On agreements in a nonrenewable resource market: a cooperative differential game approach

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Abstract

We consider a nonrenewable resource duopoly with economic exhaustion. We characterize the set of Pareto efficient equilibria. We show that when firms are sufficiently patient, there exists no Pareto efficient agreement that yields short-run gains with respect to the non-cooperative equilibrium. Given a pair of stocks, there exists a unique interior Pareto efficient agreement. We characterize the set of stocks where a Pareto efficient agreement results in larger discounted sum of profits for both players. We show that social welfare under the interior Pareto efficient agreement is smaller than under non-cooperation, despite the gains from a more cost effective extraction of the resources under an agreement.

Keywords: nonrenewable, cartel, duopoly, cooperative differential game.
JEL classification numbers: C71; C73; L13; Q30; Q32

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1 Introduction

Nonrenewable resources markets have key ingredients for the emergence of explicit cooperative agreements between oligopolists in order to control the overall quantity supplied in the market. In the case of nonrenewable resources, entry is limited by the resource endowment very few countries can own large shares of resource. Furthermore, agreements between sovereign countries are typically not subject to national antitrust laws. The Organization of Petroleum Exporting Countries (Opec) which controls around 40 percent of the current crude oil production is perhaps the most famous resource cartel. However, the oil market is not the exception: the markets of mercury, uranium, diamond, bauxite or copper have each experienced at least a period of successful cartelization[1].

In this paper we characterize the set of potential agreements, i.e, Pareto efficient equilibria, between cartel members in a nonrenewable resource duopoly.

In the literature, a cartel is often modeled as a strong and monolithic coalition. For instance, the "cartel versus fringe models", following the seminal papers of Salant (1976) and Ulph and Folie (1980), consider a coherent cartel, as one player, facing a large number of firms acting in pure competition, see also Kagan et al. (2015) for a more recent contribution. Another branch of literature, following Gilbert (1978), Newberry (1981) and more recently Groot et al. (2000, 2003), consider the case of a first-mover cartel in a Stackelberg equilibrium. However, in these papers as well, the leader or the cartel is assumed to be a coherent structure and the agreement between cartel members is not examined.

To the best of our knowledge, only two references consider a cartel as the result of a bargaining process. Hnylicza and Pindyck (1976) model two firms with asymmetric discount rates. They find a "bang-bang" solution consisting for the firms to produce in alternation. This solution may not be realistic "since the temptation to cheat would be considerable" (Hnylicza and Pindyck, 1976, p. 147). Okullo and Reynès (2016) consider a framework that captured important features that are relevant in the case of the oil market (e.g., the cartel faces a fringe with a capacity constraint, endogenous reserve development) in order to specifically study the Opec cartel. They introduce a coefficient of cooperation and select the level of the coefficient using the Nash Bargaining solution. The equilibrium under non-cooperation is an open-loop equilibrium where each player chooses a production path at time zero. In our paper, the non-cooperative equilibrium we consider is a vector of Markovian stationary strategies and therefore it is subgame perfect. Moreover in contrast with both papers where the results are established using numerical simulations, our results are shown analytically.

We use a cooperative differential game framework, see e.g., Hämäläinen et al., (1985) for a seminal application in the case of the fisheries, Leitmann (1974) and more recently Reddy and Engwerda (2013) for influential methodological contributions. We consider cooperation between two firms, each owning a private stock of a nonrenewable resource. We examine the case of

[1] See Teece et al. (1993) for case studies on oil, mercury, uranium and diamond cartels.

economic exhaustion or scarcity rather than physical exhaustion of the resource: the amount of the resource used by each firm is endogenously determined and is dictated by market conditions as well as how the extraction cost is affected by the level of the stock of the resource available. We make use of a necessary condition to Pareto optimality in cooperative differential games recently found in Reddy and Engwerda (2013) to fully characterize the set of all Pareto optimal payoffs and study group and individual rationality of the potential agreements. The non-cooperative scenario has been already described by Salo and Tahvonen (2001).

In a cooperative differential game, players negotiate to establish a cooperative agreement on how to produce through time and how to share the total payoff. The cooperative agreement must satisfy group rationality and individual rationality (Yeung and Petrosyan (2006) or Zaccour (2008)). To achieve group rationality, the outcome has to satisfy Pareto optimality. Indeed, the objective of cooperation is to reach a Pareto efficient allocation, which is, in general, unattainable in a non-cooperative market structure. To achieve individual rationality, both players' payoffs must be at least equal to what they can get in the non-cooperative equilibrium. Otherwise, the cooperation is not self-enforced. Inspired by the context of international markets of nonrenewable resources we rule out direct monetary transfers between players.

Focusing first on group rationality, we find that for each ratio of resource stocks, there exists a unique interior Pareto solution. Every other solutions are corner ones, involving that one of the firms does not produce for a period of time. This agreement has several properties which have been corroborated in empirical studies of resource cartels. First, each firm should get a share of total production exactly equal to his share of the total nonrenewable resource stock of the cartel. This result is often met as an assumption in the literature (Mason and Polasky (2005) or Wiggins and Libecap (1987)) and is highly suspected to be the policy currently run by Opec. Second, as an immediate consequence, the cooperative agreement presents a bias in favor of the small firm which also finds empirical evidence (Griffin and Xiong (1997)). Third, the rate of exhaustion of the nonrenewable resource total stock of the cartel depends on its inner symmetry.

Checking the cooperative agreements that satisfy individual rationality, we find that the interior cooperative agreement may be non-acceptable for the firm with the largest resource stock. Specifically, if firms are too asymmetric, the player with the larger stock will have a discounted sum of profits lower than in the non-cooperative equilibrium. Motivated again by the context of the countries involved in nonrenewable resource markets, we further examine the impact of an agreement on short-run profits. In many instances countries face budgetary constraints and the income derived from the natural resource sector represents a substantial share of their gross domestic product. It is therefore highly unlikely that a country would accept an agreement that involves a short-run loss of income with respect to the non-cooperative equilibrium even if this loss will be more than compensated by future gains. We call this condition the immediate individual rationality, i.e., at the initial time of the agreement instantaneous profits are required to be larger under cooperation than under non-cooperation. We show that when the discount rate is small enough, there exists no Pareto efficient agreement such that the condition of short-run individual rationality is satisfied. This is a rather surprising result that runs against the general wisdom from the Folk Theorem in game theory whereby more patience is viewed as more conducive to cooperation. In our case players are not using punishment strategies with delay of detection. In our game, when players are sufficiently patient they have an incentive to postpone the extraction of the resource since a larger stock in the ground allows to extract at a lower cost. This incentive to conserve the resource can be strong enough to imply a high initial reduction of extraction with respect to the non-cooperative equilibrium which ultimately results in smaller short-run profits.

The remainder of the paper is the following. In section 2, we expose the model and characterize a unique interior Pareto solution for any ratio of initial resource stocks. In section 3, we focus on the individual rationality of the feasible cooperative agreements. In section 4, we conduct a welfare analysis. In section 5, we make some concluding remarks.

2 The model

2.1 Basic framework

We use a standard framework nonrenewable resource duopoly with economic exhaustion (Salo and Tahvonen 2001). Two firms extract a nonrenewable resource from two stocks, denoted by S_i, $i = 1, 2$, with well-defined property rights. They face a linear inverse demand, $p = p_0 - q_1 - q_2$, where p designates the price, p_0 the choke price and q_i, $i = 1, 2$, the extraction rates of both firms. Unit extraction costs are stock-dependent, $C(S_i) = c_0 - cS_i$, $i = 1, 2$, where $c_0 > 0$ and $c > 0$. For simplicity, we assume that parameters c_0 and c are the same for both firms.

We consider the case of economic exhaustion: the cost of extracting the last unit of the resource stock exceeds the choke price, $c_0 > p_0$. As a consequence, each firm leaves a stock $\bar{S} \equiv \frac{c_0 - p_0}{c}$ unexploited. Introducing $s_i = S_i - \bar{S}$, the profitable part of the physical resource stock S_i, the unit extraction cost can be expressed as $p_0 - cs_i$.

With these specifications, firm i's present value of his flow of instantaneous profits is

$$J_i = \int_0^\infty e^{-rt} \left[(p_0 - q_1 - q_2)q_i - (p_0 - cs_i)q_i \right] dt \tag{1}$$

which simplifies into

$$J_i = \int_0^\infty e^{-rt} \left(cs_i - q_1 - q_2 \right) q_i dt \tag{2}$$

Within this framework, we study the possibility for the firms to make a binding cooperative agreement on how to share a surplus they jointly generate. Such an agreement defined a pair of extraction paths $(q_1^\star, q_2^\star)$, one for each firm. We assume that there is no possibility of side-payments between firms.

2.2 Group rationality

A cooperative agreement $(q_1^\star, q_2^\star)$ must satisfy group rationality. It is achieved when the pair of extraction paths agreed upon is Pareto optimal. To characterize the set of all the Pareto optimal controls in the context of a cooperative differential game with nontransferable payoffs, we use the following Leitmann's Lemma (Leitmann, 1974):

Lemma 1. *Let $\alpha \in (0,1)$. If $(q_1^\star, q_2^\star)$ is such that:*

$$\begin{aligned} (q_1^\star, q_2^\star) &\in arg\max_{q_1,q_2} \alpha J_1 + (1-\alpha) J_2 \\ s.t.\ \dot{s}_1 &= -q_1 \quad \dot{s}_2 = -q_2 \end{aligned} \tag{3}$$

Then $(q_1^\star, q_2^\star)$ is Pareto optimal.

Using this Lemma, we start by characterizing the cooperative agreements that yield an interior solution for each player throughout the time horizon. We call such an agreement, an interior cooperative agreement (ICA). It turns out that, given a pair of initial stocks, such a cooperative agreement is unique.

Proposition 1 (ICA)**.** *For a given pair of initial resource stocks (s_{10}, s_{20}), the ICA is unique. The solution to (25) is interior iff $\alpha = \alpha^\star$ where*

$$\alpha^\star = \frac{\left(\frac{s_{20}}{s_{10}}\right)^2}{1 + \left(\frac{s_{20}}{s_{10}}\right)^2} \tag{4}$$

Under this unique ICA, we have

$$q_1^\star(t) = -\theta^\star s_1^\star(t) \tag{5}$$

$$q_2^\star(t) = -\theta^\star s_2^\star(t) \tag{6}$$

$$s_1^\star(t) = s_{10} e^{\theta^\star t} \tag{7}$$

$$s_2^\star(t) = s_{20} e^{\theta^\star t} \tag{8}$$

where

$$\theta^\star = \frac{r}{2} - \sqrt{\left(\frac{r}{2}\right)^2 + rc\frac{\frac{s_{20}}{s_{10}}}{\left(1 + \frac{s_{20}}{s_{10}}\right)^2}}$$

Proof. See Appendix A. □

To establish the *uniqueness* of the interior solution we used the contribution of Reddy and Engwerda (2013) which provides a necessary condition to Pareto optimality and thus completes Leitmann's Lemma and ensures that we have exhaustively listed all Pareto optimal payoffs.

An implication of the above proposition is that given a pair of initial stocks, aside from the ICA, all the other cooperative agreements consist in corner solutions whereby at least one of the players' production is nil during a period of time. We provide in Proposition 2 the characterization of the set of all Pareto optimal agreements.

Proposition 2 (Corner solutions)**.** *Let initial stocks (s_{10}, s_{20}) and assume that we choose $\alpha > \alpha^\star$ with $\alpha^\star = \frac{\left(\frac{s_{20}}{s_{10}}\right)^2}{1+\left(\frac{s_{20}}{s_{10}}\right)^2}$. The cooperative agreement, solution to (3), consists of two phases:*

- *Phase 1: firm 1 extracts alone until her stock equals $\tilde{s}_{10}$ define by:*

$$\alpha = \frac{\left(\frac{s_{20}}{\tilde{s}_{10}}\right)^2}{1 + \left(\frac{s_{20}}{\tilde{s}_{10}}\right)^2}$$

- *Phase 2: both firms extract and the extraction paths correspond to the ones characterized in Proposition 1 for the initial stocks $(\tilde{s}_{10}, s_{20})$.*

Proof. See Appendix C. □

Figure 2.2 illustrates both Propositions 1 and 2. First, it shows the ICA corresponding to a ratio of initial stocks $\frac{s_{20}}{s_{10}}$ (point A). It consists in the ray going to the origin with slope $\sqrt{\frac{\alpha^\star}{1-\alpha^\star}}$ where $\alpha^\star = \frac{\left(\frac{s_{20}}{s_{10}}\right)^2}{1+\left(\frac{s_{20}}{s_{10}}\right)^2}$. Starting from point A, both stocks evolve along this line. Second, it shows the corner solutions, solution to (3) for $\alpha \neq \alpha^\star$. Starting from point A, the stocks evolve in order to reach the ICA corresponding to α. If $\alpha > \alpha^\star$ ($\alpha < \alpha^\star$), firm 1 (firm 2) has to extract alone in a fist phase. It results in a horizontal (vertical) line until the new ICA is reached.

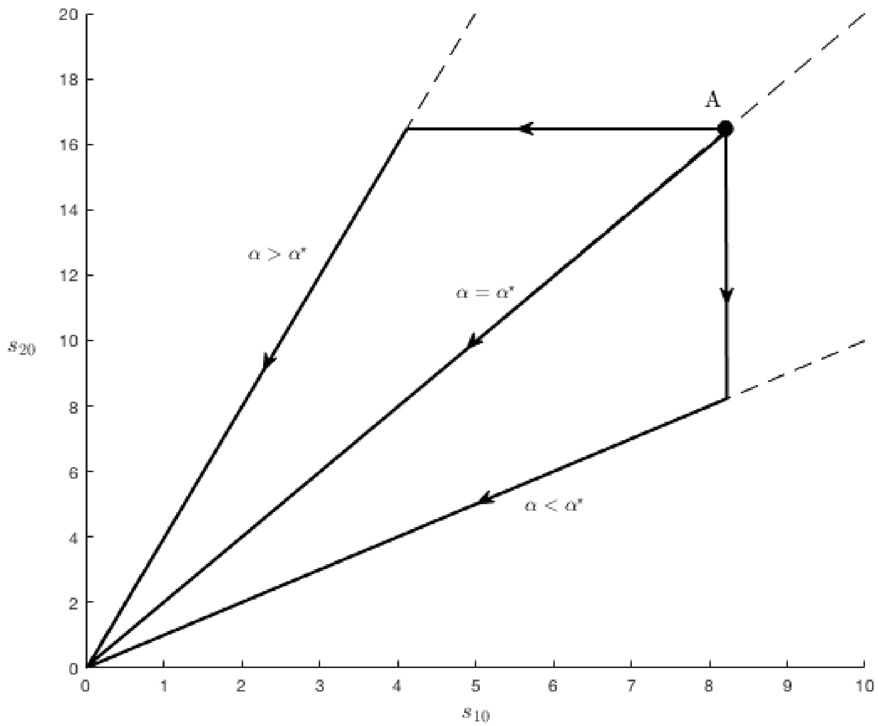

Figure 1: Phase diagram of the resource stocks ($r = 0.05$, $c = 0.08$, $\alpha^\star = 0.8$).

3 Individual Rationality (IR)

In this section we examine under what conditions a cooperative agreement $(q_1^\star, q_2^\star)$ satisfies individual rationality (IR), i.e., the payoff of each player under the agreement has to be no lower than her payoff under non-cooperation. In this case, the agreement is also said to be 'acceptable' (see definition 6.3 in Docker et al., 2000, p. 153). To this end, we provide the equilibrium payoff of each player under non-cooperation. This equilibrium is characterized in Salo and Tahvonen (2001) and reproduced in the subsection below for completeness.

3.1 The non-cooperative game: Salo and Tahvonen (2001)

Each player takes a stationary Markovian strategy of its competitor as given, and maximizes her individual payoff, subject to the state constraints. The equilibrium strategies denoted $(\bar{q}_1, \bar{q}_2)$ are given by (see, Salo and Tahvonen (2001), Proposition 1, p.678):

$$\bar{q}_1(s_1, s_2) = -\frac{\theta_1 + \theta_2}{2}\bar{s}_1 + \frac{\theta_1 - \theta_2}{2}\bar{s}_2 \tag{9}$$

$$\bar{q}_2(s_1,s_2) = \frac{\theta_1-\theta_2}{2}\bar{s}_1 - \frac{\theta_1+\theta_2}{2}\bar{s}_2 \tag{10}$$

The resulting equilibrium paths of each resource stock, denoted $\bar{s}_1(.)$ and $\bar{s}_2(.)$, are given by:

$$\bar{s}_1(t) = \frac{s_{10}-s_{20}}{2}e^{\theta_1 t} + \frac{s_{10}+s_{20}}{2}e^{\theta_2 t} \tag{11}$$

$$\bar{s}_2(t) = \frac{s_{20}-s_{10}}{2}e^{\theta_1 t} + \frac{s_{10}+s_{20}}{2}e^{\theta_2 t} \tag{12}$$

where

$$\theta_1 = \tfrac{r}{2} - \sqrt{\left(\tfrac{r}{2}\right)^2 + \tfrac{cr}{2}\tfrac{r_0(\gamma)+9}{\sqrt{63+r_0(\gamma)^2}}}$$

$$\theta_2 = \tfrac{r}{2} - \sqrt{\left(\tfrac{r}{2}\right)^2 + \tfrac{cr}{2}\tfrac{r_0(\gamma)-3}{\sqrt{63+r_0(\gamma)^2}}}$$

$$r_0(\gamma) = 3\gamma + 5 + 2\sqrt{64+60\gamma+9\gamma^2}\cos\left[\tfrac{1}{3}\arccos\left(\tfrac{404+666\gamma+270\gamma^2+27\gamma^3}{(64+60\gamma+9\gamma^2)^{3/2}}\right)\right] \quad , \quad \gamma = \tfrac{r}{c}$$

Extraction rates (9) and (10) are positive iff $\frac{\theta_1-\theta_2}{\theta_1+\theta_2} < \frac{\bar{s}_2(t)}{\bar{s}_1(t)} < \frac{\theta_1+\theta_2}{\theta_1-\theta_2}$. For instance, in the case the initial stocks are such that: $\frac{s_{20}}{s_{10}} > \frac{\theta_1+\theta_2}{\theta_1-\theta_2}$, firm 1 extracts alone until $\frac{s_{20}}{s_{10}} = \frac{\theta_1+\theta_2}{\theta_1-\theta_2}$, after what firm 2 eventually starts to extract.

3.2 Individual rationality of the ICA

If we denote by $J_i^\star$ firm i's payoff, under cooperation, and $\bar{J}_i$ firm i's payoff, under non-cooperation, the condition for the cooperative agreement to be acceptable, or individually rational (IR), is formally:

$$\textbf{(IR)} \quad \forall i \in \{1,2\}, \quad J_i^\star - \bar{J}_i \geq 0 \tag{13}$$

This condition is also sometimes referred to as overall individual rationality (Zaccour (2008)) since it is the discounted sum of profits *over the whole time horizon*, under a cooperative agreement that are compared to the non-cooperative case. In our differential game framework, using the discounted sum of profits of a player as her payoff leaves the possibility that an agreement may be rational even if during an initial phase, say $[0,\tau]$, the discounted sum of profits of one of the players is lower than it would be under non-cooperation. In the absence of side-payments, such an agreement would lead to an immediate loss for one of the players due to the agreement. While this is theoretically conceivable, incurring losses especially in the short term can be difficult to accept politically. Indeed for many of these markets, the important players are developing countries for which the revenues from their natural resources represent an important part of their gross domestic product.

Therefore, we define a second rationality criterion: the immediate individual rationality (IIR). If we denote the initial profits by $\pi_i^\star(0)$, under cooperation, and $\bar{\pi}_i(0)$, under non-cooperation, a condition for immediate individual rationality to be satisfied for player i is:

$$\textbf{(IIR)} \quad \pi_i^\star(0) > \bar{\pi}_i(0) \tag{14}$$

This is related and a weaker form of the notion of instantaneous individual rationality (Zaccour (2008)) which requires, throughout the agreement that if the game is stopped at time $t > 0$ and the agreement has been followed up-until t then we must have

$$\pi_i^\star(t) > \bar{\pi}_i(t) \quad \forall t \geq 0 \quad i = 1, 2$$

Thus, corner solutions, characterized in Proposition 2, are ruled out because they never satisfy the IIR criterion (14). The only candidates which can possibly satisfy both IR and IIR, that is (13) and (14), is the ICA characterized in Proposition 1. Furthermore, along the ICA's stock path $(s_1^\star(.), s_2^\star(.))$, any renegotiation at any date $t > 0$ will result in the same agreement, that is the value of $\alpha^\star$ remains unchanged. Thus, if condition (14) is satisfied at date $t = 0$, it will also be satisfied along the whole stock path, providing instantaneous individual rationality (Zaccour, 2008).

In the remainder of this section, we focus then on the ICA characterized in Proposition 1. Consider firm 1 (results are symmetric for firm 2), we know, from Salo and Tahovonen (2001), that her discounted sum of profits is, for a given pair of initial stocks (s_{10}, s_{20}):

$$\bar{J}_1 = \frac{c\theta_1}{2\theta_1 - r}\left(\frac{s_{10} - s_{20}}{2}\right)^2 + \frac{\theta_2(c + 2\theta_2)}{2\theta_2 - r}\left(\frac{s_{10} + s_{20}}{2}\right)^2 + \frac{c(\theta_1 + \theta_2) + 2\theta_1\theta_2}{\theta_1 + \theta_2 - r}\left(\frac{s_{10}^2 - s_{20}^2}{4}\right) \tag{15}$$

Using Proposition 1, we have for the ICA:

$$J_1^\star = \frac{\theta^\star}{2\theta^\star - r} s_{10}\left((c + \theta^\star)s_{10} + \theta^\star s_{20}\right) \tag{16}$$

Comparing payoffs (15) and (16), we first observe the following results about IR:

Lemma 2. *The IR of the ICA characterized in proposition 1 only depends upon the ratio of initial resource stocks* $\frac{s_{20}}{s_{10}}$ *and* $\gamma = \frac{r}{c}$.

Proof. This straightforward result is obtained by dividing both sides of condition (13) by cs_{10}^2. □

Proposition 3 (IR)**.** *For any* $\gamma > 0$*, there exists* $\varepsilon \in (0, 1)$*, such that the ICA associated with a pair of initial stocks* (s_{10}, s_{20}) *satisfies Individual Rationality (IR) iff* $\frac{s_{20}}{s_{10}} \in (1 - \varepsilon, 1 + \varepsilon)$

Proof. See Appendix D. □

Figure 3.2 provides an illustration of Proposition 3. In the space of payoffs (J_1, J_2), it shows the Pareto frontier, that is every reachable payoff vector corresponding to the cooperative agreements described in Proposition 1 and Proposition 2. For a given pair of initial stocks (s_{10}, s_{20}), the unique ICA is represented by a star over the frontier. The non-cooperative payoff is designated with a cross. We see that, depending on the pair of initial stocks, the star can be on the top-right region of the cross (left) or, in contrast be outside of this region (right). On the left Figure, the ICA is IR whereas on the right Figure, the ratio of stocks $\frac{s_{20}}{s_{10}}$ is too high for the ICA to be IR.

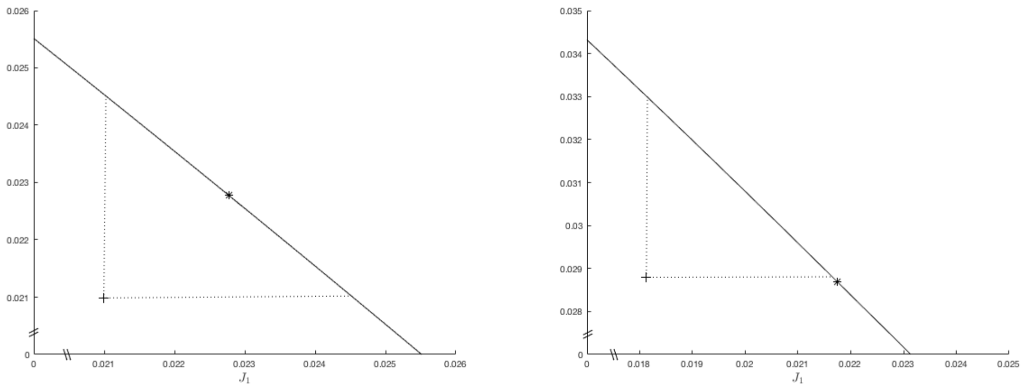

Figure 2: Pareto frontier in the space of payoffs (J_1,J_2) for $c = 0.08$, $r = 0.05$, $s_{10} = 10$, $s_{20} = 10$ (left) and $s_{20} = 11$ (right). Plain line: Pareto frontier, cross: non-cooperation, star: cooperation payoffs.

We note that, for a given pair of initial stocks, the relative weight of each player under the unique ICA only depends on $\frac{s_{20}}{s_{10}}$ and along this unique ICA the ratio of stocks $\frac{s_2(t)}{s_1(t)}$ remains constant. We can therefore conclude that the ICA remains a robust agreement if even renegotiations are reopened at a future date. This is the time-consistency property of an agreement (see Zaccour (2008) for a tutorial).

We now examine whether the ICA determined in Proposition 1 also satisfies IIR , given by (14). We find the following result a priori surprising: for any given pair of initial stocks (s_{10}, s_{20}), when γ is small enough any Pareto optimal pair of extraction paths necessarily has an initial phase where, at least one player is worse off than under non-cooperation. Surprisingly this holds even for the case of symmetric players: when γ is small enough, the ICA results in instantaneous losses for both players. More precisely we have the following proposition.

Proposition 4 (IIR)**.** *For any given pair of initial stocks (s_{10}, s_{20}), there exists $\bar{\gamma} > 0$ such that for all r, c with $\frac{r}{c} < \bar{\gamma}$, there is no Pareto optimal agreement that satisfies immediate individual rationality (14).*

Proof. See Appendix E. □

Since our IIR criterion is weaker than the instantaneous IR criterion (Zaccour 2008) we can also conclude that when the discount rate is small enough, there exists no Pareto optimal agreement that satisfies the instantaneous IR.

This result is interesting and counter-intuitive: it is when players are sufficiently patient that they can not reach any agreement that achieves immediate gains with respect to the non-cooperative outcome. This appears to run against the general wisdom from the Folk Theorem where more patience increases the range of situations where cooperation is sustainable. We should note however that in our analysis we are interested in cooperative agreements and do not examine sustaining cooperation with punishment strategies. The intuition behind our result is that, in our framework, in addition to the duopolistic competition in the output market, when players are sufficiently patient they tend to conserve the stock longer for cost savings purposes. This conservative attitude helps achieve larger long-run profits at the expense of short-run profits.

Unfortunately, both Propositions 3 and 4 do not provide exact values of the bounds of parameters $\gamma = \frac{r}{c}$ and $\frac{s_{20}}{s_{10}}$ inside which the ICA, characterized in proposition 1, is individually rational (IR) and / or immediately individually rational (IIR). We conduct a numerical analysis of these bounds and summarize the results in Figure 3 which shows two areas. The first one, delimited by plain lines, is the set of ratios of initial stocks $\frac{s_{20}}{s_{10}}$ and $\gamma = \frac{r}{c}$ for which the ICA is IR for both firms. The second area, delimited by dotted lines, is the set of ratios of initial stocks $\frac{s_{20}}{s_{10}}$ and $\gamma = \frac{r}{c}$ for which the ICA is IIR for both firms.

A decrease in r, or an increase in c, tends to enlarge the bandwidth of the area inside which the ICA is IR. It follows the intuition of the Folk Theorem, that is the more patient the firms are, the more feasible is a cooperation between them. In contrast, a decrease in r, or an increase in c, tends to reduce the bandwidth of the area inside which the ICA is IIR. For $\frac{r}{c} \leq \bar{\gamma}$, this set is even empty: no cooperative agreement is IIR below this threshold.

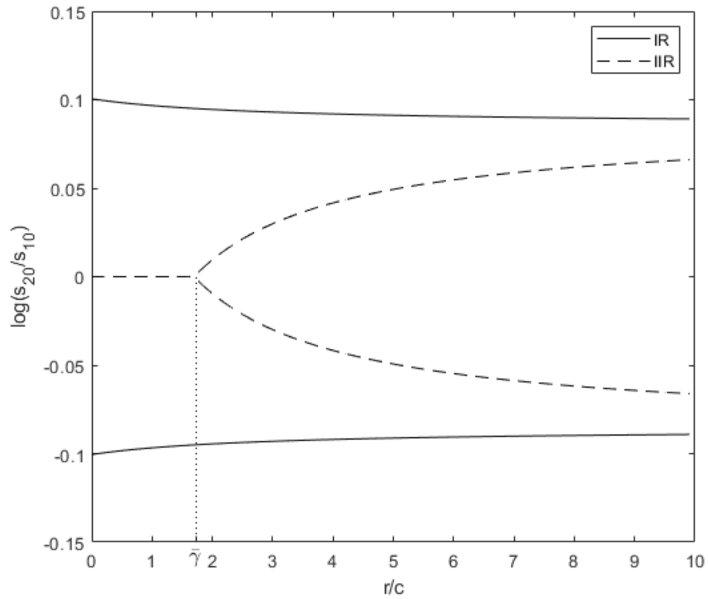

Figure 3: Areas inside which the ICA is IR and / or IIR ($r \in [0, 0.1]$, $c = 0.01$).

4 Welfare analysis

We compare social welfare, defined as the sum of consumers' surplus and firms' surplus, under the cooperative agreement obtained in Proposition 1, to social welfare under non-cooperation.

It is interesting to note that within our framework, cooperation is not a priori necessarily detrimental to social welfare. There is the usual cartelization effect whereby an initial reduction in overall output. However, in the case of asymmetric firms, there is a cost inefficiency under non-cooperation due to the fact that the owner of the smaller stock, and therefore the firm with larger marginal cost, is simultaneously extracting along the low marginal cost firm. What is the impact of cooperation on this source of inefficiency? This is an important question that does not have an a priori obvious answer. If cooperation reduces this source of inefficiency then the cooperative agreement has the potential to increase social welfare defined as the sum of consumers' surplus and firms' surplus. Our analysis reveals that despite this potential efficiency gain, cooperation of firms is detrimental to social welfare.

We first compare, for a given pair of initial stocks (s_{10}, s_{20}), the total extraction path under the associated ICA, $q^\star(.)$, and under non-cooperation, $\bar{q}(.)$. We have

$$q^\star(t) = -\theta^\star(s_{10} + s_{20})e^{\theta^\star t} \tag{17}$$

$$\bar{q}(t) = -\theta_2(s_{10} + s_{20})e^{\theta_2 t} \tag{18}$$

Initial extraction rates equal $q^\star(0) = -\theta^\star(s_{10} + s_{20})$ and $\bar{q}(0) = -\theta_2(s_{10} + s_{20})$. We show that $\theta^\star > \theta_2 > \theta_1$ (see appendix F) and, therefore, $q^\star(0) < \bar{q}(0)$. This implies an initial loss of consumers' surplus due to cooperation between the duopolists. This cooperation is in fact detrimental to the discounted sum of consumers' surplus. Indeed, this latter is given by:

$$CS = \int_0^\infty \left(p_0 q - \frac{q^2}{2} - pq\right) e^{-rt} dt = \int_0^\infty \frac{q^2}{2} e^{-rt} dt$$

Then, straightforward calculations give:

$$CS^\star = \int_0^\infty \frac{q^{\star 2}}{2} e^{-rt} dt = -\frac{\theta^{\star 2}}{2\theta^\star - r} \frac{(s_{10} + s_{20})^2}{2} \tag{19}$$

$$\overline{CS} = \int_0^\infty \frac{\bar{q}^2}{2} e^{-rt} dt = -\frac{\theta_2^2}{2\theta_2 - r} \frac{(s_{10} + s_{20})^2}{2} \tag{20}$$

Note that the function $F(\theta) \equiv -\frac{\theta^2}{2\theta - r}$ is a decreasing function of $\theta \in]-\infty, 0)$. Therefore, as we have shown that $\theta^\star > \theta_2$, we can conclude that $CS^\star < \overline{CS}$.

Cooperation on the other hand results in an increase in the industry's discounted sum of profits only for a range of initial stocks. The firms' surplus is the sum of the individual firms' surplus, given by (15), under non-cooperation, and (16), under cooperation. More precisely, we have

$$J_1^\star + J_2^\star = \frac{c\theta^\star}{2\theta^\star - r}\frac{(s_{10}-s_{20})^2}{2} + \frac{\theta^\star(c+2\theta^\star)}{2\theta^\star - r}\frac{(s_{10}+s_{20})^2}{2} \tag{21}$$

$$\bar{J}_1 + \bar{J}_2 = \frac{c\theta_1}{2\theta_1 - r}\frac{(s_{10}-s_{20})^2}{2} + \frac{\theta_2(c+2\theta_2)}{2\theta_2 - r}\frac{(s_{10}+s_{20})^2}{2} \tag{22}$$

First, recall that, according to Proposition 2, the agreement is such that $\bar{J}_1 + \bar{J}_2 > J_1^\star + J_2^\star$ for symmetric firms. Consider now asymmetric firms. Note that the function $G(\theta) \equiv \frac{c\theta}{2\theta - r}$ is a decreasing function of $\theta \in]-\infty, 0)$. Thus, the inequality $\theta^\star > \theta_1$ implies that $\frac{c\theta^\star}{2\theta^\star - r} < \frac{c\theta_1}{2\theta_1 - r}$. Moreover, we have showed, in Appendix D, that the function $G(\theta) \equiv \frac{\theta(c+2\theta)}{2\theta - r}$ reaches a maximum for $\theta = \frac{r}{2} - \sqrt{\left(\frac{r}{2}\right)^2 + \frac{rc}{4}}$, over $]-\infty, 0)$, and is decreasing hereafter. This latter value is exactly the one taken by $\theta^\star$ when stocks are equal. Therefore, we get that $\frac{\theta^\star(c+2\theta^\star)}{2\theta^\star - r} > \frac{\theta_2(c+2\theta_2)}{2\theta_2 - r}$ for symmetric firms and $\frac{\theta^\star(c+2\theta^\star)}{2\theta^\star - r} < \frac{\theta_2(c+2\theta_2)}{2\theta_2 - r}$ when firms are 'too' asymmetric. Finally, with asymmetry the discounted sum of profits ends up by being higher under non-cooperation than under cooperation.

This point is illustrated on Figure 4, the range of relative stocks where industry's discounted sum of profits increases due to the cooperative agreement is $\frac{s_{10}}{s_{20}} \in [0.72, 1.39]$. When the stocks are 'too' asymmetric, the agreement is heavily distorted towards the small firm's payoff and implies a 'very' conservative industry's extraction path. The range $[0.72, 1.39]$ clearly includes $[0.91, 1.09]$, under which the cooperative agreement is individually rational.

The social welfare is the addition of the consumers' surplus and the firms' surplus:

$$J_1^\star + J_2^\star + CS^\star = \frac{c\theta^\star}{2\theta^\star - r}\frac{(s_{10}-s_{20})^2}{2} + \frac{\theta^\star(c+\theta^\star)}{2\theta^\star - r}\frac{(s_{10}+s_{20})^2}{2} \tag{23}$$

$$\bar{J}_1 + \bar{J}_2 + \overline{CS} = \frac{c\theta_1}{2\theta_1 - r}\frac{(s_{10}-s_{20})^2}{2} + \frac{\theta_2(c+\theta_2)}{2\theta_2 - r}\frac{(s_{10}+s_{20})^2}{2} \tag{24}$$

Again, we know that $\frac{c\theta^\star}{2\theta^\star - r} < \frac{c\theta_1}{2\theta_1 - r}$. Moreover, the function $L(\theta) \equiv \frac{\theta^\star(c+\theta^\star)}{2\theta^\star - r}$ reaches a maximum $\theta = \frac{r}{2} - \sqrt{\left(\frac{r}{2}\right)^2 + \frac{rc}{2}}$, over $]-\infty, 0)$, and is decreasing hereafter. But both θ_2 and $\theta^\star$ are higher than this latter value. Therefore, $\frac{\theta^\star(c+\theta^\star)}{2\theta^\star - r} < \frac{\theta_2(c+\theta_2)}{2\theta_2 - r}$ and the social welfare is always higher under non-cooperation than under cooperation.

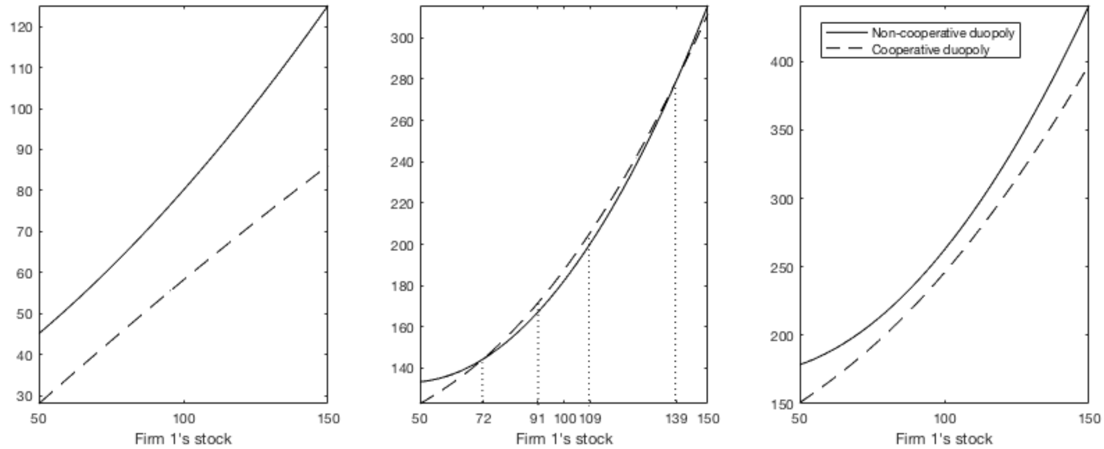

Figure 4: Consumers' surplus (left), firms' surplus (center) and social welfare (right) for $r = 0.05$, $c = 0.08$ and $s_{20} = 100$

Finally, this welfare analysis is summarized in this last Proposition:

Proposition 5. *For any pair of initial stocks* (s_{10}, s_{20})*, the discounted sum of social welfare under the ICA is smaller than under non-cooperation.*

5 Concluding remarks

In a linear quadratic differential game framework, inspired by Salo and Tahvonen (2001), and using recent results of Reddy and Engwerda (2013), we fully characterize the set of Pareto efficient cooperative agreements. We first show that, for each pair of initial stocks, there is a unique Pareto efficient agreement where the extraction paths of both firms are positive. Any other Pareto efficient agreement implies that one producer initially refrains from extracting for a period of time. The interior solution has several properties among which the fact that the rate of depletion depends on the ratio of both firms' resource stocks. The reason is that, in order to produce simultaneously, a larger weight must be given to the firm with the smallest resource stock in the overall maximization. Therefore, the more the firms are asymmetric, the more they have to exert their market power and slow extraction to push up the price.

However this interior cooperative agreement may not satisfy individual rationality. We show that there exists a neighborhood of the pure symmetric situation where the interior cooperative agreement is acceptable to both players. Thus, if a resource duopoly is not too asymmetric, cooperation is rational for both players. However when the discount rate is small enough, there exists no Pareto efficient agreement that can result in short-run gains for both players with respect to the non-cooperative equilibrium, even in the case where the players are symmetric.

Our framework is obviously stylized and some additions such as incorporating exploration decisions would be both relevant and insightful. Indeed, since agreements are very much dependent on the ratio of stocks owned by each player, examining how the possibility of an agreement affects players' incentive for exploration would be particularly insightful. This is beyond the scope of the present paper but constitutes a promising line for future research.

A Proof of Proposition 1: Interior Cooperative Agreements (ICA)

First, we have to check that the dynamical system

$$\dot{s}_1(t) = -q_1(t)$$
$$\dot{s}_2(t) = -q_2(t)$$

is controllable. Rewriting it in a canonical way, we get that

$$\begin{bmatrix} \dot{s}_1 \\ \dot{s}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}}_{A} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \underbrace{\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}}_{B} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

This system is controllable iff the controllability matrix

$$\begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}$$

has full row rank (see Theorem 3.19 in Engwerda, 2005, p. 92). This is trivially the case here: the dynamic system is therefore controllable. Under this condition, Reddy and Engwerda (2013) showed that Leitmann's Lemma is actually a necessary and sufficient condition for Pareto optimality and, thus, we can get every feasible Pareto optimal controls by maximizing the weighted sum of the payoff:

$$\begin{aligned} &\max_{q_1, q_2} \alpha J_1 + (1-\alpha) J_2 \\ s.t. \quad &\dot{s}_1 = -q_1 \quad \dot{s}_2 = -q_2 \quad \alpha \in (0,1) \end{aligned} \tag{25}$$

Let $\alpha \in (0,1)$, the Hamiltonian corresponding to (25) is:

$$\mathcal{H}(s_1, s_2, q_1, q_2, \lambda_1, \lambda_2) = \alpha(cs_1 - q_1 - q_2)q_1 + (1-\alpha)(cs_2 - q_1 - q_2)q_2 - \lambda_1 q_1 - \lambda_2 q_2 \tag{26}$$

Where (λ_1, λ_2) designate the costate variables of state constraints $(\dot{s}_1 = -q_1, \dot{s}_2 = -q_2)$.

Considering an interior solution $(q_1^\star, q_2^\star)$, the first order conditions of the maximization read:

$$\lambda_1 = \alpha c s_1 - 2\alpha q_1^\star - q_2^\star \tag{27}$$

$$\lambda_2 = (1-\alpha)c s_2 - q_1^\star - 2(1-\alpha)q_2^\star \tag{28}$$

$$\dot{\lambda}_1 = r\lambda_1 - \alpha c q_1^\star \tag{29}$$

$$\dot{\lambda}_2 = r\lambda_2 - (1-\alpha)c q_2^\star \tag{30}$$

Assume that $\alpha \neq \frac{1}{2}$ (the case $\alpha = \frac{1}{2}$ is solved separately, in Appendix B). Using (27) and (28), we can express the extraction rates with respect to the state and costate variables as

$$q_1^\star = \frac{-2\alpha(1-\alpha)cs_1 + (1-\alpha)cs_2 + 2(1-\alpha)\lambda_1 - \lambda_2}{1-4\alpha(1-\alpha)}$$
$$q_2^\star = \frac{-2\alpha(1-\alpha)cs_2 + \alpha cs_1 + 2\alpha\lambda_2 - \lambda_1}{1-4\alpha(1-\alpha)}$$

Substituting these expressions into (29) and (30), we get the following Modified Hamiltonian Dynamic System:

$$\begin{bmatrix} \dot{s}_1(t) \\ \dot{s}_2(t) \\ \dot{\lambda}_1(t) \\ \dot{\lambda}_2(t) \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{2\alpha(1-\alpha)c}{1-4\alpha(1-\alpha)} & -\frac{(1-\alpha)c}{1-4\alpha(1-\alpha)} & -\frac{2(1-\alpha)}{1-4\alpha(1-\alpha)} & \frac{1}{1-4\alpha(1-\alpha)} \\ -\frac{\alpha c}{1-4\alpha(1-\alpha)} & \frac{2\alpha(1-\alpha)c}{1-4\alpha(1-\alpha)} & \frac{1}{1-4\alpha(1-\alpha)} & -\frac{2\alpha}{1-4\alpha(1-\alpha)} \\ \frac{2\alpha^2(1-\alpha)c^2}{1-4\alpha(1-\alpha)} & -\frac{\alpha(1-\alpha)c^2}{1-4\alpha(1-\alpha)} & r - \frac{2\alpha(1-\alpha)c}{1-4\alpha(1-\alpha)} & \frac{\alpha c}{1-4\alpha(1-\alpha)} \\ -\frac{\alpha(1-\alpha)c^2}{1-4\alpha(1-\alpha)} & \frac{2\alpha(1-\alpha)^2c^2}{1-4\alpha(1-\alpha)} & \frac{(1-\alpha)c}{1-4\alpha(1-\alpha)} & r - \frac{2\alpha(1-\alpha)c}{1-4\alpha(1-\alpha)} \end{bmatrix}}_{M_\alpha} \begin{bmatrix} s_1(t) \\ s_2(t) \\ \lambda_1(t) \\ \lambda_2(t) \end{bmatrix} \tag{31}$$

The explicit expression of the eigenvalues of the Jacobian matrix M_α is known (see Kemp et al., 1993, Dockner, 1991 or Salo and Tahvonen, 2001) and equal

$$\theta^\star_{1,2,3,4} = \frac{r}{2} \pm \sqrt{\left(\frac{r}{2}\right)^2 - \frac{\delta}{2} \pm \sqrt{\left(\frac{\delta}{2}\right)^2 - \det M_\alpha}} \tag{32}$$

where $\delta = \frac{4\alpha(1-\alpha)rc}{1-4\alpha(1-\alpha)}$ and, after some transformations ($L_3 \leftarrow L_3 - \alpha c L_1$ and $L_4 \leftarrow L_4 - (1-\alpha)cL_2$):

$$\det M_\alpha = r^2 \begin{vmatrix} \frac{2\alpha(1-\alpha)c}{1-4\alpha(1-\alpha)} & -\frac{(1-\alpha)c}{1-4\alpha(1-\alpha)} \\ -\frac{\alpha c}{1-4\alpha(1-\alpha)} & \frac{2\alpha(1-\alpha)c}{1-4\alpha(1-\alpha)} \end{vmatrix} = -\frac{\alpha(1-\alpha)r^2c^2}{1-4\alpha(1-\alpha)}$$

The negative sign of the determinant $\det M_\alpha$ and the positive sign of δ imply that among the eigenvalues given by (32), only one is negative and its expression is

$$\theta^\star = \frac{r}{2} - \sqrt{\left(\frac{r}{2}\right)^2 - \frac{\delta}{2} + \sqrt{\left(\frac{\delta}{2}\right)^2 - \det M_\alpha}} \tag{33}$$

Because, of the transversality conditions on the costate variables, we only keep the negative eigenvalue and the resource stocks can be expressed as

$$s_1(t) = s_{10}e^{\theta^\star t}$$
$$s_2(t) = s_{20}e^{\theta^\star t}$$

Substituting these expressions into (27) and (28), we get that costate variables (λ_1, λ_2) equal:

$$\lambda_1(t) = (\alpha(c + 2\theta^\star)s_{10} + \theta^\star s_{20})e^{\theta^\star t} \tag{34}$$
$$\lambda_2(t) = ((1-\alpha)(c + 2\theta^\star)s_{20} + \theta^\star s_{10})e^{\theta^\star t} \tag{35}$$

Substituting again (34) and (35) into (29) and (30), and rearranging the terms, leads to

$$\alpha\left(\frac{rc - 2\theta^\star(\theta^\star - r)}{\theta^\star(\theta^\star - r)}\right) = \frac{s_{20}}{s_{10}} \tag{36}$$
$$(1-\alpha)\left(\frac{rc - 2\theta^\star(\theta^\star - r)}{\theta^\star(\theta^\star - r)}\right) = \frac{s_{10}}{s_{20}} \tag{37}$$

We verify that $\left(\frac{rc-2\theta^\star(\theta^\star-r)}{\theta^\star(\theta^\star-r)}\right) \neq 0$. Indeed:

$$\theta^\star(\theta^\star - r) = -\frac{\delta}{2} + \sqrt{\left(\frac{\delta}{2}\right)^2 - \det M_\alpha}$$

which never equals $\frac{rc}{2}$, whatever the value of $\alpha \neq \frac{1}{2}$. Finally, combining (36) and (37) yields to

$$\alpha = \frac{\left(\frac{s_{20}}{s_{10}}\right)^2}{1 + \left(\frac{s_{20}}{s_{10}}\right)^2}$$

Substituting this expression into (33), after some algebra, we get that

$$\theta^\star = \frac{r}{2} - \sqrt{\left(\frac{r}{2}\right)^2 + \frac{rc\frac{s_{20}}{s_{10}}}{\left(1 + \frac{s_{20}}{s_{10}}\right)^2}}$$

B Proof of proposition 1 for $\alpha = \frac{1}{2}$

Assume that $\alpha = \frac{1}{2}$. First-order conditions (27) and (28) give:

$$\lambda_1 - \lambda_2 = \frac{cs_1^\star - cs_2^\star}{2}$$

The derivation of this expression yields

$$\dot{\lambda}_1 - \dot{\lambda}_2 = \frac{c\dot{s}_1^\star - c\dot{s}_2^\star}{2}$$

Substituting this into (29) and (30), we get that

$$r(\lambda_1 - \lambda_2) = 0$$

and, thus, $s_1^\star(.) = s_2^\star(.)$.

In particular:

$$s_{10} = s_{20}$$

Finally, stock, extraction, and co-state paths will be the same for both firms. Consider, for instance, firm 1. Her stock variables $s_1^\star(.)$ is solution of

$$\begin{bmatrix} \dot{s}_1 \\ \dot{\lambda}_1 \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{c}{4} & \frac{1}{2} \\ -\frac{c^2}{8} & \frac{c}{4} + r \end{bmatrix}}_{M_{1/2}} \begin{bmatrix} s_1 \\ \lambda_1 \end{bmatrix}$$

Eigenvalues of the Jacobian matrix $M_{1/2}$ are:

$$\theta_{1,2}^\star = \frac{r}{2} \pm \sqrt{\left(\frac{r}{2}\right)^2 + \frac{rc}{4}}$$

Again, the transversality condition leads to keep only the negative root $\theta^\star = \frac{r}{2} - \sqrt{\left(\frac{r}{2}\right)^2 + \frac{rc}{4}}$, and

$$s_1^\star(t) = s_{10} e^{\theta^\star t} \left(= s_2^\star(t) = s_{20} e^{\theta^\star t} \right)$$

We observe that this solution is exactly what we can obtain from the general case when $s_{10} = s_{20}$.

C Proof of Proposition 2: corner solutions

Let a pair of initial stocks (s_{10}, s_{20}), we show that a corner solution happens when we solve optimal control problem (25) with α different from $\alpha^\star = \frac{\left(\frac{s_{20}}{s_{10}}\right)^2}{1+\left(\frac{s_{20}}{s_{10}}\right)^2}$.

Let $\alpha > \alpha^\star$ and define $\tilde{s}_{10}$ such that:

$$\alpha = \frac{\left(\frac{s_{20}}{\tilde{s}_{10}}\right)^2}{1 + \left(\frac{s_{20}}{\tilde{s}_{10}}\right)^2}$$

We postulate that firm 1 produces alone within a first phase whose duration is denoted by T, until her resource stock reaches $\tilde{s}_{10}$. Then, the second phase consists in an the ICA which corresponds to α, both firms extracting until infinite horizon. The optimal control problem is then equivalent to

$$\max_{q_1} \int_0^T e^{-rt}\alpha\left(cs_1q_1 - q_1^2\right)dt + e^{-rT}J_\alpha^\star(\tilde{s}_{10}, s_{20}) \tag{38}$$

where the scrap value $e^{-rT}J_\alpha^\star(\tilde{s}_{10}, s_{20})$ is the discounted payoff of the ICA for the pair of stocks $(\tilde{s}_{10}, s_{20})$, starting at $t = T$.

Hamiltonian associated with (38) is

$$\mathcal{H}(s_1, \lambda, q_1) = \alpha\left(cs_1q_1 - q_1^2\right) - \lambda q_1$$

First-order conditions are:

$$\alpha\left(cs_1 - 2q_1\right) - \lambda = 0 \tag{39}$$

$$\dot{\lambda} = r\lambda - \alpha c q_1 \tag{40}$$

Equations (39) and (40) imply the following Modified Hamiltonian Dynamic System

$$\begin{bmatrix}\dot{s}_1\\ \dot{\lambda}\end{bmatrix} = \underbrace{\begin{bmatrix}-\frac{c}{2} & \frac{1}{2\alpha}\\ -\frac{\alpha c^2}{2} & r+\frac{c}{2}\end{bmatrix}}_{M_c}\begin{bmatrix}s_1\\ \lambda\end{bmatrix}$$

Eigenvalues of the Jacobian matrix M_c are $\psi_{1,2} = \frac{r}{2} \pm \sqrt{\left(\frac{r}{2}\right)^2 + \frac{cr}{2}}$ where $\psi_1 < 0$ and $\psi_2 > 0$. Thus, the solution can be written as

$$s_1(t) = Ae^{\psi_1 t} + Be^{\psi_2 t}$$
$$\lambda_1(t) = \frac{\alpha c\psi_1}{\psi_1 - r}Ae^{\psi_1 t} + \frac{\alpha c\psi_2}{\psi_2 - r}Be^{\psi_2 t}$$

where A and B are constants to determine.

Initial and terminal conditions being $s_1(0) = s_{10}$, $s_1(T) = \tilde{s}_{10}$, we get that:

$$A = \frac{\tilde{s}_{10} - s_{10}e^{\psi_2 T}}{e^{\psi_1 T} - e^{\psi_2 T}}$$

$$B = \frac{s_{10}e^{\psi_1 T} - \tilde{s}_{10}}{e^{\psi_1 T} - e^{\psi_2 T}}$$

Last, the transversality condition is $\mathcal{H}(s_1(T), \lambda(T), q_1(T)) = rJ^\star_\alpha(\tilde{s}_{10}, s_{20})$. Equivalently, it leads to $\alpha q_1^2(T) = rJ^\star_\alpha(\tilde{s}_{10}, s_{20})$ or

$$\left(-\psi_1 \frac{\tilde{s}_{10} - s_{10}e^{\psi_2 T}}{e^{\psi_1 T} - e^{\psi_2 T}} e^{\psi_1 T} - \psi_2 \frac{s_{10}e^{\psi_1 T} - \tilde{s}_{10}}{e^{\psi_1 T} - e^{\psi_2 T}} e^{\psi_2 T}\right)^2 = \frac{r}{\alpha} J^\star_\alpha(\tilde{s}_{10}, s_{20}) \tag{41}$$

To finish the proof we have to show that this latter equation has a solution $T > 0$. We define the function

$$LHS_1(T) \equiv \left(-\psi_1 \frac{\tilde{s}_{10} - s_{10}e^{\psi_2 T}}{e^{\psi_1 T} - e^{\psi_2 T}} e^{\psi_1 T} - \psi_2 \frac{s_{10}e^{\psi_1 T} - \tilde{s}_{10}}{e^{\psi_1 T} - e^{\psi_2 T}} e^{\psi_2 T}\right)^2$$

which corresponds to the Left-Hand Side of equation (41). First, we show that there exists a unique $\bar{T}$ such that $LHS_1(\bar{T}) = 0$. Indeed,

$$LHS_1(T) = 0 \Leftrightarrow -\tilde{s}_{10}(\psi_1 e^{\psi_1 T} - \psi_2 e^{\psi_2 T}) - s_{10}(\psi_2 - \psi_1)e^{(\psi_1+\psi_2)T} = 0$$

By continuity, this equation has a unique solution $\bar{T}$ because the function $LHS_2(T) \equiv -\tilde{s}_{10}(\psi_1 e^{\psi_1 T} - \psi_2 e^{\psi_2 T}) - s_{10}(\psi_2 - \psi_1)e^{(\psi_1+\psi_2)T}$ is increasing and

$$LHS_2(0) = -(\psi_2 - \psi_1)(s_{10} - \tilde{s}_{10}) < 0 \;\; as \;\; s_{10} > \tilde{s}_{10}$$

$$LHS_2(+\infty) = +\infty$$

Note that over $[\bar{T}, +\infty)$, the function LHS_2 is positive. Therefore, it would result in a negative expression inside the brackets of LHS_1, which is impossible because $q_1(T)$ cannot be negative. As a consequence, we focus hereafter on $(0, \bar{T}]$. Over this interval, the function LHS_1 is decreasing and

$$LHS_1(0) = +\infty$$

$$LHS_1(\bar{T}) = 0$$

By continuity, (41) has then a unique $T > 0$ and this concludes the characterization of the corner solutions.

D Proof of Proposition 3: individual rationality of the ICA

Consider firm 1. Factorizing by cs_{10}^2, denoting $\gamma = \frac{r}{c}$, firm 1's non-cooperative payoff can be written as

$$\bar{J}_1(s_{10}, s_{20}) = cs_{10}^2 \underbrace{\left(\frac{\frac{\theta_1}{c}}{2\frac{\theta_1}{c} - \gamma} \left(\frac{1 - \frac{s_{20}}{s_{10}}}{2} \right)^2 + \frac{\frac{\theta_2}{c}(1 + 2\frac{\theta_2}{c})}{2\frac{\theta_2}{c} - \gamma} \left(\frac{1 + \frac{s_{20}}{s_{10}}}{2} \right)^2 + \frac{(\frac{\theta_1}{c} + \frac{\theta_2}{c}) + 2\frac{\theta_1}{c}\frac{\theta_2}{c}}{\frac{\theta_1}{c} + \frac{\theta_2}{c} - \gamma} \left(\frac{1 - \frac{s_{20}}{s_{10}}}{4} \right) \right)}_{= \bar{\phi}_1 \left(\frac{s_{20}}{s_{10}} \right)}$$

Similarly, the ICA's payoff is

$$J_1^\star(s_{10}, s_{20}) = cs_{10}^2 \underbrace{\left(\frac{\frac{\theta^\star}{c}}{2\frac{\theta^\star}{c} - \gamma} \left(\left(1 + \frac{\theta^\star}{c} \right) + \frac{\theta^\star}{c} \frac{s_{20}}{s_{10}} \right) \right)}_{= \phi_1^\star \left(\frac{s_{20}}{s_{10}} \right)}$$

Thus, studying the sign of $J_1^\star(s_{10}, s_{20}) - \bar{J}_1(s_{10}, s_{20})$ is equivalent to study those of $\phi_1^\star \left(\frac{s_{20}}{s_{10}} \right) - \bar{\phi}_1 \left(\frac{s_{20}}{s_{10}} \right)$ which only depends on the ratio of initial stocks $\frac{s_{20}}{s_{10}}$ and γ. Note that it proves Lemma 2.

The remainder of this Proof falls into three parts. First, we show that when the cartel is symmetric, the ICA is always individually rational. Assume that $\frac{s_{20}}{s_{10}} = 1$. In this case:

$$\bar{\phi}_1(1) = \frac{\frac{\theta_2}{c}\left(1 + 2\frac{\theta_2}{c}\right)}{2\frac{\theta_2}{c} - \gamma} \qquad \phi_1^\star(1) = \frac{\frac{\theta^\star}{c}\left(1 + 2\frac{\theta^\star}{c}\right)}{2\frac{\theta^\star}{c} - \gamma}$$

The function $G(x) \equiv \frac{x(1+2x)}{2x-\gamma}$ reaches a local maximum over $]-\infty, 0]$ for $x = \frac{\theta^\star}{c}$. Indeed, its derivative is:

$$G'(x) = \frac{4x^2 - 4\gamma x - \gamma}{(2x-\gamma)^2}$$

which is positive on $\left(-\infty, \frac{\gamma}{2} - \sqrt{\left(\frac{\gamma}{2}\right)^2 + \frac{\gamma}{4}}\right]$ and negative on $\left[\frac{\gamma}{2} - \sqrt{\left(\frac{\gamma}{2}\right)^2 + \frac{\gamma}{4}}, 0\right]$. Yet, the threshold $\frac{\gamma}{2} - \sqrt{\left(\frac{\gamma}{2}\right)^2 + \frac{\gamma}{4}}$ is exactly the value taken by $\frac{\theta^\star}{c}$ when $\frac{s_{20}}{s_{10}} = 1$. Finally, we can conclude that $\phi_1^\star(1) - \bar{\phi}_1(1) > 0$: the ICA is always individually rational for symmetric firms.

Second, we show that the ICA is never individually rational for $\frac{s_{20}}{s_{10}} \to 0$. Indeed, we have

$$\lim_{\frac{s_{20}}{s_{10}} \to 0} \phi_1^\star\left(\frac{s_{20}}{s_{10}}\right) = 0$$

Moreover, we know from Salo and Tahvonen (2001) that when $\frac{s_{20}}{s_{10}} \in]-\infty, \frac{\theta_1 - \theta_2}{\theta_1 + \theta_2}]$, firm 1 is initially alone on the market, for a period of time, because firm 2 is too small to enter it. At the extreme, when $\frac{s_{20}}{s_{10}} \to 0$, firm 1's payoff tends to a monopolist payoff, the duration of the first phase being infinite.

Finally, we can conclude that $\lim_{\frac{s_{20}}{s_{10}} \to 0} \left(\phi_1^\star\left(\frac{s_{20}}{s_{10}}\right) - \bar{\phi}_1\left(\frac{s_{20}}{s_{10}}\right)\right) < 0$.

Third, knowing the two previous steps, by continuity, there exists $\frac{s_{20}}{s_{10}} \in (0, 1]$ such that $\phi_1^\star\left(\frac{s_{20}}{s_{10}}\right) - \bar{\phi}_1\left(\frac{s_{20}}{s_{10}}\right) = 0$. The result being symmetric for firm 2, we can conclude that there exists $\epsilon > 0$ such that $\phi_i^\star\left(\frac{s_{20}}{s_{10}}\right) - \bar{\phi}_i\left(\frac{s_{20}}{s_{10}}\right) > 0$ $(i = 1, 2)$ for $\frac{s_{20}}{s_{10}} \in [1-\epsilon, 1+\epsilon]$.

E Proof of Proposition 4: immediate individual rationality of the ICA

Assume that firms are symmetric and consider firm 1. Writing down her initial profit and factorizing it by $c^2 s_{10}^2$, denoting $\gamma = \frac{r}{c}$, we get, under non-cooperation:

$$\bar{\pi}_1(0) = c^2 s_{10}^2 \underbrace{\left(\frac{\theta_1}{c} \left(\frac{1 - \frac{s_{20}}{s_{10}}}{2} \right)^2 + \frac{\theta_2}{c} (1 + 2\frac{\theta_2}{c}) \left(\frac{1 + \frac{s_{20}}{s_{10}}}{2} \right)^2 + (\frac{\theta_1}{c} + \frac{\theta_2}{c}) + 2\frac{\theta_1}{c}\frac{\theta_2}{c} \left(\frac{1 - \frac{s_{20}}{s_{10}}}{4} \right) \right)}_{= \bar{\chi}_1\left(\frac{s_{20}}{s_{10}}\right)}$$

Similarly, her initial profit under cooperation is

$$\pi_1^\star(0) = c^2 s_{10}^2 \underbrace{\left(\frac{\theta^\star}{c} \left(\left(1 + \frac{\theta^\star}{c} \right) + \frac{\theta^\star}{c} \frac{s_{20}}{s_{10}} \right) \right)}_{= \chi_1^\star\left(\frac{s_{20}}{s_{10}}\right)}$$

We want to show that, for $\frac{s_{20}}{s_{10}} = 1$, there exists $\bar{\gamma}$ such that

$$\begin{aligned} \chi_1^\star(1) &> \chi_1(1) \quad \textit{when } \gamma > \bar{\gamma} \\ \chi_1^\star(1) &< \chi_1(1) \quad \textit{when } \gamma < \bar{\gamma} \end{aligned}$$

The inequality $\chi_1^\star(1) > \chi_1(1)$ is equivalent to

$$\frac{\theta^\star}{c} \left(1 + 2\frac{\theta^\star}{c} \right) > \frac{\theta_2}{c} \left(1 + 2\frac{\theta_2}{c} \right)$$

The function $K(x) \equiv x(1 + 2x)$ reaches a minimum for $x = -\frac{1}{4}$ over $(-\infty, 0]$. Besides, for $\gamma \to +\infty$, we can write that:

$$\lim_{\gamma \to +\infty} \frac{\theta^\star}{c} = -\frac{1}{4}$$

$$\lim_{\gamma \to +\infty} \frac{\theta_2}{c} = -\frac{1}{2}$$

And therefore, for γ sufficiently high, $\chi_1^\star(1) > \chi_1(1)$.

In contrast, we know that

$$\lim_{\gamma\to 0}\theta_2 = 0$$

$$\lim_{\gamma\to 0}\theta^\star = 0$$

Knowing also that $\theta_2 < \theta^\star$ (see F), there exists $\bar{\gamma}$ such that for $\gamma < \bar{\gamma}$, we get $-\frac{1}{4} < \frac{\theta_2}{c} < \frac{\theta^\star}{c}$. It implies that for $\gamma < \bar{\gamma}$, $\chi_1^\star(1) < \chi_1(1)$ and, therefore, no agreement is IIR for symmetric firms. As a consequence, no cooperative agreement can be IIR for asymmetric firms either.

F Proof of the ranking $\theta_1 < \theta_2 < \theta^\star$

The property $\theta_1 < \theta_2 < \theta^\star$ is used several times in the welfare analysis. The inequality $\theta_1 < \theta_2$ has already been proved in Salo and Tahvonen (2001). In order, to prove that $\theta_2 < \theta^\star$, we can consider only symmetric stocks because, in this case,

$$\theta^\star = \frac{r}{2} - \sqrt{\left(\frac{r}{2}\right)^2 + \frac{rc}{4}}$$

which is the minimal value that $\theta^\star$ can take, for given r and c. Recall that

$$\theta_2 = \theta_2 = \frac{r}{2} - \sqrt{\left(\frac{r}{2}\right)^2 + \frac{cr}{2}\frac{r_0(\gamma)-3}{\sqrt{63+r_0(\gamma)^2}}}$$

where $r_0(\gamma) = 3\gamma + 5 + 2\sqrt{64+60\gamma+9\gamma^2}\cos\left[\frac{1}{3}\arccos\left(\frac{404+666\gamma+270\gamma^2+27\gamma^3}{(64+60\gamma+9\gamma^2)^{3/2}}\right)\right]$, $\gamma = \frac{r}{c}$.

Therefore, $\theta_2 < \theta^\star$ iff

$$\sqrt{\left(\frac{r}{2}\right)^2 + \frac{cr}{2}\frac{r_0(\gamma)-3}{\sqrt{63+r_0(\gamma)^2}}} > \sqrt{\left(\frac{r}{2}\right)^2 + \frac{rc}{4}}$$

This is equivalent to

$$\sqrt{\gamma + 2\frac{r_0(\gamma)-3}{\sqrt{63+r_0(\gamma)^2}}} > \sqrt{\gamma+1}$$

And by squaring this expression

$$(\gamma+2)(r_0(\gamma)-3)^2 > (\gamma+1)(63+r_0(\gamma)^2)$$

Or,

$$r_0(\gamma)^2 - 6r_0(\gamma)(\gamma+2) - 54\gamma - 45 > 0$$

This inequality is satisfied iff

$$r_0(\gamma) > 3\gamma + 6 + \sqrt{9\gamma^2 + 90\gamma + 81}$$

In the expression of $r_0(\gamma)$, note that

$$\cos\left[\frac{1}{3}\arccos\left(\frac{404+666\gamma+270\gamma^2+27\gamma^3}{(64+60\gamma+9\gamma^2)^{3/2}}\right)\right] > \cos\left[\frac{1}{3}\arccos\left(\frac{404}{64^{3/2}}\right)\right] \simeq 0.98$$

Therefore, for all γ

$$2\sqrt{64+60\gamma+9\gamma^2}\cos\left[\frac{1}{3}\arccos\left(\frac{404+666\gamma+270\gamma^2+27\gamma^3}{(64+60\gamma+9\gamma^2)^{3/2}}\right)\right] > 2\sqrt{64+60\gamma+9\gamma^2}\cos\left[\frac{1}{3}\arccos\left(\frac{404}{64^{3/2}}\right)\right] > \sqrt{9\gamma^2+90\gamma+81}$$

Moreover,

$$2\sqrt{64+60\gamma+9\gamma^2}\cos\left[\frac{1}{3}\arccos\left(\frac{404}{64^{3/2}}\right)\right] - \sqrt{9\gamma^2+90\gamma+81} > 16\cos\left[\frac{1}{3}\arccos\left(\frac{404}{64^{3/2}}\right)\right] - 9 > 1$$

Finally, $r_0(\gamma)$ satisfies

$$r_0(\gamma) > 3\gamma + 6 + \sqrt{9\gamma^2 + 90\gamma + 81}$$

and, therefore,

$$\theta_2 < \theta^\star$$

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