Optimal carbon taxes with social and private discounting*

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Abstract

An analytically tractable climate-economy model is extended to allow for a planner that discounts the future differently than private agents. This is relevant to the climate-policy debate: some have argued that climate policy should be designed using a discount rate lower than the rate at which individuals appear to discount their own future utilities. If a social planner discounts the future differently than private agents, laissez-faire and socially optimal rate of fossil-fuel depletion differ substantially. A planner more patient than the market wants to slow down fossil-fuel depletion a great deal, which calls for carbon taxes that fall over time, eventually turning into subsidies. Welfare losses in the event that the first-best cannot be implemented are substantially larger when discount rates differ.

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1 Introduction

Climate change is a long-term phenomenon. The benefits of fossil fuel consumption are enjoyed instantaneously, whereas the costs of the associated emissions of carbon dioxide (CO_2)—often referred to as damages—largely occur in the future. Indeed, parts of our additions of CO_2 to the atmosphere remain there for very long periods of time, if not forever. For this reason, the issue how to weigh future generations against those currently alive has been at the forefront of the climate-policy debate. Much of the discussion has revolved around the appropriate choice of the rate of pure time preference in climate-economy models. Weitzman (2007) argues that "it is not an exaggeration to say that the biggest uncertainty of all in the economics of climate change is the uncertainty about which interest rate to use for discounting. In one form or another, this little secret is known to insiders in the economics of climate change, but it needs to be more widely appreciated by economists at large."

In this paper, I investigate the consequences for optimal carbon taxation of a point made informally by Kaplow et al. (2010). These authors argue that, in principle, two quite different discounting processes are involved, and these should be separated more clearly. Beckerman and Hepburn (2007) stress the distinction between the same individual at different points in time on the one hand, and different generations on the other hand. They write: "Schelling notes that while the Ramsey and Pigou references to 'impatience' or 'myopia' might accurately describe the virtually universal preference for consumption during one's lifetime by oneself, it is absurd to apply these adjectives to the consumption of somebody one will never know in 200 years' time."

The distinction between private and social discounting may be summarized as follows. On the one hand, individuals discount their own utility at future dates (and that of their offspring, to the extent they value it), using their own preferences for intertemporal consumption smoothing. Let us refer to the time-preference parameter—a dicsount factor β, say—as a 'private' or 'market' discounting parameter. This discounting process determines consumers' savings decisions, which gives rise to capital accumulation, and also the choice of how to optimally deplete an exhaustible resource. Such discounting is not readily affected by policy, and indeed the question at what rate the 'market' discounts the future is an empirical one. On the other hand, when designing optimal climate policy, a policymaker has to aggregate the well-being of different generations into a welfare function. In doing so, she may choose to discount future generations, or indeed not at all.

What discounting practices should be employed in this setting is a normative question, and as such an ethical decision rather than an empirical matter. Ramsey (1928), for one, suggests that the only ethically defensible social discount rate is zero.

Climate-economy models such as, for example, DICE (Nordhaus, 2008) or PAGE (Hope, 2006) model consumers as infinitely-lived representative agents, which helps explain why the distinction between the two types of discounting has been blurred in much previous work: individual utility and social welfare coincide, and it is not obvious how to distinguish the two different discounting concepts. In models, such as DICE, where output and consumption are endogenous, this has necessarily led to a 'descriptive' stance on discounting: discount parameters do not only determine how to compare climate damages over time, they also govern capital accumulation and fossil fuel depletion. Imposing a low rate of time preference in such a model may lead to, e.g., savings behaviour that does not seem plausible. On the other hand, modellers that see output or consumption streams as exogenous, as in PAGE, can more easily treat discounting as a purely ethical matter.

Stern (2006) advocates a rate of time preference of 0.1% per year, chosen on ethical grounds. Nordhaus (2007) discusses this stance in detail, stressing that such a low time-preference rate, together with an assumption of logarithmic utility, results in an equilibrium interest rate that is much lower than what is observed in reality. He goes on to suggest that a Stern-type time-preference rate can be consistent with 'reasonable' interest rates, provided that consumers exhibit strong enough preferences for consumption smoothing. Put differently, if consumers make consumption and savings decisions that give rise to a consumption sequence $\{C_t\}_{t=0,\ldots,\infty}$, in order to maximize

$$\sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\eta}}{1-\eta}, \tag{1}$$

then observed market interest rates are consistent with a very high degree of patience (a high β) if consumers are also very averse to consumption differences over time (a high η). However, one cannot freely choose any combination of β and η and still get savings behaviour and equilibrium interest rates that match reality.[1]

[1] A similar argument is used in Manne et al. (1995), in the context of the MERGE model:

> Note that a lower or a zero rate of utility time preference would not provide a good description of the collective outcome of individual choices. It would also imply an unrealistically rapid increase in the near-term rate of investment and capital formation [...].

The position implicit in this line of reasoning seems to be as follows. We may study optimal taxes on fossil-fuel consumption using a model with an infinitely-lived representative agent. In that case, we should use preferences for the representative agent that correspond to observed market outcomes[2], such as interest rates and capital accumulation, and implement climate taxes that maximize the consumer's lifetime utility, as defined by these preferences. This view is restrictive, for several reasons. Sælen et al. (2008) argue that the preference structure in (1), which is standard in climate-economy models, is underspecified. This refers to the fact that the parameter η is forced to simultaneously capture risk aversion, inequality aversion, and individuals' attitudes towards intertemporal consumption smoothing. Requiring η to be such that it ensures that the model produces reasonable interest rates, given a choice of β, makes the model even more underspecified.

In addition, the distinction between the concepts of individual utility and social welfare makes it clear that we may well want to design climate policy in a way that implies a higher weight on future utilities than what is suggested by consumers' investment decisions. It is far from obvious that a social planner, when defining a social welfare function, should weight down the utilities of individuals who are not yet born simply because they will live in the future. Nordhaus's insistence on, e.g., 'reasonable' interest rates precludes such an assessment, however. Climate policy designed using 'market' discount rates forces the social planner to discount the utilities of future generations at the same rate that individuals who are alive today discount their own future utilities.

These arguments seem uncontroversial when applied to the real world, where individuals live finite lives and many who will suffer the most from a changing climate have not yet been born. In climate-economy models, however, the standard approach is to model individuals' preferences with an infinitely-lived representative agent. As alluded to earlier, this makes it unclear how to distinguish between private and social discounting.

An obvious solution to this dilemma would be to instead consider a climate-economy model with finitely-lived agents, possibly with a structure of overlapping generations

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Dasgupta (2007) does not strike a clear balance between private and social discounting, but emphasizes the point that if β is seen as an ethical parameter, then so should η. This is also a central argument in Dasgupta (2008).

[2]Schneider et al. (2010) show that a representative agent in an infinite-horizon model will appear less patient than a finitely-lived agent in an overlapping-generations model, if both agents exhibit the same preferences for intertemporal consumption smoothing (η). This is due to the fact that in an OLG model, individuals' consumption growth exceeds overall consumption growth, as long as consumers do not fully incorporate the utilities of future generations.

(OLG). Schneider et al. develop such a model, and argue that in some respects an OLG model behaves quite differently from a representative-agent model. Howarth (2000), on the other hand, shows that the standard infinitely-lived representative-agent model common in climate economics closely approximates an OLG model, in terms of optimal-policy prescriptions. In an OLG model, the planner's rate of time preference used for aggregating utilities over generations is separate from the individuals' time-preference rate, and can thus in princple be set to a different value.

In Blanchard's (1985) perpetual-youth model, which is somewhat of a hybrid between an infinite-horizon and an OLG model, consumers discount future utilities due to two separate factors. On the one hand, consumers exhibit a pure rate of time preference, θ. They also face a constant (and time-invariant) risk of death, p. Consumers end up discounting future utilities at the rate $\theta + p$. In the paper there is no detailed discussion of what discount rate a policymaker should use in this setting, but the implied default value seems to be θ. In other words, if the planner shares the consumers' pure rate of time preference, she may still discount the future at a lower rate. This is due to the fact that an individual faces a risk of dying, whereas society as a whole does not face a risk of extinction (if nonzero, that risk is much smaller than the individual risk). Indeed, similar arguments were used in the Stern review for why a planner should use a low discount rate.

Still, the standard representative-agent setup has much to recommend it in terms of simplicity, in addition to its widespread use in climate-economy models. In this paper I explore the consequences for climate policy of a difference between social and private discount rates, and I do so within a climate-economy model that is standard in the sense of considering an infinitely-lived representative agent. More precisely, I make use of the climate-economy model laid out in Golosov et al. (2011), but unlike these authors I allow for 'social' and 'market' discount factors to differ. I investigate how this affects optimal taxes on emissions of CO_2 (or, equivalently, on the consumption of carbon-based energy), as well as the welfare implications. As we shall see, the optimal rate of depletion of fossil fuels depends on the social planner's preferences only, and not on the market's discount rates. By contrast, the sequence of energy taxes required to make market participants deplete fossil-fuel reserves at this optimal rate depend on the difference between social and market discount rates. When the planner is more patient than consumers, a tax sequence that falls more quickly than in the standard case is required, eventually turning into subsidies for fossil-fuel consumption.

The model by Glolosov et al. is a simple dynamic model of the global economy and climate, much in the spirit of William Nordhaus's DICE model (e.g., Nordhaus, 2008). The main difference to DICE is that (carbon-based) energy is explicitly modelled as an input into the production process.[3] The authors first present a general model with few assumptions about functional forms.[4] A number of simplifying assumptions, some less innocuous than others, are then introduced, which allow the model to be solved analytically. These will be discussed in more detail below.

The paper is organized as follows. Section 2 looks at a two-period model, which brings out the main message of the paper in a straightforward manner: when the market discounts the future more heavily than the social planner, the time profile of energy taxes must be adjusted in order to ensure fossil fuels are depleted at an optimal rate. Section 3 considers an infinite-horizon model with similar characteristics, which brings out additional insights about the time path of optimal energy taxes. Section 4 presents results from a calibrated infinite-horizon model, and discusses welfare considerations. Section 5 concludes.

2 A Two-Period Model

First, let us consider a simple two-period version of the model laid out in Golosov et al. (2011). Time is denoted by $t \in \{1, 2\}$. Preferences are logarithmic in consumption, and period felicity functions are aggregated into lifetime utility using a discount factor β:

$$U(C_1, C_2) = \ln C_1 + \beta \ln C_2.$$

The central theme of the paper is to explore the implications of market participants using a discount factor different from that of the planner. Throughout, β is used to denote the discount factor of private agents' (consumers'), whereas the discount factor of the planner

[3] A distinction should be drawn between DICE and the related RICE model in this respect. RICE considers a world economy consisting of several regions, whereas in DICE there is only one globally representative consumer. The version of RICE in Nordhaus and Boyer, (2000) does indeed feature a production technology where energy enters explicitly as an input. In this model, however, exhaustibility of fossil fuels is assumed away.

[4] The general model already embodies assumptions that do not correspond to the most general case. For example, it is assumed that carbon based energy is essential in production, and that—except for the possibility of a so-calld backstop that makes carbon-based energy redundant—it will remain essential indefinitely. Dasgupta and Heal (1974) discuss this and many other aspects relevant to optimal fossil-fuel depletion.

is referred to as β^*.

Production takes place using a Cobb-Douglas production function in capital, K_t, labour, N_t, and fossil-fuel based energy, E_t,

$$Y_t = D_t A_t K_t^{\alpha} N_t^{1-\alpha-\nu} E_t^{\nu}.$$

Y_t denotes output, and A_t is a productivity shifter. $D_t \in [0,1]$ represents the effects of climate change on total production: a fraction $(1 - D_t)$ of total output is destroyed due to the changing climate. D_t depends on the concentration of CO_2 in the atmosphere, S_t. I follow Golosov et al. and use an exponential function $D_t = e^{-\gamma S_t}$, where the parameter γ is a measure of the strength of climate-change impacts. This functional-form assumption implies that the marginal externality damage due to an additional unit of CO_2 is independent of the atmospheric CO_2 stock, S_t. While this assumption is a simplification, Golosov et al. show that the resulting 'damage function' D_t approximates the functions used in DICE quite well.

The physical resource constraints are $Y_1 = C_1 + I_1$ and $Y_2 = C_2$ where I_1 denotes first-period investment. Capital has a standard law of motion: $K_2 = (1-\delta)K_1 + I_1$. In the interest of analytical tractability, capital fully depreciates between time periods, i.e., $\delta = 1$. This is arguably not a very extreme assumption if we consider a time period to be ten years, as is common in climate-economy models.

The natural resource constraint reads $E_1 + E_2 = R$, where R is the available amount of fossil fuel. I will assume that all fossil fuel gets used up, hence the equality.[5] Atmospheric carbon follows a simple decay structure[6] defined by a decay parameter φ:

$$S_1 = E_1; S_2 = \varphi S_1 + E_2. \tag{2}$$

In addition, it is assumed that consumers supply labour inelastically, and there is no population growth. Without loss of generality, labour supply is normalized to unity ($N_t = 1$). The supply of fossil-fuel energy takes place under perfect competition, and there are no extraction costs for oil.

[5] As will become clear, this is the interesting case in terms of the extension considered in this paper. If it is not optimal to use up all available fossil fuels, then the simple fact that discount factors differ matters very little for optimal policy. In addition, based on the calculations in Golosov et al., it seems likely that at least all globally available oil will be consumed.

[6] The infinite-horizon model discussed in Section 3 has a more sophisticated carbon cycle formulation.

2.1 The planner's problem

The planner maximizes the discounted sum of utilities, internalizing the climate externality due to fossil fuel consumption. This is captured by the presence of S_t in the objective function. Here, S_t has been substituted away using the law of motion for the carbon cycle in (2). Note that the planner uses the discount factor β^*:

$$\max_{K_2,E_1} \Big[\ln\left(e^{-\gamma E_1} A_1 K_1^\alpha E_1^\nu - K_2\right) \\ + \beta^* \ln\left(e^{-\gamma(\varphi E_1 + R - E_1)} A_2 K_2^\alpha (R - E_1)^\nu\right)\Big]$$

The first-order conditions are

$$K_2: \quad \frac{1}{C_1} = \beta^* \frac{1}{C_2}\frac{\alpha Y_2}{K_2}, \quad \text{and} \tag{3}$$

$$E_1: \quad \frac{1}{C_1}\frac{\nu Y_1}{E_1} - \frac{1}{C_1}\gamma Y_1 - \beta^* \frac{1}{C_2}\gamma\varphi Y_2 = \beta^* \frac{1}{C_2}\frac{\nu Y_2}{E_2} - \beta^* \frac{1}{C_2}\gamma Y_2. \tag{4}$$

Using the physical resource constraints $C_1 + K_2 = Y_1$ and $C_2 = Y_2$, the Euler equation in (3) gives standard consumption and investment functions

$$C_1 = \frac{1}{1+\alpha\beta^*} Y_1 \quad \text{and} \quad K_2 = \frac{\alpha\beta^*}{1+\alpha\beta^*} Y_1. \tag{5}$$

The first-order condition for energy, (4), is a Hotelling result (Hotelling, 1931). The first, positive terms on each side of the equality represent the marginal product of energy, valued at the marginal utility of consumption, and discounted. The negative terms capture the externality damage due to fossil fuel consumption. First-period energy use causes negative effects in both time periods, hence the two negative terms on the LHS, whereas period-two energy use only causes damages in the same period, hence the single negative term on the RHS. The equation states that the marginal product of energy net of damages, valued at the marginal utility of consumption and appropriately discounted over time, should be equal across time periods. We can use the expression for C_1 in (5), along with the constraint that $C_2 = Y_2$, to arrive at

$$\frac{\nu}{E_1^*} - \gamma\frac{1+\beta^*(\alpha+\varphi)}{1+\alpha\beta^*} = \frac{\beta^*}{1+\alpha\beta^*}\left(\frac{\nu}{E_2^*} - \gamma\right). \tag{6}$$

The equation in (6), together with the physical resource constraint $E_1 + E_2 = R$, uniquely determines the optimal fossil-fuel consumption over time from the planner's perspective. The notation E_t^* denotes optimal energy consumption. Note that optimal energy consumption is defined by the solution to the planner's problem only, and hence does not involve the market discount rate, β.

2.2 The decentralized economy

2.2.1 Consumers

Let consumers discount the future using a discount factor β. The consumer's problem is then

$$\max_{C_1, C_2, K_2, E_1, E_2} \ln C_1 + \beta \ln C_2,$$

subject to budget and resource constraints

$$\begin{aligned} C_1 + K_2 &= (1 - \tau_1^K) r_1 K_1 + w_1 + (p_1 - \tau_1^E) E_1 + T_1, \\ C_2 &= (1 - \tau_2^K) r_2 K_2 + w_2 + (p_2 - \tau_2^E) E_2 + T_2, \quad \text{and} \\ E_1 + E_2 &= R. \end{aligned}$$

Here, two tax instruments are introduced. τ_t^K is a tax on capital returns, and τ_t^E is a tax on energy profits.[7] It is assumed that the government budget is balanced in each time period: $\tau_t^E E_t + \tau_t^K r_t K_t = T_t$. In other words, any tax revenue is rebated back to consumers in the form of lump-sum transfers. This means that consumers' disposable income is equal to the income generated by total output: $(1 - \tau_t^K) r_t K_t + w_t N_t + (p_t - \tau_t) E_t + T_t = Y_t$.

[7] The former is formulated as a standard *ad valorem* tax. As we will see, a constant *ad valorem* capital tax (subsidy) will be required to decentralize the optimal allocation; hence this formulation is more straightforward. A per-unit capital tax (subsidy) could be used instead, with the same results. The energy tax is expressed as a per-unit tax, as in Golosov et al. The advantage of such a formulation is that optimal taxes are expressed in terms of, say, dollars per tonne of carbon, which is the standard measure for carbon taxes in the climate-policy debate (as opposed to what percentage of the gross oil price that is accounted for by taxes, which would be the interpretation of an ad-valorem tax).

The first-order conditions of the consumer's problem are

$$K_2: \quad \frac{1}{C_1} = \beta \frac{1}{C_2} r_2 (1 - \tau_2^K), \quad \text{and} \tag{7}$$

$$E_1: \quad \frac{1}{C_1}(p_1 - \tau_1^E) = \beta \frac{1}{C_2}(p_2 - \tau_2^E) \tag{8}$$

$$\Leftrightarrow \frac{(p_2 - \tau_2^E)}{(p_1 - \tau_1^E)} = r_2(1 - \tau_2^K). \tag{9}$$

The Euler equation in (7) states a standard result, and the Hotelling equation in (9) is also familiar. In equilibrium, the after-tax price on the exhaustible resource must rise at a rate which corresponds to the return on other assets. The relevant rate of return is the effective interest rate on capital, also inclusive of the (capital) tax.

2.2.2 Firms

Firms maximize profits, taking factor prices as given:

$$\max_{K_t, N_t, E_t} D_t A_t K_t^{\alpha} N_t^{1-\alpha-\nu} E_t^{\nu} - r_t K_t - w_t N_t - p_t E_t.$$

Equilibrium factor prices are determined by theh first-order conditions (FOCs) for firms,

$$K_t: \quad r_t = \alpha \frac{Y_t}{K_t},$$

$$N_t: \quad w_t = (1 - \alpha - \nu) \frac{Y_t}{N_t}, \quad \text{and}$$

$$E_t: \quad p_t = \nu \frac{Y_t}{E_t}.$$

2.2.3 Market equilibrium

Using the Euler equation in (7), the firm's FOC for K_t, and the budget constraints, we find

$$K_2 = \frac{\alpha\beta(1 - \tau_2^K)}{1 + \alpha\beta(1 - \tau_2^K)} Y_1 \quad \text{and} \quad C_1 = \frac{1}{1 + \alpha\beta(1 - \tau_2^K)} Y_1.$$

Substitute the Euler equation and the expressions for r_2 and p_t from the firms' first-order conditions into the Hotelling equation, and rewrite it as

$$\frac{(\frac{\nu}{E_2} Y_2 - \tau_2^E)}{(\frac{\nu}{E_1} Y_1 - \tau_1^E)} = \alpha \frac{Y_2}{K_2}(1 - \tau_2^K).$$

Divide through by $\frac{Y_2}{Y_1}$, and, as in Golosov et al. define $\hat{\tau}_t^E$ energy taxes as a proportion of output, $\hat{\tau}_t^E \equiv \tau_t^E / Y_t$[8]. Simplify using the physical resource constraints, and arrive at the following equation, which pins down E_1 and $E_2 = R - E_1$, for a given set of taxes $\hat{\tau}_1$ and $\hat{\tau}_2$:

$$\left(\frac{\nu}{E_1} - \hat{\tau}_1^E\right) = \frac{\beta}{1 + \alpha\beta(1 - \tau_2^K)} \left(\frac{\nu}{E_2} - \hat{\tau}_2^E\right). \tag{10}$$

2.3 Implementing the optimal allocation

Assume there exists a set of tax parameters $\left\{\tau_2^K, \tau_1^E, \tau_2^E\right\}$ that allows the planner to implement the optimal allocation.[9] Output, consumption, capital stocks and energy consumption in the decentralized equilibrium will then be identical to the planner's solution. In such a solution, both Euler euqations (3) and (7) must hold, with identical consumption levels. In other words,

$$\begin{aligned} \beta^* \frac{\alpha Y_2}{K_2} &= \beta r_2 (1 - \tau_2^K) \\ &= \beta \frac{\alpha Y_2}{K_2} (1 - \tau_2^K) \\ \Rightarrow \qquad \tau_2^K &= -\frac{(\beta^* - \beta)}{\beta}. \end{aligned}$$

To implement the optimal allocation, the capital tax must be a subsidy when $\beta^* > \beta$: a government should subsidize the return to captial such that the effective return at the desired savings rate is consistent with the consumer's private discount rate β. The decentralized Hotelling equation in (9) can then be written as

$$\frac{\nu}{E_1} - \hat{\tau}_1^E = \frac{\beta}{1 + \alpha\beta^*} \left(\frac{\nu}{E_2} - \hat{\tau}_2^E\right). \tag{11}$$

The condition in (11) pins down energy consumption $\{E_1, E_2\}$, assuming that all fossil fuel gets used ($E_1 + E_2 = R$), for a given set of energy taxes $\left\{\hat{\tau}_1^E, \hat{\tau}_2^E\right\}$. From the planner's

[8] Although $\hat{\tau}_t^E$ should properly be considered as 'per-unit energy taxes as a proportion of output', I will refer to τ_t^E and $\hat{\tau}_t^E$ interchangeably as energy taxes. The usefulness of defining the object $\hat{\tau}_t^E$ will become clear in the infinite-horizon model discussed below.

[9] Unsurprisingly, the first-period capital tax τ_1^K is redundant.

point of view, optimal oil consumption is defined by (6):

$$\frac{\nu}{E_1} - \gamma \frac{1+\beta^*(\alpha+\varphi)}{1+\alpha\beta^*} = \frac{\beta^*}{1+\alpha\beta^*}\left(\frac{\nu}{E_2} - \gamma\right)$$

Comparing (6) and (11), we notice a number of things. First, consider a situation where the optimal sequence for fossil-fuel consumption does not deplete the total available stock, R. This happens when (6) holds, with $E_1^* + E_2^* < R$, i.e., it is not optimal to increase either E_1^* or E_2^*. This means that

$$\frac{\nu}{E_1^*} = \gamma \frac{1+\beta(\alpha+\varphi)}{1+\alpha\beta} \quad \text{and} \quad \frac{\nu}{E_2^*} = \gamma.$$

Increasing fossil-fuel consumption in either period would then push down the marginal product of energy, making it fall short of the marginal externality damage. In other words, both sides of (6) will equal 0. From (11), we see that this must imply

$$\hat{\tau}_1^E = \gamma \frac{1+\beta(\alpha+\varphi)}{1+\alpha\beta} \quad \text{and} \quad \hat{\tau}_2^E = \gamma.$$

Now, the difference in discount factors plays no role: optimal energy taxes, as a proportion of output, depend on the planner's discount factor, β^*, but not on the consumers' discount factor, β. This highlights why the interesting case, for the purposes of this paper, is the one with scarcity of fossil fuels.

Let us thus consider the case with scarcity, i.e., all fossil fuel gets used up.[10] First, notice that if $\beta = \beta^*$, it is easy to see that

$$\hat{\tau}_1^E = \gamma \frac{1+\beta(\alpha+\varphi)}{1+\alpha\beta} \quad \text{and} \quad \hat{\tau}_2^E = \gamma$$

will still implement the optimal allocation, together with $\tau_2^K = 0$. These are Pigouvian taxes: simply set the tax equal to the marginal climate damage caused by the optimal energy consumption, discounted appropriately. This is not the only set of taxes that works, however—an infinite number of combinations of taxes $\left\{\hat{\tau}_1^E, \hat{\tau}_2^E\right\}$ will do. Any $\left\{\hat{\tau}_1^E, \hat{\tau}_2^E\right\}$ that satisfies (11), where $\{E_1, E_2\}$ is defined by (6), will implement the optimal energy consumption, as long as $\frac{\nu}{E_t} \geq \hat{\tau}_t^E \ \forall t$.[11]

[10] Technically, there is also a boundary case where the optimal total fossil fuel consumption $E_1 + E_2$ is exactly R. In such a situation, all oil gets used up, but there is still no scarcity.

[11] If this latter condition were not met, the marginal units of energy used would incur taxes that exceed

To see how optimal $\hat{\tau}_1^E$ and $\hat{\tau}_2^E$ relate to each other when private and social discount rates differ, fix $\hat{\tau}_2^E = \gamma$ and substitute this into (11), which gives

$$\hat{\tau}_1^E = \frac{\nu}{E_1} - \frac{\beta}{1+\alpha\beta^*}\left(\frac{\nu}{E_2} - \gamma\right).$$

Combine this expression with (6) to find

$$\begin{aligned} \frac{\nu}{E_1} &= \gamma\frac{1+\beta^*(\alpha+\varphi)}{1+\alpha\beta^*} + \frac{\beta^*}{1+\alpha\beta^*}\left(\frac{\nu}{E_2} - \gamma\right) \\ \Rightarrow \hat{\tau}_1^E &= \gamma\frac{1+\beta^*(\alpha+\varphi)}{1+\alpha\beta^*} + \frac{(\beta^*-\beta)}{1+\alpha\beta^*}\left(\frac{\nu}{E_2} - \gamma\right). \end{aligned} \tag{12}$$

If $\hat{\tau}_2^E$ is set to γ, the first-period energy tax must be set at a higher level; this is necessary in order to induce an optimal postponement of fossil-fuel consumption. Part of this is captured by the first term on the RHS of (12) (which is greater than γ, as $\varphi > 0$). The second term on the right-hand side is only relevant when social and private discount factors differ. The term is positive when $\beta^* > \beta$ ($\frac{\nu}{E_2} > \gamma$, by assumption), which can be interpreted as follows: When the planner is more patient than private agents, the tax differential between periods 1 and 2 must be even greater in order to induce an optimal postponement of fossil-fuel depletion. Note that this second term is directly proportional to the difference in discount factors, $(\beta^* - \beta)$.

3 An Infinite-Horizon Model

Let us turn to a full infinite-horizon model as in Golosov et al. Most properties of the infinite-horizon model are directly analogous to the two-period model. The main difference is the carbon cycle, which is based on Archer (2005). This study shows that large proportions of greenhouse gas (GHG) emissions will remain in the atmosphere indefinitely, a fact that is not captured by the carbon-cycle formulations in existing climate-economy models, such as DICE. This turns out to matter a great deal for how society should value the dynamic externality due to GHG emissions.

Here, it is assumed that a proportion φ_L of emissions remains in the atmosphere forever. Out of the proportion $(1-\varphi_L)$ that does not remain forever, $(1-\varphi_0)$ disappears

their product, in which case firms and consumers would be better off leaving some of the oil in the ground—which is not optimal, by assumption.

from the atmosphere immediately (within a decade). The remaining proportion, $\varphi_0(1-\varphi_L)$, disappears slowly from the atmosphere, at the rate φ per decade. This carbon cycle setup is summarized in equation (14) below. R_0 denotes the initial stock of fossil fuels, and the natural-resource constraint is then that the sum of all energy consumption equals this resource stock R_0, as given by equation (15).

The setup here differs somewhat from that of the original paper, the most important difference being the separation between the planner's and the consumers' discount factors. In addition, Golosov et al. consider uncertainty about the damage parameter γ; I abstract from that here. Finally, the original paper considers two types of fossil fuel, oil and coal, with substantially different extraction technologies. I abstract from the coal type in their paper, and treat all fossil fuel as if it were oil (i.e., all the available fossil-fuel resource stock can be extracted at zero cost).

3.1 Infinite-horizon: Planner's problem

The planner's optimization problem is

$$\max_{C_t,K_{t+1},S_t,E_t} \sum_{t=0}^{\infty} \beta^{*t} \ln(C_t),$$

subject to physical and natural resource constraints, and the dynamics of the carbon cycle:

$$C_t = e^{-\gamma S_t} A_t K_t^{\alpha} N_t^{1-\alpha-\nu} E_t^{\nu} - K_{t+1}, \tag{13}$$

$$S_t = \sum_{s=0}^{t} (\varphi_L + (1-\varphi_L)\varphi_0(1-\varphi)^s) E_{t-s}, \quad \text{and} \tag{14}$$

$$\sum_{t=0}^{\infty} E_t = R_0. \tag{15}$$

The first-order conditions from this optimization problem are similar to the ones found in the two-period model. Combining the FOCs for C_t and K_{t+1} gives us the Euler equation

$$\frac{C_{t+1}}{C_t} = \alpha\beta^* \frac{Y_{t+1}}{K_{t+1}}. \tag{16}$$

As expected, given the assumption of full depreciation, this gives a constant savings rate $\alpha\beta^*$. In other words,

$$C_t = (1 - \alpha\beta^*)Y_t \quad \text{and} \quad K_{t+1} = \alpha\beta^* Y_t \tag{17}$$

satisfy the Euler equation. The FOC for S_t can be simplified to give

$$\psi_t = \gamma \frac{1}{1 - \alpha\beta^*},$$

Where ψ_t is the multiplier on the carbon-cycle constraint (14) in period t (this result stems from the assumption of a constant γ, i.e., a marginal externality damage du to GHG emissions that is independent of S_t). Combine this with the FOC for E_t to find

$$\begin{aligned} \beta^{*t} \left(\frac{Y_t}{C_t} \frac{\nu}{E_t} - \gamma \frac{1}{1 - \alpha\beta^*} \left[\frac{\varphi_L}{1 - \beta^*} + \frac{(1 - \varphi_L)\varphi_0}{1 - \beta^*(1 - \varphi)} \right] \right) - \mu = 0 \\ \Rightarrow \beta^{*t} \left(\frac{\nu}{E_t} - \Gamma(\beta^*) \right) - \mu(1 - \alpha\beta^*) = 0, \end{aligned} \tag{18}$$

where μ is the (time-invariant) multiplier on the natural-resource constraint (15), and $\Gamma(\beta^*)$ is defined as

$$\Gamma(\beta^*) \equiv \gamma \left[\frac{\varphi_L}{1 - \beta^*} + \frac{(1 - \varphi_L)\varphi_0}{1 - \beta^*(1 - \varphi)} \right],$$

for notational convenience. $\Gamma(\beta^*)$ is expressed as a function of the social discount rate β^* to emphasize that society's valuation of the dynamic climate externality due to GHG emissions necessarily depends on the rate at which the future is discounted. $\Gamma(\beta^*)$ measures the externality damage associated with a unit of carbon emitted into the atmosphere, incorporating the immediate externality γ as well as the discounted stream of damages incurred throughout the infinite future. Using (18) in periods t and $t+1$ to eliminate the term in μ, we arrive at

$$\frac{\nu}{E_t^*} - \Gamma(\beta^*) = \beta^* \left[\frac{\nu}{E_{t+1}^*} - \Gamma(\beta^*) \right]. \tag{19}$$

As in the two-period model, E_t^* is used to denote optimal energy consumption. Together with the physical resource constraint (15), (19) defines the optimal path for fossil fuel consumption, $\{E_t^*\}_t$. As in the two-period model, optimal depletion of the exhaustible fossil-fuel resource only depends on β^*, γ and the parameters of the carbon cycle. The market discount factor, β, does not play any role.

3.2 Infinite-horizon: Decentralized equilibrium

3.2.1 Consumers

Consumers optimize using a discount factor β, according to

$$\max_{C_t, K_{t+1}, E_t} \sum_{t=0}^{\infty} \beta^t \ln(C_t)$$

subject to physical and natural resource constraints:

$$C_t = r_t(1-\tau_t^K)K_t + w_t N_t + (p_t - \tau_t^E)E_t + T_t - K_{t+1}, \quad \text{and} \tag{20}$$

$$\sum_{t=0}^{\infty} E_t = R_0. \tag{21}$$

Once more, it is assumed that the government budget is balanced in each time period,

$$\tau_t^E E_t + \tau_t^K r_t K_t = T_t,$$

which means that disposable income equals total output in each time period. Combining the first-order conditions with respect to C_t and K_{t+1} gives the Euler equation

$$\frac{C_{t+1}}{C_t} = \beta r_{t+1}(1-\tau_{t+1}^K). \tag{22}$$

To arrive at the Hotelling equation, note that the FOC for E_t in the consumer's problem can be written as

$$\beta^t \theta_t (p_t - \tau_t^E) - \xi = 0, \tag{23}$$

where θ_t is the multiplier on the physical-resource constraint (20) at t, and ξ is the multiplier on the natural-resource constraint (21). Combine (23) in periods t and $t+1$ to eliminate ξ:

$$\begin{aligned} \beta^t \theta_t (p_t - \tau_t^E) &= \beta^{t+1} \theta_{t+1} (p_{t+1} - \tau_{t+1}^E) \\ \Rightarrow r_{t+1}(1-\tau_{t+1}^K) &= \frac{(p_{t+1} - \tau_{t+1}^E)}{(p_t - \tau_t^E)}. \end{aligned} \tag{24}$$

This Hotelling equation is directly analogous to the two-period version in (9).

3.2.2 Firms

The firms' optimization problem in the infinite-horizon case is identical to the two-period setting. The firms' objective function in

$$\max_{K_t,N_t,E_t} D_t A_t K_t^{\alpha} N_t^{1-\alpha-\nu} E_t^{\nu} - r_t K_t - w_t N_t - p_t E_t, \tag{25}$$

and the first-order conditions are $r_t = \alpha \frac{Y_t}{K_t}$, $w_t = (1-\alpha-\nu)\frac{Y_t}{N_t}$, and $p_t = \nu \frac{Y_t}{E_t}$, as previously.

3.3 Infinite-horizon: Implementing the optimal allocation

The Euler equation in (22), together with the firm's FOC for K_t and the budget constraints, leads to consumption and savings functions

$$C_t = (1 - \alpha\beta(1 - \tau_2^K))Y_1 \quad \text{and} \quad K_{t+1} = \alpha\beta(1 - \tau_2^K)Y_t.$$

Once more, assume there to be a set of taxes $\left\{\tau_{t+1}^K, \tau_t^E\right\}_t$ that implements the optimal allocation. Both (16) and (22) must then hold, with identical values for consumption, output and capital, which means that

$$\alpha\beta^* \frac{Y_{t+1}}{K_{t+1}} = \alpha\beta(1 - \tau_{t+1}^K)\frac{Y_{t+1}}{K_{t+1}}$$

must hold. In other words,

$$\begin{aligned} \beta^* &= \beta(1 - \tau_{t+1}^K) \\ \Rightarrow (1 - \tau_{t+1}^K) &= \frac{\beta^*}{\beta} \Leftrightarrow \tau_{t+1}^K = -\frac{(\beta^* - \beta)}{\beta}, \end{aligned}$$

i.e., a constant capital tax implements the optimal savings rate (indeed, the same tax as in the two-period case). In the case where the planner is more patient than private agents, the capital tax is negative, i.e., a subsidy to capital returns is required to induce high enough savings.

Now we can rewrite the Hotelling equation in (24), using the expressions for r_{t+1} and p_t from the firm's first-order conditions, together with the result that $(1-\tau_{t+1}^K) = \frac{\beta^*}{\beta}$ must hold, as

$$\alpha \frac{Y_{t+1}}{K_{t+1}} \left(\frac{\beta^*}{\beta}\right) = \frac{\left(\frac{\nu}{E_{t+1}} Y_{t+1} - \tau_{t+1}^E\right)}{\left(\frac{\nu}{E_t} Y_t - \tau_t^E\right)}.$$

Divide through by $\frac{Y_{t+1}}{Y_t}$, and again define $\hat{\tau}_t \equiv \frac{\tau_t}{Y_t}$. Using this and the savings function for K_{t+1}, we arrive at

$$\alpha \frac{Y_t}{\alpha \beta^* Y_t} \left(\frac{\beta^*}{\beta}\right) = \frac{1}{\beta} = \frac{\left(\frac{\nu}{E_{t+1}} - \hat{\tau}_{t+1}^E\right)}{\left(\frac{\nu}{E_t} - \hat{\tau}_t^E\right)} \tag{26}$$

$$\Rightarrow \left(\frac{\nu}{E_t} - \hat{\tau}_t^E\right) = \beta \left(\frac{\nu}{E_{t+1}} - \hat{\tau}_{t+1}^E\right). \tag{27}$$

The condition in (27) defines how the market will choose to consume energy over time, given a sequence of energy taxes $\left\{\hat{\tau}_t^E\right\}_t$, whereas (19) defines the optimal sequence for energy consumption, given parameter values ν, β^*, and the various carbon cycle parameters. If social and market discount factors coincide ($\beta^* = \beta$), one set of taxes that implements the optimal allocation is to simply set energy taxes equal to $\Gamma(\beta^*)$ in each time period. This is the central insight in Golosov et al.—optimal taxes on fossil fuel can be set to a constant proportion of output.[12] However, with different discount factors, this elegant solution is no longer available. To explore the general case, consider the difference equation in (28) below. This equation gives a relationship between $\hat{\tau}_t^E$ and $\hat{\tau}_{t+1}^E$, for any set of energy taxes $\left\{\hat{\tau}_t^E\right\}_t$ that implements the optimal allocation (the derivation of this equation is given in Appendix 6.A below)

$$\hat{\tau}_{t+1}^E = \frac{1}{\beta} \hat{\tau}_t^E - \frac{(\beta^* - \beta)}{\beta^* \beta} \frac{\nu}{E_t^*} - \frac{(1 - \beta^*)}{\beta^*} \Gamma(\beta^*). \tag{28}$$

Let us first consider the case when $\beta^* = \beta$. The difference equation in (28) then reduces to

$$\hat{\tau}_{t+1}^E = \frac{1}{\beta} \hat{\tau}_t^E - \frac{(1 - \beta)}{\beta} \Gamma(\beta^*),$$

which is straightforward to interpret. First, note that there is a stationary point at $\Gamma(\beta^*)$, i.e., $\hat{\tau}_t^E = \Gamma(\beta^*)\ \forall t$ is a solution. Moreover, this is an unstable equilibrium: for an initial $\hat{\tau}_0^E$ that exceeds $\Gamma(\beta^*)$, $\hat{\tau}_t^E$ must increase indefinitely. For a $\hat{\tau}_0^E$ that is lower

[12] At this point, the usefulness of considering carbon taxes as a proportion of output, $\hat{\tau}_t^E$, becomes evident.

than $\Gamma(\beta^*)$, the opposite holds, i.e., taxes decrease and eventually turn into subsidies. The highest possible initial-period tax that would implement the optimal sequence for fossil-fuel consumption is $\hat{\tau}_0^E = \frac{\nu}{E_0^*}$, which constitutes a full appropriation of all surplus due to energy (see Appendix 6.B below).

When social and market discount factors differ, we must consider the more general difference equation in (28). The presence of a term involving E_t^* indicates that it is not possible to find a constant value for $\hat{\tau}_t^E$ that solves the equation. By contrast, the only tax sequences that will implement the optimal allocation are either a complete expropriation of fossil-fuel reserves, or a sequence of taxes that may increase initially, but will eventually have to start decreasing and finally turn into a subsidy. To see this, first note that the full-expropriation tax schedule is still available, as shown in Appendix 6.B. This is indeed a sequence of ever increasing taxes $\hat{\tau}_t^E$, starting out in period 0 with $\hat{\tau}_0^E = \frac{\nu}{E_0^*}$. Starting above this level is not feasible, in the sense that such a tax sequence will not implement the optimal allocation. Finally, a tax schedule that starts out at less than full expropriation, $\hat{\tau}_0^E < \frac{\nu}{E_0^*}$, is feasible if it obeys (28). Appendix 6.C shows that any such tax sequence is such that $\hat{\tau}_t^E$ must eventually turn negative, i.e., taxes must turn into subsidies over time.

4 A Calibrated Example

This section presents results from a calibrated version of the model discussed in Section 3. Most parameter values are taken from Golosov et al. The factor shares are set to $\alpha = 0.3$ and $\nu = 0.03$. The carbon-cycle parameters are chosen such that 20% of CO_2 emissions remain in the atmosphere forever ($\varphi_L = 0.2$), half of the remaining 80% has decayed after 300 years ($\varphi = 0.0228$), and half of the total emissions has decayed after 20 years ($\varphi_0 = 0.393$). The damage parameter, which is calibrated using the damage function from DICE, is set to $\gamma = 2.379 \times 10^{-5}$ (see Golosov et al. for details). The initial capital stock K_0 is arbitrarily set to a fraction $\alpha\beta$ of first-period output, and the initial TFP level A_0 is chosen to match global output in 2010, 63 trillion 2010 US dollars (World Bank). A TFP growth of 0.5% per year is assumed. This does not matter for the optimal sequence of taxes per unit of output, $\hat{\tau}_t^E$, but productivity growth of course leads to output growth and hence plays a role for τ_t^E, i.e., taxes measured as dollars per tonne of carbon. Finally, a fossil-fuel supply $R_0 = 400$ GtC is used. This corresponds to the available reserves of oil assumed in Golosov et al.

Two discount factors, a higher β^* and a lower β, are defined. In most of what follows, the social planner is assumed to discount the future using a pure rate of time preference of 0.1% per year, based on Stern (2006). Refer to this time-preference rate as $\rho^* = 0.001$, and then define the corresponding discount factor

$$\beta^* = \left(\frac{1}{1+\rho^*}\right)^{10}.$$

Note that β^* refers to the discount factor in a setting where the time step is ten years. The higher time-preference rate, ρ, is chosen to be 1.5% per year ($\rho = 0.015$), based on Nordhaus (2008), and the discount factor β is defined in a manner analogous to β^*.

4.1 Optimal energy taxes

First, let us consider the standard case, where the planner discounts the future at the same rate as consumers. Optimal and laissez-faire fossil-fuel consumption is illustrated in Figure 1 below. Panel A illustrates the case when both the planner and consumers use a discount factor β, and in Panel B both use a discount factor β^*. In both panels, optimal climate policy is used in order to postpone the consumption of fossil fuel from the laissez-faire sequence (represented by the dashed line) to the optimal sequence (given by the solid line). In Panel B, the high β^* used by the social planner implies a fossil-fuel depletion schedule that is much flatter than that in Panel A. However, this optimal schedule is not too far from the laissez-faire schedule in Panel B, given the assumption that consumers discount also using β^*.

Consider now a situation where consumers discount using β, but the planner discounts using β^*. Optimal climate policy then amounts to shifting the extraction path for fossil fuel from the dashed line in Panel A to the solid line in Panel B. This is a graphical illustration of the need for a substantially more pronounced tax differential between time periods, in order to induce the optimal postponement of fossil-fuel depletion.

Figure 2 is an attempt at comparing optimal energy taxes in the standard model, where the planner as well as consumers discount in the same way, with the extended model presented here, where consumers discount using β and the planner uses β^*. Panel A plots energy taxes as a proportion of output, $\hat{\tau}_t^E$, over time, whereas Panel B plots taxes measured as dollars per tonne of carbon, τ_t^E, for three model setups. For the standard model, the constant-$\hat{\tau}_t^E$ solution is shown, for the case when both the planner and the markets discount the future heavily (shown by the grey line), and for the situation where

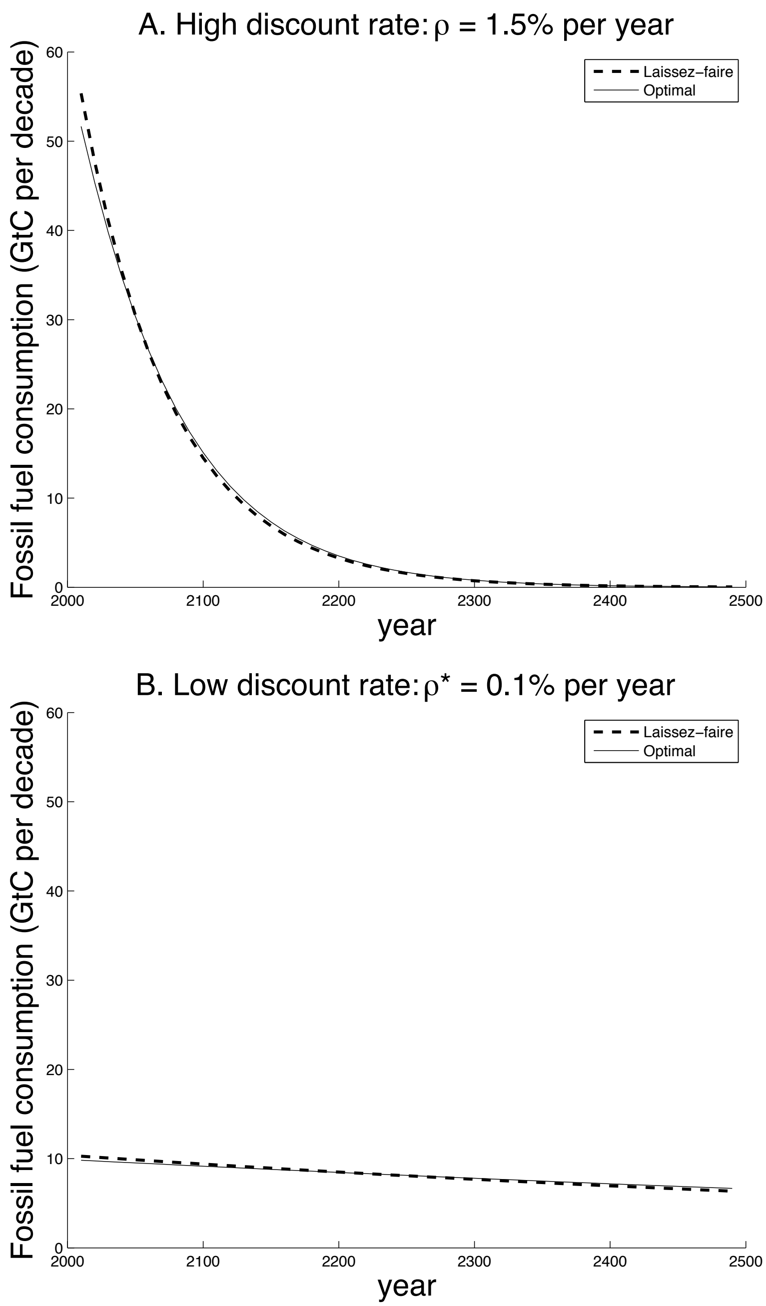

Figure 1: Fossil-fuel consumption

both the planner and the markets are more patient (the black line). The dashed line shows a tax schedule that implements the optimal allocation (together with an interest rate subsidy, as discussed above), when the planner is more patient than the market. This tax schedule is one of an infinite number of tax schedules that will implement the optimal allocation, and it is chosen such that the discounted stream of tax revenue from energy taxes is the same as in the case when both the planner and the market use $\rho^* = 0.1\%$ per year (the black solid line).

With different social and market discount rates, carbon taxes rise initially, but must eventually turn into substantial subsidies.

4.2 Welfare considerations

Let us now turn to the welfare effects of climate policy, first looking at the standard model, where both the planner and the market discount by β^*. In the standard model, the optimal allocation can be implemented using only one tax instrument, the sequence $\left\{\hat{\tau}_t^E\right\}_t$. The relevant welfare comparison in this setting is between the optimal allocation, or first best, and the laissez-faire solution. For each consumption stream $\{C_t\}_t$, define a constant consumption level $\bar{C}$, such that

$$\sum_{t=0}^{\infty} \beta^{*t} \ln(\bar{C}) = \sum_{t=0}^{\infty} \beta^{*t} \ln(C_t).$$

The difference in $\bar{C}$ between the laissez-faire outcome and the first-best outcome is a mere 0.0013%. This corresponds to the first row in Table 1 below, which summarizes the welfare comparisons discussed in this section. With a discount rate of 1.5% per year, common to both planner and market, the welfare loss in the laissez-faire outcome is even more negligible (second row in Table 1). In the extended model however, with the planner discounting with β^* and the market with β, the welfare loss in terms of $\bar{C}$ is now a substantial 6.10% (row 3 in the table).

In the extended model, two types of tax are needed in order to implement the first-best allocation. One could therefore imagine more outcomes than only the first-best and the laissez-faire solutions. For example, although a globally implemented tax on fossil-fuel consumption seems far-fetched politically, it may well be easier to implement a global energy tax only, compared to a global energy tax *and* a global subsidy to capital returns.

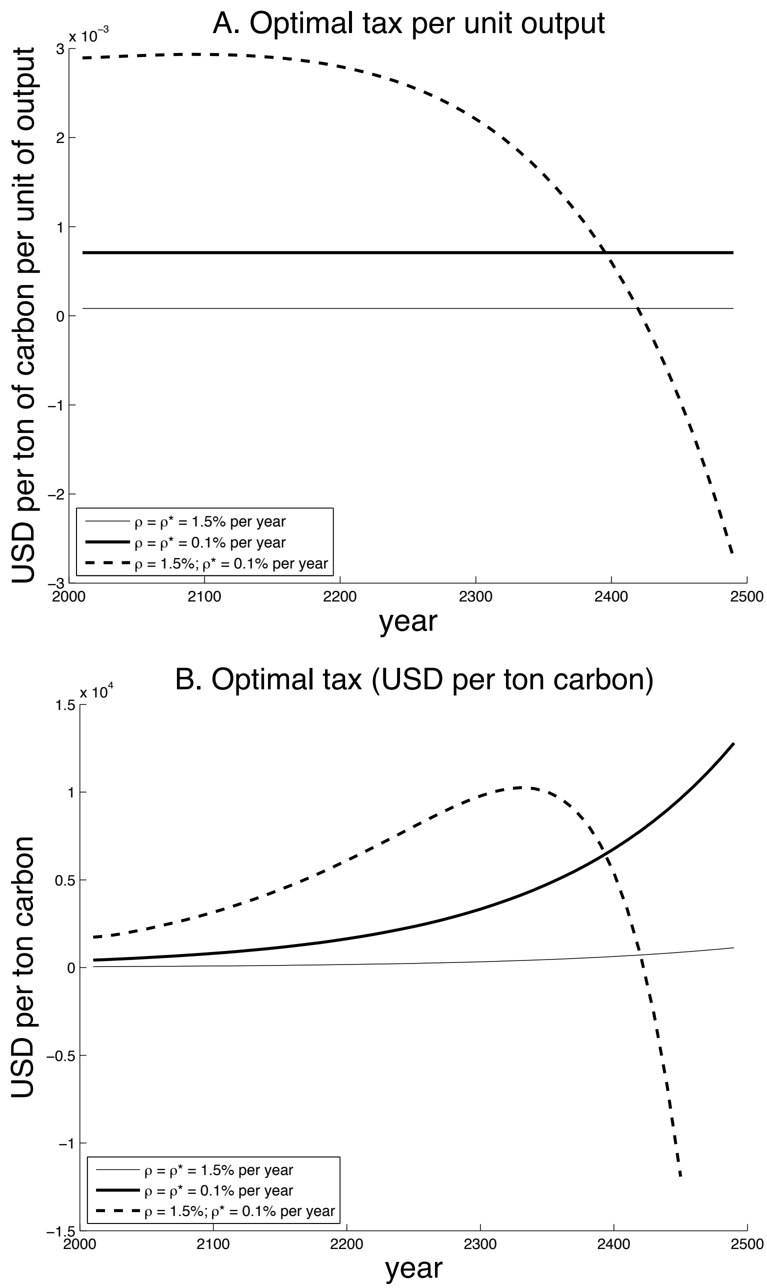

Figure 2: Optimal energy taxes

Therefore, it is interesting to consider a second-best policy of optimal energy taxes without the capital subsidy (row 4 in Table 1).[13] Such a policy would amount to a drop in $\bar{C}$ of 0.40% only, around 7% of the consumption drop in the laissez-faire solution.

We can consider two more thought experiments. First, suppose that a planner uses a discount factor β^*, and thinks that this is also the case for the market, while consumers actually discount using β. The planner would then choose a tax sequence $\hat{\tau}_t^E = \Gamma(\beta^*)\ \forall t$, hoping to implement the optimal depletion of fossil fuels. However, the sequence for fossil-fuel consumption actually implemented with this tax policy would not be the optimal one, defined by (19). Instead, it will be the sequence that satisfies (27), given the chosen tax policy $\hat{\tau}_t^E = \Gamma(\beta^*)\ \forall t$. This energy consumption profile lies in between the laissez-faire outcome (the dashed curve in Figure 1A) and the optimal outcome (the solid curve in Figure 1B). The welfare loss in such a scenario would amount to 3.29%, as measured by a drop in $\bar{C}$ (row 5 in Table 1). This situation could be thought of as 'Stern's mistake': the planner sets enery taxes based on her rate of time preference, but ignores the fact that consumers discount the future at a different rate. Climate damages are valued correctly over time, but energy taxes do not induce consumers to leave behind enough of the fossil-fuel stocks to future generations.

Finally, consider what may be referred to as 'Nordhaus's mistake'. The planner still discounts the future at a low rate, 0.1% per year, but is persuaded by the argument that carbon taxes should be set with a market discount rate in mind. Such a planner would implement a lower constant tax per unit of output $\hat{\tau}_t^E = \Gamma(\beta)$. According to row 6 in the table, such a policy leads to a welfare loss that is near the laissez-faire outcome in row 3, when the resulting consumption streams are valued using the planner's true discount rate of 0.1% per year.

5 Conclusions

It has been argued, most famously in the Stern Review (Stern, 2006), that climate policy should be designed with a high weight on future generations, higher indeed than the

[13]Note that optimal energy taxes $\left\{\hat{\tau}_t^E\right\}_t$ are the same, even if the capital subsidy is not available. To see this, notice that the optimal sequence for fossil-fuel consumption, defined by (19), only depends on parameters ν, β^*, and the carbon-cycle parameters. Therefore, optimal fossil-fuel consumption, $\{E_t^*\}_t$, is unchanged. Moreover, the market's choice of of fossil-fuel consumption, in the presence of taxes, is still defined by (27), which is also independent of capital stocks and output levels. Hence, any sequence of energy taxes that implements the first-best allocation will still implement the optimal sequence for fossil-fuel consumption if capital subsidies are not available.

Table 1: Welfare comparisons

	Consumers' discount rate	Planner's discount rate	Policy instruments	Welfare loss
1	0.1%	0.1%	none	0.0013%
2	1.5%	1.5%	none	$6.71{\times}10^{-6}$%
3	1.5%	0.1%	none	6.10%
4	1.5%	0.1%	optimal $\left\{\hat{\tau}_t^E\right\}_t$	0.40%
5	1.5%	0.1%	$\hat{\tau}_t^E = \Gamma(\beta^*)\forall t$	3.29%
6	1.5%	1.5%	$\hat{\tau}_t^E = \Gamma(\beta)\forall t$	5.88%

weight individuals currently alive seem to place on their own future selves. This may well be a very reasonable approach to the issue of climate-change externalities, but policy-makers must consider the fact that individual consumers still behave according to their own preferences, which involves discounting the future (including their own future selves) more heavily. This has some implications for optimal taxation of fossil-fuel consumption.

A policy-maker that cares for future generations must therefore not only pay attention to impacts due to climate change arising in the future, but should also induce generations currently alive to save more for the future, and leave more of the available stock of fossil fuels in the ground for future generations to enjoy. Such considerations lie behind the main result in this paper: optimal carbon taxes must in general decrease over time, and finally become subsidies, in order to induce a consumption path for fossil fuels that is optimal from the social planner's perspective.

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6 Appendix

6.A Difference equation for optimal energy taxes

Optimal depletion of fossil fuels is given by (19), as

$$\left[\frac{\nu}{E_t^*} - \Gamma(\beta^*)\right] = \beta^* \left[\frac{\nu}{E_{t+1}^*} - \Gamma(\beta^*)\right].$$

Rearrange this to find $\frac{\nu}{E_{t+1}^*}$ as a function of $\frac{\nu}{E_t^*}$,

$$\frac{\nu}{E_{t+1}^*} = \frac{1}{\beta^*}\frac{\nu}{E_t^*} - \left(\frac{1-\beta^*}{\beta^*}\right)\Gamma(\beta^*). \tag{29}$$

Now use (27), which defines how the market will consume energy in the presence of taxes, substituting out $\frac{\nu}{E_{t+1}^*}$ using (29):

$$\begin{aligned}
\left(\frac{\nu}{E_t^*} - \hat{\tau}_t^E\right) &= \beta\left(\frac{\nu}{E_{t+1}^*} - \hat{\tau}_{t+1}^E\right) \\
\Rightarrow \beta\hat{\tau}_{t+1}^E &= \hat{\tau}_t^E + \frac{\beta}{\beta^*}\frac{\nu}{E_t^*} - \frac{\nu}{E_t^*} - \frac{\beta(1-\beta^*)}{\beta^*}\Gamma(\beta^*) \\
\Rightarrow \hat{\tau}_{t+1}^E &= \frac{1}{\beta}\hat{\tau}_t^E - \frac{(\beta^*-\beta)}{\beta^*\beta}\frac{\nu}{E_t^*} - \frac{(1-\beta^*)}{\beta^*}\Gamma(\beta^*)
\end{aligned}$$

6.B Full expropriation of fossil-fuel reserves

The marginal net private benefit to using energy in production in period t is given by $(\frac{\nu Y_t}{E_t} - \tau_t^E)$. This expression will never be negative—if it were, energy use would fall, pushing up the marginal product until it reaches the level of the tax. Any sequence of taxes that implements the optimal allocation $\{E_t^*\}_t$ must therefore be such that

$$\tau_t^E \leq \frac{\nu Y_t}{E_t} \Leftrightarrow \hat{\tau}_t^E \leq \frac{\nu}{E_t}.$$

Setting $\hat{\tau}_t^E = \frac{\nu}{E_t}$ in each period taxes away all profits in the energy market. To see that this set of taxes is a solution to (28), consider a period-t tax $\hat{\tau}_t^E - \frac{\nu}{E_t^*}$. By (28), next period's tax is then

$$\begin{aligned}\hat{\tau}_{t+1}^E &= \frac{1}{\beta}\frac{\nu}{E_t^*} - \frac{(\beta^* - \beta)}{\beta^*\beta}\frac{\nu}{E_t^*} - \frac{(1-\beta^*)}{\beta^*}\Gamma(\beta^*)\\ &= \frac{1}{\beta^*}\frac{\nu}{E_t^*} - \frac{(1-\beta^*)}{\beta^*}\Gamma(\beta^*).\end{aligned}$$

From (19), we see that $\frac{\nu}{E_t^*} = \beta^* \frac{\nu}{E_{t+1}^*} + (1-\beta^*)\Gamma(\beta^*)$. Hence,

$$\begin{aligned}\hat{\tau}_{t+1}^E &= \frac{1}{\beta^*}\left(\beta^*\frac{\nu}{E_{t+1}^*} + (1-\beta^*)\Gamma(\beta^*)\right) - \frac{(1-\beta^*)}{\beta^*}\Gamma(\beta^*)\\ &= \frac{\nu}{E_{t+1}^*}.\end{aligned}$$

In other words, setting $\hat{\tau}_t^E = \frac{\nu}{E_t^*}$ for all t implements the optimal consumption path for fossil fuels, and taxes away all the surplus. Indeed, this may be considered an expropriation of all existing fossil fuel reserves.

6.C Energy taxes must turn into subsidies

Suppose we start out with a first-period tax $\hat{\tau}_0^E = \frac{\nu}{E_0^*} - \varepsilon$, and let future tax leves be defined by (28), so as to implement the optimal sequence for energy consumption, $\{E_t^*\}_t$. Energy taxes will then eventually have to turn into subsidies, for any $\varepsilon > 0$. To see this, start with $\hat{\tau}_0^E$ and iterate forward using (28), making use of (29) when necessary:

$$\begin{aligned}
\hat{\tau}_0^E &= \frac{\nu}{E_0^*} - \varepsilon \\
\Rightarrow \hat{\tau}_1^E &= \frac{1}{\beta}\left(\frac{\nu}{E_0^*} - \varepsilon\right) - \frac{\beta^* - \beta}{\beta^* \beta}\frac{\nu}{E_0^*} - \frac{1-\beta^*}{\beta^*}\Gamma(\beta^*) \\
&= \frac{1}{\beta^*}\frac{\nu}{E_0^*} - \frac{1-\beta^*}{\beta^*}\Gamma(\beta^*) - \frac{1}{\beta}\varepsilon \\
\Rightarrow \hat{\tau}_2^E &= \frac{1}{\beta}\left(\frac{1}{\beta^*}\frac{\nu}{E_0^*} - \frac{1-\beta^*}{\beta^*}\Gamma(\beta^*) - \frac{1}{\beta}\varepsilon\right) \\
&\quad - \frac{\beta^* - \beta}{\beta^* \beta}\left(\frac{1}{\beta^*}\frac{\nu}{E_0^*} - \frac{1-\beta^*}{\beta^*}\Gamma(\beta^*)\right) - \frac{1-\beta^*}{\beta^*}\Gamma(\beta^*) \\
&= \left(\frac{1}{\beta^*}\right)^2 \frac{\nu}{E_0^*} - \left(\frac{1-\beta^*}{\beta^*}\right)^2 \Gamma(\beta^*) - \left(\frac{1}{\beta}\right)^2 \varepsilon \\
\Rightarrow \hat{\tau}_t^E &= \left(\frac{1}{\beta^*}\right)^t \frac{\nu}{E_0^*} - \left(\frac{1-\beta^*}{\beta^*}\right)^t \Gamma(\beta^*) - \left(\frac{1}{\beta}\right)^t \varepsilon
\end{aligned}$$

The middle term in the above expression converges to 0 as t grows, whereas the other terms grow exponentially. When $\beta^* > \beta$, the latter term grows faster than the first term. Hence, for any $\varepsilon > 0$, $\hat{\tau}_t^E$ must eventually turn negative. In other words, for any set of taxes $\left\{\hat{\tau}_t^E\right\}_t$ that implements the optimal allocation, and that does not constitute a full expropriation of fossil fuel reserves, energy taxes must eventually turn into a subsidy.