Mineral Resources for Renewable Energy: Optimal Timing of Energy Production*

Adrien Fabre[†], Mouez Fodha[‡] and Francesco Ricci[§]

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Abstract

The production of renewable energy is much more intensive in minerals than that from fossil resources. The scarcity of certain minerals limits the potential for substituting renewable energy for declining fossil resources. However, minerals can be recycled, while fossil resources cannot. We show that the greater the recycling rate of minerals, the more the energy mix should rely on renewable energy, and the sooner should investment in renewable capacity take place. Our original mechanism affecting the optimal timing of investment in renewable is active even in the presence of other —better known— mechanisms affecting the decision.

KEYWORDS : Renewable and Non-Renewable Natural Resources, Mineral resources, Recycling, Energy Transition.

JEL CODES: Q3, Q2, Q42, Q54.

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[†]Paris School of Economics and University Paris 1Panthéon Sorbonne. adrien.fabre@psemail.eu.

[‡]Paris School of Economics and University Paris 1Panthéon Sorbonne. mouez.fodha@univ-paris1.fr.

[§]Université de Montpellier (UMR 5281 ART-Dév). francesco.ricci@umontpellier.fr.

1 Introduction

Renewable sources of energy are generally more dispersed on the territory than non renewable ones. In particular this is the case of wind or solar energy, as compared to coal or gas. It follows that a larger quantity of mineral inputs is required to produce one unit of final energy from renewable as compared to non renewable sources of energy.[1] For instance, Hertwich et al. (2015) conclude that one unit of electricity requires "11–40 times more copper for photovoltaic systems and 6–14 times more iron for wind power plants", than from conventional fossil generation, as one can see in Figure 1. Concern about mineral intensity of renewable sources of energy has been expressed in official reports and academic studies.[2]

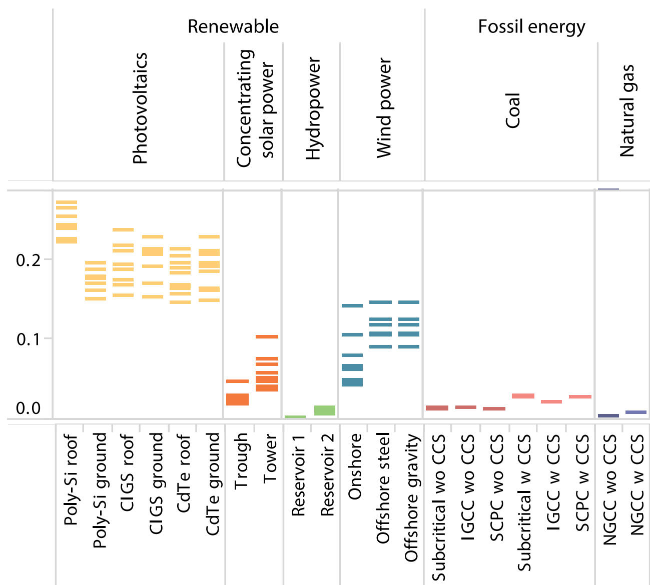

Figure 1: Copper intensity of energy technologies, kg/MWh, from Hertwich et al. (2015)

We bring along a novel argument in favor of early development of the production capacity for energy from renewable sources. Our argument for early development of renewable energy relies on the asymmetry between the types of natural resources used to produce energy ser-

[1] In the case of intermittent renewable energy, backup or storage capacity requirements exacerbate this difference in mineral intensity of the megawatt hour produced.

[2] See for instance Ali et al. (2017), Moss et al. (2013), Vidal et al. (2013), Bauer et al. (2010) and Commission (2008).

vices. When a unit of non renewable resource is directly used as fuel to supply energy services through combustion, as in the case of oil, gas and coal, that amount of resource is definitely lost. When a unit of mineral resources is embedded in the equipment and infrastructure used to produce energy from renewable sources, it supplies a flow of energy services over an interval of time and, at the end of the life cycle of the equipment, it adds to the stock of secondary mineral resources that can be recycled. Hence some part of the original unit of resource can provide services in the next period.

While the opportunity to recycle a non renewable natural resource improves the production possibilities set of the economy, it also requires time as an input in order to do so. From a technological perspective, recycling first requires to use the primary (currently extracted) resource, in order to build, with some delay, the secondary (recycled) resource. This technological constraint interacts with social preferences in determining the optimal calendar of resource extraction and use. To illustrate this, let us consider a society, where neither extraction nor recycling are costly, that wishes to maintain the level of resource use constant over a finite interval of time. If it is endowed with a non recyclable non renewable resource, it should spread it evenly over the planning horizon. If instead the resource can be partially recycled, say at a recovery rate $\delta \in (0, 1)$, with some time lag, say ten years, it should use exclusively primary resources during the first ten years, then reduce the extraction rate by $1 - \delta$ during the following decades. As compared to the former case, the intertemporal profile of resource extraction is put forward.

Together, the technological specificity of recycling mineral resources and the relative mineral intensity of renewable energy provide a rationale for developing more renewable energy infrastructure in the initial period than in subsequent ones and to choose a larger share for renewables in the energy mix, as compared to a case without recycling. Our analysis is based on a simplified description of the economic problem presented in section 2. We first consider in section 3 the benchmark model with an infinite time horizon, and then present a few extensions in section 4, within a two-period version of our model, to consider alternative mechanisms affecting the optimal decision on investment in renewable production capacity.

In our model, agents value energy services which result from a combination of energy provided by two distinct sources: the flow of renewable energy and combustion of a non renewable fossil resource. These sources are more or less good substitutes, either because of heterogeneous uses (Chakravorty & Krulce (1994)) or because of the intermittent availability of the renewable sources (Ambec & Crampes (2015)). The production of renewable energy employs specific equipment, dubbed "green" capital, embedding mineral non renewable resources. Part of the mineral resources embedded in the current period equipment can be also used in the next periods. The reserves of the two non renewable resources (fossils and minerals) are fixed. The issue is the timing of their extraction that maximizes the net present value of the utility from energy services.

The answers we obtain encompass some well known argument, as for instance that development of renewable energy should be delayed in the expectations of sufficient improvements in the productivity of green equipment capital. But the framework we consider allows us to put forward two original arguments: To the extent that mineral resources embedded in that equipment and infrastructure can be recycled, the development of renewable energy should be brought forward in time, and the energy mix should rely largely on renewable sources.

Other factors can affect the optimal decision on the timing of investment in green capital. In particular, mineral extraction should be delayed when endowment in green capital is excessive. In the two-period version of the model case, we illustrate that our original mechanism is active even if in the presence of alternative mechanisms. First, we consider expectations of fast growth in the productivity of green capital. Even though this tends to delay investment in green capital, we find that the recycling rate somewhat counteracts this effect. Moreover, we show that recyclability plays a role similar to productivity of green capital installed in the first period. We are also able to disentangle the specificity of these two parameters. Namely, holding mineral resource extraction constant in the first period, an improved productivity of renewable energy capital increases first period energy services only, while an improved recycling rate of renewable energy capital increases second period energy services only. Hence there is a difference in the optimal response of renewable energy capacity to changes in each of the parameters: first period productivity of renewable capital, and renewable capital recycling rate. Finally, we consider the case with pollution from the use of fossil resources. In the presence of damages from the use of fossil resources, early investment in green capital is commonly called for.

Our work is related to an extensive literature covering the timing and policies associated with the energy transition. Herfindahl (1967) develops the "least cost first" principle, relating the order of extraction of various deposits of non renewable resources with their cost of exploitation. Kemp & Van Long (1980) and Amigues et al. (1998) show the limits of the "least cost first" principle, while Tahvonen & Salo (2001) and Amigues et al. (2015) study the transition from nonrenewable to renewable energy. Also considering the energy transition, Chakravorty et al. (2006), Chakravorty et al. (2008) and Greiner et al. (2014) show that in certain circumstances, a large endowment in fossils delays the build up of renewable capital stock. Although these models adequately examine different aspects of the timing of the energy transition, none of them embeds the dependency of renewable production on recyclable but scarce minerals. Thus, our paper completes the literature by examining the role of minerals in the energy sector. Our approach in modeling recycling of a natural resource in a dynamic macroeconomic setting is close to growth theory models where recycling postpones the working-out of mining resources, Dasgupta & Heal (1979).

We present our model in section 2. In section 3 we give the results and the analysis of our benchmark case, with the infinite horizon and specific functional forms for the utility and the production functions. Then, we consider in section 4 further issues in a two-period model. First, we consider differences in the productivity growth across the two energy types. Second, we introduce pollution from the use of fossil resources. Third, we study the polar cases of perfect substitutability and perfect complementarity between the two energy sources. In the last section we sum up our results and give some perspectives, in particular on the choice of the recycling rate, from which we abstract in this paper.

2 The model

Let us consider a representative household, whose utility is a function of consumption of energy services q_t:[3]

$$u(q_t) \tag{2.1}$$

with $u' > 0$, $u'' \leq 0$.

Energy services combine two types of energy flow: the flow of energy from non renewable resources, x_t, and the flow of energy from renewable sources, y_t. Formally we set:

$$q_t = Q(x_t, y_t) \tag{2.2}$$

with $Q_i' \geq 0$, $Q_i'' \leq 0$ $i \in \{x, y\}$. The degree to which the two types of energy can be combined to produce energy services may vary from perfect substitutability to perfect complementarity.[4]

The energy flow x_t is produced transforming the quantity of extracted non renewable resource $f_t \geq 0$, which we dub fossil resources, according to the linear production function:

$$x_t = A_t f_t \tag{2.3}$$

where A_t is the exogenous productivity index. The quantity of fossil resources is limited, it

[3] For the moment we abstract from any influence on the household's utility from the energy system. In section 4 we assume that utility also depends on the types of energy sources that are used to produce energy services, namely that the use of one source also generates disutility.

[4] Two approaches can be considered. Either firms sell energy services by using the two types of energy. This is the case, for instance, of a power company generating electricity out of a differentiated portfolio of power stations, some based on conventional fossil resources, others on wind and solar power. Alternatively, one can consider that households directly consume the two resources. For instance, a household endowed of a solar thermal panel and a gas fuelled heater to heat water, can use the two sources of energy as imperfect substitutes due to the intermittent nature of the former.

is initially available in a finite stock F and is directly reduced by extraction:[5]

$$F \geq \sum_{t \geq 0} f_t. \tag{2.4}$$

The flow of energy y_t is produced employing a specific stock of capital K_t, which we dub "green" capital, according to the linear technology:

$$y_t = B_t K_t \tag{2.5}$$

where B_t is the exogenous productivity index. Green capital is built out of minerals. Specifically, the capital stock at date t is the sum of minerals extracted at date t —the primary resource $m_t \geq 0$— and the stock of secondary minerals recycled from previous period's green capital δK_{t-1}. The exogenous parameter $\delta \in [0, 1]$ measures the rate at which minerals embedded in the capital stock can be recycled from one period to the next. In this paper, we implicitly assume perfect substitutability between primary and recycled mineral resources, and the possibility of infinite recycling.[6] Defining $K_{-1} \geq 0$ as the stock of minerals embedded in the capital stock before date 0, and assuming a constant recycling rate, the history of mineral extraction determines the stock of green capital:[7]

$$K_t = K_{-1}\delta^{t+1} + \sum_{\tau=0}^{t} m_\tau \delta^{t-\tau}. \tag{2.6}$$

[5]We assume that extraction of the non renewable resources is cost-less. We will discuss in forthcoming work how the results change when resource extraction is costly, with convex costs.

[6]These conditions do not take into account the cost of waste recovery and processing and the lower quality of recycled resources. We restrict our analysis to the case of an exogenous recycling rate, like much of the literature (e.g. Dasgupta & Heal (1979)).

[7]In practice, both types of energy sources require specific capital embedding some mineral resources. Our focus is the asymmetry in mineral intensity between the specific capital for each energy source. We therefore adopt the extreme assumption that only one energy source relies on the specific capital, so as to simplify the analysis, without loosing in the qualitative features of our model. Similarly, we do not consider alternative uses of mineral resources, although in practice, the demand of mineral for investment in green capital is only a share of total mineral consumption. Our simplifying assumption according to which there is a limited amount of minerals available for investment in green capital, simplifies the analysis, but the framework can be readily extended to consider competition in the use of the global supply of minerals between investment in green capital and other uses.

Minerals are non renewable resources, initially available in a finite stock M. Primary extraction is constrained over time by

$$M \geq \sum_{t \geq 0} m_t. \tag{2.7}$$

In our framework the distinction between fossil and renewable sources of energy hinges on the recycling rate of minerals δ. If minerals were perfectly recyclable, i.e. $\delta = 1$, it would be possible to produce forever a flow $B_t M$ of renewable energy, once the specific equipment had been installed at its maximum potential. If minerals were not recyclable, i.e. $\delta = 0$, they could not be used twice —just as fossil resources— and the two types of resources would be analogous.

We analyze optimal trajectories, assuming that a benevolent planner chooses the path of resource extraction that maximizes intertemporal discounted utility of the representative household, subject to technology constraints and resource dynamics. It applies a social pure discount rate $\rho > 0$ and solves the following problem

$$(\mathcal{P}): \quad \max_{(f_t, m_t)_t} \sum_{t \geq 0} \frac{1}{(1+\rho)^t} u(q_t)$$

$$\text{subject to } (2.2) - (2.7) \text{ and } f_t, m_t \geq 0$$

$$\text{with } M, F, K_{-1} \text{ given.}$$

3 Optimal energy production in the benchmark case

In this section, we further specify the production and utility functions, in order to be able to characterize the optimal policy by closed-form solutions. Specifically, we assume a unitary elasticity of substitution between fossil and renewable energy

$$Q(x_t, y_t) = x_t^{\alpha} y_t^{1-\alpha} \tag{3.1}$$

with $\alpha \in (0,1)$. We also assume a utility function with a constant elasticity of intertemporal substitution of consumption[8]

$$u(q_t) = \frac{1}{1-\varepsilon} q_t^{1-\varepsilon} \tag{3.2}$$

with $\varepsilon > 0$. Moreover we restrict the analysis to the case of constant and equal productivity, and set $\forall t \; A_t = B_t = 1$.

These assumptions imply that the extraction of fossil resources is always positive, i.e. $\forall t,\ f_t$. In fact, if $f_t = 0$ at some t, $q_t = 0$, which is suboptimal since the marginal utility of q is infinite at $q = 0$. The reasoning applies to green capital, so that $\forall t,\ K_t > 0$. The same argument applies to the extraction of mineral resources in absence of recycling, that is $\forall t,\ m_t > 0$ if $\delta = 0$. In this special case the economy relies on the use of two non-renewable resources as imperfect substitutes for consumption. Along the optimal path, the input ratio is held constant and equal to the relative resource endowment, i.e. $f_t/m_t = F/M$. The extraction of the two non renewable resources, as well as the production of renewable energy and consumption, decline at the common pace dictated by the factor $(1+\rho)^{-\frac{t}{\varepsilon}}$.[9]

When instead the equipment for the production of renewable energy is recyclable, i.e. if $\delta > 0$, the argument does not apply to the extraction of minerals. In fact, the production of renewable energy could be positive, i.e. $y_t > 0$, at some date t even in the absence of contemporaneous extraction of primary mineral resource, i.e. even if $m_t = 0$, to the extent that the specialized capital stock was positive in the previous period, $K_{t-1} > 0$, and it would be precisely equal to $y_t = \delta K_{t-1} > 0$.

There are two distinct potential reasons for shutting down the mine at some date, that is for choosing $m_t = 0$, when $\delta > 0$. First of all, the opportunity to recycle minerals embedded

[8]The elasticity of intertemporal substitution of consumption equals $1/\varepsilon$. For $\varepsilon = 1$, $u(q_t) = \ln q_t$.

[9]See Appendix A for the analysis of this sub-case.

in capital introduces an incentive to put forward the extraction date. To see this, consider the extreme case of a 100% recycling rate, i.e. $\delta = 1$. In this case, given our assumption of costless extraction, there is no gain from leaving any mineral resource underground for future use. It is clearly optimal to choose $m_0 = M$ and $m_t = 0$ for any $t \geq 1$. In our analysis we take into account the possibility that along the optimal path extraction comes to an end in finite time, and denote by $\bar{t}$ the last period during which extraction is positive. Second, there may be situations where it is preferable to initially keep mines closed and begin extracting only at some later date. This is the case when the economy is endowed of a large initial capital stock, but only a relatively small stock of primary mineral resources. By choosing $m_t = 0$ over an initial interval $[0, \underline{t})$, one can delay the use of the limited resource stock M to periods $t \geq \underline{t}$, while keeping the renewable energy input for consumption at rate $y_t = K_{-1}\delta^{t+1}$ for $t < \underline{t}$.

We therefore search for the extraction paths of the two resources, such that $\forall t \; f_t > 0$, $\forall t \in [\underline{t}, \bar{t}] \; m_t > 0$ and otherwise $m_t = 0$, where periods $\underline{t}$ and $\bar{t}$ have to be chosen.[10] The

[10] In our deterministic framework, the optimal policy rules out any path with intermittent extraction of minerals. This is demonstrated in Appendix C.1, but intuitively, the Bellman principle of optimality implies that if along the optimal path extraction comes to an end at $\bar{t}$, it is not efficient to open again the mine at some later period $\tilde{t} > \bar{t}$. Suppose in fact that it is optimal to chose $m_{\tilde{t}} > 0$. It makes sense to keep $m_{\tilde{t}-1} = 0$ at $\tilde{t} - 1$ only if the capital stock $K_{\tilde{t}}$ is considered too large given the remaining stocks of resources $F - \sum_{\tau=0}^{\tilde{t}-1} f_\tau$ and $M - \sum_{\tau=\underline{t}}^{\tilde{t}-1} m_\tau$. But these stocks are optimal, since they result of the extraction paths f_t and m_t up to date $\tilde{t} - 1$, assumed to be optimal. Hence, $K_{\tilde{t}}$ cannot be considered excessive. This contradiction shows that our premise, according to which it is optimal to chose $m_{\tilde{t}} > 0$ when $m_{\tilde{t}-1} = 0$ is optimal, is wrong. *Mutatis mutandis* the argument holds for the interval $[0, \underline{t})$.

planner's problem is

$$
\begin{aligned}
\max \sum_{t=0}^{\underline{t}-1} \left(\frac{1}{1+\rho}\right)^t & \left(\frac{f_t^\alpha \left(\delta^{t+1} K_{-1}\right)^{1-\alpha}}{1-\varepsilon}\right)^{1-\varepsilon} \\
+ \sum_{t=\underline{t}}^{\bar{t}} \left(\frac{1}{1+\rho}\right)^t & \left(\frac{f_t^\alpha \left(\delta^{t+1} K_{-1} + \sum_{\tau=\underline{t}}^{t} \delta^{t-\tau} m_\tau\right)^{1-\alpha}}{1-\varepsilon}\right)^{1-\varepsilon} \\
+ \sum_{t=\bar{t}+1}^{\infty} \left(\frac{1}{1+\rho}\right)^t & \left(\frac{f_t^\alpha \left(\delta^{t+1} K_{-1} + \delta^{t-\bar{t}} \sum_{\tau=\underline{t}}^{\bar{t}} \delta^{\bar{t}-\tau} m_\tau\right)^{1-\alpha}}{1-\varepsilon}\right)^{1-\varepsilon} \\
+ \lambda \left(F - \sum_{t=0}^{\infty} f_t\right) & + \nu \left(M - \sum_{\tau=\underline{t}}^{\bar{t}} m_\tau\right)
\end{aligned}
\tag{3.3}
$$

where λ, $\nu \geq 0$ are the values of the fossil and mineral resource stocks respectively.

The optimal policy is characterized by the following.

Proposition 1. *The unique trajectories solving problem (3.3), are of three types depending on initial capital and resource stocks, and on preference and technological parameters.*

1. *If the technological efficiency of recycling is above the social discount factor, i.e. if $\delta \geq r := (1+\rho)^{-\frac{1}{\varepsilon}}$, the mineral resource is exhausted in the first period, i.e. $\underline{t} = \bar{t} = 0$, while the fossil resource is extracted at an exponentially declining rate*

$$
f_t = F\left(1-R\right) R^t \tag{3.4}
$$

   where $R := \left(\frac{\delta^{(1-\alpha)(1-\varepsilon)}}{1+\rho}\right)^{\frac{1}{1-\alpha(1-\varepsilon)}}$, for all $t \in \mathbb{N}$.

2. *If instead $\delta < r$, both mineral and fossil resources are exhausted over the infinite horizon. There are two distinct types of trajectories in this sub-case.*

   (a) *If the stock of primary mineral resources is abundant relatively to the stock of secondary resources available in the first period, i.e. if $\frac{M}{\delta K_{-1}} \geq \frac{r-\delta}{1-r}$, both resources are extracted at all periods, i.e. $\underline{t} = 0$ and $\bar{t} = \infty$, and from the second period* *onward their extraction falls at a common exponential rate, dictated by the social discount factor r, i.e.* $f_{t+1} = f_t r$ *and* $m_{t+1} = m_t r$ $\forall t \geq 1$. *While fossil resource extraction declines from the first to the second period according to factor r, i.e.* $f_1 = f_0 r$, *the extraction of the mineral resource between the first and second period follows* $m_1 = (r - \delta)(m_0 + \delta K_{-1}) = (r - \delta) K_0$. *Initial optimal extraction is*

$$f_0 = (1 - r) F \tag{3.5}$$

$$m_0 = (1 - r) \frac{M}{1 - \delta} \left(1 - \frac{r - \delta}{1 - r} \frac{\delta K_{-1}}{M}\right) \tag{3.6}$$

(b) *If instead* $\frac{M}{\delta K_{-1}} < \frac{r-\delta}{1-r}$, *extraction of the mineral resource is delayed, i.e.* $\underline{t} \geq 1$ *and* $\overline{t} = \infty$. *The optimal* $\underline{t}$ *is the lowest non-negative integer at or above the value* $\ln\left(\frac{M}{\delta K_{-1}} \frac{1-r}{r-\delta}\right) / \ln \delta$. *Over the first interval of time fossil resource extraction declines according to the factor R. From* $\underline{t} + 1$ *onward, the extraction of both resources falls at the common rate r. Between period* $\underline{t}$ *and* $\underline{t} + 1$ *the extraction of fossil resources declines at rate r while that of minerals follows* $m_{\underline{t}+1} = \left(m_{\underline{t}} + \delta^{\underline{t}+1} K_{-1}\right)(r - \delta)$. *In this case*

$$f_0 = \left(\frac{1 - R^{\underline{t}}}{1 - R} + \Gamma(\underline{t}) \frac{r^{\underline{t}}}{1 - r}\right)^{-1} F \tag{3.7}$$

$$f_{\underline{t}} = \left(\Gamma(\underline{t}) \frac{1 - R^{\underline{t}}}{1 - R} + \frac{r^{\underline{t}}}{1 - r}\right)^{-1} F r^{\underline{t}} \tag{3.8}$$

$$m_{\underline{t}} = (1 - r) \frac{M}{1 - \delta} \left(1 - \delta^{\underline{t}} \frac{r - \delta}{1 - r} \frac{\delta K_{-1}}{M}\right) \tag{3.9}$$

where $\Gamma(\underline{t}) := \left(\frac{r\delta(1-\delta)K_{-1}}{(1-r)(M+\delta^{\underline{t}+1}K_{-1})}\right)^{\frac{(1-\alpha)(1-\varepsilon)}{1-\alpha(1-\varepsilon)}}$.

Proof. The detailed proof is in Appendix B and C.

□

Let us explain the optimal trajectory of resource extraction and energy production specified in Proposition 1 and comment on them.

First, notice that the Hotelling principle for the efficient management of non renewable resources applies to our framework. When the optimal policy maintains a constant input ratio, consumption falls at the same rate as the common rate driving the decline in resource extraction. Say that q declines at a factor $g \in (0,1)$, i.e. $q_{t+1} = gq_t$. Then the value of a marginal unit of the resource mix increases at rate $p_{t+1}/p_t = g^{-\varepsilon}$. Along the optimal trajectory from period 1 onward in case (*2.a*), or from period $\underline{t}+1$ onward in trajectory (*2.b*), the optimal path of resource extraction implies $g = r$, therefore $p_{t+1}/p_t = r^{-\varepsilon} = 1 + \rho$: the value of a marginal unit of resource increases at the social pure discount rate.[11] This is the rule found by Hotelling (1931) applies to our context.

Second, the asymmetry between the two types of resources, concerning the possibility to recycle them, implies a difference in their optimal extraction paths. To see this let us focus on the case of moderate recycling ($\delta < r$) and no endowment of green capital ($K_{-1} = 0$), a sub-case of (*2.a*) in Proposition 1. In this case, the initial ratio of resource extraction f_0/m_0 equals the initial input ratio f_0/K_0. As previously argued, without recyclability, it is optimal to choose $f_0/m_0 = F/M$, according to the relative resource endowment.[12] When green capital can be recycled, we see from (3.6) that the extraction and input ratios are initially biased toward more intensive use of mineral $f_0/m_0 = (1-\delta)\, F/M$. This first period choice is the same as the one made in an economy endowed of a larger stock of non renewable and non recyclable resources of size $\tilde{M} := M/(1-\delta)$. The stock $\tilde{M}$ measures the maximum feasible amount of mineral inputs that can be used in the production of renewable energy over time, i.e. $\tilde{M} = \sum_{t=0}^{\infty} \delta^t M$ obtained by extracting all minerals in the first period ($\underline{t} = 0$ and $\bar{t} = 0$). This observation points to the fact that the possibility of recycling the mineral resource embedded in green capital is tantamount to an endowment of a larger stock of mineral resources. Since, due to $\delta > 0$, mineral resources are relatively more abundant, the constant input ratio f_t/K_t is optimally chosen larger. However, the ratio of resource extraction f_t/m_t

[11] p_t is the marginal value of energy services, to which the marginal values of mineral and fossil resources extracted are proportional.

[12] From (3.6) with $K_{-1} = \delta = 0$.

can only be kept constant from period 1 onward, as mineral extraction is adjusted at 0 to account for the absence of recycled resources at that date. In so doing, the input ratio f_t/K_t remains constant. As a consequence the ratio of resource extraction is increased after the initial period.[13] The following statement summarizes this analysis.

Proposition 2. *When $K_{-1} = 0$ and $\delta < r := (1+\rho)^{-\frac{1}{\varepsilon}}$, the solution of problem (3.3) implies that the larger is the recycling rate $\delta \in [0, r)$, the more intensive in renewable energy is the constant input ratio, the greater is the extraction of minerals in the first period and green capital at every period, the more are extracting activities concentrated on minerals, and more so in the first than the following periods.*

$$\forall t \geq 0 \ \frac{x_t}{y_t} = \frac{f_t}{K_t} = (1-\delta)\frac{F}{M} \quad ; \quad m_0 = \frac{1-r}{1-\delta}M \quad ; \quad \forall t \geq 1 \ \frac{f_t}{m_t} = \frac{r}{r-\delta}(1-\delta)\frac{F}{M} \ . \tag{3.10}$$

Proof. The value of m_0 is an application of (3.6) in Proposition 1. We have $\frac{\partial m_0}{\partial \delta} = \frac{m_0}{1-\delta} > 0$, $\frac{dK_t}{d\delta} = \frac{dm_0}{d\delta} r^t > 0$ and $\frac{dm_t}{d\delta} = -\left(\frac{1-r}{1-\delta}\right)^2 M r^t < 0$. The result on the input ratio holds because, as argued in the main text f_t and K_t grow at the same rate r at any date. Applying results for the case (*2.a*) in Proposition 1, we get the ratio of resource extraction for $t \geq 1$. Thus $\frac{\partial f_t/m_t}{\partial \delta} = r\frac{1-r}{(1-\delta)^2}\frac{F}{M} > 0$ and $\frac{f_0}{m_0} < \frac{f_t}{m_t}$ for $t \geq 1$. □

Figure 2 illustrates the optimal paths of extraction of mineral and fossil resources, of green capital and consumption. It represents the cases of two economies differing by the recycling rate δ under the assumption of case (*2.a*) where $r > \delta$ and no green capital endowment. We can see that the dynamics of mineral resource extraction is qualitatively affected, while that of consumption and green capital is not (though their levels shift upwards with the rate of recycling).

The results in Proposition 2 have relevant policy implications. On the one hand, the

[13] $K_{-1} = 0$ implies $f_0/K_0 = f_0/m_0$. From case (*2.a*) in Proposition 1 $\forall t \geq 1$, $f_t/m_t = f_{t+1}/m_{t+1}$, and $f_1/m_1 = rf_0/((r-\delta)K_0)$. Hence the upward jump in the extraction ratio from period 0 to period 1: $f_1/m_1 > f_0/m_0$.

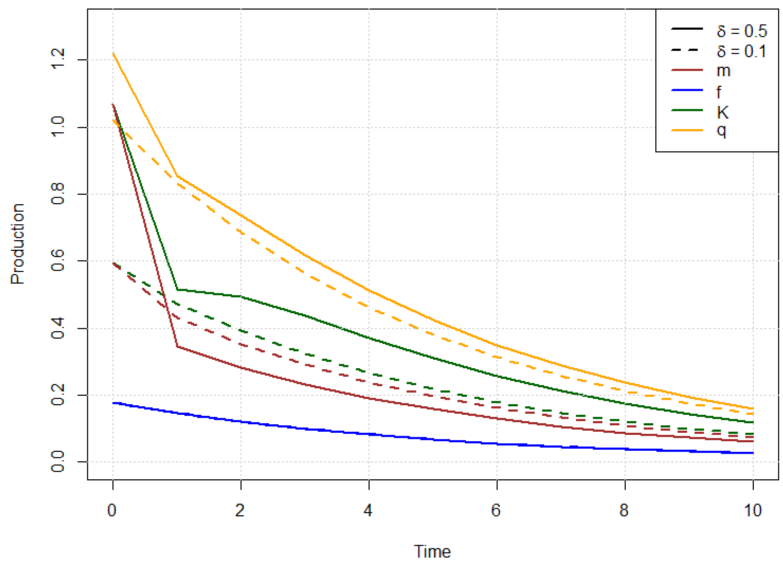

Figure 2: Resource extraction, green capital and consumption paths for a different recycling rates. For the calibrations, we used $\rho = .04$, $\alpha = .7$, $\varepsilon = .2$, $K_{-1} = 0$, $M = 2$, $F = 1$, $\frac{q}{f^{\alpha}K^{1-\alpha}} = 4$.

empirically grounded observation that the production of renewable energy relies on the use of specific non renewable mineral resources, namely minerals, suggests that the economy is poorer than it would be if the renewable energy could be produced out of non exhaustible inputs. From this point of view, the observation points to a limitation of renewable energy as a factor to overcome the limits to growth. In terms of our framework, this argument is represented by the lower value of welfare *ceteris paribus* when $\delta < 1$ than when $\delta = 1$. The observation provides an argument stating that the potential production of renewable energy is more limited than generally thought. We label this argument as the *pessimistic stance*.

On the other hand, our analysis illustrates that the possibility to recycle minerals embedded in green capital makes it preferable to choose an energy mix composed of more renewable energy and less conventional fossil resources, as compared to a choice made without taking into account the asymmetry in the possibility to recycle fossil versus mineral resources. Hence, adding a plausible assumption on the recycling technology to the same empirical observation, we provide a *pro renewable energy* argument, partially countering the *pessimistic stance*.

Moreover, we present an original argument in favor of a *pro active* renewable energy policy. In fact, we show that for a given amount of mineral resources to be devoted to the production of renewable energy, we should skew extraction toward the present the greater the recyclability of minerals. In other words, because minerals are recyclable and fossil resources are not, we should develop as soon as possible the green capital embedding the minerals, that allows us to produce renewable energy and to substitute for conventional fossil energy. This is found in Proposition 2, as well as in the extreme in case (*1.*) of Proposition 1 where minerals are entirely embedded in green capital from $t = 0$. Notice that this original *pro active* argument is grounded on the same empirical observation underlying the *pessimistic stance*. It relies on the flexibility in scheduling resource use typical of the management of non renewable resources. In fact, putting forward the potential of future production of renewable energy is off the production possibility set in commonly used models with renewable and non renewable sources of energy (e.g. Moreaux & Ricci (2005)).

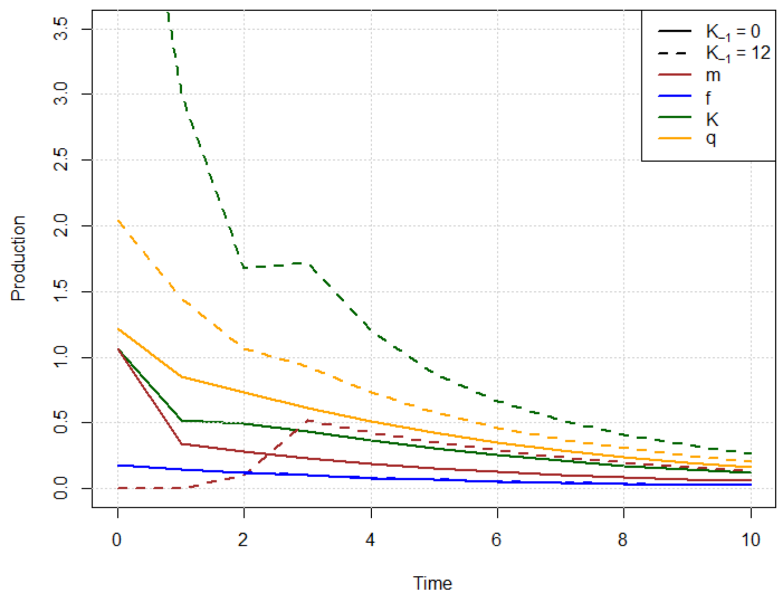

Figure 3: Resource extraction, green capital and consumption paths for a different endowments in green capital. We used the same calibrations as in Figure 2, and $\delta = .5$.

Our discussion above abstracts from several potential reasons for putting forward or for delaying investment in green capital. In the next section we review a few of them. The framework in Proposition 1 provides already a possible reason for delaying. It could be that at the start of the planning horizon, the economy has inherited of a large stock of green capital. If previous investment decisions were not optimal, and inefficiently biased toward renewable resources, the resulting stock of green capital, and thus of secondary mineral resource available in the first period, could exceed the desirable initial stock of capital for the first period. This corresponds to case (*2.b*) in Proposition 1. Figure 3 shows how the optimal paths of four endogenous variables -extraction of fossil and mineral resources, green capital, and consumption- vary with the endowment of green capital. The level of this endowment is chosen to represent the qualitative features of cases (*2.a*) and (*2.b*) in Proposition 1. In the latter case it is optimal to delay the extraction of mineral resources (as $\underline{t} > 0$). Nevertheless, in this case, production of renewable energy is initially quite high, and actually higher than socially desirable. In practice, this case may be of little relevance.

Two further remarks on case (*2.b*) are worthwhile. First, over the interval of time $[0, \underline{t})$ the stock of green capital declines at rate δ instead of the socially desired rate $r > \delta$. Though abundant, green capital is still valuable because productive, and it is therefore used at full capacity. As a result though, over this interval of time, the rate at which fossil extraction decreases is adjusted and differs from the one prevailing in presence of mineral extraction.[14] A similar adjustment to the extraction of fossil resources applies in case (*1.*) of a sufficiently efficient recycling technology. Second, the marginal effect of an increase in the recycling rate δ is more complex in the case (*2.b*) of Proposition 1 than in the case (*2.a*) treated in Proposition 2. On the one hand the same forces presented in Proposition 2 apply. Yet now, a countervailing effect operates through the fact that more secondary mineral resources are made available by the increase in δ. The beginning of extraction may be delayed, and the

[14] More precisely, we deduce from Appendix D that the optimal rate of decay for fossil extraction is the closest one between r and R from the discount factor $\frac{1}{1+\rho}$.

initial extraction of mineral resources may decline with δ.[15] Though, in general, the higher the recycling rate, the higher the optimal initial extraction of minerals, and the lesser the subsequent ones, unless reserves M are very low as compared to endowment in green capital. Indeed, the higher the recycling rate, the more abundant are resources in the future. This weakens the trade-off between present and future consumption, allowing for earlier extraction. However, absent sufficient reserves for the future, another effect dominates: the higher the recycling rate, the more one benefits from past investment in green capital, and the less one needs to extract minerals in the future. In practice, we argue that recoverable resources M is several order of magnitudes larger than green capital K_{-1} for base metals, so we can reasonably assume that the realistic case is the case (*2.a*),[16] with $K_{-1} \approx 0$ and $s = 0$.

4 Extensions in a two-period model

In this section we consider further factors affecting the optimal timing of energy production and in particular m_0. In order to develop these extensions in a clear and tractable way, we consider the two-period version of the model presented in section 2, with $t \in \{0; 1\}$, for which the results of the benchmark specification hold. Within this simplified framework, we first study how climatic damages from the use of fossil resources impact the optimal path. Then we consider alternative assumptions on preferences, namely substitutability between resources, by exploring the extreme cases of perfect and no substitutability. The former case allows us to illustrate the mechanism running through different rates of technological progress in the production of the two types of energy.

[15] We have $\frac{dm_{\underline{t}}}{d\delta} > 0 \iff \forall t > \underline{t}$, $\frac{dm_t}{d\delta} < 0 \iff M > \delta^{\underline{t}} K_{-1} \left(\frac{(r-\delta)(1-\delta)}{1-r} (\underline{t}+1) - \delta \right)$ and $\frac{d\underline{t}}{d\delta} = \frac{1}{(r-\delta)\ln(\delta)} - \frac{\ln\left(\frac{M}{K_{-1}} \frac{1-r}{r-\delta}\right)}{\delta \ln^2(\delta)}$. One can check that, even in the range of parameters of case (*2.b*) —$\delta < r$ and $\frac{M}{\delta K_{-1}} < \frac{r-\delta}{1-r}$— both signs are possible for each of these derivatives.

[16] Indeed, $\delta \geq r$ does not seem realistic. An upper credible value for the discount factor ρ is 0.05, while the inverse of the elasticity of inter-temporal substitutionε can reasonably be assumed higher than 0.5. Combining these conservative figures gives a low estimate for r: 0.9. Taking more common values for ρ and ε would yield an even higher threshold r, so that for any realistic value of the recycling rate δ, it is extremely likely to have $\delta < r$ and to be in the case where the optimal path is an endless extraction.

To begin with, let us set the stage and point out an analogy between the recycling rate and the initial productivity of green capital.

The intertemporal constraints on natural resources are:

$$F \geq f_0 + f_1 \tag{4.1}$$

$$M \geq m_0 + m_1 \tag{4.2}$$

The social planner's program now writes:

$$\max_{x_0, y_0, x_1, y_1} u\left(Q\left(x_0, y_0\right)\right) + \frac{1}{1+q} u\left(Q\left(x_1, y_1\right)\right)$$

As $u' > 0$, it is optimal to consume all available resources. Using the intertemporal availability constraints (4.1) and (4.2), the problem can be rewritten only in terms of the two quantities of resources extracted in $t = 0$:

$$\max_{\{f_0, m_0\}} u\left(Q\left(A_0 f_0, B_0 m_0\right)\right) + \frac{1}{1+\rho} u\left(Q\left(A_1\left(F - f_0\right), B_1\left(M - (1-\delta) m_0\right)\right)\right)$$

The first order conditions are:

$$u'\left(q_0\right) Q'_x\left(x_0, y_0\right) A_0 = \frac{1}{1+\rho} u'\left(q_1\right) Q'_x\left(x_1, y_1\right) A_1 \tag{4.3}$$

$$u'\left(q_0\right) Q'_y\left(x_0, y_0\right) B_0 = \frac{1}{1+\rho} u'\left(q_1\right) Q'_y\left(x_1, y_1\right) B_1 (1-\delta) \tag{4.4}$$

The marginal benefits of each resource are equated at both dates. Conditions (4.3) and (4.4) hold when the resources are extracted at both periods. In this case, they can be combined to obtain the following trade-off condition:

$$\frac{Q'_x\left(x_0, y_0\right)}{Q'_y\left(x_0, y_0\right)} \frac{A_0 (1-\delta)}{B_0} = \frac{Q'_x\left(x_1, y_1\right)}{Q'_y\left(x_1, y_1\right)} \frac{A_1}{B_1} \tag{4.5}$$

This is the well known equality between marginal rates of transformation at each period, for the production function $v_t(f_t, m_t) = Q(A_t f_t, B_t m_t)$.

Proposition 3. *The timing of extraction depends on the ratio $B_0/(1-\delta)$. It implies an equivalence between an increase in the lifetime of the minerals extracted in $t=0$ through δ, and a higher mineral productivity in the first period through B_0.*

The possibility of recycling the mineral resource influences the optimal extraction rule in the same way as the productivity index of the mineral resource extracted in $t=0$.

4.1 Damages from fossils

In this subsection, we apply our two-period model with the combination of a Cobb-Douglas and a CRRA utility of section 2. We introduce a period specific convex damage of fossil resources use $\frac{d_i}{\theta} f_i^\theta$, with $\theta > 1$. Defining $\phi = \alpha(1-\varepsilon)$ and $\mu = (1-\alpha)(1-\varepsilon)$, the program writes

$$\max \frac{f_0^\phi m_0^\mu}{1-\varepsilon} - \frac{d_0}{\theta} f_0^\theta + \frac{1}{1+\rho}\left(\frac{f_1^\phi (m_1 + \delta m_0)^\mu}{1-\varepsilon} - \frac{d_1}{\theta} f_1^\theta\right) + \lambda(F - f_0 - f_1) + \nu(M - m_0 - m_1)$$

We provide hereafter the optimal solution when it is optimal to leave some fossil resources in the ground, i.e. $f_0 + f_1 < F$.

Proposition 4. *When it is optimal to choose $f_0 + f_1 < F$,the optimal solution is:*

$$m_0 = \frac{M}{\left(\frac{1+\rho}{1-\delta}\right)^{\frac{\theta-\phi}{1-\varepsilon-\theta}} \left(\frac{d_1}{d_0}\right)^{\frac{\phi}{1-\varepsilon-\theta}} + 1 - \delta}$$

$$f_0 = \left(\frac{d_0}{\alpha}\right)^{\frac{1}{\phi-\theta}} \left(\frac{1}{M}\left(\left(\frac{1+\rho}{1-\delta}\right)^{\frac{\theta-\phi}{1-\varepsilon-\theta}} \left(\frac{d_1}{d_0}\right)^{\frac{\phi}{1-\varepsilon-\theta}} + (1-\delta)\right)\right)^{\frac{\mu}{\phi-\theta}}$$

$$m_1 = \frac{\left(\frac{1+\rho}{1-\delta}\right)^{\frac{\theta-\phi}{1-\varepsilon-\theta}} \left(\frac{d_1}{d_0}\right)^{\frac{\phi}{1-\varepsilon-\theta}} - \delta}{\left(\frac{1+\rho}{1-\delta}\right)^{\frac{\theta-\phi}{1-\varepsilon-\theta}} \left(\frac{d_1}{d_0}\right)^{\frac{\phi}{1-\varepsilon-\theta}} + 1 - \delta} M$$

$$f_1 = \left(\frac{d_1}{\alpha}\right)^{\frac{1}{\phi-\theta}} \left(\frac{1}{M}\left(1 + (1+\rho)^{\frac{\phi-\theta}{1-\varepsilon-\theta}} (1-\delta)^{\frac{\theta\mu}{1-\varepsilon-\theta}} \left(\frac{d_0}{d_1}\right)^{\frac{\phi}{1-\varepsilon-\theta}}\right)\right)^{\frac{\mu}{\phi-\theta}}$$

The sign of dependency of the solutions on the parameters, summarized in Table 1.

Table 1: Signs of derivatives of the solutions with respect to the parameters

	m_0	m_1	f_i
δ	+	-	$+ \iff \varepsilon < 1$
d_j	$+ \iff \varepsilon > 1$		$+ \iff i \neq j$

Proof. See Appendix E. To sign the marginal effects in Table 1 we use the fact that the sums of resources from both periods are constant and that $1 - \varepsilon - \theta < 0$ and $\phi - \theta < 0$. □

Notice that the higher the recycling rate, the sooner the extraction of minerals and the investment in green capital.

Let us turn to the effect of an increase in the relative damage of use fossil resources in the first rather than in the second period, i.e. d_0/d_1. This may represent a short cut for cumulative nature of damages in pollution problems such as climate change. We find that an increase in this ratio does not necessarily put forward investment in green capital. In fact, for higher damage rate from fossils in the first period (and fixed damage rate in the second), the intuitive effect is that less fossil is used in the first period. As a consequence, the marginal utility of consumption in the first period is higher, making m_0 more valuable. This first effect calls for increasing m_0. However, a second effect, related to complementarity in consumption, comes from the fact that the marginal value of one resource is increasing in the other resource. This calls for shifting the use of minerals to the second period. The balance between these two effects is solved according to the willingness of the representative agent to shift utility across time: if the elasticity of intertemporal substitution is low (i.e. for high smoothing of consumption corresponding to $\varepsilon > 1$) the first effect dominates and m_0 increases with d_0, and vice versa.

4.2 Impact of productivities

We now study the two-period model in the extreme fully linear case in order to give salience to the role of productivities. We assume a linear utility function $u(q) = q$, and consider the case where both types of energy are perfect substitutes:

$$q_t = x_t + y_t$$

Given $\rho > 0$, extraction and consumption in $t = 0$ are a priori preferred. They can be delayed if and only if resource productivity increases sufficiently. Conditions (4.3) and (4.4) do not generally hold, as extraction takes place over a single period. To determine the period of extraction, we compare both sides of the conditions (4.3)-(4.4). As one can see in Table 2, the optimal timing of extraction depends on the evolution of the productivity of each resource relative to the rate of time preference, adjusted by the recycling rate δ in the case of minerals.

Table 2 : Optimal timing of extraction in the fully linear case

	$B_1/B_0 < (1+\rho)/(1-\delta)$	$B_1/B_0 > (1+\rho)/(1-\delta)$
$A_1/A_0 < 1+\rho$	$f_0 = F$ $f_1 = 0$ $m_0 = M$ $m_1 = 0$	$f_0 = F$ $f_1 = 0$ $m_0 = 0$ $m_1 = M$
$A_1/A_0 > 1+\rho$	$f_0 = 0$ $f_1 = F$ $m_0 = M$ $m_1 = 0$	$f_0 = 0$ $f_1 = F$ $m_0 = 0$ $m_1 = M$

Proposition 5. *In the fully linear case, a resource is optimally extracted at only one period.*

When productivities remain constant ($A_0 = A_1 = A$ and $B_0 = B_1 = B$), all resources are extracted in the first period and the maximised objective reaches $AF + \left(1 + \frac{\delta}{1+\rho}\right) BM$. Extraction takes place in the second period if and only if resource productivity increases enough. More precisely, minerals are extracted in second period if and only if $B_1/B_0 > (1+\rho)/(1-\delta)$ while fossils are extracted in second period if and only if $A_1/A_0 > 1+\rho$.

We therefore identify cases where it is optimal to delay extraction of mineral resources and investment in green capital. However our original mechanism is still at work. In fact, the optimality of postponing investment in green capital depends on the specific technological progress relative to a threshold, and this threshold is an increasing function of the recycling rate.

4.3 Leontief production of energy

Let us now turn to the polar extreme case where the representative consumer cannot substitute between the two types of energy:

$$q_t = \min\{\theta x_t, y_t\}$$

Since resource use can be shifted across time at no cost, one of the two resource constraints will be binding: if mineral (respectively fossil) resources are abundant, fossil (resp. mineral) resources are exhausted and additional mineral (resp. fossil) resources are not valuable. All other parameters given, one can identify specific values of M and F such that both resources are exhausted.

Consider the case of abundant mineral resources. Consumption is $q_0 = \theta A_0 f_0$, and $q_1 = \theta A_1 (F - f_0)$. The problem is essentially one where there is only one resource. The optimal allocation of scarce fossil resources requires that:

$$\frac{u'(\theta A_0 f_0^*)}{u'(\theta A_1 (F - f_0^*))} = \frac{1}{1+\rho}\frac{A_1}{A_0} \tag{4.6}$$

Assuming a concave utility function with $u'(q) \underset{q \to 0}{\longrightarrow} +\infty$, a unique value of $f_0^* \in (0, F)$ is defined since the left-hand-side of (4.6) is decreasing from $+\infty$ for $f_0 = 0$, down to 0 for $f_0 = F$. Discounting fosters early extraction (and for constant A, $f_0^* > F/2$ follows from the concavity of u). Moreover, f_0^* is decreasing (and f_1^* is increasing) in A_0, and increasing (while f_1^* is decreasing) in A_1, if the elasticity of intertemporal substitution in consumption is low, and vice versa (see Appendix F):

$$\frac{df_0^*}{dA_0} \begin{cases} > 0 \text{ if } \varepsilon \equiv -\frac{u''(q^*)q^*}{u'(q^*)} < 1 \\ < 0 \text{ if } \varepsilon \equiv -\frac{u''(q^*)q^*}{u'(q^*)} > 1 \end{cases}$$

Improved productivity of fossil resources in the first period relaxes the constraint, exerting an income effect and a substitution effect. The income effect tends to increase consumption in both periods through the reallocation of fossil resource use to the second period (and mineral use too, since $m_0^* = \frac{\theta A_0}{B_0} f_0^*$). A productivity improvement in the first period, however, also exerts a substitution effect, by increasing the relative cost of consuming fossil resources in the second rather than the first period, favoring earlier extraction. The balance between these two effects depends on the elasticity of intertemporal substitution of consumption in the utility function.

In the extreme case of a linear instantaneous utility function (i.e. $u' = 1$), when the representative household does not care about smoothing consumption over time, the solution is $f_0^* = F$ if $A_0 > A_1/(1+\rho)$ and $f_0^* = 0$ if $A_0 < A_1/(1+\rho)$.

Mutatis mutandis, when fossil resources are abundant, consumption is $q_0 = B_0 m_0$, and $q_1 = B_1 (M - (1-\delta) m_1)$. Again the problem is essentially, as one where there is only one resource and the solution satisfies:

$$\frac{u'(B_0 m_0^*)}{u'(B_1 (M - (1-\delta) m_0^*))} = \frac{1-\delta}{1+\rho} \frac{B_1}{B_0} \tag{4.7}$$

In this case the quantity of fossil resource used in each period is obtained as: $f_0^* = \frac{B_0}{\theta A_0} m_0^*$,

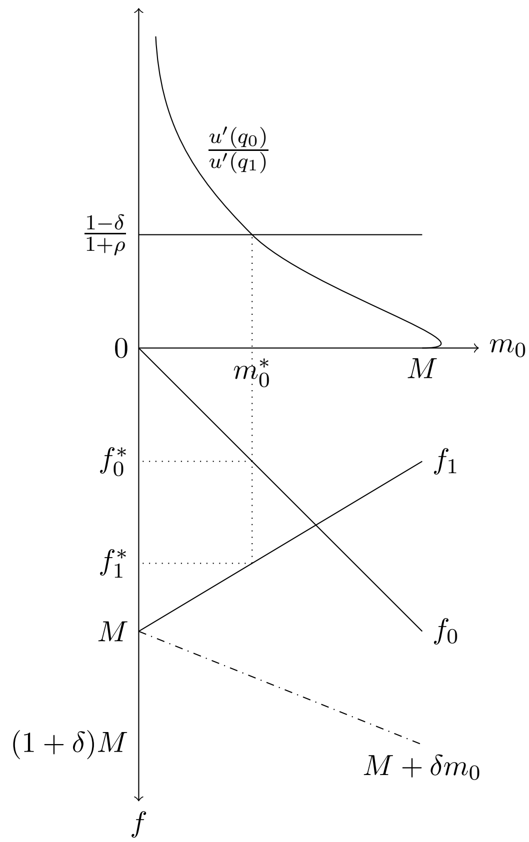

Figure 4 : Mineral and fossil extraction when energy services are perfect complements and fossil resources are abundant.

$f_1^* = \frac{B_1}{\theta A_1}(M - (1-\delta)\, m_0^*)$. Of course this is compatible with the case of abundant fossil resource only if $f_0^* + f_1^* < F$, for the solution m_0^* of (4.7), that is $m_0^* \left(\frac{B_0}{A_0} - \frac{B_1}{A_1}(1-\delta) \right) < \theta F - \frac{B_1}{A_1} M$. Therefore a sufficient condition for it to be satisfied is $\theta F > \left(\frac{B_0}{A_0} + (1+\delta) \frac{B_1}{A_1} \right) M$, for the stock of fossil resources be sufficient to cover the required quantity of fossil resource to be used when mineral resources are exhausted in the first period. See Figure 4 for an illustration where it is assumed that $\alpha = 1$ and $A = B$.

The case where the two resource constraints are simultaneously binding is peculiar. We should have (4.6)-(4.7) holding together with $m_0^* = \left(F - \frac{B_1}{\theta A_1} M \right) \left(\frac{B_0}{\theta A_0} - (1-\delta) \frac{B_1}{\alpha A_1} \right)^{-1}$. The first two conditions, with $\theta A_0 f_0^* = B_0 m_0^*$ and $\theta A_1 (F - f_0^*) = B_1 (M - (1-\delta)\, m_0^*)$, imply

$\frac{B_1}{B_0}(1-\delta) = \frac{A_1}{A_0}$, which is an improbable restriction on parameters.[17]

Figure 5 illustrates the impact of an increase in the recycling rate. We can see that the higher the share of recycled minerals, the earlier their use m_0^*.[18] We have the following :

Proposition 6. *When energy services from the two types of resources are perfect complements, an improvement in the recycling rate δ*

1. *has no effect on the optimal mineral consumption plan, if mineral resources are abundant relative to fossil resources ;*
2. *fosters early extraction of mineral resources, i.e. $\frac{dm_0^*}{d\delta} > 0$, and of fossil resources, if fossil resources are abundant relative to mineral resources, while its effect on fossil use in the second period is ambiguous, and depends on preferences with respect to the intertemporal profile of consumption ;*
3. *can change the nature of the economy from one with relatively abundant fossil resources to one where they are relatively rare.*

Démonstration. See Appendix F.2. □

An increase in the productivity of the mineral resource in the first period B_0 exerts an ambiguous effect on first period mineral extraction (see Appendix F.3). The solution m_0^* of (4.7), is subject to two countervailing effects. In terms of Figure 5, holding m_0 constant, the left hand side of (4.7) falls at the same time as its right hand side shifts downwards. The income effect gives incentives to postpone mineral extraction in order to be able to increase consumption in both periods. The substitution effect gives incentives to fasten extraction in order to produce more global output. Hence the decision reflects the willingness to smooth consumption overtime characterizing the representative household preferences. This is equi-

[17] For instance, in the parametric specification used to draw the figure or in the previous footnote, where $A_t = B_t$ and $\alpha = 1$, the parametric restriction does not hold as soon as $\delta \neq 0$.

[18] Indeed, the downward sloping schedule on the the left-hand-side of (4.7) shifts upwards, while the line on its right-hand-side shifts downwards in the (m_1, value) space.

valent to the effect of improved first period productivity of the fossil resource, when mineral resources are abundant.

Let us underscore that, when fossil resources are abundant, there is an asymmetry between the consequence of an improvement in first period productivity of the mineral resource B_0, and that of the recycling rate δ. Both events exert a positive income effect, since production possibilities expand, for a given amount of raw mineral resource. However, the means through which it is possible to shift consumption to the second period differ radically. In the case B_0 increases, if one keeps first period extraction constant, it entails an increase in first period consumption only. One can transfer part of the potential gain in consumption from the first to the second period by delaying extraction. In the case δ increases, instead, if one keeps first period extraction constant, it entails an increase in the second period consumption only. One can transfer part of the potential gain in consumption from the second to the first period by anticipating extraction.

Hence, this analysis allows us to show that, although an improved recycling rate is similar to an improved productivity of first period mineral resources (cf. Proposition 3) , it is distinct because the improvement in recycling requires time to deliver its fruits. This distinction between the two forms of technological progress has implications for the optimal timing of mineral resource extraction.

5 Conclusion

Some observers argue that renewable energy is not *manna from heaven*, as it requires specific equipment that relies on intensive use of exhaustible and finite mineral resources. Noticing that mineral embedded in green capital can be recycled, as opposed to fossil resources burned for energy production, we obtain original policy implications in favor of abundant and early investment in green capital for the production of renewable energy.

Our analysis has focused on the role of recycling in determining the optimal path of

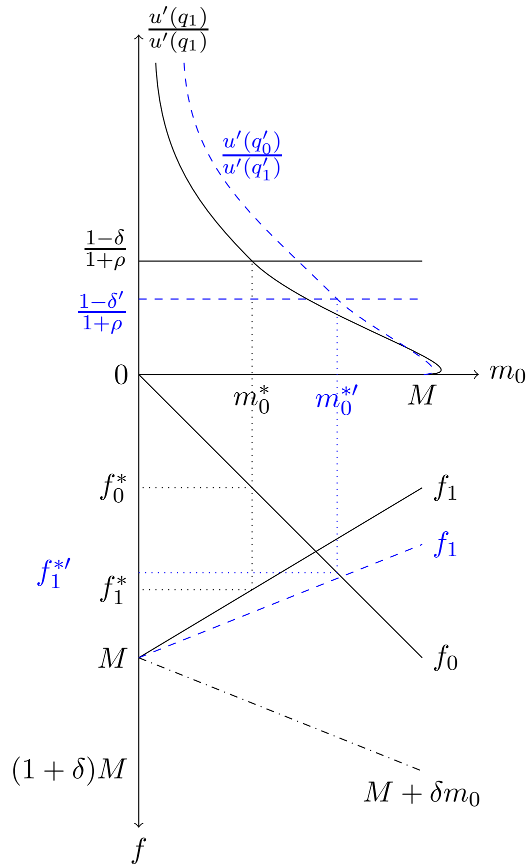

FIGURE 5 : Impact of increased recycling rate δ on mineral and fossil extraction when energy services are perfect complements and fossil resources are abundant.

extraction of fossil and mineral resources, and the investment in green capital. However, we have considered a constant, costless and exogenous recycling process. It would be relevant to check how robust our argument is to relaxing these assumptions. On its own the issue of the optimal choice of the recycling rate is interesting, and more so in our context as it could affect the timing of investment in green capital.

Moreover, we have adopted the normative approach of the benevolent social planner. However, it can be argued that market failures would lead to inefficient equilibria. Some market failures concern imperfect competition, both in the resource market and in the secondary market for resources, when there is recycling (see Ba & Mahenc (2016)). Other potential failures concern the thinness of markets for specific minerals and the joint production of several mineral resources (Fizaine (2015)). Under these conditions, public intervention can take several forms. We plan to explore these extensions in future work.

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A Benchmark case when the equipment is not recyclable

This appendix presents a special case of the problem analyzed in B. It is presented separately, as the reader may find it useful as an initial stage, and underpins the results presented in the main text early on in section 3.

Consider the case $\delta = 0$. As argued in the text $\forall t\ f_t, m_t > 0$, so that the non negativity constraints can be ignored. The planner problem is :

$$\max \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho}\right)^t \frac{\left(f_t^{\alpha} m_t^{1-\alpha}\right)^{1-\varepsilon}}{1-\varepsilon} + \lambda \left(F - \sum_{t=0}^{\infty} f_t\right) + \mu \left(M - \sum_{t=0}^{\infty} m_t\right) \tag{A.1}$$

The first order conditions are

$$\frac{\left(f_t^{\alpha} m_t^{1-\alpha}\right)^{1-\varepsilon}}{f_t} = \frac{\lambda}{\alpha} (1+\rho)^t \tag{A.2}$$

$$\frac{\left(f_t^{\alpha} m_t^{1-\alpha}\right)^{1-\varepsilon}}{m_t} = \frac{\mu}{1-\alpha} (1+\rho)^t \tag{A.3}$$

which imply a constant optimal input ratio

$$\frac{f_t}{m_t} = \frac{\mu}{\lambda} \frac{\alpha}{1-\alpha} \tag{A.4}$$

The marginal utility of consumption can therefore be written as $\left(f_t^{\alpha} m_t^{1-\alpha}\right)^{-\varepsilon} = \left(\frac{\mu}{\lambda}\frac{\alpha}{1-\alpha}\right)^{-\alpha\varepsilon} m_t^{-\varepsilon}$. Combining with (A.2)-(A.3) we get

$$f_t = \left(\frac{\alpha}{\lambda}\right)^{1+\frac{\alpha(1-\varepsilon)}{\varepsilon}} \left(\frac{1-\alpha}{\mu}\right)^{\frac{1-\alpha(1-\varepsilon)}{\varepsilon}-1} \left(\frac{1}{1+\rho}\right)^{\frac{t}{\varepsilon}} \tag{A.5}$$

$$m_t = \left(\frac{\alpha}{\lambda}\right)^{\frac{\alpha(1-\varepsilon)}{\varepsilon}} \left(\frac{1-\alpha}{\mu}\right)^{\frac{1+\alpha(1-\varepsilon)}{\varepsilon}} \left(\frac{1}{1+\rho}\right)^{\frac{t}{\varepsilon}} \tag{A.6}$$

Notice that consumption q_t declines over time given $\rho > 0$, at a slower pace the larger is ε, the parameter determining the willingness to smooth consumption over time.

The value of the resource constraints are pinned down by the conditions of full exhaustion in infinite time, i.e. (2.4) and (2.7) with the equality holding:

$$F = \left(\frac{\alpha}{\lambda}\right)^{1+\frac{\alpha(1-\varepsilon)}{\varepsilon}} \left(\frac{1-\alpha}{\mu}\right)^{\frac{1-\alpha(1-\varepsilon)}{\varepsilon}-1} \frac{1}{1-\left(\frac{1}{1+\rho}\right)^{\frac{1}{\varepsilon}}} \tag{A.7}$$

$$M = \left(\frac{\alpha}{\lambda}\right)^{\frac{\alpha(1-\varepsilon)}{\varepsilon}} \left(\frac{1-\alpha}{\mu}\right)^{\frac{1-\alpha(1-\varepsilon)}{\varepsilon}} \frac{1}{1-\left(\frac{1}{1+\rho}\right)^{\frac{1}{\varepsilon}}} \tag{A.8}$$

handing

$$\frac{\lambda}{\mu} = \frac{M}{F}\frac{\alpha}{1-\alpha} \tag{A.9}$$

This implies that the input ratio in (A.4) is always chosen

$$\frac{f_t}{m_t} = \frac{F}{M} \tag{A.10}$$

Using this in (A.6) we have the value

$$\lambda = \alpha F^{\alpha(1-\varepsilon)-1} M^{(1-\alpha)(1-\varepsilon)} \left(1-\left(\frac{1}{1+\rho}\right)^{\frac{1}{\varepsilon}}\right)^{-\varepsilon} \tag{A.11}$$

$$\mu = (1-\alpha) F^{\alpha(1-\varepsilon)} M^{(1-\alpha)(1-\varepsilon)-1} \left(1-\left(\frac{1}{1+\rho}\right)^{\frac{1}{\varepsilon}}\right)^{-\varepsilon} \tag{A.12}$$

Combining (A.3), (A.10), (A.11) and (A.12), we get the optimal path of resource extraction

$$f_t = F\left(1-(1+\rho)^{-\frac{1}{\varepsilon}}\right)(1+\rho)^{-\frac{t}{\varepsilon}} \tag{A.13}$$

$$m_t = M\left(1-(1+\rho)^{-\frac{1}{\varepsilon}}\right)(1+\rho)^{-\frac{t}{\varepsilon}} \tag{A.14}$$

thus

$$q_t = F^{\alpha} M^{1-\alpha}\left(1-(1+\rho)^{-\frac{1}{\varepsilon}}\right)(1+\rho)^{-\frac{t}{\varepsilon}} \quad . \tag{A.15}$$

B Solutions of the infinite horizon

In the general case where we do not assume that periods $\underline{t}$ to $\bar{t}$ are those with positive mineral extraction, the maximization program writes:

$$\max_{f_x,\, m_x} \sum_{x\geq 0} \frac{(1+\rho)^{-x}}{1-\varepsilon} \left(f_x^{\alpha} \left(K_{-1}\delta^{t+1} + \sum_{u=0}^{t} m_u \delta^{x-u} \right)^{1-\alpha} \right)^{1-\varepsilon}$$
$$+\lambda \left(F - \sum_{x\geq 0} f_x \right) + \nu \left(M - \sum_{x\geq 0} m_x \right) + \sum_{x\geq 0} \lambda_x f_x + \sum_{x\geq 0} \nu_x m_x$$

In the following, we simplify the notations by introducing:

$$\phi := \alpha\,(1-\varepsilon)$$
$$\mu := (1-\alpha)\,(1-\varepsilon)$$

To solve the program, we first assume in subsection B.1 that the positivity constraints always hold after a certain date $\underline{t}$, i.e. $\forall t \geq \underline{t},\ \lambda_t = \nu_t = 0$, which corresponds to an endless extraction of resources. Then in subsection B.2, we derive the optimal solution in the case where minerals are depleted at the initial period: $\forall t > 0,\ m_t = 0$. We show in Appendix C that these solutions are optimal under the conditions given in Proposition 1.

B.1 Endless extraction

We assume a positive extraction of both resources starting at a date $\underline{t}$, before which only fossils are extracted. The social planner's program rewrites:

$$\max \sum_{x=0}^{\underline{t}-1} \frac{(1+\rho)^{-x}}{1-\varepsilon} f_x^{\phi} \left(\delta^{x+1}K_{-1}\right)^{\mu} + \sum_{x\geq \underline{t}} \frac{(1+\rho)^{-x}}{1-\varepsilon} f_x^{\phi} K_x^{\mu} + \lambda \left(F - \sum_{x\geq 0} f_x \right) + \nu \left(M - \sum_{x\geq \underline{t}} m_x \right)$$

In the computations, we assume $\delta > 0$, but the solution extends to the limit cases $\delta = 0$. The log case $\varepsilon = 1$ is covered by the computations (only the program writes differently in this case).

The f.o.c.s are:

$$\begin{cases} (\partial f_t)_{t<s} & \alpha f_t^{\phi-1} \left(\delta K_{-1}\right)^{\mu} = \lambda \left(\frac{1+\rho}{\delta^{\mu}}\right)^{t} \\ (\partial f_t)_{t\geq s} & \alpha f_t^{\phi-1} K_t^{\mu} = \lambda \left(1+\rho\right)^{t} \\ (\partial m_t)_{t\geq s} & \sum\limits_{x\geq t} f_x^{\phi} K_x^{\mu-1} \left(\frac{\delta}{1+\rho}\right)^{x} = \frac{\nu}{1-\alpha}\delta^{t} \end{cases}$$

The f.o.c. on m_{t+1} can be expressed as follows:

$$\sum_{x\geq t} f_x^{\phi} K_x^{\mu-1}\left(\frac{\delta}{1+\rho}\right)^x - f_t^{\phi} K_t^{\mu-1}\left(\frac{\delta}{1+\rho}\right)^t = \frac{\nu}{1-\alpha}\delta^{t+1}$$

Then, subtracting the f.o.c. on m_{t+1} from the f.o.c. on m_t, we have:[19]

$$f_t^{\phi} K_t^{\mu-1} = \frac{\nu}{1-\alpha}(1-\delta)(1+\rho)^t \tag{B.1}$$

so that,

$$\begin{aligned} &\forall t < \underline{t},\ K_t = \delta^{t+1} K_{-1} \\ &\forall t \geq \underline{t},\ K_t = \left(\frac{\nu}{1-\alpha}(1-\delta)(1+\rho)^t\right)^{\frac{1}{\mu-1}} f_t^{\frac{\phi}{1-\mu}} \end{aligned} \tag{B.2}$$

Injecting this into the f.o.c. on f_t:

$$\begin{aligned} &\forall t < \underline{t},\ \lambda = \left(\frac{1+\rho}{\delta^{\mu}}\right)^{-t} \alpha f_t^{\phi-1}(\delta K_{-1})^{\mu} \\ &\forall t \geq \underline{t},\ \lambda = (1+\rho)^{-t} \alpha f_t^{\phi-1}\left(\frac{\nu}{1-\alpha}(1-\delta)(1+\rho)^t\right)^{\frac{\mu}{\mu-1}} f_t^{\frac{\phi\mu}{1-\mu}} \end{aligned}$$

Given that $\phi+\mu-1=-\varepsilon<0$:[20]

$$\begin{cases} \lambda = \alpha f_0^{\phi-1}(\delta K_{-1})^{\mu} \\ \forall t < \underline{t},\ f_t = \left(\frac{\lambda}{\alpha}\left(\frac{1+\rho}{\delta^{\mu}}\right)^t (\delta K_{-1})^{-\mu}\right)^{\frac{1}{\phi-1}} = f_0\left(\frac{1+\rho}{\delta^{\mu}}\right)^{\frac{t}{\phi-1}} \\ \forall t \geq \underline{t},\ f_t = \left(\frac{\lambda}{\alpha}(1+\rho)^{\frac{t}{1-\mu}}\left(\frac{\nu}{1-\alpha}(1-\delta)\right)^{\frac{\mu}{1-\mu}}\right)^{\frac{1-\mu}{\phi+\mu-1}} \end{cases}$$

Defining $r := (1+\rho)^{\frac{1}{\phi+\mu-1}} = (1+\rho)^{-1/\varepsilon} < 1$ and $R := \left(\frac{1+\rho}{\delta^{\mu}}\right)^{\frac{1}{\phi-1}}$, this system gives:

[19] One can retrieve this result by re-expressing the program in function of $(K_t)_t$ (instead of $(m_t)_t$), avoiding the implicit hypothesis on the convergence of the series in the f.o.c.s on m_t (which turns out to be true given the solution).

[20] In the case $\varepsilon = 0$ of a linear utility function, we have $\lambda = (1+\rho)^{\frac{1}{\mu-1}t}\alpha\left(\frac{\nu}{1-\alpha}(1-\delta)\right)^{\frac{\mu}{\mu-1}}$, which is impossible as $\nu > 0$. In this case, the only admissible solution is indeed the immediate exhaustion studied in subsection B.2.

$$\forall t < \underline{t},\ f_t = f_0 R^t$$
$$\forall t \geq \underline{t},\ f_t = f_S r^t \tag{B.3}$$

Combining the f.o.c.s of $f_{\underline{t}}$ and $f_{\underline{t}-1}$:

$$\frac{\alpha f_S^{\phi-1} r^{(\phi-1)\underline{t}} K_{\underline{t}}^{\mu}}{(1+\rho)^{\underline{t}}} = \alpha f_0^{\phi-1} R^{(\phi-1)(\underline{t}-1)} \left(\delta K_{-1}\right)^{\mu} \left(\frac{1+\rho}{\delta^{\mu}}\right)^{1-\underline{t}}$$
$$f_S = f_0 \left(r\delta \frac{K_{-1}}{K_{\underline{t}}}\right)^{\frac{\mu}{\phi-1}}$$

so that

$$\begin{aligned}
\sum_{t\geq 0} f_t &= f_0 \frac{1-R^{\underline{t}}}{1-R} + f_S \frac{r^{\underline{t}}}{1-r} \\
&= f_0 \left(\frac{1-R^{\underline{t}}}{1-R} + \left(r\delta \frac{K_{-1}}{K_{\underline{t}}}\right)^{\frac{\mu}{\phi-1}} \frac{r^{\underline{t}}}{1-r}\right) \\
&= f_S \left(\left(r\delta \frac{K_{-1}}{K_{\underline{t}}}\right)^{\frac{\mu}{1-\phi}} \frac{1-R^{\underline{t}}}{1-R} + \frac{r^{\underline{t}}}{1-r}\right)
\end{aligned}$$

The constraint on recoverable resource of fossils gives

$$f_0 = F \left(\frac{1-R^{\underline{t}}}{1-R} + \left(r\delta \frac{K_{-1}}{K_{\underline{t}}}\right)^{\frac{\mu}{\phi-1}} \frac{r^{\underline{t}}}{1-r}\right)^{-1}$$
$$f_S = F \left(\left(r\delta \frac{K_{-1}}{K_{\underline{t}}}\right)^{\frac{\mu}{1-\phi}} \frac{1-R^{\underline{t}}}{1-R} + \frac{r^{\underline{t}}}{1-r}\right)^{-1}$$

Turning to the minerals, we have from (B.2) and (B.3):

$$
\begin{aligned}
m_{\underline{t}} = K_{\underline{t}} - \delta^{\underline{t}+1} K_{-1} &= \left(\frac{\nu}{1-\alpha}(1-\delta)\right)^{\frac{1}{\mu-1}} f_S^{\frac{\phi}{1-\mu}} r^{\underline{t}} - \delta^{\underline{t}+1} K_{-1} \\
\forall t > \underline{t},\; m_t = K_t - \delta K_{t-1} &= \left(\frac{\nu}{1-\alpha}(1-\delta)(1+\rho)^t\right)^{\frac{1}{\mu-1}} f_S^{\frac{\phi}{1-\mu}} r^{\frac{\phi t}{1-\mu}} \\
&\quad - \delta\left(\frac{\nu}{1-\alpha}(1-\delta)(1+\rho)^{t-1}\right)^{\frac{1}{\mu-1}} f_S^{\frac{\phi}{1-\mu}} r^{\frac{\phi(t-1)}{1-\mu}} \\
&= \left(\frac{\nu}{1-\alpha}(1-\delta)\right)^{\frac{1}{\mu-1}} f_S^{\frac{\phi}{1-\mu}} r^{t-1}(r-\delta) =: K_s r^{t-\underline{t}-1}(r-\delta)
\end{aligned}
$$

Lastly, $K_{\underline{t}}$ is determined by the transversality condition on $(m_t)_t$:

$$
\begin{aligned}
M = \sum_{t \geq \underline{t}} m_t &= K_{\underline{t}} - \delta^{\underline{t}+1} K_{-1} + \sum_{t > \underline{t}} K_{\underline{t}} r^{t-\underline{t}-1}(r-\delta) \\
&= K_{\underline{t}} - \delta^{\underline{t}+1} K_{-1} + K_{\underline{t}} \frac{r-\delta}{1-r} = K_{\underline{t}} \frac{1-\delta}{1-r} - \delta^{\underline{t}+1} K_{-1}
\end{aligned}
$$

i.e.

$$
K_{\underline{t}} = \frac{1-r}{1-\delta}\left(M + \delta^{\underline{t}+1} K_{-1}\right)
$$

Finally, we obtain:

$$\begin{aligned}
r &= (1+\rho)^{-\frac{1}{\varepsilon}} \\
R &= \left(\frac{1+\rho}{\delta^{\mu}}\right)^{\frac{1}{\phi-1}} \\
f_0 &= \left(\frac{1-R^{\underline{t}}}{1-R} + \left(\frac{r\delta\left(1-\delta\right)K_{-1}}{\left(1-r\right)\left(M+\delta^{\underline{t}+1}K_{-1}\right)}\right)^{\frac{\mu}{\phi-1}} \frac{r^{\underline{t}}}{1-r}\right)^{-1} F \\
f_S &= \left(\left(\frac{r\delta\left(1-\delta\right)K_{-1}}{\left(1-r\right)\left(M+\delta^{\underline{t}+1}K_{-1}\right)}\right)^{\frac{\mu}{1-\phi}} \frac{1-R^{\underline{t}}}{1-R} + \frac{r^{\underline{t}}}{1-r}\right)^{-1} F \\
\forall t < s, \quad f_t &= f_0 R^t \\
\forall t \geq s, \quad f_t &= f_S r^t \\
\forall t < s, \quad m_t &= 0 \\
m_s &= \frac{1-r}{1-\delta}M - \frac{r-\delta}{1-\delta}\delta^{\underline{t}+1}K_{-1} \\
\forall t > s, \quad m_t &= \frac{1-r}{1-\delta}\left(M+\delta^{\underline{t}+1}K_{-1}\right)\left(1-\frac{\delta}{r}\right) r^{t-\underline{t}}
\end{aligned}$$

The positivity constraints hold for $\delta < r$ and for $\underline{t}$ such that $M \geq \delta^{\underline{t}+1}K_{-1}\frac{r-\delta}{1-r}$.

B.2 Immediate exhaustion

In this case, $\forall t > 0,\ m_t = 0$. We also assume that $\forall t,\ f_t > 0$ (see subsection C.1 for the justification). As $\forall t \geq 0,\ K_t = m_0\delta^t$, the program writes:

$$\max \sum_{t\geq 0} (1+\rho)^{-t} \frac{m_0^{\mu}}{1-\varepsilon} f_t^{\phi} \delta^{\mu t} + \lambda \left(F - \sum_{t\geq 0} f_t\right) + \nu\left(M - m_0\right)$$

The objective is increasing in m_0, so it should be set to its maximum: $m_0 = M$.

Then, the f.o.c. on f_t writes:

$$\alpha f_t^{\phi-1} M^{\mu} \delta^{\mu t} = \lambda\left(1+\rho\right)^t$$

i.e.

$$f_t = \left(\frac{\lambda}{\alpha M^{\mu}}\left(\frac{1+\rho}{\delta^{\mu}}\right)^t\right)^{\frac{1}{\phi-1}}$$

Defining $R := \left(\frac{1+\rho}{\delta^\mu}\right)^{\frac{1}{\phi-1}}$ and $f_0 := \left(\frac{\lambda}{\alpha M^\mu}\right)^{\frac{1}{\phi-1}}$, we have:

$$\forall t \geq 0, \quad f_t = f_0 R^t$$

To conclude, we will assume that $R < 1$, which is satisfied as long as $\delta \geq r$ (except in the degenerate case $\delta = 1 + \rho = R = 1$ for which there is no solution because the supremum of the objective is infinite and cannot be attained).[21]

The transversality condition must be saturated, as the program is increasing in f_t for all t. This gives:

$$F = \sum_{t \geq 0} f_t = \frac{f_0}{1-R}$$

i.e. $f_0 = F(1-R)$, and we obtain:

$$\forall t \geq 0, \quad f_t = F(1-R)R^t$$

C Optimality of the solutions

In this section, we demonstrate the results of Proposition 1.We show that it is never optimal to interrupt the extraction when $\delta < r$ in C.1. Then we derive the solution when minerals are depleted in a finite time and show that it is sub-optimal in C.2. Finally, we use all this to prove Proposition 1 in the case $\delta < r$ in C.3, and we treat the case $\delta \geq r$ in C.4.

C.1 Interruption of extraction

It is never optimal to let K or f be nil at any period because the marginal welfare goes to $+\infty$ when consumption is nil.

Let us now show that for $\delta < r$, it is never optimal to interrupt mineral extraction, i.e. $\delta < r \implies \exists \underline{t}, \bar{t} \geq \underline{t},\ m_t > 0 \iff t \in [\underline{t};\bar{t}]$.

Let $(m_t, f_t)_{t \geq 0}$ be an optimal solution and let $\bar{t}$ be such that $m_{\bar{t}} > 0$ and such that $\{t | m_t > 0\} \neq \emptyset$. We define $\tau := \min_{t > \bar{t}} \{t | m_t > 0\}$ in order to prove that $\tau = \bar{t} + 1$, i.e. that interruption of mineral extraction is suboptimal. Let us assume *ad absurdo* that $\tau \neq \bar{t} + 1$, so that $m_{\bar{t}+1} = 0$ and $m_{\tau-1} = 0$.

[21] Indeed, $\delta \geq r \implies \delta^{\phi-1} \leq \frac{r^{\mu+\phi-1}}{\delta^\mu} \implies 1 > \delta \geq R$

Then, as $\phi < 1$ and $\varepsilon > 0$, we deduce from $\delta < r$:

$$1 > \left(\frac{\delta}{r}\right)^{\frac{\varepsilon}{1-\phi}} > \left(\frac{\delta^{\tau-1-\bar{t}}}{r}\right)^{\frac{\varepsilon}{1-\phi}} = (1+\rho)^{\frac{1}{1-\phi}} \left(\frac{m_{\tau-1}}{K_{\bar{t}}} + \delta^{\tau-1-\bar{t}}\right)^{\frac{\varepsilon}{1-\phi}}$$
$$= (1+\rho)^{1-\frac{\phi}{\phi-1}} \left(\frac{K_{\tau-1}}{K_{\bar{t}}}\right)^{\frac{\mu\phi}{\phi-1}+1-\mu} = (1+\rho) \left(\frac{K_{\bar{t}}}{K_{\tau-1}}\right)^{\mu-1} \left(\frac{f_{\bar{t}}}{f_{\tau-1}}\right)^{\phi} \tag{C.1}$$

where we used the f.o.c.s on $f_{\bar{t}}$ and $f_{\bar{t}+1}$ to find the last equality. We thus have:

$$(1-\alpha)(1+\rho)^{-\bar{t}} f_{\bar{t}}^{\phi} K_{\bar{t}}^{\mu-1} < (1-\alpha)(1+\rho)^{-(\tau-1)} f_{\tau-1}^{\phi} K_{\tau-1}^{\mu-1}$$

Besides, taking into account the non-negativity constraints $\nu_{\tau-1} \geq 0$ and $\nu_{\bar{t}+1} \geq 0$ in equation B.1, we have:

$$(1-\alpha)(1+\rho)^{-\bar{t}} f_{\bar{t}}^{\phi} K_{\bar{t}}^{\mu-1} = (1-\delta)\nu + \delta\nu_{\bar{t}+1} \geq (1-\delta)\nu - \nu_{\tau-1} = (1-\alpha)(1+\rho)^{-(\tau-1)} f_{\tau-1}^{\phi} K_{\tau-1}^{\mu-1}$$

These two inequalities contradict, so we deduce that $\tau = \bar{t}+1$, i.e. that
$\nexists \underline{t}, \tau > \underline{t}\; m_{\underline{t}} > 0 \wedge m_{\underline{t}+1} = 0 \wedge m_{\tau} = 0$, Q.E.D.

C.2 Exhaustion in a finite time

Let $\bar{t} \in \mathbb{N}^*$ be the last period at which minerals are extracted. We assume in this subsection that extraction takes place from the initial period on. The program can be decomposed in two eras, during and after the extraction of minerals:

$$\max \sum_{x=0}^{\bar{t}} \frac{(1+\rho)^{-x}}{1-\varepsilon} f_x^{\phi} K_x^{\mu} + \sum_{x>\bar{t}} \frac{(1+\rho)^{-x}}{1-\varepsilon} f_x^{\phi} K_{\bar{t}}^{\mu} \delta^{\mu(x-\bar{t})} + \lambda \left(F - \sum_{x\geq 0} f_x\right) + \nu \left(M - \sum_{x\geq 0} m_x\right)$$

f.o.c.s:

$$\begin{cases} (\partial f_t)_{t\leq\bar{t}} & \alpha f_t^{\phi-1} K_t^{\mu}(1+\rho)^{-t} = \lambda \\ (\partial f_t)_{t>\bar{t}} & \alpha f_t^{\phi-1} K_{\bar{t}}^{\mu}\delta^{\mu(t-\bar{t})}(1+\rho)^{-t} = \lambda \\ (\partial m_t)_{t\leq\bar{t}} & (1-\alpha)\sum_{x=t}^{\bar{t}}(1+\rho)^{-x} f_x^{\phi} K_x^{\mu-1}\delta^{x-t} + (1-\alpha)\sum_{x>\bar{t}}(1+\rho)^{-x} f_x^{\phi} K_{\bar{t}}^{\mu-1}\delta^{\mu(x-\bar{t})+\bar{t}-t} = \nu \end{cases}$$

The f.o.c. on m_{t+1} can be expressed as follows for $t < \bar{t}$:

$$\frac{1-\alpha}{\delta}\sum_{x=t}^{\bar{t}}(1+\rho)^{-x} f_x^{\phi} K_x^{\mu-1}\delta^{x-t} - (1-\alpha)(1+\rho)^{-t}\frac{f_t^{\phi}K_t^{\mu-1}}{\delta} + \frac{1-\alpha}{\delta}\sum_{x>\bar{t}}(1+\rho)^{-x} f_x^{\phi} K_{\bar{t}}^{\mu-1}\delta^{\mu(x-\bar{t})+\bar{t}-t} = \nu$$

Then, $\forall t < \bar{t}$, $\frac{(\partial m_t)}{\delta} - (\partial m_{t+1})$ yields:

$$(1-\alpha)(1+\rho)^{-t} f_t^{\phi} K_t^{\mu-1} = \nu(1-\delta)$$

so that

$$\forall t < \bar{t}, \quad K_t = \left(\frac{\nu}{1-\alpha}(1-\delta)(1+\rho)^t\right)^{\frac{1}{\mu-1}} f_t^{\frac{\phi}{1-\mu}} \tag{C.2}$$

Furthermore, $(\partial m_{\bar{t}})$ gives $(1-\alpha)\sum_{x\geq\bar{t}}(1+\rho)^{-x} f_x^{\phi} K_{\bar{t}}^{\mu-1}\delta^{\mu(x-\bar{t})} = \nu$, *i.e.*

$$K_{\bar{t}} = \left(\frac{1-\alpha}{\nu}\sum_{x\geq\bar{t}}(1+\rho)^{-x} f_x^{\phi}\delta^{\mu(x-\bar{t})}\right)^{\frac{1}{1-\mu}} \tag{C.3}$$

Injecting (C.2) into the f.o.c. on f_t yields $\forall t < \bar{t}$,

$$\begin{aligned}\lambda &= \alpha f_t^{\phi-1+\frac{\phi\mu}{1-\mu}}\left(\frac{\nu}{1-\alpha}(1-\delta)\right)^{\frac{\mu}{\mu-1}}(1+\rho)^{\frac{1}{\mu-1}t} \\ &= \alpha f_0^{\phi-1+\frac{\phi\mu}{1-\mu}}\left(\frac{\nu}{1-\alpha}(1-\delta)\right)^{\frac{\mu}{\mu-1}}\end{aligned}$$

i.e.

$$f_t = \left(\frac{\lambda}{\alpha}(1+\rho)^{\frac{1}{1-\mu}t}\left(\frac{\nu}{1-\alpha}(1-\delta)\right)^{\frac{\mu}{1-\mu}}\right)^{\frac{1-\mu}{\phi+\mu-1}}$$

Defining $r := (1+\rho)^{\frac{1}{\phi+\mu-1}} < 1$ and $f_< := \left(\frac{\lambda}{\alpha}\right)^{\frac{1-\mu}{\phi+\mu-1}} \left(\frac{\nu}{1-\alpha}(1-\delta)\right)^{\frac{\mu}{\phi+\mu-1}}$, we have:

$$\forall t < \bar{t}, \quad f_t = f_< r^t \tag{C.4}$$

For $t > \bar{t}$, the f.o.c. on f_t yields:[22]

$$f_t = \left(\frac{\lambda \delta^{\mu \bar{t}}}{\alpha K_{\bar{t}}^{\mu}} \left(\frac{1+\rho}{\delta^{\mu}}\right)^t\right)^{\frac{1}{\phi-1}}$$

Defining $R := \left(\frac{1+\rho}{\delta^\mu}\right)^{\frac{1}{\phi-1}}$ and $f_> := \left(\frac{\lambda \delta^{\mu \bar{t}}}{\alpha K_{\bar{t}}^{\mu}}\right)^{\frac{1}{\phi-1}}$, we have:

$$\forall t > \bar{t}, \quad f_t = f_> R^t \tag{C.5}$$

In the following, we assume that $R < 1$.

Combining the f.o.c. on $f_{\bar{t}}$ with the f.o.c. on $f_{\bar{t}+1}$:

$$f_{\bar{t}}^{\phi-1}(1+\rho) = f_{\bar{t}+1}^{\phi-1}\delta^\mu = f_>^{\phi-1} R^{(\phi-1)(\bar{t}+1)}\delta^\mu$$

This gives $f_{\bar{t}}$:

$$f_{\bar{t}} = f_> R^{\bar{t}} \tag{C.6}$$

The transversality condition on fossils gives:

$$\begin{aligned} F &= f_< \sum_{t=0}^{\bar{t}-1} r^t + f_> \sum_{t \geq \bar{t}} R^t \\ &= f_< \frac{1-r^{\bar{t}}}{1-r} + f_> \frac{R^{\bar{t}}}{1-R} \end{aligned} \tag{C.7}$$

Injecting (C.4) into (C.2) for $t < \bar{t}$:

$$K_t = \left(\frac{\nu}{1-\alpha}(1-\delta)(1+\rho)^t\right)^{\frac{1}{\mu-1}} f_<^{\frac{\phi}{1-\mu}} r^{\frac{\phi}{1-\mu}t}$$

[22]For $\delta = 0$, marginal welfare goes to infinity for each $t > \bar{t}$, as $K_t = 0$. Hence, it is obviously suboptimal not to extract at every period in this case. In the following, we assume $\delta > 0$.

So that,

$$K_0 = \left(\frac{\nu}{1-\alpha}(1-\delta)\right)^{\frac{1}{\mu-1}} f_<^{\frac{\phi}{1-\mu}} \tag{C.8}$$

Hence

$$\forall t < \bar{t}, \quad K_t = K_0 (1+\rho)^{\frac{t}{\mu-1}} r^{\frac{\phi}{1-\mu}t} = K_0 (1+\rho)^{\frac{1}{\phi+\mu-1}t} = K_0 r^t \tag{C.9}$$

which gives

$$\begin{aligned}\forall t \in [1; \bar{t}-1], \quad m_t &= K_t - \delta K_{t-1} \\ &= K_0 \left(1 - \frac{\delta}{r}\right) r^t\end{aligned}$$

The transversality condition on minerals gives:

$$\begin{aligned}M = \sum_{t=0}^{\bar{t}} m_t &= K_0 - \delta K_{-1} + \sum_{t=1}^{\bar{t}-1} K_0 r^t \left(1 - \frac{\delta}{r}\right) + m_{\bar{t}} \\ m_{\bar{t}} &= M + \delta K_{-1} - K_0 \left(1 + (r-\delta)\frac{1-r^{\bar{t}-1}}{1-r}\right)\end{aligned}$$

Hence,

$$\begin{aligned}K_{\bar{t}} = m_{\bar{t}} + \delta K_{\bar{t}-1} &= M + \delta K_{-1} - K_0 \left(1 + (r-\delta)\frac{1-r^{\bar{t}-1}}{1-r}\right) + \delta K_0 r^{\bar{t}-1} \\ &= M + \delta K_{-1} - K_0 \frac{1-r^{\bar{t}}}{1-r}(1-\delta) = \bar{M} - aK_0 \end{aligned} \tag{C.10}$$

with $a := \frac{1-r^{\bar{t}}}{1-r}(1-\delta)$ and $\bar{M} := M + \delta K_{-1}$.

Using (C.4) in the f.o.c. on f_0 together with the f.o.c. on $f_{\bar{t}+1}$:

$$f_<^{\phi-1} K_0^{\mu} = \delta^{-\mu\bar{t}} f_>^{\phi-1} K_{\bar{t}}^{\mu}$$

Injecting (C.7) and (C.10) into this:[23]

$$
\begin{aligned}
f_<^{\frac{\phi-1}{\mu}} f_>^{-\frac{\phi-1}{\mu}} K_0 &= \delta^{-\bar{t}}\left(\bar{M} - aK_0\right) \\
f_<^{\frac{\phi-1}{\mu}} \left(\left(F - f_< \frac{1-r^{\bar{t}}}{1-r}\right)\frac{1-R}{R^{\bar{t}}}\right)^{-\frac{\phi-1}{\mu}} &= \delta^{-\bar{t}}\left(\frac{\bar{M}}{K_0} - a\right) \qquad \text{(C.11)}
\end{aligned}
$$

The f.o.c. on $m_{\bar{t}}$ gives:

$$
\begin{aligned}
(1-\alpha)(1+\rho)^{-\bar{t}} f_{\bar{t}}^{\phi} K_{\bar{t}}^{\mu-1} + (1-\alpha)\sum_{x>\bar{t}} (1+\rho)^{-x} f_x^{\phi} K_{\bar{t}}^{\mu-1}\delta^{\mu(x-\bar{t})} &= (1-\alpha)\sum_{x\geq\bar{t}} R^x f_>^{\phi} K_{\bar{t}}^{\mu-1}\delta^{-\mu\bar{t}} \\
&= (1-\alpha) f_>^{\phi} K_{\bar{t}}^{\mu-1}\delta^{-\mu\bar{t}}\frac{R^{\bar{t}}}{1-R} = \nu
\end{aligned}
$$

Using $\frac{K_0^{\mu-1} f_<^{\phi}}{1-\delta} = \frac{\nu}{1-\alpha}$ from the (C.8), we have from last equation:

$$
f_>^{\phi} K_{\bar{t}}^{\mu-1}\delta^{-\mu\bar{t}}\frac{R^{\bar{t}}}{1-R} = \frac{K_0^{\mu-1} f_<^{\phi}}{1-\delta} \qquad \text{(C.12)}
$$

Injecting (C.7) and (C.10) into this:

$$
\left(\left(F - f_< \frac{1-r^{\bar{t}}}{1-r}\right)\frac{1-R}{R^{\bar{t}}}\right)^{\frac{\phi}{\mu-1}} \left(\frac{\bar{M}}{K_0} - a\right) = \left(\frac{f_<^{\phi}}{1-\delta}\frac{1-R}{R^{\bar{t}}}\delta^{\mu\bar{t}}\right)^{\frac{1}{\mu-1}} \qquad \text{(C.13)}
$$

Combining this with (C.11) we have an equation in $f_<$:

$$
\begin{aligned}
f_<^{\frac{\phi-1}{\mu}-\frac{\phi}{\mu-1}} \left(\left(F - f_< \frac{1-r^{\bar{t}}}{1-r}\right)\frac{1-R}{R^{\bar{t}}}\right)^{\frac{\phi}{\mu-1}-\frac{\phi-1}{\mu}} &= \delta^{-\bar{t}}\left(\frac{\delta^{\mu\bar{t}}}{1-\delta}\frac{1-R}{R^{\bar{t}}}\right)^{\frac{1}{\mu-1}} \\
\left(\left(\frac{F}{f_<} - \frac{1-r^{\bar{t}}}{1-r}\right)\frac{1-R}{R^{\bar{t}}}\right)^{\frac{\mu+\phi-1}{\mu(\mu-1)}} &= \left(\frac{\delta^{\bar{t}}}{1-\delta}\frac{1-R}{R^{\bar{t}}}\right)^{\frac{1}{\mu-1}} \\
\left(\frac{F}{f_<} - \frac{1-r^{\bar{t}}}{1-r}\right)\frac{1-R}{R^{\bar{t}}} &= b^{\mu} \qquad \text{(C.14)}
\end{aligned}
$$

where $b := \left(\frac{\delta^{\bar{t}}}{1-\delta}\frac{1-R}{R^{\bar{t}}}\right)^{\frac{1}{\mu+\phi-1}}$.

$$
f_< = \left(b^{\mu}\frac{R^{\bar{t}}}{1-R} + \frac{1-r^{\bar{t}}}{1-r}\right)^{-1} F = \frac{1-\delta}{a+c} F \qquad \text{(C.15)}
$$

[23] In the log case, $\varepsilon = 1$, $R = r < 1$, $f_< = f_>$ and the solution derived below extends to this case.

where:

$$c := b^{\mu} \frac{R^{\bar{t}}}{1-R}(1-\delta) = \left(\frac{1-\delta}{\delta^{\bar{t}}} \frac{R^{\bar{t}}}{1-R}\right)^{\frac{-\mu}{\mu+\phi-1}} \frac{R^{\bar{t}}}{1-R}(1-\delta) = \left((1-\delta)\frac{R^{\bar{t}}}{1-R}\right)^{\frac{\phi-1}{\mu+\phi-1}} \delta^{\frac{\mu\bar{t}}{\mu+\phi-1}}$$

Injecting (C.14) into (C.11) (at the second line), we deduce K_0:

$$\begin{aligned} K_0 &= \bar{M}\left(f_<^{\frac{\phi-1}{\mu}}\left(\left(F - f_< \frac{1-r^{\bar{t}}}{1-r}\right)\frac{1-R}{R^{\bar{t}}}\right)^{-\frac{\phi-1}{\mu}} \delta^{\bar{t}} + a\right)^{-1} \\ &= \frac{\bar{M}}{b^{1-\phi}\delta^{\bar{t}} + a} \\ &= \frac{\bar{M}}{a+c} \end{aligned} \tag{C.16}$$

Indeed,

$$b^{1-\phi}\delta^{\bar{t}} = \left(\frac{\delta^{\bar{t}}}{1-\delta}\frac{1-R}{R^{\bar{t}}}\right)^{\frac{1-\phi}{\mu+\phi-1}} \delta^{\bar{t}} = \left((1-\delta)\frac{R^{\bar{t}}}{1-R}\right)^{\frac{\phi-1}{\mu+\phi-1}} \delta^{\frac{\mu\bar{t}}{\mu+\phi-1}} = c$$

Injecting (C.16) into (C.10), we deduce $K_{\bar{t}}$:

$$K_{\bar{t}} = \frac{c\bar{M}}{a+c} = cK_0 = b^{1-\phi}\delta^{\bar{t}}K_0 \tag{C.17}$$

We get $f_>$ from (C.7) and (C.15):

$$\begin{aligned} f_> &= \frac{1-R}{R^{\bar{t}}}\left(F - f_< \frac{1-r^{\bar{t}}}{1-r}\right) \\ &= b^{\mu} f_< \end{aligned} \tag{C.18}$$

Finally, the inter-temporal welfare writes:

$$\begin{aligned}
W_{\bar{t}} &= \frac{f_0^{\phi} m_0^{\mu}}{1-\varepsilon} + \sum_{x=1}^{\bar{t}-1} (1+\rho)^{-x} \frac{f_x^{\phi} K_x^{\mu}}{1-\varepsilon} + (1+\rho)^{-\bar{t}} \frac{f_{\bar{t}}^{\phi} K_{\bar{t}}^{\mu}}{1-\varepsilon} + \sum_{x>\bar{t}} (1+\rho)^{-x} \frac{f_x^{\phi} K_{\bar{t}}^{\mu}}{1-\varepsilon} \delta^{\mu(x-\bar{t})} \\
&= \frac{1}{1-\varepsilon} \left(f_<^{\phi} K_0^{\mu} + \sum_{x=1}^{\bar{t}-1} (1+\rho)^{-x} f_<^{\phi} r^{\phi x} K_0^{\mu} r^{\mu x} + (1+\rho)^{-\bar{t}} f_>^{\phi} R^{\phi \bar{t}} K_{\bar{t}}^{\mu} + \sum_{x>\bar{t}} (1+\rho)^{-x} f_>^{\phi} R^{\phi x} K_{\bar{t}}^{\mu} \delta^{\mu(x-\bar{t})} \right) \\
&= \frac{1}{1-\varepsilon} \left(f_<^{\phi} K_0^{\mu} \frac{1-\left(\frac{r^{\phi+\mu}}{1+\rho}\right)^{\bar{t}}}{1-\frac{r^{\phi+\mu}}{1+\rho}} + (1+\rho)^{-\bar{t}} b^{\mu\phi} f_<^{\phi} R^{\phi\bar{t}} b^{\mu(1-\phi)} \delta^{\mu\bar{t}} K_0^{\mu} + f_>^{\phi} K_{\bar{t}}^{\mu} \delta^{-\mu\bar{t}} \frac{R^{\bar{t}+1}}{1-R} \right) \\
&= \frac{1}{1-\varepsilon} \left(f_<^{\phi} K_0^{\mu} \left(\frac{1-\left(\frac{r^{\phi+\mu}}{1+\rho}\right)^{\bar{t}}}{1-\frac{r^{\phi+\mu}}{1+\rho}} + b^{\mu} \delta^{\mu\bar{t}} \left(R^{\phi\bar{t}} (1+\rho)^{-\bar{t}} + \delta^{-\mu\bar{t}} \frac{R^{\bar{t}+1}}{1-R} \right) \right) \right) \\
&= \frac{1}{1-\varepsilon} \left(F^{\phi} \bar{M}^{\mu} \left(\frac{1-\delta}{a+c} \right)^{\phi} (a+c)^{-\mu} \left(\frac{1-r^{\bar{t}}}{1-r} + b^{\mu} \frac{R^{\bar{t}}}{1-R} \right) \right) \\
&= \frac{1}{1-\varepsilon} \left(F^{\phi} \bar{M}^{\mu} (1-\delta)^{\phi-1} (a+c)^{1-\phi-\mu} \right)
\end{aligned}$$

To show that extraction in a finite time is not optimal, we derive welfare with respect to the last period of extraction:

$$\begin{aligned}
\frac{dW}{d\bar{t}} &= W \frac{1-\phi-\mu}{a+c} \frac{d(a+c)}{d\bar{t}} \\
\frac{a+c}{1-\phi-\mu} \frac{1}{W} \frac{d\ln W}{d\bar{t}} &= \frac{da}{d\bar{t}} + \frac{dc}{d\bar{t}} \\
&= -\ln(r)\, r^{\bar{t}} \frac{1-\delta}{1-r} + c \left(\ln\left(R^{\frac{\phi-1}{\mu+\phi-1}} \right) + \ln\left(\delta^{\frac{\mu}{\mu+\phi-1}} \right) \right) \\
&= -\ln(r)\, r^{\bar{t}} \frac{1-\delta}{1-r} + r^{\bar{t}} \left(\frac{1-\delta}{1-R} \right)^{\frac{\phi-1}{\mu+\phi-1}} \left(\ln\left(\frac{r}{\delta^{\frac{\mu}{\mu+\phi-1}}} \right) + \ln\left(\delta^{\frac{\mu}{\mu+\phi-1}} \right) \right) \\
&= -\ln(r)\, r^{\bar{t}} \left(\frac{1-\delta}{1-r} - \left(\frac{1-\delta}{1-R} \right)^{\frac{\phi-1}{\mu+\phi-1}} \right)
\end{aligned}$$

It is optimal to delay the exhaustion of minerals if and only if $\frac{d\ln W}{dt} > 0$. Notice that

$$\begin{aligned}
\frac{1}{W}\frac{d\ln W}{d\bar{t}} &> 0 \\
\iff \frac{1-\delta}{1-r} &> \left(\frac{1-\delta}{1-R}\right)^{\frac{\phi-1}{\mu+\phi-1}} \\
\iff 1-R &> (1-\delta)^{\frac{-\mu}{\phi-1}}(1-r)^{\frac{\mu+\phi-1}{\phi-1}} \\
\iff 1-\frac{r^{\frac{\mu+\phi-1}{\phi-1}}}{\delta^{\frac{\mu}{\phi-1}}} &> \left(\frac{1-\delta}{1-r}\right)^{\frac{\mu}{1-\phi}}(1-r)
\end{aligned}$$

Defining $e := \frac{\mu}{1-\phi}$ and $g_r(\delta) := 1-\left(\frac{\delta}{r}\right)^e r - \left(\frac{1-\delta}{1-r}\right)^e (1-r)$, we have:

$$\frac{1}{W}\frac{d\ln W}{d\bar{t}} > 0 \iff g_r(\delta) > 0$$

Yet,

$$g_r^{'}(\delta) = e\left(\left(\frac{1-\delta}{1-r}\right)^{e-1} - \left(\frac{\delta}{r}\right)^{e-1}\right)$$

For $e \in (0;1)$: $g_r^{'}(\delta) > 0 \iff \frac{1-\delta}{1-r} < \frac{\delta}{r} \iff r < \delta$

while for $e < 0$, the inverse is true: $g_r^{'}(\delta) > 0 \iff \delta < r$.

In addition, $0 < \varepsilon < 1 \implies (1-\varepsilon) - \alpha(1-\varepsilon) < 1 - \alpha(1-\varepsilon) \implies e = \frac{(1-\alpha)(1-\varepsilon)}{1-\alpha(1-\varepsilon)} \in (0;1)$ while $\varepsilon > 1 \implies e < 0$ (in the limit case $\varepsilon = 1,\ g_r = 0$).

For $\varepsilon < 1$, as $g_r(r) = 0$ and g_r is strictly decreasing below r and strictly increasing above r, we deduce that $\forall r,\ g_r \geq 0$ and that $\forall r, \forall \delta \neq r\ g_r(\delta) > 0$.

For $\varepsilon > 1$, the same reasoning shows that $\forall r,\ g_r \leq 0$ and that $\forall r, \forall \delta \neq r\ g_r(\delta) < 0$.

Given that $W > 0 \iff \varepsilon < 1, \varepsilon \neq 1 \implies \forall \delta \neq r,\ \frac{d\ln W}{dt} > 0$.

The solutions extend to the log case $\varepsilon = 1$, but the formula of intertemporal welfare does not. Let us compare in this case $W_{\bar{t}+1}$ and $W_{\bar{t}}$.

$$W_{\bar{t}+1} - W_{\bar{t}} = \sum_{t=0}^{\bar{t}} (1+\rho)^{-t} \ln\left(\left(\bar{M}\frac{1-r}{1-\delta}\right)^{1-\alpha} F^{\alpha} r^{t}\right) + \sum_{t>\bar{t}} (1+\rho)^{-t} \ln\left(\left(\bar{M}\delta^{t-\bar{t}-1} r^{\bar{t}+1}\right)^{1-\alpha} F^{\alpha} r^{\alpha t}\right)$$
$$- \sum_{t=0}^{\bar{t}-1} (1+\rho)^{-t} \ln\left(\left(\bar{M}\frac{1-r}{1-\delta}\right)^{1-\alpha} F^{\alpha} r^{t}\right) - \sum_{t\geq\bar{t}} (1+\rho)^{-t} \ln\left(\left(\bar{M}\delta^{t-\bar{t}} r^{\bar{t}}\right)^{1-\alpha} F^{\alpha} r^{\alpha t}\right)$$
$$\frac{W_{\bar{t}+1} - W_{\bar{t}}}{1-\alpha} = r^{\bar{t}} \ln\left(\frac{1-\delta}{1-r}\right) + \sum_{t>\bar{t}} r^{t} \ln\left(\frac{\delta}{r}\right) = r^{\bar{t}}\left(\ln\left(\frac{1-r}{1-\delta}\right) + \frac{r}{1-r}\ln\left(\frac{r}{\delta}\right)\right)$$

Hence, for $\varepsilon = 1$, $W_{\bar{t}+1} > W_{\bar{t}} \iff h_r(\delta) := (1-r)\ln\left(\frac{1-r}{1-\delta}\right) + r\ln\left(\frac{r}{\delta}\right) > 0$. Yet, $h_r'(\delta) = \frac{1-r}{1-\delta} - \frac{r}{\delta} > 0 \iff \delta > r$ and $h_r(r) = 0$, so that $\forall \delta \neq r,\ h_r(\delta) > 0$.

As a consequence, whatever the value of ε, it is always optimal to delay the end of mineral extraction and it is never optimal to exhaust minerals at a date $\bar{t} > 0$. Indeed, in the only case for which it is optimal to do so, $\delta = r$, all candidate solutions conflate to immediate exhaustion.

C.3 Case $\delta < r$

In this subsection, we call $\bar{t}$ the first period for which an optimal program's mineral extraction has a positive value: $\bar{t} := \min_{t\geq 0}\{m_t > 0\}$. Let us prove by induction on $\bar{t}$ that for all optimal solutions $(m_t,\ f_t)_{t\geq 0}$ such that $\bar{t}$ is the first period with positive mineral extraction, $t < \bar{t} \iff m_t = 0$. $t < \bar{t} \implies m_t = 0$ being true by definition, we only need to prove the reciprocal.

In the base case $\bar{t} = 0$, $m_t = 0 \implies t \in \emptyset \implies t < \bar{t} = 0$ comes from the results of the three previous subsections that it is never optimal to stop or interrupt extraction.

Then we turn to the inductive step, and we assume that the proposition has been proven for all $\bar{t} \leq n$, to show it in the case $\bar{t} = n+1$. Let $(m_t,\ f_t)_{t\geq 0}$ be a solution of the original, constrained program (we thus have $\forall t,\ m_t \geq 0$) such that $\bar{t} = n+1$.

Necessarily, $(m_t,\ f_t)_{t\geq 1}$ is optimal solution of the unconstrained and constrained program starting at *1* with stock of resources $(M - m_0,\ F - f_0)$. Applying the induction on $(m_t,\ f_t)_{t\geq 1}$, we know that $\forall t \geq 1,\ (m_t = 0 \implies t < n+1)$. In addition, by definition of $\bar{t}$, $m_0 = 0$, so that $\forall t \geq 0,\ t < \bar{t} \implies m_t = 0$, which achieves the proof.

C.4 Case $\delta \geq r$

For $\delta > r$, and using the f.o.c.s on f_t and f_{t+1}[24], we have:

$$\forall t, \quad 1 < \left(\frac{\delta}{r}\right)^{\frac{\varepsilon}{1-\phi}} \leq (1+\rho)^{\frac{1}{1-\phi}} \left(\frac{m_{t+1}}{K_t} + \delta\right)^{\frac{\varepsilon}{1-\phi}}$$

$$= (1+\rho)^{1-\frac{\phi}{\phi-1}} \left(\frac{K_{t+1}}{K_t}\right)^{\frac{\mu\phi}{\phi-1}+1-\mu} = (1+\rho)\left(\frac{K_t}{K_{t+1}}\right)^{\mu-1}\left(\frac{f_t}{f_{t+1}}\right)^{\phi}$$

$$\forall t, \quad (1+\rho)^{-t} f_t^{\phi} K_t^{\mu-1} > (1+\rho)^{-(t+1)} f_{t+1}^{\phi} K_{t+1}^{\mu-1}$$

Suppose *ad absurdo* that the optimal path $(m_t,\, f_t)_{t\geq 0}$ is such that there exists $T > 0$ such that $m_T > 0$. Let $\left(\tilde{m_t},\, \tilde{f_t}\right)_{t\geq 0}$ be an alternative path defined by $\forall t,\ \tilde{f_t} = f_t,\ \forall t \notin \{0;\, T\},\ \tilde{m_t} = m_t$, $\tilde{m_0} = m_0 + \eta,\ \tilde{m_T} = m_T - \eta$, for an arbitrary $\eta \in (0; m_T)$. Let us compare the welfares W_η and W given by $\left(\tilde{m},\, \tilde{f}\right)$ and $(m,\, f)$, respectively.

$$\tilde{W}_\eta = \frac{1}{1-\varepsilon}\sum_{t\geq 0}(1+\rho)^{-t}\,\tilde{f_t}^{\phi}\tilde{K}_t^{\mu}$$

$$= \frac{1}{1-\varepsilon}\left(\sum_{t<T}(1+\rho)^{-t} f_t^{\phi}\left(K_t + \eta\delta^t\right)^{\mu} + \sum_{t\geq T}(1+\rho)^{-t} f_t^{\phi}\left(K_t + \eta\left(\delta^t - \delta^{t-T}\right)\right)^{\mu}\right)$$

$$\underset{\eta\to 0^+}{=} \frac{1}{1-\varepsilon}\left(\sum_{t<T}(1+\rho)^{-t} f_t^{\phi}K_t^{\mu}\left(1+\mu\eta\frac{\delta^t}{K_t}\right) + \sum_{t\geq T}(1+\rho)^{-t} f_t^{\phi}K_t^{\mu}\left(1+\mu\eta\frac{\delta^t-\delta^{t-T}}{K_t}\right)\right) + o(\eta)$$

$$\tilde{W}_\eta - W_\eta \underset{\eta\to 0^+}{=} \eta\alpha\left(\sum_{t\geq 0}(1+\rho)^{-t} f_t^{\phi}K_t^{\mu-1}\delta^t - \sum_{t\geq T}(1+\rho)^{-t} f_t^{\phi}K_t^{\mu-1}\delta^{t-T}\right) + o(\eta)$$

$$\underset{\eta\to 0^+}{=} \eta\alpha\left(\sum_{t\geq 0}\left((1+\rho)^{-t} f_t^{\phi}K_t^{\mu-1} - (1+\rho)^{-t-T} f_{t+T}^{\phi}K_{t+T}^{\mu-1}\right)\delta^t\right) + o(\eta)$$

From above, we know that $\forall t,\ (1+\rho)^{-t} f_t^{\phi}K_t^{\mu-1} > (1+\rho)^{-t-T} f_{t+T}^{\phi}K_{t+T}^{\mu-1}$, so that $\tilde{W} > W$.[25] This contradicts the optimality of $(m,\, f)$. We deduce that $\delta > r \implies \forall T > 0,\ m_T = 0$. Observing that for $\delta = r$ the unconstrained solution gives $\forall t > 0,\ m_t = 0$ concludes the proof.

[24] To equate the f.o.c.s on f_t we used the fact the $\forall t,\ f_t > 0$ shown in subsection C.1.

[25] The argument does not rely on $\varepsilon \neq 1$, the limit case $\varepsilon = 1$ has not been presented for simplicity.

D Relations between the different rates

Lemma 1. *For $\varepsilon < 1$, there are only three possible exclusive cases:*

$$\delta < R < r < \frac{1}{1+\rho} \quad \vee \quad r < R < \frac{1}{1+\rho} < \delta \quad \vee \quad \delta = R = r > \frac{1}{1+\rho}$$

whereas for $\varepsilon > 1$, the three possible cases are:

$$\delta < \frac{1}{1+\rho} < r < R \quad \vee \quad \frac{1}{1+\rho} < R < r < \delta \quad \vee \quad \delta = R = r < \frac{1}{1+\rho}$$

Finally, $\varepsilon = 1$ entails $r = R = \frac{1}{1+\rho} < 1$, with no relation on δ.

As $\delta < 1$, it follows that $\delta \geq r \implies R < 1$.

Proof. We always have $\delta < R \iff \delta^{\phi-1} > \frac{r^{\mu+\phi-1}}{\delta^{\mu}} \iff \delta < r$.

For $\varepsilon < 1$, $\mu = (1-\alpha)(1-\varepsilon) > 0$, so that $r < R \iff r^{\phi-1} > \frac{r^{\mu+\phi-1}}{\delta^{\mu}} \iff \delta > r$.

Ad absurdo, we also have that $\delta < R \implies R < r$ for $\varepsilon < 1$. In effect, suppose that $\delta < R$ and $r \leq R$. From the inequalities above (which are also valid as non-strict inequalities), we deduce two contradictory properties: $\delta < r$ and $\delta \geq r$.

Reciprocally, $r \leq R \implies R \leq \delta$. In addition, as $(\delta = r \iff r = R) \implies (\delta = r \iff R = \delta)$, there are only three possible exclusive cases in the case $\varepsilon < 1$: $\delta < R < r \quad \vee \quad r < R < \delta \quad \vee \quad \delta = R = r$.

Using a similar reasoning for $\varepsilon > 1$, and given that in this case $r < R \iff \delta < r$, we have:

$\varepsilon > 1 \implies \delta < r < R \quad \vee \quad R < r < \delta \quad \vee \quad \delta = R = r$.

Finally, observing that

$R < \frac{1}{1+\rho} \iff \frac{1+\rho}{\delta^{\mu}} > (1+\rho)^{1-\phi} \iff (1+\rho)^{\phi} > \delta^{\mu} \iff (1+\rho)^{\alpha(1-\varepsilon)} > \delta^{(1-\alpha)(1-\varepsilon)} \iff \varepsilon < 1$

and $r < \frac{1}{1+\rho} \iff (1+\rho)^{1-\frac{1}{\varepsilon}} < 1 \iff \varepsilon < 1$ concludes the proof.

□

E Damages from fossils

Assuming that the resource constraint on fossils is not binding, the program writes:

$$\max \frac{f_0^\phi m_0^\mu}{1-\varepsilon} - \frac{d_0}{\theta} f_0^\theta + \frac{1}{1+\rho}\left(\frac{f_1^\phi \left(m_1 + \delta m_0\right)^\mu}{1-\varepsilon} - \frac{d_1}{\theta} f_1^\theta\right) + \lambda \left(F - f_0 - f_1\right) + \nu \left(M - m_0 - m_1\right)$$

The f.o.c.s are:

$$\begin{cases} (\partial f_0) & \alpha f_0^{\phi-1} m_0^\mu = d_0 f_0^{\theta-1} \\ (\partial f_1) & \alpha f_1^{\phi-1} \left(m_1 + \delta m_0\right)^\mu = d_1 f_1^{\theta-1} \\ (\partial m_0) & (1-\alpha) f_0^\phi m_0^{\mu-1} + \frac{\delta(1-\alpha)}{1+\rho} f_1^\phi \left(m_1 + \delta m_0\right)^{\mu-1} = \nu \\ (\partial m_1) & \frac{1-\alpha}{1+\rho} f_1^\phi \left(m_1 + \delta m_0\right)^{\mu-1} = \nu \end{cases}$$

The f.o.c. on f_0 gives:

$$m_0 = \left(\frac{d_0}{\alpha} f_0^{\theta-\phi}\right)^{\frac{1}{\mu}} \tag{E.1}$$

Combining the f.o.c.s on m_i, we have: $(1-\alpha) f_0^\phi m_0^{\mu-1} = \nu (1-\delta)$, so that:

$$f_0 = \left(\frac{\nu (1-\delta)}{(1-\alpha) m_0^{\mu-1}}\right)^{\frac{1}{\phi}} \tag{E.2}$$

Plugging (E.1) into (E.2) yields:

$$f_0 = \left(\frac{\nu (1-\delta)}{1-\alpha}\right)^{\frac{1}{\phi}} \left(\frac{\phi}{d_0}\right)^{\frac{\mu-1}{\mu\phi}} f_0^{(\phi-\theta)\frac{\mu-1}{\mu\phi}}$$

Re-arranging this and defining $e := \phi - \theta (1-\mu)$:

$$f_0 = \left(\frac{\nu (1-\delta)}{1-\alpha}\right)^{\frac{\mu}{e}} \left(\frac{\phi}{d_0}\right)^{\frac{\mu-1}{e}} \tag{E.3}$$

Let us notice that $e = \alpha (1-\varepsilon) - \theta (1 - (1-\alpha)(1-\varepsilon)) = 1 - \varepsilon - \theta < 0$, because $\varepsilon > 0$ and $\theta > 1$. Plugging (E.3) into (E.1) gives in turn m_0:

$$m_0 = \left(\frac{\nu (1-\delta)}{1-\alpha}\right)^{\frac{\theta-\phi}{e}} \left(\frac{d_0}{\phi}\right)^{\frac{\phi}{e}}$$

Combining the f.o.c. on m_1 and f_1, we have $\alpha f_1^{\phi-1}\left(\frac{\nu f_1^{-\phi}}{1-\alpha}(1+\rho)\right)^{\frac{\mu}{\mu-1}} = d_1 f_1^{\theta-1}$, so that:

$$f_1 = \left(\frac{d_1}{\alpha}\right)^{\frac{1-\mu}{e}} \left(\frac{\nu}{1-\alpha}(1+\rho)\right)^{\frac{\mu}{e}} \tag{E.4}$$

We can then retrieve m_1 from the f.o.c. on f_1, using the formulas for m_0 and f_1:

$$m_1 = \left(\frac{\nu}{1-\alpha}\right)^{\frac{\theta-\phi}{e}} \left((1+\rho)^{\frac{\theta-\phi}{e}} \left(\frac{d_1}{\alpha}\right)^{\frac{\phi}{e}} - \delta(1-\delta)^{\frac{\theta-\phi}{e}} \left(\frac{d_0}{\alpha}\right)^{\frac{\phi}{e}}\right)$$

As the objective is strictly increasing in m_0 and m_1, all resources are extracted at the optimum:

$$M = m_0 + m_1 = \left(\frac{\nu}{1-\alpha}\right)^{\frac{\theta-\phi}{e}} \left((1+\rho)^{\frac{\theta-\phi}{e}} \left(\frac{d_1}{\alpha}\right)^{\frac{\phi}{e}} + (1-\delta)^{\frac{\theta\mu}{e}} \left(\frac{d_0}{\alpha}\right)^{\frac{\phi}{e}}\right)$$

so that

$$\left(\frac{\nu}{1-\alpha}\right)^{\frac{1}{e}} = \left(\frac{1}{M}\left((1+\rho)^{\frac{\theta-\phi}{e}} \left(\frac{d_1}{\alpha}\right)^{\frac{\phi}{e}} + (1-\delta)^{\frac{\theta\mu}{e}} \left(\frac{d_0}{\alpha}\right)^{\frac{\phi}{e}}\right)\right)^{\frac{1}{\phi-\theta}}$$

Finally

$$e = 1 - \varepsilon - \theta < 0$$

$$m_0 = \frac{(1-\delta)^{\frac{\theta-\phi}{e}} \left(\frac{d_0}{\alpha}\right)^{\frac{\phi}{e}}}{(1+\rho)^{\frac{\theta-\phi}{e}} \left(\frac{d_1}{\alpha}\right)^{\frac{\phi}{e}} + (1-\delta)^{\frac{\theta\mu}{e}} \left(\frac{d_0}{\alpha}\right)^{\frac{\phi}{e}}} M = \frac{M}{\left(\frac{1+\rho}{1-\delta}\right)^{\frac{\theta-\phi}{e}} \left(\frac{d_1}{d_0}\right)^{\frac{\phi}{e}} + 1 - \delta}$$

$$f_0 = (1-\delta)^{\frac{\mu}{e}} \left(\frac{d_0}{\alpha}\right)^{\frac{1-\mu}{e}} \left(\frac{1}{M}\left((1+\rho)^{\frac{\theta-\phi}{e}} \left(\frac{d_1}{\alpha}\right)^{\frac{\phi}{e}} + (1-\delta)^{\frac{\theta\mu}{e}} \left(\frac{d_0}{\alpha}\right)^{\frac{\phi}{e}}\right)\right)^{\frac{\mu}{\phi-\theta}}$$

$$= \left(\frac{d_0}{\alpha}\right)^{\frac{1}{\phi-\theta}} \left(\frac{1}{M}\left(\left(\frac{1+\rho}{1-\delta}\right)^{\frac{\theta-\phi}{e}} \left(\frac{d_1}{d_0}\right)^{\frac{\phi}{e}} + (1-\delta)\right)\right)^{\frac{\mu}{\phi-\theta}}$$

$$m_1 = \frac{(1+\rho)^{\frac{\theta-\phi}{e}} \left(\frac{d_1}{\alpha}\right)^{\frac{\phi}{e}} - \delta(1-\delta)^{\frac{\theta-\phi}{e}} \left(\frac{d_0}{\alpha}\right)^{\frac{\phi}{e}}}{(1+\rho)^{\frac{\theta-\phi}{e}} \left(\frac{d_1}{\alpha}\right)^{\frac{\phi}{e}} + (1-\delta)^{\frac{\theta\mu}{e}} \left(\frac{d_0}{\alpha}\right)^{\frac{\phi}{e}}} M = \frac{\left(\frac{1+\rho}{1-\delta}\right)^{\frac{\theta-\phi}{e}} \left(\frac{d_1}{d_0}\right)^{\frac{\phi}{e}} - \delta}{\left(\frac{1+\rho}{1-\delta}\right)^{\frac{\theta-\phi}{e}} \left(\frac{d_1}{d_0}\right)^{\frac{\phi}{e}} + 1 - \delta} M$$

$$f_1 = (1+\rho)^{\frac{\mu}{e}} \left(\frac{d_1}{\alpha}\right)^{\frac{1-\mu}{e}} \left(\frac{1}{M}\left((1+\rho)^{\frac{\theta-\phi}{e}} \left(\frac{d_1}{\alpha}\right)^{\frac{\phi}{e}} + (1-\delta)^{\frac{\theta\mu}{e}} \left(\frac{d_0}{\alpha}\right)^{\frac{\phi}{e}}\right)\right)^{\frac{\mu}{\phi-\theta}}$$

$$= \left(\frac{d_1}{\alpha}\right)^{\frac{1}{\phi-\theta}} \left(\frac{1}{M}\left(1 + (1+\rho)^{\frac{\phi-\theta}{e}} (1-\delta)^{\frac{\theta\mu}{e}} \left(\frac{d_0}{d_1}\right)^{\frac{\phi}{e}}\right)\right)^{\frac{\mu}{\phi-\theta}}$$

F Results with Leontief production

F.1 Influence of the productivity of fossils

$$\frac{u'(\theta A_0 f_0^*)}{u'(\theta A_1 (F - f_0^*))} - \frac{1}{1+\rho}\frac{A_1}{A_0} = 0$$

$$\frac{u''(\theta A_0 f_0^*)\, u'(\theta A_1 (F - f_0^*))\left(\theta f_0^* + \theta A_0 \frac{df_0^*}{dA_0}\right) + u'(\theta A_0 f_0^*)\, u''(\theta A_1 (F - f_0^*))\, \theta A_1 \frac{df_0^*}{dA_0}}{\left[u'(\alpha_2 A_1 (F - f_0^*))\right]^2} + \frac{1}{1+\rho}\frac{A_1}{A_0^2} = 0$$

$$\frac{\theta A_0 u''(q_0^*)\, u'(q_1^*) + \theta A_1 u'(q_0^*)\, u''(q_1^*)}{\left[u'(q_1^*)\right]^2}\frac{df_0^*}{dA_0} + \frac{u''(q_0^*)\, u'(q_1^*)\, \theta f_0^*}{\left[u'(q_1^*)\right]^2} + \frac{1}{1+\rho}\frac{A_1}{A_0^2} = 0$$

$$\frac{\theta A_0 u''(q_0^*)\, u'(q_1^*) + \theta A_1 u'(q_0^*)\, u''(q_1^*)}{\left[u'(q_1^*)\right]^2}\frac{df_0^*}{dA_0} + \frac{u''(q_0^*)\, q_0}{u'(q_0^*)}\frac{1}{1+\rho}\frac{A_1}{A_0^2} + \frac{1}{1+\rho}\frac{A_1}{A_0^2} = 0$$

$$\frac{\theta A_0 u''(q_0^*)\, u'(q_1^*) + \theta A_1 u'(q_0^*)\, u''(q_1^*)}{\left[u'(q_1^*)\right]^2}\frac{df_0^*}{dA_0} + \frac{1}{1+\rho}\frac{A_1}{A_0^2}\left(1 + \frac{u''(q_0^*)\, q_0}{u'(q_0^*)}\right) = 0$$

$$\frac{df_0^*}{dA_0} = -\frac{1}{1+\rho}\frac{A_1}{A_0^2}\left(1+\frac{u''(q_0^*)\,q_0}{u'(q_0^*)}\right)\frac{[u'(q_1^*)]^2}{\theta A_0 u''(q_0^*)\,u'(q_1^*)+\theta A_1 u'(q_0^*)\,u''(q_1^*)}$$

F.2 Proof of proposition 6

Proof. Part (a) follows from the previous analysis: m_0^* is determined by the choice of f_0^* which does not depend on the amount of mineral resource available, nor on its relative productivity. For (b) we compute directly:[26]

$$\frac{dm_0^*}{d\delta} = \frac{1}{1+\rho}\frac{B_1}{B_0}\left(\frac{u''(q_1^*)\,q_1^*}{u'(q_1^*)}\frac{(1-\delta)\,m_0^*}{(M-(1-\delta)\,m_0^*)}-1\right)\frac{[u'(q_1^*)]^2}{u''(q_0^*)\,u'(q_1^*)\,B_0+u'(q_0^*)\,u''(q_1^*)\,B_1(1-\delta)}$$

which is positive for $u''<0$. The result on f_0^* follows from complementarity. Two countervailing effects are exerted on $f_1^* = \frac{B_1}{\theta A_1}(M-(1-\delta)\,m_0^*)$. First, a higher δ tends to increases the demand for f_1. Second, the induced shift of mineral extraction to the first period, leaves less mineral resources to be extracted in the second period (see the term m_0^*), moderating demand for f_1. Finally, (c) comes from the application of the envelope theorem to the fact that fossils are abundant relative to minerals if and only if $m_0^*\left(\frac{B_0}{A_0}-\frac{B_1}{A_1}(1-\delta)\right) < \theta F - \frac{B_1}{A_1}M$.

□

[26]

$$\frac{u'(\beta_0 B_0 m_0^*)}{u'(\beta_1 B_1 (M-(1-\delta)\,m_0^*))} - \frac{1}{1+\rho}\frac{\beta_1 B_1}{\beta_0 B_0}(1-\delta) =$$

$$\frac{u''(\beta_0 B_0 m_0^*)\,\beta_0 B_0 \frac{dm_0^*}{d\delta} u'(\beta_1 B_1(M-(1-\delta)\,m_0^*)) - u'(\beta_0 B_0 m_0^*)\,u''(q_1^*)\left(\beta_1 B_1 m_0^* - \beta_1 B_1 (1-\delta)\frac{dm_0^*}{d\delta}\right)}{[u'(\beta_1 B_1 (M-(1-\delta)\,m_0^*))]^2} + \frac{1}{1+\rho}\frac{\beta_1 B_1}{\beta_0 B_0} =$$

$$\frac{\beta_0 B_0 u''(q_0^*)\,u'(q_1^*) + u'(q_0^*)\,u''(q_1^*)\,\beta_1 B_1 (1-\delta)}{[u'(q_1^*)]^2}\frac{dm_0^*}{d\delta} + \frac{1}{1+\rho}\frac{\beta_1 B_1}{\beta_0 B_0}(1-\delta)\frac{-u''(q_1^*)\,q_1^*}{u'(q_1^*)}\frac{m_0^*}{(M-(1-\delta)\,m_0^*)} + \frac{1}{1+\rho}\frac{\beta_1 B_1}{\beta_0 B_0} =$$

$$\frac{\beta_0 B_0 u''(q_0^*)\,u'(q_1^*) + u'(q_0^*)\,u''(q_1^*)\,\beta_1 B_1 (1-\delta)}{[u'(q_1^*)]^2}\frac{dm_0^*}{d\delta} + \frac{1}{1+\rho}\frac{\beta_1 B_1}{\beta_0 B_0}\left(1 - \frac{u''(q_1^*)\,q_1^*}{u'(q_1^*)}\frac{(1-\delta)\,m_0^*}{(M-(1-\delta)\,m_0^*)}\right) =$$

F.3 Influence of the productivity of minerals

Recalling that $\varepsilon \equiv -\frac{u''(q^*)q}{u'(q^*)}$ and proceeding as in subsection F.1 for (4.7), one gets:

$$\frac{dm_0^*}{dB_0} = \frac{1-\delta}{1+\rho}\frac{\beta_1 B_1}{\beta_0 (B_0)^2}\left(1+\frac{u''(q_0^*)\,q_0}{u'(q_0^*)}\right)\frac{\left[u'(q_1^*)\right]^2}{\beta_0 B_0 u''(q_0^*)\,u'(q_1^*)+\beta_1 B_1(1-\delta)u'(q_0^*)\,u''(q_1^*)} \quad \begin{cases} < 0 \text{ if } \varepsilon < 1 \\ > 0 \text{ if } \varepsilon > 1 \end{cases}$$

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