# The Elasticity of Substitution between Clean and Dirty Inputs in the Production of Electricity[†]

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**Abstract**

I obtain a calibrated estimate of the elasticity of substitution between *clean* and *dirty* inputs in the production of electricity for 21 countries. To perform the calibration, I extend the endogenous growth model with directed technical change developed in Acemoglu et al. (2009) to a multi-sector setting. In the model, the elasticity of substitution determines the relative size of two effects – the *price effect* and the *market size effect* – which in turn determines the direction of technical change towards *clean* or *dirty* technologies. The threshold value of 1 defines an interval where a switch to *clean* inputs is impossible (<1) and one where it can be attained with reasonable subsidies to research and a carbon tax (>1). I calibrate an average elasticity of 0.51 for all the 21 countries taken into consideration, thanks to the hypothesis of perfect capital mobility between *clean* and *dirty* production within sectors. The complementarity of these inputs makes it theoretically impossible for the electricity sector to reach a tipping point if left to its own devices. Moreover, the strong complementarities characterizing it allow to generalize the prediction to the economy as a whole. A complete switch to clean technologies seems to be difficult to attain unless growth in the electricity sector comes to a complete stop.

JEL classification: O13, O31, Q20, Q55.

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# 1 Introduction

Can a new zero-emission technology supplant the actual one, while keeping the economy growing? When is this going to happen? Is the government supposed to play a major role in the change? A big debate surrounds these questions in environmental economics. In spite of the variety of answers, the literature seems to agree that some level of state intervention is unavoidable.[1] I use an endogenous growth model with directed technical change to contribute to this discussion. This paper shows that, due to the lack of substitutability between green and polluting inputs, it may be more complex than previous literature thought to keep growing while stop polluting, regardless of government intervention.

The main goal of this paper is to obtain an estimate of the elasticity of substitution between *clean* and *dirty* sources of electricity generation. The precision of this estimate is critically important for two reasons. First, because electricity is a primary source of energy in modern economies, errors in the estimation of this elasticity may have large consequences for the accuracy of policy advice. Second, the conclusions of climate-economy models with a disaggregated energy sector like WITCH (Bosetti et al., 2006), ENTICE (Popp, 2004), GTAP-E (Burniaux and Truong, 2002), DEMETER (van der Zwaan and Gerlagh, 2009), or GREEN (Lee and Oliveira-Martins, 1994), are heavily dependant on it. Yet, to date this kind of Computable General Equilibrium (CGE) models are calibrated using *ad-hoc* values, typically ranging between 0.25 and 10, whose main virtue is their plausibility, although there is little empirical or theoretical basis against which to gauge this plausibility.

This paper stems from the endogenous growth model developed in Acemoglu et al. (2009) [henceforth AABH]. AABH's model is structured as follows. An aggregate final good is produced using a mix of intermediate goods coming from two intermediate-production sectors: a clean one and a dirty one. The direction of technical change – toward a more intensive utilization of clean or dirty inputs – is determined by three forces.[2] The first one, the *direct productivity effect*, leads innovation toward the sector with higher productivity. The second one, the *price effect* pushes it towards the sector with higher prices (naturally

[1] Conte et a., 2010; Newell, 2009; Newell, 2008; Aghion et al., 2006; Goulder, 2004; Jaffe et al., 2003

[2] The pairs of words clean/dirty and polluting/non-polluting are used as synonyms throughout the paper.

the relatively less developed sector). Finally, the direction dictated by the *market size effect* depends crucially on the degree of substitutability of production inputs. If they are substitutes, it encourages innovation in the sector with greater employment, hence more firms and where a particular patent might generate more revenue. Vice-versa if they are complements. The relative weights of these forces are functions of the elasticity of substitution between clean and dirty inputs, $\varepsilon$, and determine the direction taken by innovation. If $\varepsilon < 1$ (i.e. the *price effect* and the *market size effect* go in the same direction), the two inputs are complements and hence both are essential for production. Indeed, in the long run both sectors will grow at the same rate. If $\varepsilon > 1$ (i.e. the *market size effect* goes in the opposite direction and is stronger than the *price effect*), the two inputs are substitutes and hence neither is requisite for production. Thus, growth is stronger in the dirty and – by assumption – more advanced sector. Eventually, only the dirty sector will experience growth. In this case a complete switch toward clean innovation is achievable via subsidies and a carbon tax, while in the former case a switch is theoretically impossible.

In this paper, I aim at obtaining a calibrated estimate of the parameter $\varepsilon$, i.e. the elasticity of substitution between clean and dirty inputs. In order to obtain it I extend the model of AABH to a multi-sector version, in which every sector produces a different consumption good. In this version of the model, each sector is composed by two separate production branches. One branch uses dirty inputs and the other one uses clean inputs. Each sector's final good is produced using a mix of intermediate goods coming from the two sector-specific branches. I focus especially on the calibration of the elasticity of substitution in the electricity sector.

In addition to the intrinsic importance of the energy sector, the choice of power production for this study is dictated by two other reasons. First, in the case of electricity the distinction between polluting and non-polluting inputs is straightforward. In other sectors this distinction would be more difficult and involve additional hypotheses. Take for instance any kind of industrial production in which green technologies are used, it is not clear how to disentangle them from the polluting ones. Second and crucially, because of the strong complementarities characterizing the electricity sector. Electricity is a complement to a large variety of other consumption goods – both durable and non-durable – and, at the moment, there is no substitute for it. These two particular features make of electricity the best subject for this study.[3] Aghion et al. (2011) – also building upon AABH – investigate the automobile sector using patent data in order to distinguish between clean and dirty innovation. The aim of their paper is to investigate the existence of path dependency in technical innovation and indeed they find evidence of it and of the fact that higher taxes and/or subsidies stimulate clean innovation.

The elasticity of substitution is first identified for the US. Identification is achieved by adding perfect capital mobility between the two branches (clean and dirty) within a sector. This new hypothesis implies equality of the marginal productivity of capital in the two branches, which allows to obtain a closed form solution for the elasticity (results are robust to the relaxation of this hypothesis). I then calibrate the value of the elasticity of substitution for other 20 countries. This is presented as a second step because less data is available for these additional countries. The equalization of the US relative price between clean and dirty intermediates – computed using the previously calibrated value of the US's elasticity of substitution – to the price ratio in another country allows me to identify the elasticity of substitution for the other country.

I find an average value for the elasticity of 0.51 across all the countries in the sample, identifying the two inputs as complements. The worse case scenario considered in AABH is $\varepsilon = 3$. In this case permanent green research subsidies are needed in order to switch toward a non-polluting production. Nevertheless, this would involve temperatures increasing for another 55 years and a complete switch toward clean innovation only in 45 years. Subsidies would need to be coupled with a high carbon tax, which would have to continue increasing for almost 300 years. Therefore, the implications of a value lower than 1 are profound.

I argue that the low value of the elasticity found for the production of electricity has two interpretations. The first derives from the lack of a proper storage technology for electricity and the need for supply to constantly meet demand, the failure of which would

[3] The case of electricity presents one problem concerning the availability of data. Series on quantities produced using clean and dirty inputs as well as on prices for electricity produced with dirty inputs are easy to find. Yet, data on prices of electricity produced using clean inputs do not exist.

result in a regional blackout.[4] Given that electricity cannot yet be stored efficiently for long time periods (more than a day), the only way to ensure a smooth supply is by storing the resources needed to produce it. Yet, while storing dirty resources is relatively easy, it is impossible to store the two main sources of renewable energy (sun and wind) which are highly intermittent. The second interpretation of the result arises from the fact that – at least at an early stage of development of new clean technologies – energy coming from dirty sources is needed in order to produce the tools allowing the exploitation of clean inputs.

The result of this paper, the complementarity between clean and dirty inputs, implies that, for the electricity sector, a complete switch toward clean inputs is theoretically impossible – even with government intervention. In this framework, state intervention at a first stage is justified by the public nature of the environment and the climate, which causes two market failures to occur simultaneously (Conte et al., 2010). First, environmental externalities lead to too little spontaneous demand for emission-reducing technologies. On the one hand, it is not possible to fully capture the benefits of a mitigation action and on the other hand, firms and households do not have to pay for the climate damage they impose by using polluting technologies. Second, companies lack incentives to invest in non-polluting technologies, because of the *appropriability effect* associated with the expected post-innovation rents (Aghion et al., 2006). Companies anticipate that, given society preferences, there could be pressure for a large scale dissemination of the outputs of green innovation. This would impede them to fully capture the market value of their investments in green R&D. Therefore, they reduce their contribution to green innovations (Jaffe et al., 2004; Newell, 2009). This problem tends to worsen the more basic and long term is the research.

The remainder of the paper is organized as follows. Section 2 introduces the multi-sector version of AABH's model. Section 3 describes the data, illustrating the differences in data availability across countries and the ensuing need of different identification strategies. Section 4 presents the hypotheses that allow me to identify an estimable expression for the elasticity of substitution between clean and dirty inputs. Section 5 outlines the results, first for the US and then for the rest of the world. Section 6 presents robustness checks for the

[4]Electricity outages cost the US approximately USD79 billion a year (APS, 2007)

two identifying assumptions. Section 7 concludes.

## 2 The Model

This paper focuses on the electricity market. First, because of the simplicity of defining clean and dirty inputs and second, because of the specificity of electricity as a consumption good: it enters consumption as a complement to other goods.[5] Electricity is particularly interesting because it is a direct complement of a large variety of durable, semi-durable and non-durable goods. This singularity, together with the lack of a substitute for it, makes of electricity an extremely interesting sector to study in this context. In order to capture a sectorial elasticity of substitution, I extend the baseline model developed in AABH to a multi-sector setting.[6]

The model consists in an infinite-horizon discrete-time set-up. The economy is inhabited by a continuum of households comprising workers, entrepreneurs and scientists. Let $t$ be a time subscript. $C$ and $S \in [0, \bar{S}]$ respectively represent consumption of the unique final good and the quality of the environment. $\bar{S}$ represents the maximum environmental quality – defined as its pre-industrial level – implying that an additional improvement in it does not ameliorate utility. Instead, $S$ reaching 0 would have severe implications for utility.[7] Henceforth, for simplicity, I call these severe consequences an environmental disaster. All households have the following utility function

$$U = \sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} u(C_t, S_t) \tag{1}$$

where $\rho > 0$ represents the discount rate.

In the US, 75% of the electricity produced is used as a final consumption good, while

[5] Some exceptions exist. For example electricity and gasoline are imperfect substitutes to run a car.

[6] I refer the reader interested in the complete developments and proofs of the baseline version of the model to the original paper Acemoglu et al. (2009)

[7] This is implied by the following Inada-type conditions: $\lim_{C\downarrow 0} \frac{\partial u(C,S)}{\partial C} = \infty$, $\lim_{S\downarrow 0} \frac{\partial u(C,S)}{\partial S} = \infty$ and $\lim_{S\downarrow 0} u(C,S) = -\infty$ and the following assumption $\frac{\partial u(C,\bar{S})}{\partial S} = 0$.

only 25% is used as an intermediate good.[8] Therefore, for the sake of simplicity, I introduce electricity in the model only as one of the consumption goods, and not as a production input.

The final consumption good is an aggregate of final goods produced in a variety of sectors $i$ with $i = 1, ..., n$. Electricity is one of these sectors. Given its particular features, I allow for it to be a possible complement to all other goods via a CES aggregation between electricity – indicated by subscript $n$ – and any other good $j \in (1, m)$[9]

$$Z_{tj} = \left(Y_{tn}^{\psi_j} + Y_{tj}^{\psi_j}\right)^{\frac{1}{\psi_j}} \tag{2}$$

I then aggregate all these composite final goods, $Z_{tj}$, using a simple Cobb-Douglas aggregation, in order to obtain the final consumption good

$$Y_t = \Pi_{j=1}^{m} \left(Z_{tj}\right)^{\zeta_j} \tag{3}$$

where $\sum_{j=1}^{m} \zeta_j = 1$. In each sector $i$ the final good is assembled using a mix of intermediate goods produced with clean and dirty inputs. Intermediates are aggregated using a CES production function

$$Y_{ti} = (Y_{cti}^{-\sigma_i} + Y_{dti}^{-\sigma_i})^{-\frac{1}{\sigma_i}} \tag{4}$$

where $-\sigma_i = \frac{\varepsilon_i - 1}{\varepsilon_i}$, and $\varepsilon_i$ represents the value of the elasticity of interest for sector $i$. Each intermediate good is produced using capital (a continuum of sector-specific machines) and labor in the following way

$$Y_{fti} = (L_{fti} A_{fti})^{1-\alpha_i} \int_0^1 x_{fi}(\nu, t)^{\alpha_i} d\nu \tag{5}$$

where index $f \in \{c, d\}$ identifies a variable as being clean/non-polluting, if associated with $c$ and dirty/polluting if otherwise. $A_{fti}$ and $x_{fi}(\nu, t)$ represent respectively the branch-

[8] Source: US Energy Information Administration

[9] where $i \in (1, n)$ represents all sectors, including the electricity one, while $j \in (1, m)$ represents all sectors, except the electricity one.

specific stock of technology and the quantity of machines.[10] $\alpha_i$ denotes the share of capital (i.e. machines) used in production. I allow for this share to vary across different sectors. It could also vary across countries, as this would not affect the calibration of $\varepsilon$. Yet, in the case of the electricity market, it is probable that this share is fairly constant across countries. For instance, the capital to labor ratio in a nuclear power plant is likely to be extremely similar in France and in the US.

The market for machines is monopolistically competitive. Every year, each scientist decides whether to do research in the clean or in the dirty branch of her sector-$i$. A patent for the machine is then issued to the successful scientist for the following year. Her probability of success is of $\eta_{fi} \in [0, 1]$. Consequently, the branch-specific stock of technology, i.e. the quality of machines, in branch-$f$ of sector-$i$ evolves in the following way

$$A_{fti} = (1 + \gamma\eta_{fi}s_{fti})A_{f(t-1)i} \tag{6}$$

where $\gamma$ and $s_{fti}$ represent respectively the increase in quality if the innovation is successful and the number of branch-specific scientists in sector-$i$.[11] The model has the classic market clearing conditions, for labor $\sum_i (L_{cti} + L_{dti}) \leq 1$ and for scientists $s_{cti} + s_{dti} \leq 1$.

Environmental quality evolves as follow

$$S_{t+1} = -\xi \sum_i Y_{dti} + (1 + \delta)S_t \tag{7}$$

where $\xi$ and $\delta$ represent respectively the rate of environmental degradation resulting from production using dirty inputs and the rate of regeneration of the environment (coming for instance from forests and oceans).[12]

The next subsection is devoted to the solution of the laissez-faire equilibrium of this model. It is kept at the minimum necessary in order to show the implications of the results

[10] While AABH supposes technology to be machine-specific, I assume it to be branch-specific. This change of hypothesis allows the obtention of the calibrated estimate of $\varepsilon$ and, given the multi-sector feature of the model, is not too restrictive.

[11] For simplicity I assume $\gamma_i = \gamma \ \forall i$.

[12] Therefore, the environmental disaster is reached if the production of dirty intermediates across sectors reaches $\sum_i Y_{dti} = (1 + \delta)\bar{S}\xi^{-1}$.

of this paper.

## 2.1 Laissez Faire Equilibrium

In this model the direction of technical change is dictated by profit incentives. Scientists decide to do research in the branch with higher expected profits. The relative profits from undertaking research in the polluting versus non-polluting branch of sector-$i$ are

$$\frac{\pi_{cti}}{\pi_{dti}} = \frac{\eta_{ci}}{\eta_{di}} \underbrace{\left(\frac{p_{cti}}{p_{dti}}\right)^{\frac{1}{1-\alpha_i}}}_{\text{Price E.}} \underbrace{\frac{L_{cti}}{L_{dti}}}_{\text{Market Size E.}} \underbrace{\frac{A_{c(t-1)i}}{A_{d(t-1)i}}}_{\text{Dir. Prod. E.}} \tag{8}$$

If this ratio is above 1, it is more profitable to develop machines for clean inputs. If instead the ratio is between 0 and 1, it is more profitable to develop machines for dirty inputs. The composition of this ratio directly shows the three forces shaping the direction of technical change

- *Price Effect*, encourages innovation in the branch with higher prices;
- *Market Size Effect*, encourages innovation in the branch with greater employment, if the two inputs are substitutes;
- *Direct Productivity Effect*, encourages innovation in the branch with higher productivity.

$\frac{\eta_{ci}}{\eta_{di}}$ simply represents the ratio of the probability of a successful discovery between the two branches. I now focus on the first two effects. The *market size effect* may create a force toward innovation in the more advanced branch – hence with a bigger market – or toward the other branch, while the *price effect* produces a force toward innovation in the more backward branch – hence with higher prices. These two effects are determined by relative productivities and the elasticity of substitution ($\varepsilon$). The higher the degree of substitutability among the two inputs the stronger and opposite is the *market size effect* with respect to the *price effect*. Therefore, when $\varepsilon_i > 1$ (inputs are substitutes) the *market* *size effect* is stronger and opposite to the *price effect*, and vice-versa when $\varepsilon_i < 1$ (inputs are complements).

Assuming that at the starting point, in all sectors, the more developed branch is the dirty one, the model concludes that if $\varepsilon_i > 1$ innovation will be stronger in that branch. This will reinforce the pattern by increasing the gap between technology in the clean and the dirty branch. In the long term only $A_{di}$ will grow and the growth rate of the sector will be of $\gamma\eta_{di}$. This process can be reversed via the introduction of subsidies ($q_{ti}$) for research in the non-polluting branch

$$\frac{\pi_{cti}}{\pi_{dti}} = (1+q_{ti})\frac{\eta_{ci}}{\eta_{di}}\left(\frac{p_{cti}}{p_{dti}}\right)^{\frac{1}{1-\alpha_i}}\frac{L_{cti}}{L_{dti}}\frac{A_{c(t-1)i}}{A_{d(t-1)i}} \tag{9}$$

Subsidies will make investing in the clean branch of the sector more profitable. In the case in which $\varepsilon_i > \frac{1}{1-\alpha_i}$ (strong substitutes), once clean technology has caught up with dirty technology the switch will have occurred and it will be irreversible, and the government can stop subsidizing the clean branch. When $1 < \varepsilon_i < \frac{1}{1-\alpha_i}$ (weak substitutes), subsidies need to be permanent. However, in both cases, an environmental disaster can be avoided, and the growth rate of the sector in the long run is $\gamma\eta_{ci}$.

Instead, if $\varepsilon_i < 1$ both inputs are required in order to produce the final good of sector $i$. When $\varepsilon_i < 1$, the *price effect* is stronger than the *market size effect*, therefore innovation is more intense in the more backward branch of the sector, in our case the non-polluting one. Innovation will happen more intensively in that branch until it catches up with the polluting one, after that point, both branches will continue to grow at the same rate. In this case, an environmental disaster can be avoided only by stopping economic growth. If there would be no stop, the long run growth rate of sector $i$ would be $\gamma\tilde{\eta}_i$, where $\tilde{\eta}_i \equiv \frac{\eta_{di}\eta_{ci}}{\eta_{di}+\eta_{ci}}$.

The extension of the model presented here, allows for each sector $i$ to be characterized by a different elasticity of substitution. It is particularly interesting to observe what happens if only the elasticity of substitution in the electricity sector is assumed to be smaller than 1.

**Proposition 2.1** *If all sectors-i experience substitutability between clean and dirty inputs* *in production and a sufficient flow of subsidies $q_{ti}$ is provided, scientists in all sectors will only perform clean research. In this case an environmental disaster is avoided. Yet, if one sector, producing a good which is itself a complement to other consumption goods, does not show substitutability but complementarity, the only way in which to avoid an environmental disaster is to stop growth in that particular sector.*

**Proof** Taking logs and time derivatives of equations (2) and (3) obtains the aggregate growth rate of production in the economy[13]

$$\frac{\dot{Y}_t}{Y_t} = \sum_{j=1}^{m} \zeta_j \left[ \delta_j \frac{\dot{Y}_{tn}}{Y_{tn}} + (1-\delta_j)\frac{\dot{Y}_{tj}}{Y_{tj}} \right] \tag{10}$$

where $\delta_j \equiv \frac{Y_{tn}^{\psi_j}}{Y_{tn}^{\psi_j}+Y_{tj}^{\psi_j}}$ and $\sum_{j=1}^{m} \zeta_j = 1$. Three cases are relevant in order to understand the behavior of the economy. One in which we observe substitutability in all sectors, $\varepsilon_j > 1 \ \forall j$ *and* $\varepsilon_n > 1$. One in which we observe complementarity in all sectors, $\varepsilon_j < 1 \ \forall j$ *and* $\varepsilon_n < 1$. And a last one, in which we observe substitutability in all sectors except one, $\varepsilon_j > 1 \ \forall \ j$ *and* $\varepsilon_n < 1$.

*Case I: Substitutability in all sectors,* $\varepsilon_j > 1 \ \forall j$ *and* $\varepsilon_n > 1$

If there is substitutability between clean and dirty inputs in production within each sector of the economy, and a sufficient flow of subsidies is provided, the long run growth rate in each of them is $\gamma\eta_{ci}$, as seen before. Therefore the long run growth rate of aggregate production equals $\frac{\dot{Y}_t}{Y_t} = \gamma \sum_{j=1}^{m} \zeta_j \left[\delta_j \eta_{cn} + (1-\delta_j)\eta_{cj}\right]$. The aggregate growth rate does not depend on $\eta_{dj}$ or $\eta_{dn}$, this implies that all the scientists are allocated to research in the clean branch of each sector. Therefore the overall growth rate of the dirty branch is equal to zero and an environmental disaster is avoided.

*Case II: Complementarity in all sectors,* $\varepsilon_j < 1 \ \forall j$ *and* $\varepsilon_n < 1$

As a consequence of complementarity, inputs coming from both the clean and the dirty branch are essentials in every sector $i$ to produce the final good. Implying that $s_{ci} = \frac{\eta_{di}}{\eta_{di}+\eta_{ci}} > 0$ and $s_{di} = \frac{\eta_{ci}}{\eta_{di}+\eta_{ci}} > 0 \ \forall \ i$, i.e. a positive proportion of scientists is working

[13] As before, subscripts $n$ and $j \in (1, m)$ denote respectively the electricity sector and all other sectors, while subscript $i \in (1, n)$ designates all sectors.

in the dirty inputs branch.[14] In this case the growth rate in every sector is $\gamma\tilde{\eta}_i$ with $\tilde{\eta}_i \equiv \frac{\eta_{di}\eta_{ci}}{\eta_{di}+\eta_{ci}}$. Therefore, the long run growth rate of aggregate production equals $\frac{\dot{Y}_t}{Y_t} = \gamma\sum_{j=1}^{m}\zeta_j\left[\delta_j\tilde{\eta}_n + (1-\delta_j)\tilde{\eta}_j\right]$. This growth rate depends on $\eta_{dj}$, $\eta_{dn}$, $\eta_{cj}$ and $\eta_{cn}$. Therefore, the growth rate of clean and dirty intermediate goods is positive. As a consequence the economy will reach in a finite time $\sum_i Y_{id} = (1+\delta)\bar{S}\xi^{-1}$, i.e. an environmental disaster. The only way to avoid it is to stop growth in all sectors.

*Case III: Substitutability in all sectors except one, $\varepsilon_j > 1$ $\forall j$ and $\varepsilon_n < 1$*

We now observe substitutability between clean and dirty production inputs in all sectors except one, in which we observe complementarity, sector $n$. Moreover sector $n$ is itself a complement to many other sectors, hence it cannot be substituted away. This implies that a fraction of scientists within sector $n$ is allocated to dirty research, $s_{dn} = \frac{\eta_{cn}}{\eta_{dn}+\eta_{cn}} > 0$. Therefore, the overall growth rate of production of intermediate goods using dirty inputs is small but positive (coming from one single sector). In this case, the long run growth rate of aggregate production equals $\frac{\dot{Y}_t}{Y_t} = \gamma\sum_{j=1}^{m}\zeta_j\left[\delta_j\tilde{\eta}_n + (1-\delta_j)\eta_{cj}\right]$. The aggregate growth rate depends on $\eta_{cj}$, $\eta_{cn}$ and $\eta_{dn}$. This implies that in a finite time (longer than in the previous case) the economy reaches $\sum_i Y_{id} = (1+\delta)\bar{S}\xi^{-1}$, or, more precisely, $Y_{nd} = (1+\delta)\bar{S}\xi^{-1}$, and thus an environmental disaster. The only way to avoid it is to stop growth in that particular sector. ∎

## 3 Data

I evaluate the elasticity of substitution for the following countries: United States, Germany, France, Italy, Spain, United Kingdom, Norway, Sweden, Switzerland, EU16, Australia, Japan, Turkey, New Zealand, Mexico, South Korea, Canada, Brazil, Chile, China, Russia, India and finally an aggregate for the whole world. Data on $Y_{ctn}$, $Y_{dtn}$, $x_{ctn}$ and $x_{dtn}$ are available for the US. If need be, it would then be easy – thanks to the assumptions of the

[14] The expressions for $s_{ci}$ and $s_{di}$ are found by rewriting equation (8) using (6), $\frac{\pi_{cti}}{\pi_{dti}} = \frac{\eta_{ci}}{\eta_{di}}\left(\frac{1+\gamma\eta_{ci}s_{cit}}{1+\gamma\eta_{di}s_{dit}}\right)^{-\varphi_i-1}\left(\frac{A_{cit-1}}{A_{dit-1}}\right)^{-\varphi_i}$, where $\varphi_i \equiv (1-\alpha_i)(1-\varepsilon_i)$. In the long run $\frac{\pi_{cti}}{\pi_{dti}} = 1$ and innovation grows in both sectors, implying $A_{ci} = A_{di}$. Therefore, $\frac{1+\gamma\eta_{ci}s_{cit}}{1+\gamma\eta_{di}s_{dit}} = 1$. Using this and the market clearing condition for scientists, it is straightforward to obtain the expressions needed.

model – to retrieve data on labor in efficient units, which would be equal to

$$L_{ftn}A_{ftn} = \left(\frac{Y_{ftn}}{x_{ftn}^{\alpha_n}}\right)^{\frac{1}{1-\alpha_n}} \tag{11}$$

Instead, for the other countries in the sample, data exists only on $Y_{ctn}$ and $Y_{dtn}$, but not on $x_{ctn}$ and $x_{dtn}$. Therefore, a different identification has to be devised for these countries.

The data used in the simulation of the elasticity comes from the *U.S. Energy Information Administration* for the US and from *Eurostat* for the European Union. Data for the other countries included in the study and for the world aggregate come from the *International Energy Agency.* The computations cover data for a variety of types of energy productions. Data for the US are available yearly from 1949 to 2009, for Europe only from 1996 to 2006 and for other countries and the world aggregate only for 2007 (therefore for these countries I will provide only point calibrations).

All data are expressed in *btu*, i.e. British Thermal Units, which is a standard measure enabling the comparison of different sources of energy using their potential power.[15] This measure allows to sum the production of energy using different sources, and at the same time to know how much energy is lost in production. Hence the data used do not express a volume, but the quantity of heat produced. For example a barrel of oil can be aggregated with a ton of coal, because one knows exactly how much heat each of them may produce.

The polluting branch includes energy produced from the following sources : coal, petroleum, natural gas and other gases. The non-polluting branch includes the following: hydroelectric, biomass (wood and waste), geothermal, solar/photovoltaic, wind and nuclear.

# 4 Identification of the Elasticity of substitution

Based on US data availability and the model, an additional hypothesis is required in order to identify $\varepsilon_i$. An estimable expression may be obtained by assuming perfect capital mobility

[15] It is approximately the amount of energy needed to heat 1 pound (0.454 kg) of water by 1 °F (0.556 °C). And can be translated in kilowatts with the following transformation rule, 1 $Kilowatthour = 3'412$ *btu*.

between branches within a sector. This hypothesis can then be relaxed, as I show in a robustness check in section 6.

Using equation (5), the expression for aggregate production, (4), writes

$$Y_{ti} = \left[ \left( (L_{cti}A_{cti})^{1-\alpha_i} \int_0^1 x_{ci}(\nu,t)^{\alpha_i} d\nu \right)^{-\sigma_i} + \left( (L_{dti}A_{dti})^{1-\alpha_i} \int_0^1 x_{di}(\nu,t)^{\alpha_i} d\nu \right)^{-\sigma_i} \right]^{-\frac{1}{\sigma_i}} \quad (12)$$

Define

$$\int_0^1 x_{fi}(\nu,t) d\nu \equiv x_{fti} \quad (13)$$

as the total amount of capital in branch-$f$ of sector-$i$. The new hypothesis implies equality among the marginal product of capital in both branches, leading to the following ratio

$$\frac{MPK_{ci}}{MPK_{di}} = 1 = \left( \frac{L_{cti}A_{cti}}{L_{dti}A_{dti}} \right)^{\sigma_i(\alpha_i - 1)} \left( \frac{x_{cti}}{x_{dti}} \right)^{-\alpha_i\sigma_i - 1} \quad (14)$$

where $MPK_{ci}$ and $MPK_{di}$ denote respectively the marginal product of capital in the clean and in the dirty branch of sector-$i$. Therefore $\varepsilon_i$ equals

$$\varepsilon_i = \frac{\alpha_i(X_i - N_i) + N_i}{\alpha_i(X_i - N_i) + N_i - X_i} \quad (15)$$

where $N_i \equiv \ln\left(\frac{L_{cti}A_{cti}}{L_{dti}A_{dti}}\right)$ and $X_i \equiv \ln\left(\frac{x_{cti}}{x_{dti}}\right)$ are respectively the logarithm of the ratio between efficient labor and capital used in the clean with respect to the dirty branch.

Using equation (11) – and hence exploiting the Cobb-Douglas assumption on the production of intermediate goods – and abstracting from the sectorial index $i$, $N$ writes as

$$N = \ln\left(\frac{L_{ct}A_{ct}}{L_{dt}A_{dt}}\right) = \frac{1}{1-\alpha} \ln\left(\frac{Y_{ct}}{Y_{dt}}\right) - \frac{\alpha}{1-\alpha} \ln\left(\frac{x_{ct}}{x_{dt}}\right) \quad (16)$$

Using expression (16), I simplify equation (15) to

$$\varepsilon = \frac{\ln\left(\frac{Y_{ct}}{Y_{dt}}\right)}{\ln\left(\frac{Y_{ct}}{Y_{dt}}\right) - \ln\left(\frac{x_{ct}}{x_{dt}}\right)} \tag{17}$$

Note that the elasticity does not depend on the value of $\alpha$. From expression (17), the data needed to define $\varepsilon_i$ are $Y_{cti}$, $Y_{dti}$, $x_{cti}$ and $x_{dti}$. As seen in the previous section, such detailed data on the production of electricity are available only for the US. Therefore, in order to calibrate the elasticity of substitution for a full array of countries, I need an alternative identification.

In what follows the elasticity of substitution for the US is taken as given. In order to compute the elasticity for every other country, I follow AABH and assume that the price ratio of intermediate goods coming from the two branches equalizes among countries, i.e. $\frac{p^u_{cti}}{p^u_{dti}} = \frac{p^v_{cti}}{p^v_{dti}}$, where $u$ and $v$ denote two different countries. This implies for example that the price ratio between electricity produced with photovoltaic panels and coal in France is equal to the one in Germany. This assumption follows directly from the competitive market hypothesis for intermediate goods. Section 6 verifies the robustness of the results and shows that they hold even when this assumption is relaxed. Final goods are also produced competitively, hence the relative price of the two inputs takes the following form

$$p_{ti} \equiv \frac{p_{cti}}{p_{dti}} = \left(\frac{Y_{cti}}{Y_{dti}}\right)^{-\sigma_i - 1} \tag{18}$$

The price ratio for the US is computed thanks to the value of the elasticity obtained before

$$p^{US}_{ti} = \left(\frac{Y^{US}_{cti}}{Y^{US}_{dti}}\right)^{-(\sigma^{US}_i + 1)} \tag{19}$$

and – using the assumption of equalization of price ratios between countries – equalized to the price ratio for country $v$

$$p^{US}_{ti} = \left(\frac{Y^{v}_{cti}}{Y^{v}_{dti}}\right)^{-(\sigma^{v}_i + 1)} \tag{20}$$

the expression for the elasticity of substitution of country $v$ follows

$$\varepsilon_i^v = \frac{1}{2 + \frac{P_i^{US}}{Y_i^v}} \tag{21}$$

where $P_i^{US} \equiv \ln\left(p_{ti}^{US}\right)$ is the ratio of the price of the intermediate good produced with clean relative to dirty inputs in the US, and $Y_i^v \equiv \ln\left(\frac{Y_{cti}^v}{Y_{dti}^v}\right)$ is the ratio of production of intermediate goods using non-polluting with respect to polluting inputs in country $v$. Note that in order to compute the elasticity of substitution for other countries, only data on $Y_{cti}^v$, $Y_{cti}^v$ and $\varepsilon_i^{US}$ are needed.

# 5 Results

## 5.1 The US

This section gives first a brief description of the data at hand, and then moves to the main result of the paper: the complementarity between polluting and non-polluting inputs in the production of electricity. Results are followed by their interpretation and a discussion of the lessons which can be drawn from the model for the economy as a whole.

Starting from 1949 and up to the beginning of the 70s, the share of electricity produced using clean inputs has known a constant decline, see the first graph in Figure (2). Afterwards, and up to the mid 90s, this proportion started recovering, probably due to the appearance of nuclear production. While it seems that in the latest years the trend is becoming negative again. The trend reversal observed in the 70s is also observable in the diagram depicting the evolution of clean versus dirty production, second graph in Figure (2). Notwithstanding both curves are growing, production using clean inputs was almost flat up to the 70s, when it started growing more significantly. The increase in the production of electricity using both polluting and non-polluting inputs has been enormous, led by the increasing energy needs of society. Both types of production underwent an increase in capital-efficiency in production through the years. This inference is based on the observation of the third graph in Figure (2), which shows that the proportion of capital used in the production of both types of energies decreased over the years.

The value of the elasticity of substitution for the US is clearly below the threshold of 1, with a time average of 0.513. Calibrated estimates of the elasticity of substitution between clean and dirty inputs in the production of electricity are practically constant over the last 60 years, see Figure (1).[16] This means that the two inputs may be qualified as complements, yet not as perfect complements.[17] The explanation for the low value of the elasticity is twofold, coming from the two meanings of complementarity of inputs. First, production of the final good requires both inputs. Second, an increase in the utilization of one of the two inputs entails an increase in the utilization of the other one. In the case of electricity both properties of complementarity can be interpreted.

The explanation of the first property resides in the difficulty of storage of renewable resources. As known, the demand for electricity at low/medium frequencies is extremely smooth. In order to meet such demand, production needs to be uninterrupted. Two reasons are behind the necessity of a steady production. First, electricity cannot be stored, hence production has to constantly meet demand. Second, a failure of production to meet demand results in an immediate regional blackout, i.e. all the power distribution grid shuts down. With present technology, storing electricity is extremely difficult, and involves important power losses, making it costly. There are some possibilities of storing electricity produced by wind or solar power, but only for a short time (from minutes to a handful of hours). These storage facilities are ideal for price arbitrage on the electricity market. Nevertheless, electricity cannot be stored there for longer periods in order to hedge out seasons - or even days - in which, for example, the wind is not blowing.[18] The only way to alleviate this

[16]The 5% confidence interval is constructed by bootstrap with a 1000 repetitions.

[17]The elasticity of substitution between two production inputs represents the convexity of the isoquants. $\varepsilon = 0$ implies Leontief -type isoquants, $\varepsilon = 1$ implies Cobb-Douglas-type isoquants ans $\varepsilon = \infty$ implies isoquants in the shape of downward sloping straight lines.

[18]See for example Bathurst and Strbac (2003) and DeCarolis and Keith (2006). Research in storage technology is crucially important. Improving this technology has three major benefits: (i) reducing the need for reserve power plants, which usually run on dirty fuels (ii) cutting the cost of power failures, which approximate $79 billion a year in the US (iii) enabling renewable energy. Wind and sun are the two largest sustainable sources of carbon free power, yet both are highly intermittent. Currently seven different and promising storage technologies are under study – pumped hydropower, compressed air energy storage, batteries, flywheels, superconducting magnetic energy storage, electrochemical capacitors and power electronics – yet none of them can yet provide efficient storability for periods going beyond 24 hours. (APN, 2007)

problem is to store the resource. In this way it can be transformed in electricity when needed. Yet, storing oil or coal is just a question of space and organization, while storing wind or sunlight is impossible, at least under present technological knowledge. This explains why the two inputs are not substitutes. Yet, it seems that technology could have an impact on the value of the elasticity. For the time being, polluting inputs have a definite advantage over non-polluting ones, they can be stored and transformed in electricity when the market needs it.

The second property of complementarity implies that upon an increase in the use of clean resources, the use of dirty ones has to increase as well. In order to understand why this property is satisfied, we have to take into account both, the direct and the indirect use of polluting resources in the production of electricity. The direct use is the easier to understand, we burn coal in order to produce electricity. The indirect use is more subtle and theoretically decreases with time. It occurs for instance in the consumption of electricity used for the construction of the first photovoltaic panel. It is obvious that this energy cannot come from a photovoltaic panel but it will come from a dirty source. Subsequently, the energy produced by the first solar panel is not sufficient in order to produce the second one, and so on until production reaches a critical mass able to reproduce itself independently. The same can be said for the production of aeolic turbines and other means of exploitation of clean resources. Therefore, in order to increase capacity in the non-polluting branch, production of energy in the polluting one needs to be increased.

According to these results, at no point in the near future we will be able to produce electricity using exclusively clean resources. The complementarity result implies that this is true even if the government steps in with heavy subsidies to clean research. From the model, subsidies work only if inputs are substitutable. From Proposition 2.1, some conclusions for the economy as a whole may be drawn. The proposition argues that if one good – which is itself a complement to other consumption goods – presents complementarity between the two production inputs, an environmental disaster cannot be avoided with taxes and/or subsidies, even if all other sectors present substitutability. The only way in which it can be avoided is by stopping growth in the production of that particular good.

## 5.2 Other Countries

Figure (3) shows the trends of production using non-polluting resources in the eight european countries studied. The proportion $Y_{ctn}/Y_{tn}$ does not evolve in every country as in the US. Over the period 1996-2006 the ratio decreases in the United Kingdom, Spain and Italy. Yet, it increases in Germany and Sweden, while it stays constant in France, Norway and Switzerland. Norway almost exclusively uses hydroelectric production. Together with energy produced with other clean resources it makes up for more than 99% of the country electricity production. Sweden, like Norway uses clean resources much more than the average european country, 90% (50% from nuclear and 40% from other clean sources) against an average of 12%. Switzerland also relies heavily on hydroelectric power generation, while France depends mostly on nuclear power. These statistics are not reported for the other countries in the sample because data are available for one year only.

The value of the elasticity for all these additional countries is inferred using equation (21). The expression takes the value of the elasticity for the US as given and needs data only on $Y^v_{cti}$ and $Y^v_{dti}$. Elasticity calibrated estimates for all countries analyzed in this paper roughly align. As specified in the data section, the elasticity for European countries is computed over the period 1996-2006. The results are presented in Figure (4). The value obtained for all the European countries studied - Germany, France, United Kingdom, Italy, Spain, Norway, Sweden, Switzerland and an aggregate for the EU16 - are in line with those for the US.[19] Only Germany and Spain seem to behave differently. Yet, while the value of the elasticity for Germany and Spain seems to have an erratic path, the time averages for both countries are lower than one and in line with the other, as reported in Table 1. The table reports simple time averages of the elasticity for each European country. From this table we see that the average elasticity of substitution between clean and dirty inputs is quite similar to the one observed for the US. Interpreting the small differences in the value

[19] Electricity trade within Europe is important, yet it is not taken into account by the data used, which simply represent a country's production level. For this reason I compute the elasticity also for the EU16 aggregate. The problem of controlling for electricity trade is thus avoided by interpreting Europe as a single country. The value found for the EU16 aligns with the value found for individual European countries, therefore it seems that electricity trade does not affect the results.

of elasticity across countries goes beyond the scope of this paper. The main message is that for all the countires studied the value found are below the critical value of 1.

The last sub-sample of countries includes Australia, Japan, Turkey, New Zealand, Mexico, the Republic of Korea, Canada, Brazil, Chile, China, the Federation of Russia, India and an aggregate for the whole world. For these countries I have data only for one year, i.e. 2007. Results are reported in Table (2). These numbers are similar to those found for the US and for European countries. The values reported are point estimates and not time averages.

A potential problem which may arise when taking into account countries other than the US is to know whether a model of domestic endogenous technical innovation is applicable to them, especially when treating with emerging countries. While a definite answer to this question lies outside the reach of this paper, I provide some evidence that all the countries considered are actively engaged in research and development. Figure 5 shows data on the share of R&D in GDP for all the countries considered. What we can observe, is that all the countries included in my sample did engage in some kind of R&D over the last 10 years. This allows me to rule out the possibility of applying the model to a country not doing any R&D.

Overall, I obtained results for a wide variety of developed and developing countries, and they are consistent all around the world. The interpretation given in the previous section about the United States seems to be confirmed by these results. The two-sided problem of complementarity – on the one side the need to provide a smooth flow of electricity and, on the other side the necessity of an indirect consumption of dirty intermediates in order to have clean production – seems to be common to all countries.

# 6 Robustness Checks

## 6.1 The Perfect Capital Mobility Hypothesis

In section 4, I used the hypothesis of perfect capital mobility between the clean and the dirty branch of a sector in order to identify the elasticity of subtitution. Yet, this hypothesis does not perfectly represent reality. Capital/machines in the two branches are, at best, imperfect substitutes. The focus of this section is on relaxing this hypothesis. This is done in an extremely simple way, by assuming that, when capital is moved from one branch to the other, part of it is lost. This loss is modeled as an iceberg transport cost, $\tau_i$, on the mobility of capital between the two branches, i.e. when capital is transferred from one branch to the other a proportion $\tau_i$ of it is lost. The ratio of marginal productivities becomes

$$\frac{MPK_{ci}}{MPK_{di}} = (1 - \tau_i) = \left(\frac{L_{cti}A_{cti}}{L_{dti}A_{dti}}\right)^{\sigma_i(\alpha_i - 1)} \left(\frac{x_{cti}}{x_{dti}}\right)^{-\alpha_i\sigma_i - 1} \tag{22}$$

if transport costs affect capital in the clean branch, and $\frac{1}{(1-\tau_i)}$ if they affect capital in the dirty one. In order to understand why it is sufficient to let $\tau_i$ vary between 0 and 1, let us assume a different price for reproducible capital in the two branches ($p^K_{ci}$ for clean capital and $p^K_{di}$ for dirty capital, while $p_{ci}$ remains the price for the clean intermediate good and $p_{di}$ for the dirty ones). In this case, frictionless capital markets imply indifference between investing in one branch or in the other one (Caselli and Feyrer, 2007), i.e.

$$\frac{p_{ci}MPK_{ci}}{p^K_{ci}} = \frac{p_{di}MPK_{di}}{p^K_{di}} \tag{23}$$

If this does not hold, than we would not observe investments in both branches of production. This equality implies that

$$\frac{p_{ci}}{p_{di}}\frac{MPK_{ci}}{MPK_{di}} = \frac{p^K_{ci}}{p^K_{di}} \tag{24}$$

and, using equation (22) that

$$\frac{p_{ci}}{p_{di}}(1 - \tau_i) = \frac{p^K_{ci}}{p^K_{di}} \qquad \text{and analogously} \qquad \frac{p_{ci}}{p_{di}}\frac{1}{(1 - \tau_i)} = \frac{p^K_{ci}}{p^K_{di}} \tag{25}$$

This means that when $\tau_i$ varies from 0 to 1, it creates a wedge between the price ratio of intermediate goods and the price ratio of the two different types of capital varying from

0 to $\infty$ and covering all possible cases, even the most improbable ones. Therefore, with $\tau_i \in (0.01; 0.99)$, I consider the robustness check performed exhaustive.

Solving equation (22) for $\varepsilon_i$ obtains

$$\varepsilon_i = \frac{\alpha_i(X_i - N_i) + N_i}{\ln(1-\tau_i) + \alpha_i(X_i - N_i) + N_i - X_i}$$

It is easy to compute the expression for the elasticity for the case in which transport costs affect capital in the dirty branch.

As illustrated in Figure (6), increasing the value of the transport cost from 1% to 99% does not significantly change the value of the elasticity. The value is almost always below 1, leaving the main result of the paper unchanged. As it is shown, for relatively high values of $\tau_i$, i.e. a big fraction of capital goes to waste when moving it from one branch to the other, the value of $\varepsilon_i$ goes above 1. Yet, this happens only in some countries, and the elasticity goes above 1 only if the relatively high values of $\tau_i$ affect directly dirty capital, and not in the opposite case. The implication of this being that the clean branch needs to become extremely more productive than the polluting one for the elasticity to go above 1. In some cases, for high values of $\tau_i$ – implying an improbable wedge between price ratios – I obtain counter-intuitive results.

## 6.2 CES Production Function for Intermediates

This section investigates the consequences of the unit elasticity of substitution between labor and capital in the production of intermediates implied by the Cobb-Douglas production function. I study what happens if instead of modelling intermediate production using a Cobb-Douglas production function I do it with a CES production function, therefore generalizing the model. In this case, the elasticity of substitution between clean and dirty inputs would take the following form

$$\varepsilon_i = \frac{\theta_i \ln\left(\frac{Y_{cti}}{Y_{dti}}\right)}{\ln\left(\frac{Y_{cti}}{Y_{dti}}\right) - \ln\left(\frac{x_{cti}}{x_{dti}}\right)} \tag{26}$$

where $\theta_i$ represents the elasticity of substitution between capital and labor in sector-$i$'s intermediate production. All the computation can be found in *Appendix I.*

Intuitively, the elasticity of substitution between labor and capital ($\theta_i$) should play a major role. This because the model assumes homogeneous labor but branch-specific capital. Therefore, the more we can substitute homogeneous labor to branch-specific capital, the more the two intermediate goods will be substitutables. It would thus be easier to achieve a complete switch toward clean inputs. In fact, the elasticity of substitution between clean and dirty inputs is directly proportional to the elasticity of substitution between labor and capital ($\varepsilon \propto \theta$).

In order to know in which direction this would affect my results, I have to take estimates of $\theta$ from the literature. These estimates, at the sectorial and at the aggregate level (see for example Arrow et al., 1961; Behrman, 1972 and Pessoa et al., 2005) tend to agree on values between 0 and 1. Therefore, not altering the result of this paper but making it stronger – using this specification would further reduce the value of $\varepsilon$.

### 6.3 Equality of Price Ratio of Intermediate Goods Between Countries

In order to compute the value of $\varepsilon_i$ for countries other than the US, I equalized the price ratio of intermediate goods within a sector between countries. In reality, these price ratios may differ, especially in the case of countries separated by water, for instance the US and Germany. Electricity is usually exported to neighboring countries yet, the further away the countries the less efficient the transport. For instance, it is not efficient to export electricity across the Atlantic. In this section I explore what happens if this hypothesis is relaxed. I model this by introducing a price ratio differential – positive and negative – between the two countries, identified by $\lambda \in (-0.9; 0.9)$

$$p_{ti}^{US}(1+\lambda) = \left(\frac{Y_{cti}^{v}}{Y_{dti}^{v}}\right)^{-(\sigma_i^v+1)} \tag{27}$$

Obtaining a new expression for $\varepsilon_i^v$

$$\varepsilon_i^v = \frac{1}{2 + \frac{P_i^{US} + \ln(1+\lambda)}{Y_i^v}} \tag{28}$$

as before $P_i^{US} \equiv \ln\left(p_{ti}^{US}\right)$ and $Y_i^v \equiv \ln\left(\frac{Y_{cti}^v}{Y_{dti}^v}\right)$. Setting $\lambda \in (-0.9; 0.9)$ takes into account a wide range of potential differences in price ratios. This differential affects the results, but does not change their magnitude. The results are still always below the critical value of 1. The only exceptions are Sweden and France, a big negative price differential in these two countries pushes the value of the elasticity over the critical threshold of 1. Results for six European countries can be found in Figure (7), Figure (8) and Figure (9).

## 7 Conclusion

In this paper I extend the model of AABH to a multi-sector setting. The extended model is then used to obtain a calibrated estimate of the value of the elasticity of substitution between clean and dirty inputs in the production of electricity in 21 countries. The focus on electricity is dictated by two of its characteristics. First, the identification of clean and dirty inputs is clear-cut. Second, electricity is a complement to a large variety of other consumption goods, both durable and non-durable, and hence cannot be easily substituted away. The multi-sector version of the model tells us that it is sufficient to have complementarity between clean and dirty inputs for a single good of this kind in order for the economy, in the long run, to head toward an environmental disaster, unless growth in the production using that input is not stopped. In order to obtain an estimable expression for the elasticity, I add to the model the assumption of perfect capital mobility between the clean and the dirty branch within a sector and the hypothesis of equality of price ratios between clean and dirty intermediates between countries. Both of these assumptions are used for identification purposes and do not need to hold exactly.

The results of the paper are threefold. First, the value found is low, around 0.51, implying complementarity between inputs. Second, the value is almost constant across countries. And third, in countries with time series data, the value stays constant through time. In light of Acemoglu et al. (2009), these results are worrying enough. On the one hand, according to the model, if we keep producing electricity using dirty inputs, we head toward an environmental disaster. On the other hand, looking at the empirical results, it seems impossible to stop producing electricity with polluting resources. The policy implication of this paper thus, seems to be that we need more important subsidies to research, as fast as possible, and high carbon taxes combined with a complete halt of the growth rate of the production of electricity. In this way, according to the model, we may be able to avoid an environmental disaster.

The results presented in this paper and the interpretations offered suggest that further research in this relatively new field is needed. The model fails to take into account some important factors, especially when dealing with electricity production. For example, the model should take into account new developments in storage technology. The results seem to suggest that this complementarity is likely to disappear once a critical mass of electricity will be produced with clean inputs. A new version of the model should take this into account. Another interesting development for future research would be to use the methodology developed here in order to estimate the elasticity of substitution between clean and dirty inputs for other sectors. It would be interesting to see if other sectors are characterized by complementarity as well, or if this result is specific to the electricity sector.

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## Figures

Figure 1: Yearly elasticity of substitution in the US, 1949-2009. 5% confidence interval constructed by bootstrap

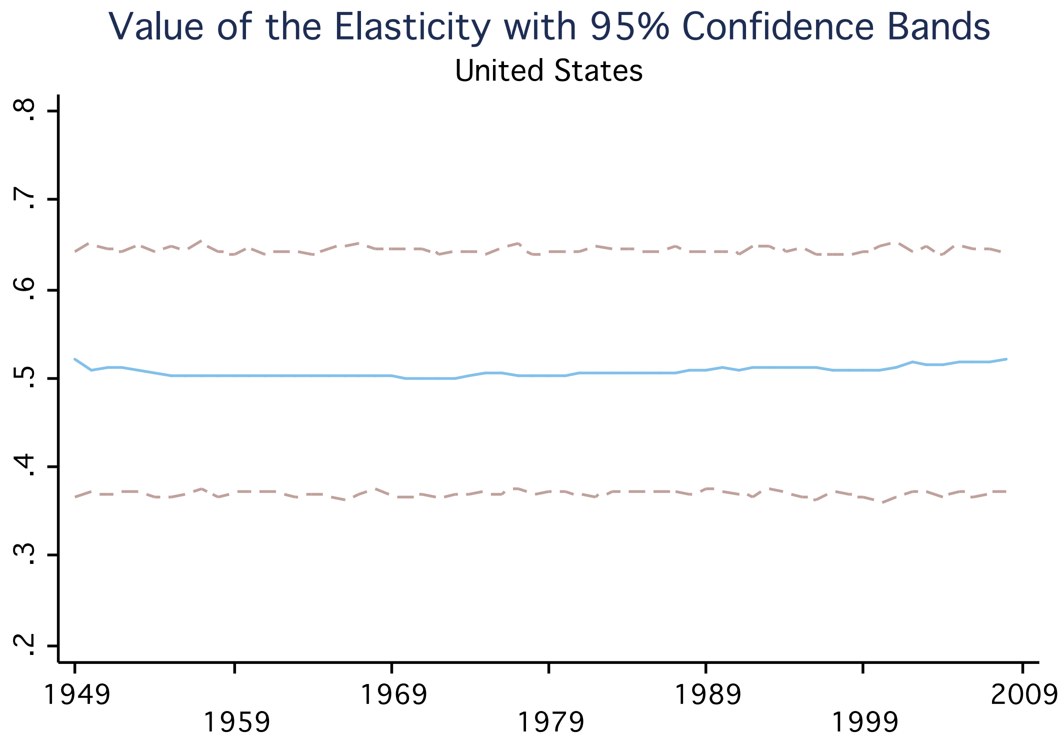


Figure 2: Proportion of production of electricity using clean inputs over total electricity production in the US (top left), production of electricity using clean and dirty inputs in the US (top right) and proportion of capital in electricity production using clean and dirty inputs in the US (bottom left), 1949-2009

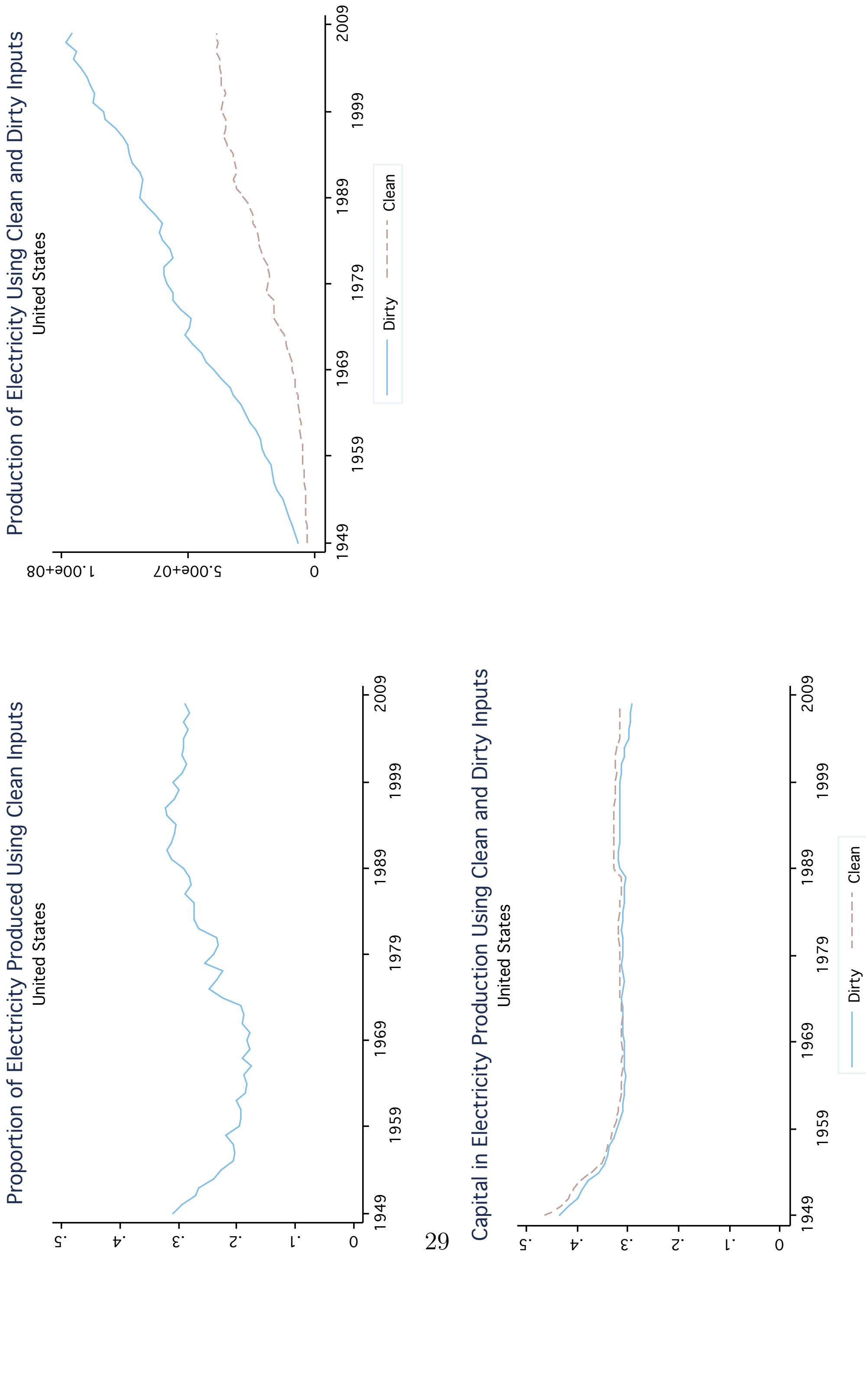


Figure 3: Proportion of production of electricity using clean inputs over total electricity production in European Countries, 1996-2006

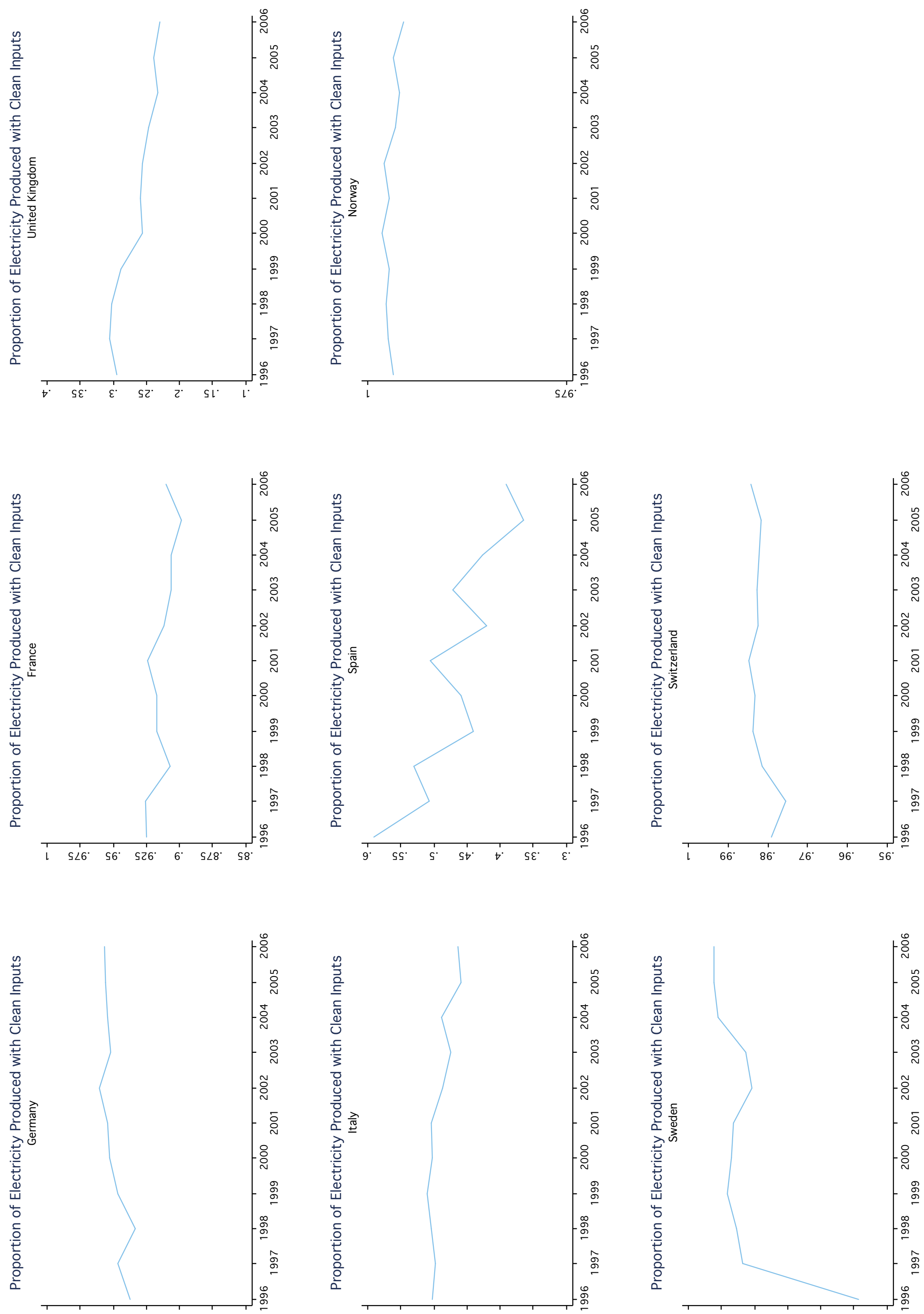


Figure 4: Yearly elasticity of substitution for 8 european countries compared to the US, 1996-2006

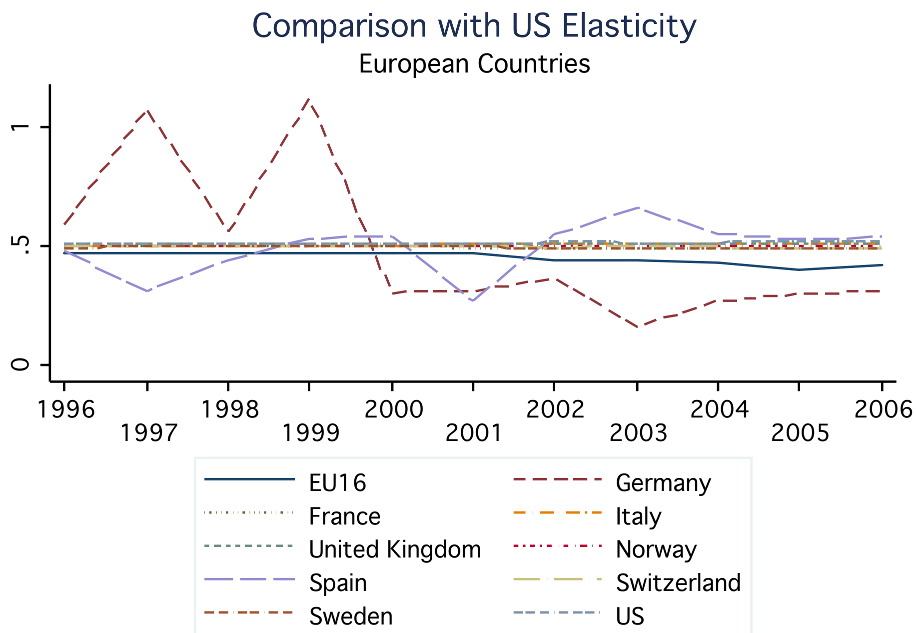
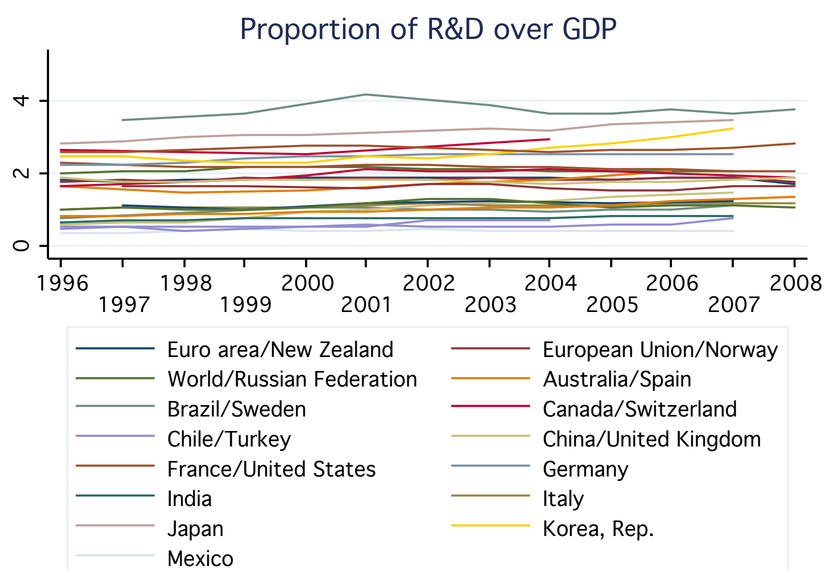
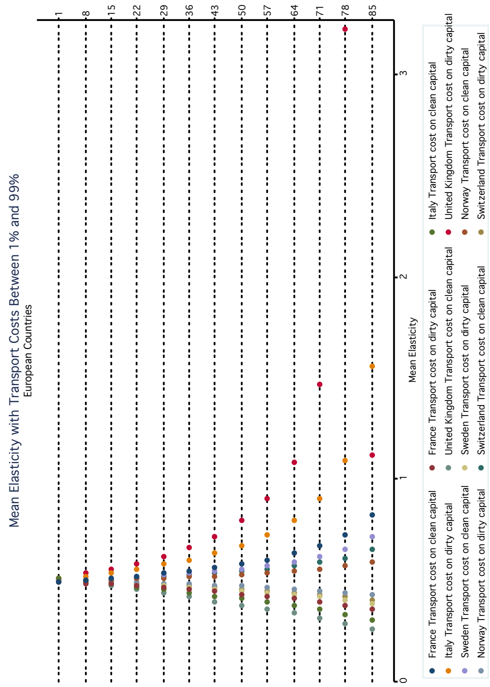


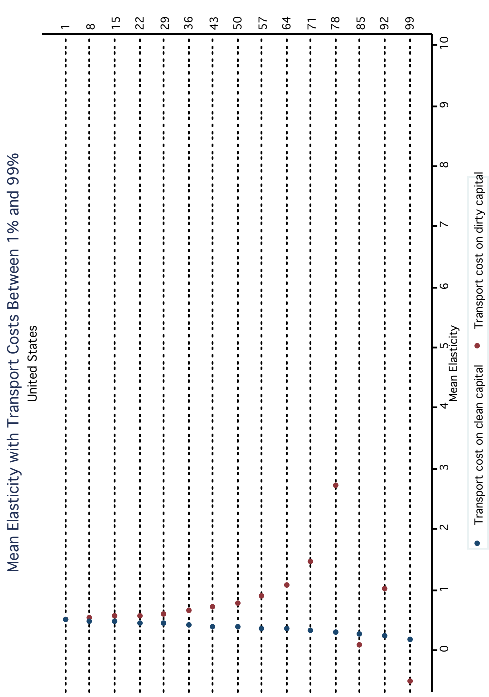

Figure 5: Evolution of the R&D/GDP ratio, over the period 1996-2008.




Source: World Development Indicators, World Bank

Figure 6: Average elasticity of substitution for the US, European countries and other countries with varying iceberg transport costs (from 1% to 99%) on capital mobility between sectors



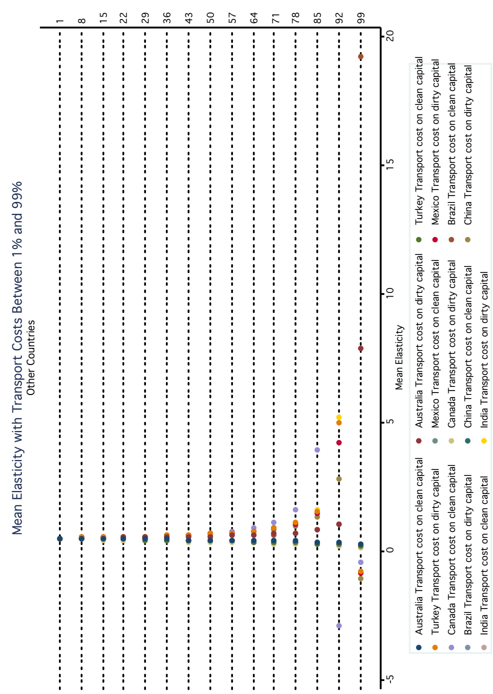



Figure 7: Yearly elasticity of substitution in France and in Italy taking into a account a positive and a negative price ratio differential with the US, 1996-2006

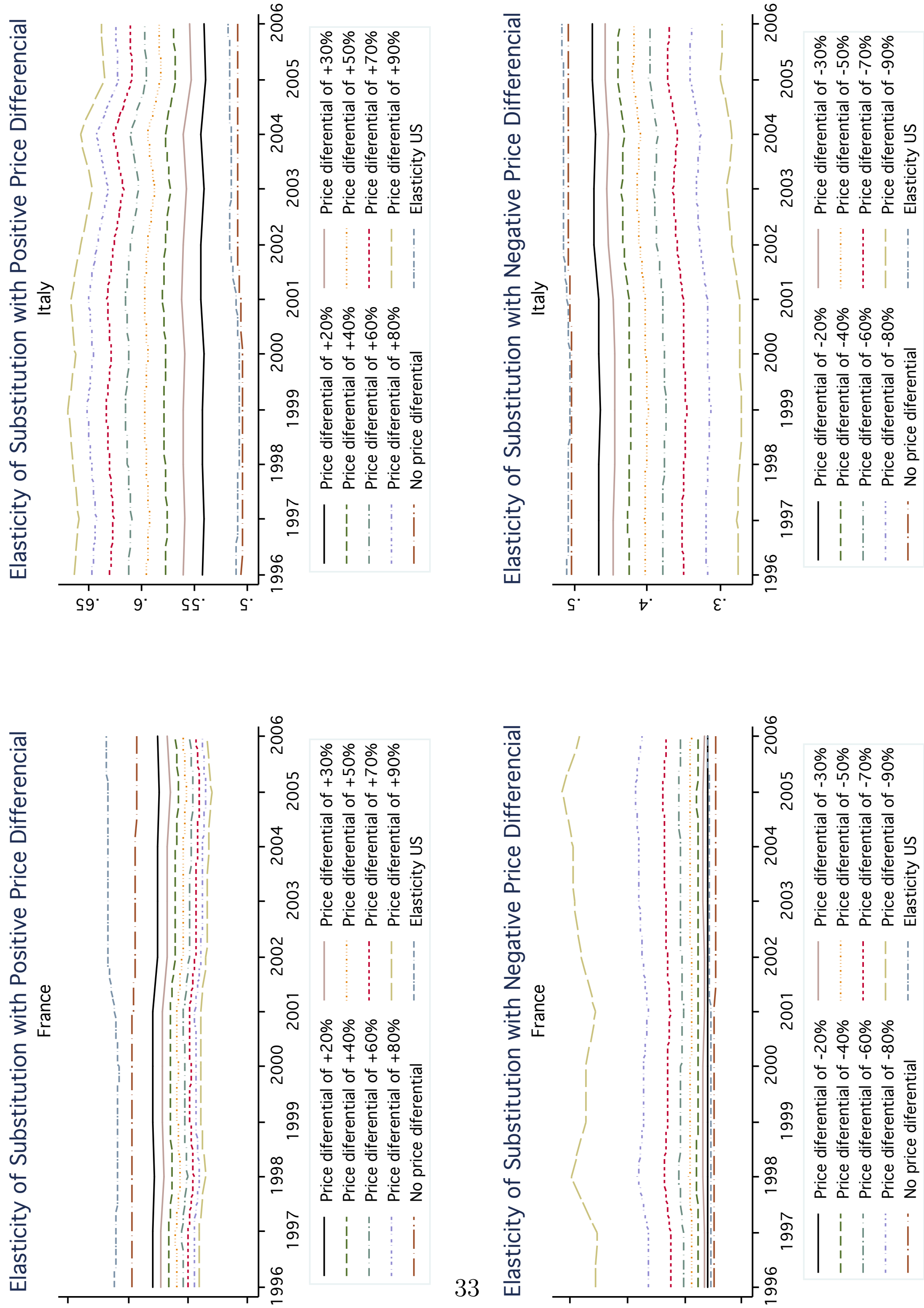


Figure 8: Yearly elasticity of substitution in Norway and Sweden taking into a account a positive and a negative price ratio differential with the US, 1996-2006

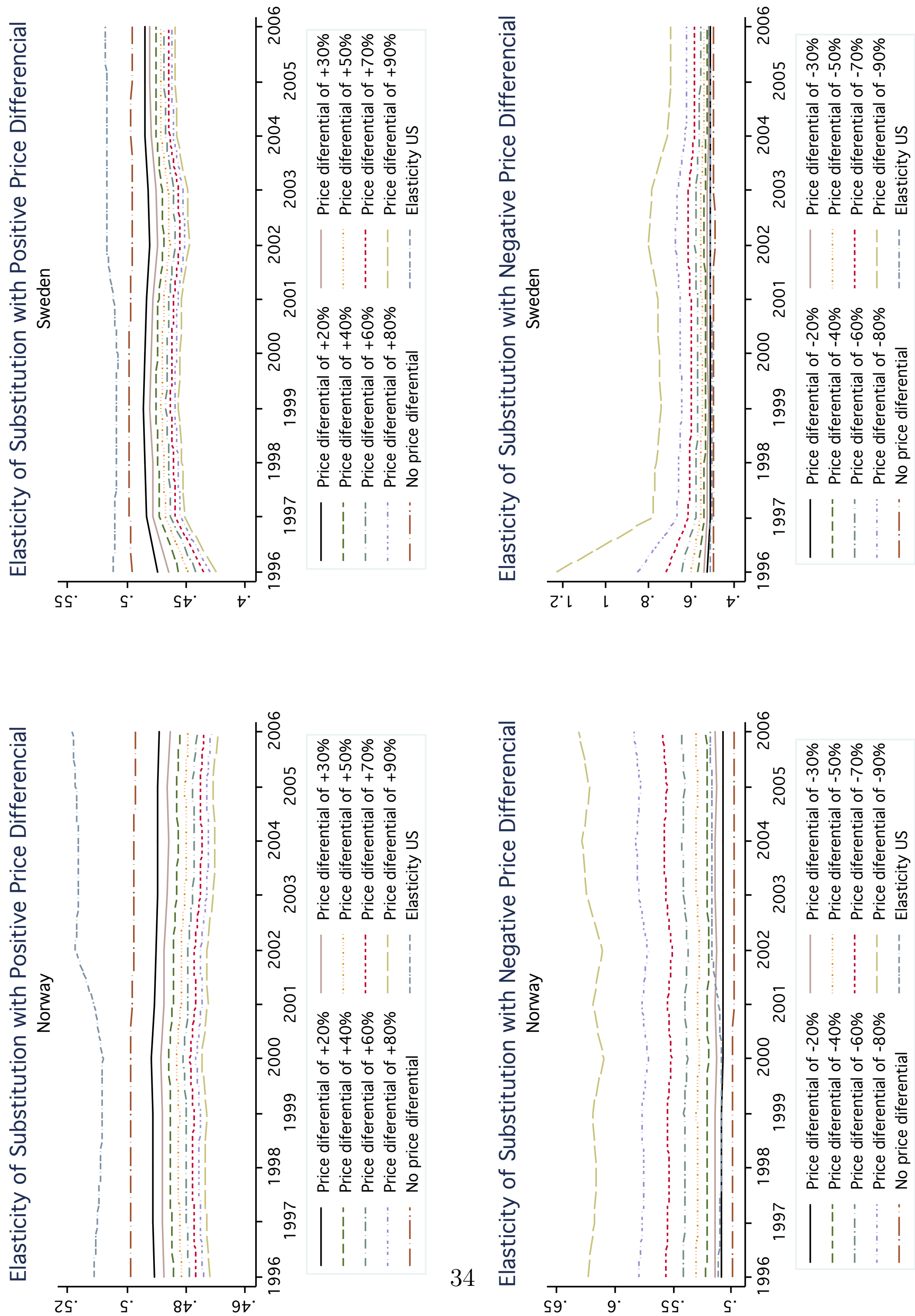
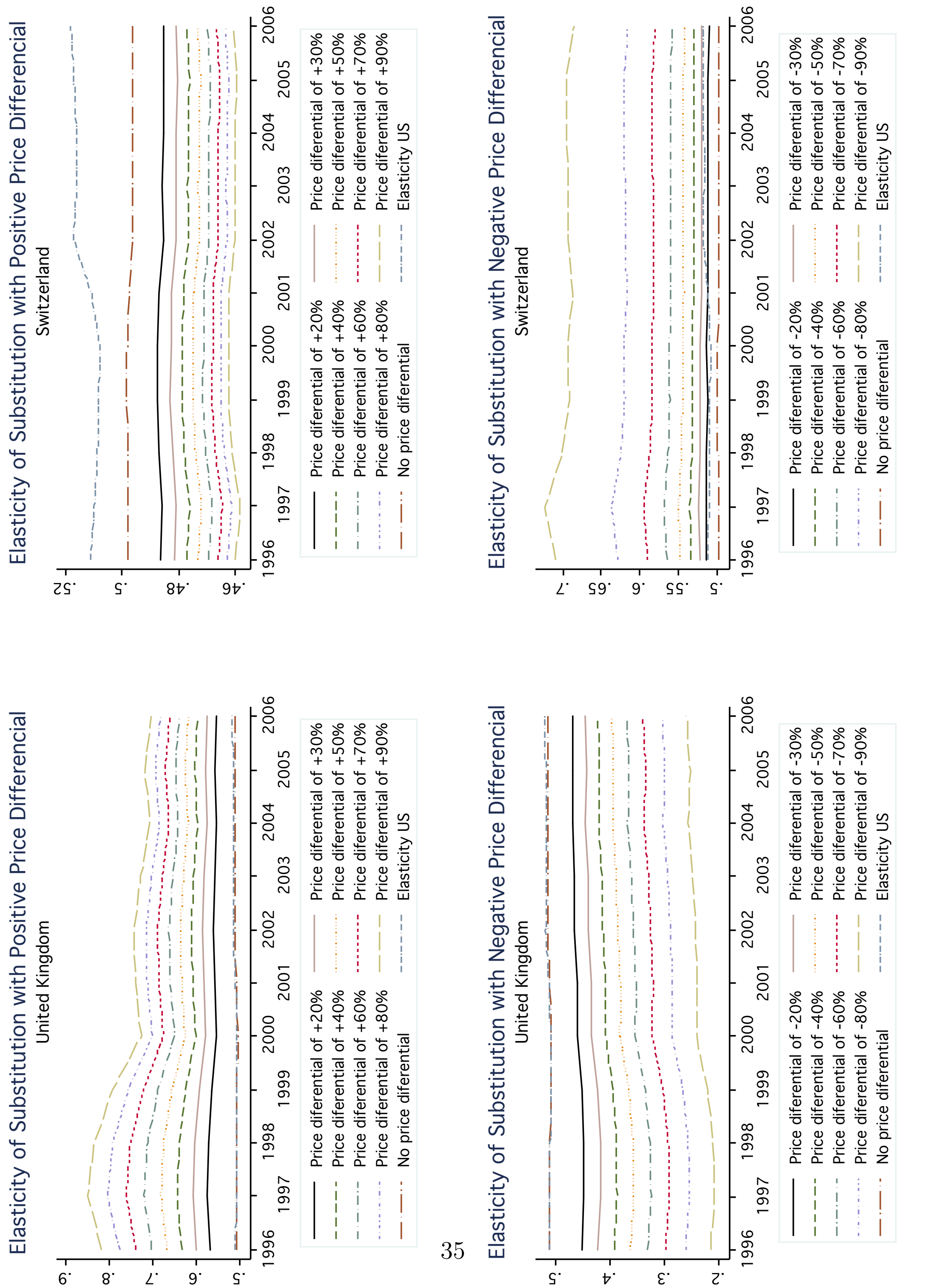


Figure 9: Yearly elasticity of substitution in Switzerland and in the United Kingdom taking into a account a positive and a negative price ratio differential with the US, 1996-2006



## Tables

Table 1: Summary Statistics of the Elasticities of Substitution

| **Country** | **Mean elasticity** | **Std. Dev.** |
|---|---|---|
| US | 0.513 | 0.004 |
| EU16 | 0.453 | 0.026 |
| Germany | 0.490 | 0.324 |
| France | 0.495 | 0.002 |
| Italy | 0.508 | 0.002 |
| Spain | 0.492 | 0.112 |
| UK | 0.511 | 0.003 |
| Norway | 0.498 | 0.001 |
| Sweden | 0.497 | 0.001 |
| Switzerland | 0.497 | 0.001 |

Table 2: Summary statistics of the Elasticity of Substitution of Other Countries

| **Country** | **Elasticity 2007** |
|---|---|
| Australia | 0.507 |
| Japan | 0.527 |
| Turkey | 0.513 |
| New Zealand | 0.474 |
| Mexico | 0.512 |
| Republic of Korea | 0.530 |
| Canada | 0.484 |
| Brazil | 0.492 |
| Chile | 0.602 |
| China | 0.511 |
| Federation of Russia | 0.527 |
| India | 0.513 |
| World | 0.525 |

# Appendix I : Dependence of $\varepsilon_i$ on $\alpha$ in the CES case

In this appendix the production of intermediate goods is generalized by replacing the Cobb-Douglass production function by a CES production function of the following form (for clarity sectorial subscripts $i$ are omitted)

$$Y_{ft} = \left[(1-\alpha)(A_{ft}L_{ft})^{\frac{\theta-1}{\theta}} + \alpha x_{ft}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{29}$$

Therefore production of the final good equals

$$Y_t = \left\{\left[(1-\alpha)(A_{ct}L_{ct})^{\frac{\theta-1}{\theta}} + \alpha x_{ct}^{\frac{\theta-1}{\theta}}\right]^{-\frac{\sigma\theta}{\theta-1}} + \left[(1-\alpha)(A_{dt}L_{dt})^{\frac{\theta-1}{\theta}} + \alpha x_{dt}^{\frac{\theta-1}{\theta}}\right]^{-\frac{\sigma\theta}{\theta-1}}\right\}^{-\frac{1}{\sigma}} \tag{30}$$

Consequently the respective $MPK_f$s equal

$$\begin{aligned} MPK_f &= \frac{\partial Y_t}{\partial x_{ft}} \\ &= -\frac{1}{\sigma}\left\{\left[(1-\alpha)(A_{ct}L_{ct})^{\frac{\theta-1}{\theta}} + \alpha x_{ct}^{\frac{\theta-1}{\theta}}\right]^{-\frac{\sigma\theta}{\theta-1}} + \left[(1-\alpha)(A_{dt}L_{dt})^{\frac{\theta-1}{\theta}} + \alpha x_{dt}^{\frac{\theta-1}{\theta}}\right]^{-\frac{\sigma\theta}{\theta-1}}\right\}^{\frac{\sigma-1}{\sigma}} \\ &\quad \left(-\frac{\sigma\theta}{\theta-1}\right)\left[(1-\alpha)(A_{ft}L_{ft})^{\frac{\theta-1}{\theta}} + \alpha x_{ft}^{\frac{\theta-1}{\theta}}\right]^{-\frac{\sigma\theta}{\theta-1}-1}\frac{\theta-1}{\theta}\alpha x_{ft}^{-\frac{1}{\theta}} \end{aligned} \tag{31}$$

Taking now the ratio of the two

$$\frac{MPK_c}{MPK_d} = \left[\frac{(1-\alpha)(A_{ct}L_{ct})^{\frac{\theta-1}{\theta}} + \alpha x_{ct}^{\frac{\theta-1}{\theta}}}{(1-\alpha)(A_{dt}L_{dt})^{\frac{\theta-1}{\theta}} + \alpha x_{dt}^{\frac{\theta-1}{\theta}}}\right]^{\frac{-\sigma\theta-\theta+1}{\theta-1}}\left(\frac{x_{ct}}{x_{dt}}\right)^{-\frac{1}{\theta}} \tag{32}$$

and equalizing it to 1

$$\begin{aligned} \left(\frac{x_{ct}}{x_{dt}}\right)^{\frac{1}{\theta}} &= \left[\frac{(1-\alpha)(A_{ct}L_{ct})^{\frac{\theta-1}{\theta}} + \alpha x_{ct}^{\frac{\theta-1}{\theta}}}{(1-\alpha)(A_{dt}L_{dt})^{\frac{\theta-1}{\theta}} + \alpha x_{dt}^{\frac{\theta-1}{\theta}}}\right]^{\frac{-\sigma\theta-\theta+1}{\theta-1}} \\ \frac{1}{\theta}\ln\left(\frac{x_{ct}}{x_{dt}}\right) &= \frac{-\sigma\theta-\theta+1}{\theta-1}\ln\left(\frac{(1-\alpha)(A_{ct}L_{ct})^{\frac{\theta-1}{\theta}} + \alpha x_{ct}^{\frac{\theta-1}{\theta}}}{(1-\alpha)(A_{dt}L_{dt})^{\frac{\theta-1}{\theta}} + \alpha x_{dt}^{\frac{\theta-1}{\theta}}}\right) \end{aligned}$$

Knowing that

$$\begin{aligned} \ln(Y_{ft}) &= \ln\left(\left[(1-\alpha)(A_{ft}L_{ft})^{\frac{\theta-1}{\theta}} + \alpha x_{ft}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}\right) \\ \frac{1}{\theta}\ln(Y_{ft}) &= \frac{1}{\theta-1}\ln\left((1-\alpha)(A_{ft}L_{ft})^{\frac{\theta-1}{\theta}} + \alpha x_{ft}^{\frac{\theta-1}{\theta}}\right) \end{aligned}$$

Therefore

$$
\begin{aligned}
\frac{1}{\theta}\ln\left(\frac{x_{ct}}{x_{dt}}\right) &= (-\sigma\theta-\theta+1)\frac{1}{\theta}\ln\left(\frac{Y_{ct}}{Y_{dt}}\right) \\
\sigma &= \frac{(1-\theta)\ln\left(\frac{Y_{ct}}{Y_{dt}}\right)-\ln\left(\frac{x_{ct}}{x_{dt}}\right)}{\theta\ln\left(\frac{Y_{ct}}{Y_{dt}}\right)} \qquad (33)
\end{aligned}
$$

And

$$
\varepsilon = \frac{\theta\ln\left(\frac{Y_{ct}}{Y_{dt}}\right)}{\ln\left(\frac{Y_{ct}}{Y_{dt}}\right)-\ln\left(\frac{x_{ct}}{x_{dt}}\right)} \qquad (34)
$$

The elasticity of substitution between capital and labor in the intermediate goods production, $\theta$, has a direct effect on the elasticity of substitution among clean and dirty inputs. In the case in which $\theta = 1$, and hence the intermediate production function is Cobb-Douglass, we obtain equation (11).

# Data Appendix

- Electricity production in the US
  - Source: *US Energy Information Administration*: Table 8.2b, Electricity Net Generation: Electric Power Sector, 1949-2009.
  - Description: The table contains annual data expressed in thousands of Kilowatthours. The data are transformed in British Thermal Units (btu) using the standard conversion $1Kilowatthour = 3'412btu$. $Y_{dt}$, i.e. intermediate production using dirty inputs, includes electricity produced with coal, petroleum, natural gas and other gasses; while $Y_{ct}$, i.e. intermediate production using clean inputs, includes electricity produced with nuclear, conventional hydroelectric power, biomass (wood and waste), geothermal, solar/photovoltaic and wind.
- Capital used in electricity production in the US
  - *US Energy Information Administration*: Table 8.4b, Consumption for Electricity Generation by Energy Source: Electric Power Sector, 1949-2009.
  - Description: The table contains annual data expressed in billion of British Thermal Units. $x_{dt}$, i.e. capital used in the intermediate production using dirty inputs, includes electricity produced with coal, petroleum, natural gas and other gasses; while $x_{ct}$, i.e. capital used in the intermediate production using clean inputs, includes electricity produced with nuclear, conventional hydroelectric power, biomass (wood and waste), geothermal, solar/photovoltaic and wind.
- Electricity production in european countries

  Source: Eurostat, and more precisely
  - Dirty Inputs
    - Coal: http://epp.eurostat.ec.europa.eu/tgm/table.do?tab=table&init=1&plugin=1 &language=en&pcode=ten00088

    * Petroleum: http://epp.eurostat.ec.europa.eu/tgm/table.do?tab=table&init=1&plugin=1 &language=en&pcode=ten00089
    * Gas: http://epp.eurostat.ec.europa.eu/tgm/table.do?tab=table&init=1&plugin=1 &language=en&pcode=ten00090
  - Clean Inputs
    * Nuclear: http://epp.eurostat.ec.europa.eu/tgm/table.do?tab=table&init=1&plugin=1 &language=en&pcode=ten00091
    * Hydroelectric: http://epp.eurostat.ec.europa.eu/tgm/table.do?tab=table&init=1&plugin=1 &language=en&pcode=ten00092
    * Wind: http://epp.eurostat.ec.europa.eu/tgm/table.do?tab=table&init=1&plugin=1 &language=en&pcode=ten00093

  All the preceding data are annual and available for the years 1996-2006. The series are expressed in Gigawatthour and are therefore transformed in British Thermal Units (btu) using the standard conversion $1Kilowatthour = 3'412btu$.

- Electricity production in other countries
  - Source: *International Energy Agency*: Country Statistics.
  - Description: Data are available for 2007 only. Data are expressed in Gigawatthour and are therefore transformed in British Thermal Units (btu) using the standard conversion $1Kilowatthour = 3'412btu$. $Y_{dt}$, i.e. intermediate production using dirty inputs, includes electricity produced with coal, petroleum and natural gas; while $Y_{ct}$, i.e. intermediate production using clean inputs, includes electricity produced with nuclear, conventional hydroelectric power, biomass (wood and waste), geothermal, solar/photovoltaic and wind.