Fossil Fuel Extraction and Climate Policy:  
a Review of the Green Paradox with Endogenous Resource Exploration  *
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Abstract
Policies aimed at reducing emissions from fossil fuels may increase climate damages. This “Green Paradox” emerges if resource owners increase near-term extraction in reaction to stricter policy measures in the future. An increasing resource tax may generate this paradox within a basic resource extraction model based on Hotelling (1931). This article shows that the emergence of the Green Paradox within this framework relies on the non-existence of a backstop technology and fixed fossil fuel resources. In doing this, it initially presents a basic exhaustible resource model which includes a backstop technology and shows that the implementation of a specific sales tax path is effective in mitigating global warming. Secondly, it considers the case of costly exploration activities being introduced within the basic model and accounts for the real world condition that the location of fossil fuels is unknown. Under this condition, an increasing cash flow tax is effective in dealing with climate change if policy makers commit to a high initial tax level and to a specific range of growth rates.

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1 Introduction

Public policies aimed at limiting global warming have been introduced by a considerable number of countries ever since the beginning of the Nineties (Fullerton et al. 2010, 427 and Anderson 2001, 16). Their main target has been the reduction of the use of fossil fuels as they represent a major contributor to the increase of atmospheric greenhouse gas concentrations, the principal driver of global warming (IPCC 2007, 36 and 39). Standard policy measures implemented on the national and international level generally consist of a tax, a cap-and-trade system, a subsidy or a regulation to promote low carbon or highly-efficient technologies (Fölster and Nyström 2010, 224). In the light of the scientific evidence concerning global warming, the efforts undertaken are reasonable and have been insufficient. This emerges from the latest Assessment Report issued by the Intergovernmental Panel of Climate Change that forecasts a temperature increase between 1.8 and 4.0°C at the end of the century, if no further policies are introduced, which would have unpredictable consequences for human and natural systems (IPCC 2007, 44/45 and 48-54).

Economists have raised several concerns regarding the effectiveness of the standard policy approach in terms of worldwide emission control. One important argument was formulated in 1992 by Sinclair and focused on a carbon tax. He argued that a carbon tax may be redundant or even damaging for climate change issues. The main insight of this comes from the theory of exhaustible resource extraction (Sinclair 1992, 41). Within a standard Hotelling model, the author derived that a carbon tax is only effective if it is decreasing over time. The reason for this is that fossil fuel owners have a finite quantity of resources and hence their decision of how much to extract today depends not only on the current tax rate, but on the whole time path of the tax.

In a more recent contribution, Hans-Werner Sinn (2008) used the Hotelling model to analyze the effect of the current climate policy approach. Most of the climate policies that are currently being enforced are aimed at reducing the demand for fossil fuel resources - such as a tax on fuel consumption or a subsidy for low carbon
energy supply – and thereby lead to a decline in fossil fuel prices (ibid., 388). Sinn (2008) warned that if fossil fuel owners expect climate policies to become stricter over time and hence expect prices in the farer future to decline more than in the present and nearer future, they may decide to bring forward the sales of their fossil fuels. Consequentially, the accumulation of atmospheric greenhouse gases occurs faster and worsens the problem of climate change – a phenomenon that he named a “Green Paradox” (ibid., 380). Within a Hotelling model with a given fossil fuel stock, stock-dependent extraction costs and no backstop technology, Sinn (2008) analyzed different tax schemes and concluded that an increasing cash flow tax as well as a sales tax that increases over time by more than the interest rate generates a Green Paradox. While total fossil fuel supply is not impacted by these policy measures, their implementations lead to a change in the expected (producer) price path such that it is optimal for fossil fuel owners to extract faster. Sinn (2008) applied the result of the policy analysis regarding the sales tax to a range of policies that lead to a decrease of fossil fuel demand based on the argument that they constitute an equivalent effect on the producer price. On the basis of this rationale, Sinn (ibid., 388) concluded that “measures to reduce carbon demand, ranging from taxes on fossil fuel consumption to the development of alternative energy sources [...] will not mitigate the problem of global warming.”

The emergence of the Green Paradox within the model of Sinn (2008) is dependent upon the non-existence of a backstop technology and the assumption that firms are endowed with the resource stock. The latter assumption of endowments of all of the resource stock is unrealistic since fossil fuels are stored underground, and their identification through exploration activities is necessary (Bhattacharyya 2011, 191). This article extends the framework of Sinn (2008) by providing two models that respectively allow for the existence of a backstop technology and exploration activities. In both models, the rationale for the emergence of a Green Paradox is weakened.

Initially, the article adopts an extraction model that only differs to the one employed by Sinn (2008) in so far that a backstop technology exists. Within this model I derive the effect of a sales tax that grows with the interest rate. The effect of such a tax significantly changes, if a backstop technology exists compared to the
results obtained by Sinn (2008). In particular, while the policy is neutral in the scenario without a backstop technology, in this case, the policy measure leads to a lower amount of resources extracted at each point in time compared to the baseline scenario and hence represents an effective instrument for climate change mitigation.

Second, the article’s main contribution is to derive analytically the effect of standard policy measures in an extended Hotelling framework to include the characteristic that firms have to invest in exploration activities. In general, tools to identify fossil fuels include geographical studies and exploratory drilling. These activities involve significant costs and a high risk regarding the success of finding fossil fuel resources (Bhattacharyya 2011, 191). The implementation of a tax or any demand reducing policy measure decreases the profits obtained from fossil fuel sales. I argue that this lowers the incentive to invest in exploration activities as their expected return decreases and thereby shrinks the total amount of fossil fuels available. This is expected to have a positive impact on the climate as overall emissions are reduced.

To show this formally, I propose a model that utilises an extraction-exploration framework in line with the Hotelling model employed by Sinn (2008) in the sense that a representative firm extracts its resource stock over time in a competitive economy without a backstop technology. While the extraction process within Sinn (2008) involves costs, the extraction-exploration model assumes that there are no extraction costs for the sake of mathematical convenience, but without a loss of generality. Furthermore, it includes an endogenous resource base which depends on the firm’s exploration efforts and this assumption is drawn from the literature. As in Lasserre (1991, 104-107) and Heaps and Helliwell (1985, 452/453), a representative firm must invest during an initial phase of exploration and, in a second phase, extract the amount of available resources that depends on the investment in exploration previously incurred. The firm within this model aims to maximize total discounted profits from extraction net of cumulative expenditures in exploration. Within this framework, I explore the impact of an increasing cash flow tax on the supply decision.

A key result of the analysis is that an increasing cash flow tax tends to reduce
climate damages when exploration is modeled endogenously. This clearly stands
in contrast to the well-known effect of an increasing cash flow tax within a basic
Hotelling framework with an exogenous resource base. While total extraction and
hence total emissions are not impacted by the tax in a framework with an exogenous
resource stock, it decreases in the exploration-extraction framework. An increasing
cash flow tax may still result in increased short-term emissions, but the reduction
in total emissions is likely to reduce climate change damages compared to the
business-as-usual scenario. The results show that short-term emissions are likely
to increase, the lower the initial tax burden and the higher the growth rate of the
tax. In the long-run, the risk of the appearance of a Green Paradox is still present,
but can be avoided by setting a high initial tax level and/or a very high or very
low growth rate of the tax\textsuperscript{1}. By deriving the conditions for the appearance of a
Green Paradox, I show that a specific tax scheme is more likely to be effective,
the lower the sensitivity of climate damages to emissions in early periods, and
the higher the strength of the absorptive capacity of emissions from the environment.

The result obtained in this contribution is in line with the literature as it has
long been recognized that the need for capital expenditures modifies the impact
of taxation on fossil fuel supply behavior compared to when the fossil fuel stock
is exogenous. Dasgupta, Heal, and Stiglitz (1980, 18) emphasized that a tax re-
duces profits from fossil fuel sales and thereby lowers the overall return on capital
employed in the fossil fuel industry. In turn, this negatively affects the incentive
to allocate funds for exploration and development activities for new mines/wells
which decreases extractable fossil fuel resources.

The literature provides several simulations to assess the effect of different tax
schemes within the standard extraction-exploration model provided by the Pindyck

\textsuperscript{1}The level of the growth rate has an ambiguous impact on the climate. A very low growth rate
leads to a small temporal redistribution such that a Green Paradox is not generated. However, if
the growth rate is too high, the level of the tax increases significantly and leads to a reduction
of expected profits from extraction such that the investments in exploration activities shrink
significantly. The reduction of the total amount of extraction may be sufficiently high to offset
the increase in initial extraction generated by a high growth rate.
model (1978) (refer to Chakravorty et al. 2010, Deacon 1993, Kunce et al. 2003, Yücel 1986). In this extended Hotelling model a competitive firm has to decide at each point in time how much to extract and how much to invest in exploratory activity across a finite time horizon. Exploration is the means to maintain or increase the resource base from which the firm can extract and it further affects extraction costs as they have a negative relationship with the size of known reserves. While most of these studies simulate the impact of a state’s taxes on the national supply (and hence treat the price path as exogenous), only the contribution of Yücel (1986) examines the impact of a worldwide tax on global fossil fuel supply. To do this, he analysed the effect of a constant severance tax on the supply path of a representative competitive firm. Yücel (1986) shows that both, extraction and exploration activity, are reduced at any point in time compared to the business-as-usual scenario (Yücel 1986, 205).

Venables (2011) recently extended the standard Hotelling model (with no backstop technology) to incorporate costly exploration and development activities. Firms can add new discoveries to their fossil fuel stock by opening new fields, hence incurring capital expenses. The firm has to decide how much to extract at each point in time and when to open new fields. Venables (ibid., 17) showed that a permanent decrease in demand (equivalent to the implementation of a sales tax) leads to a smaller amount of field openings and a decrease in extraction activities at all points in time. Applying his result to the Green Paradox discussion, it can be stated that a demand decrease within this framework does not lead to a Green Paradox, instead it has a positive impact on the state of the climate as emissions at any time point are below the baseline level. However, a constant demand decrease has also positive effects on the climate in a basic Hotelling model with extraction costs, so it will be interesting to extend the framework of Venables to derive the effect of a demand decrease that is becoming heavier over time. In addition, he derived the impact of a permanent reduction in the rate of growth of the expected price path, a situation comparable to an increasing fossil fuel tax. The result is a temporary increase in fossil fuel extraction, but a decrease in total extraction, hence the overall effect on the climate is ambiguous.
The layout of the article is the following. Chapter 2 presents a review of contributions that employ a similar Hotelling framework to the one within Sinn (2008), except for the inclusion of a backstop technology, to assess climate policy measures. Subsequently, the framework presented by Hoel (2010) is used to assess the impact of an increasing sales tax on emissions. Chapter 3 presents a model that is similar to the one within Sinn (2008) except for the inclusion of costly exploratory activity. The inclusion of exploration has been based on the approach of Lasserre (1991, 104-106). In this framework, the effect of an increasing cash flow tax is analyzed and compared to the result derived by Sinn (2008) who showed that an increasing cash flow tax undoubtedly leads to a Green Paradox. Chapter 4 provides the main conclusions of this article.

2 The existence of a backstop technology and the Green Paradox

The emergence of the Green Paradox within the model proposed by Sinn (2008) is dependent upon the non-existence of a backstop technology. According to Nordhaus (1973, 532) who coined the term, a backstop is a technology that constitutes a perfect substitute to the fossil fuel energy resource and is not constrained by exhaustibility. The existence of a backstop technology together with increasing extraction costs is expected to lead to an economic exhaustion of fossil fuels, rather than a physical exhaustion of the resource (Perman et al. 2003, 524/525). Perman et al. (2003) noted that this is due to the existence of a choke price that coincides with the cost for producing the backstop technology. Upon reaching this price, demand for fossil fuels will be completely replaced by the backstop technology.

The existence of a backstop technology in an economy such as that considered in Sinn (2008) where extraction costs increase with cumulative extraction implies that the fossil fuel stock will not be exhausted completely. Instead, fossil fuels that can only be extracted by incurring costs that are higher than the choke price, are left in the ground. Several recent contributions have assessed the effect of various climate policy measures on extraction behavior while allowing for a backstop technology.
in a Hotelling framework of resource extraction. The next section reviews some of these contributions, followed by the presentation of a Hotelling framework as employed in Sinn (2008), but with the existence of a backstop technology. Using this model, the impact of an increasing sales tax on the climate is analyzed.

2.1 Literature review

The impact of a climate policy which decreases the costs of a backstop technology has been analyzed by Gerlagh (2011) and Van der Ploeg and Withagen (2011). From the introductory explanation to this chapter it is clear that any decrease in the cost to produce the backstop technology means that the choke price decreases and hence the fossil fuel price for the firm in the terminal period will decrease as well. According to Neher (1990, 323), in the terminal period production ceases, that is, the firm’s optimal decision is to extract nothing given by the optimality condition to produce a quantity for which the choke price just covers the marginal costs of extraction. From this condition, the ‘cutoff’ grade can be determined, that is, the amount of fossil fuels remaining in the ground (ibid.). This is true because in the last period, the price equals marginal extraction costs and the size of the marginal extraction costs determines the amount of fossil fuel remaining in the ground as they are decreasing in the stock. Hence, any decrease in the backstop price leads to a decrease of optimal marginal extraction costs in the last period and in turn determines a higher ‘cut-off’ grade; that is, a lower total amount of fossil fuels is extracted.

The literature shows that a decrease in the backstop price may not only generate a volume effect, but also a change in the intertemporal distribution of extraction and thereby potentially increasing initial extraction. While in the framework of Sinn (2008) an increase in the initial extraction level was undoubtedly harmful for the the climate, however, with a backstop this is not clear-cut because of the volume effect. This led Gerlagh (2011, 82) to distinguish between a weak and a strong Green Paradox. The weak Green Paradox refers to the short term effect of the policy measure and is generated if the current and near future extraction activity (and hence emissions) increase compared to the business-as-usual case. However, the increase in current emissions may not be a substantial concern in the
long run. If extraction in the future significantly decreases, long-term damages may be less severe compared to the business-as-usual scenario. While the weak Green Paradox refers to an immediate effect – the short-term increase in emissions – the strong Green Paradox measures the total damage on the climate via a damage function. In a framework similar to Sinn (2008), Gerlagh (2011) explored the appearance of both paradoxes with a policy induced decrease in the backstop price. He concluded that in a competitive economy with stock dependent extraction costs and a backstop technology that is supplied competitively with constant marginal costs, any decrease in the backstop price leads to a weak Green Paradox, but not to a strong Green Paradox (ibid., 89-92). In addition, Gerlagh (ibid., 85-88) showed that a weak and strong Green Paradox is generated by a cheaper backstop technology if extraction costs are constant.

The contribution of Strand (2007, 130) shows that a technology treaty to enhance systematic global efforts to develop a low-carbon backstop technology may generate a Green Paradox. Once developed, the alternative technology would be available at a constant marginal cost that is lower than the constant extraction cost of the fossil fuel resource and hence make them redundant. In a Hotelling framework, he shows that the signature of the treaty leads to an acceleration of extraction and hence to an increase in near term emissions. Once the backstop is developed, extraction activities stop. The further in the future the backstop technology will be developed (if at all), the more probable that the treaty worsens climate change.

The contribution of Hoel (2010) provides a Hotelling model as employed by Sinn (2008), expect that it incorporates a backstop technology, to analyze the effect of different carbon tax schemes for climate issues. Hoel (2010) derived that the implementation of a carbon tax leads to lower total emissions no matter what time profile the carbon tax has (ibid., 8). On this basis, a carbon tax that increases by more than the interest rate [and would have a clear negative effect on the climate within the framework of Sinn (2008)] may lead to an increase in near-term emissions if the growth rate is significantly high, but may also limit climate change as it reduces total emissions (Hoel 2010, 9). A carbon tax that grows with the interest rate over time [and hence would have no effect on the extraction path within
the framework of Sinn (2008)] leads to a positive effect on the climate. While his contribution focuses on the implementation of a carbon tax, the following section provides an equivalent framework as employed by Hoel (2010) in order to derive the effect of an increasing sales tax as it is this tax scheme that has been the main focus of Sinn (2008).

2.2 The impact of an increasing sales tax on the climate

The model developed in this section builds on Sinn (2008) in the sense that a representative competitive firm possesses a fixed and known stock of the size $S_0$, extraction costs, $g(S_t)$, depend negatively on the stock remaining in the ground and the amount of resource extracted, $R_t$, can be sold for the market price, $P_t$. The only different feature is that it assumes the existence of a backstop technology. This new component draws on the contributions of Hoel (2010) and Gerlagh (2011). Assume that the choke price for which fossil fuel demand becomes zero is given by $\hat{P}_t$. The sales tax rate is given by $\tau_t = \tau_0 \cdot e^{\hat{\tau}_t}$ with $\hat{\tau}_t = r - \hat{P}_t$ with $\hat{P}_t = \frac{\hat{P}_T}{P_T}$. This tax scheme does not affect extraction decisions in a model where no backstop technology exists (Sinn (2008, 379).

By choosing an optimal extraction amount at each point in time, and an optimal extraction horizon that is finite (with $T$ being the terminal period), the firm solves

$$\max \int_0^T \left[ P_t - \tau_t \cdot P_t - g(S_t) \right] \cdot R_t \cdot e^{-rt} dt,$$  \hspace{1cm} (1)

subject to the resource constraint

$$\dot{S}_t = -R_t, \ R_t \geq 0, \text{ with } S_0 \text{ given.}$$ \hspace{1cm} (2)

In line with the literature (see for example Lasserre 1991, 91), it is assumed that the extraction cost function has properties such that complete exhaustion of the mine is uneconomic for the representative firm, formally, $S_T > 0$.

Appendix A derives formally the optimal price paths for the firm in the business-
as-usual scenario and the policy scenario. They are both given by

\[ \dot{P}_t = r \cdot [P_t - g(S_t)]. \]  

(3)

These relative price paths are equivalent to the one derived within the framework of Sinn (2008) for the business-as-usual scenario. However, the transversality condition to solve the firm’s optimization problem within the model presented in this section is considerably different to the one resulting in Sinn (2008). In particular, the existence of the backstop technology requires that the extraction activity ceases in finite time, exactly when the price of the fossil fuel reaches the choke price as it is in this moment that the demand for it becomes zero. The transversality condition is given by

\[ \lambda_T \cdot S_T \cdot e^{-rT} = 0. \]  

(4)

This is satisfied by \( \lambda_T = 0 \) because \( S_T > 0 \) by assumption. Note that the maximum constraint [refer to equation (29) in appendix A] requires the equalization of the marginal profit to the shadow price at any point in time \( t \). As extraction becomes increasingly expensive and the fossil fuel price converges to the choke price, the shadow value decreases over time. In the final period \( T \), when the price equals the choke price, the firm has to satisfy the maximum principle given by (see Levhari and Liviatan 1977, 191),

\[ \overline{P}_T \cdot (1 - \tau_T) = g(S_T). \]  

(5)

In the final period the price net of taxes equals marginal extraction costs. The higher the tax rate, the lower the revenue that can be used to cover marginal extraction costs because the choke price remains unchanged after implementing the tax. As extraction costs are increasing in cumulative extraction, a lower \( g(S_T) \) is associated to a higher \( S_T \) indicating a higher “cut-off grade” (Neher 1990, 323), that is, more fossil fuels remain unextracted. From this it follows that the higher the rate of the sales tax in the terminal period, the more fossil fuels remain in the ground.

Let us now review the extraction levels. Remember from equation (3) that the implementation of the tax scheme does not impact the timing of extraction as the relative price path is unchanged. However, the sales tax decreases total extraction
and thereby impacts absolute extraction levels. Consider the possibility that the price starts from a lower level when a tax is implemented in comparison to the business-as-usual scenario. With an identical growth of prices for both scenarios, this would indicate higher initial extraction and higher extraction levels in all subsequent periods for a longer time frame (because the price takes longer time to rise until $P_t$). This would indicate more overall extraction compared to the business-as-usual scenario which is not admissible because total extraction is lower in the policy scenario. Also, an identical initial price level is not possible because – following the same rationale – it would require that the total amount of extraction is equal in both scenarios. Instead, the optimal initial price level in the policy scenario will be higher. In this case, the level of extraction would be lower for all periods than under the business-as-usual scenario, the choke price would be reached earlier, and total extraction would be reduced. Hence, the policy measure has a positive effect on the climate because greenhouse gas emission at any time is lower than in the baseline case over an identical extraction horizon.

In conclusion, it has been shown that the result of Sinn (2008) regarding the impact of a sales tax that is increasing over time with $(r - \hat{P}_t)$ changes if a backstop technology is available. The result drawn by Sinn (2008) regarding the neutrality of this specific sales tax scheme (or, equivalently, a demand reducing policy measure that has the same effect on the price path) changes because it loses its neutral character. In effect, it has been shown that while the total amount of fossil fuels extracted decreases, the optimal relative price path is not impacted and the tax leads to lower extraction at any point in time compared to the business-as-usual scenario. The tax scheme therefore represents an effective instrument for climate change issues.

3 Endogenous Exploration and the Green Paradox

The model proposed in this chapter builds on Sinn (2008) in the sense that a representative firm extracts its resource stock over time in a competitive economy without a backstop technology. While the extraction process within Sinn (2008) involves costs, the following analysis assumes no extraction costs. Also, it includes
an endogenous resource base which depends on the firm’s (costly) exploration efforts. This new component is drawn from the contributions by Lasserre (1991, 104-107) and Heaps and Helliwell (1985, 452/453). Under these assumptions, the optimal supply path with and without an increasing cash flow tax are derived and compared in order to assess the policy’s effectiveness for climate change mitigation. The focus is on the impact of an increasing cash flow tax because this tax scheme undoubtedly leads to a Green Paradox within the framework of Sinn (2008). Also, because the framework does not consider extraction costs, the cash flow tax is equivalent to a carbon tax. It can be shown that an increasing cash flow tax reduces total emissions, but the impact on the climate depends on the specific tax scheme applied. To evaluate the short and long-term effects of the tax on the climate, a concept drawn from Gerlagh (2011) is employed.

3.1 Assumptions

The assumptions regarding the timing of extraction and exploration activities as well as the exploration process are based on the contribution of Lasserre (1991, 105). Consider a representative competitive fossil fuel firm that undertakes costly exploration activities to accumulate a stock of fossil fuels in a first phase. The duration of the exploration phase is optimally chosen by the firm, as well as the size of exploration expenditure in each period. Extraction starts as soon as the exploration process is over. The beginning of the extraction activity can be fixed at the date $t = 0$ without any loss of generality. Cumulative exploration expenses can be expressed by $C(S_0)$ with $S_0$ being the total amount of fossil fuels discovered during the exploration period. It is intuitive that the total amount discovered is denominated as $S_0$ as it represents the total extractable amount of fossil fuels available at $t = 0$. According to Lasserre (ibid.), total discovery costs can be expressed by

$$C(S_0) = \min \int_{-T_x}^{0} e^{-rt} c(s_{-t}, S_{-t}) \, dt$$

subject to

$$\dot{S}_{-t} = s_{-t}.$$  \hfill (6)

Note that the subscripts refer to time periods. The time $-T_x$ denotes the initial
period of the exploration process with \(-T_x < 0\). At the starting point, there are no discoveries given that \(S_{-T_x} = 0\). With no initial resource allocation, the firm invests at any time an amount of \(c(s_{-t}, S_{-t})\) to build up a natural capital stock. The interest rate must be added to the amount invested in the natural capital at each point in time to take account of an alternative investment strategy, namely investments in the capital market, in order to capture the total expenses weighted at time 0 when extraction starts. Discovery costs in any period increase with both the amount of discoveries, \(s_{-t}\), and the total amount of discoveries made in previous periods, \(S_{-t}\). This reflects the standard assumption of the exploration literature that marginal discoveries at any time decrease with the level of exploratory effort (measured by the number of wells drilled or the drilling footage of depths) and cumulative discoveries from previous times (refer to Pindyck 1978, 844). From this it follows that \(C(S_0)\) is convex and rising. The firm minimizes the discovery costs to obtain the initial amount of fossil fuels, \(S_0\), and due to equation (6) it is assumed that these costs can be captured by the following function

\[
C(S_0) = \beta \cdot S_0^\alpha
\]  

with \(\alpha > 1\), and \(\beta > 0\).

Extraction activities start as soon as the exploration process is terminated. The optimal extraction decisions within this model depend on assumptions in line with the framework proposed by Sinn (2008). However, Sinn’s approach is simplified by the removal of extraction costs. Also, a stock dependent extraction cost function with the properties assumed in Sinn (2008) might be misspecified according to Livernois and Uhler (1987), Livernois (1987), and Swierzbinski and Mendelsohn (1989) if it is used in extraction models where the size of the resource stock depends on discoveries\(^2\). All other assumptions remain valid. That is, the extraction amount at

\(^2\)Livernois and Uhler (1987, 195/196) point out that in these models convex and decreasing stock dependent cost function looses its validity if new discoveries do not have characteristics that lower extraction costs. For example, it does not capture extraction costs if deposits that can be depleted with lower extraction costs tend to be found first. In this case the employment of such a cost function leads to the wrong conclusion that any increase of the resource base decreases extraction costs.
time $t$ is denoted by $R_t$. The firm can sell a unit of fossil fuel for the competitive price $P_t$ that leads at each point in time to a market equilibrium. Demand for fossil fuels is given by $D(P_t)$ with $D'(P_t) < 0$ and the elasticity of demand is bounded from above as $R_t$ goes to zero to reflect the non existence of a backstop technology. To conduct the analysis below in a simpler manner, at this point the properties of demand are further specified by introducing a specific demand function that reflects that the elasticity of demand is constant for any $R_t$ and given by

$$D(P_t) = P_t^{-\gamma},$$

with $\gamma > 0$, [refer to Dasgupta and Heal (1979, 161)] who employed this demand function to express fossil fuel demand in a simple Hotelling framework). The property of the market demand leads to an optimal time horizon for extraction that is equal to infinity. An increasing cash flow tax is imposed with a factor given by $\theta_t = \theta_0 \cdot e^{\hat{\theta} t}$ with $\hat{\theta} < 0$. There exists a limited amount of fossil fuels in the ground representing the real world physical finiteness of fossil fuel resources. However, the complete discovery of the resource is assumed to not be economically profitable.

In an extended profit maximizing problem the firm selects the optimal investment in exploration in an initial phase and, in a second phase, its optimal extraction plan, so that total discounted profits from extraction (net of eventual taxes minus cumulative expenditures in exploration) are maximized. Formally this is shown as

$$\max \int_0^\infty P_t \cdot R_t \cdot \theta_t \cdot e^{-rt} \, dt - C(S_0) \text{,}$$

s.t.

$$\dot{S}_t = -R_t \text{ with } R_t \geq 0, \quad S_0 \text{ endogenous} \text{,}$$

and

$$C(S_0) \text{ as defined by equation (6).}$$

It will be shown that the overall impact of an increasing cash flow tax on the climate is not clear because the tax exerts two countervailing effects on fossil fuel supply. On the one hand, it modifies the temporal distribution of extraction by making it
more profitable to extract the fossil fuel stock faster. On the other hand, it reduces the total amount extracted because exploration incentives are lowered. The results of section 3.2 will show that these effects may lead to an extraction path with (1) higher or (2) lower initial extraction compared to the business-as-usual scenario. Following Gerlagh (2011, 82), an extraction path as described in (1) establishes the conditions for a weak Green Paradox. As already stated in section 2.1 above, a weak Green Paradox captures the short term impact of the tax on extraction decisions and the climate. Denote the extraction amount in the business-as-usual scenario and tax scenario respectively as $R^\text{bau}_t$ and $R^\text{tax}_t$ at $t$.

**Definition 3.1** A weak Green Paradox arises when the implementation of an increasing cash flow tax augments current and near term extraction compared to what would have been optimal with no tax, formally, if $R^\text{tax}_0 > R^\text{bau}_0$.

The extraction scenario described in (2) implies that the increasing cash flow tax has a positive impact on the climate because extraction levels are lower at any point in time compared to the business-as-usual scenario. In the opposite, an optimal extraction path with an initially higher extraction level compared to the business-as-usual scenario (1) does not have a clear impact on the climate in the long run. It is possible that the negative impact on the climate due to higher early emissions are offset by the reduction in total emissions. The long term effect on the climate is captured by the concept of the strong Green Paradox and occurs if the policy lead to higher climate damages compared to the business-as-usual scenario.

**Definition 3.2** A strong Green Paradox arises if an increasing cash flow tax leads to higher cumulative net present value climate damages due to fossil fuel extraction than without a tax, formally, if $\Gamma^\text{tax} > \Gamma^\text{bau}$.

Climate change damages are captured through a shadow price on emissions, $\chi_t$ with $\chi_t = \chi_0 \cdot e^{\hat{\chi} t}$ reflecting an increasing shadow price over time. This assumption relies on the expectation that the greenhouse gas concentration in the atmosphere increases over time and this leads to increasing marginal damages from emissions. The net present value at $t = 0$ of climate change damages is given according to Gerlagh (ibid., 87) by

$$\Gamma = \int_0^\infty e^{-\delta t} \cdot \chi_t \cdot R_t \, dt.$$  \hspace{1cm} (12)
In line with Sinn’s Green Paradox discussion, an important assumption Gerlagh (ibid.) implements is that early extraction (emissions) cause higher net present value damages than delayed emissions, thus $e^{-\delta t} \chi_t$ decreases over time. That is, marginal damage from extraction increases by a lower amount than the discount rate such that $\hat{\chi} < \delta$.

Note that a strong Green Paradox represents the issue raised by Sinn (2008). It captures the overall effect of a climate policy and is manifested in the case where the policy worsens the climate change problem. The concept of the weak Green Paradox captures the short-term effect of a climate measure. It arises when a climate policy leads to an initial increase of emissions. The policy may seem paradoxical in this sense, however, it does not necessarily lead to an overall contradiction of the policy intention.

3.2 Results

This section presents the conditions for a weak and a strong Green Paradox. To derive these conditions, the fossil fuel extraction paths that emerge under the business-as-usual scenario and the tax scenario must be derived.

3.2.1 The optimal supply decision

The differences in the optimal supply paths are crucial to assessing the impact of an increasing cash flow tax on the climate. The differences in the extraction levels are determined by three factors. First, the total amount of fossil fuels that are extracted over the time horizon, second, the temporal distribution of extraction activities and, ultimately, the optimal time frame of extractive activities (Hotelling 1931, 283/284). While the optimal extraction horizon is infinity, the remaining two determinants are derived in appendix B. It can be shown that the optimal temporal distributions are given in the business-as-usual scenario by the Hotelling rule,

$$\dot{P}_{t}^{baus} = r \cdot P_{t}^{baus},$$

(13)
and in the policy scenario by,

\[ \dot{P}_t^{\text{tax}} = (r - \hat{\theta}) \cdot P_t^{\text{tax}}. \]  (14)

It clearly follows that the growth rate of the price is greater in the policy scenario because \((r - \hat{\theta}) > r\). This in turn leads to a steeper extraction path via the price mechanism. For given initial fossil fuel stocks, \(S_0^{\text{bau}}\) and \(S_0^{\text{tax}}\) (and an identical extraction horizon), this indicates that the firm extracts a higher fraction of the respective initial fossil fuel stock in the policy in the near future compared to the fraction that is optimal in the business-as-usual scenario. This is intuitive as a notable growth rate of the tax indicates a higher tax burden in the farer future compared to the present and nearer future such that is optimal to reallocate extraction activities to earlier periods.

Next, appendix B derives the absolute extraction paths as a function of the (endogenous) initial amount of fossil fuels for the business-as-usual scenario,

\[ R_t^{\text{bau}} = \frac{\gamma \cdot r \cdot S_0^{\text{bau}}}{e^{\gamma rt}}, \]  (15)

and for the policy scenario

\[ R_t^{\text{tax}} = \frac{\gamma \cdot (r - \hat{\theta}) \cdot S_0^{\text{tax}}}{e^{\gamma (r - \hat{\theta}) t}}. \]  (16)

The initial extraction levels are respectively given by

\[ R_0^{\text{bau}} = \gamma \cdot r \cdot S_0^{\text{bau}}, \]  (17)

and

\[ R_0^{\text{tax}} = \gamma \cdot (r - \hat{\theta}) \cdot S_0^{\text{tax}}. \]  (18)

Recall from definition 3.1 that a weak Green Paradox is manifested if initial extraction is higher in the policy scenario than in the business-as-usual scenario. The result obtained here indicates that the higher the growth rate of the tax, \(\hat{\theta}\), the more likely the appearance of a weak Green Paradox. However, \(\hat{\theta}\) might also impact
exploration incentives and hence extraction levels. In effect, appendix B shows that
the implementation of an increasing cash flow tax leads to lower investments in
exploration thereby lowering the amount of fossil fuels discovered compared to the
business-as-usual scenario, formally, $S_{0}^{\text{bau}} > S_{0}^{\text{tax}}$.

### 3.2.2 Conditions for a weak Green Paradox

A *weak* Green Paradox captures the short term effect of an increasing cash flow
tax. It arises when initial emissions from fossil fuel extraction are higher in the
policy scenario compared to the business-as-usual scenario, formally, if $R_{0}^{\text{tax}} > R_{0}^{\text{bau}}$. Appendix B derives that a *weak* Green Paradox is generated if

$$\theta_{0} > \left(1 - \frac{\hat{\theta}}{r}\right)^{1-\alpha}.$$  \hspace{1cm} (19)

The short term effect on the climate is detrimental with increased emissions in early
periods if the initial tax factor is greater than $(1 - \frac{\hat{\theta}}{r})^{1-\alpha}$. The condition depicts
clearly the drivers of a *weak* Green Paradox. The higher the initial tax factor $\theta_{0}$
equivalently, the lower the initial tax rate) and the higher the growth rate of the
cash flow tax (expressed by a high negative value of $\hat{\theta}$), the more probable a *weak*
Green Paradox. Equivalently, an increase of $\theta_{0}$, ceteris paribus, increases the left
hand side of (19) and hence makes a *weak* Green Paradox more probable. The
same effects holds for an increase of the growth rate of the tax, ceteris paribus.

The following two graphs emphasize the role of the two components of the tax
scheme in generating a *weak* Green Paradox. Consider figure 1. It depicts the two
possible effects of an increasing cash flow tax associated to a high and a low growth
rate of the tax. It can be shown, that the low growth rate (tax scenario I) does
not lead to a *weak* Green Paradox, while the high growth rate (tax scenario II),
ceteris paribus, generates a *weak* Green Paradox. On the x-axis, the amount of
fossil fuels discovered, $S_{0}$, is plotted. The y-axis shows possible courses for $C'(S_{0})$,
$P_{0}$ and $\lambda_{0}$ as functions of $S_{0}$ for both scenarios. The first decreasing line from
above depicts the optimal price and shadow value paths depending on the initial
fossil fuel stock, $S_{0}$, in the business-as-usual scenario. The two dotted lines below
graph the shadow values associated to a low $\theta$ and a high negative $\theta$ respectively, while the two remaining lines depict the associated initial prices for the two policy scenarios. The only increasing line represents marginal exploration costs $C'(S_0)$.

Consider that under business-as-usual assumptions, the optimal initial fossil fuel stock, $S_{bau}^{0}$, is obtained by the value on the x-axis with which the line that depicts the initial shadow value as a function of $S_0$ intersects with the marginal exploration cost line. This satisfies the optimal exploration condition stated in appendix B in equation (56). Because the initial price path is equal to the shadow value path, the value on the y-axis states the respective optimal initial price associated to the optimal $S_{bau}^{0}$. It is denominated as $P_{bau}$ in the diagram. The optimal initial values for the two tax scenarios are obtained in an equivalent manner. It is given by the value on the x-axis for which the respective shadow values for both scenarios intersect the marginal exploration cost function. It can be clearly seen that the imposition of the increasing cash flow tax leads to a decline in the optimal initial fossil fuel stock. The initial price associated to the two tax scenarios are given by the respective values on the y-axis of the respective price curves for the respective optimal initial fossil fuel stock, denoted in figure 1 as $S_{taxI}$ and $S_{taxII}$.
Also, the probability of a weak Green Paradox rises, the higher the initial tax factor. Recall that a high initial tax factor corresponds to a low initial tax burden (and hence also to a lower level of absolute tax burden in all subsequent periods). Graph 2 shows two tax scenarios where a high $\theta_0$ (tax scenario I) lead to a weak Green Paradox, whereas a low $\theta_0$ (tax scenario II), ceteris paribus, does not. The intuition is that a low level of $\theta_0$ does significantly reduce $S_{0}^{bau}$ and as such extraction levels at all times have to decrease (associated to an increase in the price level) in order to satisfy the resource constraint. The first decreasing line from above again depicts the optimal price and initial shadow value path in the business-as-usual scenario.

Figure 2: The impact of the initial tax rate

Under equation (19) there is no value for the growth rate of the tax that excludes the possibility of the tax scheme generating a weak Green Paradox. This becomes clear because for any $\hat{\theta} > 0$, the right hand side of (19) never becomes greater than 1. However, if the government chooses a value of $\theta_0$ between 0 and 1 for which the left hand side of (19) is greater than the right hand side, the appearance of a weak Green Paradox is avoided. From this it follows that there is no admissible
value for the initial tax rate, $\theta_0$, that excludes the possibility of a weak Green Paradox because for any value it assumes, there are values for $\hat{\theta}$ that generate a weak Green Paradox.

Finally, the exploration cost function enters the condition for a weak Green Paradox and as such there are a few words to say. A higher $\alpha$ is associated to greater marginal exploration costs in absolute values. This leads to a smaller relative decrease of total discoveries for a given tax scheme (that in turn leads to an absolute decrease in the shadow value) compared to what would have been respectively optimal in the business-as-usual scenario. Obviously, the lower the relative decrease in the fossil fuel stock, the lower the absolute increase in the price level. A lower increase in the price level is associated to a lower decrease of extraction levels, hence making a weak Green Paradox more probable.

3.2.3 Conditions for a strong Green Paradox

A strong Green Paradox occurs when net climate damages in the policy scenario [refer to equation (67) in appendix B] are greater than climate net damages arising in the business-as-usual scenario [equation (66)]. As derived in appendix B, this occurs, if

\[
\theta_0 > \left(1 - \frac{\gamma \cdot \hat{\theta}}{\delta - \hat{\chi} + \gamma r}\right)^{(\alpha - 1 + \frac{1}{\gamma})} \left(1 - \frac{\hat{\theta}}{r}\right)^{1 - \alpha}.
\]

An increasing cash flow tax is more likely to generate a strong Green Paradox, the lower the initial tax burden (the higher $\theta_0$) with given values of $\hat{\theta}$, $r$, $\delta$, $\gamma$, $\hat{\chi}$ and $\alpha$. This is intuitive as for a given growth rate of the tax, the temporal distribution of extraction remains the same, but a lower initial tax level (and hence a lower tax level in all subsequent periods) assures a higher level of investments in exploratory activities and hence a higher total amount of extractable fossil fuels. In turn, the greater the fossil fuel stock, the higher the extraction levels at any time $t$ and hence the more emissions enter the atmosphere at any time.

The level of the growth rate of the tax has an ambiguous impact on the climate. A very low growth rate (expressed by a small negative value of $\hat{\theta}$) leads to a small
temporal redistribution such that a strong Green Paradox is not generated. However, if the growth rate is too high, the level of the tax increases significantly and leads to a reduction of expected profits from extraction such that the investments in exploration activities shrink significantly. In particular, the higher the growth rate the higher the tax rates at any point in time for a given initial tax rate and hence the lower the initial shadow value of the fossil fuels. The reduction of the total amount of extraction may be sufficiently high to offset the increase in initial extraction generated by a high growth rate. In conclusion, there is a specific range of growth rates that generate a strong Green Paradox, ceteris paribus. A very tiny growth rate, as well as a very high growth rate do not lead to a strong Green Paradox. Also, the higher the initial tax burden, the more probable that a high growth rate does not generate a strong Green Paradox because the total tax burden is high such that total extraction (emissions) decreases sufficiently to offset the increased speed of extraction.

The right hand side of (20) increases in $\delta$ which reflects the sensitivity of the climate damage to emissions in early periods. A higher $\delta$, ceteris paribus, leads to a smaller value on the right hand side and hence makes a strong Green Paradox more probable. Recall from the climate change damage function stated in (12) that the higher $\delta$, the greater the climate damage associated to the use of a unit of fossil fuel in early periods compared to later periods. This implies that a given tax scheme that decreases total extraction but causes faster extraction, in early periods is more harmful to the climate, due to the higher $\delta$ and hence a strong Green Paradox is more probable.

The right hand term of (20) decreases with $\hat{\chi}$ which reflects the strength of the globe’s absorptive capacity. The higher the absorptive capacity, the lower the greenhouse gas concentration in the atmosphere in later periods, thereby reducing marginal damages from emissions. A strong Green Paradox is more probable the lower the value of $\hat{\chi}$. To show that, consider the consequences of the highest possible value for it, with $\hat{\chi}$ approaching $\delta$ (recall that $\delta > \hat{\chi}$ by assumption). This would imply that the climate damage $e^{-\delta \cdot t} \cdot e^{\hat{\chi} \cdot t} \cdot \hat{\chi} \cdot 0$ of a unit of fossil fuels, in present value terms remains almost constant over time. In this case, (when $\hat{\chi}$ is at is largest possible value) a significant increase in initial extraction due to the tax scheme
hardly impacts climate change damages. Instead, the decrease in total extraction caused by the tax leads to a decrease in climate change damages. In this case it can be stated that any increasing cash flow tax actually generates environmental benefits. This leads to the general conclusion that the higher $\hat{\chi}$, the smaller the increase in climate damages associated to higher initial extraction, hence a strong Green Paradox becomes less probable. On the other hand, a small $\hat{\chi}$ has the same effect as a higher $\delta$ because it means that the climate damage of a unit of fossil fuels is decreasing over time, making initial extraction more harmful for the climate relative to later extraction.

The exploration cost function also enters the condition for the strong Green Paradox. The higher the value of $\alpha$, the smaller is the left hand side of equation (20), making a strong Green Paradox more possible. As already discussed in the section where the condition for a weak Green Paradox has been derived, a high $\alpha$ is associated to an exploration cost function with high marginal exploration costs. With higher values of $\alpha$, any given decrease in the optimal marginal exploration cost due to the implementation of a tax results in a smaller decrease in the absolute amount of fossil fuel discovered. Hence, total discoveries are lowered, but by a smaller fraction. This results in a lower reduction in climate change damages under the tax scenario due to smaller decreases in exploration and hence makes a strong Green Paradox more probable.

3.2.4 Summary of the results

The results obtained in this section show that an increasing cash flow tax tends to reduce climate damages when exploration is modelled endogenously. This weakens the result obtained by Sinn (2008) which is derived in a similar framework without exploration and finds that an increasing cash flow tax worsens climate change.

The results show that when costly exploratory activities are considered, the size of the initial fossil fuel stock is endogenous and depends on the expected profit from extraction activities. Under these assumptions, any tax imposed on extraction profits reduces the cash flow obtained by a given amount of fossil fuels. In response,
firms will reduce exploratory activities, thereby lowering the size of the initial stock available. This is clear because the optimal exploration decision requires that marginal expenses for exploration equal the marginal revenue obtained for it. Because any cash flow tax decreases the latter, the optimality condition for exploratory activities is no longer satisfied if the firm sticks with the investment decision previously determined to the tax announcement. The firms then revise their exploration decision by decreasing investments in exploratory activities. This is the crucial element for the change in the result obtained compared to Sinn (2008). In Sinn (2008), the representative firm is endowed with the fossil fuel resource and hence an increasing cash flow tax does not impact the total amount extracted. An increasing cash flow tax within the framework of Sinn (2008) exerts a change in the optimal timing of extraction decisions, while the volume is not modified. Since the total amount of fossil fuels extracted does not change, and it is more profitable to extract in periods in the near future, the firms will extract their stock at a faster rate. Under an exploration-extraction framework, the imposition of a tax has two effects: a reduction in the total amount extracted and a change in the temporal distribution of extraction. The impact of the tax on the temporal distribution and the volume has two countervailing consequences for the effectiveness of the tax. While the tax results in the extraction of a higher fraction of the fossil fuel stock in the near term future compared to the no tax scenario, the volume effect leads to a decrease in overall extraction. Hence, the absolute extraction levels are ambiguous and depend on the formulation of the exact tax scheme.

Figure 3 shows the possible optimal extraction paths for the business-as-usual scenario in the framework presented in Sinn (2008) as well as in this chapter, depicted by the unbroken line. They are identical because it is assumed that the firm chooses to explore in the extraction-exploration framework an optimal initial fossil fuel stock that equals the fossil fuel stock given in Sinn (2008). The dotted line depicts the extraction path that may arise under the assumptions made by Sinn (2008) when an increasing cash flow tax is implemented. The dashed line graphs the optimal extraction path that may arise due to the same policy measure within the framework presented in this chapter.
Initially, in both frameworks, supply increases (and hence, a weak Green Paradox is manifested). However, the increase induced under the assumptions made by Sinn (2008) is higher than in the extraction-exploration scenario where the stock is endogenous. At some point in time, extraction levels fall under the business-as-usual paths in both frameworks. The time point with which this occurs is earlier in the scenario with an endogenous fossil fuel stock. The increase of initial extraction levels clearly lead to a Green Paradox in the framework of Sinn (2008) because total extraction are not changed. The initial increase in extraction in the alternative scenario leads to a negative short term effect on the climate due to higher initial emissions and thus creates a weak Green Paradox. However, due to the decrease in overall extraction, the total impact on the climate is not clear. A strong Green Paradox will depend on the specific tax scheme \( \theta_0, \hat{\theta} \), on the sensitivity of climate.
damage to emissions in early periods, $\delta$, the relative strength of the absorptive capacity, $\hat{\chi}$, exploration cost, $\alpha$ [as represented in (20)].

It can be concluded that to guarantee the effectiveness of an increasing cash flow tax it is important to choose the 'right' initial level and growth rate of the tax. In fact, by deriving the condition for a strong Green Paradox [refer to equation (20)], it has been shown that the higher the initial tax rate, the higher the probability that climate damages are reduced by implementing this specific tax scheme. Also, a tax with a very low and very high growth rate is with a higher probability effective for climate change issues. Equation (20) further shows that for specific levels of the growth rate, a strong Green Paradox can be excluded independently of the initial tax rate when the right hand side of the term becomes greater than 1 because, by definition, the left hand side can only assume a value smaller than 1.

3.3 Discussion

By including costly exploration activity in the model of Sinn (2008), it has been shown that an increasing cash flow tax will quite likely be an effective instrument to control global warming. It induces the economic actors to use less fossil fuels over the whole time horizon and hence reduce total emissions. If the reduction in emissions is high enough to outweigh the emissions from faster extraction induced by an increase in the tax over time, a positive impact on the climate is achieved. This requires a specific tax scheme to generate desirable results in terms of climate change mitigation. In particular, the higher the initial tax rate and the closer the growth rate to its two extremes (given by a very low and a very high rate), the more probable that climate damages will be reduced compared to the business-as-usual extraction path.

The result obtained in this section can be applied to an assessment of the climate policy approach of Kyoto countries. Their efforts are aimed at shrinking fossil fuel demand and thereby lowering the expected profit from extraction. This leads to a decrease in the value of any mining project, and as such, the incentive to invest in these projects are decreased. Through the reduction of total emissions, a Kyoto
consistent approach is likely to have a positive effect on the climate in the long-term if short-term emissions do not rise substantially.

The exploration cost function described above can also capture the huge investments necessary for utilising the newly discovered fossil fuel stock after successful exploration. This becomes clear from Bohi and Toman’s (1983, 928) description of the real world supply process that necessitates both, exploration and development expenditures for a single stock before any extraction process can start. Development of the resource includes gaining access to the resource through sinking mine shafts or drilling wells and installing surface equipment for extraction.

There are two main drawbacks regarding the approach utilised in this article to incorporate exploration within the model presented by Sinn (2008). First, real world facts suggest that extraction and exploration activities occur simultaneously. Second, discoveries are modelled as depending on a continuous exploration cost function which is associated to resource discoveries being the outcome of a continuous variable, exploration activities. This is not realistic as in reality exploration costs are sunk and huge investment amounts are necessary to explore a stock. The interesting question is how the results obtained change when these two elements are considered. It is argued that the inclusion of these assumptions would not change the basic result. An increasing cash flow tax would decrease the total amount extracted and change the temporal distribution in a similar way as derived above. Beside the shortcomings of the framework mentioned, it is able to capture an important feature of the fossil fuel supply process; the necessity to invest capital to build up a fossil fuel stock from which to extract.

Another shortcoming of the framework proposed is that extraction costs are zero. This assumption allows to derive the absolute extraction paths in the business-as-usual scenario and policy scenario. Moreover, it is argued that including extraction costs (such as constant extraction costs) does not change the main results because the tax base is the profit from extraction and extraction costs does not affect total extraction. However, the non existence of extraction costs does not allow for the separate analysis of the effect of different taxes on emissions because
without extraction costs, a cash flow tax coincides with a carbon tax and a sales tax.

From the analysis undertaken in chapter 3, there are two important conclusions for climate policy design. First, while an increasing cash flow tax may be effective for climate change issues, it is important to select the most appropriate tax scheme. Second, when policy-makers announce the tax scheme, they need to be credible because investment in exploration activities depend on the expected price path of the fossil fuels. If policy-makers are not credible when announcing the tax, the exploratory activity may be not impacted and hence overall extraction may not decrease.

4 Conclusion

Considerable efforts have been undertaken to formulate policy to tackle climate change on a national and international level. Recently, Hans-Werner Sinn (2008, 388) noted that the standard climate policy approach – the implementation of measures aimed at reducing carbon demand (such as taxes on fuels and subsidies for low-carbon and high-efficient technologies) – may not mitigate the problem of global warming. This result was built upon a Hotelling (1931) model of optimal resource extraction with stock-dependent extraction costs and no backstop technology. Within this framework, Sinn (2008) analyzed different tax schemes and showed that the imposition of an increasing cash flow tax and a sales tax which increases by more than a specific rate leads resource owners to extract their resources faster in order to avoid the higher tax burden in the farer future compared to the tax burden today and the nearer future. In the light of these considerations, Sinn (2008) regarded an effective policy measure as one that flattens the extraction path, and leads fossil fuel owners to extract their resources at a slower pace.

The emergence of the Green Paradox within the model of Sinn (2008) is dependent upon the non-existence of a backstop technology and the assumption that firms are endowed with the resource stock. This article develops the framework of Sinn (2008) by providing two models that respectively allow for the existence of a backstop technology and exploration activities. First, it analyzes how the existence of a
backstop technology changes the result obtained by Sinn (2008) regarding the effect of a sales tax that increases over time by the factor \((r - \frac{\dot{P}_t}{P_t})\) with \(\dot{P}_t = \frac{\partial P_t}{\partial t}\). It has been shown in section 2.2 that the result drawn by Sinn (2008) regarding the neutrality of this specific tax scheme significantly changes if a backstop technology exists because it loses its neutral character. In effect, if a backstop technology is available, the policy measure leads to a decrease in the total amount of fossil fuels extracted, while the optimal relative price path is not impacted. In consequence, the amount extracted is lower at any point in time compared to the baseline scenario and hence the tax scheme is considered as an effective instrument for climate change mitigation. This result can applied to the implementation of any demand reducing policy measure as they have an equivalent effect on the expected price path for fossil fuels.

Secondly, the article extends the model of Sinn (2008) to take into account of the reality that firms are not “endowed” with fossil fuels, but instead have to incur costs in exploration activity before any extraction starts. To incorporate exploration, the approach of Heaps and Helliwell (1985, 452/453) and Lasserre (1991, 104-107) is applied. Both contributions focus on the endogeneity of the reserve base and the exploration efforts that are undertaken prior to the extraction phase. The results obtained in chapter 3 show that an increasing cash flow tax tends to reduce climate damage when exploration is modelled endogenously. The tax leads to a temporal redistribution of extraction activities because of the increasing tax burden over time and hence gives an incentive for the firm to extract their resources faster. However, overall extraction is reduced because the incentive to explore is reduced following the decrease in expected profits from extraction. Hence, the total impact on the climate depends on the specific tax scheme and the climate damage function that specifies the temporal extraction decisions’ impact on the climate. In particular, the effectiveness of an increasing cash flow tax depends on the specific tax scheme chosen, the sensitivity of climate damage to emissions in early periods, the relative strength of the absorptive capacity of the environment, and the exploration costs incurred before extraction. The findings of this article weaken the result obtained by Sinn (2008) who abstracts from costly exploration and finds that an increasing cash flow tax leads to a Green Paradox and hence worsens climate change.
The result obtained within the extraction-exploration framework presented in chapter 3 indicates that it is important to choose the most appropriate tax scheme for an increasing cash flow tax to be effective. The cash flow tax will likely be more effective in combating climate change, the higher the initial level of the tax rate and the more extreme the growth rate of the tax (either higher or lower).

References


Appendix A derives the solution to the firm’s optimal extraction decision under an increasing sales tax within the scenario presented in chapter 2.2 in the main text. Consider a representative competitive firm that possesses a fixed and known stock of homogeneous non-renewable resource reflecting fossil fuel energy supplies, denoted by $S_0$. The firm’s objective is to maximize the discounted profit from extracting the stock. Profits at each point in time $t$ are obtained by extracting an amount of fossil, $R_t$, and selling them for the market price, $P_t$. The price is determined by the market demand function $R(P_t)$ with $\frac{\partial R(P_t)}{\partial P_t} < 0$ representing the normal property of demand decreasing with the price level. Extraction costs $C_t = C(R_t, S_t)$ are independent of the current rate of extraction, $R_t$, thus unit (and marginal) extraction costs are constant within a given period. It is assumed that unit extraction costs depend only on the size of the remaining stock (the amount of resources remaining after extraction occurred at all previous points in time). In particular, the unit cost of extraction in $t$ is higher, the smaller the remaining stock. Total cost of extraction at time $t$ can hence be expressed by $C_t = g(S_t) \cdot R_t$ with $g'(S_t) < 0$. A real-world example of stock-dependency determining recovery costs is provided by Krautkraemer (1998, 2069) with natural pressure within an oil field declining as an oil stock is depleted. Denote the tax rate of the sales tax by $\tau_t = \tau_0 \cdot e^{\hat{\tau}t}$ with $\hat{\tau} = r - \hat{P}_t$ with $\hat{P}_t = \frac{\dot{P}_t}{P_t}$.

The evolution of the fossil fuel resource stock over time is only dependent on resource extraction decisions and can thus be described by the state equation $\dot{S}_t = -R_t$, with $\dot{S}_t = \frac{dS_t}{dt}$. For simplicity, it is assumed that fossil fuels once extracted are not able to be stored for subsequent periods. This implies that extraction of fossil fuels directly determines the use of fossil fuels and thus is proportional to the amount of greenhouse gases emitted via the combustion process.

So far, the assumptions are in line with the model employed by Sinn (2008). However, there exists a choke price for which fossil fuel demand becomes zero given by $\bar{P}_t$, reflecting the existence of a backstop technology.
By choosing \( R_t \) and \( T \) (\( T \) being the terminal period), the competitive firm solves

\[
\max \int_0^T [P_t - \tau_t \cdot P_t - g(S_t)] \cdot R_t \cdot e^{-rt} dt,
\]

subject to the resource constraint

\[
\dot{S}_t = -R_t,
\]

\( R_t \geq 0, S_0 \) given, and \( \lim_{t \to \infty} S_t \geq 0 \).

The latter expression represents the physical constraint given by the fact that over the whole time horizon, the firm cannot extract more than the fixed initial stock. This constrained dynamic decision problem can be solved by applying the maximum principle. The current value Hamiltonian function is given by

\[
H_c = [P_t - g(S_t)] \cdot R_t - \tau_t \cdot P_t \cdot R_t - \lambda_t \cdot R_t,
\]

with \( \lambda_t = \mu_t \cdot e^{rt} \) representing the co-state variable for the fossil fuel stock.

First, the business-as-usual extraction path is derived by setting \( \tau_0 = 0 \). The resulting necessary conditions include the static efficiency condition,

\[
\frac{\partial H_c}{\partial R_t} = P_t - g(S_t) - \lambda_t = 0,
\]

the dynamic efficiency condition,

\[
\dot{\lambda}_t = r \cdot \lambda_t - \frac{\partial H_c}{\partial S_t} = r \cdot \lambda_t + g'(S_t) \cdot R_t,
\]

and the transversality condition,

\[
\lambda_T \cdot S_T \cdot e^{-rt} = 0.
\]

Differentiate the static efficiency condition by time yields (under the assumption that \( g''(S_t) = 0 \)),

\[
\dot{\lambda}_t = \dot{P}_t + g'(S_t) \cdot R_t.
\]
The maximum principle requires that at each point in time, the marginal profit equals the shadow value of the resource. Differentiating the condition by time, solving it by $\dot{\lambda}$ and equalizing the expression to the dynamic condition leads to the optimal price path

$$\dot{P}_t = r \cdot [P_t - g(S_t)].$$ \hspace{1cm} (28)

Now assume that a sales tax with a growth rate $\hat{\tau} = r - \dot{P}$ is implemented. It is argued that it is optimal for the firm to stick to the extraction decision undertaken in the business-as-usual scenario. This is proved in the following. Express the price path that is optimal in the policy scenario as $\dot{P}_t^* = r \cdot [P_t^* - g(S_t^*)]$. To show that sticking to this price path is optimal, the necessary conditions has still to be satisfied. These are given by the static efficiency condition,

$$[P_t^* - g(S_t^*)] - \tau_t \cdot P_t^* = \lambda_t,$$ \hspace{1cm} (29)

the dynamic efficiency condition,

$$\dot{\lambda}_t = r \cdot \lambda_t + g'(S_t^*) \cdot R_t^*,$$ \hspace{1cm} (30)

and the transversality condition,

$$e^{-rT} \cdot S_T \cdot \lambda_T = 0.$$ \hspace{1cm} (31)

Differentiating the static efficiency condition to time $t$ yields,

$$\dot{\lambda}_t = \dot{P}_t^* + g'(S_t^*) \cdot R_t^* - (\dot{P}_t^* \cdot \tau_t + \hat{\tau}_t \cdot P_t^*).$$ \hspace{1cm} (32)

Inserting the static efficiency condition into the dynamic efficiency condition yields a dynamic efficiency condition given by

$$\dot{\lambda}_t = r \cdot [P_t^* - g(S_t^*)] - r \cdot \tau_t \cdot P_t^*.$$ \hspace{1cm} (33)

The necessary conditions are yielding an optimal extraction path that is identical
to the business-as-usual price path only if,

\[ r \cdot \tau_t \cdot P_t^* = (\dot{P}_t^* \cdot \tau_t + \hat{\tau}_t \cdot P_t^*). \]  

(34)

In words, the discounted amount of tax paid at each point in time must be constant and this requires that the growth rate of the tax, \( \hat{\tau}_t \), is equal to \( (r - \dot{P}_t) \). q.e.d.

B Appendix

This appendix first provides the derivation of the optimal supply paths for the business-as-usual and policy scenarios of the firm for the dynamic optimization problem as presented in chapter 3. Subsequently, the conditions for a weak and a strong Green Paradox are obtained.

B.1 The optimal supply decision

The absolute supply paths for both scenarios are determined by three factors. First, by the total amount of fossil fuels that are extracted over the time horizon, second, the temporal distribution of extraction activities and, ultimately, the optimal time frame of extractive activities (Hotelling 1931, 283/284). While the optimal extraction horizon is infinity, the remaining two determinants are derived in the following.

1\textsuperscript{st} step: Optimal temporal distribution

The optimal temporal distribution captures the relative distribution of extraction amounts of a given stock over the time horizon of extraction. The temporal distribution is optimal if the fossil fuel firm cannot increase its total discounted profit by reallocating extraction amounts from one period to another. The optimal rule for the firm is denominated according to Hotelling (ibid.) and states that the marginal profit of extraction is equal in each period. In the following, the conditions for the optimal relative distributions of extraction amounts are determined for the two scenarios. To do this, the discounted value Hamiltonian function is formulated,
\[ H_t = P_t \cdot R_t \cdot \theta_t - \lambda_t \cdot R_t. \] (35)

The necessary conditions are given by the maximum principle,

\[ P_t \cdot \theta_t = \lambda_t \] (36)

the dynamic constraint,

\[ \dot{\lambda}_t = r \cdot \lambda_t. \] (37)

and the transversality condition,

\[ \lim_{t \to \infty} S_t \cdot \lambda_t e^{rt} = 0. \] (38)

The necessary conditions for the business-as-usual scenario are given by setting \( \theta_t = 1 \) in equation (35). The transversality condition together with the dynamic constraint formally confirm what has been implicitly assumed beforehand; that it is optimal to completely exploit the initially available fossil fuel stock over time in both scenarios. Note that the dynamic constraint for both scenarios display a shadow value that is increasing over time with the interest rate. When time approaches infinity, the shadow value gets infinitely high such that the transversality condition is only satisfied if the extractable stock in the final period becomes zero. Hence, the initial fossil fuel stock available is equal to the total amount extracted, formally, \( S_0 = \int_0^\infty R_t \, dt \).

Differentiating the maximum principle (36) by time, equalizing it to the dynamic constraint (37), allows for the derivation of the optimal price path in the business-as-usual scenario given by the Hotelling condition,

\[ \dot{P}_t^{bau} = r \cdot P_t^{bau}, \] (39)

and in the policy scenario,

\[ \dot{P}_t^{tax} = (r - \hat{\theta}) \cdot P_t^{tax}. \] (40)
Using equations (39) and (40) provide the respective price levels at \( t \) as a function of the respective initial price levels,

\[
P_t^{\text{bau}} = P_0^{\text{bau}} \cdot e^{rt}.
\]

\[
P_t^{\text{tax}} = P_0^{\text{tax}} \cdot e^{(r-\hat{\theta})t},
\]

In the next step, the optimal relative extraction paths that correspond to these price paths are derived.

\textbf{2\textsuperscript{nd} step: Derivation of the optimal extraction paths}

At any point in time, the fossil fuel market is in equilibrium following from the assumption that the economy is competitive. This requires that

\[
R_t = D(P_t) = P_t^{-\gamma} \quad t \in [0; \infty].
\]

Inserting here equations (41) and (42) yield the optimal extraction path as a function of the initial price levels,

\[
R_t^{\text{bau}} = P_0^{\text{bau}}^{-\gamma} \cdot e^{\gamma rt},
\]

and

\[
R_t^{\text{tax}} = P_0^{\text{tax}}^{-\gamma} \cdot e^{\gamma (r-\hat{\theta})t}.
\]

The rate of extracting the fossil fuel resource in the policy scenario falls at a higher constant percentage rate, \( \gamma \cdot (r - \hat{\theta}) \), compared to the business-as-usual scenario, \( \gamma \cdot r \),

\[
\dot{R}_t^{\text{bau}} = -\gamma \cdot r \cdot R_t^{\text{bau}},
\]

and

\[
\dot{R}_t^{\text{tax}} = -\gamma \cdot (r - \hat{\theta}) \cdot R_t^{\text{tax}}.
\]

As stated above, it is optimal for the firm to completely extract the extractable resource stock, \( S_0 \), over an infinite time horizon in both scenarios. Applying (43),
this is satisfied if
\[ \int_0^\infty P_{t_{\text{bau}}}^{\beta_{\text{bau}}} \, dt = S_{0_{\text{bau}}}, \tag{48} \]
and
\[ \int_0^\infty P_{t_{\text{tax}}}^{\beta_{\text{tax}}} \, dt = S_{0_{\text{tax}}}. \tag{49} \]
Inserting (42) and (41) respectively in these stock constraints allow the solutions of \( P_{0_{\text{tax}}} \) and \( P_{0_{\text{bau}}} \):
\[ P_{0_{\text{bau}}} = \left[ \frac{1}{\beta \cdot r \cdot S_{0_{\text{bau}}}} \right]^{\frac{1}{\beta}}, \tag{50} \]
and
\[ P_{0_{\text{tax}}} = \left[ \frac{1}{\beta \cdot (r - \hat{\theta}) \cdot S_{0_{\text{tax}}}} \right]^{\frac{1}{\beta}}. \tag{51} \]
According to the demand function this yields optimal initial extraction levels of
\[ R_{0_{\text{bau}}} = \beta \cdot r \cdot S_{0_{\text{bau}}}, \tag{52} \]
and
\[ R_{0_{\text{tax}}} = \beta \cdot (r - \hat{\theta}) \cdot S_{0_{\text{tax}}}. \tag{53} \]
According to the optimal price paths stated in (42) and (41), the price paths as a function of the initial level can be derived. The corresponding extraction amounts for these price paths are obtained via the demand function and are given by
\[ R_{t_{\text{bau}}} = \frac{\beta \cdot r \cdot S_{0_{\text{bau}}}}{e^{\gamma r t}}. \tag{54} \]
and
\[ R_{t_{\text{tax}}} = \frac{\beta \cdot (r - \hat{\theta}) \cdot S_{0_{\text{tax}}}}{e^{\gamma (r - \hat{\theta}) t}}. \tag{55} \]
To shed further light on the initial extraction level and the complete extraction path, the optimal fossil fuel stock for both scenarios are derived in the following.

\textbf{3rd step: Optimal exploratory decision}
Consider first the optimal level of exploratory activity in the business-as-usual scenario. According to Lasserre (1991, 106), the firm’s optimal investment in exploration must satisfy the following transversality condition at $t = 0$,

$$ C'(S^\text{bau}_0) = \lambda^\text{bau}_0 = P^\text{bau}_0. $$

(56)

The transversality condition requires that at $t = 0$ the cost of exploring a marginal unit of fossil fuel, given by $C'(S^\text{bau}_0)$, must be equal to its additional benefit that is measured by its shadow value, $\lambda^\text{bau}_0$ (both expressed in discounted value terms). This is intuitive as $\lambda^\text{bau}_0$ measures the value of a unit of resource *in situ* at the beginning of period 0 that arises if the fossil fuel owners extracts its stock optimally (Perman et al. 2003, 499). In effect, the shadow value for any $t$ measures the sensitivity of the total discounted maximum profit to the fossil fuel resource stock, that is, to which extent a marginal unit of fossil fuel in the ground augment the total profit at $t$ (Chiang 1992, 206). How is the optimal exploration decision impacted by the implementation of an increasing cash flow tax? The transversality condition required in the policy scenario yields insight:

$$ C'(S^\text{tax}_0) = \lambda^\text{tax}_0 = P^\text{tax}_0 \cdot \theta_0. $$

(57)

The implementation of an increasing cash flow tax leads to lower investments in exploration and hence to a lower total amount of fossil fuels discovered. However, the effect on the initial price level is unclear. This will be shown in the following, starting with the decrease in the optimal exploration activity.

Note that the tax does not affect the exploration cost function; the left-hand side of (57) remains unaffected. The right hand side is impacted by the cash flow tax twofold due to a change of the initial optimal price and the appearance of the tax factor $\theta_0$. Assume the firm would stick to the exploration decision that is optimal under business-as-usual conditions, that is, $C'(S^\text{tax}_0) = C'(S^\text{bau}_0)$, which implies the same initial amount of fossil fuels available. In turn, the optimal initial price would be lower in the policy scenario [to see this compare equations (50) and (51)] thereby decreasing the initial shadow value. The latter is further diminished by
a fraction corresponding to the initial tax rate of the initial price. This makes it clear that $\lambda_{0}^{\text{tax}} < \lambda_{0}^{\text{bau}}$ such that maintaining the same exploratory effort in the policy scenario does not satisfy the transversality condition. It must follow that the optimal exploratory investment is diminished in the policy scenario, as is the total extractable fossil fuel stock as a lower $C'(S_0)$ is linked to a lower level of $S_0$.

**B.2 Condition for a weak Green Paradox**

Recall from definition 3.1 that a weak Green Paradox captures the short term effect of an increasing cash flow tax. It arises when initial emissions from fossil fuel extraction are higher in the policy scenario compared to the business-as-usual scenario, formally, if $R_{0}^{\text{tax}} > R_{0}^{\text{bau}}$. Inserting the expressions obtained in (52) and (53) yield

$$\gamma (r - \hat{\theta}) \cdot S_{0}^{\text{tax}} > r \cdot S_{0}^{\text{bau}}. \quad (58)$$

While $r$ and $\hat{\theta}$ are constants, the respective initial reserve levels are endogenous depending on the optimal investment in exploration and are derived in the following. Inserting the optimal initial price levels expressed in (51) and (50) into the respective condition for optimal exploration decisions and rearranging obtains

$$S_{0}^{\text{bau}} = \frac{1}{\gamma \cdot r \cdot C'(S_{0}^{\text{bau}})^{\gamma}} \quad (59)$$

and

$$S_{0}^{\text{tax}} = \frac{\theta_{0}^{\gamma}}{\gamma \cdot (r - \hat{\theta}) \cdot C'(S_{0}^{\text{tax}})^{\gamma}} \quad (60)$$

The expression for the optimal exploration cost function can be replaced with the first derivation of the exploration cost function specified in (8), that is, $C'(S_0) = \alpha \cdot \beta \cdot S_0^{\alpha - 1}$. This yields an expression for the optimal fossil fuel stock in both scenarios which depends on the constants:

$$S_{0}^{\text{bau}} = \left( \frac{1}{\gamma \cdot r \cdot (\alpha \beta)^{\gamma}} \right)^{\frac{1}{\gamma(\alpha - 1) + 1}} \quad (61)$$

$$S_{0}^{\text{tax}} = \left( \frac{\theta_{0}^{\gamma}}{\gamma \cdot (r - \hat{\theta}) \cdot (\alpha \beta)^{\gamma}} \right)^{\frac{1}{\gamma(\alpha - 1) + 1}}. \quad (62)$$
Conditions (58), (61) and (62) reveal the effect of the components of the tax schemes in terms of generating a weak Green Paradox. That is, the higher $\theta_0$, the higher the optimal fossil fuel stock in the policy scenario and hence the higher the initial extraction level. However, the growth rate of the tax causes two countervailing impacts. On the one hand, (58) shows clearly that for a higher $\hat{\theta}$, ceteris paribus, the initial amount of fossil fuels decreases. On the other hand, the initial extraction level is defined by (62). It becomes clear that a higher $\hat{\theta}$ decreases the initial extraction stock and hence initial extraction levels rise [refer to (58)]. Note also that equations (61) and (62) prove formally what has been said beforehand, that is, that the optimal initial fossil fuel stock in the policy scenario is lower compared to the business-as-usual scenario.

Finally, inserting equations (61) and (62) in equation (58) provides the condition for a weak Green Paradox:

$$\theta_0 > \left(1 - \frac{\hat{\theta}}{r}\right)^{1-\alpha}. \quad (63)$$

### B.3 Condition for a strong Green Paradox

This section derives the formal conditions under which an increasing cash flow tax produces a strong Green Paradox. According to definition 3.2, this occurs if the implementation of an increasing cash flow tax leads to higher climate change damage compared to the business-as-usual scenario, formally given by $\Gamma^{\text{tax}} > \Gamma^{\text{bau}}$.

To solve the climate damage function for both scenarios, it is necessary to derive the complete extraction paths. Replacing $S_0^{\text{bau}}$ and $S_0^{\text{tax}}$ in (55) and (54) with the functions given in (61) and (62) obtain

$$R_t^{\text{bau}} = (\gamma \cdot r) \left(1 - \frac{1}{\gamma (\alpha-1+\frac{1}{\gamma})}\right) \cdot (\alpha \cdot \beta) \left(-\frac{1}{\gamma (\alpha-1+\frac{1}{\gamma})}\right) \cdot e^{-\gamma r t} \quad (64)$$

and,

$$R_t^{\text{tax}} = [\gamma \cdot (r - \hat{\theta})] \left(1 - \frac{1}{\gamma (\alpha-1+\frac{1}{\gamma})}\right) \cdot (\alpha \cdot \beta) \left(-\frac{1}{\gamma (\alpha-1+\frac{1}{\gamma})}\right) \cdot e^{-\gamma (r-\hat{\theta}) t} \cdot \theta_0 \left(\frac{1}{\alpha-1+\frac{1}{\gamma}}\right) \quad (65)$$
Inserting these equations into the net present value of climate change damages expressed in (12), yield climate damages of

\[
\Gamma_{bau} = \frac{1}{\delta - \bar{\chi} + \gamma r} c \cdot (\gamma \cdot r) \left(1 - \frac{1}{\gamma \left( (\alpha - 1 + \frac{1}{\gamma}) \right)} \right) \cdot (\alpha \cdot \beta) \left( -\frac{1}{(\alpha - 1 + \frac{1}{\gamma})} \right) \quad (66)
\]

and,

\[
\Gamma_{tax} = \frac{1}{\delta - \bar{\chi} + \gamma (r - \hat{\theta})} c \cdot [\gamma \cdot (r - \hat{\theta})] \left(1 - \frac{1}{\gamma \left( (\alpha - 1 + \frac{1}{\gamma}) \right)} \right) \cdot (\alpha \cdot \beta) \left( -\frac{1}{(\alpha - 1 + \frac{1}{\gamma})} \right) \cdot \frac{1}{(\alpha - 1 + \frac{1}{\gamma})} \quad (67)
\]

A strong Green Paradox occurs when net climate damages in the policy scenario [equation (67)] are greater than climate net damages arising in the business-as-usual scenario [equation (66)]. This occurs, if

\[
\theta_0 > \left( 1 - \frac{\gamma \cdot \hat{\theta}}{\delta - \bar{\chi} + \gamma r} \right) \left( (\alpha - 1 + \frac{1}{\gamma}) \right) \cdot \left( 1 - \frac{\hat{\theta}}{r} \right)^{1-\alpha}. \quad (68)
\]