An oligopoly fringe non renewable resource game in the presence of a renewable sunstitute*

On the importance of the order of extraction

Hassan Benchekroun
McGill University
CIREQ

Gerard van der Meijden♯
Vrije Universiteit Amsterdam
Tinbergen Institute

Cees Withagen
IPAG Business School (Paris)
Vrije Universiteit Amsterdam
Tinbergen Institute

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Abstract

We model the oil market as an oligopoly facing a fringe as well as competition from renewable resources. Within such framework we fully characterise, i.e., for all vectors of stocks, the equilibrium extraction of the fringe and the oligopolists. We show that there always exist a phase of simultaneous supply of the olgopolists and the fringe, morever this phase occurs at the beginning of the game.**JEL codes**: Q31, Q42, Q54, Q58

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♯Corresponding author: Department of Spatial Economics, Vrije Universiteit Amsterdam, De Boelelaan 1105, 1081 HV Amsterdam, The Netherlands, Phone: +31-20-598-2840, e-mail: g.c.vander.meijden@vu.nl.

1 Introduction

In this paper we build a model that incorporates three important features of the oil market: market power, a competitive fringe and competition of a renewable substitute. We allow market power to vary by considering an arbitrary number of countries with market power. We characterize the equilibrium in such a market.

Our paper is related to the literature on resource use under imperfect competition. Important contributions to this field were made by **?** on monopoly, **?** on oligopoly, **?** and **?** on dominant firms. More recently, **?**, **??** and **?** have developed cartel-fringe models of the resource market. Here we model OPEC as an important player on the market, but allow its market power to be diminished. To do this we follow **?**, p. 144 which concludes that "OPEC's behavior is best described as Cournot competition in the face of a competitive fringe constituted by non-OPEC producers." We model the market as a situation with a large number of price-taking mining firms and a set of oligopolists, which reduces to the cartel-fringe model if the number of oligopolists equals unity. We take account of the existence of renewables that provide perfect substitutes for oil and that can be produced in unlimited amounts. This raises the possibility of a limit-pricing strategy by fossil fuel suppliers in equilibrium (see, e.g., **??** and **?** for recent work and **?**, **? ?** for early contributions).

We show by construction the existence of an open-loop Nash-Cournot equilibrium on the oil market. Our main findings are as follows.

First, the oligopolists and the fringe start out supplying simultaneously to the market, despite their constant but differing unit extraction costs. If the initial stock of the fringe is large relative to the oligopolists', the stocks of the oligopolists are depleted before the stock of the fringe, i.e. the phase with simultaneous supply will be followed by a phase during which only the fringe is active. In this case, limit-pricing does not occur. When the initial stock of the oligopolists is relatively large, the phase with simultaneous supply is the followed by a period during which only the oligopolists are supplying.

Whether during this period limit-pricing occurs immediately after the simultaneous supply phase depends on the sign of marginal profits and the size of the stock of the oligopolists. More precisely, the oligopolists either choose to price strictly below the price of renewables, in which case the price increases over time, or to perform a limit-pricing strategy of marginally undercutting the renewables price, in which case the price is constant over time. If marginal profits in a limit-pricing regime are non-positive, oligopolists will start with limit pricing as soon as the fringe's stock is depleted. However, if marginal profits during limit pricing are positive, the oligopolists will only start limit pricing after the fringe's stock is depleted and their own remaining stock is smaller than a certain threshold.

Our problem is a hybrid version of the well-known cartel-fringe framework where the cartel announces a price path and the fringe chooses an extraction path, and the oligopoly framework where each player chooses an extraction strategy. In the next section, we first introduce the equilibrium concept when instead of a cohesive cartel we have several oligopolists. After describing the different extraction phases, we provide a full characterization of the oligopoly-fringe equilibrium.

The remainder of the paper is structured as follows. Section 2 outlines the model. Section 3 characterizes the open-loop Nash-Cournot equilibrium. Finally, Section 4 offers concluding remarks.

2 The model

We consider the markets for perfectly substitutable renewable and non-renewable resources. The non-renewable resource is supplied by a price-taking fringe and a group of n $(< \infty)$ suppliers with market power, referred to as oligopolists. The fringe owns an aggregate initial stock S_0^f and extracts at constant per unit extraction cost k^f. All oligopolists have the same initial stock $S_{0i}^o = S_0^o/n = \sum_{j=1}^{n} S_{0j}^o/n$. The superscript o refers to the oligopolists. The per unit extraction cost of each oligopolist is constant and denoted by k^o. [1] Extraction rates at time $t \geq 0$ by the aggregate fringe and oligopolist i are $q^f(t)$ and $q_i^o(t)$, respectively. We write aggregate supply by the oligopolists as $q^o(t) \equiv \sum_i q_i^o(t)$. Inverse consumer demand for the non-renewable resource is linear and given by $p(t) + \tau = \alpha - \beta(q^f(t) + q^o(t))$, with $\alpha > 0$ and $\beta > 0$, and where τ denotes a constant specific tax on non-renewable resource consumption and p is the producer price, i.e., the price received by the suppliers of the resource. Hence, $p + \tau$ is the consumer price for non-renewables. The perfect substitute for the non-renewable resource can be produced at marginal cost $b > 0$ and consumption of renewables is subsidized at a constant rate σ.[2] Hence, demand for the non-renewable resource

[1] Asymmetry of oligopolists can deliver interesting insights, but would obscure the source behind the novelty of the results of the paper.

[2] The constancy of the tax can be motivated by constant marginal damages of emissions. Constancy of the renewables subsidy is convenient for the results' exposition.

vanishes for $p+\tau > b-\sigma$. In that case demand for renewables x is given by $b-\sigma = \alpha-\beta x$. We abstract from technological progress (cf. **?**), as well as from set-up costs. The prevailing interest rate r is constant.

The aggregate fringe maximizes its discounted profits,

$$\int_0^\infty e^{-rt}(p(t)-k^f)q^f(t)dt, \tag{1}$$

taking the price *path* as given, subject to its resource constraint

$$\dot{S}^f(t) = -q^f(t),\ q^f(t) \geq 0,\ S^f(t) \geq 0 \text{ for all } t \geq 0, \text{ and } S^f(0) = S_0^f. \tag{2}$$

We consider an open-loop Nash equilibrium where each oligopolist i takes the time *paths* of q^f and q_j^o ($j \neq i$) as given and maximizes

$$\int_0^\infty e^{-rt}(\alpha - \beta(q^f(t) + \sum_{j=1}^{n} q_j^o(t)) - \tau - k^o)q_i^o(t)dt, \tag{3}$$

subject to its resource constraint

$$\dot{S}_i^o(t) = -q_i^o(t),\ q_i^o(t) \geq 0,\ S_i^o(t) \geq 0 \text{ for all } t \geq 0, \text{ and } S_i^o(0) = S_{0i}^o. \tag{4}$$

The existence of perfectly substitutable renewables effectively implies an upper limit on the price oligopolists can ask, yielding the additional constraint

$$\alpha - \beta(q^f(t) + q^o(t)) \leq b - \sigma. \tag{5}$$

We make the following assumption that enables us to restrict attention to cases that we think are empirically relevant.

Assumption

1. $k^o < k^f$.
2. $k^f + \tau < b - \sigma < \alpha$.
3. $k^f < (\alpha - \tau + nk^o)/(1+n)$.

Part (1) says that the unit extraction cost of the oligopolists are smaller than that of the fringe members. Part (2) ensures that the tax-inclusive marginal production costs of the non-renewable resource are lower than the after-subsidy marginal production costs of renewables, and that there is demand for renewables after exhaustion of the non-renewable resource. Part (3) makes sure that the marginal extraction cost of the fringe is below the profit-maximizing price of the oligopolists. To see this, consider the extreme case with an infinitely large S_0^o, implying a zero scarcity rent. Instantaneous marginal profits of the oligopolists (if $q^f = 0$) are then given by $\alpha - \tau - \beta q^o(1+1/n) - k^o$. Hence, the profit-maximizing price is $p^* = (\alpha - \tau + nk^o)\frac{1}{1+n}$. Condition (3) thus implies $k^f < p^*$.

An equilibrium is defined as follows.

Definition 1 *A vector of functions $q \equiv (q_1^o, ..., q_n^o, q^f)$ with $q(t) \geq 0$ for all $t \geq 0$ is an Open-Loop Oligopoly-Fringe Equilibrium (OL-OFE) if*

(i) each extraction path of the vector q satisfies the corresponding resource constraint,

(ii) for all $i = 1, 2, ..., n$

$$\int_0^\infty e^{-rs}\left[\alpha - \beta\left(q^o(s) + q^f(s)\right) - \tau - k^o\right] q_i^o(s)ds$$
$$\geq \int_0^\infty e^{-rs}(\alpha - \beta(\sum_{j\neq i} q_j^o(s) + \hat{q}_i^o(s) + q^f(s)) - \tau - k^o)\hat{q}_i^o(s)ds,$$

for all $\hat{q}_i^o$ satisfying the resource constraint, and

(iii)

$$\int_0^\infty e^{-rs}\left[p\left(s\right) - k^o\right] q^f(s)ds \geq \int_0^\infty e^{-rs}\left[p\left(s\right) - k^o\right] \hat{q}^f(s)ds,$$

where $p\left(s\right) = \alpha - \tau - \beta\left(q^o(s) + q^f(s)\right)$, for all $\hat{q}^f$ satisfying the resource constraint.

We use optimal control to characterize an OL-OFE. The Hamiltonian associated with the fringe's problem reads

$$\mathcal{H}^f = e^{-rt}(p(t) - k^f)q^f(t) + \lambda^f(t)[-q^f(t)]. \tag{6}$$

The necessary conditions include

$$p(t) = \alpha - \tau - \beta(q^f(t) + q^o(t)) \leq k^f + \lambda^f(t)e^{rt}, \tag{7a}$$

If $p(t) = \alpha - \tau - \beta(q^f(t) + q^o(t)) < k^f + \lambda^f(t)e^{rt}$ then $q^f(t) = 0$, (8)

$$\dot{\lambda}^f(t) = 0 \tag{9}$$

Here, λ^f is the fringe's shadow price of the resource stock. Hence, the conditions say that in an equilibrium with positive supply of the fringe, Hotelling's rule is satisfied: The net price, $p - k^f$, increases over time at a rate equal to the rate of interest.

The Lagrangian associated with oligopolist i's problem reads

$$\mathcal{L}_i^0 = e^{-rt}(\alpha - \beta(q^f(t) + \sum_{j=1}^{n} q_j^o(t))) - \tau - k^o)q_i^o(t) + \lambda_i^o[-q_i^o(t)]$$
$$+ \mu_i^o(t)(b - \sigma - \alpha + \beta(q^f(t) + \sum_{j=1}^{n} q_j^o(t)). \tag{10}$$

Since all oligopolists are equal we can focus on the conditions that characterize an equilibrium where the extraction paths of the oligopolists are identical, $q_i^o = q^o/n$, $\lambda_i^o = \lambda^o$, $\mu_i^o = \mu^o$. The necessary conditions then include

$$\alpha - \tau - \beta(q^f(t) + (1 + \frac{1}{n})q^o(t)) \leq k^o + \lambda^o(t)e^{rt} - \mu^o(t)\beta e^{rt}, \tag{11a}$$

If $\alpha - \tau - \beta(q^f(t) + (1 + \frac{1}{n})q^o(t)) < k^o + \lambda^o(t)e^{rt} - \mu^o(t)\beta e^{rt}$ then $q^o(t) = 0,$ (12)

$$b - \sigma - \alpha + \beta(q^f(t) + q^o(t) \leq 0, \tag{13}$$

If $b - \sigma - \alpha + \beta(q^f(t) + q^o(t) < 0$ then $\mu^o(t) = 0,$ (14)

$$\dot{\lambda}^o(t) = 0, \tag{15}$$

where λ^o denotes the shadow price of the resource stock of the oligopolists and μ^o is the Lagrange multiplier associated with restriction (5). Hence, the conditions imply that as long as $p < b - \sigma - \tau$ (i.e., as long as restriction (5) is non-binding) and $q_i^o > 0$, marginal profit of the oligopolists increases over time at the rate of interest. This is the Hotelling rule in monopoly. Finally, the oligopolists are free to choose the moment T of depletion of their stocks, so that in equilibrium the Hamiltonian vanishes at that date, implying

$$\big(p(T) - k^o - \lambda^o e^{rT}\big)\,\frac{q_i^o(T)}{n} = 0. \tag{16}$$

2.1 Phases of resource extraction

In the OL-OFE, different phases of resource extraction exist. By F, O, S and L we denote phases with only the fringe supplying, only the oligopolists supplying at a consumer price strictly below $b-\sigma$, simultaneous supply, and supply by the oligopolists at a consumer price $b-\sigma$ (i.e., limit pricing), respectively. In the sequel T^F, T^O, T^S and T^L will indicate the moments at which each of these phases comes to an end. We first summarize the necessary conditions that hold in each phase and then proceed by eliminating specific sequences of phases. Subsequently, we characterize the equilibrium of the game.

Lemma 1

Along F we have

$$p(t) = \alpha - \tau - \beta q^f(t), \tag{17}$$

with

$$p(t) = k^f + \lambda^f e^{rt}, \tag{18a}$$

$$p(t) \leq k^o + \lambda^0 e^{rt}, \tag{19}$$

implying

$$q^f(t) = \frac{1}{\beta}(\alpha - \tau - k^f - \lambda^f e^{rt}). \tag{20}$$

Along S we have

$$p(t) = \alpha - \tau - \beta(q^f(t) + q^o(t)), \tag{21}$$

with

$$p(t) = k^f + \lambda^f e^{rt}, \tag{22a}$$

$$p(t) = k^o + \lambda^0 e^{rt} + \beta \frac{q^o(t)}{n}, \tag{23}$$

implying

$$q^f(t) = \frac{1}{\beta} \left(\alpha - \tau - (n+1)(k^f + \lambda^f e^{rt}) + n(k^o + \lambda^o e^{rt}) \right), \tag{24}$$

$$\frac{q^o(t)}{n} = \frac{1}{\beta} \left(k^f + \lambda^f e^{rt} - k^o - \lambda^o e^{rt} \right). \tag{25}$$

Along O we have

$$p(t) = \alpha - \tau - \beta q^o(t), \tag{26}$$

with

$$p(t) \leq k^f + \lambda^f e^{rt}, \tag{27a}$$

$$p(t) = k^o + \lambda^o e^{rt} + \beta \frac{q_o(t)}{n}, \tag{28}$$

implying

$$\frac{q^o(t)}{n} = \frac{1}{\beta} \frac{1}{n+1} (\alpha - \tau - k^o - \lambda^0 e^{rt}). \tag{29}$$

Along L we have

$$p(t) = \alpha - \tau - \beta q^o(t), \tag{30}$$

with

$$p(t) + \tau = b - \sigma, \tag{31a}$$

$$b - \sigma - \tau - \frac{\alpha - (b - \sigma)}{n} \leq k^o + \lambda^o e^{rt}, \tag{32}$$

implying

$$q^o(t) = \frac{\alpha - (b - \sigma)}{\beta}. \tag{33}$$

Proof. This is straightforward from the application of the Maximum Principle to the problem of each oligopolist and the fringe and using symmetry. □

The next lemma restricts transitions from one regime to another.

Lemma 2.

In an OL-OFE

Lemma 1 *(i) L can only occur as a final regime.*

(ii) A direct transition from O to F or vice versa is excluded.

(iii) The final regime is not O.

(iv) The initial regime is not O.

(v) A direct transition from F or O to S is excluded.

(vi) A direct transition from F to L is excluded.

Proof.

(1) As long as the fringe is producing the producer price $p = k^f + e^{rt}\lambda^f$ is increasing. Hence, the fringe is not producing in a limit pricing phase L, where the price is constant. Also in O the producer price is increasing, because we see from (29) that supply is decreasing. Moreover, along the equilibrium the price is continuous, in view of the continuity of the Hamiltonians. Hence, once there is limit pricing, no transition to other regimes is possible.

(2). Since along O and F the producer price is increasing, a direct transition can only take place at a moment T where the producer price is below $b - \sigma - \tau$. Moreover, since the price is continuous at such a T it holds from (18a), (28) and (26) that

$$\begin{aligned} p(T) &= \alpha - \tau - \beta(q^f(T) + q^o(T)) = k^f + \lambda^f e^{rT} \\ &= \frac{1}{n+1}(\alpha - \tau + n(k^o + \lambda^o e^{rT})). \end{aligned}$$

If we have $F \to O$ then it follows from (18a) and (19) that $k^f + \lambda^f e^{rT} \leq k^o + \lambda^o e^{rT}$. Hence $((n+1)(k^f + \lambda^f e^{rT}) - (\alpha - \tau)\frac{1}{n}) \geq k^f + \lambda^f e^{rT}$, implying $p(T) \geq \alpha - \tau$, so that there is no demand for the non-renewable resource, a contradiction. The proof to exclude $O \to F$ is similar.

(3). Suppose the final regime is O. At the moment of exhaustion of the oligopolists, T^o, the price equals $p(T^o) = b - \sigma - \tau$, because the price is continuous and renewables take over. Also $\beta q^o(T^o) = \alpha - (b - \sigma)$ from continuity. But, from (16) we have $p(T^o) = k^o + \lambda^o e^{rT^o}$, so that from (29) $\beta q^o(T^o) = \frac{n}{n+1}(\alpha - (b - \sigma))$, contradicting $n < \infty$.

(4) Suppose the initial regime is O. Then it follows from (26), (27a) and (28) that $\alpha - \tau + nk^o - (n+1)k^f \leq ((n+1)\lambda^f - n\lambda^o)e^{rt}$ for all t in an initial interval of time. No transition to F is possible (part 2). A transition to L would imply that the fringe is not producing at all, since L can only be a final regime (part 1). Hence there must be a transition to S, say at T. So, $\alpha - \tau + nk^o - (n+1)k^f = ((n+1)\lambda^f - n\lambda^o)e^{rT}$ because $q^f(T+) \geq 0$.(see (23)). Since $\alpha - \tau + nk^o - (n+1)k^f > 0$ by assumption and O starts before S, we have $(n+1)\lambda^f - n\lambda^o > 0$, so that $((n+1)\lambda^f - n\lambda^o)e^{rt}$ is increasing over time, yielding a contradiction.

(5) A direct transition from O to S has been excluded in the proof of (4). Along F we have $k^f + \lambda^f e^{rt} \leq k^o + \lambda^o e^{rt}$ ((18a) and (19)), which implies $k^f - k^o \leq (\lambda^o - \lambda^f)e^{rt}$. At the transition from F to S at say T we have from the continuity of the price $k^f - k^o = (\lambda^o - \lambda^f)e^{rT}$. Because $k^f > k^o$ by assumption, we have $\lambda^o - \lambda^f$. Hence, $(\lambda^o - \lambda^f)e^{rt}$ is growing over time. However, since F precedes S, $(\lambda^o - \lambda^f)e^{rt}$ is larger than $k^f - k^o$ before T and equal to $k^f - k^o$ at T, yielding a contradiction.

(6) Suppose it is optimal to have $F \rightarrow L$ and assume the transition takes place at T. Then for $0 \leq t \leq T$ we have $\beta q^f(t) = \alpha - \tau - k^f - \lambda^f e^{rt}$ and $\beta q^f(T) = \alpha - (b - \sigma)$,because of price continuity. Hence $\lambda^f e^{rT} = b - \tau - \sigma - k^f$ and $\beta q^f(t) = \alpha - \tau - k^f - (b - \sigma - \tau - k^f)e^{rt-rT}$. The oligopolists should not want to supply before T so that for $0 \leq t \leq T$ we have

$$\alpha - \tau - \beta[\frac{\alpha - \tau - k^f - (b - \tau - \sigma - k^f)e^{rt-rT)}}{\beta}] \leq k^o + \lambda^o e^{rt} \quad (34)$$

$$= k^o + (b - \tau - \sigma - k^o)e^{rt-rT^L}, \quad (35)$$

from (16), or $k^f(1 - e^{r(t-T)}) - k^o(1 - e^{r(t-T^L)}) \leq (b - \tau - \sigma)e^{rt-rT^L}(1 - e^{rT^L - rT})$. Take the limit for t approaching T. Then the condition boils down to $(b - \tau - \sigma - k^o)(1 - e^{rT - rT^L}) \leq 0$, a contradiction.

Marginal instantaneous profit of the oligopolists during a limit pricing are

$$\hat{\Pi} = b - \sigma - \tau - k^o - \frac{1}{n}(\alpha - (b - \sigma)),$$

because instantaneous profits made by the oligopolists are $(\alpha - \beta(q^f + q^o) - k^o - \tau)q^o$ and marginal profits in this case are to be evaluated at $q^f = 0$ and $\beta q^o = \alpha - (b - \sigma)$.

An important distinction is between positive and negative $\hat{\Pi}$.

Lemma 3

Suppose $\hat{\Pi} \leq 0$. Then $O = \varnothing$.

Proof

If $\hat{\Pi} \leq 0$ then $b - \sigma - \tau - k^o - \frac{1}{n}(\alpha - (b - \sigma)) < e^{rt}\lambda^o$ for all $t \geq 0$. Using this in (**??**) and subsequently in (30) we find $p(t) > b - \sigma - \tau$ along O. But at a transition to L we have $p(t) = b - \sigma - \tau$ so that the price has been decreasing in O, which is not possible, as can be seen from (**??**) according to which q^o is decreasing in O.

2.2 Characterization of an OL-OFE

Our strategy to characterize an OL-OFE is to take a given stock S_0^f and use the stock S_0^o as a pivotal variable. Other ways to characterize the equilibrium are possible of course, but our approach yields also nice interpretations. It follows from lemmata 1, 2 and 3 that there are only four candidates for an equilibrium:

1. $F \to S$,
2. S,
3. $S \to L$,
4. $S \to O \to L$.

We will consider each candidate in detail.

Suppose the equilibrium reads S. Let S_0^f be given. Along S we have (24) and (25). Moreover, with final time T we have $p(T) = k^o + \lambda^o e^{rT}$. Also (23) holds. Integration of the extraction rates yields

$$r\beta S_0^f = (\alpha - \tau + nk^o - (n+1)k^f)(rT - 1 + e^{-rT}) + (\alpha - (b - \sigma))(1 - e^{-rT}) \quad (36)$$

Due to our assumption T is well-defined for any positive S_0^f. Moreover, $T = 0$ for $S_0^f = 0$ and T is increasing in S_0^f, with $T \to \infty$ as $S_0^f \to \infty$ as $T \to \infty$. We write $T(S_0^f)$. Next define $g(S_0^f)$ by

$$r\beta g(S_0^f) = n(k^f - k^o)(rT(S_0^f) - 1 + e^{-rT(S_0^f)}). \quad (37)$$

Also g is well-defined, and increasing in S_0^f. We have $g(0) = 0$ and $g(\infty) = \infty$.The two equations (36) and (37) have to hold in case of S being an equilibrium.

Suppose the equilibrium reads $S \to F$. Along S we have (24) and (25). Along F we have (17). Furthermore, $\lambda^f = (b - \sigma - \tau - k^f)e^{-rT^F}$ at the final time T^F. Also $\lambda^o = (k^f - k^o)e^{-rT^S} + \lambda^f = (k^f - k^o)e^{-rT^S} + (b - \sigma - \tau - k^f)e^{-rT^F}$ at the transition time T^S from S to F. Integration of the extraction rates yields after some manipulations

$$r\beta S_0^f = -n(k^f - k^o)(rT^S - 1 + e^{-rT^S}) \tag{38a}$$

$$+ (b - \sigma - \tau - k^f)(rT^F - 1 + e^{-rT^F}) \tag{38b}$$

$$+ (\alpha - (b - \sigma))rT^F. \tag{38c}$$

$$r\beta S_0^o = n(k^f - k^o)(rT^S - 1 + e^{-rT^S}) \tag{39}$$

Suppose the equilibrium reads $S \to L$. Then the same procedure as before yields

$$r\beta S_0^f = \left(\alpha - \tau + nk^o - (n+1)k^f\right) rT^S + n(b - \sigma - \tau - k^o)e^{rT^S - rT^L}(1 - e^{-rT^S}) \tag{40a}$$

$$- (n+1)(b - \sigma - \tau - k^f)(1 - e^{-rT^S}), \tag{40b}$$

$$r\beta S_0^o = n(k^f - k^o)(rT^S - 1 + e^{-rT^S}) \tag{40c}$$

$$+n(1 - e^{-rT^S})(1 - e^{rT^S - rT^L})(b - \sigma - \tau - k^o) \tag{40d}$$

$$+(\alpha - (b - \sigma))(rT^L - rT^S). \tag{40e}$$

A special case of $S \to L$ occurs when the length of the limit pricing phase is maximal. At the transition T^S from S to L we have $p(T^S) = k^f + \lambda^f e^{rT^S} = b - \sigma - \tau$. Also $p(T^L) = b - \sigma - \tau = k^o + \lambda^o e^{rT^L}$, from (16). Finally, the duration of the limit pricing phase is longest if $q^f(T^S) = 0$. That implies from (24) that

$$\frac{1}{\beta}\left(\alpha - \tau - (n+1)(k^f + \lambda^f e^{rT^S}) + n(k^o + \lambda^o e^{rT^S})\right)$$

Then we get

$$e^{rT^S - rT^L} = \frac{b - \sigma - \tau - k^o - \frac{1}{n}(\alpha - (b - \sigma))}{b - \sigma - \tau - k^o}. \tag{41}$$

Note that this makes sense only if $\hat{\Pi} > 0$. We now define $h(S_0^f)$ as the solution S_0^o of with $e^{rT^S - rT^L}$ inserted in the expressions for the stocks in $S \to L$. Then

$$r\beta S_0^f = (\alpha - \tau + nk^o - (n+1)k^f)(rT^S - 1 + e^{-rT^S}) \tag{42a}$$

and

$$\begin{aligned} r\beta S_0^o &= n(k^f - k^o)(rT^S - 1 + e^{-rT^S}) && \text{(42b)} \\ &+(\alpha - (b - \sigma))(1 - e^{-rT^S}) && \text{(42c)} \\ &+(\alpha - (b - \sigma))(rT^L - rT^S). && \text{(42d)} \end{aligned}$$

For $S_0^f = 0$ we have a positive T^L from (41) if $\hat{\Pi} > 0$. Moreover, h is increasing in T^S and therefore in S_0^f.Also $h(0) > 0$ and $h(\infty) = \infty$.It is easily seen that for a given positive S_0^f the end of the simultaneous phase occurs later in the $S \to \hat{L}$ equilibrium, where $\hat{L}$ denotes he limit pricing phase of the specific duration just derived. Therefore the oligopolists' stock required for this regime is higher, stated differently

$$h(S_0^f) > g(S_0^f). \tag{43}$$

We can now fully characterize the equilibrium of the $OL - OLF$ game for the case of negative marginal profits in case of limit pricing: $\hat{\Pi} \leq 0$. In that case we do not have to deal with $S \to O \to L$ as an equilibrium candidate.

Proposition 1.

Suppose $\hat{\Pi} \leq 0$. Let $S_0^f > 0$ be given.

(1) The equilibrium reads S if and only if $S_0^o = g(S_0^f)$.

(2) The equilibrium reads $S \to F$ if and only if $S_0^o < g(S_0^f)$.

(3) The equilibrium reads $S \to L$ if and only if $S_0^o > g(S_0^f)$.

Proof.

(1) Clearly, S implies $S_0^o = g(S_0^f)$. Suppose then that $S_0^o = g(S_0^f)$ but the equilibrium is not S. If the equilibrium would read $S \to F$ then T^S for S(i.e., the T^S following from (37)) would have to be equal to the T^F for $S \to F$. This is so because with the given initial stock of the oligopolists we need the same T^S. This yields a contradiction (compare (36) and (**??**)). Suppose the equilibrium would be $S \to L$. Then the $S-$phase

must last shorter than in a pure S equilibrium. The initial price should therefore be higher, implying a higher λ^f. But total extraction by the oligopolists in the $S-$phase must be smaller, so that also λ^o is higher (see (25)). Since the final price is the same, (16) says that the final instant of time must be smaller, a contradiction. Hence $S_0^o = g(S_0^f)$ implies S.

(2) We first show that $S \to F$ implies $S_0^o < g(S_0^f)$. Suppose $S_0^o > g(S_0^f)$ and the equilibrium reads $S \to F$. For the given S_0^o the transition time T^S is uniquely determined by (39). Hence, from (**??**) the duration of the $F-$phase is a strictly increasing function of S_0^f. Taking $S_0^o > g(S_0^f)$ fixed we increase S_0^f and thereby T^F. But once we reach the manifold where the equilibrium is purely S the length of the $F-$regime collapses to zero, a contradiction. Next, we prove that $S_0^o < g(S_0^f)$ implies $S \to F$. To this end suppose $S_0^o < g(S_0^f)$ and the equilibrium is not $S \to F$. Clearly, we cannot have pure S in this case. Hence, we have $S \to L$. Keep $S_0^o < g(S_0^f)$ fixed. There exists $\hat{S}_0^f < S_0^f$ such that $S_0^o = g(\hat{S}_0^f)$.With these initial stocks the equilibrium reads S and the corresponding $\hat{T}^S$ follows from

$$r\beta\hat{S}_0^f = (\alpha - \tau + nk^o - (n+1)k^f)(r\hat{T}^S - 1 + e^{-r\hat{T}^S}) + (\alpha - (b-\sigma))(1 - e^{-r\hat{T}^S}). \quad (44)$$

But along $S \to L$ we have

$$\begin{aligned} r\beta S_0^f = & \left(\alpha - \tau + nk^o - (n+1)k^f\right)\left(rT^S - 1 + e^{-rT^S}\right) + (\alpha - (b-\sigma))(1 - e^{-rT^S}) \\ & n\{(b - \sigma - \tau - k^o)e^{rT^S - rT^L} - (b - \sigma - \tau - k^f)\}(1 - e^{-rT^S}). \end{aligned} \quad (45a)$$

In the limit for $T^S \to T^L$ we find $S_0^f > \hat{S}_0^f$, a contradiction.

(3). We first show that $S \to L$ implies $S_0^o > g(S_0^f)$. Suppose $S_0^o < g(S_0^f)$ and the equilibrium reads $S \to L$. From here the proof follows the proof of the second part of (2). It remains to be shown that $S_0^o > g(S_0^f)$ implies $S \to L$ as the equilibrium. If this would not hold, the equilibrium is not S. So the equilibrium must be $S \to F$. But it has been shown above that then $S_0^o < g(S_0^f)$, a contradiction.

Next we move on to the case $\hat{\Pi} > 0$. Then we have to take into account that the equilibrium could read $S \to O \to \hat{L}$, with O possibly collapsing. Indeed, in this equilibrium with O we have indeed $\hat{L}$, meaning that the limit pricing phase has a length that does not depend on the initial stock. This can be seen as follows. At the start of the $L-$phase we have $p(T^L) = b - \sigma - \tau = k^o + \lambda^o e^{rT^L}$, from (16). At the end of the

$O-$phase we have because of continuity $\frac{q^o(t)}{n} = \frac{1}{\beta}\frac{1}{n+1}(\alpha - \tau - k^o - \lambda^0 e^{rt}) = \frac{\alpha-(b-\sigma)}{\beta}$.

Proposition 2

Suppose $\hat{\Pi} > 0$. Let $S_0^f > 0$ be given.

(1) The equilibrium reads S if and only if $S_0^o = g(S_0^f)$,

(2) The equilibrium reads $S \to F$ if and only if $S_0^o < g(S_0^f)$,

(3) The equilibrium reads $S \to \hat{L}$ if and only if $S_0^o = h(S_0^f)$,

(4) The equilibrium reads $S \to L$ if and only if $S_0^o \in (g(S_0^f), h(S_0^f))$,

(5) The equilibrium reads $S \to O \to \hat{L}$ if and only if $S_0^o > h(S_0^f)$.

Proof.

The proofs of (1) and (2) are equal to the proofs for the case $\hat{\Pi} \leq 0$.

(3). Clearly, $S \to \hat{L}$ implies $S_0^o = h(S_0^f)$. Suppose then that $S_0^o = h(S_0^f)$ and the equilibrium does not read $S \to \hat{L}$. We first observe that $S \to \hat{L}$ can be realized while satisfying the necessary conditions. If the equilibrium would be S or $S \to F$ then the $S-$phase would be shorter than in the case of $S \to \hat{L}$. This implies a higher λ^f, but also a higher λ^0 because somewhere in the $S-$phase there must be a smaller supply from the oligopolists. But that implies from (16) an earlier final instant of time. Hence, not all of the resource is extracted. If the equilibrium would be $S \to O \to \hat{L}$ then it can be shown that the transition to O occurs at the same time as the transition to $\hat{L}$. Moreover, the lengths of the limit pricing phases are equal. Therefore $S \to O \to \hat{L}$ cannot be an equilibrium.

(4) Suppose we have $S \to L$ and $S_0^o < g(S_0^f)$. This has been excluded in the previous proposition. Suppose we have $S \to L$ and $S_0^o > h(S_0^f)$. Keep S_0^o fixed and let S_0^f increase up to the point $\hat{S}_0^f$ where $S_0^o = h(\hat{S}_0^f)$. At that point we should have

$$r\beta\hat{S}_0^f = (\alpha - \tau + nk^o - (n+1)k^f)(rT^S - 1 + e^{-rT^S}).$$

In the limit as $S_0^f \to \hat{S}_0^f$ it holds that

$$e^{rT^S - rT^L} = \frac{b - \sigma - \tau - k^o - \frac{1}{n}(\alpha - (b - \sigma))}{b - \sigma - \tau - k^o}.$$

It this is inserted in the expression for S_0^f in the $S \to L$ equilibrium we get

$$r\beta\hat{S}_0^f = (\alpha - \tau + nk^o - (n+1)k^f)(rT^S - 1 + e^{-rT^S}) + (1 - e^{-rT^s})(\alpha - (b - \sigma)).$$

This yields a contradiction. We next prove that $S_0^o \in (g(S_0^f), h(S_0^f))$ implies $S \to L$. Clearly, the equilibrium is not S or $S \to F$ or $S \to \hat{L}$. So, we only need to exclude $S \to O \to \hat{L}$. We can then repeat the argument used in (3): If the equilibrium would be $S \to O \to \hat{L}$ then it can be shown that the transition to O occurs at the same time as the transition to $\hat{L}$. Moreover, the lengths of the limit pricing phases are equal. Therefore $S \to O \to \hat{L}$ cannot be an equilibrium.

(5) The proof is straightforward and follows from the necessary and sufficient conditions discussed before.

3 Concluding remarks

We have characterized the equilibrium when several resource owners with market power compete in the presence of a competitive fringe and a renewable substitute. The features of the equilibrium is that, both the fringe and the oligopolists simultaneously supply initially. Whether the fringe or the oligopolists exhaust the resource first depends on their relative stocks. Moreover, when the oligopolists exhaust their resources first there always exist a limit pricing phase. This limit pricing phase may be preceded by a phase where oligopolists supply the market with a price strictly below the limit price if their resource stock is large enough.

The existence of the simultaneous supply phase is a source inefficiency for two reasons, the stock of the fringe is typically considered not only more expensive to extract but also more dirty than the stock of the oligopolists.

The existence of the simultaneous supply phase can therefore have important implications of the strength of the cartel formed by the oligopolists. Indeed while grey social welfare, that is social welfare that does not take into account environmental damages caused by oil, is maximum when the OPEC members behave as price takers, when environmental damages are taken into account, there is a potential gain from OPEC members forming a successful cartel which is the reduction in environmental damages. If indeed one views the number of oligopolists as a proxy for the OPEC's cartelization, it is then possible that social welfare defined as the grey social welfare minus environmental damages is a non monotonic function of the number of oligopolists, that is of

OPEC's success in cartelization.

Another potential area of application of the equilibrium characterized is the examination of different climate policies, such as a tax on carbon or a subsidy on renewables on the extraction paths of the cartel members as well as on the extraction path of the fringe. These questions are related to the Green Paradox literature (cf. **????**), in which it is shown that under perfect competition the announcement of stringent future climate policies (such as carbon taxes or subsidies for renewable energy) may cause an increase in current fossil fuel supply and therefore leads to an acceleration rather than a mitigation of global warming. Using the equilibrium characterized above, one can add to this active research field by highlighting the importance of the number of oligopolists and their impact on how both the level of current fossil use, and also the mix between relatively clean and dirty fuels react to climate policies.