U_i\) subject to the technology, income, and resource constraints faced by country \(i\) at given international prices.

At the mathematical level, the derivation of the conditionally efficient allocation (\(CE\)-allocation, hereafter) for each country is very similar to the standard method of welfare maximization in closed-economy models of endogenous growth. However, conditional efficiency and optimality are quite different concepts. In a closed economy, the centralized problem is solved by a hypothetical social planner and the resulting allocation is socially optimal because the planner is endowed with full control over all the elements of the allocation. In the present two-country setting, instead, the \(CE\)-allocation in country \(i\) implies the maximization of domestic utility \(U_i\) at given international prices. On the one hand, this implies that conditional efficiency in one single country does not imply optimality at the world level. On the other hand, there is no general presumption that national governments actually wish to implement conditional efficiency since this depends on the assumed information set of policymakers. If country \(i\)'s government actually takes international prices as given, implementing the \(CE\)-allocation in country \(i\) is desirable. But if country \(i\)'s government, instead, could infer all the consequences of implementing a given allocation in country \(i\) for the world equilibrium, the allocation maximizing domestic utility \(U_i\) is generally not the \(CE\)-allocation. In other words, a well-informed policymaker may rationally choose to deviate from conditional efficiency by implementing inefficient policies that raise national welfare to the detriment of the other country. In the next subsection we show that this is indeed the case for the Home country. These considerations clarify the nature of our interest in conditional efficiency: it is a benchmark state of affairs starting from which we can study the welfare effects of discretionary national policies in isolation from the inefficiencies induced by domestic market failures (i.e., monopolies and spillovers).

We denote variables evaluated in conditionally efficient allocations by tildas. In Home, the \(CE\)-allocation is represented by the paths of imported resource flows and expenditures (in consumption, intermediates’ production and R&D activity), that maximize Home’s indirect utility subject to the final-good technology, the intermediate-good technology, the R&D technology, and Home’s expenditure constraint:

\[
\{ \tilde{R}_h, \tilde{E}_c^h, \tilde{E}_x^h, \tilde{E}_d^h \}_{t=0}^\infty = \arg \max \ U_h \ \text{s.t.} \ (1), (11), (23)
\]

where \(U_h\) in (18) is maximized taking prices as given and the R&D externality is fully taken into account through constraint (11). In Foreign, the \(CE\)-allocation is represented by the extraction paths of both imported and exported resource flows, and the paths of expenditures \((E_c^h, E_x^h, E_d^h)\) that maximize utility in Foreign subject to the technology constraints, the
aggregate expenditure constraint, and the exhaustible resource constraint:

$$\{ \dot{R}_h, \dot{R}_f, \dot{E}_f^c, \dot{E}_f^d \}_{t=0}^{\infty} = \arg\max U_f \text{ s.t. } (1), (11), (23) \text{ and } \dot{Q} = -R_h - R_f.$$  

The two maximization problems are fully described in the Appendix. The first result is that if both economies exhibit conditionally efficient allocations, the trade equilibrium is characterized by a constant efficient level of relative resource use $\tilde{\theta}$ equal to

$$\tilde{\theta} = \frac{\epsilon}{1 - \epsilon}.$$  \hspace{1cm} (40)  

Result (40) shows that the efficient level of relative resource use $\tilde{\theta}$ is exclusively determined by preference parameters, without any role for technology. As a consequence, it differs from (coincides with) the laissez-faire level (39) when the technological parameters governing R&D productivity, $\varphi_h$ and $\varphi_f$, are different (equal). Obviously, this does not mean that laissez-faire equilibria are efficient under homogeneous technologies: when $\varphi_h = \varphi_f$ and all taxes and subsidies are zero, the level of $\tilde{\theta}$ coincides with the efficient one but the competitive equilibrium is still inefficient since domestic output is misallocated among its competing uses – i.e., consumption, intermediates’ production and R&D investment – by virtue of the existing market failures. In fact, the second result (see Appendix for details) is that if governments wish to decentralize the $CE$-allocation in the competitive economy, they must implement an efficient policy consisting of the following subsidies and taxes

$$\tilde{a}_i = \frac{\varphi_i (1 - \alpha) - \rho}{\rho (1 - \alpha)} > 0,$$  \hspace{1cm} (41)  

$$\tilde{b}_i = \frac{\varphi_i \alpha (1 - \alpha) - \rho}{\rho} > 0,$$  \hspace{1cm} (42)  

$$\tilde{\tau}_i = \frac{\varphi_i (1 - \alpha) - \rho}{\rho} > 0.$$  \hspace{1cm} (43)  

The role of subsidies to R&D investment is clearly that of internalizing knowledge spillovers: research activity generates positive externalities and must therefore be encouraged by public authorities through $\tilde{a}_i > 0$. The fact that private agents exhibit inefficiently low saving rates implies inefficiently high consumption propensities and, consequently, excessive demand for the inputs employed in final production.\footnote{The private marginal benefits from resource use and intermediates equal the respective marginal productivities of these inputs in final production. The social marginal benefits, instead, equal the un-invested fractions of the respective marginal productivities: see equations (??)-(??) in Appendix. The slight asymmetry between the levels of the resource tax and the intermediates’ tax is explained in footnote ?? below.} Since the equilibrium quantities of resources and intermediates exceed socially desirable levels, efficiency is restored by imposing positive taxes on resource use and intermediates’ purchases.\footnote{Since the excess demand for inputs originates in inefficiently low saving rates, each input should in principle be taxed at the same rate. In the present model, however, the efficient tax on resources (42) is higher than the efficient tax on intermediates’ purchases (42). The reason is that, in the competitive economy, the excess demand for intermediates is already partially contrasted by the monopolistic behavior of producers, which restricts supply. This is a general characteristic of endogenous growth models with R&D externalities. As shown by Valente (2009) in a closed-economy model without resources, the excess-demand effect (induced by inefficiently-low saving rates) always dominates the contrasting effect of restricted supply (induced by monopoly rents) when preferences are logarithmic. Consequently, intermediates’ purchases must be taxed.} It is easily verified from (36) and (37) that
if the governments of both countries implement the efficient policy \((\tilde{a}_i, \tilde{b}_i, \tilde{\tau}_i)\), the equilibrium relative resource use \(\tilde{\theta}\) coincides with the efficient level in (40). Moreover, little algebra shows that decentralizing the CE-allocation in a market economy implies faster physical productivity growth relative to laissez-faire conditions. We now exploit this and the previous results in order to address the issue of strategic taxation.

4.3 Welfare and Resource Taxation

Do the governments of Home and Foreign have particular incentives to implement inefficient taxes? In this section, we analyze the marginal effects of national resource taxes on welfare for each country. These welfare-tax relationships provide the basis for studying the potential incentives that national governments have to implement discretionary policies: if fiscal authorities could take into account all the general equilibrium effects of national taxes and, hence, calculate the final effect on domestic welfare levels, they might rationally choose to deviate from laissez-faire equilibria or efficient allocations. While this is a general argument, we restrict the present discussion to the analysis of potential welfare gains stemming from variations in national taxes on domestic resource use.

The reaction of utility levels to variations in domestic resource taxes is represented by two welfare-tax relationship that we derive in explicit form. As shown in the Appendix, present-value utilities for each consumer in the two countries equal

\[
U_h = \kappa_h + \frac{1}{\rho} \ln \left\{ Y_h(0) \cdot \left[ \frac{P^h_Y(0)}{P^f_Y(0)} \right]^{1-\epsilon_c} \cdot \bar{\sigma}^c_h \right\},
\]

\[
U_f = \kappa_f + \frac{1}{\rho} \ln \left\{ Y_f(0) \cdot \left[ \frac{P^f_Y(0)}{P^h_Y(0)} \right]^\epsilon \cdot \bar{\sigma}^c_f \right\},
\]

where \(\kappa_i\) is a constant term representing a weighted average of physical productivity growth rates in the two countries, and is therefore independent of resource taxes; the constant \(\bar{\sigma}^c_i = E^c_i / (P^h_i Y_i)\) is the ratio between consumption expenditures and final output in country \(i\), equal to

\[
\bar{\sigma}^c_h = 1 - \tilde{\gamma}_h - I_h \quad \text{and} \quad \bar{\sigma}^c_f = 1 + \tilde{\gamma}_f \tilde{\theta} - I_f,
\]

where \(I_h\) and \(I_f\) are independent of resource taxes, as shown in (36). To simplify notation, we will henceforth denote the price ratio by \(p_0 = P^h_Y(0) / P^f_Y(0)\). Starting from (44)-(45), utility levels can be represented as functions of the respective national taxes. As shown below, the welfare curves \(U_h(\tau_h)\) and \(U_f(\tau_f)\) are concave and each exhibits a unique maximum associated with a finite level of the domestic resource tax, denoted as \(\tau^\text{max}_i\) for country \(i\). The interpretation of \(\tau^\text{max}_i\) is that of a resource tax yielding maximal utility in country \(i\) for a given state of affairs – especially, for a given resource tax implemented – in the other country.

\textit{Welfare-tax relation in Home.} Considering Home’s utility in (44), a ceteris paribus increase in \(\tau_h\) affects the components of \(U_h\) as follows. First, resource use in Home declines and this reduces physical output: \(\partial Y_h(0) / \partial \tau_h < 0\). Second, the price index \(p_0\) increases in order to compensate for the decline in Home’s relative physical output: \(\partial p_0 / \partial \tau_h > 0\). Third, from Proposition 3, the increase in the resource tax generates rent transfer and implies an increase in Home’s income share: this implies \(\partial \sigma^c_h / \partial \tau_h > 0\) since Home residents can increase the ratio between consumption expenditures and domestic final output other things being equal. This third effect will be shown to be crucial for all the results presented in this section. By virtue
of these mechanisms, the welfare curve \( U_h(\tau_h) \) is concave: as shown in the Appendix, the first order condition \( dU_h/d\tau_h = 0 \) is associated to a unique maximum characterized by a finite level of the resource tax, \( \tau_h = \tau_h^{\max} \), and by an equilibrium level of relative resource use, \( \bar{\theta} = \theta_h^{\max} \), exhibiting an important characteristic:

\[
\frac{\partial U_h}{\partial \tau_h} = 0 \iff \bar{\theta} = \theta_h^{\max} < \frac{\epsilon}{1 - \epsilon}.
\]

Recalling (40), the efficient equilibrium level of relative resource equals \( \frac{\epsilon}{1 - \epsilon} \). Hence, result (47) shows that the welfare-maximizing resource tax in Home is always associated to an inefficient equilibrium where Home’s resource use is relatively low.

**Welfare-tax relation in Foreign.** Considering Foreign residents’ utility in (45), a ceteris paribus increase in \( \tau_f \) has the following effects. First, resource use in Foreign declines and this reduces physical output: \( \partial Y_f(0)/\partial \tau_f < 0 \). Second, the price index \( p_0 \) declines in order to compensate for the rise in Home’s relative physical output, and this induces a favourable terms-of-trade effect on welfare since the relative price of Foreign final goods increases: \( \partial p_0^{-1}/\partial \tau_f > 0 \). Third, differently from Home, an increase in the Foreign resource tax leaves the consumption-to-output ratio unchanged: recalling Proposition 4, no rent transfer between countries implies that national income shares are independent of \( \tau_f \), and so is the consumption-to-output ratio in Foreign (\( \partial \sigma_c/\partial \tau_f = 0 \)). The first two mechanisms nonetheless imply the concavity of \( U_f(\tau_f) \): as shown in Appendix, the condition \( dU_f/d\tau_f = 0 \) is associated to a finite tax level \( \tau_f = \tau_f^{\max} \) and to a specific equilibrium level of relative resource use, \( \bar{\theta} = \theta_f^{\max} \), exhibiting the following property:

\[
\frac{\partial U_f}{\partial \tau_f} = 0 \iff \bar{\theta} = \theta_f^{\max} = \frac{\epsilon}{1 - \epsilon}.
\]

From (48), the welfare-maximizing resource tax in Foreign is always associated to an equilibrium where relative resource use coincides with the efficient level. Note, however, that this does not mean that \( \tau_f^{\max} \) is always associated to an efficient equilibrium, since relative resource use may be equal to \( \bar{\theta} = \frac{\epsilon}{1 - \epsilon} \) also in inefficient equilibria.\(^{13}\)

Results (47)-(48) imply that if both national governments fully recognize all the general-equilibrium effects of the respective taxes on resource use, the independent pursuit of domestic welfare maximization results into conflicting objectives as each government seeks a different equilibrium level of relative resource use. This is indeed a very general conclusion since neither (47) nor (48) assume that the two economies are starting from a specific world competitive equilibrium. As noted before, this result originates in the asymmetric effects of national taxes on the consumption-to-output ratio: as noted in Appendix, if \( \sigma_h^{\ast} \) were independent of \( \tau_h \), Home welfare \( U_h(\tau_h) \) would be maximized in association with an efficient level of relative resource use exactly as Foreign welfare. The economic mechanism behind (47)-(48) is thus the rent transfer effect generated by Home resource taxes – which allows Home residents to increase the ratio between consumption expenditures and domestic final output all other things being equal – that does not arise, instead, when Foreign governments modify \( \tau_f \). The next two Propositions apply results (47)-(48) to specific contexts where the initial states of affairs

\(^{13}\)For example, when \( \varphi_h = \varphi_f \), the laissez-faire equilibrium implies \( \bar{\theta} = \frac{\epsilon}{1 - \epsilon} \) (see equation (39) above) but is inefficient due to the market failures induced by R&D externalities and monopolistic competition in the two economies.
are the benchmark regimes previously analyzed: symmetric $CE$-allocations and laissez-faire equilibria.

**Deviations from Efficiency.** Suppose that, in the initial equilibrium, both governments implement conditionally efficient allocations by means of the efficient taxes (41)-(42)-(42). Given this state of affairs in Foreign, Home would gain from raising a positive resource tax. The opposite is not true since Foreign would not gain from deviating from the initial state of affairs:

**Proposition 5** If both countries start from $CE$-allocations, $\partial U_f / \partial \tau_f = 0$ and $\partial U_h / \partial \tau_h > 0$.

Proposition 5 establishes that, in an efficient equilibrium, the Foreign government has no incentive to vary $\tau_f$ because $\bar{\theta} = \bar{\theta}$ already guarantees maximal domestic welfare given this state of affairs; the Home government, instead, has an incentive to deviate because $\bar{\theta} = \bar{\theta} > \theta_h^{max}$ implies that Home welfare is ceteris paribus improved if $\tau_h$ is increased up to $\tau_h^{max} > \bar{\tau}_h$. This result suggests that if domestic welfare represents the payoff of each government in a political game, inefficient equilibria may well be the final outcome. Pursuing this argument obviously requires an extended analysis of game-theoretic issues, which is beyond the scope of this paper. We only point out that, in a one-shot sequential game where Home deviates from conditional efficiency by implementing $\tau_h^{max} > \bar{\tau}_h$ and Foreign reacts by implementing the welfare-maximizing tax $\tau_h^{max}$ determined by the new state of affairs, Home’s welfare at the end of the game is still higher than in the initial $CE$-allocation because the inefficiencies created by both taxes ultimately fall on Foreign residents (see Appendix).

**Deviations from Laissez-Faire.** The previous results do not necessarily hold if the benchmark equilibrium is a world laissez-faire allocation, because productivity differences come into play:

**Proposition 6** If both countries start from laissez-faire allocations, then (i) $\varphi_h = \varphi_f$ implies $\partial U_f / \partial \tau_f = 0$ and $\partial U_h / \partial \tau_h > 0$; (ii) $\varphi_h > \varphi_f$ implies $\partial U_f / \partial \tau_f < 0$ and $\partial U_h / \partial \tau_h > 0$; (iii) $\varphi_h < \varphi_f$ implies $\partial U_f / \partial \tau_f > 0$ while $\partial U_h / \partial \tau_h \geq 0$.

If R&D technologies are identical in the two countries, results are similar to the case of efficient allocations. Given laissez-faire in Foreign, Home would gain from raising a positive resource tax; given laissez-faire in Home, Foreign would not gain from deviating from laissez-faire. If R&D productivity is higher in Home, relative resource use exceeds the efficient level: from (39), $\varphi_h > \varphi_f$ implies $\bar{\theta} > \bar{\theta}$. In this case, both countries have an incentive to deviate, since Foreign would gain from subsidizing domestic resource use whereas Home would gain from raising a resource tax. Finally, if R&D productivity is higher in Foreign, relative resource use falls short of the efficient level: Foreign would gain from raising a resource tax; Home would gain by implementing either a resource tax or a subsidy depending on how big is the productivity gap.

In general, our interest in welfare-tax relationships stems from the fact that oil taxes are extensively used in oil-importing economies and are often interpreted as strategic policies. The theoretical literature made a first recognition of the topic after the oil shocks of 1970s, and suggests that the rent-extraction effect of oil taxes is virtually unbounded (Bergstrom, 1982).
Our results may be linked to those of a related paper by Brander and Djajic (1983) showing that oil importers tend to impose strategic tariffs in response to monopolistic behavior of oil producers. In this respect, our results are different in four respects. First, we show that oil-poor economies have an incentive to raise the domestic tax starting from both laissez-faire and efficient allocations, not just in response to Foreign actions. Second, in Brander and Djajic (1983), the oil-rich economy tends to restrict the resource supply to the oil-poor country whereas, in our model, the Foreign government does not have incentives to deviate from the efficient level of relative resource use. Third, if Home begins a tax war in our model, Foreign does not react by restricting its oil supply to Home, but rather by reducing its domestic oil demand: if Home enacts a strategic increase of $\tau_h$ which reduces $\theta$ below the efficient level, the response of Foreign is to increase $\tau_f$ because this increases $\theta$ and brings it towards the efficient level. Fourth, the only case in which Foreign has an independent incentive to subsidize domestic oil consumption is when there is a structural productivity gap in favor of Home: this result is specific to the present model and, to our knowledge, is novel to the literature.

5 Conclusion

Since 1980, the aggregate income of oil-exporting countries relative to that of oil-poor countries has been remarkably constant, despite structural gaps in labor productivity growth rates. We rationalized this behavior in a two-country model of asymmetric trade where growth rates are endogenously determined by R&D activity, and productivity differences between countries are compensated by terms-of-trade dynamics. The model predictions regarding the basic determinants of income shares are supported by empirical evidence: the share of each oil-poor economy is positively (negatively) related to the domestic (average foreign) investment rate and to the national tax on domestic resource use. Our results regarding the marginal effects of taxation are also relevant for the current policy debate. If the initial state of affairs is an efficient equilibrium in which all domestic market failures are internalized, the government of an oil-importing country could increase domestic welfare by raising the national resource tax, whereas this potential incentive does not arise for the government of the oil-exporting country. If the initial state of affairs is a laissez-faire equilibrium, instead, incentives are crucially affected by productivity differences: if productivity grows faster in the oil-importing country, the government of the oil-exporting country has an incentive to subsidize domestic resource use.

A Appendix

Monopoly rents and derivation of (6). Maximization of $\Pi_i (m_i) = (P^i_{X(m_i)} - \zeta P^i_Y) X_i (m_i)$ subject to (4) yields

$$X_i (m_i) = X_i = \left[ \frac{\alpha^2 (v_i L_i)^{\beta} R^i_i}{\zeta (1 + b_i)} \right]^{\frac{1}{1-\alpha}}, \quad (A.1)$$

$$\Pi_i (m_i) = \Pi_i = (1 - \alpha) P^i_X X_i. \quad (A.2)$$

Plugging (A.1) back in (4) yields (5). From (1), symmetric varieties imply $Y_i = M_i X^i \alpha (v_i L_i)^{\beta} R^i_i$, where we can substitute (A.1) to obtain (6).
Optimal resource extraction in Foreign: derivation of (13)-(14). Given the objective (12) and constraint $\dot{Q} = -R$, the current value Hamiltonian associated with the extraction problem is $P_R(t) R(t) - \chi(t) R(t)$, where $\chi(t)$ is the dynamic multiplier. The conditions for optimality in extraction read

\begin{align*}
P_R(t) &= \chi(t), \quad (A.3) \\
\dot{\chi}(t) &= r_f(t) \chi(t), \quad (A.4) \\
\lim_{t \to \infty} \chi(t) Q(t) e^{-\int_{i}^{\infty} r_f(u) du} &= 0, \quad (A.5)
\end{align*}

Plugging (A.3) in (A.4) we have Hotelling's rule (13). Integrating (A.4) and substituting the resulting expression in the transversality condition (A.5) yields

\begin{align*}
\lim_{t \to \infty} Q(t) = 0,
\end{align*}

Integrating the resource constraint between $t = 0$ and infinity, and imposing $\lim_{t \to \infty} Q(t) = 0$ yields (14).

Static consumer problem. Maximizing the utility index (16) subject to the expenditure constraint (17) for each country $i = f, h$, we obtain

\begin{align*}
c_i^f / c_i^h &= \frac{1 - \epsilon}{\epsilon} (P_i^h / P_i^f), \quad (A.6) \\
E_i^c / L_i &= \frac{1}{\epsilon} P_i^h c_i^h \quad \text{and} \quad E_i^c / L_i = \frac{1}{1 - \epsilon} P_i^f c_i^f, \quad (A.7) \\
\bar{u}_i &= \ln \left\{ \frac{\epsilon (1 - \epsilon)^{\frac{1}{\epsilon} - \epsilon}}{(P_i^h)^{\epsilon} (P_i^f)^{1 - \epsilon}} \right\} \left( E_i^c / L_i \right), \quad (A.8)
\end{align*}

where (A.6) is the first order condition in each country, (A.7) results from plugging (A.6) back in constraint (17), and (A.8) is the indirect utility function obtained from substituting the optimal quantities (A.7) back in the utility index (16). Denoting the term in curly brackets in (A.8) as $\omega = \omega(P_i^h, P_i^f)$, we can write $\bar{u}_i = \ln [\omega \cdot (E_i^c / L_i)]$. For future reference, notice that (A.7) imply

\begin{align*}
P_i^h L_i c_i^f = \epsilon E_i^c f \quad \text{and} \quad P_i^f L_i c_i^h = (1 - \epsilon) E_i^c h, \quad (A.9)
\end{align*}

Dynamic consumer problem: derivation of (21)-(22). The current-value Hamiltonian of the dynamic consumer problem is

\begin{align*}
\ln(\omega \cdot (E_i^c / L_i)) + \lambda_i \cdot \left[ r_i n_i + P_i^L - (E_i^c / L_i) - F_i \right],
\end{align*}

where $\lambda$ is the dynamic multiplier associated to the wealth constraint; we neglect resource rents for Foreign residents ($i = f$) since this does not affect the solution. The optimality conditions read

\begin{align*}
L_i / E_i &= \lambda_i \quad \text{and} \quad \dot{\lambda}_i = \lambda_i (\rho - r_i), \quad (A.10) \\
0 &= \lim_{t \to \infty} \lambda_i(t) n_i(t) e^{-\rho t}, \quad (A.11)
\end{align*}
Equation (21) directly follows from (A.10).

**Aggregate constraints: derivation of (23)-(24).** Substituting \( n_i = (V_i M_i)/L_i \) and \( V_i r_i = \Pi_i + \dot{V}_i \) from (9) in (19) and multiplying both sides by \( L_i \) we obtain

\[
V_h \dot{M}_h = \Pi_h M_h + P^h_L L_h - E^h_h - F_h L_h.
\]

Plugging \( V_i \dot{M}_i = P^i_Y Z_i \) from (7)-(8) and \( \Pi_i M_i = M_i X_i \left(P^i_X - \zeta P^i_Y\right) \) from (A.2), we obtain

\[
P^h_Y Z_h + E^h_h + P^h_Y \zeta M_h X_h = M_h P^h_X X_h + P^h_L L_h - F_h L_h.
\]

Plugging \( F_i L_i = a_i P^i_Y Z_i - b_i M_i P^i_X X_i - \tau_i P_R R_i \) from (15), we have

\[
P^h_Y (1 + a_h) + E^h_h + P^h_Y \zeta M_h X_h = M_h P^h_X X_h (1 + b_h) + P^h_L L_h + \tau_h P_R R_h
\]

From conditions (2) and (4), we can substitute \( P^i_Y L_i = \beta P^i_Y Y_i \) and \( M_i P^i_X X_i (1 + b_i) = \alpha P^i_Y Y_i \) to obtain

\[
E^h_h + P^h_Y Z_h (1 + a_h) + P^h_Y \zeta M_h X_h = (\alpha + \beta) P^h_Y Y_h + \tau_h P_R R_h,
\]

where we can plug \( \alpha + \beta = 1 - \gamma \) together with condition (3) to obtain

\[
E^h_h + P^h_Y Z_h (1 + a_h) + P^h_Y \zeta M_h X_h = P^h_Y Y_h - P_R R_h.
\]

(A.12)

Substituting \( E^d_h \equiv P^h_Y Z_h (1 + a_h) \) and \( E^e_h \equiv P^h_Y \zeta M_h X_h \) we obtain (23). Repeating the above steps for the Foreign economy with constraint (20), and recalling that \( R - R_f = R_h \), we obtain (24).

**Equilibrium interest rates: derivation of (25).** In equilibrium, the profit to patent-value ratio equals

\[
\frac{\Pi_i}{V_i} = \phi_i \frac{(1 + a_i) (1 - \alpha) P^i_X X_i}{P^i_Y} = \varphi_i \cdot \frac{(1 + a_i) (1 - \alpha) \alpha}{1 + b_i},
\]

(A.13)

where the central term is obtained from the ratio between profits in (A.2) and patent-value in (8); the last term is then obtained by substituting \( \phi_i \) with (10) and plugging \( \left(P^i_X M_i X_i\right)/\left(P^i_Y Y_i\right) = \alpha/(1 + b_i) \) from (4). Equations (8) and (10) also imply \( V_i = \left(P^i_Y Y_i\right) / [\varphi_i \cdot M_i (1 + a_i)] \) so that

\[
\frac{\dot{V}_i(t)}{V_i(t)} = \frac{\dot{P}^i_Y}{P^i_Y} + \frac{\dot{Y}_i}{Y_i} - \frac{\dot{M}_i}{M_i}.
\]

(A.14)

Substituting (A.13) in (9), the interest rate in country \( i \) equals

\[
r_i = \varphi_i \alpha (1 - \alpha) \cdot \frac{1 + a_i}{1 + b_i} + \frac{\dot{P}^i_Y}{P^i_Y} + \frac{\dot{Y}_i}{Y_i} - \frac{\dot{M}_i}{M_i}.
\]

(A.15)

Time-differentiating (6) we obtain

\[
\frac{\dot{Y}_i}{Y_i} = \frac{\dot{M}_i}{M_i} + \frac{\beta}{1 - \alpha} n_i + \frac{\gamma}{1 - \alpha} \dot{R}_i.
\]

(A.16)

Plugging (A.16) in (A.15) we obtain equation (25) in the text.
Proof of Proposition 1. Proposition 1 hinges on a preliminary result that is related to expenditures allocation in Home. Define the expenditure-to-output ratios in country $i$ as

$$ \sigma_i^c \equiv E_i^c / (P_i^c Y_i) \quad \text{and} \quad \sigma_i^d \equiv E_i^d / (P_i^d Y_i) \quad \text{and} \quad \sigma_i^x \equiv E_i^x / (P_i^x Y_i). $$  \hspace{1cm} (A.17)

From (5) we have $\sigma_i^c = (P_i^c Z_i) / (P_i^c Y_i) = \alpha \left[ (P_i^c M_i X_i) / (P_i^c Y_i) \right]$, where we can substitute $(P_i^c M_i X_i) / (P_i^c Y_i) = \alpha / (1 + a_i)$ from (4) to obtain

$$ \sigma_i^x = \frac{E_i^x}{P_i^x Y_i} = \frac{\alpha^2}{1 + b_i} \quad \text{in each} \quad i = h, f. \hspace{1cm} (A.18) $$

From (21), the growth rate of $\sigma_i^c \equiv E_i^c / (P_i^c Y_i)$ equals

$$ \frac{\dot{\sigma}_i^c}{\sigma_i^c} = r_i(t) - \rho - \frac{\dot{P_i^c}}{P_i^c} - \frac{\dot{Y}_i}{Y_i} = \varphi_i \frac{(1 - \alpha)(1 + a_i)}{1 + b_i} - \frac{\dot{M_i}}{M_i} - \rho; $$

where we have substituted $r_i(t)$ by (A.15). Plugging $E_i^d \equiv P_i^d Z_i (1 + a_i)$ in (11) and using $\sigma_i^d \equiv E_i^d / (P_i^d Y_i)$, the growth rate of varieties equals $\dot{M_i}/M_i = \varphi_i \sigma_i^d$, which can be substituted in the above expression to obtain

$$ \frac{\dot{\sigma}_i^c}{\sigma_i^c} = \varphi_i \frac{(1 - \alpha)(1 + a_i)}{1 + b_i} - \varphi_i \sigma_i^d - \rho. \hspace{1cm} (A.19) $$

For future reference, notice that (A.18)-(A.19) are valid in both economies. Now consider country Home. Exploiting (A.17), and defining the tax-adjusted resource elasticity $\bar{\gamma}_h \equiv \gamma (1 + \tau_h)^{-1}$, the aggregate constraint (23) can be written as

$$ \sigma_h^c + \sigma_h^d + \sigma_h^x = 1 - (P_R R_h) / (P_h^h Y_h) = 1 - \bar{\gamma}_h, $$

where the last term follows from (3). From (A.20), we can substitute $\sigma_h^d = 1 - \bar{\gamma}_h - \sigma_h^c - \sigma_h^x$ in (A.19), and eliminate $\sigma_h^x$ by (A.18), obtaining

$$ \frac{\dot{\sigma}_h^c}{\sigma_h^c} = \varphi_h \sigma_h^c + \varphi_h \frac{(1 - \alpha)(1 + a_h)}{1 + b_h} + \sigma_h^x - \varphi_h (1 - \bar{\gamma}_h) - \rho, \hspace{1cm} \text{in each} \quad h. \hspace{1cm} (A.21) $$

where all terms to the right hand side except $\sigma_h^c$ are constant. Since $\varphi_h > 0$, equation (A.21) is globally unstable around the unique stationary point: ruling out by standard arguments explosive dynamics – which would be associated to unbounded dynamics in the propensity to consume – we thus have

$$ \sigma_h^c = (1 - \bar{\gamma}_h) - \frac{\varphi_h \left[ (1 - \alpha)(1 + a_h) + \alpha^2 \right] - \rho (1 + b_h)}{\varphi_h (1 + b_h)} \quad \text{in each} \quad t. \hspace{1cm} (A.22) $$

By (A.18) and (A.22), constant values of $\sigma_h^c$ and $\bar{\sigma}_h^x$ imply constant $\sigma_h^d$ which, from (A.20), equals

$$ \sigma_h^d = 1 - \bar{\gamma}_h - \sigma_h^c - \sigma_h^x = \frac{\varphi_h \alpha (1 - \alpha)(1 + a_h) - \rho (1 + b_h)}{\varphi_h (1 + b_h)} \hspace{1cm} (A.23) $$
Constant expenditure-output ratios imply that the growth rate of \( P_h Y_h \) coincides with the balanced growth rate of all expenditure shares \( \hat{E}_h / E_h = \hat{E}_h / E_h = \hat{E}_h / E_h \), which is determined by the Kenyes-Ramsey rule (21). Moreover, since (23) and (3) imply that the ratio
\[
E_h / P_h Y_h = (1 - \tilde{\gamma}_h)
\]
is constant, the balanced growth rate in Home is
\[
\frac{\dot{E}_h}{E_h} = \frac{\dot{P}_h}{P_h} + \frac{\dot{Y}_h}{Y_h} = r_h - \rho.
\]
Now substitute (A.9) in (22) to obtain
\[
P_R R_h + (1 - \epsilon) E_h^c = \epsilon E_f^c.
\]
Substituting \( P_R R_h = \tilde{\gamma}_h P_h Y_h \) from (3) and \( E_h^c = \tilde{\sigma}_h P_h Y_h \) from (A.22) in (A.26) we get
\[
E_f^c = \frac{1}{\epsilon} \left[ \tilde{\gamma}_h + (1 - \epsilon) \tilde{\sigma}_h^c \right] \cdot P_h Y_h,
\]
where all the terms in square brackets are constant, implying that the ratio \( E_f^c / (P_h Y_h) \) is constant over time. Since \( P_h Y_h \) grows at the same rate as \( E_h^c \) by (A.25), we have \( \dot{E}_f^c / E_f^c = \dot{E}_h^c / E_h^c \). By the Keynes-Ramsey rules (21) this implies equal interest rates in the two countries, \( r_h = r_f \). Imposing \( r_h = r_f \) in (26) yields (27).

**Proof of Proposition 2.** Using the definition \( R_h = \theta R_f \) and condition (3) for country \( i = f \), constraint (24) implies
\[
E_f = P_f Y_f + P_R R_h = P_f Y_f + \theta P_R R_f = P_f Y_f (1 + \tilde{\gamma}_f \theta) .
\]
Recalling definitions (A.17), result (28) and the central term in (24) imply \( \tilde{\sigma}_f^c + \tilde{\sigma}_f^c + \tilde{\sigma}_f^c = 1 + \tilde{\gamma}_f \theta \), where we can substitute \( \tilde{\sigma}_f^c = \alpha^2 (1 + b_f)^{-1} \) from (A.18) to obtain
\[
\tilde{\sigma}_f^d = 1 + \tilde{\gamma}_f \theta - \frac{\alpha^2}{1 + b_f} - \tilde{\sigma}_f^c.
\]
Plugging (A.29) in (A.19) for country \( i = f \) we obtain
\[
\frac{\tilde{\sigma}_f}{\tilde{\sigma}_f^c} = \varphi_f \frac{\alpha (1 - \alpha) (1 + a_f)}{1 + b_f} - \varphi_f \left[ 1 + \tilde{\gamma}_f \theta - \frac{\alpha^2}{1 + b_f} - \tilde{\sigma}_f^c \right] - \rho.
\]
Now divide both sides of (A.27) by \( P_f Y_f \) and solve for \( \tilde{\sigma}_f^c \equiv E_f^c / (P_f Y_f) \) to obtain
\[
\tilde{\sigma}_f = \frac{1}{\epsilon} \left[ \tilde{\gamma}_h + (1 - \epsilon) \tilde{\sigma}_h^c \right] \cdot \frac{P_f Y_h}{P_f Y_f} = \frac{1}{\epsilon} \left[ \tilde{\gamma}_h + (1 - \epsilon) \tilde{\sigma}_h^c \right] \cdot \frac{\tilde{\gamma}_f}{\tilde{\gamma}_h},
\]
where we have used (28) to get the last term. Now define
\[
\chi = \frac{1}{\epsilon} - \frac{1}{\epsilon} \cdot \frac{\tilde{\sigma}_h^c}{\tilde{\gamma}_h} > 1.
\]
Since \( \tilde{\sigma}_h^\gamma \) is constant by (A.22), \( \chi \) is also constant and (A.31) implies

\[
\tilde{\sigma}_f^\gamma = \chi \tilde{\gamma}_f \theta \quad \text{and} \quad \frac{\dot{\tilde{\sigma}}_f^\gamma}{\tilde{\sigma}_f^\gamma} = \frac{\dot{\theta}}{\theta}.
\]  
(A.33)

Substituting the second expression in (A.33) into (A.30) we obtain

\[
\frac{\dot{\theta}}{\theta} = \varphi_f (\chi - 1) \tilde{\gamma}_f \cdot \theta + \frac{\varphi_f [\alpha (1 - \alpha) (1 + a_f) + \alpha^2] - (\varphi_f + \rho) (1 + b_f)}{1 + b_f},
\]  
(A.34)

where all terms to the right hand side except \( \theta \) are constant. Since \( \varphi_f (\chi - 1) \tilde{\gamma}_f > 0 \), equation (A.34) is globally unstable around the unique stationary point: ruling out by standard arguments explosive dynamics – which would be associated to unbounded dynamics in the propensity to consume in Foreign – we thus have \( \theta(t) = \hat{\theta} \) in each \( t \in [0, \infty) \), where \( \hat{\theta} \) is the steady-state level (??) obtained by imposing \( \dot{\theta} = 0 \) in (A.34):

\[
\hat{\theta} = \frac{(\varphi_f + \rho) (1 + \tau_K^f) - \varphi_f [\alpha (1 - \alpha) (1 + a_f) + \alpha^2]}{\tilde{\gamma}_f (\chi - 1) \varphi_f (1 + \tau_K^f)}.
\]  
(A.35)

From (28), a constant \( \theta \) implies

\[
\frac{\dot{P}_Y^h}{P_Y^h} - \frac{\dot{P}_Y^f}{P_Y^f} = \frac{\dot{Y}_f}{Y_f} - \frac{\dot{Y}_h}{Y_h},
\]  
(A.36)

where we can substitute (27) and (??) to obtain (29). Since \( P_R R_h = \tilde{\gamma}_h P_Y^h Y_h \), the Hotelling rule \( \dot{P}_R/P_R = r_h \) in (13) and result (A.25) imply that \( P_R R_h \) grows at the rate \( r_h - \rho \), so that \( \dot{R}_h/R_h = -\rho \) because \( \theta = R_h/R_f \) is constant, which proves (30). From (A.28), a constant \( \theta \) also implies that \( E_f \) grows at the same rate as \( P_Y^f Y_f \), which coincides with the growth rate of \( E_h \) and \( P_Y^h Y_h \) by (A.36) and (A.25). We thus have (31). From (A.33), we can substitute \( \tilde{\sigma}_f^\gamma = \chi \tilde{\gamma}_f \hat{\theta} \) in (A.29) to obtain \( \tilde{\sigma}_f^d = 1 - \frac{\alpha^2}{1 + \alpha} - (\chi - 1) \tilde{\gamma}_f \hat{\theta} \), where we can eliminate \((\chi - 1) \tilde{\gamma}_f \hat{\theta}\) by means of (A.35) to obtain

\[
\tilde{\sigma}_i^d = \frac{\varphi_i \alpha (1 - \alpha) (1 + a_i) - \rho (1 + b_i)}{\varphi_i (1 + b_i)}
\]  
(A.37)

in country \( i = f \). But (A.23) implies that (A.37) is also valid for country \( i = h \). From (11), both countries exhibit \( \dot{M}_i/M_i = \varphi_i \tilde{\sigma}_i^d \), so that result (A.37) implies (32). Finally, from (23) and (24), Home’s income share can be written as \( s_h = \frac{P_Y^h Y_h (1 - \gamma_h)}{P_Y^h Y_h + P_Y^h Y_f} \), where we can plug (28) to obtain (31).

**Closed form solution.** Equation (6) and result (29) imply that physical final output in country \( i \) equals

\[
Y_i(t) = \frac{(\alpha^2/\varsigma)_{\frac{\alpha}{1 + b_i}}}{1 + b_i} \cdot M_i(0) (v_i(0) L_i)_{\frac{\beta}{\gamma}} (R_i(0))_{\frac{\gamma}{\alpha}} \cdot e^{(\Omega_i - \rho) t},
\]  
(A.38)
where $M_i(0)$ and $v_i(0)$ are exogenously given. The only jump variable is the initial resource use $R_i(0)$, which can be determined from the solution of the optimal extraction problem:\footnote{Since $R = R_h + R_f$ and $\theta = \bar{\theta}$, the intertemporal resource constraint (14) can be written as $Q_0 = \int_0^{\infty} R_f(t) (1 + \bar{\theta}) \, dt$ and directly integrated to obtain $R_f(0)$ in (A.39), from which $R_h(0)$ can be obtained as $\frac{\partial R_f}{\partial \bar{\theta}}$.}

\[
R_h(0) = \frac{\bar{\theta}}{1 + \bar{\theta}} \rho Q_0 \quad \text{and} \quad R_f(0) = \frac{1}{1 + \bar{\theta}} \rho Q_0.
\]

Equations (A.38)-(A.39) yield explicit solutions for the time paths of physical output in the two countries and, in particular, imply that Home’s physical output relative to Foreign is a concave function of relative resource use in each point in time:

\[
\frac{Y_h(t)}{Y_f(t)} = \frac{\bar{\theta} - \frac{\gamma}{h}}{\gamma_h} \cdot \psi_0 \cdot e^{(\Omega_h - \Omega_f)t},
\]

where we have defined $\psi_0 \equiv \left[ \frac{M_h(0)}{M_f(0)} \left( \frac{1 + b_h}{1 + b_h} \right) \left( \frac{v_h(0) L_h}{v_f(0) L_f} \right) \right]^{\frac{\beta}{\alpha - \beta}}$, which is given at $t = 0$.

**Derivation of (35)-(36)-(37).** Substituting the definition $E^c_f = \bar{\sigma}^c_f P_Y^c Y_f$ in (A.27) we have

\[
\frac{P^h_Y Y_h}{P^f_Y Y_f} = \left( 1 - \frac{\bar{\sigma}^d_h}{1 - \bar{\sigma}^d_f} \right)^{\frac{1}{1 - \bar{\sigma}^d_h}}.
\]

Substituting $\bar{\sigma}^c_f = 1 + \bar{\gamma}_f \bar{\theta} - \bar{\sigma}^x_f - \bar{\sigma}^d_f$ from (A.29), $\bar{\sigma}^c_h = 1 - \bar{\gamma}_h - \bar{\sigma}^x_h - \bar{\sigma}^d_h$ from (A.20), and

\[
\frac{P^h_Y Y_h}{P^f_Y Y_f} = \frac{\bar{\gamma}_h}{1 - \bar{\gamma}_h} \bar{\theta}
\]

from (28), we obtain

\[
\bar{\gamma}_h \bar{\theta} = \frac{\epsilon}{(1 - \bar{\sigma}^x_f - \bar{\sigma}^d_f) + (1 - \bar{\sigma}^x_h - \bar{\sigma}^d_h) + \epsilon},
\]

which can be solved for $\bar{\theta}$ to get

\[
\bar{\theta} = \frac{\bar{\gamma}_h}{1 - \bar{\gamma}_h} \cdot \frac{\epsilon}{(1 - \bar{\sigma}^x_f - \bar{\sigma}^d_f) + (1 - \bar{\sigma}^x_h - \bar{\sigma}^d_h) + \epsilon}.
\]

Plugging $\frac{P^h_Y Y_h}{P^f_Y Y_f} = \frac{\bar{\gamma}_h \bar{\theta}}{1 - \bar{\gamma}_h}$ back into (A.42) yields (35), where $I_i \equiv \bar{\sigma}^c_i + \bar{\sigma}^d_i$ and (36) follows directly from (A.18) and (A.37). Since $\bar{\gamma}_i = \gamma(1 + \tau_i)^{-1}$, result (A.42) also implies (37).

**Proof of Proposition 3.** From (37) we have $\frac{\partial \bar{Y}_h}{\partial \bar{\theta}} > 0$ and $\frac{\partial \bar{Y}_f}{\partial \bar{\theta}} < 0$ and therefore $\frac{\partial \bar{Y}_h}{\partial \bar{\tau}_h} = \frac{\partial \bar{Y}_h}{\partial \bar{\theta}} \frac{\partial \bar{\theta}}{\partial \bar{\tau}_h} < 0$ and $\frac{\partial \bar{Y}_f}{\partial \bar{\tau}_h} = \frac{\partial \bar{Y}_f}{\partial \bar{\theta}} \frac{\partial \bar{\theta}}{\partial \bar{\tau}_h} > 0$. These results in turn imply $\frac{\partial (\bar{Y}_h/\bar{Y}_f)}{\partial \bar{\tau}_h} < 0$. From (35), we have $\frac{\partial (P^h_Y Y_h)/(P^f_Y Y_f)}{\partial \bar{\tau}_h} = 0$. Plugging this result into (34), we have

\[
\frac{\partial \bar{Y}_f}{\partial \bar{\tau}_h} = \frac{1}{1 + (P^h_Y Y_h)/(P^f_Y Y_f)} \cdot \frac{\partial (P^h_Y Y_h)/(P^f_Y Y_f)}{\partial \bar{\tau}_h} (1 - \bar{\gamma}_h) > 0
due to $\frac{\partial \bar{Y}_h}{\partial \bar{\tau}_h} < 0$. This implies

\[
\frac{\partial \bar{Y}_f}{\partial \bar{\tau}_h} = \frac{\partial (1 - \bar{\gamma}_h)}{\partial \bar{\tau}_h} < 0.
\]

**Proof of Proposition 4.** From (37) we have $\frac{\partial \bar{Y}_h}{\partial \bar{\theta}} > 0$. Equations (A.38)-(A.39) imply $\frac{\partial \bar{Y}_h}{\partial \bar{\theta}} > 0$ and $\frac{\partial \bar{Y}_f}{\partial \bar{\theta}} < 0$ and therefore $\frac{\partial \bar{Y}_h}{\partial \bar{\tau}_f} = \frac{\partial \bar{Y}_h}{\partial \bar{\theta}} \frac{\partial \bar{\theta}}{\partial \bar{\tau}_f} > 0$ and $\frac{\partial \bar{Y}_f}{\partial \bar{\tau}_f} = \frac{\partial \bar{Y}_f}{\partial \bar{\theta}} \frac{\partial \bar{\theta}}{\partial \bar{\tau}_f} < 0$. From (35), we have $\frac{\partial (P^h_Y Y_h)/(P^f_Y Y_f)}{\partial \bar{\tau}_f} = 0$, which also implies $\frac{\partial \bar{Y}_h}{\partial \bar{\tau}_f} = 0$ because $\bar{\gamma}_h$ is independent of $\tau_f$, and therefore $\frac{\partial \bar{Y}_f}{\partial \bar{\tau}_f} = 0$. \qed
Conditional efficiency in Home. By definition, the CE-allocation in Home solves

\[
\max_{\{E^c_h,E^x_h,E^d_h,R_h\}} \int_0^\infty e^{-\rho t} \cdot \ln((\omega/L_h) \cdot E^c_h) dt \text{ subject to }
\]

\[
Y_h = M_h X^\alpha_h (v_h L_h)\beta \ R_h^\gamma ,
\]

\[
E^x_h = P^b_Y \varsigma M_h X_h ,
\]

\[
P^b_Y Y_h = E^c_h + E^d_h + E^x_h + P_R R_h ,
\]

\[
\dot{M}_h = M_h \varphi_h \cdot \left[ E^d_h / (P^b_Y Y_h) \right] ,
\]

where \( \omega = \omega(P^b_Y, P^f_Y) \) is taken as given and symmetry across varieties is already imposed without any loss of generality. The first constraint is the final-good technology (1), the second is the intermediate-good technology with linear cost, the third is (23), the fourth is the R&D technology (11) with knowledge spillovers taken into account. Recalling that \( \sigma^d_h \equiv E^d_h / (P^b_Y Y_h) \) and combining the first three constraints, the problem becomes

\[
\max_{\{E^c_h,X_h,\sigma^d_h,R_h\}} \int_0^\infty e^{-\rho t} \cdot \ln((\omega/L_h) \cdot E^c_h) dt \text{ subject to }
\]

\[
P^b_Y M_h X^\alpha_h (v_h L_h)\beta R_h^\gamma \left( 1 - \sigma^d_h \right) = E^c_h + P^b_Y \varsigma M_h X_h + P_R R_h , \quad (A.43)
\]

\[
\dot{M}_h = M_h \varphi_h \sigma^d_h , \quad (A.44)
\]

where the controls are \( \{E^c_h,X_h,\sigma^d_h,R_h\} \) and the only state variable is \( M_h \). The current-value Hamiltonian is

\[
\ln \left( (\omega_h / L_h) \cdot E^c_h \right) + \mu'_h \cdot M_h \varphi_h \sigma^d_h + \\
+ \mu''_h \cdot \left[ P^b_Y M_h X^\alpha_h (v_h L_h)\beta R_h^\gamma \left( 1 - \sigma^d_h \right) - E^c_h - P^b_Y \varsigma M_h X_h - P_R R_h \right]
\]

where \( \mu'_h \) is the dynamic multiplier associated to (A.44) and \( \mu''_h \) is the static multiplier attached to (A.43). The optimality conditions read

\[
\frac{\partial}{\partial E^c_h} = 0 \rightarrow \frac{1}{E^c_h} = \mu''_h , \quad (A.45)
\]

\[
\frac{\partial}{\partial X_h} = 0 \rightarrow \left( 1 - \sigma^d_h \right) \alpha P^b_Y Y_h = P^b_Y \varsigma M_h X_h , \quad (A.46)
\]

\[
\frac{\partial}{\partial \sigma^d_h} = 0 \rightarrow \mu'_h M_h \varphi_h = \mu''_h P^b_Y Y_h , \quad (A.47)
\]

\[
\frac{\partial}{\partial R_h} = 0 \rightarrow \left( 1 - \sigma^d_h \right) \gamma P^b_Y Y_h = P_R R_h \quad (A.48)
\]

\[
\rho \mu'_h - \dot{\mu}'_h = \frac{\partial}{\partial M_h} \rightarrow \rho \mu'_h - \dot{\mu}'_h = \mu'_h \varphi_h \sigma^d_h + \mu''_h P^b_Y \left[ \frac{Y_h}{M_h} \left( 1 - \sigma^d_h \right) - \varsigma K_h \right] . (A.49)
\]
and imply  
\[ \dot{E}_h = \left[ 1 - \gamma \left( 1 - \sigma^d_h \right) \right] \cdot P^h_Y Y_h, \]  
(A.50) 
\[ \dot{E}^x_h = \alpha \left( 1 - \sigma^d_h \right) \cdot P^h_Y Y_h, \]  
(A.51) 
\[ \dot{E}^c_h = \beta \left( 1 - \sigma^d_h \right) \cdot P^h_Y Y_h, \]  
(A.52) 
\[ E^d_h = \sigma^d_h \cdot P^h_Y Y_h. \]  
(A.53) 

Substituting (A.46) and (A.47) in (A.49) we have 
\[ \frac{\dot{\mu}_h}{\mu_h} = \rho - \varphi_h \left[ 1 - \alpha \left( 1 - \sigma^d_h \right) \right]. \]  
(A.54) 

Time-differentiating (A.47) and using (A.54) we have 
\[ \frac{\dot{\mu}''_h}{\mu''_h} = \rho - \varphi_h (1 - \alpha) \left( 1 - \sigma^d_h \right) - \frac{\dot{P}^h_Y Y_h}{P^h_Y Y_h}, \]  
where we can substitute \( \mu''_h = 1/E^c_h \) from (A.45) to obtain 
\[ \frac{\dot{E}^c_h}{E^c_h} - \frac{\dot{P}^h_Y Y_h}{P^h_Y Y_h} = \varphi_h (1 - \alpha) \left( 1 - \sigma^d_h \right) - \rho. \]  
(A.55) 

From (??) we have 
\[ \frac{\dot{E}^x_h}{E^x_h} = \frac{\dot{P}^h_Y Y_h}{P^h_Y Y_h} = -\frac{\sigma^d_h}{1-\sigma^d_h} \]  
which can be combined with (A.55) to get 
\[ \sigma^d_h = \rho \frac{1 - \sigma^d_h}{\varphi_h (1 - \alpha)} - \varphi_h (1 - \alpha) \left( 1 - \sigma^d_h \right)^2. \]  
(A.56) 

Equation (A.56) is globally unstable around its unique steady state: ruling out explosive dynamics by standard arguments, the conditionally-efficient rate of investment in R&D is 
\[ \sigma^d_h = \frac{\varphi_h (1 - \alpha) - \rho}{\varphi_h (1 - \alpha)} \]  
\[ \text{and} \quad 1 - \sigma^d_h = \frac{\rho}{\varphi_h (1 - \alpha)}, \]  
(A.57) 

in each point in time. Substituting (A.57) in (A.51)-(A.52) we obtain 
\[ \tilde{\sigma}^x_h = \frac{\alpha \rho}{\varphi_h (1 - \alpha)} \]  
\[ \text{and} \quad \tilde{\sigma}_h = \frac{\beta \rho}{\varphi_h (1 - \alpha)}. \]  
(A.58) 

**Conditional efficiency in Foreign.** Following the same preliminary steps of the Home problem, the CE-allocation in Foreign solves 
\[ \max_{\{E^h, Xf, \sigma^d_f, R_h, R_f\}} \int_0^\infty e^{-\rho t} \cdot \ln((\omega/L_f) \cdot E^f_t) dt \]  
subject to 
\[ P^f_Y Mf X^f_j (v_f L_f) \beta R_f \gamma \left( 1 - \sigma^d_f \right) = E^f_c + P^f_Y \varsigma Mf X_f - P_R R_h, \]  
(A.59) 
\[ \dot{M}_f = M_f \varphi_f \sigma^d_f, \]  
(A.60) 
\[ \dot{Q} = -R_h - R_f \]  
(A.61)

\[ ^{15} \text{Plugging (A.48) in constraint (23) we have (A.50). Plugging (A.46) in technology } E^h = P^h_Y \varsigma M_h K_h \text{ yields (A.51). Plugging (A.46) and (A.48) in (A.43) we have (A.52). Equation (A.53) is determined residually by } E^d_h = \dot{E}_h - \dot{E}_h - \dot{E}_h. \]
where (A.59) follows from (24) and, differently from Home, we have the resource constraint (A.61) and also exported resources \( R_h \) as an additional control. The state variables are \( M_f \) and the resource stock \( Q \). The Hamiltonian is
\[
\ln \left( \frac{\omega}{L_f} \cdot E^c_f \right) + \mu'_f \cdot M_f \varphi_f \sigma^d_f + \\
+ \mu''_f \cdot \left[ P^f_Y M_f X_f^\alpha (v_f L_f)^\beta R_f^\gamma \left( 1 - \sigma^d_f \right) - E^c_f - P^f_Y \varsigma M_f X_f + R_h R_f \right] + \\
+ \mu''_f \cdot (-R_h - R_f)
\]
where \( \mu'_f \) is the dynamic multiplier associated to (A.60), \( \mu''_h \) is the Lagrange multiplier attached to (A.59), and \( \mu''_f \) is the dynamic multiplier associated to (A.61). The first order conditions read
\[
\frac{\partial}{\partial E^c_f} = 0 \rightarrow \frac{1}{E^c_f} = \mu''_f, \tag{A.62}
\]
\[
\frac{\partial}{\partial X_f} = 0 \rightarrow \left( 1 - \sigma^d_f \right) \alpha P^f_Y Y_f = P^f_Y \varsigma M_f X_f, \tag{A.63}
\]
\[
\frac{\partial}{\partial \sigma^d_f} = 0 \rightarrow \mu'_f M_f \varphi_f = \mu''_f P^f_Y Y_f, \tag{A.64}
\]
\[
\frac{\partial}{\partial R_h} = 0 \rightarrow \mu''_f \cdot P_R = \mu''_f R_f, \tag{A.65}
\]
\[
\frac{\partial}{\partial R_f} = 0 \rightarrow \mu'_f \cdot \left( 1 - \sigma^d_f \right) \gamma P^f_Y Y_f = \mu''_f R_f, \tag{A.66}
\]
\[
\rho \mu'_f - \mu'_f = \frac{\partial}{\partial M_f} \rightarrow \rho \mu'_f - \mu'_f = \mu'_f \varphi_f \sigma^d_f + \mu''_f P^f_Y \left[ \frac{Y_f}{M_f} \left( 1 - \sigma^d_f \right) - \varsigma K_f \right], \tag{A.67}
\]
\[
\rho \mu''_f - \mu''_f = \frac{\partial}{\partial Q} \rightarrow \rho \mu''_f - \mu''_f = 0. \tag{A.68}
\]
Notice that, from (A.65)-(A.66) and definition \( R_h = \theta R_f \), we have
\[
P_R \tilde{R}_f = \left( 1 - \sigma^d_f \right) \gamma P^f_Y \tilde{Y}_f, \tag{A.69}
\]
\[
P_R \tilde{R}_h = \left( 1 - \sigma^d_f \right) \gamma \tilde{\theta} \cdot P^f_Y \tilde{Y}_f, \tag{A.70}
\]
so that expenditures equal\(^{16}\)
\[
\tilde{E}_f = \left[ 1 + \left( 1 - \sigma^d_f \right) \gamma \tilde{\theta} \right] \cdot P^f_Y \tilde{Y}_f, \tag{A.71}
\]
\[
\tilde{E}^x_f = \alpha \left( 1 - \sigma^d_f \right) \cdot P^f_Y \tilde{Y}_f, \tag{A.72}
\]
\[
\tilde{E}^c_f = \left( 1 - \alpha + \gamma \tilde{\theta} \right) \left( 1 - \sigma^d_f \right) \cdot P^f_Y \tilde{Y}_f, \tag{A.73}
\]
\[
\tilde{E}^d_f = \sigma^d_f \cdot P^f_Y \tilde{Y}_f. \tag{A.74}
\]
\(^{16}\)Plugging (A.70) in (24) yields (A.71). Plugging (A.63) in technology \( E^k_f = P^f_Y \varsigma M_f K_f \) yields (A.72). Plugging (77) and (A.70) in (A.59) we have (A.73). Equation (A.74) is determined residually by \( \tilde{E}^d_f = \tilde{E}_f - \tilde{E}^k_f - \tilde{E}^c_f \).
Before deriving the explicit value of $\tilde{\sigma}_f^d$ we show that the efficient relative resource use $\bar{\theta}$ is constant over time. From the balanced trade condition (A.26), we have $P_R R_h + (1 - \epsilon) E_h^c = \epsilon E_f^c$ where we can use (A.52) and (A.73) to eliminate $E_h^c$ and $E_f^c$, respectively, and also use (A.48) to eliminate $P_R R_h$, obtaining

$$\frac{1 - \tilde{\sigma}_h^d}{1 - \tilde{\sigma}_f^d} = \frac{P_Y^h Y_h}{P_Y^f Y_f} = \frac{\epsilon \left(1 - \alpha + \gamma \bar{\theta}\right)}{\gamma + (1 - \epsilon) \beta},$$

(A.75)

where tildas denote conditionally-efficient values. Taking the ratio between (A.48) and (A.70) we have $\bar{\theta} = \frac{1 - \tilde{\sigma}_h^d}{1 - \tilde{\sigma}_f^d} = \frac{P_Y^h Y_h}{P_Y^f Y_f}$.

Combining (A.76) with (A.75) we obtain

$$\bar{\theta} = \frac{\epsilon}{1 - \epsilon} \cdot \frac{1 - \alpha}{\gamma + \beta} = \frac{\epsilon}{1 - \epsilon}.$$  

(A.77)

Result (A.77) implies that $\bar{\theta}$ is constant. Now go back to (A.67) and substitute (A.63)-(A.64) to re-write it as

$$\frac{\dot{\mu}_f^d}{\mu_f^d} = \rho - \varphi_f \left[1 - \alpha \left(1 - \tilde{\sigma}_f^d\right)\right].$$  

(A.78)

Time-differentiating (A.64) and substituting (A.62)-(A.60), we obtain

$$\frac{\dot{\mu}_f^d}{\mu_f^d} = -\frac{\dot{E}_f^c}{E_f} + \frac{P_Y^f Y_f}{P_Y Y_f} - \varphi_f \tilde{\sigma}_f^d$$

which can be combined with (A.78) to obtain

$$\frac{\dot{E}_f^c}{E_f} - \frac{P_Y^f Y_f}{P_Y Y_f} = \varphi_f \left[1 - \alpha \left(1 - \tilde{\sigma}_f^d\right)\right] - \rho.$$  

(A.79)

Since $\bar{\theta}$ is constant by (A.77), time-differentiation of (A.73) yields $\frac{\dot{E}_f^c}{E_f} - \frac{P_Y^f Y_f}{P_Y Y_f} = -\frac{\dot{\sigma}_f^d}{1 - \tilde{\sigma}_f^d}$.

Plugging this result in (A.80) we obtain the usual equilibrium relation (see (A.56) above for Home) which can be solved for the steady-state level

$$\tilde{\sigma}_f^d = \frac{\varphi_f (1 - \alpha) - \rho}{\varphi_f (1 - \alpha)}$$
or $$1 - \tilde{\sigma}_f^d = \frac{\rho}{\varphi_f (1 - \alpha)}.$$  

(A.80)

Substituting (A.80) in (A.72)-(A.74) we obtain

$$\tilde{x}_f = \frac{\alpha \rho}{\varphi_f (1 - \alpha)}$$ and $$\tilde{\sigma}_f^d = \frac{\rho \left(1 - \alpha + \gamma \bar{\theta}\right)}{\varphi_f (1 - \alpha)}.$$  

(A.81)

**Derivation of (40).** Equation (40) is proved in (A.77).
Derivation of (41)-(43). Efficient taxes are obtained by equalizing efficient and equilibrium values of \((\sigma_i^e, \sigma_i^d, \sigma_i^c)\). First, results (A.58) and (A.81) imply \(\tilde{\sigma}_i = \frac{\alpha \rho}{\varphi_i(1-\alpha)}\) in both countries. Imposing the equality between the efficient values \(\tilde{\sigma}_i^e\) and the competitive-equilibrium values \(\tilde{\sigma}_i^e = \frac{\alpha^2}{1+b_i}\) derived in (A.18), we obtain the efficient tax on intermediates’ purchases \(\tilde{b}_i\) in (42). Second, results (A.57) and (A.80) imply \(\tilde{\sigma}_i^d = \frac{\varphi_i(1-\alpha) - \rho}{\varphi_i(1-\alpha)}\) in both countries. Imposing the equality between \(\tilde{\sigma}_i^d\) and the competitive-equilibrium values \(\tilde{\sigma}_i^d = \frac{\rho(1-\alpha)(1+\alpha) - \rho(1+b_i)}{\varphi_i(1+b_i)}\) derived in (A.37), and substituting \(\tilde{b}_i\) by (42), we obtain the efficient subsidy \(\tilde{a}_i\) in (41). Now consider Home: from (A.58) we have \(\tilde{\sigma}_h^c = \frac{\beta \rho}{\varphi_h(1-\alpha)}\) whereas (A.23) implies \(\tilde{\sigma}_h^c = 1 - \tilde{\gamma}_h - \tilde{\sigma}_h^d - \tilde{\sigma}_h^c\). Setting \(\tilde{\sigma}_h^c = \tilde{\sigma}_h^c\) and imposing that \(\tilde{\sigma}_h^e = \tilde{\sigma}_h^c\) and \(\tilde{\sigma}_h^d = \tilde{\sigma}_h^d\) by virtue of (41)-(42), we obtain

\[
\tilde{\gamma}_h = \frac{\varphi_h(1-\alpha) - \rho}{\rho}.
\]

where the last term follows from \(\tilde{\sigma}_h^d\) and \(\tilde{\sigma}_h^c\) derived in (A.57) and (A.58). Rearranging terms and solving for \(\tilde{\gamma}_h\) we obtain \(\tilde{\gamma}_h = \frac{\beta \rho}{\varphi_h(1-\alpha)}\), which implies the efficient resource tax for Home

\[
\tilde{\tau}_h = \frac{\varphi_h(1-\alpha) - \rho}{\rho}.
\]

The optimal resource tax in Foreign \(\tilde{\tau}_f\) then follows from (A.76). Since \(1 + \tilde{\tau}_h = 1 - \tilde{\sigma}_h^d\) by (A.82), the only way to satisfy \(\tilde{\theta} = \tilde{\theta}\) in equations (28) and (A.76) is to set \(1 + \tilde{\tau}_h = 1 - \tilde{\sigma}_h^d = \frac{1}{\rho} \varphi_h(1-\alpha)\), which proves (43). It can be easily verified that, residually, (41)-(43) imply \(\tilde{\sigma}_f^c = \tilde{\sigma}_f^d\).

Derivation of (44)-(46). Defining the constant \(\tilde{\varepsilon}_i \equiv (\epsilon/L_i) (\frac{1-\epsilon}{\epsilon})^{1-\epsilon}\) and recalling that \(E^c_i = \tilde{\sigma}_i^c P_i Y_i\) by definition (A.17), present-value utility (18) reads

\[
U_i = \int_0^\infty e^{-\rho t} \cdot \ln \left[ \tilde{\varepsilon}_i \left( \frac{\tilde{\sigma}_i^c P_i Y_i}{(P_i^h)^{\epsilon}(P_i^Y)^{1-\epsilon}} \right) \right] dt.
\]

Plugging the respective country indices, we obtain

\[
U_h = \int_0^\infty e^{-\rho t} \cdot \ln \left[ \tilde{\varepsilon}_h \left( \frac{P_i^h}{P_i^Y} \right) \tilde{\sigma}_h^c \right] dt \quad \text{and} \quad U_f = \int_0^\infty e^{-\rho t} \cdot \ln \left[ \tilde{\varepsilon}_f \left( \frac{P_i^f}{P_i^Y} \right) \tilde{\sigma}_f^c \right] dt.
\]

Substituting \(P_i^h(t)/P_i^Y(t) = [P_i^h(0)/P_i^Y(0)] e^{(\Omega_f - \Omega_i) t}\) and \(Y_i(t) = Y_i(0) e^{(\Omega_i - \rho) t}\) and collecting the terms to isolate \(P_i^h(0), P_i^f(0), Y_i(0), Y_f(0)\) and \((\tilde{\sigma}_h^c, \tilde{\sigma}_f^c)\), we can define the constants

\[
\kappa_h \equiv \int_0^\infty e^{-\rho t} \cdot \ln \left[ e^{(\Omega_f - \Omega_i - \rho)(\Omega_f - \Omega_i) t} \right] dt + \frac{1}{\rho} \ln \tilde{\varepsilon}_h,
\]

\[
\kappa_f \equiv \int_0^\infty e^{-\rho t} \cdot \ln \left[ e^{(\Omega_f - \Omega_i - \rho)(\Omega_f - \Omega_i) t} \right] dt + \frac{1}{\rho} \ln \tilde{\varepsilon}_f,
\]

and rewrite \(U_h\) and \(U_f\) as in (44)-(45). The first expression in (46) follows from (A.22) and \(I_h \equiv \tilde{\sigma}_h^e + \tilde{\sigma}_h^d\). Rewriting (A.29) as \(\tilde{\sigma}_f = 1 + \tilde{\gamma}_f \theta - \tilde{\sigma}_h^e - \tilde{\sigma}_h^d\) and substituting \(I_f \equiv \tilde{\sigma}_f^e + \tilde{\sigma}_f^d\) we obtain the last expression in (46).
Utility-Tax relationship in Home. Setting $t = 0$ in (28) and solving for $p_0 = P_Y^h(0)/P_Y^f(0)$ we obtain
\[
p_0 = \theta \cdot \frac{1 + \tau_h Y_f(0)}{1 + \tau_f Y_h(0)} = \frac{\epsilon}{1 - \epsilon} \cdot \frac{1 - I_f}{1 - I_h} \cdot \psi_0^{-1} \cdot \bar{\theta}^{-\frac{\gamma}{1 - \alpha}}, \tag{A.84}
\]
where the last term follows from (A.40). Equation (A.84) implies
\[
\frac{\partial p_0}{\partial \tau_h} \cdot \frac{1}{p_0} = -\frac{\gamma}{1 - \alpha} \cdot \frac{\partial \bar{\theta}}{\partial \tau_h} \cdot \frac{1}{\bar{\theta}}. \tag{A.85}
\]
From (A.38)-(A.39) we also have
\[
\frac{\partial Y_h(0)}{\partial \tau_h} \cdot \frac{1}{Y_h(0)} = \frac{\gamma}{1 - \alpha} \cdot \frac{\partial \bar{\theta}}{\partial \tau_h} \cdot \frac{1}{\bar{\theta}}. \tag{A.86}
\]
Differentiating (44) we have $\rho \frac{\partial U_h}{\partial \tau_h} = \frac{\partial \bar{\theta}}{\partial \tau_h} \frac{1}{\sigma_h} + (1 - \epsilon) \frac{\partial p_0}{\partial \tau_h} \frac{1}{p_0} + \frac{\partial Y_h(0)}{\partial \tau_h} \frac{1}{Y_h(0)}$, where we can plug $\frac{\partial \bar{\theta}}{\partial \tau_h} \frac{1}{\sigma_h} = -\frac{\partial \gamma_h}{\partial \tau_h} \frac{1}{\sigma_h}$ from (46) together with (A.85)-(A.86) to obtain
\[
\rho \frac{\partial U_h}{\partial \tau_h} = -\frac{\partial \gamma_h}{\partial \tau_h} \frac{1}{\gamma_h} + \frac{\gamma}{1 - \alpha} \cdot \frac{\partial \bar{\theta}}{\partial \tau_h} \cdot \frac{1}{\bar{\theta}} \left( \frac{1 - \epsilon}{1 + \bar{\theta}} - 1 \right). \tag{A.87}
\]
From (37) we have $\frac{\partial \bar{\theta}}{\partial \tau_h} \frac{1}{\gamma_h} = \frac{\partial \gamma_h}{\partial \tau_h} \frac{1}{\gamma_h}$ and (A.87) can be written as
\[
\rho \frac{\partial U_h}{\partial \tau_h} = -\frac{\partial \gamma_h}{\partial \tau_h} \frac{1}{\gamma_h} \cdot \left( \frac{\gamma_h}{\sigma_h} - \frac{\gamma}{1 - \alpha} \cdot \left( \frac{1 - \epsilon}{1 + \bar{\theta}} - 1 \right) \right). \tag{A.88}
\]
Recalling that $\frac{\partial \gamma_h}{\partial \tau_h} \frac{1}{\gamma_h} = -\frac{1}{1 + \tau_h}$ and substituting $\sigma_h$ by (46) in the above expression, we obtain
\[
\rho \frac{\partial U_h}{\partial \tau_h} = \frac{\gamma}{1 + \tau_h} \left\{ \frac{1}{(1 + \tau_h)(1 - \gamma_h - I_h)} - \frac{1}{1 - \alpha} \cdot \left( \frac{1 - \epsilon}{1 + \bar{\theta}} - 1 \right) \right\}. \tag{A.89}
\]
Define the two terms in curly brackets as $\Upsilon^a(\tau_h) \equiv 1/[(1 + \tau_h)(1 - \gamma_h - I_h)]$ and $\Upsilon^b(\tau_h) \equiv \frac{1}{1 - \alpha} \cdot \left( \frac{1}{1 + \bar{\theta}} - (1 - \epsilon) \right)$, respectively. We have
\[
\rho \frac{\partial U_h}{\partial \tau_h} = \frac{\gamma}{1 + \tau_h} \left[ \Upsilon^a(\tau_h) - \Upsilon^b(\tau_h) \right], \tag{A.90}
\]
where $\Upsilon^a(\tau_h)$ is strictly increasing and $\Upsilon^b(\tau_h)$ is strictly decreasing in $\tau_h$. Moreover, $\lim_{\tau_h \to \infty} \bar{\theta} = 0$ implies that
\[
\lim_{\tau_h \to \infty} \Upsilon^a(\tau_h) = 0 \quad \text{and} \quad \lim_{\tau_h \to \infty} \Upsilon^b(\tau_h) = \frac{\epsilon}{1 - \alpha} > 0. \tag{A.91}
\]
Result (A.91) combined with $\partial \Upsilon^a/\partial \tau_h < 0$ and $\partial \Upsilon^b/\partial \tau_h > 0$ implies two results. First, there can only exist a unique finite value $\bar{\tau}_h$ satisfying $\Upsilon^a(\bar{\tau}_h^h) = \Upsilon^b(\bar{\tau}_h^h)$. Second, the value $\bar{\tau}_R^h$ satisfying $\Upsilon^a(\bar{\tau}_R^h) = \Upsilon^b(\bar{\tau}_R^h)$ is associated with a maximum in $U_h$ because (i) $\Upsilon^a = \Upsilon^b$ implies $\frac{\partial U_h}{\partial \tau_h} = 0$, and (ii) $\partial \Upsilon^a/\partial \tau_h < 0$ and $\partial \Upsilon^b/\partial \tau_h > 0$ imply that $\frac{\partial U_h}{\partial \tau_h} > 0$ for any $\tau_h < \bar{\tau}_R^h$ as well as $\frac{\partial U_h}{\partial \tau_h} < 0$ for any $\tau_h > \bar{\tau}_R^h$. 

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Utility-Tax relationship in Foreign and derivation of (48). From (37), $\tilde{\gamma}_f \tilde{\theta}$ is independent of $\tau_f$. As a consequence,

$$\frac{\partial \tilde{\sigma}_f}{\partial \tau_f} \frac{1}{\tilde{\sigma}_f} = \frac{\partial \tilde{\gamma}_f}{\partial \tau_f} \frac{1}{\tilde{\sigma}_f} = 0.$$  (A.92)

Result (A.84) implies

$$\frac{\partial p_0}{\partial \tau_f} \cdot \frac{1}{p_0} = -\frac{\gamma}{1-\alpha} \cdot \frac{\tilde{\theta}}{\tilde{\sigma}_f} \cdot \frac{1}{\tilde{\theta}}.$$  (A.93)

From (A.38)-(A.39) we have

$$\frac{\partial Y_f(0)}{\partial \tau_f} \cdot \frac{1}{Y_f(0)} = -\frac{\gamma}{1-\alpha} \cdot \frac{\tilde{\theta}}{1+\tilde{\theta}} \cdot \frac{1}{\tilde{\theta}}.$$  (A.94)

Differentiating (45) we have

$$\rho \frac{\partial U_f}{\partial \tau_f} = \frac{\partial \varphi_f}{\partial \tau_f} \frac{1}{\sigma_f} - \epsilon \frac{\partial p_0}{\partial \tau_f} \frac{1}{p_0} + \frac{\partial Y_f(0)}{\partial \tau_f} \frac{1}{Y_f(0)},$$

where we can plug (A.92), (A.93) and (A.94) to obtain

$$\rho \frac{\partial U_f}{\partial \tau_f} = \frac{\gamma}{1-\alpha} \cdot \frac{\tilde{\theta}}{1+\tilde{\theta}} \cdot \frac{1}{\tilde{\theta}} \left( \epsilon - \frac{\tilde{\theta}}{1+\tilde{\theta}} \right).$$  (A.95)

From (37) we have $\frac{\partial \tilde{\theta}}{\partial \tau_f} \cdot \frac{1}{\tilde{\theta}} > 0$, which implies that $U_f(\tau_f)$ is concave in $\tau_f$. Setting to zero the term in brackets in (A.95), the maximum is characterized by condition (48) in the text.

**Proof of Proposition 5.** In a symmetric CE-allocation we have $\tilde{\theta} = \tilde{\theta} = \frac{\epsilon}{1-\epsilon}$. In Home, this implies $\partial U_h/\partial \tau_h > 0$ because $\Upsilon^a (\tau_h) > \Upsilon^b (\tau_h)$ in (A.90). In Foreign, this implies $\partial U_f/\partial \tau_f = 0$ from (A.95).

**Proof of Proposition 5.** In a laissez-faire equilibrium, $\tilde{\theta}$ is given by (39). If $\varphi_h = \varphi_f$ we have $\tilde{\theta} = \frac{\epsilon}{1-\epsilon}$ and $\partial U_f/\partial \tau_f = 0$ and $\partial U_h/\partial \tau_h > 0$ are proved exactly as in Proposition 5. If $\varphi_h > \varphi_f$, we have $\tilde{\theta} > \frac{\epsilon}{1-\epsilon}$, which implies $\partial U_h/\partial \tau_h > 0$ because $\Upsilon^a (\tau_h) > \Upsilon^b (\tau_h)$ in (A.90), and $\partial U_f/\partial \tau_f < 0$ from (A.95). If $\varphi_h < \varphi_f$, we have $\tilde{\theta} < \frac{\epsilon}{1-\epsilon}$, which implies $\partial U_f/\partial \tau_f > 0$ from (A.95) whereas $\partial U_h/\partial \tau_h$ is generally ambiguous in (A.90).

**References**


