Intertemporal Abatement Decisions under Ambiguity Aversion in a Cap and Trade

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Abstract
Cap-and-trade systems are subject to considerable uncertainty, especially of a regulatory nature, which can disrupt intertemporal efficiency and undermine the long-term price signal. We argue that the prevalence of such uncertainty can be characterized by a situation of ambiguity and study intertemporal abatement decisions by an ambiguity averse firm covered under a market for permits, where ambiguity is assumed to bear on both the future permit price and firm’s permit demand. Ambiguity aversion drives equilibrium choices away from intertemporal efficiency by inducing two effects: a pessimistic distortion of the firm’s beliefs overemphasizing detrimental outcomes and a shift in its discount rate. Intertemporal arbitrage does not solely depend on expected future permit prices but also on expected future market positions, e.g. pessimism leads firms to overabate (resp. underabate) early on relative to intertemporal efficiency when they expect to be short (resp. long) in the future. In general, pessimism provides an incentive for early overabatement which is more pronounced under auctioning than free allocation and forward contracts cannot restore intertemporal efficiency.

Keywords: Intertemporal emissions trading; Regulatory uncertainty; Ambiguity aversion.

JEL Classification codes: D81; D92; Q58.

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1 Introduction

Most Emissions Trading Systems allow for some degree of temporal flexibility through banking and borrowing of permits.\(^1\) As firms cost minimize over time, the opportunity to bank (resp. borrow) permits for (resp. from) future compliance periods implies an arbitrage condition between present and discounted future permit prices. That is, the permit price is the vehicle that balances long-term supply and demand of permits and, in a competitive permit market where firms have perfect foresight and information, the aggregate abatement effort is efficiently spread across time, i.e. the least discounted cost solution obtains (Rubin, 1996; Cronshaw & Kruse, 1996; Kling & Rubin, 1997).\(^2,3\) Furthermore, because permits are an exhaustible resource and have negligible storage costs, the cost-of-carry price must be congruent with the spot price grown at the interest rate. Therefore, the current permit price should reflect the net present value of the last permit surrendered for compliance in the system and the optimal price path should grow at the interest rate (Hotelling, 1931).

In practice, however, ETSs are subject to considerable uncertainty. Indeed, these systems do not function in a vacuum and are thus affected by external factors and policies outside their perimeters, e.g. macroeconomic conditions, the usage of offset credits to partly acquit compliance obligations and the reach of complementary policies (Newell et al., 2013; de Perthuis & Trotignon, 2014; Borenstein et al., 2016; Chèze et al., 2016; Ellerman et al., 2016). Policy overlap can be fortuitous as (deplored) in the EUETS or explicitly built into an open regulatory system as in California where the ETS operates in conjunction with a set of complimentary measures, thereby constituting a safety net ensuring that the state-wide target is attained and that no low-cost abatements are left behind.\(^4\) In turn, the above factors can erode the cap stringency – the so-called waterbed effect – as there exists significant uncertainty about baseline emission levels which Borenstein et al. (2016, 2017) estimate to be «at

\(^{1}\) In general, there is lenient or no restriction at all on banking whereas individual borrowing is severely limited or simply prohibited, lest it be incompatible with the attainment of the environmental target or that firms may shirk. See Chevallier (2012) and Hasegawa & Salant (2015) for a literature review on intertemporal emissions trading and Holland & Moore (2013), PMR & ICAP (2016) and ICAP (2017) for details on existing market designs, e.g. holding limits in California or implicit year-on-year borrowing in the EUETS.

\(^{2}\) It is common practice to establish emissions caps that are descending over time so that it is rational for cost-minimizing firms to emit less than the cap at the start, accumulating a ‘permit bank’ and then drawing it down as the cap gets more stringent. Such cap profiles can be seen as market-wide borrowing mechanisms, as compared to an even apportionment of the overall abatement effort across compliance periods.

\(^{3}\) Under conditions of uncertainty, intertemporal trading allows the smoothing of abatement efforts through time, thereby reducing the costs associated with otherwise unresponsive, i.e. vertical, short-run permit supply curves first identified in the seminal contributions by Roberts & Spence (1976) and Weitzman (1978).

\(^{4}\) Notice that policy overlap is not limited to climate change and energy related regulations. For instance, Schmalensee & Stavins (2013) underline the impact of railway deregulation on the US SO\(_2\) trading program.
least as large as uncertainty about the effect of abatement measures».\(^5\)

As it turned out, baselines have been lower than anticipated in most ETSs and, in general, permit prices have declined and now keep hovering at low levels or just above price floors when in place (Tvinnereim, 2014; Lecuyer & Quirion, 2016). If agents are rational, however, lower-than-expected price levels should be inconsequential for intertemporal cost efficiency. Indeed, a positive (resp. negative) permit demand shock would shift the optimal price path upwards (resp. downwards), but its slope would be preserved and it would remain efficient. In other words, even if abatement costs were subject to random shocks, the cap would still be achieved at least discounted expected cost (Schennach, 2000; Salant, 2016). Moreover, in a rational expectations market equilibrium, permit carry-over arbitrage conditions would solely depend on expected future permit prices (Samuelson, 1971).

However, it is difficult to ascertain whether low prices are attributable to external demand shocks or to intrinsic market and regulatory imperfections (Acworth et al., 2017). Prices may thus not be ‘right’ in that they may not reflect expected intertemporal marginal abatement costs. As a case in point, Hintermann et al. (2016) review the empirical literature on the price determinants in the EUETS and find that «there is a complex interaction between BAU emissions, abatement quantities and allowance prices». Put otherwise, our understanding of these determinants is limited and, for instance, Koch et al. (2014) find that abatement-related fundamentals can explain only a small fraction of variations in permit prices. Present market and regulatory imperfections, low prices can be of consequence for intertemporal efficiency: interactions between such imperfections and exogenous shocks could impair price formation and undermine the long-term price signal conveyed by the system.

In the EUETS, a scan of the literature indicates that three types of imperfections are at the center of attention, viz. limited foresight, excessive discounting and regulatory uncertainty.\(^6\),\(^7\)

First, if agents have truncated time horizons and permits are relatively more abundant today than they will be in the future, prices will not reflect the long-term permit scarcity and will be lower than they ought to, which raises overall compliance costs (Ellerman et al., 2015). Second, it can be that discount rates higher than the interest rate are applied for banking

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\(^5\)In particular, Borenstein et al. (2016, 2017) show that significant baseline uncertainty coupled with little price elasticity is likely to generate either very high or very low price levels with high price volatility.

\(^6\)Hintermann et al. (2016) note that the alleged wedge between permit prices and intertemporal marginal abatement costs can also be sustained by other factors, e.g. transaction costs or market power.

\(^7\)Relative to ‘standard’ markets a specificity of permit markets is that the supply of permits is exogenously set by the regulatory authority, i.e. permits are not natural commodities with intrinsic value. Rather, they can be limited, modified or simply cancelled. Hence our analysis could also apply to other types of ‘insecure, ill-defined’ natural resource tradable property rights, see e.g. Grainger & Costello (2014).
strategies outside a ‘hedging corridor’ due to institutional or corporate constraints (Neuhoff et al., 2012; Schopp et al., 2015). Additionally, when future abatement costs are uncertain, risk premia can be associated with holding permits when firms are risk averse (Kollenberg & Taschini, 2016). In turn, these ‘too high’ discount rates would encourage lower banking and sustain lower prices today, as compared to the cost-effective price path. As a case in point, Bredin & Parsons (2016) find that the EUETS has been in contango since early Phase II in 2008. That is, futures prices have been higher than cost-of-carry prices with implied premiums (i.e., negative convenience yields) of significant sizes.

Third, regulatory uncertainty about permit supply adjustments has shown to bear on price formation. With a focus on the EUETS, Salant (2016) shows how, in a rational expectations market equilibrium, regulatory uncertainty weighs on price formation. Empirical support is provided by Koch et al. (2016) and Creti & Joëts (2017) who find the EUETS responsive to political events and announcements. In other words, not only the announced cap level will hold sway over price formation, but also speculation about the attendant regulatory commitment may cause prices to fluctuate. Additionally, Koch et al. (2016) present evidence that market participants tend to belittle, if not ignore, announcements that should be indicative of higher future prices, which also suggests that firms cannot entirely hedge against regulatory risks. Moreover, low price levels have sparked short-term regulatory interventions (e.g., ex post supply management) as well as structural design reforms (e.g., price or quantity collars) in all existing ETSs, which further feeds regulatory uncertainty.

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8Firms can bank permits (or purchase forward contracts) at the opportunity cost of capital as part of their hedging strategies. Beyond this ‘hedging corridor’ higher rates (up to 15%) should apply because holding permits is regarded as a risky investment, i.e. speculation. However, Neuhoff et al. (2012) and Schopp et al. (2015), who report results from a dozen interviews with representatives of the European power sector, do not provide a normative explanation for why such a high rate of return would be required. Note that banking can be also limited at the institutional level, e.g. in South Korea and via holding limits in California.

9Interestingly, Bredin & Parsons (2016) note that this term structure reflects a ‘sort of fear’ that is not consistent with the types of reforms discussed at the EU level. For instance, changes in the permit supply via the Market Stability Reserve or an adjustment of the annual cap-decreasing factor should shift the term structure as a whole, not just its slope. A change in permit bankability, by contrast, could shift the slope.

10Examples are many. For instance, the price rise in early 2016 in the New Zealand ETS is attributable to the announcement that the 2:1 compliance rule should be abolished. Similarly, downward pressure on prices in Chinese pilots results from regulatory uncertainty about the transition to a national market, especially regarding the carry-over provision for pilot permits into the national market. Also, prices in RGGI increased when the 45% slash in the cap was under discussion, but before it was actually passed and implemented.

11Positive prices in oversupplied Phase II indicate banking as firms expect the emission constraint to be binding in the future and also reflect «their awareness of regulatory uncertainty» (Hintermann et al., 2016).

12This distorts the intertemporal optimality of agents’ decisions and other desirable elements of the system may possibly be impaired. In the best case, this may be a necessary consequence of a policy update, i.e. when agents fully understand the objectives of the regulator (Kydland & Prescott, 1977). However, this may be a needless cost when agents regard regulatory actions as somewhat random (Salant & Henderson, 1978), i.e. when the policy update is poorly managed or worse, when speculation is unfounded.
Faced with such uncertainty, we argue that firms lack confidence and/or relevant information to assign a probability measure uniquely describing the stochastic nature of their decision problems. This corresponds to a situation characterized by ambiguity while, by contrast, risk refers to situations where such distributions are perfectly known and unique. This paper thus examines intertemporal abatement decisions by a risk neutral ambiguity averse firm to take account of the prevalence of such non-probabilizable uncertainty. Ambiguity neutrality is our natural benchmark, in which the firm’s optimal abatement stream is congruent with the least discounted expected cost solution (Schennach, 2000) and permit carry-over arbitrage conditions solely depend on expected future prices (Samuelson, 1971).

We consider a firm covered under a two-period ETS that starts at the beginning of date 1 and terminates at the end of date 2. We assume that the firm is already compliant at date 1 but can still undertake additional abatement and bank permits into date 2 in anticipation of (higher) date-2 requirements. At date 1, however, both the date-2 market permit price and the firm’s demand for permits are ex-ante ambiguous and exogenous to the firm. This reflects that regulatory uncertainty directly bears on price formation (Salant, 2016) and that there is significant uncertainty about permit demands, e.g. via direct or indirect policy overlaps (Borenstein et al., 2016, 2017). One could expect regulatory uncertainty to bear on the firm’s endowment of permits. However we choose to keep permit allocation as a parameter in the model to be able to measure its influence on intertemporal abatement decisions. Indeed, we show that neutrality of allocation does not hold under ambiguity aversion.

Ambiguity resolves at the start of date 2. We solve the firm’s intertemporal cost minimization program by backward induction and compare the optimal level of date-1 abatement (i.e., banking) under ambiguity aversion relative to ambiguity neutrality. We consider a smooth ambiguity model of choice à la Klibanoff et al. (2005, 2009) in which the firm is confronted with a set of possible scenarios about the future regulatory framework, i.e. objective probabilities for the related permit price and demand forecasts, and has subjective beliefs over the materialization of these scenarios. Ambiguity neutrality ensures linearity between ob-

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13For instance, Hoffmann et al. (2008) define regulatory uncertainty as «an individual’s perceived inability to predict the future state of the regulatory environment» where the term ‘uncertainty’ is deliberately used in the Knightian sense (Knight, 1921), which can also be referred to as ambiguity.

14For accuracy in terminology, the firm displays aversion toward model uncertainty Marinacci (2015).

15Ultimately, it is the firm’s gross effort of abatement (i.e., baseline minus allocation) that is impacted by regulatory uncertainty, which we already capture by letting the baseline be ambiguous.

16Permit allocation is not neutral as soon as one of the assumptions sustaining the market equilibrium solution of Montgomery (1972) is relaxed. See Hahn & Stavins (2011) for a literature review.

17Typically, consider for instance that these objective scenarios are provided by expert groups, e.g. BNEF, Energy Aspects, ICIS-Tschach, Point Carbon, diverse academic fora or think tanks, etc.
jective and subjective lotteries, thereby collapsing to a Savagian framework (Savage, 1954). Attitudes toward ambiguity originate in the relaxation of such linearity and ambiguity aversion corresponds to the (additional) aversion (w.r.t. risk aversion) to being unsure about the probabilities. This will incite the firm to favor abatement streams that reduce ambiguity.

Ambiguity aversion drives equilibrium abatement stream choices away from intertemporal cost efficiency. Before examining joint market price and firm’s baseline ambiguities we consider each source of ambiguity in isolation. This enables us to separate out two ambiguity aversion induced effects. First, with risky price and individual baseline ambiguity, we note that from the perspective of the risk neutral firm the cap and trade can be assimilated to an emissions tax where the tax rate is set at the expected permit price. Ambiguity aversion induces an upward (resp. downward) shift in the firm’s discount factor when it exhibits Decreasing (resp. Increasing) Absolute Ambiguity Aversion. Therefore, early overabatement (resp. underabatement) occurs relative to the benchmark under DAAA (resp. IAAA).

Second, with price ambiguity and risky individual baseline, ambiguity aversion induces another effect by which the firm pessimistically distorts its subjective beliefs and overweights detrimental scenarios. Intuitively, when the firm expects to be net short (resp. long) in the future it overemphasizes scenarios where high (resp. low) prices are relatively more likely. This raises (resp. lowers) the firm’s estimate of the future price relative to the benchmark and raises (resp. lowers) its incentive for early abatement accordingly. Relative to the benchmark the ambiguity averse firm does not solely base its present abatement decisions on the expected future price but also on its expected future market position. This ultimately hinges upon permit allocation and we identify allocation thresholds below (resp. above) which pessimism unconditionally leads the firm to overabate (resp. underabate) early on.

Third, with both price and individual baseline ambiguities, we show that early overabatement occurs when the conditions for early overabatement under sole price ambiguity obtain and, additionally, high-price scenarios coincide with high-baseline scenarios. We then extend the model and consider a continuum of firms identical but for allocation where the permit price is endogenously determined on the market and depends on market-wide baseline ambiguity. We can refine the threshold condition on permit allocation and we show that pessimism generates early overabatement under symmetric allocation of permits. This can be a behavioral factor contributing to the formation of a permit bank, e.g. in the EUETS.

The two ambiguity aversion induced effects can be aligned or countervailing, the direction and

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18 We thus define DAAA as ambiguity prudence following Berger (2014) and Gierlinger & Gollier (2017).
magnitude of which depend on the degree of ambiguity aversion and allocation. In particular, a higher degree of ambiguity aversion is not necessarily conducive to a larger adjustment in early abatement (in absolute terms).19 With a parametrical example we numerically show that early abatement decreases with allocation volume and that the magnitude of the distortion due to pessimism is generally greater than that due to the shift in discounting. This shows that, under ambiguity aversion, early abatement is higher under auctioning than free allocation and suggests that the distortion away from intertemporal efficiency should be greater under a cap and trade than an emissions tax.20

Finally, we consider three extensions to the model in the Appendices. First, we show that introducing forward contracts cannot restore intertemporal efficiency, which suggests that regulatory risks cannot be entirely hedged. Although forwards have potential to partially mitigate pessimism, the shift in discounting always persists. Second, we show that the volume of trade is reduced when allocation is sufficiently asymmetrical across ambiguity averse firms. Third, we show that the equilibrium in a market populated by a mix of ambiguity neutral and averse firms should be brought further away from intertemporal efficiency.

The remainder is organized as follows. Section 2 reviews the related literature. Section 3 presents the modelling framework. Section 4 analyzes intertemporal abatement decisions under ambiguity aversion relative to ambiguity neutrality. In particular, Section 4.1 considers the case of pure firm-level baseline ambiguity and Section 4.2 that of pure market price ambiguity. The case of joint price and baseline ambiguities is next presented in Section 4.3 and that of market-wide demand ambiguity with endogenous permit price is considered in Section 4.4. Section 5 illustrates the results numerically. Section 6 concludes. An Appendix contains the analytical derivations, proofs (A) and some extensions to the model (B), considers alternative representation theorems under ambiguity (C), provides additional details on both the two ambiguity aversion induced effects (D) and the case of joint price and baseline ambiguities (E), and finally considers the special case of binary price ambiguity (F).

19 An increase in ambiguity aversion always increases the magnitude of the pessimistic distortion in the sense of a monotone likelihood ratio deterioration (Gollier, 2011) and we show that it can increase that of ambiguity prudence only when ambiguity prudence is not too strong relative to ambiguity aversion.

20 We merely compare how price or quantity controls affect intertemporal decisions in terms of distortion magnitude. See e.g. Pizer & Prest (2016) for a normative approach to comparing price and intertemporally tradable quantity regulations under uncertainty: they exploit the (often under-appreciated) feature of the latter in equating current and expected future prices (as compared to the former that cannot do so) to establish the first best in all periods by an adequate selection of a sequence of policy updates. Present exogenous noises in policy updates (i.e., a de facto analog of regulatory uncertainty) they show that intertemporally tradable quantities welfare-dominate prices provided that the noise variance is low enough.
2 Related literature

The paper combines two strands of literature, namely (dynamic) abatement and investment incentives under environmental regulations and decision-making under ambiguity aversion.

**Dynamic abatement and investment incentives.** First, we extend the results of Baldursson & von der Fehr (2004) to ambiguity aversion. Similarly, Baldursson & von der Fehr show that risk averse firms that expect to be short (resp. long) on the permit market over-invest (resp. under-invest) in abatement technology relative to risk neutrality.\(^{21}\) However, we show that both price and quantity regulations deteriorate under ambiguity aversion while a price instrument remains intertemporally efficient under risk aversion. Additionally, in our setup firms expecting to be net long (resp. short) can still overabate (resp. underabate) when they exhibit DAAA (resp. IAAA).\(^{22}\) In particular, the DAAA (resp. IAAA) induced increase (resp. decrease) in the discount factor can create a downward pressure on future (resp. present) prices. In our framework, the hypothesis of ‘excessive discounting’ found in the literature to account for the current price depreciation in the EUETS (either due to a hedging corridor (Neuhoff et al., 2012; Schopp et al., 2015) or due to risk aversion (Kollenberg & Taschini, 2016)) coincides with IAAA, i.e. ‘imprudence toward ambiguity’.\(^{23}\) However, our results do not imply that IAAA-firms necessarily under-bank permits as pessimism can dominate the shift in discounting depending on their permit endowment. Finally, we also extend the work of Chevallier et al. (2011) who examine the impacts of risky permit allocations on banking decisions and find that banking increases consecutive to an increase in risk if, and only if, the third derivative of the firm’s production function is positive.\(^{24}\)

We follow the literature on dynamic investment incentives under environmental regulations

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\(^{21}\)With a different modelling framework, we note that Ben-David et al. (2000) find similar results.

\(^{22}\)With reference to Phase I of the EUETS, Ellerman et al. (2010) (especially in Chapter 5) note that there is an asymmetry between long and short entities since the former are under no compulsion to sell and can adopt a passive wait-and-see attitude as long as uncertainty is high and experience is being gained.

\(^{23}\)More precisely, Kollenberg & Taschini (2016) study the continuum of convex combinations between a pure price and pure quantity instruments in a dynamic framework where there is uncertainty about abatement costs. When firms are risk averse, firms’ risk premium is the highest (resp. lowest) under a pure quantity (resp. price) instrument because the burden to adjust to shocks is entirely borne by firms (resp. the regulator). They find that aggregate compliance costs are minimized somewhere in between these two polar cases, which reflects a trade-off between the inability to take advantage of the differences in realized marginal abatement costs through time via intertemporal trading (under a tax) and significant costs for firms of having to adjust their strategies in response to shock realizations (under a permit market).

\(^{24}\)Chevallier et al. (2011) describe how banking can be a risk-management tool, and, provided that firms are able to pool risks, define optimal risk-sharing rules between firms. Relatedly, Hennessy & Roosen (1999) show that when pollution is random, firms subject to a permit market have an incentive to merge (i.e., consolidate) for permit management purposes under both risk neutrality and risk aversion.
in that it generally considers exogenous shocks on permit prices and firms’ demands – see Requate (2005) for a review.\textsuperscript{25} In general, partial equilibrium models tend to favor taxes over ETSs essentially because in the latter the permit price comprises a real option value and thus deviates from marginal abatement costs – see e.g. Xepapadeas (2001) with permit price uncertainty and Chao & Wilson (1993) with aggregate demand uncertainty. This literature further distinguishes between irreversible and reversible investments and generally shows that the former tend to decrease with uncertainty (Blyth et al., 2007) while the latter can be used as a hedge and tend to increase with uncertainty (Chen & Tseng, 2011).\textsuperscript{26} For instance, Zhao (2003) finds that, in a general equilibrium model, irreversible investments decrease in the level of abatement cost uncertainty, but more so under a tax than an ETS. Finally, Albrizio & Costa (2014) explicitly analyze the effects of policy uncertainty on irreversible and reversible investments by ETS-liable firms in a model where the cap set by the regulator is observed only once firms have made their investment decisions.

**Decision-making under ambiguity.** Since Ellsberg’s seminal article in 1961 it has been well documented that most individuals treat ambiguity differently than objective risk and prefer gambles with known rather than unknown probabilities.\textsuperscript{27} There exist alternatives to Subjective Expected Utility (Savage, 1954) – see Etner et al. (2012) and Machina & Siniscalchi (2014) for a review. These models of choice differ in their treatments of objective and subjective probabilities and preferences are no longer linear in probabilities. They can roughly be grouped into three categories. The first category represents non-additive beliefs, i.e. the probability of an outcome depends on its ranking among all possible outcomes (Schmeidler, 1989; Chateauneuf et al., 2007). The second category considers that agents have a set of multiple subjective priors. Gilboa & Schmeidler (1989) provided behavioral foundations for Multiple-priors (or Maximin) Expected Utility (MEU) preferences. Ghirardato et al. (2004) later axiomatized the $\alpha$-maxmin model of choice which considers a convex combination of maximal and minimal expected utilities over the set of priors.

The third category corresponds to Recursive Expected Utility models, in which agents have a second-order subjective prior over a set of first-order objective measures and are EU-maximizers over the two layers of uncertainty (Klibanoff et al., 2005, or KMM). A KMM

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\textsuperscript{25}Economic theory suggests that there is an option value to postpone investments under uncertainty (Dixit & Pindyck, 1994). Note that Dorsey (2017) empirically validates that regulatory uncertainty increases compliance costs by delaying investments in the context of EPA’s Clean Air Interstate Rule.

\textsuperscript{26}In the words of Laffont & Tirole (1996) low-emission investment are a ‘bypass’ of permit markets.

\textsuperscript{27}Ellsberg (1961) showed that rational decision-makers behaved in ways incongruent with the Savagian axiomatization, and especially with the sure-thing principle.
model of choice has the advantage of disentangling ambiguity itself (or ‘beliefs’) from attitudes (or ‘tastes’) toward ambiguity. It also comes with nice comparative statics and tractability properties to which the decision-making under risk machinery readily applies, can be embedded in a dynamic framework (Klibanoff et al., 2009) and nests other models of choice under ambiguity aversion as special cases.  

Ambiguity aversion has been applied to a variety of fields in economics, such as finance (Gollier, 2011; Gierlinger & Gollier, 2017), formation of precautionary savings (Berger, 2014), self-insurance and self-protection (Alary et al., 2013; Berger, 2016) or health (Treich, 2010; Berger et al., 2013), and can explain otherwise unaccounted for empirical facts such as the equity premium puzzle (Collard et al., 2016) or the negative correlation between asset prices and returns (Ju & Miao, 2012). Closer to our model is the theory of the competitive firm à la Sandmo (1971) under ambiguity aversion (Wong, 2015a) and the integration of risk and model uncertainty in Integrated Assessment Models (Millner et al., 2013; Berger et al., 2017). There is also evidence that individuals tend to display ambiguity aversion and especially DAAA, see e.g. Berger & Bosetti (2016) and references therein.

We develop a two-period model to analyze what is fundamentally a fully-fledged dynamic problem. This is sufficient to capture the essence of the two ambiguity aversion induced effects and simplifies the problem at hand in two respects. First, considering more than two periods is technically difficult. For instance, Collard et al. (2016) assume CAAA to simplify Euler equations but this means that solely pessimistic distortions are considered while shifts in levels are abstracted away. Second, another difficulty relates to the incorporation of new information to update beliefs and preferences. This issue is mechanically absent in our two-period model. Millner et al. (2013) opt for two polar exogenous learning scenarios: one where ambiguity resolves after the first period, the other with persistent and unchanged ambiguity throughout. Ju & Miao (2012) and Guerdjikova & Sciubba (2015) consider two similar types of learning structures, one where the true scenario is determined in the first period, another where the ‘hidden’ scenario is a Markov process and cannot never be identified. Alternatively, Gierlinger & Gollier (2017) and Traeger (2014) use a one-step-ahead formulation consisting of nested sets of identical ambiguity structures.

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28When \( \phi \) displays CAAA with \( \phi(x) = e^{-\alpha x} \), Klibanoff et al. (2005) show that, under some conditions, the KMM model approaches the MEU criterion when the ambiguity aversion coefficient \( \alpha \) tends to infinity.

29Note that Klibanoff et al. (2009) are able to retain dynamic consistency by defining preferences recursively, assuming ‘rectangularity’ of subjective beliefs together with prior-by-prior Bayesian updating, but this does not accommodate the dynamic three-color-urn Ellsberg example in Epstein & Schneider (2003).
3 Modelling framework

We consider a polluting firm regulated under a permit market. There are two dates \( t = 1, 2 \). At date 1, the date-2 permit price \( \tau \) and the firm’s date-2 baseline level of emissions \( b \) (or production output) are ambiguous to the firm in a sense that will be defined below. Ambiguity resolves at the beginning of date 2.\(^{30} \) The firm’s date-2 abatement depends on its date-1 abatement as well as on price and baseline realizations. We analyze the firm’s optimal level of date-1 abatement under ambiguity aversion relative to ambiguity neutrality.

The economic environment. Regulation is effective at both dates and terminates at the end of date 2. As in Chevallier et al. \( (2011) \) we assume date-1 compliance is effective and that all inter-firm trading opportunities are exhausted. The firm can still undertake additional date-1 abatement \((a_1 \geq 0)\) in the perspective of more stringent date-2 requirements, which frees up a corresponding amount of permits that are banked into date 2.\(^{31} \) This assumption ensures that the Rubin-Schennach banking condition is always satisfied and assumes corner solutions away \((Rubin, 1996; Schennach, 2000)\). There are two alternative descriptions of this framework where regulation is effective at date 2 only: \( a_1 \) may also correspond to investments in abatement technology in anticipation of future regulation or ‘early reduction permits’ handed out to the firm for its early abatements.\(^{32} \)

Given an abatement stream \((a_1; a_2)\) the firm’s date-2 emission level is \( b - a_1 - a_2 \). Letting \( \omega \) denote the firm’s endowment of permits at date 2, a positive (resp. negative) value for \( b - a_1 - a_2 - \omega \) corresponds to a short (resp. long) market position, i.e. the amount of permits it buys (resp. sells) on the market at date 2. Abatement cost functions are given by twice continuously differentiable functions \( C_1 \) and \( C_2 \). Abatement is said to have long-term effect in the sense that \( C_2 \) also depends on the level of date-1 abatement, i.e. \( C_2 \equiv C_2(a_1, a_2) \).

Therefore, the marginal cost of date-1 abatement is \( \partial a_1(C_1 + C_2) \). Abatement costs are assumed to be strictly increasing and convex on \([0; \infty)\) with no fixed cost, i.e. \( C'_1, C''_1 > 0 \) with \( C_1(0) = C'_1(0) = 0 \) and \( \partial a_2 C_2, \partial^2 a_2 a_2 C_2 > 0 \) with \( C_2(·, 0) = 0 \). The firm also faces decreasing abatement opportunities, i.e. \( \partial^2 a_1 a_2 C_2 \geq 0 \) \((Bréchet & Jouvet, 2008)\). This is compensated by a

\(^{30}\)Learning is perfect and exogenous to the firm because it can readily observe the prevailing market price and its own demand at date 2, and cannot influence the extent of learning by its date-1 actions.

\(^{31}\)In the case of the EUETS presented in the Introduction, date 1 corresponds to Phase II with a non-binding constraint on emissions and date 2 to Phase III and beyond with an expected permit scarcity.

\(^{32}\)These interpretations are equivalent provided that a given level of abatement or investment cuts down emissions by a corresponding amount, and that date-1 abatement or investment reduces both date-1 and date-2 emissions by the same amount. In this respect, note that Slechten \( (2013) \) underlines a ‘partial’ substitutability between banked abatements and low-carbon technology investments.
positive learning-by-doing effect which is captured by assuming that $\frac{\partial^2}{\partial a_1 \partial a_1} (C_1 + C_2) \geq \frac{\partial^2}{\partial a_1 \partial a_2} C_2$ and $\frac{\partial^2}{\partial a_2 \partial a_2} C_2 \geq \frac{\partial^2}{\partial a_1 \partial a_2} C_2$ (Slechten, 2013). When we want to derive analytical results, we will assume that abatement cost functions are equipped with the following quadratic specification, where

$$C_1(a_1) = c_1 a_1^2/2 \quad \text{and} \quad C_2(a_1, a_2) = c_2 a_2^2/2 + \gamma a_1 a_2,$$

for all $a_1, a_2 \geq 0$, with $c_1, c_2 > 0$ and $c_2 > \gamma$ for our assumptions on cost functions to obtain. Note that a quadratic specification is a usual and mild assumption (Newell & Stavins, 2003). Here, this also allows us to single out the effects of ambiguity aversion on optimal abatement streams as it guarantees intertemporal efficiency under ambiguity neutrality (see Proposition 4.3). Note that $1/c_t$ measures the firm’s flexibility in abatement at date $t$ and $\gamma$ denotes the long-term abatement effect coefficient. For tractability, we will sometimes need to assume that there is no long-term effect of abatement, i.e. $\partial a_1 C_2 \equiv 0$ or $\gamma = 0$.

The firm’s objective under uncertainty. We consider a partial equilibrium model and focus solely on the firm’s abatement and permit trading decisions. The model ignores both the interactions with the goods’ market and the firm’s production decisions. Denote by $\zeta_t > 0$ the firm’s net profits on the goods’ market at date $t$ that are independent of the firm’s volume of emissions. To solve for the firm’s optimal abatement stream we proceed in two steps using backward induction. At date 2 the firm observes the realized permit price $\tau$ and individual baseline $b$. Given its date-1 abatement $a_1 \geq 0$, the firm maximizes its date-2 profits

$$\max_{a_2 \geq 0} \pi_2(a_1, a_2; \tau, b) = \zeta_2 - C_2(a_1, a_2) - \tau(b - a_1 - a_2 - \omega). \quad (2)$$

Date-2 optimality requires that $\partial a_2 C_2(a_1, a_2^*) = \tau$, where the optimal date-2 abatement is implicitly defined by $a_2^* \equiv a_2^*(a_1; \tau)$. With cost specification (1) it comes

$$a_2^*(a_1; \tau) = (\tau - \gamma a_1)/c_2. \quad (3)$$

At date 1, however, both the date-2 permit price and baseline emissions are uncertain. Let the price risk $\tilde{\tau}$ be described by the objective cumulative distribution $G^0$ supported on $T = [\underline{\tau}; \bar{\tau}]$ with $0 < \underline{\tau} < \bar{\tau} < \infty$. Let also the baseline risk $\tilde{b}$ be described by the objective cumulative

[33]This is a restrictive yet usual assumption. It can be justified if firms produce different goods and/or belong to different sectors. While an interaction between the goods’ market and environmental policy undoubtedly exists, its direction and magnitude are uncertain. For instance, Martin et al. (2014) show that the UK carbon tax has reduced both energy use and intensity, but find no evidence of impacts on employment or production. See e.g. Requate (1998) for a treatment of the interaction between permit and output markets.
distribution \( L^0 \) with support on \( B = [b; \bar{b}] \) with \( 0 < b < \bar{b} < \infty \). These two risks are assumed to be independent, i.e. there is no connection between the prevailing market price and the firm’s baseline.\(^{34}\) This parallels a frequent assumption in the literature on firms’ decisions under uncertainty that price and production shocks are independent stochastic variables (Viaene & Zilcha, 1998; Dalal & Alghalith, 2009). We consider that the firm is risk neutral. The firm’s date-1 optimal abatement decision thus satisfies

\[
\bar{a}_1 = \arg \max_{a_1 \geq 0} \left\{ \pi_1(a_1) + \beta \mathbb{E}_{G^0, L^0} \left\{ \pi_2(a_1, a_2^*(a_1; \bar{\tau}); \bar{\tau}, \tilde{b}) \right\} \right\}, \tag{4}
\]

where \( \beta \in [0; 1] \) is the firm’s discount factor and \( \pi_1(a_1) = \zeta_1 - C_1(a_1) \) is the date-1 profit (note the absence of trade terms). Combining optimality conditions at both dates yields

\[
C'_1(\bar{a}_1) + \beta \mathbb{E}_{G^0} \{ \partial_{a_1} C_2(\bar{a}_1, a_2^*(\bar{a}_1; \bar{\tau})) \} = \beta \langle \bar{\tau} \rangle = \beta \mathbb{E}_{G^0} \{ \partial_{a_2} C_2(\bar{a}_1, a_2^*(\bar{a}_1; \bar{\tau})) \}, \tag{5}
\]

where \( \langle \bar{\tau} \rangle = \mathbb{E}_{G^0} \{ \bar{\tau} \} \) is the expected permit price. Intertemporal efficiency obtains in expectations since expected marginal abatement costs are equated at both dates. For a price realization \( \tau \in \mathbb{T} \) the abatement stream \( (\bar{a}_1; a_2^*(\bar{a}_1; \tau)) \) coincides with the Rubin-Schennach least discounted cost solution (Rubin, 1996; Schennach, 2000). Note that Equation (5) is independent of both the firm’s baseline risk and permit allocation \( \omega \) and with cost specification (1) it comes

\[
\bar{a}_1 = \frac{(c_2 - \gamma) \beta \langle \bar{\tau} \rangle}{c_1 c_2 - \beta \gamma^2}. \tag{6}
\]

With quadratic cost functions the optimal level of date-1 abatement under uncertainty \( \bar{a}_1 \) is invariant to any mean-preserving spread in \( \bar{\tau} \), cf. Proposition 4.3. It is also clear from Equation (6) that \( \bar{a}_1 \) is solely dictated by the discounted expected date-2 permit price and does not depend on the expected market position at date 2.

**Introduction of ambiguity.** Ambiguity is introduced in the sense of Klibanoff et al. (2005), i.e. the firm is uncertain about \( G^0 \) and \( L^0 \). Formally, the firm is confronted with a set of objective probability measures for both \( \bar{\tau} \) and \( \tilde{b} \) and is uncertain about which of those truly govern the two risks. For each realization \( \theta \) (called \( \theta \)-scenario) of the random variable \( \tilde{\theta} \), let \( G(\cdot; \theta) \) and \( L(\cdot; \theta) \) denote the objective probability measures for \( \bar{\tau}_\theta \) and \( \tilde{b}_\theta \), the

\(^{34}\)The cap stringency is determined by the difference between the aggregate permit demand and the cap. Independence can be justified if the permit market is competitive. Alternatively, it may reflect that «there is a complex interaction between BAU emissions, abatement quantities, and allowance prices» (Hintermann et al., 2016). In Section 4.4 we consider the case of an endogenous price that solely reflects the cap stringency.
\( \theta \)-scenario price and baseline risks, respectively. Ambiguity is represented by a second-order subjective probability distribution for \( \hat{\theta} \) denoted \( F \) with support on \( \Theta = [\theta; \bar{\theta}] \). The measure \( F \) represents the firm’s beliefs about which scenario it feels will materialize. While we consider that \( G \) and \( L \) are second-order dependent across \( \theta \)-scenarios we assume for consistency with the uncertain case that \( G \) and \( L \) are first-order independent given a \( \theta \)-scenario, i.e.

\[
E_{G,L}\{\cdot|\theta\} \equiv E_G\{\cdot|\theta\} E_L\{\cdot|\theta\}.
\]

An ambiguity neutral firm compounds first and second order lotteries, i.e. it is a Savagian expected profit maximizer w.r.t. the compound risk measures \( \bar{G} \equiv E_F\{G(\cdot; \hat{\theta})\} \) and \( \bar{L} \equiv E_F\{L(\cdot; \hat{\theta})\} \). We assume there is no bias in the ambiguity neutral firm’s beliefs, i.e. \( \bar{G} \equiv G^0 \) and \( \bar{L} \equiv L^0 \). That is, an ambiguity neutral firm is not affected by the introduction of ambiguity nor a shift in the level of ambiguity.

**Ambiguity aversion.** Attitudes towards ambiguity originate in the relaxation of the reduction of compound first and second order lotteries. The construction of the firm’s objective can be decomposed into three steps. First, in any given \( \theta \)-scenario the firm computes its expected date-2 profits w.r.t. \( G(\cdot; \theta) \) and \( L(\cdot; \theta) \). Second, each \( \theta \)-scenario first-order expected date-2 profits are transformed by an increasing function \( \phi \). Third, the firm’s second-order expected date-2 profits obtain by taking the expectation of the \( \phi \)-transformed first-order expected date-2 profits w.r.t. \( F \). Ambiguity aversion is characterized by a concave function \( \phi \). As defined in Equation (9), denote by \( V(a_1; \theta) \) the firm’s expected profit at date 2 in scenario \( \theta \in \Theta \) when it abates \( a_1 \) at date 1. Under ambiguity aversion, Jensen’s inequality yields

\[
\phi^{-1}\left( E_F\{\phi(V(a_1; \hat{\theta}))\} \right) \leq E_F\{V(a_1; \hat{\theta})\}.
\]

The left-hand side of Inequality (7) is the date-2 \( \phi \)-certainty equivalent expected profit and the right-hand side corresponds to ambiguity neutrality (\( \phi \) is linear) since expectations is taken w.r.t. compound probability distributions. In words, the ambiguity averse firm dislikes any mean-preserving spread in the space of second-order expected profits. Finally note that since the firm is taken to be risk neutral the function \( \phi \) actually characterizes aversion towards model uncertainty (Marinacci, 2015).\(^{35}\) Because ambiguity aversion requires stronger aversion towards model uncertainty than towards risk our assumption leads to an overesti-

\(^{35}\)See Guetlein (2016) for comparative static results on risk aversion under smooth ambiguity aversion. That firms are risk neutral is a standard assumption as they should be able to diversify risk. Firms can still exhibit ambiguity aversion which is a different psychological trait. Note that there is empirical evidence of ambiguity aversion for actuaries (Cabantous, 2007). Brunette et al. (2015) also show that individuals are less risk averse but more ambiguity averse in a group than alone. Seeing firms as groups of individuals making joint decisions may help underpin our assumption of a risk-neutral ambiguity-averse firm.
mation of the effects of ambiguity aversion (Berger & Bosetti, 2016).36

The firm’s objective under ambiguity. Note that date-2 optimality and Equation (3) hold irrespective of both the presence of ambiguity and the firm’s attitude towards ambiguity. However the optimal date-1 abatement decision under ambiguity aversion hinges upon the ambiguity level, as perceived from date 1, in conjunction with the degree of ambiguity aversion. We use the recursive smooth ambiguity model of choice of Klibanoff et al. (2009).37

Because ambiguity is resolved at the beginning of date 2 the firm’s program writes

$$\max_{a_1 \geq 0} \pi_1(a_1) + \beta \phi^{-1}\left(\mathbb{E}_F\{\phi(V(a_1; \tilde{\theta}))\}\right),$$

(8)

where the $\theta$-scenario-expected profitability from date-1 abatement $V(a_1; \theta)$ satisfies

$$V(a_1; \theta) = \mathbb{E}_{G,L}\{V(a_1; \theta)\} \text{ with } V(a_1; \theta) = \max_{a_2} \pi_2(a_1, a_2; \tilde{\tau}_\theta, \tilde{b}_\theta),$$

(9)

with $\mathbb{E}_F$ denoting expectation taken w.r.t. $F$ conditional on all relevant information available to the firm at date 1. Similarly $\mathbb{E}_{G,L}\{\cdot|\theta\}$ denotes expectation taken w.r.t. $G(\cdot; \theta)$ and $L(\cdot; \theta)$ conditional on the true scenario being $\theta$. Notice, Program (8) is well-defined provided that ambiguity tolerance $-\phi'/\phi''$ is concave (Berger, 2016; Gierlinger & Gollier, 2017). Because this condition is satisfied for the $\phi$ functions we use for numerical simulations in Section 5 we assume that it holds throughout. By the Envelop Theorem applied to $\tilde{V}$, it comes, for all $a_1 \geq 0$ and $\theta \in \Theta$

$$\tilde{V}_{a_1}(a_1; \theta) = \bar{\tau}_\theta - \partial_{a_1} C_2(a_1, a_2^*(a_1; \bar{\tau}_\theta)).$$

(10)

With cost specification (1) the $\theta$-scenario-expected marginal profitability from date-1 abatement reads

$$V_{a_1}(a_1; \theta) = ((c_2 - \gamma)\bar{\tau}_\theta + \gamma^2 a_1)/c_2,$$

(11)

where $\bar{\tau}_\theta = \mathbb{E}_G\{\tilde{\tau}_\theta|\theta\}$ denotes the expected permit price at date 2 in scenario $\theta \in \Theta$. By construction $V_{a_1}$ is positive, i.e. the firm always have an incentive to bank at date 1.

36However this assumption allows us to derive clear analytical results. If the firm was risk averse and maximized the utility of its profits at each date, joint conditions on both the utility and ambiguity functions would emerge to determine the direction of the date-1 abatement adjustment, see e.g. Gierlinger & Gollier (2017), Berger (2016) and Wong (2015a) and the criterion to sign pessimism in Proposition 4.4 would have to be restated. In our case this also renders the firm’s optimization program ill-defined.

37Note that in Klibanoff et al. (2009) the scenario space $\Theta$ is finite. Here we consider its continuous extension with a continuous subjective distribution $F$. Note also that the KMM axiomatization is based on acts rather than probability distribution on the outcome spaces $T$ and $B$. 

15
4 Abatement decisions under ambiguity aversion

4.1 Tax regime: Cap and trade under firm’s baseline ambiguity

First consider that $\theta \partial G(\cdot; \theta) \equiv 0$. Since the firm is risk neutral the situation can be assimilated to a tax regime where the date-2 proportional tax rate on emissions is $\mu \equiv E\bar{G}\{\tilde{\tau}\}$. In this case, $\omega$ can be interpreted as a tax-threshold liability where the tariff is charged only on the difference between emissions and the threshold (Pezzey & Jotzo, 2013). Note that the $\theta$-scenario-expected marginal profitability from date-1 abatement satisfies $V_{a_1}(a_1; \theta) = \mu - \partial a_1 C_2(a_1, a_2^*(a_1; \mu)) > 0$ where both $\mu$ and $\partial a_1 C_2$ are deterministic. Hence $V_{a_1}$ is deterministic and does not depend on the $\theta$-scenario considered.

**Ambiguity neutrality.** With $\phi$ linear the necessary first-order condition for Program (8) defines the optimal level of date-1 abatement under ambiguity neutrality by

$$- C_1'(\bar{a}_1^\mu) + \beta V_{a_1}(\bar{a}_1^\mu) = 0. \quad (12)$$

Combining optimality conditions at both dates then yields

$$C_1'(\bar{a}_1^\mu) + \beta \partial a_1 C_2(\bar{a}_1^\mu, a_2^*(\bar{a}_1^\mu; \mu)) = \beta \mu = \beta \partial a_2 C_2(\bar{a}_1^\mu, a_2^*(\bar{a}_1^\mu; \mu)). \quad (13)$$

The (aggregate) marginal date-1 abatement cost is equated to the marginal date-2 abatement cost, i.e. intertemporal efficiency obtains. With cost specification (1) the optimal abatement stream is $(\bar{a}_1^\mu; a_2^*(\bar{a}_1^\mu; \mu))$, where $\bar{a}_1^\mu$ obtains from Equation (6) with $\mu = \langle \tilde{\tau} \rangle$. With no long-term dependency (i.e., $\gamma = 0$), $a_2^*$ is independent of $a_1$ and the firm’s overall level of abatement under ambiguity neutrality is $\bar{a}_1^\mu + a_2^*(\mu) = \beta \mu / c$, where $1/c = 1/c_1 + 1/(\beta c_2)$ is the firm’s aggregate flexibility in abatement over the two dates, and is efficiently apportioned between the two dates

$$\bar{a}_1^\mu = \frac{c}{\beta c_1} \left( \frac{\beta \mu}{c} \right) \quad \text{and} \quad a_2^*(\mu) = \frac{c}{\beta c_2} \left( \frac{\beta \mu}{c} \right), \quad (14)$$

that is in proportion to each date abatement flexibility. The ambiguity neutral benchmark corresponds to a decision under risk — here for a risk neutral firm. Under risk aversion, Baldursson & von der Fehr (2004) show that intertemporal efficiency continues to hold in a tax regime. As exposed below, however, this does not carry over to ambiguity aversion.

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38 The date-2 tax rate is thus certain and exogenously given. The tax regime is such that the date-1 tax rate is zero. This is without loss of generality and roughly captures that tax rates generally rise over time.
Ambiguity Aversion. With $\phi$ concave the necessary first-order condition for Program (8) defines the optimal level of date-1 abatement under ambiguity aversion by

$$- C'_1(\hat{a}_1^\mu) + \beta \mathcal{A}((\hat{a}_1^\mu)\nu_{a_1}(\hat{a}_1^\mu)) = 0, \quad (15)$$

where the shift in levels $\mathcal{A}$ is a function defined by

$$\mathcal{A}(a_1) \doteq \frac{\mathbb{E}_F\{\phi'(\nu(a_1;\tilde{\theta}))\}}{\phi' \circ \phi^{-1}(\mathbb{E}_F\{\phi(\nu(a_1;\theta)))\}}. \quad (16)$$

Proposition 4.1 characterizes the impact of ambiguity aversion on the firm’s optimal date-1 abatement decision relative to ambiguity neutrality.

**Proposition 4.1.** Ambiguity aversion is conducive to higher (resp. lower) date-1 abatement than under ambiguity neutrality if, and only if, the firm displays Decreasing (resp. Increasing) Absolute Ambiguity Aversion. Under Constant Absolute Ambiguity Aversion, the introduction of ambiguity aversion does not affect the firm’s date-1 abatement decision.

*Proof.* Relegated to Appendix A.1.

Except when the firm displays CAAA the tax regime is not intertemporally efficient under ambiguity aversion. This suggests that the relative merits of price versus quantity instruments showed by Baldursson & von der Fehr (2004) under risk aversion would tend to fade away under ambiguity aversion. Moreover, Proposition 4.1 is in line with the literature on the formation of precautionary saving under ambiguity aversion, e.g. Osaki & Schlesinger (2014) and Gierlinger & Gollier (2017). Because the firm overabates at date 1 relative ambiguity neutrality i.f.f. it exhibits DAAA, we follow Berger (2014) and Gierlinger & Gollier (2017) in assimilating DAAA with prudence towards ambiguity.\(^{39}\) With this definition,

**Corollary 4.2.** The firm overabates at date 1 relative to ambiguity neutrality, i.e. forms precautionary date-1 abatement, if, and only if, it displays prudence towards ambiguity.

\(^{39}\)We note that this definition is presently unsettled. For instance, Baillon (2017) defines ambiguity prudence by the less demanding condition that $\phi'''$ be positive (DAAA $\Rightarrow$ $\phi''' > 0$). This definition parallels that of risk prudence under Expected Utility and can be defined in terms of lotteries. However, $\phi''' > 0$ is not sufficient to guarantee the formation of precautionary banking with the KMM certainty equivalent representation theorem we use. For instance, adopting an approach similar to Kimball (1990), Osaki & Schlesinger (2014) show that only under DAAA is the ambiguity precautionary premium bigger than the ambiguity premium. DAAA is thus the ‘natural’ definition for ambiguity prudence in our analysis. Finally note that there is empirical evidence for DAAA (Berger & Bosetti, 2016).
Comparing optimality conditions under ambiguity neutrality (Equation (12)) and ambiguity aversion (Equation (15)) we see a shift in the firm’s discount factor from $\beta$ to $\beta A$. A value higher than unity for function $A$ indicates ambiguity prudence and the discount factor is shifted up (resp. down) when the firm exhibits DAAA (resp. IAAA). In words, ambiguity prudence puts relatively more weight on date-2 profits than under ambiguity neutrality — lowering impatience, as it were — which leads to date-1 overabatement.\footnote{Another interpretation is that DAAA intensifies the importance of any date-2 profit risk and can be assimilated to a «preference for an earlier resolution of uncertainty», see e.g. Theorem 4 in Strzalecki (2013).}

### 4.2 Cap and trade under pure permit price ambiguity

Now let $\partial_{\theta}L(\cdot;\theta) \equiv 0$. This corresponds to a cap and trade under pure price ambiguity, i.e. ambiguity is extrinsic to the firm and transmitted via the permit price only.

**Ambiguity neutrality.** With $\phi$ linear the necessary first-order condition for Program (8) defines the optimal level of date-1 abatement under ambiguity neutrality by

\[ -C_1'(\bar{a}_1) + \beta \mathbb{E}_F \{ V_{a_1}(\bar{a}_1; \tilde{\theta}) \} = 0. \] (17)

Since the ambiguity neutral firm compounds lotteries and we assume its beliefs are unbiased, Equation (17) coincides with the first-order condition for Program (4) under uncertainty. Intertemporal efficiency hence obtains in expectations, see Equation (5). As is standard the effects of uncertainty on optimal decisions relate to the third derivative of the profit (in general, utility) function (Leland, 1968; Kimball, 1990).

**Proposition 4.3.** Assume time separability, i.e. $\partial_{a_1} C_2 \equiv 0$. Then, in the face of an increase in risk in the sense of a mean-preserving spread (Rothschild & Stiglitz, 1971), the ambiguity neutral firm overabates at date 1 if, and only if, $C''_2 > 0$.

*Proof.* Relegated to Appendix A.2

Therefore, date-1 overabatement under ambiguity neutrality is conditional on the positivity of the third derivative of the abatement cost function. Note again from Equation (6) that $\bar{a}_1$ does not depend on the expected market position at date 2 and that, with the quadratic abatement costs, it is also invariant to any mean-preserving spread in $\tilde{\tau}$. 
**Ambiguity aversion.** With $\phi$ concave the necessary first-order condition for Program (8) defines the optimal level of date-1 abatement under ambiguity aversion by

$$- C'_1(\hat{a}_1) + \beta \mathbb{E}_F \{ \phi'(\mathcal{V}(\hat{a}_1; \tilde{\theta})) \mathcal{V}_{a_1}(\hat{a}_1; \tilde{\theta}) \} = 0. \quad (18)$$

Normalizing and decomposing the fraction in Equation (18) into two terms yields

$$- C'_1(\hat{a}_1) + \beta \mathcal{A}(\hat{a}_1) \mathbb{E}_F \{ \mathcal{D}(\hat{a}_1; \tilde{\theta}) \mathcal{V}_{a_1}(\hat{a}_1; \tilde{\theta}) \} = 0, \quad (19)$$

where function $\mathcal{A}$ is defined in Equation (16) and $\mathcal{D}$ is a distortion function satisfying, for all $a_1 \geq 0$ and $\theta \in \Theta$,

$$\mathcal{D}(a_1; \theta) = \frac{\phi'(\mathcal{V}(a_1; \theta))}{\mathbb{E}_F \{ \phi'(\mathcal{V}(a_1; \theta)) \}}. \quad (20)$$

In addition to the shift in levels $\mathcal{A}$, ambiguity aversion distorts the subjective prior $F$ via $\mathcal{D}$. By concavity of $\phi$ the distortion function $\mathcal{D}$ overweights those $\theta$-scenarios with low-$\mathcal{V}$ values. This can be interpreted as pessimism in the sense of a monotone likelihood ratio (MLR) deterioration (Gollier, 2011; Gierlinger & Gollier, 2017) and the pessimistically distorted second-order subjective measure $H$ is such that, for all $a_1 \geq 0$ and $\theta \in \Theta$,

$$H(a_1; \theta) = \int_2^\theta \mathcal{D}(a_1; X) dF(X) = \frac{\mathbb{E}_F \{ \phi'(\mathcal{V}(a_1; X)) | X \leq \theta \}}{\mathbb{E}_F \{ \phi'(\mathcal{V}(a_1; \theta)) \}} F(\theta), \quad (21)$$

with $H(.; \theta) = 0, H(.; \tilde{\theta}) = 1$ and $\partial_\theta H(.; \theta) > 0$. By concavity of the objective function,

$$\hat{a}_1 \geq \bar{a}_1 \iff \mathcal{A}(\bar{a}_1) \mathbb{E}_H \{ \mathcal{V}_{a_1}(\bar{a}_1; \tilde{\theta}) \} \geq \mathbb{E}_F \{ \mathcal{V}_{a_1}(\hat{a}_1; \tilde{\theta}) \}. \quad (22)$$

Controlling for the shift in levels $\mathcal{A}$, introducing ambiguity in the ambiguity averse firm’s decision is identical to a shift in the ambiguity neutral firm’s subjective beliefs from $F$ to $H$, where $H$ overemphasizes low-profit $\theta$-scenarios relative to $F$. Intuitively, this will incite the ambiguity averse firm to overabate at date 1 provided that these low-profit scenarios have high marginal profitabilities from date-1 abatement.

**Proposition 4.4.** Under CAAA, pessimism raises date-1 abatement relative to ambiguity neutrality if, and only if, $(\mathcal{V}(\tilde{a}_1; \theta))_\theta$ and $(\mathcal{V}_{a_1}(\tilde{a}_1; \theta))_\theta$ are anticomonotone. Under DAAA (resp. IAAA) ambiguity aversion is conducive to higher (resp. lower) date-1 abatement than under ambiguity neutrality only if anticomonotonicity (resp. comonotonicity) holds.

**Proof.** Relegated to Appendix A.3.
In words, anticomonotonicity requires that low-$\mathcal{V}$ scenarios coincide with high-$\mathcal{V}_{a_1}$ scenarios. Controlling for the shift in levels $\mathcal{A}$, this ensures that the firm overabates at date 1 relative to ambiguity neutrality. Note that similar (anti)comonotonicity criteria obtain with other representation theorems to sign the effects of ambiguity aversion — see Appendix C for MEU and $\alpha$-maxmin preferences. The underlying relation between (anti)comonotonicity and pessimism is further illustrated in Examples 4.5 and 4.6.

**Example 4.5.** Let $\Theta = \{\theta_1, \theta_2\}$, $F = (q, \theta_1; 1 - q, \theta_2)$ with $0 \leq q \leq 1$ and $\phi$ exhibit CAAA ($\mathcal{A} \equiv 1$). Assume that $\mathcal{V}(\cdot; \theta_2) \geq \mathcal{V}(\cdot; \theta_1)$. Pessimism thus overweights scenario $\theta_1$ relative to $\theta_2$, i.e. $H = (\hat{q}, \theta_1; 1 - \hat{q}, \theta_2)$ with $q \leq \hat{q} \leq 1$.

Then, under ambiguity neutrality, date-1 abatement with the subjective prior $H(\hat{a}_1, H)$ is higher than with $F(\hat{a}_1, F)$ i.f.f.

$$\hat{q} \mathcal{V}_{a_1}(\hat{a}_1; \theta_1) + (1 - \hat{q}) \mathcal{V}_{a_1}(\hat{a}_1; \theta_2) \geq q \mathcal{V}_{a_1}(\hat{a}_1; \theta_1) + (1 - q) \mathcal{V}_{a_1}(\hat{a}_1; \theta_2),$$

which is true when anticomonotonicity holds, i.e. $\mathcal{V}_{a_1}(\cdot; \theta_1) \geq \mathcal{V}_{a_1}(\cdot; \theta_2)$.

**Figure 1:** The effect of pessimism under anticomonotonicity (Example 4.6)

Note: $\mathcal{V}(\cdot; \theta)$ and $\mathcal{V}_{a_1}(\cdot; \theta)$ (and thus $\Upsilon(\cdot; \theta)$ and $\Upsilon_{a_1}(\cdot; \theta)$) are anticomonotonic w.r.t. $\theta$-scenarios; $a_{1,i}$ denotes the optimal level of date-1 abatement for a risk neutral firm solely considering scenario $\theta_i$.

**Example 4.6.** Let $\Theta = \{\theta_1, \theta_2, \theta_3\}$ and $\Upsilon(a_{1}; \theta_i)$ denote the net intertemporal expected profit from date-1 abatement $a_1 \geq 0$ in scenario $\theta_i$, i.e. $\Upsilon(a_{1}; \theta_i) = \pi_1(a_1) + \beta \mathcal{V}(a_1; \theta_i)$. Let $\Theta$ be ordered such that $\Upsilon(\cdot; \theta_i)$ is increasing in $i$. Assume that anticomonotonicity holds, i.e. $\Upsilon_{a_1}(\cdot; \theta_i)$ is decreasing in $i$, as depicted in Figure 1 where $a_{1,i}$ is the optimal date-1 abatement in scenario $\theta_i$. Anticomonotonicity implies that $a_{1,i}$ is decreasing with $i$ and that the higher date-1 abatement the narrower the spread in $\Upsilon(\cdot; \theta)$ across $\theta$-scenarios.

41Note that $\hat{q} = 1$ with MEU preferences. This illustrates that a KMM model of choice converges to MEU in the limiting case of infinite ambiguity aversion (Klibanoff et al., 2005).
The ambiguity averse firm dislikes any mean-preserving spread in the space of second-order expected profit. Pessimism thus adjusts date-1 abatement in the direction of a reduced spread in $\mathcal{V}(:, \theta)$ across $\theta$-scenarios. We note that anticomonotonicity is quite demanding a condition – it requires that $\mathcal{V}(:, \theta)$ do not cross between $\theta$-scenarios – and could be relaxed somewhat. It might be sufficient that the discrepancy in $\mathcal{V}(\cdot; \theta)$ across $\theta$-scenarios diminishes with date-1 abatement in some rough sense for pessimism to induce overabatement.\footnote{We could not get there analytically but this is illustrated with numerical simulations in Section 5. In this respect, note that Berger et al. (2017) transform the anticomonotonicity criterion into a ‘convergence effect’ between scenarios. They are able to do so because they use a binary structure, i.e. a good and a bad state, and ambiguity bears solely on the chances that these two states occur.}

Proposition 4.4 also indicates that the two ambiguity aversion induced effects can be aligned or countervailing. To further account for the shift in levels $A$, momentarily assume for clarity that there is no long-term effect of abatement, i.e. $\partial_{a_1} C_2 \equiv 0$. Then, Condition (22) rewrites

$$\hat{a}_1 \geq \bar{a}_1 \Leftrightarrow A(\bar{a}_1)(\langle \tilde{\tau} \rangle + \mathcal{P}(\bar{a}_1)) \geq \langle \tilde{\tau} \rangle,$$

(23)

where $\mathcal{P}$ can be interpreted as a pessimism-only price distortion function satisfying, for all $a_1 \geq 0$,

$$\mathcal{P}(a_1) = \frac{\text{Cov}_\theta \{ \phi'(\mathcal{V}(a_1; \tilde{\theta})); \mathcal{V}_{a_1}(a_1; \tilde{\theta}) \}}{\mathbb{E}_F \{ \phi'(\mathcal{V}(a_1; \tilde{\theta})) \}}.$$

(24)

Note that anticomonotonicity is equivalent to a non-negative $\mathcal{P}$. In other words, the ambiguity averse firm adjusts date-1 abatement upwards (resp. downwards) when its pessimistically-distorted estimate of the date-2 permit price is higher (resp. lower) than under ambiguity neutrality. Additionally, it directly follows from Equation (23) that

**Proposition 4.7.** Let $\partial_{a_1} C_2 \equiv 0$. Then, the following equivalence conditions obtain

(i) When $\phi$ displays CAAA, $\hat{a}_1 \geq \bar{a}_1$ if, and only if, $\mathcal{P}(\bar{a}_1) \geq 0$;

(ii) When $\phi$ displays DAAA, $\hat{a}_1 \geq \bar{a}_1$ if, and only if, $\mathcal{P}(\bar{a}_1) \geq (1 - A(\bar{a}_1)) \langle \tilde{\tau} \rangle / A(\bar{a}_1) < 0$;

(iii) When $\phi$ displays IAAA, $\hat{a}_1 \leq \bar{a}_1$ if, and only if, $\mathcal{P}(\bar{a}_1) \leq (1 - A(\bar{a}_1)) \langle \tilde{\tau} \rangle / A(\bar{a}_1) > 0$.

Proposition 4.7 characterizes the conditions about the relative strengths and directions of the pessimistic price distortion $\mathcal{P}$ and the shift in levels $A$ in determining the direction of the date-1 abatement adjustment under ambiguity aversion. Note that these two effects can be aligned or countervailing. For an instance of the latter case, let $\phi$ display DAAA and assume that $\mathcal{P}(\bar{a}_1) \in [(1 - A(\bar{a}_1)) \langle \tilde{\tau} \rangle / A(\bar{a}_1); 0]$. Anticomonotonicity does not hold and pessimism only would lead to date-1 underabatement. However the upward shift in the firm’s discount factor is large enough for precautionary date-1 abatement to form overall.
Figure 2: Joint effects of pessimism and shift in levels under DAAA

(a) Aligned effects
(b) Countervailing effects

Note: \( \Theta = \{\theta_1, \theta_2\}, \mathcal{V}_{a_1}(\cdot; \theta_1) \geq \mathcal{V}_{a_1}(\cdot; \theta_2) \) and \( F = (.5, \theta_1; .5, \theta_2) \). Fig. 2a: anticomonotonicity holds so that \( H \) overweights \( \theta_1 \) relative to \( \theta_2 \) as compared to \( F \), and the two effects are aligned. Fig. 2b: comonotonicity holds so that \( H \) overweights \( \theta_2 \) relative to \( \theta_1 \) relative to \( F \), and the two effects are countervailing. In this case, the shift in levels dominates pessimism in terms of adjustment magnitude.

Figure 2 graphically depicts the joint effects of pessimism and shift in levels under DAAA where for clarity we let \( H(\cdot; \theta) \) and \( A \) be constant functions of date-1 abatement — see Appendix D when they are allowed to vary. Note that Figure 2 separates the pessimism effect \((\bar{a}_1 = \bar{a}_{1,F} \rightarrow \bar{a}_{1,H})\) from the shift in levels \((\bar{a}_{1,H} \rightarrow \hat{a}_1)\) in terms of date-1 abatement adjustment. Pessimism operates a vertical translation of the \( F \)-averaged expected marginal profitability from date-1 abatement within the \( \mathcal{V}_{a_1}(\cdot; \theta_2) - \mathcal{V}_{a_1}(\cdot; \theta_1) \) band in the direction of the lower \( \mathcal{V} \)-value scenario. The DAAA-induced shift in levels then increases the slope of the \( H \)-averaged expected marginal profitability from date-1 abatement.

While the anticomonotonicity criterion is intuitively appealing, it also lacks some concreteness. Proposition 4.8 provides more tangible conditions under which this criterion holds.

**Proposition 4.8.** Let \( \phi \) exhibit CAAA and abatement costs be quadratic as in (1). Then, the ambiguity averse firm overabates at date 1 relative to ambiguity neutrality if, and only if

(i) it expects to be in a net short position at date 2 under the abatement stream \((\bar{a}_1; a_2^*(\bar{a}_1; \tau^*_\theta))\) in all \( \theta \)-scenarios where \( \tau^*_\theta \doteq \int x \partial_{\theta} G(x; \theta) dx / \int \partial_{\theta} G(x; \theta) dx \);

(ii) for a given date-2 permit allocation \( \omega \), it abates too little at date 1 under ambiguity neutrality \( \bar{a}_1 \leq \min_\theta \mathbb{E}_C \{ a_1(\theta) = b - \omega - a_2^*(\bar{a}_1; \tau^*_\theta) \} \) with \( b = \mathbb{E}_L \{ \tilde{b} \} \), or reciprocally,

(iii) its date-2 allocation is relatively small \( \omega \leq \omega^* \doteq \min_\theta \mathbb{E}_C \{ \omega^*_\theta = b - \bar{a}_1 - a_2^*(\bar{a}_1; \tau^*_\theta) \} \).

**Proof.** Relegated to Appendix A.4. \( \square \)
Proposition 4.8 shows that pessimism can alternatively lead to overabatement or underabatement at date 1 relative to ambiguity neutrality, which depends on the expected market position at date 2. This ultimately relates to the firm’s endowment of permits, which is thus non neutral under ambiguity aversion. By contrast, the optimal level of date-1 abatement under ambiguity neutrality is solely driven by the $\tilde{G}$-expected permit price. While we can intuitively appreciate that the level of date-1 abatement under ambiguity aversion should be decreasing with permit allocation, Appendix A.8 shows that no clear results of comparative statics obtain. Section 5 will confirm this in a numerical example.

Notice, date-1 overabatement occurs only in those ‘unfavorable’ situations where the firm expects to be a net buyer of permits under abatement streams $(\bar{a}_1; \bar{a}^*_2(\bar{a}_1; \tau^*_\theta))$ in all $\theta$-scenarios. In these situations pessimism overweights those $\theta$-scenarios in which high future prices are relatively more likely, which, in turn, inflates the firm’s own estimate of the future price and thus leads to overabatement. Put otherwise, evaluated at $a_1 = \bar{a}_1$, the marginal benefit of date-1 abatement (i.e., a lowering of the likelihood of effectively being net short and of the volume of permit purchases) outweighs the marginal cost of date-1 abatement for sure. Symmetrically, in those ‘favorable’ situations where the firm expects to be net long in all $\theta$-scenarios pessimism overemphasizes low-price $\theta$-scenarios, which leads to a smaller price estimation than in the benchmark and in turn to underabatement. Otherwise, when the firm is net long in some $\theta$-scenarios and net short in others we cannot conclude a priori.

Anticomonotonicity translates into threshold criteria on initial conditions, i.e. $\bar{a}_1$ or $\omega$. Similarly, Berger (2016) obtains threshold conditions in translating anticomonotonicity in the case of self-insurance and self-protection under ambiguity aversion, in the specific case where ambiguity is concentrated on one state. Note that pessimism acts in line with a ‘two-sided’ precautionary principle. Indeed, if date-2 permit allocation is sufficiently low (resp. high) for the firm to expect to be net short (resp. long) in all $\theta$-scenarios $\omega < \omega^*$ (resp. $\omega > \omega^*$) then the pessimistic firm will overabate (resp. underabate) at date 1.

Corollary 4.9. Under DAAA, conditions (i-iii) in Proposition 4.8 are sufficient, but not necessary, for ambiguity aversion to raise date-1 abatement relative to neutrality.

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43Clear comparative statics results under ambiguity aversion are hard to come by. One difficulty is that $H$ and $A$ values are endogenous to the optimization program, which ultimately hinges upon initial conditions.
44Note the parallel between permit banking and self-insurance or self-protection. Banking is costly but reduces the likelihood of being in a net short position at date 2 (self-protection) and, for a given date-2 net position, it increases date-2 profits by either increasing sales or reducing purchases of permits (self-insurance).
45Again, this suggests that anticomonotonicity might actually be too strong a criterion to sign pessimism.
46The characterization of the cut-off allocation volume $\omega^*$ will be refined in Section 4.4. Appendix F shows that when price ambiguity is binary the conditions to sign pessimism are milder although not unequivocal as is the case in Snow (2011), Alary et al. (2013), Wong (2015a) and Berger (2016).
In other words, ambiguity prudence (DAAA) is in line with a ‘one-sided’ precautionary principle whereby a sufficiently high allocation \((\omega > \omega^*)\) does not guarantee that there is underabatement while overabatement always occurs when allocation is low enough \((\omega < \omega^*)\).

**Increase in ambiguity aversion.** In the sense of Klibanoff et al. (2005) firm 2 is more ambiguity averse than firm 1 if firm 2’s ambiguity function \(\phi_2\) writes as an increasing, concave transformation of that of firm 1, \(\phi_1\). Denote by \(A_i\), \(D_i\) and \(\hat{a}_i\) firm \(i\)’s shift in levels, distortion function and optimal date-1 abatement level. By concavity of the firms’ objective functions

\[
\hat{a}_2 \geq \hat{a}_1 \Leftrightarrow A_2(\hat{a}_1)\mathbb{E}_F\{D_2(\hat{a}_1; \tilde{\theta})V_{a_1}(\hat{a}_1; \tilde{\theta})\} \geq A_1(\hat{a}_1)\mathbb{E}_F\{D_1(\hat{a}_1; \tilde{\theta})V_{a_1}(\hat{a}_1; \tilde{\theta})\}. \tag{25}
\]

Proposition 4.10 separates out two effects consecutive to an increase in ambiguity aversion.

**Proposition 4.10.** Consider two ambiguity averse firms 1 and 2 and assume that there exists a function \(\psi\) such that \(\phi_2 = \psi \circ \phi_1\) with \(\psi' > 0\) and \(\psi'' \leq 0\). Then,

\(\text{(i)}\) (Gollier, 2011) firm 2 is more pessimistic than firm 1 in the sense of a monotone likelihood ratio deterioration;

\(\text{(ii)}\) assuming that \(\psi\) is almost quadratic (i.e., \(\psi''' \simeq 0\)) a necessary condition for a larger upward shift in levels for firm 2 is that firm 1’s ambiguity prudence is not too strong relative to ambiguity aversion, that is \(-\phi_1''/\phi_1' \leq -\phi_2''/\phi_2' \leq -3\phi_1''/\phi_1'\).

**Proof.** Relegated to Appendix A.5.

First, point (i) states that an increase in ambiguity aversion induces an increase in pessimism, i.e. a relatively more concave \(\phi_2\) places relatively more weight on those low-profit scenarios than \(\phi_1\). Therefore, assuming CAAA on the part of both firms, an increase in ambiguity aversion is always conducive to a larger adjustment date-1 abatement (in absolute terms). Second, point (ii) only provides a necessary condition regarding the direction of the shift in levels because it is difficult to characterize when \(A_2\) is uniformly larger than \(A_1\). Moreover, \(\psi\) must be equipped with an additional property and we impose the simplest one, namely \(\psi''' = 0\). In words, point (ii) states that when the ambiguity prudence effect for firm 1 is already relatively strong, increasing ambiguity aversion might not further increase the upward shift in levels (that of firm 2). Note that Guerdjikova & Sciubba (2015) also find a cut-off condition on the strength of ambiguity prudence in a market survival context.\(^{47}\) This result motivates further work on higher-order ambiguity prudence (Baillon, 2017).

\(^{47}\)Consider a market populated by both ambiguity neutral (i.e., SEU-maximizers) and ambiguity averse agents. The latter tend to disappear with time because they form ‘wrong beliefs’ as compared to SEU-
4.3 Cap and trade under price and firm’s baseline ambiguities

This section considers the case where the two first-order independent ambiguities on the firm’s baseline and the market permit price are simultaneously present. Note that the baseline ambiguity can be interpreted as a multiplicative background risk.

Proposition 4.11. Let \( \phi \) exhibit CAAA. Then, the ambiguity averse firm overabates at date 1 relative to ambiguity neutrality if, and only if, its date-2 permit allocation is relatively small
\[
\omega \leq \min_{\theta \in \Theta} \left( \bar{b}_\theta - \bar{a}_1 - a_2^*(\bar{a}_1; \tau_\theta^*) \right) \text{ and } \text{Cov}_\theta \{ G, L \} \geq 0.
\]


The first difference with Proposition 4.8 relates to the definition of the allocation threshold which now comprises \( \bar{b}_\theta = E_L \{ \tilde{b}_\theta | \theta \} \), the \( \theta \)-scenario expected baseline. The second difference is the additional covariance criterion. It states that \( \theta \) must rank \( G(\cdot; \theta) \) and \( L(\cdot; \theta) \) in the same order in the sense of first-order stochastic dominance. In words, pessimism triggers overabatement when allocation is low enough, i.e. the firm expects to be net short, and those \( \theta \)-scenarios where high prices are more likely coincide with those \( \theta \)-scenarios where high firm-level demand for permits is more likely. Symmetrically, pessimism triggers underabatement when allocation is high enough, i.e. the firm expects to be net long, and high-price scenarios coincide with low firm-level demand scenarios. When neither of the above holds it is difficult to determine a clear-cut condition to sign pessimism for sure.\(^{48}\)

4.4 Cap and trade under pure market-wide baseline ambiguity

Consider a continuum \( S \) of infinitesimally small and competitive firms indexed by \( s \). The mass of firms is \( S \). All firms have the same abatement technology \((C_1, C_2)\), subjective beliefs \( F \) and ambiguity functions \( \phi \).\(^{49}\) Therefore, firms are identical but for their initial allocation \( \omega(s) \) which is one key determinant of the date-1 abatement adjustment under ambiguity aversion. Firms are subject to individual baseline ambiguity and we consider that the market-wide ambiguity on firms’ baselines is the sole determining factor of the permit price ambiguity which endogenously emerges on the market.

\(^{48}\)Appendix E contains numerical simulations with joint market price and firm’s demand ambiguities.

\(^{49}\)It is difficult to define the market equilibrium when firms have heterogeneous attitudes towards ambiguity and subjective beliefs, see e.g. Danan et al. (2016). In Appendix B we consider the case of a permit market populated by a mix of (equally) ambiguity averse and neutral firms.
To be able to derive clear analytical results we let abatement cost functions be time separable
and the $\theta$-scenario firm-level baseline uncertainty $\tilde{b}_\theta(s)$ be equipped with a specific structure
such that for all $\theta \in \Theta$ and $s \in S$, $\tilde{b}_\theta(s) = \bar{b}_\theta + \tilde{\epsilon}_\theta(s)$.\(^{50}\) That is, individual baselines
comprise a first term $\bar{b}_\theta$ common to all firms but specific to any given $\theta$-scenario, and an
idiosyncratic term $\tilde{\epsilon}_\theta(s)$ such that for all $\theta \in \Theta$, $(\tilde{\epsilon}_\theta(s))_{s \in S}$ are i.i.d. with $\mathbb{E}_L \{\tilde{\epsilon}_\theta(s)|\theta\} = 0$
and finite variance. Now fix a $\theta$-scenario. By the Law of Large Numbers for a continuum of
i.i.d. variables the $\theta$-scenario aggregate level of baseline emissions level is deterministic and
given by
\[
\int_S \tilde{b}_\theta(s)ds = \int_S \bar{b}_\theta ds + \int_S \tilde{\epsilon}_\theta(s)ds = S\bar{b}_\theta.
\]
(26)

Fix an aggregate emission cap $\Omega$ and an allocation profile $(\omega(s))_{s \in S}$. Date-2 optimality
requires that all firms abate up to the realized market price $\tau$ whatever their baseline real-
izations, i.e. $C'_2(a_2^*\theta(A_1;\bar{b}_\theta)) = \tau$ for all $s \in S$. All firms abate by the same amount $a_2^* = a_2^*(s)$, for
all $s \in S$. Date-2 market closure in the considered $\theta$-scenario thus yields
\[
\int_S (\tilde{b}_\theta(s) - a_1(s) - a_2(\theta) - \omega(s))ds = 0 \Rightarrow a_2^*(A_1;\bar{b}_\theta) = \bar{b}_\theta - \frac{A_1 + \Omega}{S},
\]
(27)

where $A_1$ is the aggregate date-1 abatement volume carried into date 2. The resulting $\theta$-
scenario permit price is $\tau_\theta = C'_2(a_2^*(A_1;\bar{b}_\theta)) > 0$ and thus deterministic. Noting that individual
date-1 abatement decisions have no influence on the date-2 permit price, i.e. $\partial_{a_1}\tau_\theta = 0$, it
follows that for all $a_1 \geq 0$ and $\theta \in \Theta$, $\mathcal{V}_{a_1}(a_1;\theta) = \tau_\theta$. Denote by $\Psi(s;\theta) \equiv \bar{a}_1 + a_2^*(\bar{A}_1;\bar{b}_\theta) + \omega(s) - \bar{b}_\theta$ firm $s$’ expected net position on the market in scenario $\theta$ under ambiguity neutral-
ity. Proposition 4.12 refines the cut-off condition for the formation of precautionary date-1
abatement under ambiguity aversion.

**Proposition 4.12.** Let $\partial_{a_1}C_2 \equiv 0$. Pessimism will incite firm $s \in S$ to overabate at date 1
provided that $\min_{\theta \in \Theta} \Psi(s;\theta) < C'_2(a_2^*(\bar{A}_1;\bar{b}_\theta))/C''_2(a_2^*(\bar{A}_1;\bar{b}_\theta))$. This is always the case under
symmetric allocation of permits.

**Proof.** Relegated to Appendix A.7.

First, given that firms are identical, symmetric allocation of permits coincides with grandfa-
thering, in which case sole pessimism triggers date-1 overabatement. Controlling for shifts in
levels, this result is suggestive of a general behavioral tendency towards precautionary permit

\(^{50}\)With long-term dependency our results carry over if we suppose symmetric allocation of permits. This
ensures that all firms abate the same at both dates. However no trade occurs in equilibrium since firms are
identical along all relevant dimensions.
banking which can contribute to the observed formation of permit surpluses in existing ETSs. Second, note that the anticomonotonicity criterion is laxer than under pure price ambiguity since net long positions under abatement streams \((\bar{a}_1; a^*_2(\bar{A}_1; \bar{b}_0))\) can be sufficient to trigger date-1 overabatement (provided that these positions are not too big).

5 Numerical illustration

For clarity we ignore long-term effects of abatement \((\gamma = 0)\) and assume the firm has the same abatement technology at both dates which we normalize to unity, i.e. \(c_1 = c_2 = 1\) and \(\beta = 1\). When the firm exhibits CAAA (resp. DAAA) we take \(\phi(x) = \frac{e^{-\alpha x}}{\alpha}\) (resp. \(\phi(x) = \frac{x^{1-\alpha}}{1-\alpha}\)) where \(\alpha > 0\) (resp. \(\alpha > 1\)) is the coefficient of absolute ambiguity aversion. If \(\hat{a}^\alpha_1\) denotes the optimal date-1 abatement when the degree of ambiguity aversion is \(\alpha\), it solves the implicit equation

\[
\hat{a}^\alpha_1 = A(\hat{a}^\alpha_1)(\langle \bar{\tau} \rangle + \mathcal{P}(\hat{a}^\alpha_1)).
\]

By extension, let \(\hat{a}_1^\infty\) denote the optimal date-1 abatement with the MEU representation theorem and \(\bar{a}_1 = \bar{a}_1\) under CAAA. Similarly \(\hat{a}_1^1 = \bar{a}_1\) under DAAA.

We take a discrete scenario space \(\Theta = [\bar{\theta}; \bar{\theta}]\) and assume that \(F\) is uniform over \(\Theta\). For all scenario \(\theta \in \Theta\), \(G(\cdot; \theta)\) is uniform over \(T_{\theta} = [\bar{\tau} + \theta; \bar{\tau} + \theta]\) where \(0 < \bar{\theta} < \bar{\tau}\) and \(\bar{\tau} > \bar{\tau} > 0\). Similarly, \(L(\cdot; \theta)\) is uniform over \(B_{\theta} = [\bar{b} + \theta; \bar{b} + \theta]\) where \(0 < \bar{\theta} < \bar{b}\) and \(\bar{\theta} < \bar{b} > 0\). The parameters are set such that \(\bar{\tau} = 10\), \(\bar{\tau} = 30\), \(\bar{b} = 50\), \(\bar{b} = 150\) and \(\bar{\theta} = 9\). Date-2 permit allocation is such that \(\omega \in [0; 120]\). By construction, for all \(\theta \in \Theta\), \(\bar{\tau}_\theta = \langle \bar{\tau}\rangle + \theta\) and \(\bar{b}_\theta = \langle \bar{b}\rangle + \theta\) where \(\langle \bar{\tau}\rangle = (\bar{\tau} + \bar{\tau})/2 = 20\) and \(\langle \bar{b}\rangle = (\bar{b} + \bar{b})/2 = 100\). Note that \(V_{a_1}(a_1; \theta) = \bar{\tau}_\theta = \langle \bar{\tau}\rangle + \theta\), i.e. the \(\theta\)-scenario expected marginal profitability from date-1 abatement is constant. Below we consider cap-and-trade regimes under pure price ambiguity (see Appendix E for joint market price and firm’s demand ambiguities). In this case, anticomonotonicity holds provided that, for all \(\theta \in \Theta\),

\[
\partial_\omega V(a_1; \theta) \leq 0 \iff \omega \leq \langle \bar{b}\rangle - a_1 - \langle \bar{\tau}\rangle - \theta.
\]

Evaluated at \(a_1 = \bar{a}_1 = \langle \bar{\tau}\rangle\), anticomonotonicity holds i.f.f. \(\omega \leq \omega^* = \langle \bar{b}\rangle - 2\langle \bar{\tau}\rangle - \bar{\theta} = 51\). Symmetrically, comonotonicity at \(a_1 = \bar{a}_1\) holds i.f.f. \(\omega \geq \langle \bar{b}\rangle - 2\langle \bar{\tau}\rangle + \bar{\theta} = 69\).

Cap-and-trade regime under CAAA. Equation (28) simplifies to \(\hat{a}_1^\alpha = \langle \bar{\tau}\rangle + \mathcal{P}(\hat{a}_1^\alpha)\). Figure 3a depicts the variations of \(\hat{a}_1^\alpha\) w.r.t. \(\alpha\) and \(\omega\). Since the codomain of the pessimistic price distortion \(\mathcal{P}\) is bounded to \([-\bar{\theta}, \bar{\theta}]\), \(\hat{a}_1^\alpha\) is confined within the range \([\langle \bar{\tau}\rangle - \bar{\theta}; \langle \bar{\tau}\rangle + \bar{\theta}]\).
Figure 3: Cap-and-trade regime under CAAA

(a) \( \hat{a}_1^\alpha = f(\omega) \) for different \( \alpha \)

(b) Optimal abatement variability

The dotted line represents the optimal date-1 abatement under ambiguity neutrality \( \hat{a}_0^0 \) and is independent of the allocation. The solid line characterizes the optimal date-1 abatement level with the MEU representation theorem \( \hat{a}_1^\infty \). It is a step function of the allocation: if \( \omega < \bar{\omega} = 60 \), \( \hat{a}_1^\infty = \langle \tau \rangle + \bar{\theta} \); otherwise, \( \hat{a}_1^\infty = \langle \tau \rangle - \bar{\theta} \). Other curves depict \( \hat{a}_1^\alpha \) for various ambiguity aversion degrees \( \alpha \). First note that the KMM representation describes the continuum between the two polar cases of ambiguity neutrality and MEU. In particular, \( \hat{a}_1^\alpha \) unambiguously decreases with \( \omega \) with a clear threshold \( \bar{\omega} = 60 \) below (resp. above) which overabatement (resp. underabatement) occurs for all ambiguity aversion degrees. It is noteworthy that this condition is laxer than anticomonotonicity since \( \omega^* < \bar{\omega} \).\(^{51}\) Second, for any given permit allocation the magnitude of the date-1 abatement adjustment \( |\hat{a}_1^\alpha - \bar{a}_1| \) increases with \( \alpha \). For instance when \( \omega < \bar{\omega} \), \( \hat{a}_1^\alpha \)-lines are ordered by increasing \( \alpha \) and never cross each other, i.e. an increase in ambiguity aversion always leads to higher date-1 abatement.\(^{52}\) Note also that the bigger \( \alpha \), the more sensitive the variations in \( \hat{a}_1^\alpha \) w.r.t. \( \omega \) around \( \bar{\omega} \). In particular, for \( \alpha = .25 \), \( \hat{a}_1^\alpha \) has already converged to its upper (resp. lower) limit when \( \omega \) reaches 30 (resp. 90). Figure 3b depicts the variability of the date-1 abatement adjustment w.r.t. \( \omega \) for various ambiguity aversion degrees.\(^{53}\) The bigger \( \alpha \), the quicker \( \hat{a}_1^\alpha \) reacts to \( \omega \) in a smaller \( \bar{\omega} \)-centred range. For lower \( \alpha \), the incentive to adjust date-1 abatement is smaller and more evenly spread over the allocation range.

\(^{51}\)Again, this suggests that anticomonotonicity might be too strong a requirement to sign pessimism. From the simulations we can infer that \( \bar{\omega} = \mathbb{E}_F \{ \omega^*_b \} = \langle \tilde{b} \rangle - 2\langle \tau \rangle \). That is, ambiguity aversion raises date-1 abatement relative to ambiguity neutrality i.f.f. anticomonotonicity holds in expectations over \( \Theta \) w.r.t. \( F \).

\(^{52}\)Under CAAA, only point (i) in Proposition 4.10 holds. The effects of an increase in \( \alpha \) are thus clear.

\(^{53}\)Figure 3b plots \( \mathcal{P}(\bar{a}_1) - \mathcal{P}(\hat{a}_1^\alpha) \) as a function of \( \omega \). From Equation (23) and injecting the first-order condition for \( \hat{a}_1^\alpha \), overabatement occurs i.f.f. \( \hat{a}_1^\alpha - \bar{a}_1 + \mathcal{P}(\hat{a}_1) - \mathcal{P}(\hat{a}_1^\alpha) > 0 \). That is, \( \mathcal{P}(\bar{a}_1) - \mathcal{P}(\hat{a}_1^\alpha) \) can be interpreted as a proxy of the incentive to increase \( \hat{a}_1^\alpha \) relative to \( \bar{a}_1 \).
Tax regime under DAAA. Equation (28) rewrites $\hat{a}_1^\alpha = A(\hat{a}_1^\alpha) \langle \tilde{\tau} \rangle$ and $\nu_{a_1}(a_1; \theta) = \langle \tilde{\tau} \rangle$. Figure 4 depicts the variations of $\hat{a}_1$ w.r.t. $\alpha$ and $\omega$ and isolates the effects of the shift in

Figure 4: Tax regime under DAAA

levels $A$. For all $\alpha > 1$, $\hat{a}_1^\alpha$ unambiguously decreases with $\omega$ and is always above $\hat{a}_1^1$. That is, $A$ is a decreasing function of allocation and has steeper variations for smaller $\omega$. Note that for a standard tax regime, i.e. $\omega = 0$, higher ambiguity aversion degrees do not guarantee higher date-1 abatement levels. In particular, there exists a threshold $\bar{\alpha}$ such that $\hat{a}_1^\alpha$ increases (resp. decreases) with $\alpha$ provided that $\alpha$ is below (resp. above) $\bar{\alpha}$. Numerically we find $\bar{\alpha} \approx 11.5$. For $\omega$ high enough, however, note that $\hat{a}_1^\alpha$ is ranked by increasing ambiguity aversion degrees. Note also that the ratio $\hat{a}_1^\alpha/\bar{a}_1 > 1$ is relatively smaller than for a cap and trade under CAAA. This suggests that the magnitude of the shift in levels $A$ is relatively smaller than the pessimistic distortion $P$.

Cap-and-trade regime under DAAA. This case combines the joint effects of $A$ and $P$ and $\hat{a}_1^\alpha$ solves Equation (28). Figure 5 depicts the variations of $\hat{a}_1^\alpha$ w.r.t. $\alpha$ and $\omega$. Figure 5a is similar to Figure 3a save for small disruptions due to the upward shift in level $A$. It is noteworthy that this upward shift is asymmetric w.r.t. allocation. When $\omega > \bar{\omega}$, $\hat{a}_1^\alpha$ is pushed up towards the $\bar{a}_1$-line, although without breaching it, and the lower limit $\langle \tau \rangle - \tilde{\theta}$ is never reached. When $\omega < \bar{\omega}$, date-1 abatement is further adjusted upwards. For relatively low allocation levels the upper limit $\langle \tau \rangle + \tilde{\theta}$ can be exceeded. As in the tax regime, for $\omega$ low enough, higher ambiguity aversion degrees do not guarantee higher date-1 abatements. More precisely, as Figure 5c shows, the magnitude of the $A$-adjustment is more pronounced for low $\alpha$ when $\omega$ is small. Note that the upward shift is relatively smaller when $\omega$ is big enough. Note also that within the $[40; 80]$ band, the $A$-adjustment is very small and ordered
by increasing $\alpha$. That $\hat{a}_1^\alpha$-lines may cross each other when $\omega$ is low enough substantiates Proposition 4.10, i.e. the shift in levels $A$ may disrupt the $P$-adjustment. By contrast, no such crossings exist when $\omega > \bar{\omega}$, i.e. there is an asymmetry in the $A$-adjustment. Relative adjustments in abatement attributable to $A$ and $P$ are illustrated in Figure 5b. It is clear that the $A$-adjustment is more pronounced for lower than bigger $\omega$ and that it is almost nil within the $[40; 80]$ band. Except for low allocation levels, this further suggests that pessimism is the principal determinant of the deviation in date-1 abatement relative to ambiguity neutrality.

6 Conclusion and supplemental results

Summary. Emissions Trading Systems are subject to considerable uncertainty, especially of a regulatory nature, which can disrupt intertemporal cost efficiency and undermine the long-term price signal conveyed by these systems. As a novel approach, this paper has introduced
ambiguity and ambiguity aversion on the part of covered firms as a way to account for the prevalence and influence of such large uncertainty on market functioning. We note that this analysis could apply to other commodity markets under similar circumstances.

Considering exogenous ambiguity about the future permit price and the firm’s permit demand, we analyze the impacts of ambiguity aversion on the firm’s intertemporal decisions relative to the case of ambiguity neutrality (i.e., rational expectations). Ambiguity aversion distorts intertemporal efficiency and induces two effects, namely a pessimistic distortion of the firm’s beliefs and a shift in the firm’s discount factor. These two effects can be aligned or countervailing, the direction and magnitude of which depend on the firm’s degree of ambiguity aversion and its expected future market position. In particular, pessimism leads the firm that expects to be short (resp. long) on the market in the future to overabate (resp. underabate) early on relative to intertemporal efficiency because it overemphasizes detrimental situations where high (resp. low) future permit prices are relatively more likely.

Furthermore, we show that under certain conditions, pessimism creates a general incentive to overabate early on that is more pronounced under auctioning than free allocation. This can be a behavioral factor that contributes to the formation of permit surpluses in existing ETSs that is adding to the ‘physical’ factors discussed in the Introduction. In our setup, the hypothesis of excessive discounting found in the literature to account for the current permit price depreciation in the EUETS coincides with ‘imprudence toward ambiguity’.

As a final note, we underline that price collars (i.e., price-triggered contingent adjustments to the supply of permits) can reduce the range of ambiguity, but not the effects of ambiguity aversion itself.\footnote{In this respect, note that we make a first step toward characterizing the effects of an increase in the range of ambiguity in the simpler case of binary price ambiguity in Appendix F.} However, because agents are forward-looking and will anticipate these adjustments, price collars will affect their intertemporal decisions even though they turn out to be non binding, what Salant et al. (2017) call an ‘action at a distance’. As underlined by these authors, more research is needed to understand better how price containment mechanisms interact with the intricate intertemporal permit trading incentives.

**Supplemental results in Appendix B.** We consider three extensions to the model. First, we introduce forward contracts and show that the possibility to trade forwards can mitigate the magnitude of the pessimism-induced effects. However, only under the assumption that forwards are fairly priced will pessimism completely vanish. Additionally, the shift in the firm’s discount factor always persists. Therefore, this suggests that the introduction of for-
wards cannot (i) restore intertemporal cost efficiency and (ii) render a cap and trade and an emissions tax equivalent under ambiguity aversion on the part of firms.

Second, we show that if permit allocation is sufficiently asymmetrical across firms and firms are ambiguity averse, then the equilibrium volume of trade will be reduced relative to the benchmark.\footnote{This can contribute to what Ellerman (2000) calls ‘autarkic compliance’ in nascent systems where traded volumes are thin and covered entities tend to cling on to their permit endowments. This is currently the case in South Korea where the government recently decided to limit the bankability of permits to force permit holders to sell in a bid to ensure there is enough supply on the market and avoid price spikes.} Third, we show that the equilibrium in a market populated by a mix of ambiguity neutral and averse firms should be brought further away from intertemporal efficiency. Although we cannot properly solve for the resulting equilibrium, we take our approach as a reasonable first pass. This also motivates further work toward the definition of a market equilibrium in which agents have heterogeneous beliefs and attitudes toward ambiguity.

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References


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Appendices & Supplemental Material

A Collected proofs

A.1 Proof of Proposition 4.1

By concavity of the objective function, \( \hat{\alpha}_1 \geq \bar{\alpha}_1 \) i.f.f. 
\[ -C_1'(\bar{\alpha}_1) + \beta A(\bar{\alpha}_1) V_a(\bar{\alpha}_1) \geq 0, \]
which is here equivalent to \( A(\bar{\alpha}_1) \geq 1 \). The proof follows if we establish the following claim:

\[
\text{DAAA (resp. IAAA, CAAA)} \iff E\{\phi'(\cdot)\} \geq \text{(resp.} \leq, =) \phi' \circ \phi^{-1}(E\{\phi(\cdot)\})
\]

\[
\iff A \geq \text{(resp.} \leq, =) 1
\]

Let \( \phi \) be thrice differentiable. An agent is said to display Decreasing Absolute Ambiguity Aversion (DAAA) i.f.f. its Arrow-Pratt coefficient of absolute ambiguity aversion \( -\phi''/\phi' \) is non-increasing. This is the case when \( -\phi'''\phi' + \phi''^2 \leq 0 \) or, upon rearranging, when \( -\phi''/\phi' \geq -\phi''/\phi' \). This is equivalent to \( -\phi' \) being more concave than \( \phi \), i.e. absolute prudence w.r.t. ambiguity exceeds absolute ambiguity aversion. In terms of certainty equivalent this translates into \( \phi^{-1}(E\{\phi(\cdot)\}) \geq (-\phi')^{-1}(-E\{\phi'(\cdot)\}) \). Applying \( -\phi' \) on both sides proves the claim. See also Osaki & Schlesinger (2014) and Guerdjikova & Sciubba (2015) for a proof based on the concepts of ambiguity premium and ambiguity precautionary premium extending similar notions under risk (Pratt, 1964; Kimball, 1990).

A.2 Proof of Proposition 4.3

For a probability measure \( G^i \), define the function \( O^i \) by

\[
0 = -C_1'(\bar{\alpha}_1) + \beta E_{G^i}\{C_2'(a_2^*(\bar{\tau}))\} \equiv O^i(\bar{\alpha}_1), \tag{A.1}
\]

where \( \bar{\alpha}_1 \) is the date-1 optimal abatement when the price risk is distributed according to \( G^i \) and \( a_2^* \) does not depend on \( a_1 \) since we assume time separability. Let the measure \( G^j \) be a mean-preserving spread of \( G^i \), i.e. an increase in risk relative to \( G^i \) in the sense of Rothschild & Stiglitz (1971). Concavity of the firm’s objective function then yields

\[
\bar{\alpha}_1 \geq \bar{\alpha}_1 \iff O^j(\bar{\alpha}_1) \geq O^i(\bar{\alpha}_1) = 0 \iff E_{G^j}\{C_2'(a_2^*(\bar{\tau}))\} \geq E_{G^i}\{C_2'(a_2^*(\bar{\tau}))\}. \tag{A.2}
\]

By Jensen’s inequality, the last inequality in Equation (A.2) holds true i.f.f. \( C_2' \) is convex.
A.3 Proof of Proposition 4.4

By concavity of the objective function, $\hat{a}_1 \geq \bar{a}_1$ is equivalent to

$$
\mathbb{E}_F \{ \phi'(\mathcal{V}(\bar{a}_1; \hat{\theta}))\mathcal{V}_{a_1}(\bar{a}_1; \hat{\theta}) \} \geq \phi' \circ \phi^{-1}(\mathbb{E}_F \{ \phi(\mathcal{V}(\bar{a}_1; \hat{\theta})) \}) \mathbb{E}_F \{ \mathcal{V}_{a_1}(\bar{a}_1; \hat{\theta}) \}. \tag{A.3}
$$

With $\phi$ displays DAAA, a sufficient condition for Inequality (A.3) to hold is

$$
\mathbb{E}_F \{ \phi'(\mathcal{V}(\bar{a}_1; \hat{\theta}))\mathcal{V}_{a_1}(\bar{a}_1; \hat{\theta}) \} \geq \mathbb{E}_F \{ \phi'(\mathcal{V}(\bar{a}_1; \hat{\theta})) \} \mathbb{E}_F \{ \mathcal{V}_{a_1}(\bar{a}_1; \hat{\theta}) \}. \tag{A.4}
$$

This is exactly $\text{Cov}_\theta \{ \phi'(\mathcal{V}(\bar{a}_1; \hat{\theta})) \}; \mathcal{V}_{a_1}(\bar{a}_1; \hat{\theta}) \} \geq 0$. Noting that $\phi'$ is non-increasing concludes. The above holds with equality (resp. reverses) when $\phi$ is CAAA (resp. IAAA).

A.4 Proof of Proposition 4.8

The proof consists in signing $\text{Cov}_\theta \{ \mathcal{V}(\bar{a}_1; \hat{\theta}); \mathcal{V}_{a_1}(\bar{a}_1; \hat{\theta}) \}$ and identifying under which conditions it is non-positive. With the quadratic cost specification (1), for all $\theta \in \Theta$, $\mathcal{V}_{a_1}(\bar{a}_1; \theta)$, $\bar{a}_1$, and $a^*_2$ are given in Equations (11), (6) and (3), respectively. Differentiating $\mathcal{V}_{a_1}(\bar{a}_1; \theta)$ w.r.t. $\theta$ and then integrating by parts yields

$$
\partial_\theta \mathcal{V}_{a_1}(\bar{a}_1; \theta) = \frac{c_2 - \gamma}{c_2} \int_T x \partial_\theta g(x; \theta) dx = \frac{\gamma - c_2}{c_2} \int_T G_\theta(x; \theta) dx, \tag{A.5}
$$

where $G_\theta(\cdot; \theta) \doteq \partial_\theta G(\cdot; \theta)$. Similarly, by the Envelop Theorem and differentiation w.r.t. $\theta$,

$$
\partial_\theta \mathcal{V}(\bar{a}_1; \theta) = -\int_T c_2(\bar{a}_1, a^*_2(\bar{a}_1; x)) + x(b - \bar{a}_1 - a^*_2(\bar{a}_1; x) - \omega) \partial_\theta g(x; \theta) dx
$$

$$
= -\int_T x(b - \omega - \left(1 - \frac{\gamma}{c_2}\right)\bar{a}_1 - \frac{x}{2c_2}) - \frac{\gamma^2 \bar{a}_1}{2c_2} \partial_\theta g(x; \theta) dx
$$

$$
= \int_T (b - \omega - \left(1 - \frac{\gamma}{c_2}\right)\bar{a}_1 - \frac{x}{c_2}) G_\theta(x; \theta) dx, \tag{A.6}
$$

where the third equality obtains by integration by parts. For all $x \in T$, let $k : x \mapsto b - \omega - \left(1 - \frac{\gamma}{c_2}\right)\bar{a}_1 - \frac{x}{c_2}$. We assume that $T < c_2(b - \omega - \bar{a}_1) < \tilde{r}$. Notice that $k$ changes sign over $T$ and by continuity there exists $\tau_0 \in T$ such that $k(\tau_0) = 0$, i.e. $\tau_0 = c_2(b - \omega) - (c_2 - \gamma)\bar{a}_1$. For all $\theta \in \Theta$, let

$$
\Gamma(\tau_0; \theta) \doteq \frac{1}{c_2} \int_T (\tau_0 - x) G_\theta(x; \theta) dx. \tag{A.7}
$$
Differentiating w.r.t. \( \tau_0 \) yields \( \Gamma'_\theta (\tau_0) = \frac{1}{c_2} \int_T G_\theta(x; \theta)dx \). When \( G_\theta > 0, \partial_\theta \Gamma(\cdot; \theta) > 0 \) so that \( \Gamma(T; \theta) < 0 \) and \( \Gamma(T; \theta) > 0 \). Symmetrically, when \( G_\theta < 0, \partial_\theta \Gamma(\cdot; \theta) < 0 \) so that \( \Gamma(T; \theta) > 0 \) and \( \Gamma(T; \theta) < 0 \). In both cases, \( \forall \theta \in \Theta \), by continuity of \( \Gamma(\cdot; \theta) \) there exists a duple \((\tau^*_\theta; a_{1, \theta})\) such that \( \tau^*_\theta = c_2(b - \omega) - (c_2 - \gamma)a_{1, \theta} \) defined by \( \Gamma(\tau^*_\theta; \theta) = 0 \). By definition,

\[
\int_T (\tau^*_\theta - x) G_\theta(x; \theta)dx = 0 \Rightarrow a_{1, \theta} = \frac{c_2}{c_2 - \gamma} \left( b - \omega - \frac{\int_T xG_\theta(x; \theta)dx}{\int_T G_\theta(x; \theta)dx} \right). \tag{A.8}
\]

For a given \( \omega, a_{1, \theta} \) corresponds to the required date-1 abatement in scenario \( \theta \) when the permit price prevailing at date 2 is \( \tau^*_\theta \triangleq \frac{\int_T x\partial_\theta G(x; \theta)dx}{\int_T \partial_\theta G(x; \theta)dx} \), i.e. when date-2 abatement is \( a^*_2(\tilde{a}_1; \tau^*_\theta) \). Two cases arise depending on the monotonicity of \( G \) w.r.t. \( \theta \).

1. \( G_\theta > 0 \): \( \forall \theta \in \Theta, \partial_\theta \mathcal{V}_{a_1}(\tilde{a}_1; \theta) < 0 \) and \( \partial_\theta \mathcal{V}(\tilde{a}_1; \theta) > 0 \) i.f.f. \( x \frac{\partial_\theta}{\partial_\theta} \left( b - \omega - \left( 1 - \frac{c_2}{c_2} \right) \tilde{a}_1 \right) > \tau^*_\theta \), that is i.f.f. \( \tilde{a}_1 < a_{1, \theta} \).

2. \( G_\theta < 0 \): \( \forall \theta \in \Theta, \partial_\theta \mathcal{V}_{a_1}(\tilde{a}_1; \theta) > 0 \) and \( \partial_\theta \mathcal{V}(\tilde{a}_1; \theta) < 0 \) i.f.f. \( x \frac{\partial_\theta}{\partial_\theta} \left( b - \omega - \left( 1 - \frac{c_2}{c_2} \right) \tilde{a}_1 \right) > \tau^*_\theta \), that is i.f.f. \( \tilde{a}_1 < a_{1, \theta} \).

In both cases, \( \tilde{a}_1 > \tilde{a}_1 \) i.f.f. \( \tilde{a}_1 > a_{1, \theta} \) for all \( \theta \in \Theta \), that is i.f.f. \( \tilde{a}_1 < \min_{\theta \in \Theta} a_{1, \theta} \), which proves \((ii)\). Points \((i)\) and \((iii)\) follow straightforwardly.

### A.5 Proof of Proposition 4.10

Assume that \( \mathcal{V}(\cdot; \tilde{\theta}) \) and \( \mathcal{V}_{a_1}(\cdot; \tilde{\theta}) \) are anticomontone, i.e. both firms form precautionary date-1 abatement. For all \( \theta \) in \( \Theta \) it holds that

\[
\frac{\mathcal{D}_2(\tilde{a}_1; \theta)}{\mathcal{D}_1(\tilde{a}_1; \theta)} = \psi' \circ \phi(\mathcal{V}(\tilde{a}_1; \theta)) \frac{\mathbb{E}_F \{ \phi'_1(\mathcal{V}(\tilde{a}_1; \tilde{\theta})) \}}{\mathbb{E}_F \{ \phi'_2(\mathcal{V}(\tilde{a}_1; \tilde{\theta})) \}} \propto \psi' \circ \phi(\mathcal{V}(\tilde{a}_1; \theta)). \tag{A.9}
\]

W.l.o.g. let \( \mathcal{V}(\tilde{a}_1; \theta) \) be non-decreasing in \( \theta \). By definition \( \psi' \circ \phi(\mathcal{V}(\tilde{a}_1; \theta)) \) and thus \( \mathcal{D}_2/\mathcal{D}_1 \) are non-increasing in \( \theta \). That is, firm 2 displays a stronger pessimism than firm 1 in the sense that it overemphasises low-\( \mathcal{V} \) scenarios even further. Since we assume anticomontonotic\nity \( \mathcal{V}_{a_1}(\tilde{a}_1; \theta) \) is non-increasing in \( \theta \). Therefore, it holds that

\[
\mathbb{E}_F \{ \mathcal{D}_2(\tilde{a}_1; \tilde{\theta}) \mathcal{V}_{a_1}(\tilde{a}_1; \tilde{\theta}) \} \geq \mathbb{E}_F \{ \mathcal{D}_1(\tilde{a}_1; \tilde{\theta}) \mathcal{V}_{a_1}(\tilde{a}_1; \tilde{\theta}) \}. \tag{A.10}
\]

Comparing Equations \((25)\) and \((A.10)\), it is always true that \( \tilde{a}_2 \geq \tilde{a}_1 \) provided that \( \mathcal{A}_2(\tilde{a}_1) \geq \mathcal{A}_1(\tilde{a}_1) \). However it is not easy to determine when \( \mathcal{A}_2 \geq \mathcal{A}_1 \). We note that a necessary condition for this to hold is that firm 2’s coefficient of absolute ambiguity prudence be higher
than that of firm 1, i.e. \(-\phi''_2/\phi''_2 \geq -\phi''_1/\phi''_1\). Assuming \(\psi''' = 0\) then yields
\[
\phi''_2 = (\psi'' \circ \phi_1) \phi''_1 + (\psi' \circ \phi_1) \phi'''_1, \text{ and } \phi'''_2 = 3 (\psi'' \circ \phi_1) \phi''_1 + (\psi' \circ \phi_1) \phi'''_1. \tag{A.11}
\]
Noting that \(-\phi'''_2/\phi''_2 \geq -\phi'''_1/\phi''_1\) rewrites \(-\phi'''_1/\phi''_1 \leq -3\phi''_1/\phi'_1\) concludes.

### A.6 Proof of Proposition 4.11

When both the date-2 market price and the firm’s baseline are ambiguous, the \(\theta\)-scenario expected profitability from date-1 abatement writes
\[
\mathcal{V}(a_1; \theta) = \int_B \int_T (\zeta_2 - C_2(a_1, a_2^*(a_1; x)) - x(y - a_1 - a_2^*(a_1; x) - \omega)) g(x; \theta) l(y; \theta) dx dy. \tag{A.12}
\]
With quadratic cost specification (1), because \(G\) and \(L\) are first-order independent, differentiating Equation (A.12) w.r.t. \(\theta\), integrating by parts and evaluating at \(a_1 = \bar{a}_1\) gives
\[
\partial_\theta \mathcal{V}(\bar{a}_1; \theta) = \int_T \left( \bar{b}_\theta - \omega - \left(1 - \frac{\gamma}{c_2} \right) \bar{a}_1 - \frac{x}{c_2} \right) G_\theta(x; \theta) dx + \bar{\tau}_\theta \int_B L_\theta(y; \theta) dy, \tag{A.13}
\]
where \(\bar{b}_\theta \doteq \mathbb{E}_L \{\bar{b}_\theta|\theta\}\) and \(L_\theta(\cdot; \theta) \doteq \partial_\theta L(\cdot; \theta)\). Note that \(\partial_\theta \mathcal{V}_{a_1}(\bar{a}_1; \theta)\) is given by Equation (A.5). By the same token as in Proof A.4 anticomonotonicity holds when \(G_\theta > 0\) (resp. \(G_\theta < 0\)) if the allocation threshold condition is satisfied and \(L_\theta > 0\) (resp. \(L_\theta < 0\)).

### A.7 Proof of Proposition 4.12

Assume quadratic abatement cost specification (1). All ambiguity neutral firms abate by the same amount at date 1 \(\bar{a}_1 = \beta \langle \tau_\theta \rangle / c_1\) where
\[
\langle \tau_\theta \rangle \doteq \mathbb{E}_F \{\tau_\theta\} = c_2 \mathbb{E}_F \{a_2^*(\bar{A}_1; \bar{b}_\theta)\} = c_2 \left( \langle \bar{b} \rangle - (\bar{A}_1 + \Omega)/S \right). \tag{A.14}
\]
Noting that \(\bar{A}_1 = S \bar{a}_1\) then gives
\[
\bar{a}_1 = c(\langle \bar{b} \rangle - \Omega/S)/c_1 \text{ and } a_2^*(\bar{A}_1; \bar{b}_\theta) = \bar{b}_\theta - c(\langle \bar{b} \rangle/c_1 - c\Omega/(\beta c_2 S)). \tag{A.15}
\]
Note that the aggregate emission constraint is satisfied in every \(\theta\)-scenario
\[
\int_S (\bar{b}_\theta(s) - \bar{a}_1 - a_2^*(\bar{A}_1; \bar{b}_\theta)) ds = \Omega. \tag{A.16}
\]
Note also that a positive permit price in each $\theta$-scenario requires that, when $A_1 = \bar{A}_1$,

$$\Omega(c_1 - c) > S \left( c_1 \max_{\hat{\theta} \in \Theta} \bar{b}_{\hat{\theta}} - c(b) \right),$$

(A.17)

which we assume is the case. Let us now sign $\text{Cov}_\theta \{ \mathcal{V}(a_1; \hat{\theta}); \mathcal{V}_{a_1}(a_1; \hat{\theta}) \}$. We have

$$\mathcal{V}(\bar{a}_1; \theta) = \zeta_2 - C_2(a_2^*(\bar{A}_1; \bar{b}_\theta)) - \tau_\theta \left( \bar{b}_\theta - \bar{a}_1 - a_2^*(\bar{A}_1; \bar{b}_\theta) - \omega(s) \right)$$

(A.18a)

$$\partial_\theta \mathcal{V}_{a_1}(\bar{a}_1; \theta) = \partial_\theta \tau_\theta = C_2''(a_2^*(\bar{A}_1; \bar{b}_\theta)) \partial_\theta a_2^*(\bar{A}_1; \bar{b}_\theta) = C_2''(a_2^*(\bar{A}_1; \bar{b}_\theta)) \partial_\theta \bar{b}_\theta,$$

(A.18b)

$$\partial_\theta \mathcal{V}(\bar{a}_1; \theta) = \left( C_2''(a_2^*(\bar{A}_1; \bar{b}_\theta)) \Psi(s; \theta) - C_2''(a_2^*(\bar{A}_1; \bar{b}_\theta)) \right) \partial_\theta \bar{b}_\theta,$$

(A.18c)

since $\partial_\theta \bar{A}_1 = \partial_\theta \bar{a}_1 = 0$ (both $\bar{A}_1$ and $\bar{a}_1$ are decided ex ante) and where $\Psi(s; \theta) \doteq \bar{a}_1 + a_2^*(\bar{A}_1; \bar{b}_\theta) + \omega(s) - \bar{b}_\theta$ is firm $s'$ expected net position on the market in scenario $\theta$ under ambiguity neutrality. Anticomonotonicity holds provided that for all $\theta \in \Theta$, $\Psi(s; \theta) < \frac{C_2''(a_2^*(\bar{A}_1; \bar{b}_\theta))}{C_2''(a_2^*(A_1; b_\theta))}$. Note that this allows a net long (i.e., positive) market position which was not the case under pure price ambiguity. Injecting Equation (A.15) gives $\Psi(s; \theta) = \omega(s) - \frac{\Omega}{S}$ which is nil for a symmetric allocation plan. Therefore, when allocation is symmetric anticomonotonicity holds unconditionally. Assume for simplicity that the ratio of abatement technology between the two dates is unitary, i.e. $c_1 = \beta c_2$. Then,

$$\Psi(s; \theta) < \frac{C_2''(a_2^*(\bar{A}_1; \bar{b}_\theta))}{C_2''(a_2^*(A_1; b_\theta))} \Leftrightarrow \omega(s) < \min_{\theta \in \Theta} \left( \omega_\theta \doteq (\Omega/S + 2\bar{b}_\theta - \langle \bar{b} \rangle)/2 \right),$$

(A.19)

Noting from Equation (A.17) that $\omega_\theta > \Omega/S$ for all $\theta \in \Theta$ concludes.

### A.8 Comparative statics w.r.t. permit allocation

With $\phi$ CAAA and no long-term effect of abatement, Equation (18) rewrites

$$- C_1'(\hat{a}_1) + \beta \mathbb{E}_F \left\{ \phi'(\mathcal{V}(\hat{a}_1; \hat{\theta})) \mathcal{V}_{a_1}(\hat{a}_1; \hat{\theta}) \right\} = 0.$$

(A.20)

Taking the total differential of Equation (A.20) yields

$$\frac{d\hat{a}_1}{d\omega} = \frac{\beta \Phi(\hat{a}_1)}{C_1'(\hat{a}_1) - \beta \Phi(\hat{a}_1)},$$

(A.21)
where, since $V_\omega = V_{a_1} = \tau_\theta$, and omitting arguments so as to avoid cluttering,

$$\Phi(a_1) = \frac{\mathbb{E}_F\{V_1^2\phi''(V)\} \mathbb{E}_F\{\phi'(V)\} - \mathbb{E}_F\{V_1\phi'(V)\} \mathbb{E}_F\{V_1\phi''(V)\}}{\mathbb{E}_F\{\phi'(V)\}^2}.$$  

(A.22)

In particular, note that $\frac{\partial \Phi}{\partial a_1} \in [-1; 0]$ i.f.f. $\Phi(\hat{a}_1) < 0$. We can show that

$$\Phi(\hat{a}_1) \propto \text{Cov}_\theta \{V_{a_1}; V_{a_1}\phi''(V)\} \mathbb{E}_F\{\phi'(V)\} - \text{Cov}_\theta \{V_{a_1}; \phi'(V)\} \mathbb{E}_F\{V_{a_1}\phi''(V)\}$$

$$\propto \mathcal{P}(\hat{a}_1) - \mathcal{P}_2(\hat{a}_1) = \frac{\text{Cov}_\theta \{V_{a_1}; \phi'(V)\}}{\mathbb{E}_F\{\phi'(V)\}} - \frac{\text{Cov}_\theta \{V_{a_1}; \phi''(V)\}}{\mathbb{E}_F\{V_{a_1}\phi''(V)\}},$$  

(A.23)

where $\mathcal{P}$ is the pessimism-only price distortion and $\mathcal{P}_2$ can be interpreted as a second-order pessimism-only price distortion. These two distortions have positive values when anticomonitoricity holds in which case $\Phi(\hat{a}_1) \leq 0$ i.f.f. $\mathcal{P}_2(\hat{a}_1) \geq \mathcal{P}(\hat{a}_1)$. It is difficult to determine the variations of $\hat{a}_1$ w.r.t. $\omega$ because it is hard to sign $\mathcal{P}_2(\hat{a}_1) - \mathcal{P}(\hat{a}_1)$ in general. In line with intuition numerical simulations in Section 5 show that the level of optimal date-1 abatement unambiguously decreases with allocation, with intensities depending on the degree of ambiguity aversion and the allocation volume itself. This would suggest that $\mathcal{P}_2$ is larger than $\mathcal{P}$. Again, this calls for studying higher orders for ambiguity prudence.

### B Model extensions and supplemental results

This appendix first extends our model by allowing for trades of forward contracts. It then analyses the impacts of (i) ambiguity aversion on the equilibrium volume of permit trade; (ii) having a mix of ambiguity averse and neutral firms on the market for permits.

#### Forward trading.

It is natural to investigate to which extent the introduction of a forwards market can diminish the effects of ambiguity aversion and restore intertemporal efficiency. In practice, ETS-liable firms liable have recourse to forward contracts for hedging purposes, e.g. power companies in the EUETS. We consider that firms now have the possibility to trade permits in a forward market at date 1. Let $a_f$ and $p_f$ denote the volume of permits contracted in the forward market and the forward price, respectively. Note that this does not change the optimal abatement decision at date-2. Then, the firm’s recursive program rewrites

$$\max_{a_1 \geq 0, a_f} \left\langle \zeta_1 - C_1(a_1) - p_f a_f + \beta \phi^{-1}\left(\mathbb{E}_F\{\phi(V(a_1, a_f; \tilde{\theta}))\}\right) \right\rangle,$$  

(B.1)
where \( \mathcal{V}(a_1, a_f; \theta) = \mathbb{E}_G \{ \zeta_2 - C_2(a_1, a_f^1(a_1; \tilde{\tau}_\theta)) - \tilde{\tau}_\theta (b - a_1 - a_f - a_f^2(a_1; \tilde{\tau}_\theta) - \omega) | \theta \} \) for all \( \theta \in \Theta \). The two necessary first-order conditions for \( \hat{a}_1 \) and \( \hat{a}_f \) are given by

\[
-C_1'(\hat{a}_1) + \beta \frac{\partial}{\partial a_1} \mathbb{E}_F \{ \phi'(\mathcal{V}(\hat{a}_1, \hat{a}_f; \hat{\theta})) \mathbb{V}_{a_1}(\hat{a}_1, \hat{a}_f; \hat{\theta}) \} = 0, \tag{B.2a}
\]

and

\[
-p_f + \beta \frac{\partial}{\partial a_1} \mathbb{E}_F \{ \phi'(\mathcal{V}(\hat{a}_1, \hat{a}_f; \hat{\theta})) \mathbb{V}_{a_1}(\hat{a}_1, \hat{a}_f; \hat{\theta}) \} = 0. \tag{B.2b}
\]

By the Envelop, \( \mathcal{V}_{a_f}(\hat{a}_1, \hat{a}_f; \theta) = \tilde{\tau}_\theta \geq \mathbb{V}_{a_1}(\hat{a}_1, \hat{a}_f; \theta) = \tilde{\tau}_\theta - \mathbb{E}_G \{ \partial_{a_1} C_2(\hat{a}_1, a_f^2(\hat{a}_1; \tilde{\tau}_\theta)) | \theta \} > 0 \), where \( \tilde{\tau}_\theta = \mathbb{E}_G \{ \tilde{\tau}_\theta | \theta \} \). Thus, absent long-term effect of abatement, intertemporal efficiency in expectations is restored since it holds \( \beta \langle \tilde{\tau} \rangle = C'_1(\hat{a}_1) = p_f = C'_1(\hat{a}_1) \) provided that \( p_f \in \mathcal{T} \)

Assume forward contracts are fairly priced, i.e. the forward price is unbiased \( p_f \equiv \beta \langle \tilde{\tau} \rangle \). For any \( a_1 \geq 0 \), the optimal forward volume \( a_f^*(a_1) \) solves \( \langle \tilde{\tau} \rangle = A(a_1, a_f^*(a_1)) \mathbb{E}_F \{ \mathcal{D}(a_1, a_f^*(a_1); \hat{\theta}) \tau_\theta \} \). Therefore, \( \hat{a}_1 \geq \bar{a}_1 \) holds if, and only if,

\[
\mathbb{E}_G \{ \partial_{a_1} C_2(\bar{a}_1, a_f^*(\bar{a}_1; \tau_\theta)) \} \geq \mathcal{A}(\bar{a}_1, a_f^*(\bar{a}_1)) \mathbb{E}_F \{ \mathcal{D}(\bar{a}_1, a_f^*(\bar{a}_1); \hat{\theta}) \mathbb{E}_G \{ \partial_{a_1} C_2(\bar{a}_1, a_f^*(\bar{a}_1; \tilde{\tau}_\theta)) | \theta \} \}. \tag{B.4}
\]

With quadratic cost specification (1), Inequality (B.4) is equivalent to

\[
\langle \tilde{\tau} \rangle + \gamma (A(\bar{a}_1, a_f^*(\bar{a}_1)) - 1) \bar{a}_1 \geq A(\bar{a}_1, a_f^*(\bar{a}_1)) \mathbb{E}_F \{ \mathcal{D}(\bar{a}_1, a_f^*(\bar{a}_1); \hat{\theta}) \tau_\theta \}, \tag{B.5}
\]

which, under the fair price assumption, is equivalent to \( A(\bar{a}_1, a_f^*(\bar{a}_1)) \geq 1 \). In summary,

**Proposition B.1.** Consecutive to the introduction of forward contracts,

(i) absent long-term effect of abatement, intertemporal efficiency in expectations is restored irrespective of how forward contracts are priced;

(ii) present long-term effect of abatement and assuming that forward contracts are fairly priced, intertemporal efficiency in expectations obtains only under CAAA. In particular, under DAAA (resp. IAAA), date-1 overabatement (resp. underabatement) persists.

Absent long-term effect of abatement, the optimal level of date-1 abatement level does not depend on the underlying ambiguity level nor on the firm's attitude towards ambiguity. This is in line with recent extensions of the separation theorem under smooth ambiguity
aversion (Wong, 2015b, 2016; Osaki et al., 2016).\textsuperscript{56} Present long-term effect of abatement, the introduction of a fairly-priced market for forward contracts only corrects for pessimism but not for the shift in levels $\mathcal{A}$.\textsuperscript{57} As far as date-1 abatement decisions are concerned a cap-and-trade regime with fairly-priced forward contracts is hence akin to a tax regime. When forwards are not priced fairly, however, pessimism does not (completely) vanish.

**Equilibrium volume of trade.** We investigate the impact of ambiguity aversion on the part of firms on the overall volume of trade. Assume CAAA for clarity. Then, when firm $s$ (resp. $l$) is allocated less (resp. more) than $\min_{\theta \in \Theta} \omega^*_\theta$ (resp. $\max_{\theta \in \Theta} \omega^*_\theta$) it expects to be net short (resp. long) in all $\theta$-scenarios under the abatement stream $(\hat{a}_1; a^*_2(\hat{a}_1); \tau^*_\theta)$. That is, $\hat{a}_1(s) \geq \hat{a}_1 \geq \hat{a}_1(l)$. At date 2, all firms equate their date-2 marginal abatement costs $\partial a_2 C_2(a_1; a^*_2)$ to the observed permit price $\tau$. With quadratic cost specification (1), total abatements for the three types of firms rank such that

\[ a^*_2(\hat{a}_1(s); \tau) + \hat{a}_1(s) = (\tau + (c_2 - \gamma)\hat{a}_1(s))/c_2 \geq a^*_2(\hat{a}_1(s); \tau) + \hat{a}_1 \geq a^*_2(\hat{a}_1(s); \tau) + \hat{a}_1(l). \quad (B.6) \]

Since the net buying (resp. selling) firm $s$ (resp. $l$) abates more (resp. less) and buys (resp. sells) less permits on the market than under ambiguity neutrality, one has

**Proposition B.2.** Let permits be non-symmetrically distributed such that at least some firms are endowed with $\omega \notin [\min_{\theta \in \Theta} \omega^*_\theta; \max_{\theta \in \Theta} \omega^*_\theta]$. Then, the equilibrium volume of trade is lower when firms are ambiguity averse than when they are ambiguity neutral.

Similarly, Baldursson \& von der Fehr (2004) find that risk aversion reduces the equilibrium volume of trade relative risk neutrality. Ambiguity and risk aversions may thus contribute to what Ellerman (2000) calls «autarkic compliance» in nascent ETSs, i.e. traded volumes are thin (e.g., presently in the Korean ETS or the Chinese pilots). Because covered entities are waiting for increased price discovery and due to high regulatory uncertainty they tend to hold on to their allocation so that trades are scarce. For instance, the volume of trades (both in spot EUAs and futures) increased steadily over Phase I of the EUETS as uncertainty gradually vanished, see e.g. Chapter 5 in Ellerman et al. (2010).

\textsuperscript{56}In the presence of pure price ambiguity for a risk-averse ambiguity-averse competitive firm, see Wong (2015b). In the presence of price ambiguity and additive background risk for a risk-neutral and ambiguity-averse competitive firm, see Osaki et al. (2016). In the presence of price ambiguity and additive or multiplicative background risk for a risk-averse ambiguity-averse competitive firm, see Wong (2016).

\textsuperscript{57}This contrasts with Wong (2015b), Wong (2016) and Osaki et al. (2016) in that they use the static KMM formulation, hence without the shift in levels $\mathcal{A}$. 

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Different tastes for ambiguity. Consider a permit market populated by both ambiguity averse and neutral firms where \( \varepsilon \in [0; 1] \) denotes the share of ambiguity averse firms. Assume \( \partial_a C_2 = 0 \). For any \( \varepsilon \in (0; 1) \) denote by \( \hat{a}_1^\varepsilon \) and \( \bar{a}_1^\varepsilon \) the optimal date-1 abatement levels for the ambiguity averse and neutral firms, respectively. Suppose also that ambiguity averse firms are allocated \( \omega \leq \min_{\theta \in \Theta} \omega^*_\theta \) so that, in a market that contains either only ambiguity averse or ambiguity neutral firms, optimal date-1 abatement levels satisfy \( \hat{a}_1^\varepsilon = \hat{a}_1 \geq \bar{a}_1 \) and \( \hat{a}_1 = S\hat{a}_1 \geq \bar{A}_1 = S\bar{a}_1 \). For any mix \( \varepsilon \), assume that market closure at date 2 gives the \( \theta \)-scenario permit price by \( \tau^\varepsilon_{\theta} = C_2' \left( \bar{b}_\theta - (\varepsilon \hat{A}_1 + (1 - \varepsilon)\bar{A}_1 + \Omega) / S \right) \). Denoting by \( \bar{\tau}^\varepsilon_{\theta} \) and \( \hat{\tau}^\varepsilon_{\theta} \) the \( \theta \)-scenario permit price when \( \varepsilon = 0 \) and \( \varepsilon = 1 \), respectively, we have \( \bar{\tau}^\varepsilon_{\theta} \leq \tau^\varepsilon_{\theta} \leq \hat{\tau}^\varepsilon_{\theta} \). Symmetrically, when ambiguity averse firms receive a large allocation \( \omega \geq \max_{\theta \in \Theta} \omega^*_\theta \), \( \bar{a}_1 \leq \hat{a}_1 \), we have \( \bar{\tau}^\varepsilon_{\theta} \leq \tau^\varepsilon_{\theta} \leq \hat{\tau}^\varepsilon_{\theta} \). By comparing the necessary first-order conditions for \( \bar{a}_1 \) and \( \hat{a}_1^\varepsilon \) on the one hand, and for \( \hat{a}_1 \) and \( \bar{a}_1^\varepsilon \) on the other hand, the following holds

**Proposition B.3.** Let \( \varepsilon \in (0; 1) \) denote the share of ambiguity averse firms. Then,

(i) when they are allocated \( \omega < \min_{\theta \in \Theta} \omega^*_\theta \), \( \bar{a}_1^\varepsilon < \hat{a}_1 < \bar{a}_1 < \hat{a}_1^\varepsilon \);

(ii) when they are allocated \( \omega > \max_{\theta \in \Theta} \omega^*_\theta \), \( \bar{a}_1^\varepsilon > \hat{a}_1 > \bar{a}_1 > \hat{a}_1^\varepsilon \).

With a mix of ambiguity averse and neutral firms where the former are endowed with a high or low number of permits should thus bring the market further away from intertemporal efficiency. Notice, this also alters abatement decisions of ambiguity neutral firms.

### C MEU preferences & anticomonotonicity

The anticomonotonicity criterion is robust in the sense that it obtains with other models of choice under ambiguity. This appendix considers the \( \alpha \)-maxmin representation theorem (Gilboa & Schmeidler, 1989; Ghirardato et al., 2004). We stick to our interpretation of \( \Theta \) as the set of possible objective probability distributions. The firm thus grants a weight \( \alpha \in [0; 1] \) to the worst \( \theta \)-scenario in \( \Theta \) and the complementary weight to the best \( \theta \)-scenario.

**Proposition C.1.** Let the firm exhibit MEU preferences. The ambiguity averse firm over-abates at date 1 relative to SEU preferences if, and only if, the sequences \( (\mathcal{V} (\bar{a}_1; \theta))_\theta \) and \( (\mathcal{V} (\hat{a}_1; \theta))_\theta \) are anticomonotone, where \( \bar{a}_1 \) denotes the SEU-optimal date 1-abatement.

**Proof.** For the purpose of the proof, let \( \Theta \) be a discrete finite set of cardinality \( k = |\Theta| \) and ordered such that \( \theta_1 \leq \cdots \leq \theta_k \). Let \( (q_i)_{i=1, \ldots, k} \) be the subjective prior where \( q_i \) denotes the...
firm’s subjective probability that the \( \theta_i \)-scenario will materialize and \( \sum_i q_i = 1 \). W.l.o.g. let the sequence \((\mathcal{V}(\bar{a}_1; \theta_i))_i\) be non-decreasing in \( i \). We have

\[
\bar{a}_1 \doteq \arg \max_{a_1 \geq 0} \left\{ \Upsilon_{\text{SEU}}(a_1) = \pi_1(a_1) + \beta \sum_{i=1}^{k} q_i \mathcal{V}(a_1; \theta_i) \right\}. \tag{C.1}
\]

The \( \alpha \)-maxmin objective function reads

\[
\Upsilon_{\alpha}(a_1) = \pi_1(a_1) + \beta \left( \alpha \min_{\theta \in \Theta} \mathcal{V}(a_1; \theta) + (1 - \alpha) \max_{\theta \in \Theta} \mathcal{V}(a_1; \theta) \right) = \pi_1(a_1) + \beta \left( \alpha \mathcal{V}(a_1; \theta_1) + (1 - \alpha) \mathcal{V}(a_1; \theta_k) \right), \tag{C.2}
\]

and let \( \hat{a}_1^\alpha \) be the unique maximizer of \( \Upsilon_{\alpha} \). By concavity of \( \Upsilon_{\alpha} \),

\[
\hat{a}_1^\alpha \geq \bar{a}_1 \iff \alpha \mathcal{V}_{a_1}(\bar{a}_1; \theta_1) + (1 - \alpha) \mathcal{V}_{a_1}(\bar{a}_1; \theta_k) \geq \sum_{i=1}^{k} q_i \mathcal{V}_{a_1}(\bar{a}_1; \theta_i). \tag{C.3}
\]

By virtue of ambiguity aversion it holds \( \Upsilon_{\alpha} \leq \Upsilon_{\text{SEU}} \). That is, for all \( a_1 \geq 0 \),

\[
\alpha \mathcal{V}(a_1; \theta_1) + (1 - \alpha) \mathcal{V}(a_1; \theta_k) \leq \sum_{i=1}^{k} q_i \mathcal{V}(a_1; \theta_i). \tag{C.4}
\]

Rearranging Equation (C.4) gives

\[
(\alpha - q_1) \mathcal{V}(a_1; \theta_1) \leq \sum_{i=2}^{k-1} q_i \mathcal{V}(a_1; \theta_i) + (\alpha + q_k - 1) \mathcal{V}(a_1; \theta_k) \leq \left( \alpha + \sum_{i=2}^{k} q_i - 1 \right) \mathcal{V}(a_1; \theta_k) = (\alpha - q_1) \mathcal{V}(a_1; \theta_k) \tag{C.5}
\]

since \( \mathcal{V}(a_1; \theta_i) \) is non-decreasing in \( i \) and \( \sum_i q_i = 1 \). Since \( \mathcal{V}(\cdot; \theta_k) \geq \mathcal{V}(\cdot; \theta_1) > 0 \), note that \( \alpha \geq q_1 \) is a sufficient condition for \( \Upsilon_{\alpha} \leq \Upsilon_{\text{SEU}} \) to hold. Then,

\[
\hat{a}_1^\alpha \geq \bar{a}_1 \iff (\alpha - q_1) \mathcal{V}_{a_1}(\bar{a}_1; \theta_1) \geq \sum_{i=2}^{k-1} q_i \mathcal{V}_{a_1}(\bar{a}_1; \theta_i) + (\alpha + q_k - 1) \mathcal{V}_{a_1}(\bar{a}_1; \theta_k). \tag{C.6}
\]

Finally note that it is sufficient for Inequality (C.6) this to hold that \( (\mathcal{V}_{a_1}(\bar{a}_1; \theta_i))_i \) be non-increasing in \( i \) since this would guarantee that

\[
\sum_{i=2}^{k-1} q_i \mathcal{V}_{a_1}(\bar{a}_1; \theta_i) + (\alpha + q_k - 1) \mathcal{V}_{a_1}(\bar{a}_1; \theta_k) \geq \left( \alpha + \sum_{i=2}^{k} q_i - 1 \right) \mathcal{V}_{a_1}(\bar{a}_1; \theta_2) = (\alpha - q_1) \mathcal{V}_{a_1}(\bar{a}_1; \theta_2). \tag{C.7}
\]

This concludes the proof. \( \square \)
W.l.o.g. fix $\alpha = 1$, i.e. MEU collapses to Wald’s minimax criterion. An increase in the level of ambiguity correspond to an increase in the cardinality of $\Theta$, say from $|\Theta|$ to $|\Theta'| \geq |\Theta|$. Note that this also corresponds to an increase in the degree of ambiguity aversion since

$$\min_{\theta \in \Theta'} \langle \max_{a_1 \geq 0} \mathcal{V}(a_1; \theta) \rangle \leq \min_{\theta \in \Theta} \langle \max_{a_1 \geq 0} \mathcal{V}(a_1; \theta) \rangle \Leftrightarrow |\Theta'| \geq |\Theta|. \quad (C.8)$$

That is, ‘beliefs’ and ‘tastes’ are not disentangled (this is attributable to the min operator).

By linearity of the objective function, Proposition C.1 also applies to the $\epsilon$-contamination model of choice (Eichberger & Kelsey, 1999) which corresponds to a convex combination between a SEU criterion with a confidence degree or weight $\epsilon \in [0; 1]$ and Wald’s criterion with weight $1 - \epsilon$. See also Gierlinger & Gollier (2017) for a treatment of multiplier preferences from robust control theory (Hansen & Sargent, 2001; Strzalecki, 2011).

**D The two ambiguity aversion induced effects**

With numerical simulations this appendix illustrates the decomposition of the two ambiguity aversion induced effects when $H$ and $A$ are now allowed to vary with $a_1$. There are only two scenarios $\Theta = \{\theta_1 = +5, \theta_2 = -5\}$ with equal probability under the subjective prior $F = (q_1, \theta_1; q_2, \theta_2)$, i.e. $q_1 = q_2 = .5$. We assume $\partial_{a_1} C_2 \equiv 0$ so that for all $\theta \in \Theta$ and $a_1 \geq 0$, $\mathcal{V}_{a_1}(a_1; \theta) = \langle \hat{\tau} \rangle + \theta$ where the $F$-expected price is $\langle \hat{\tau} \rangle = 20$. This means that $\mathcal{V}_{a_1}$-lines will be flat while they were upward-sloping in Figure 2. The pessimistically-distorted prior $H = (\hat{q}_1, \theta_1; \hat{q}_2, \theta_2)$ and shift in levels $A$ satisfy, for all $a_1 \geq 0$ and $i = \{1, 2\}$

$$\hat{q}_i(a_1) = \frac{q_i \phi' (\mathcal{V}(a_1; \theta_i))}{q_1 \phi' (\mathcal{V}(a_1; \theta_1)) + q_2 \phi' (\mathcal{V}(a_1; \theta_2))}, \quad (D.1a)$$

and

$$A(a_1) = \frac{q_1 \phi' (\mathcal{V}(a_1; \theta_1)) + q_2 \phi' (\mathcal{V}(a_1; \theta_2))}{\phi' \circ \phi^{-1} (q_1 \phi (\mathcal{V}(a_1; \theta_1)) + q_2 \phi (\mathcal{V}(a_1; \theta_2)))}. \quad (D.1b)$$

The necessary-first order condition for $\hat{a}_1$ in Equation (19) rewrites

$$- C'_1(\hat{a}_1) + \beta A(\hat{a}_1) \big(\langle \hat{\tau} \rangle + \hat{q}_1(\hat{a}_1)\theta_1 + \hat{q}_2(\hat{a}_1)\theta_2\big) = 0, \quad (D.2)$$

and is graphically depicted in Figure 6 for different combinations of $\alpha$ and $\omega$. In this numerical example Figure 6 illustrates that the bulk of the variation in date-1 abatement level under ambiguity aversion relative to ambiguity neutrality is driven by pessimism. However note that the relative effects of ambiguity prudence can be relatively significant especially when
Figure 6: Separation of pessimism and ambiguity prudence

\[ (a) \; \alpha = 5 \text{ and } \omega = 20 \]

\[ (b) \; \alpha = 10 \text{ and } \omega = 20 \]

\[ (c) \; \alpha = 75 \text{ and } \omega = 55 \]

\[ (d) \; \alpha = 75 \text{ and } \omega = 65 \]

\[ (e) \; \alpha = 75 \text{ and } \omega = 80 \]

\[ (f) \; \alpha = 5 \text{ and } \omega = 90 \]

Note: The upward-sloping grey solid line is \( C_1' \). The two flat grey dotted lines are \( V_{a_1}(a_1; \theta_i) \). The flat dark dashed line is \( E \{ V_{a_1}(a_1; \tilde{\theta}) \} \). The curved black solid line is \( A(a_1)E \{ V_{a_1}(a_1; \tilde{\theta}) \} \). The curved black dotted line is \( A(a_1)E \{ V_{a_1}(a_1; \tilde{\theta}) \} \). The intersection between \( C_1' \) and \( A(a_1)E \{ V_{a_1}(a_1; \tilde{\theta}) \} \) gives \( \hat{a}_1 \).

\( \alpha \) is low (Figs. 6a and 6b). Figures 6c and 6d highlight the high sensibility of \( \hat{a}_1 \) around the threshold \( \bar{\omega} = 60 \) for relatively high \( \alpha \). Figures 6e and 6f indicate that the pessimistic prior distortion is more pronounced when \( \alpha \) is high. Finally, Figures 6b and 6e underline that when \( \omega \) is outside of the \([40 - 80]\) band and \( \alpha \) is relatively high pessimism redistributes almost all the weight to the worst scenario, i.e. \( \theta_1 \) (resp. \( \theta_2 \)) when \( \omega \) is small (resp. high).

E Joint market price and firm’s demand ambiguities

As in Section 5 consider that \( T_{\theta} = [\bar{T} + \theta; \bar{T} + \theta] \) and \( B_{\theta} = [\bar{b} + \theta; \bar{b} + \theta] \), i.e. high-price scenarios coincide with high-demand scenarios. In this case it holds that \( \partial_{\bar{\theta}} V_{a_1}(\bar{a}_1; \bar{\theta}) = \langle \tilde{\tau} \rangle + \theta \) and

\[
\partial_{\bar{\theta}} V(\bar{a}_1; \bar{\theta}) \leq 0 \iff \omega \leq \langle \tilde{\bar{b}} \rangle - \bar{a}_1 + \theta.
\]  

(E.1)

Anticomonotonicity (resp. comonotonicity) thus holds for sure if \( \omega \leq 71 \) (resp. \( \omega \geq 89 \)). In expectations over \( \Theta \), anticomonotonicity (resp. comonotonicity) holds i.f.f. \( \omega \leq \) (resp. \( \geq \)) 80.
The situation is depicted in Figure 7a. Now consider that \( T_\theta = [\bar{\tau} + \theta; \bar{\tau} + \theta] \) and \( B_\theta = [\bar{\tau} - \theta; \bar{\tau} - \theta] \), i.e. high-price scenarios coincide with low-demand scenarios. In this case it holds that \( \partial_\theta V_{\alpha_1}(\bar{\alpha}_1; \theta) = \langle \tilde{\tau} \rangle + \theta \) and

\[
\partial_\theta V(\bar{\alpha}_1; \theta) \leq 0 \Leftrightarrow \omega \leq \langle \tilde{b} \rangle - 2\langle \tilde{\tau} \rangle - \bar{\alpha}_1 - 3\theta. \tag{E.2}
\]

Anticomonotonicity (resp. comonotonicity) thus holds for sure if \( \omega \leq 13 \) (resp. \( \omega \geq 67 \)).

Figure 7: Cap-and-trade regime under CAAA with joint price and demand ambiguities

(a) \( \bar{\alpha}_1^o = f(\omega) \) with \( \text{Cov}_\theta\{G, L\} > 0 \)

(b) \( \bar{\alpha}_1^o = f(\omega) \) with \( \text{Cov}_\theta\{G, L\} < 0 \)

expectations over \( \Theta \), anticomonotonicity (resp. comonotonicity) holds i.f.f. \( \omega \leq (\text{resp. } \geq) 40 \).

The situation is depicted in Figure 7b. Comparing Figures 7a and 7b, we see that date-1 overabatement occurs for a wider allocation range as the allocation threshold is higher (resp. lower) when \( \text{Cov}_\theta\{G, L\} > (\text{resp. } <) 0 \) as compared to pure price ambiguity (Figure 3a). Note also that the variability of the adjustment in date-1 abatement increases (resp. decreases) around the threshold when \( \text{Cov}_\theta\{G, L\} > (\text{resp. } <) 0 \).

\section*{F Special case: Binary price ambiguity}

This appendix considers the case of binary price ambiguity, i.e. in all \( \theta \)-scenarios \( \bar{\tau}_\theta \) either takes the value \( \bar{\tau} > 0 \) with probability \( p(\theta) \in [0; 1] \) or \( \bar{\tau} \) with complementary probability and \( \Delta \tau = \bar{\tau} - \tau > 0 \). Let the underlying objective price lottery be \( (p, \bar{\tau}; 1-p, \bar{\tau}) \). The no-ambiguity bias requires that \( p = \mathbb{E}_F\{p(\bar{\tau})\} \) and thus \( \langle \tilde{\tau} \rangle = p\bar{\tau} + (1-p)\bar{\tau} \). W.l.o.g. assume for clarity that abatement cost functions are time separable. Let \( \Upsilon(\cdot; \theta) \) denote the \( \theta \)-scenario
expected net intertemporal revenue from date-1 abatement. For all \( a_1 \geq 0 \) and \( \theta \in \Theta \),

\[
\Upsilon(a_1; \theta) = \pi_1(a_1) + \beta \Upsilon(a_1; \theta)
\]

\[
= \zeta - C_1(a_1) - \beta p(\theta)(C_2(a_2^*(\bar{\tau})) + \bar{\tau}(b - a_1 - a_2^*(\bar{\tau}) - \omega))
- \beta(1 - p(\theta))(C_2(a_2^*(\bar{\tau})) + \bar{\tau}(b - a_1 - a_2^*(\bar{\tau}) - \omega)),
\]

where \( \zeta = \zeta_1 + \beta \zeta_2 \). With quadratic cost specification (1) Equation (F.1) rewrites

\[
\Upsilon(a_1; \theta) = \zeta - C_1(a_1) + \beta (p(\theta) \Delta \tau(b - a_1 - \omega - \langle \tau \rangle/c_2) - \bar{\tau}(b - a_1 - \omega - \bar{\tau}/(2c_2))),
\]

where \( \langle \tau \rangle \equiv (\bar{\tau} + \bar{\tau})/2 \) denotes the date-2 average price when \( p = .5 \). Differentiating Equation (F.2) and evaluating it at \( a_1 = \bar{a}_1 = \beta \langle \bar{\tau} \rangle/c_1 \) gives

\[
\Upsilon_{a_1}(\bar{a}_1; \theta) = -C_1'(\bar{a}_1) + \beta \Upsilon_{a_1}(\bar{a}_1; \theta) = -C_1'(\bar{a}_1) + \beta (\bar{\tau} - p(\theta) \Delta \tau),
\]

which is decreasing in \( \theta \) i.f.f. \( p(\theta) \) is increasing in \( \theta \). By optimality under ambiguity neutrality \( \Upsilon_{a_1}(\bar{a}_1; \theta) = 0 \) when \( p(\theta) = p \). It follows that \( \Upsilon_{a_1}(\bar{a}_1; \theta) \) changes sign from positive to negative at \( p(\theta) = p \). Intuitively we see from Equation (F.2) that when the firm expects to be net short under the abatement stream \( (\bar{\tau}; \langle \tau \rangle/c_2) \), \( \Upsilon(\bar{a}_1; \theta) \) is relatively high (resp. low) when \( p(\theta) \) is relatively large (resp. small). Therefore, for those \( \theta \)-scenarios such that \( p(\theta) < p \) where \( \Upsilon(\bar{a}_1; \theta) \) is relatively low and \( \Upsilon_{a_1}(\bar{a}_1; \theta) > 0 \), an increase in \( a_1 \) will increase \( \Upsilon(a_1; \theta) \). Conversely, for those \( \theta \)-scenarios such that \( p(\theta) > p \) where \( \Upsilon(\bar{a}_1; \theta) \) is relatively high and \( \Upsilon_{a_1}(\bar{a}_1; \theta) < 0 \), an increase in \( a_1 \) will decrease \( \Upsilon(a_1; \theta) \). In these two cases the spread in expected profits across \( \theta \)-scenarios is reduced. Formally,

**Proposition F.1.** Let \( \phi \) exhibit CAAA and assume abatement cost functions are quadratic and time separable. Under binary price ambiguity, the prevalence of ambiguity aversion raises date-1 abatement relative to ambiguity neutrality if, and only if,

(i) the objective probability associated with the low-price scenario \( p \) is above the threshold \( \bar{p} = (\beta c_2 \bar{\tau} + c_1 \langle \tau \rangle - c_1 c_2 (b - \omega))/\beta c_2 \Delta \tau \in [0; 1] \); or equivalently,

(ii) the firm expects to be net buyer of permits under the abatement stream \( (\bar{\tau}; \bar{a}_2) \) where \( \bar{a}_1 = \beta \langle \bar{\tau} \rangle/c_1 \) and \( \bar{a}_2 = \langle \tau \rangle/c_2 \); or equivalently,

(iii) the firm’s allocation \( \omega \) is below the threshold \( \bar{\omega} = b - \bar{a}_1 - \bar{a}_2 \).
Proof. By differentiation w.r.t. \( \theta \) we have, for all \( a_1 \geq 0 \) and \( \theta \in \Theta \),

\[
\partial_\theta V(\bar{a}_1; \theta) = p'(\theta)\Delta \tau (b - \omega - \beta \langle \hat{\tau} \rangle / c_1 - \langle \tau \rangle / c_2),
\]
and

\[
\partial_\theta V_{a_1}(\bar{a}_1; \theta) = -p'(\theta)\Delta \tau.
\]

Therefore, anticomonotonicity holds i.f.f. \( b - \omega - \beta \langle \hat{\tau} \rangle / c_1 - \langle \tau \rangle / c_2 > 0 \), i.e. the firm is net short of permits when it abates \((\bar{a}_1; \bar{a}_2)\). Note that by definition, \( \langle \hat{\tau} \rangle = \bar{\tau} - p\Delta \tau \), which is decreasing with \( p \). Anticomonotonicity thus holds i.f.f.

\[
2\beta c_2 (\bar{\tau} - p\Delta \tau) + c_1 (\bar{\tau} + \bar{\tau}) < 2c_1 c_2 (b - \omega), \tag{F.5}
\]

that is, i.f.f. \( p > \bar{p} \). For \( \bar{p} \) to be admissible we need that \( \beta \bar{\tau} \leq c (b - \omega) \leq \beta \bar{\tau} \). To see why it makes sense to have such a price range, note that when the permit price is \( c (b - \omega) \) the firm’s gross abatement effort \( b - \omega \) is optimally apportioned between the two dates. \( \square \)

Initial allocation continues to dictate the direction of the date-1 abatement adjustment. This contrasts with results in Snow (2011), Alary et al. (2013), Wong (2015a) and Berger (2016) where the effect of pessimism is clear under a binary ambiguity structure. However the condition for anticomonotonicity to hold is milder than in Proposition 4.8. The ambiguity averse firm must expect to be net short under the sole the abatement stream \((\bar{a}_1; \bar{a}_2)\) — not across all \( \theta \)-scenarios — for it to overabate. Note that this is akin to a situation where the ambiguity averse firm has no idea about the future price at all and thus considers the equiprobable price scenario. By contrast the ambiguity neutral firm is not affected by (the introduction of) ambiguity.

A novel insight from Proposition F.1 is Condition (i). An explicit \( \bar{p} \)-threshold allows us to characterize the effects of an increase in the ambiguity level, here proxied by the price range \( \Delta \tau \), for given degree of ambiguity aversion. To do so we determine the infinitesimal shift \( \delta p \) in \( \bar{p} \) consecutive to an infinitesimal increase \( \delta \tau > 0 \) in \( \Delta \tau \). For an upward shift in \( \Delta \tau \), i.e. \( \bar{\tau} \) increases by \( \delta \tau \) with \( \tau \) fixed, \( \bar{p} \) reacts such that

\[
2\beta c_2 (\delta \tau - \bar{p}\delta \tau - \bar{\tau}\delta p - \delta p\delta \tau + \tau\delta p) + c_1 \delta \tau = 0, \quad \text{i.e. } R^\dagger = \frac{\delta p}{\delta \tau} = \frac{2\beta c_2 (1 - \bar{p}) + c_1}{2\beta c_2 \Delta \tau} > 0, \tag{F.6}
\]

where \( \delta p\delta \tau \simeq 0 \) in the first order and \( R^\dagger \) denotes the rate of increase in \( \bar{p} \) consecutive to an
increase in \( \bar{\tau} \) by \( \delta\tau \). For a downward shift in \( \Delta\tau \), i.e. \( \bar{\tau} \) decreases by \( \delta\tau \) with \( \bar{\tau} \) fixed,

\[
2\beta c_2 (\bar{\tau} \delta\tau + \bar{\tau} \delta p + \delta p \delta\tau - \bar{\tau} \delta p) + c_1 \delta\tau = 0, \quad \text{i.e.} \quad R^\downarrow = -\frac{\delta p}{\delta\tau} = \frac{2\beta c_2 \bar{\tau} + c_1}{2\beta c_2 \Delta\tau} > 0, \quad \text{(F.7)}
\]

where \( \delta p \delta\tau \simeq 0 \) again and \( R^\downarrow \) denotes the rate of decrease in \( \bar{p} \) consecutive to a decrease in \( \bar{\tau} \) by \( \delta\tau \), in absolute terms. Therefore, it follows that

\[
R^\uparrow - R^\downarrow = (1 - 2\bar{p})/\Delta\tau > 0 \iff \bar{p} < 1/2. \quad \text{(F.8)}
\]

Consider a symmetric price range increase from \( \Delta\tau \) to \( \Delta\tau + 2\delta\tau \) which preserves \( \langle \tau \rangle \) as well as the ambiguity neutral firm’s price estimate \( \langle \tilde{\tau} \rangle \). The ambiguity averse firm’s price estimate, however, shifts from \( \langle \tilde{\tau} \rangle_{\Delta\tau} \) to \( \langle \tilde{\tau} \rangle_{\Delta\tau+2\delta\tau} \) where

\[
\langle \tilde{\tau} \rangle_{\Delta\tau+2\delta\tau} = \langle \tilde{\tau} \rangle_{\Delta\tau} + \delta\tau (1 - 2p) \geq \langle \tilde{\tau} \rangle_{\Delta\tau} \iff p \leq 1/2 \iff \langle \tilde{\tau} \rangle_{\Delta\tau} \leq \langle \tau \rangle. \quad \text{(F.9)}
\]

An increase in the range of price ambiguity hence always brings the ambiguity averse firm’s price estimate (resp. \( \bar{p} \)) closer to \( \langle \tau \rangle \) (resp. 1/2). This can be likened to a precautionary principle. More precisely under CAAA,

1. When \( \bar{p} > 1/2 \), the ambiguity averse firm overabates at date 1 i.f.f. \( p \geq \bar{p} > 1/2 \), i.e. i.f.f. \( \langle \tilde{\tau} \rangle \leq \langle \tau \rangle \). That is, ambiguity aversion raises date-1 abatement when the ambiguity neutral firm foresees a price below \( \langle \tau \rangle \) and does not abate enough relative to the \( \langle \tau \rangle \)-price scenario. Both \( \bar{p} \) and \( \langle \tilde{\tau} \rangle \) decrease consecutive to a symmetric increase in \( \Delta\tau \), which makes the anticomonotonicity criterion relatively laxer.

2. When \( \bar{p} < 1/2 \), note that the ambiguity averse firm overabates i.f.f. \( p \in [\bar{p}; 1/2] \), i.e. even though \( \langle \tilde{\tau} \rangle > \langle \tau \rangle \) and the ambiguity neutral firm already abates more at date 1 than under the \( \langle \tau \rangle \)-price scenario. Both \( \bar{p} \) and \( \langle \tilde{\tau} \rangle \) increase consecutive to a symmetric increase in \( \Delta\tau \), which makes the anticomonotonicity criterion relatively more restrictive.

In other words, when the condition for pessimism to raise date-1 abatement relative to ambiguity neutrality is relatively demanding (resp. lax), an increase in the ambiguity range makes it laxer (resp. more demanding), which is in line with a precautionary principle.