Closing the Loop in a Circular Economy: Saving Resources or Suffocating Innovations?

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Abstract

A circular economy with closed loops and built-in reusability may provide a “win-win” solution, which alleviates environmental problems and, because of more efficient resource use, promotes economic growth at the same time. Yet enhanced recycling may also hamper innovation, which is essential for long-term reduction in material demand. We complement the extant literature and develop a two-sector endogenous growth model with Schumpeterian innovation. The primary sector is innovative: monopolists continuously develop new products and use primary resources in production. The secondary sector refurbishes retired products. It requires no primary resource for production, but also does not develop new products (and thus does not promote innovations). We use this model to study the tradeoff between resource use, refurbishing, and economic growth. We find that increased refurbishing results in more resource use and increases consumption when equilibrium relative resource prices are non-vanishing. In an equilibrium with ever declining relative resource prices, refurbishing reduces innovation and increases cumulative resource use. Further, the welfare impact follows a U-curve; increased refurbishing benefit economies only after refurbishing exceeds a certain threshold.

Keywords: Circular economy, refurbishing, innovation, creative destruction, resource market

JEL classification: Q55, O30, O41, Q30

1
1 Introduction

The circular economy has gained momentum as an economically desirable remedy for multiple environmental issues. It proposes to adopt closed-loop product designs with built-in reusability. It turns waste into economic inputs, preserves material, and reuses parts and even products for as long as possible. The concept has the appeal of a “win-win” solution that alleviates environmental stress caused by resource use, and improves living standards. As suggested by a case study on five countries (Finland, France, the Netherlands, Spain and Sweden) released by the Club of Rome before the Paris COP21 in November 2015, a move towards a circular economy could cut carbon emission by up to 70% by 2030 while at the same time create a significant amount of additional jobs (Wijkman and Skånberg, 2015). According to the European Commission (2015), measures that support a circular economy, such as waste prevention, ecodesign and re-use, could bring net savings of €600 billion annually for businesses in the EU, while reducing total annual greenhouse gas emissions by 2-4%. As a concrete example, the same memo suggests that a shift from recycling to refurbishing light commercial vehicles could save material inputs by €6.4 billion and energy costs by 140 million per year, while reducing greenhouse gas emissions by 6.3 million tonnes CO$_2$eq.\footnote{We follow IPCC convention to write GHG emissions in CO$_2$ equivalents.}

The promising perspective attracts interest and attention from both policy makers and the private sector. In December 2015, the European Commission adopted a Circular Economy Package including revised legislative Waste Proposals for increasing recycling and reducing landfilling, as well as an Action Plan that seeks measures to “close the loop” along the entire product lifecycle (European Commission, 2015). In 2016, Finland adopted a national circular economy roadmap, and the Netherlands issued a plan for a circular economy by 2050. In March 2017, the European Commission hosted the first Circular Economy Stakeholder Conference, and later launched the European Circular Economy Stakeholder Platform. In June, the first World Circular Economy Forum was held in Helsinki, attracting 1,500 experts and policymakers from 90 countries (Sormunen and Tilikainen, 2017).

Academic research on the circular economy has lagged behind the interest from policy makers and the private sector, and within that literature, the focus has been on recycling, mostly neglecting refurbishing or reuse. The IO literature on the topic originates in the judicial complexities of the Alcoa case in 1945, and focuses on the erosion of monopoly power through competitive recycling (Gaskins, 1974; Martin, 1982; Swan, 1980; Grant, 1999). Gaskins (1974) find that a secondhand market undercuts the monopolist’s revenues, and can lead to higher
prices and lower output in the short run. Martin (1982) emphasizes the role of vertical integration, and shows that in the long run consumers always (weakly) benefit from recycling, with strict benefits if scrap recovery is independent. Swan (1980) studies the role of endogenous scrap rates and the scrap market: whether scrap is sold or discarded by consumers. He finds ambiguous welfare effects with potentially inefficiently high recycling rates. Grant (1999) connects prior studies through a more detailed description of material flows. His empirical estimates suggest that competitive recycling decreases welfare.

De Beir et al. (2010) explore the material balance, noting that the supply of recycled goods is limited by virgin goods produced in the previous period. They use these dynamic connections to better describe the correlations between the recycling industry and the business cycle. They have a model with a competitive virgin goods sector and a public recycling firm. Fodha and Magris (2015) extend their analysis using an overlapping generation model with endogenous labor supply.

Another literature connects recycling to long-run resource markets and scarcity. Hoel (1978) studies recycling as a substitute for resource extraction with less negative environmental impacts. Di Vita (2001) explores transitionary dynamics and differences between developing and developed countries. Considering waste as a valuable production input, Pittel et al. (2010) characterize market inefficiencies when recycling markets are incomplete, and identify policy measures for correcting the market failures to achieve optimal material recycling.

Most of the above-mentioned studies are static and abstract from innovation as an alternative mechanism to reduce future material demand and alleviate scarcity. We complement the literature, adding dynamics and innovation, and connecting the IO implications of the circular economy to resource depletion effects. While a circular economy directly reduces resource depletion, it may indirectly increase future resource use through reduced innovation and technological progress. In particular, if firms that reuse materials and components have a lesser tendency to introduce new products and resource-saving processes, compared to ’original’ producers, or if innovation is driven by short product cycles and creative destruction, than a circular economy may reduce the level of innovation and its complementary resource saving potential. The contribution of our analysis is thus that we investigate the balance between direct resource efficiency gains brought by reuse, against potential future resource use increases brought by reduced technological progress. A second contribution is that we consider refurbishing rather than recycling, which is becoming an important sector higher up in the value-added chain vis-a-vis recycling.

We develop a two-sector endogenous growth model with Schumpeterian innovation. The primary sector is innovative – monopolists continuously develop new products – and it uses primary resources in production. We assume that new prod-
ucts create more value per labor input and per resource input, where the increase in resource efficiency exceeds that in labor efficiency; innovations are resource saving for given labor supply. The secondary sector refurbishes retired products, and it requires no primary resource for production. Importantly, although innovation in the primary sector spills over to the secondary sector and raises productivity in refurbishing, the secondary sector does not develop new products (and thus does not promote innovations). We use this model to study the tradeoff between resource use, refurbishing, and economic growth. As in Pittel et al. (2010), we focus on the scarcity alleviating effect of refurbishing and abstract from negative environmental externalities. We now preview our main results.

We first consider an equilibrium without an abundant supply of resources, in which supply still exceeds demand if resource prices are zero. We assume that refurbishing is competitive because of lower labor production costs compared to the primary sector; the size of the secondary sector is constrained by technological feasibility of refurbishing. We study the comparative statics of an improvement in the refurbishing rate. Increased refurbishing reduces short-term resource use and increases short-run consumption as the secondary sector is more efficient. Increased refurbishing also reduces the incentives for innovators in the primary sector to develop new products, since the fringe of refurbishing firms capture a bigger share of their market. As the rate of innovation falls, not only economic growth falls but also resource-saving technological change occurs at a slower pace. We find that this effect outweighs the direct resource savings benefits: a higher rate of refurbishing leads to higher cumulative resource use.

We then consider an equilibrium in which cumulative demand equals supply at a positive equilibrium resource price. In this equilibrium, higher resource prices imply lower wages, decreasing the comparative costs of labor-intensive innovation. The positive effect of resource prices on innovation leads to the following result: resource prices endogenously adjust and support a (long-run) level of innovation that is independent of the refurbishing rate. An increase in refurbishing can support an increase in consumption throughout, with a decrease in cumulative resource use. For a given resource stock, with resource pricing, refurbishing strictly increases welfare.

Further, by analyzing the transition and the steady state after a shock that permanently raises the refurbishing rate, we find that an unequal inter-temporal distribution of the welfare gains. Most gains appear in the short run, especially when the initial level of refurbishing is low.

The rest of the paper is organized as follows. Section 2 introduces the model, and Section 3 solves the model for its steady state and transitional dynamics, and investigates the impact of refurbishing. Section 4 analyzes the effect of certain parameter shocks. And finally, Section 5 concludes.
2 The Model

2.1 Final Good Producer

There is one final good, which is produced using a continuum of modules indexed by $i$. Production of the final good is essentially an assembly process of the various modules, and is subject to perfect competition:

$$y = \left[ \int_0^1 x_i^{\frac{\epsilon - 1}{\epsilon}} di \right]^{\frac{1}{\epsilon - 1}}$$  \tag{1}

$$p_{x_i} = p_y \left( \frac{x_i}{y} \right)^{-\frac{1}{\epsilon}},$$  \tag{2}

where $\epsilon > 1$ is the elasticity of substitution between the different modules, $p_{x_i}$ is the price of module $i$, and $p_y$ is the price of the final good (or the ideal price index, as $p_y = \left[ \int_0^1 p_x^{\frac{1}{\epsilon}} di \right]^{-\frac{1}{\epsilon}}$), respectively. Each module $i$ can be either produced from raw material ($x_{Ni}$) or refurbished from old modules ($x_{Ri}$). The new and refurbished modules are perfect substitute so that $x_i = x_{Ni} + x_{Ri}$.

2.2 New Module Producer

New modules of type $i$ are produced in the primary sector by a monopolist, who employs the newest technology for that type. The production of the new modules employs a Leontief technology making use of labor and raw material:

$$x_{Ni} = \min \left\{ A_i^{\mu_L} L_{Ni}, A_i^{\mu_R} a_i^{-(\mu_R - \mu_L)} R_{Ni} \right\},$$

where $L_{Ni}$ and $R_{Ni}$ represent production labor and raw material used in new module production, $A_i$ is firm $i$’s technology stock, and $a_i \equiv A_i^{1/\lambda}$ is an inverse measure of the technology distance of firm $i$ to the technology frontier $A^*$. Technology is both labor and resource augmenting, with $\mu_L$ and $\mu_R$ denoting the respective strength of the two effects.\footnote{Throughout the paper, subscript $i$ denotes module type $i$, asterisk denotes the module type at the technology frontier, subscript $N$ and $R$ denote the new module and secondary sectors, respectively. We also omit the time subscript whenever it does not cause confusion.} The term $a_i^{-(\mu_R - \mu_L)}$ means that firm $i$’s resource efficiency is also affected by a positive technology spillover from the technology frontier.\footnote{As an example, one can think of CPUs. While old and new CPUs with the same computing capacity are perfect substitutes, due to innovation over time, new CPUs can be produced using less material for the same computing capacity. The assumption on the existence and the form of the spillover is not essential for the qualitative result of the paper, but greatly simplifies the analysis.}
This spillover effect captures the notion that resource saving innovations tend to be easier to adopt by imitators and thus spread out faster. In line with the literature emphasizing the role of learning and “absorptive capacity” in determining firm productivity (see Griffith and Redding, 2003, Griffith et al., 2004, Cameron et al., 2005, Acemoglu et al., 2003, Acemoglu et al., 2006, Vandenbussche et al., 2006, among others), the further behind firm i lagged from the technology frontier (the lower $a_i$), the stronger its “absorptive capacity” and the technology spillover.

It is easy to recognize that the production function can also be written as

$$x_{N_i} = A_i^{\mu_k} \min \left\{ L_{N_i}, (A^*)^{\mu_R} R_{N_i} \right\},$$  \hspace{1cm} (3)

where $A_i^{\mu_k}$ is essentially the total factor productivity of firm $i$. Given the cost structure, the marginal cost of a new module of type $i$ is given by

$$mc_{N_i} = A_i^{-\mu_R} w + A_i^{-\mu_R} a_i^{\mu_R - \mu_L} p_R = a_i^{-\mu_L} mc_N^*,$$  \hspace{1cm} (4)

where $w$ is the wage of workers, $p_R \geq 0$ is the resource price or scarcity rent, and $mc_N^* \equiv [(A^*)^{-\mu_L} w + (A^*)^{-\mu_R} p_R]$ is the marginal cost of the frontier sector.

### 2.3 Refurbisher

In the secondary sector, each unit of refurbished module is produced out of one unit of retired, old module using labor. A refurbished module is perfect substitute to a new module. We make the following assumption concerning the relative efficiency of refurbishing compared to producing a new module:

**Assumption 1.** Refurbishing is always – independent of technological advance – the more cost efficient way of supplying modules.$^5$

This assumption basically reflects the fact that refurbishing is a much simpler process than producing a module from scratch. As technological progress makes it ever easier to produce a new module, such increased efficiency is also reflected in the refurbishing process. The production function of a refurbisher is then given by

$$x_{R_i} = \min \left\{ \delta^{-1} A_i^{\mu_L} L_{R_i}, m_i \right\},$$  \hspace{1cm} (5)

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$^5$In the IO literature surveyed in last section, increasing marginal recycling costs (or the substitutability between scrap and other recycling input, such as effort/labor) together with constant marginal production cost of virgin material leads to the result that the competitive recycling sector overproduces (i.e. too much recycling). Similar result could occur here with decreasing $mc$ of new modules and constant/increasing $mc$ of refurbishing, albeit in the very long run.
where $L_{R_i}$ and $m_i$ are production labor and retired old modules used in module refurbishing, and $\delta \leq 1$ reflects the relative efficiency of refurbishing compared to producing new modules.\(^6\)

The secondary sector is subject to perfect competition and free entry. Consequently, the following relation always holds:

$$p_{m_i} = p_{x_i} - \delta_i A_i^{-\mu L} w,$$

where $p_{m_i}$ is the price for retired, old modules.

Concerning the supply of retired modules, we make the following simplifying assumption:

**Assumption 2.** A governmental agency collects retired final goods, and sells the reusable modules to refurbishers through open auctions. At any instant of time, the amount of retired, old modules of type $i$ supplied to refurbishers is given by $m_i = \beta x_i$, where $0 \leq \beta \leq 1$ is the share of final goods that are reusable.\(^7\).

In the above assumption, $\beta = 0$ corresponds to the familiar case of zero reusability in the standard economics literature that reflects the “take-make-consume-dispose” mindset, while $\beta = 1$ is a utopia scenario of complete resource independence. The case of $0 < \beta < 1$ is the more realistic case of circular economy that we focus on, where there is still some material loss through physical deterioration or by destructive use. As $\beta$ essentially captures the durability of material, it is the central indicator of circular economy in our model and can be seen as representing the degree of circularity of the model economy. Assumption 2 makes sure that on one hand all reusable old modules will be available for reuse and there is no loss in the supply of old modules, and on the other hand there will not be a stock of reusable old modules in the economy. In the rest of the paper, we refer to $\beta$ as the refurbishing rate.

By assumption 2, the supply share of refurbished modules is the same for all module types and corresponds to the refurbishing rate $\beta$, that is $\frac{x_{R_i}}{x_i} = \beta$ for all $i$. Given that both new and refurbished modules are sold at the same price, this also means that the market share of the secondary sector is the same for all module types and is equal to $\beta$.

\(^6\)It is obvious that while innovation is resource-augmenting for the new module producers, the resource-augmenting benefit only arrives in the secondary sector with delay. Given the infinitely fast cycle in the model, however, the duration of delay approaches zero.

\(^7\)To be more precise, this continuous time equation is the limit case of the discrete time version with the discrete period length $dt$ goes to zero, that is, $m_{it} = \beta X_{i(t-dt)}$
2.4 Research Sector

As in Aghion and Howitt (1998), we assume that there is a different research sector for each module type. The Poisson arrival rate of innovation per unit of time for each module type is given by $\lambda L_A$, where $\lambda > 0$ is a research productivity parameter and $L_A$ is the amount of labor devoted to research in type $i$. Innovations all draw on the same pool of social knowledge, represented by the leading edge technology $A^*$. Each successful innovation enables the innovator to become the new monopolist in that sector, and the sector itself to become the technology frontier.

As innovation in each sector is an independent process, the aggregate flow arrival rate of innovation depends on the aggregate research effort $L_A$. With a large number of sectors, by the law of large number, the technology frontier $A^*$ expands gradually according to

$$\dot{A}^* = \lambda L_A \ln \gamma A^*, \quad (7)$$

where $\gamma > 1$ represents the size of technology improvement of each innovation and $L_A \equiv \int_0^1 L_{Ai} \, di$ is the aggregate research effort. Although the distribution of the technology stocks across sectors, $A_i$, changes over time, the distribution of the relative technology distance, $a_i$, is stationary in the long run with the following cumulative distribution function:

$$H(a) \equiv a^{1/\ln \gamma}, \quad 0 \leq a \leq 1. \quad (8)$$

Since the prospective payoff to research is the same for all module types, in equilibrium innovators are indifferent with respect to which type to target. We focus on the symmetric equilibrium in the analysis, in which all sectors have the same equilibrium flow of research labor, that is, $L_{Ai} = L_A$. The free entry condition in the research sector is then given by:

$$\lambda V^* \leq w \perp L_A \geq 0 \quad (9)$$

where $V^*$ denotes the value of a patent for the frontier technology.

2.5 Households

Household supplies inelastically one unit of labor ($L_N + L_R + L_A = 1$), and maximizes the lifetime utility $U_0 = \int_0^\infty e^{-\rho t} \ln c_t \, dt$ subject to an intertemporal budget constraint $b_t = r_t b_t + w_t + T_t - p_{gt} c_t$, where $T_t = \int_0^1 p_{mit} m_{it} \, di$ is a lump sum transfer made by the balanced budget government via the sales of retired modules.
2.6 Material Balance and Resource Extraction

Material exists either in form of a low entropy raw material or as a high entropy waste. The raw material needed for producing the new modules is extracted from a non-renewable resource stock $S$, while the waste is generated by consumption and is deposited in a waste stock $W$. The total quantity of material is accounted for in the two stocks and must satisfy Lavoisier’s law of mass conservation. The equations of motion for the two stocks are given by:

\begin{align}
\dot{S} &= -R_N, \\
\dot{W} &= R_N,
\end{align}

where both $R_N$ is a flow variable measured in mass per units of time (e.g. kg/s).

There is assumed to be a functioning market for raw material. Resource demand comes from the new module monopolists only with the aggregate resource demand given by $R_N = \int_0^1 R_N_i \, di = \int_0^1 A_i^{-\mu_R} a_i^{\mu_R} x_{N_i} \, di$. The secondary sector does not demand resource directly as production input, but has an embedded resource flow of $R_R = \int_0^1 \frac{\beta}{1-\beta} R_N_i \, di = \frac{\beta}{1-\beta} R_N$. Consequently, refurbishing effectively scales up the effective resource use by a multiplier of $\frac{1}{1-\beta}$:

$$R_{\text{Eff}} = R_N + R_R = \frac{1}{1-\beta} R_N.$$  \hfill (12)

Finally, resource extraction is assumed to have zero marginal extraction costs. The raw material price $p_R$ thus follows the classic Hotelling rule$^9$: $\hat{p}_R = r$.

3 Analysis

3.1 Equilibrium Conditions

New module monopolists are assumed to be myopic and do not internalize their impact on future supply of refurbished modules. They take the supply of refurbished modules as exogenous and demand for their own goods as a residual demand from (2). They maximize their flow profit $\pi_i = (p_{x_i} - mc_{N_i})(x_i - x_{R_i})$ by the following pricing rule:

$$p_{x_i} = \frac{\epsilon}{\epsilon - 1 + x_{R_i}/x_i} mc_{N_i}.$$  \hfill (13)

$^8$The fact that there is repeated refurbishing and that not only new goods, but also refurbished goods are re-used means that in the extreme case of maximum refurbishing ($\beta = 1$), closing the cycle can bring resource use to zero. The more realistic case of $\beta < 1$ means there are limited number of times a module can be reused.

$^9$Throughout the paper, the hat notation denotes growth rates.
Notice that if \( x_{Ri}/x_i = 0 \), \( p_{x_i} = \frac{\epsilon}{\epsilon-1} mc_N \), and we have the standard monopolistic markup. If \( x_{Ri}/x_i > 0 \), however, \( p_{x_i} < \frac{\epsilon}{\epsilon-1} mc_N \). Competition from the secondary sector thus drives down the monopolistic price for all module types. By Assumption 2, the equilibrium goods market share for refurbishes is the same across all module types \( (x_{Ri}/x_i) = \beta \). The following lemma follows immediately:

**Lemma 1.** In equilibrium, \( x_{Ri}/x_i = \beta \), and there is a uniform markup \( \xi = \frac{\epsilon}{\epsilon-1+\beta} \) for all sectors.

Due to competitive pressure, a myopic monopolist lowers the markup in order to boost the residual demand for her goods, while failing to recognize that increased sales will only backfire in future as more intensified competition from refurbishers.\(^{10}\) Due to the uniform refurbishing rate across all module types, Lemma 1 also implies that refurbishing does not affect the intersectoral allocation among module types.

The uniform markup suggests that monopolist’s profit for any module type \( i \), \( \pi_i \), is proportional to that of the frontier type, \( \pi^* \). Combining equations (4) and (2), monopolist’s profit for module \( i \) can be written as

\[
\pi_i = \frac{(\xi - 1)(1 - \beta)}{\xi} p_{x_i} \frac{(\epsilon - 1)(\xi)}{\xi} = a_i^{\mu_t(\epsilon-1)} \pi^* ,
\]

where \( \pi^* = (\xi - 1)(mc^* N^*) \). As \( \epsilon > 1 \), the closer a sector’s technology is to the frontier (higher \( a_i \)), the higher the profit.

We now derive how this monopolist profit translates into the value of innovation and incentives for R&D. For a sector that succeeds in innovating at time \( t \), the flow profit of the new monopolist in that sector at any subsequent time \( s \) is given by

\[
\pi_{t,s} = (A_t^*)^{\mu_t(\epsilon-1)}(A_s^*)^{-\mu_L(\epsilon-1)} \pi^*_s
\]

That is, at any time \( s \) after the innovation, the monopolist’s profit consists of two components: one that depends on her own knowledge stock \( A_t^* \), which is the leading edge technology at time \( t \), and another that is changing with the technology frontier. Given this flow profit, the value of an innovation created at time \( t \) is then

\[
V_t^* = \int_t^\infty e^{-\int_t^s (\mu_L + \lambda L_{A,u}) du} \pi_{t,s} ds = (A_t^*)^{\mu_t(\epsilon-1)} \pi_t^*,
\]

\(^{10}\)A farsighted monopolist will see that a fixed share of her current sales always goes back immediately as refurbished modules, and her market share will always remain \( 1 - \beta \) in equilibrium, unaffected by her pricing strategy. A farsighted monopolist will then rationally uses the same markup as in the absence of competitive refurbishing.
where \( \overline{w} = \int_t^\infty e^{-\int_t^u(r_u + \lambda L_{A,u})du} (A^*_u)^{-\mu_L(\epsilon-1)} \pi^*_u ds \) is the same for all active monopolistic firms independent of the time of their patent creation. From the free entry condition (9), we further have

\[
\frac{V^*_t}{w_t} = \frac{(A^*_t)^{\mu_L(\epsilon-1)} \overline{w}}{w_t} = \lambda^{-1}.
\]

Thus for all innovations, the real value of the innovation at the time of its creation as evaluated by wage at that instant of time is always the same. Differentiating both sides of the above equation with respect to \( t \), and realizing that \( \hat{v}_t = r_t + \lambda L_{A,t} - \mu_L(\epsilon-1) \ln(\gamma) L_{A,t} \), we have

\[
r_t = \hat{w}_t + \frac{\pi^*_t}{V^*_t} - \lambda (1 + \mu_L(\epsilon - 1) \ln(\gamma)) L_{A,t},
\]

which essentially provides the return on investment that can be generated by holding a patent and becoming the next frontier type monopolist.

We have so far established the monopolist profit and value of innovation for an individual module type. For innovation incentives at the economy level, we now move on to an aggregate view of the economy. First denote by \( \phi_{L_{N_i}} \equiv \frac{wL_{N_i}}{TC_{N_i}} = \frac{A_i^{\mu_L} w}{mc_{N_i}} \) the labor cost share for module type \( i \) and \( \phi_{L_N} \equiv \frac{wL_N}{TC_N} \) the aggregate labor cost share in new module production, where \( TC \) stands for total cost. Due to the spillover effect, the labor cost share is the same for all module types, that is,

\[
\phi_{L_{N_i}} = \phi_{L_N} = \phi_{L_N}^*,
\]

which is directly visible from equation (4).

Let us now define \( \theta_i \equiv \frac{p_{x_i} x_i}{p_y y} \) as the market share of module \( i \). Uniform markup together with equations (2) and (4) indicate that the market share of each module type is relative to the frontier type market share always depends on the technological distance only. In particular, \( \theta_i = a_i^{\mu_L(\epsilon-1)} \theta^* \). This relation of the market share of different module types also implies that the production input use for new module type \( i \) is also proportional to that for the frontier type such that \( L_{N_i} = a_i^{\mu_L(\epsilon-1)} L_N^* \) and \( R_{N_i} = a_i^{\mu_L(\epsilon-1)} R_N^* \). Since by definition the market shares of all module types add up to 1, that is, \( \int_0^1 \theta_i di = 1 \), the frontier type market share is given by

\[
\theta^* = \left[ \int_0^1 a_i^{\mu_L(\epsilon-1)} di \right]^{-1}.
\]

As \( a_i \) follows a stationary distribution specified by (8), \( \theta^* \) is essentially the following constant:

\[
\theta^* = \left[ \int_0^1 h(a) a^{\mu_L(\epsilon-1)} da \right]^{-1} = 1 + \mu_L(\epsilon - 1) \ln(\gamma).
\]
That is, the market share of the frontier type is independent of the level of the technology, and depends instead only on the distribution of the technology distance across sectors. Given a stationary distribution of the relative technology distance, the market share of the frontier type does not change over time, only the identity of the frontier type changes with creative destruction.

Consequently, the aggregate production input use in new module production is the input use for the frontier type by its market share, that is, $L_N = (\theta^*)^{-1}L_N^*$ and $R_N = (\theta^*)^{-1}R_N^*$. Thus also the relative aggregate input use is the same as in the frontier sector, and it evolves with the frontier technology:

$$\frac{R_N}{L_N} = \frac{R_N^*}{L_N^*} = (A^*)^{-(\mu_R - \mu_L)}. \tag{22}$$

As the frontier technology increases due to innovation, depending on whether technology is unbiased ($\mu_R = \mu_L$), relatively more resource ($\mu_R > \mu_L$) saving or labor saving ($\mu_R < \mu_L$), the relative demand for the two factors either stays constant, falls or rise over time.

Considering the total labor use in production, we also need to account for the secondary sector. Given the same market share of the secondary sector across module types ($\beta$), the total production labor is then

$$L_X = \int_0^1 L_{N_i} \, di + \int_0^1 L_{R_i} \, di = \zeta L_N = \zeta(\theta^*)^{-1}L_N^* \tag{23}$$

where $\zeta = 1 + \frac{\beta}{1 - \beta}$ a composite parameter representing the inverse of the labor share in new module production.

Using the aggregate relations, in particular, by plugging (19), (23) and (20) into (18), the return on investment provided by innovators is

$$r = \hat{w} + \frac{\pi^*}{V^*} - \lambda \theta^* L_A = \hat{w} + \lambda \theta^* \left[ (\xi - 1) \frac{L_N}{\phi_{LN}} - 1 + \zeta L_N \right], \tag{24}$$

which gives the capital demand by the innovators.

Turning to the consumer side, households’ intertemporal optimization leading to the standard Ramsey rule, $r = \hat{p}_y + \hat{y} + \rho$. Again by Lemma 1, the total value of final output is proportional to the aggregate monopolistic profits, since $x_{N_i} = \beta x_i$ and $mc_{N_i} = \xi^{-1}p_{x_i}$ for all module types. Thus, from the Ramsey rule the required return on households’ saving is then

$$r = \hat{w} + \hat{L}_N - \phi_{LN} + \rho, \tag{25}$$

which gives the capital supply by the households.
The capital demand and supply together establish the capital market equilibrium. Together with the optimal resource extraction, which is warranted by the Hotelling rule:

\[ \hat{p}_r = r, \]  

(26)

this establishes the dynamic equilibrium in the model economy. On the other hand, at any instant of time, the static equilibrium involves the product market clearing \( y = c \), and the clearing of the two factor markets. The labor market clearing is implicit in (24), while the relative factor price \( \frac{w}{\hat{p}_r} \) is implicit in the labor cost share

\[
\hat{\phi}_{LN} = \frac{\phi^*_{LN}}{1 - \phi^*_{LN}} = (A^*)^{\mu_R - \mu_L} \frac{w}{\hat{p}_r},
\]

(27)

which gives

\[
\hat{\phi}_L = (1 - \phi_L) \left[ \Delta(1 - \zeta L_N) + \hat{w} - \hat{p}_R \right],
\]

(28)

where \( \Delta \equiv (\mu_R - \mu_L) \lambda \ln \gamma \) measures the potential for resource saving technological change. Innovation is resource-saving if \( \Delta > 0 \). Thus, we see that the labor cost share of new module production increases as the relative price of labor \( \frac{w}{\hat{p}_r} \) increases, but also with technological progress if technology increases the efficiency of resource more than labor productivity (i.e. if \( \mu_R > \mu_L \)). This latter effect is further augmented by the productivity of the research sector \( \lambda \) and the size of each innovation \( \gamma \).

The above analysis yields a system of reduced form differential equations that summarize the dynamic equilibrium of the model, as given by the following lemma. The proof for this lemma, as for all other lemmas and propositions, is provided in the Appendix A.

**Lemma 2.** The equilibrium in its reduced form can be expressed by the differential equation system of the variables \( \phi_{LN}, L_N, \) and \( E \):

\[
\begin{align*}
\hat{\phi}_{LN} &= (1 - \phi_{LN})(\Delta + \lambda \theta^*) \left[ 1 - \zeta \left( 1 + \frac{\lambda \theta^* \chi}{\Delta + \lambda \theta^* \phi_{LN}} \right) L_N \right] \\
\hat{L}_N &= [\Delta - \rho - (\Delta + \lambda \theta^*) \phi_{LN}] - \zeta [\Delta - \lambda \theta^* \chi - (\Delta + \lambda \theta^*) \phi_{LN}] L_N \\
\hat{E} &= \Delta - (\Delta \zeta + E^{-1}) L_N
\end{align*}
\]

(29)(30)(31)

where \( E \equiv (A^*)^{\mu_R - \mu_L} S \) is the effective resource stock, \( \Delta \equiv (\mu_R - \mu_L) \lambda \ln \gamma \) is a measure of technology bias, and \( \chi \equiv \frac{\xi - 1}{\zeta} \) is the (factor-adjusted) share of monopolist rents in the productive economy.

From the Lemma we can immediately construct a phase diagram to analyse the equilibrium dynamics. The dynamics of \( \phi_{LN} \) and \( L_N \) are independent of the level of
so that we can first build a two-dimensional phase diagram in the $\phi_{LN}, L_N$ plane; see the righthand side panels of the Figures below. Similarly, the dynamics of $E$ are independent of $\phi_{LN}$, and we build a second phase diagram in the $E, L_N$ plane; see lefthand-side panels. Depending on where the zero-motion loci are located in the phase diagram, we can identify four regimes, which we will characterize below in the first proposition.

In the reduced form differential equation system, the two composite parameters $\Delta$ and $\chi$ play a critical role in determining the dynamic behavior of the system. The former controls how much more resource-saving innovations are, and thus determines the relative resource use potential. The second composite parameter $\chi$, on the other hand, is directly linked to entrepreneurs’ incentives for innovation, and steers the dynamics of labor allocation between production and innovation. As we show in the next section, conditions on the two composite parameters will determine what kind of long-run equilibrium arises.

3.2 How history and the refurbishing rate determine equilibrium resource scarcity

With an exhaustible resource being an essential input, long-run consumption and welfare crucially depend on whether innovation and technical change can make up for the increasing resource scarcity. The existence of steady states with non-falling consumption thus requires a positive rate of innovation in the long run. The next lemma links innovation incentives to the key parameter of our model circular economy, the refurbishing rate $\beta$, and specifies the condition for positive steady state innovation for different ranges of the $\beta$ values.

**Lemma 3.** The monopolist profit share in the productive economy, $\chi$, decreases with the refurbishing rate $\beta$. Denote by $\chi^{-1}$ the inverse function of $\chi$, there exists $0 \leq \bar{\beta} < \bar{\beta} < 1$, where\(^{12}\)

\[
\tilde{\beta} = \chi^{-1}\left(\frac{\rho}{\lambda \theta^*}\right) \quad (32)
\]

\[
\bar{\beta} = \max\left\{0, \chi^{-1}\left(\frac{\rho}{\lambda \theta^*} \frac{\Delta + \lambda \theta^*}{\Delta - \rho}\right)\right\}, \quad (33)
\]

such that, for $\Delta > \rho$.

\(^{11}\)The phase diagram in Figure 3 is strictly speaking only for $\beta > \bar{\beta}$. At $\beta = \bar{\beta}$ the $L_N = 0$ locus is given by the vertical line $\phi_{LN} = \phi_{LN}$. Since the dynamics is the same as in the case of $\beta > \bar{\beta}$, for the sake of brevity, we do not separate these two cases.

\(^{12}\)Since $\theta^*$ is monotonically increasing in $\epsilon - 1$, while $\chi(\epsilon) = \frac{1}{\epsilon - 1}$ is monotonically decreasing in $\epsilon - 1$, it can be shown that there exists a $\bar{\epsilon} > 1$ such that $\bar{\beta} = 0$, and for $1 < \epsilon < \bar{\epsilon}$, $\beta$ is strictly positive.
Figure 1: Low Refurbishing Rate ($\beta \leq \bar{\beta}$)

Figure 2: Moderate Refurbishing Rate ($\bar{\beta} < \beta < \bar{\beta}$)

Figure 3: High Refurbishing Rate ($\beta \geq \bar{\beta}$)
• if $\bar{\beta} \leq \beta < 1$ (high refurbishing), positive steady state innovation is only possible with a positive resource price ($p_R > 0$ and $\phi_{LN} < 1$);

• if $\bar{\beta} < \beta < \bar{\beta}$ (moderate refurbishing), positive steady state innovation is possible both with zero or positive resource price ($p_R \geq 0$ and $\phi_{LN} \leq 1$);

• and if $0 \leq \beta \leq \bar{\beta}$ (low refurbishing), positive steady state innovation is only possible with zero or negligible resource price ($p_R = 0$ and $\phi_{LN} = 1$).

Lemma 3 points to the intimate relation between competitive refurbishing and innovation. As refurbishing takes away part of the market from the monopolists of new modules, monopolist profits are squeezed out and so are the expected reward of a successful innovation. By increasing the refurbishing rate and thus lowering the market share for monopolists, it becomes increasingly difficult for innovation to occur. Considering that innovation increases the productivity of both labor and resource, the prevalence of a positive resource price should in principle lead to higher innovation incentives, since the resource saving feature of innovation is only rewarded by the market if there is a positive resource price. In this light, it is intuitive that while at low refurbishing rate it is still profitable for entrepreneurs to innovate despite a zero resource price, this is no longer possible when the competitive refurbishing becomes too strong and the expected market share becomes too small.

Apart from the refurbishing rate, also the endowment of resources matters for the scarcity conditions. Intuitively, both a large initial stock of resources and a high initial productivity level make the resource effectively abundant and lower resource prices, with consequences for the costs and benefits of innovation. The next proposition characterizes how refurbishing rate $\beta$ and initial effective resource endowment $E(0)$, which is given by history, determine scarcity conditions in terms of equilibrium prices and innovation incentives.

**Proposition 1** (Characterization of Equilibrium).

For each combination of initial effective resource stock $E(0) > 0$ and circularity parameter $\beta \in [0, 1]$, there is a unique saddle-point stable equilibrium.

1. ("no scarcity") If $\beta < \bar{\beta}$ and the initial effective resource exceeds a positive critical level $E^0$, the equilibrium resource price is zero ($\phi_{LN} = 1$) and innovation labor is positive and constant at level $L_A^0$ for all $t > 0$, with

\[
L_A^0 = \frac{\chi - \rho/\lambda\theta^*}{\chi + 1},
\]

\[
E^0 = \frac{1 + \rho/\lambda\theta^*}{\Delta\kappa(\chi - \rho/\lambda\theta^*)}.
\]
2. ("vanishing production") If $\Delta < \rho$ and the condition stated in 1 is violated, the equilibrium converges monotonically to a steady-state equilibrium with vanishing wage share and vanishing production, $\lim_{t \to \infty} \phi_{LN} = \lim_{t \to \infty} L_N = \lim_{t \to \infty} L_X = 0$.

3. ("vanishing scarcity") If $\Delta > \rho$, $0 < E(0) < E^0$, and $0 \leq \beta \leq \bar{\beta}$ equilibrium wages grow faster than resource prices; the labor share $\phi_{LN}$ grows, the effective resource stock $E$ grows, and research labor declines, with these variables converging to 1, $E^0$, and $L_A^0$, respectively.

4. ("convergence to balanced growth") If $\Delta > \rho$ and either (i) $0 < E(0) < E^0$ and $\bar{\beta} < \beta < \bar{\beta}$ or (ii) $E(0) > 0$ and $\bar{\beta} \leq \beta < 1$, the labor share $\phi_{LN}$, the effective resource stock $E$, and research labor $L_A$ converge monotonically to

$$\phi^1_{LN} = \frac{\lambda_\theta^* \chi (\Delta - \rho)}{(\Delta + \lambda^* \rho)}$$  \hspace{1cm} (36)

$$L_A^1 = \frac{\rho}{\Delta}$$  \hspace{1cm} (37)

$$E^1 = \frac{L_N^1}{(\Delta L_A^1)} = \frac{\Delta - \rho}{\Delta \zeta \rho}$$  \hspace{1cm} (38)

respectively. Convergence is monotonic, with $E$ and $\phi_{LN}$ rising if $L_A$ falls and vice versa.

The first insight from the above proposition is that only if the refurbishing rate is not too high, an effectively high endowment of resources results in zero resource prices. In the resulting "no scarcity" regime, cumulative demand for resources is lower than total supply because innovation offsets depletion, where innovation can be sufficiently high because innovators see little of their market being stolen away by refurbishers. In contrast, if the refurbishing rate is high and the resource-saving potential of innovation is low relative to the depletion incentive (i.e. if $\Delta < \rho$), innovation incentives are low and ineffective in creating or keeping high resource abundance. At zero resource prices, cumulative demand would exceed supply so that the equilibrium resource price becomes positive. Moreover, because of low innovation incentives, effective resource supply quickly falls. In the resulting "vanishing production" regime, scarcity of resources ultimately drives all labor out of production, while innovation is at maximum speed but still incapable of offsetting depletion.

The second insight from the proposition is that even with scarcity of resources and positive resource prices, the economy can converge to a balanced growth steady state in which wages and resource prices grow at the same rate. Innovation offsets depletion. This intuitively requires that innovation is effective in resource saving.
relative to depletion incentives ($\Delta > \rho$). Even when the refurbishing rate is high, and the innovators see market share go to refurbishers, equilibrium innovation effort can still stay at $\frac{\rho}{\Delta}$, which is independent of refurbishing rate $\beta$. The reason is that more refurbishing allows for a cost reduction: the same amount of goods can be produced with less labor. This lowers the wage and frees up labor for R&D. This implies that two opposite forces are at work when the refurbishing rate is higher: innovators employ less labor but also face a lower cost of R&D, so that on balance long-run R&D effort is independent of refurbishing rate $\beta$.

Finally, a closer inspection of the steady states values of the key variables reveals that the comparative statics with respect to the parameters are very different for the two steady states. Table 1 provides the comparative static results concerning the two possible steady states. We are particularly interested in the effect of changing $\beta$ on various variables on steady state. We shall return to this issue in Section 3.3.

Another interesting point to note is that to reach the same steady state, economies with different initial endowment will have very different transition trajectories. While a resource rich economy can afford to have high consumption and lower investment into technology along the transition, a resource poor economy must accumulate technology faster than the rate it extracts the resource. This difference in transition translates to difference in the patterns of changing economic composition and structure. The prediction is thus if otherwise identical economies

\begin{table}[h]
\centering
\begin{tabular}{lcccccccc}
\hline
 & $E$ & $\frac{R_N}{\bar{S}}$ & $\phi_{LN}$ & $L_N$ & $L_{\Lambda}$ & $g$ & $c$ & $\frac{(A^*)^p}{L}$ \\
\hline
SS$^0$ & $\mu_R$ & $-$ & $+$ & $0$ & $0$ & $0$ & $0$ & $0$ \\
(0 $\leq$ $\beta$ $<$ $\bar{\beta}$) & $\gamma$ & $-$ & $+$ & $0$ & $-$ & $+$ & $+$ & $-$ \\
 & $\lambda$ & $-$ & $+$ & $0$ & $-$ & $+$ & $+$ & $-$ \\
 & $\rho$ & $+$ & $-$ & $0$ & $+$ & $-$ & $-$ & $+$ \\
 & $\beta$ & $+$ & $-$ & $0$ & $+$ & $-$ & $-$ & $+$ & $-$ \\
 & $\delta$ & $+$ & $-$ & $0$ & $-$ & $-$ & $-$ & $-$ \\
\hline
SS$^1$ & $\mu_R$ & $+$ & $0$ & $+$ & $+$ & $-$ & $-$ & $+$ \\
(\bar{\beta} $<$ $\beta$ $\leq$ 1) & $\gamma$ & $+$ & $0$ & $+$ & $+$ & $-$ & $0$ & $+$ \\
 & $\lambda$ & $+$ & $0$ & $+$ & $-$ & $-$ & $0$ & $+$ \\
 & $\rho$ & $-$ & $+$ & $-$ & $-$ & $+$ & $+$ & $-$ \\
 & $\beta$ & $-$ & $0$ & $-$ & $-$ & $0$ & $-$ & $+$ & $-$ \\
 & $\delta$ & $-$ & $0$ & $-$ & $-$ & $0$ & $-$ & $-$ & $-$ \\
\hline
\end{tabular}
\caption{Comparative Statics ($\Delta > \rho$ assumed)}
\end{table}
with different initial endowment were to set the same refurbishing rate, the time path for its labor division and relative resource price should be very different. However, as what matters for the initial endowment is both natural resource stock $S$ and the economy’s technology level $A^*$, resource richness is not to be interpreted in the conventional sense where only natural resource is considered. We thus might observe a highly technologically advanced economy with little natural resource and another with abundant natural resource but low technology level to behave in similar fashion.

### 3.3 The Impact of Refurbishing

The last two sections identified domains for the refurbishing rate $\beta$ with rather different transitional and steady state properties. The comparative statics also vary between the domains. We start with a corollary of Proposition 1, which serves to emphasize the fundamental role of the circularity parameter $\beta$. We find the counter-intuitive result that increasing refurbishing tends to increase resource scarcity as measured by its price.

**Corollary 1.** For a fixed level of initial technology level $A_0$ and $\Delta > \rho$, the impact of a non-marginal (sufficiently large) increase in the refurbishing rate $\beta$ is as follows.

1. If the initial effective resource stock $E_0$ is sufficiently high, then the economy moves from (a) a "no scarcity" equilibrium without transitional dynamics and zero resource price, to (b) an equilibrium converging to a "balanced growth" steady state with decreasing growth rates and positive resource prices.

2. For a sufficiently low resource stock $E_0$, the economy will move from (a) a "vanishing scarcity" equilibrium with positive but vanishing relative resource prices to (b) a "balanced growth" equilibrium converging to a steady state with positive resource prices and lower growth.

3. For in-between initial resource-stock levels $E_0$, the equilibrium will move from (a) a "no scarcity" steady-state equilibrium with zero resource prices, to (b) a "vanishing scarcity" equilibrium with positive but vanishing resource prices, to (c) an equilibrium converging to a "balanced growth" steady state with positive resource prices.

While in regime 1.a, 2.a, 3.a and 3.b, economic growth declines with increasing $\beta$. In regimes 1.b, 2.b, 3.c, short-run economic growth declines but long-run economic growth does not change with $\beta$. 

19
Figure 4: Steady State Growth Rate and Effective Resource Stock ($\Delta > \rho$)

The corollary is illustrated in Figure 4. First, the thresholds value of $\bar{\beta}$ divides the refurbishing rates in two regions, where for $\beta \leq \bar{\beta}$ the “0” steady state (“no or vanishing scarcity”) with zero resource prices is the only stable steady state (green solid line) and for $\beta > \bar{\beta}$ the “1” steady state (“balanced growth”) with positive resource prices is the only stable steady state. Depending on in which of the regions the refurbishing rate is, the economy converges to either vanishingly small, or strictly positive long-run resource prices. Second, the value of $\beta$ also changes the steady state size of the effective resource stock and thus the “resource richness” of the initial state. In particular, the steady state effective resource stock, on the one hand, is increasing in $\beta$ at the zero-resource-price steady state, while decreasing at the positive-resource-price steady state. That is, an increase in $\beta$ can either decrease or increase the current effective resource stock relative to the steady state, changing a “resource rich” into a “resource poor” economy (for low $\beta$), or the other way around (for high $\beta$).

The next Lemma characterizes the steady-state effects of higher refurbishing rates, for fixed technology level $A^*$.

**Lemma 4.** Comparing two economies in steady state with the same technology level $A^*$ but with different refurbishing rates $\beta$, the effect of a higher refurbishing
rate \( \beta \) on the steady state consumption depends on whether or not a resource market exists and on the relative efficiency of the secondary sector \( \delta \):

1. Across steady states with zero resource prices, the economy with a higher refurbishing rate \( \beta \) has a higher cumulative resource use, but lower innovation and lower growth. With higher \( \beta \), consumption is always lower in the very long run, and higher (lower) in the medium run if \( \delta < \xi^2 \) (\( \delta > \xi^2 \)).

2. Across steady states with positive resource prices, the economy with a higher refurbishing rate \( \beta \) has a lower (instantaneous and cumulative) resource use. Innovation and the growth rate are the same across these steady states. With higher \( \beta \), steady state consumption at any point in time is higher (lower) in the high-\( \beta \) steady state if \( \delta < 1 \) (\( \delta > 1 \)).

A first insight from Lemma 4 is that the welfare impact of higher circularity depends on the existence of functioning resource markets. This might appear surprising at first. Promoting a circular economy is considered an alternative ‘soft’ regulation to alleviate resource scarcity, addressing the scarcity externality even in the absence of resource markets. While this can be correct for a static economy, we find that the interaction with innovation turns the effects of refurbishing upside down. In terms of short run welfare impact, the above proposition shows that under reasonable conditions on the relative productivity of the primary and secondary sectors, more refurbishing tends to increase short run output and consumption. Looking at the long run, the welfare impact of higher refurbishing becomes more pessimistic in the absence of a resource market. As higher refurbishing lowers innovation incentives, long run growth is harmed by a larger secondary sector. Thus, without resource markets, any short run welfare gain will be outweighed by the lowered long run consumption, and increased cumulative resource use. Proposition ?? generally backs up the claims of the circular economy proponents in terms of short-run economic benefit, but does not confirm the “win-win” scenario if not supported by a resource market.

With a functioning resource market that correctly reflects the changing scarcity and rewards resource saving innovation, however, a larger secondary sector will not harm long run growth and there is a clear welfare gain both from lowered resource use and increased consumption. This case supports the optimistic view of a “win-win” circular economy strategy.

Lemma 4 focuses on the comparative statics of the steady state with the same initial technology level \( A^*_0 \). It does not address explicitly the transitional dynamics if a shock in the refurbishing rate pulls the economy out of steady state. Now we analyze the impact of a shock to an economy at steady state that permanently raises its refurbishing rate. As Lemma 4 already makes clear, starting from a
steady state with zero resource price, any increase in the refurbishing rate always mean lower growth and lower long run consumption. This is thus a less interesting scenario to study for a refurbishing rate shock. Instead, we focus on the case where an economy is initially at a steady state with positive resource price. The result is summarized in the next proposition.

**Proposition 2.** Consider an economy in steady state with efficient refurbishing \( \delta \leq 1 \) and an initial refurbishing rate \( \beta > \bar{\beta} \) hit by a shock that permanently raises the refurbishing rate to \( \beta' > \beta \). We find that immediately following the shock, economy-wide production increases but the primary sector shrinks (\( y \uparrow, L_X \uparrow, L_N \downarrow \)), while research effort and growth drop (\( L_A, g \downarrow \)). Along the transition, production labor and labor cost share decline (\( L_N, L_X, \phi_{LN} \downarrow \)), while relative resource price, research effort and growth rise (\( p_R/w, L_A, g \uparrow \)). The economy converges to a new steady state with the same growth rate. Compared to the counter-factual scenario of no shock, the technology level is lower, while short-run consumption is higher. For sufficiently high \( \beta \), consumption is higher throughout the transition and in the long run relative to the counter-factual.

For an economy that is at the steady state, one immediate effect of the shock is crowding out innovation. As the increase of the size of the secondary sector reduces the market share of the primary sector and its innovation incentives, the economy now grows at a slower pace. Since steady state growth rate is unaffected, this means technology level (\( A^* \)) after the shock is always lower than the counterfactual. In some sense, being more thrifty with resource (more reuse and higher \( \beta \)) and being more efficient in using resource (higher \( A^* \)) are strategic substitutes.

In terms of consumption and welfare, an increase of the size of the secondary sector brings an immediate consumption boost. However, as consumption grows at a slower pace during the transition than before the shock, the relative consumption benefit as compared to the counterfactual is declining over time during the transition. We cannot analytically solve whether long-run consumption increases or decreases with a change in \( \beta \), apart from noting that for \( \beta \) close to 1, consumption will increase throughout the transition and in the long run.

The results show that increasing refurbishing is not very attractive for economies with no or low refurbishing, as the crowding out of innovation due to higher refurbishing outweighs the resource saving benefit. At the other end, economies that have achieved high refurbishing will benefit from further advances; innovation is crowded out less. This unequal gains from increased refurbishing hints at potential difficulties when integrating economies at different stages into a global circular economy.
4 Investment and Resource Shocks

In this section we look at the short run and long run effect of different parameter shocks to an economy at the steady state. We consider only the saddle point stable steady states, that is, the zero resource price steady state if $\beta \leq \bar{\beta}$ or the positive resource price steady state if $\beta > \bar{\beta}$. For each of the parameters considered here, we analyze the impact for the shocks in one direction only. The outcome of shocks in the opposite direction follows by reversing the direction of change.

4.1 Shock of Impatience $\rho$

A point of interest often mentioned in recent climate change policy discussions is the discount rate. Let us consider an economy in steady state. Suddenly, at time $t_0$, the agents become more impatient leading to a higher discount rate so that $\rho' > \rho$. The impact of such a preference shock is summarized in the following proposition.

**Proposition 3.** Starting from a steady state with initial degree of impatience $\rho$, consider a preference shock that raises the degree of impatience to $\rho'$.

1. In a steady state with zero resource prices ($\beta \leq \bar{\beta}$), the shock leads to sudden jump in the relative resource price ($p_R/w$); production immediately drops ($L_N, L_X \downarrow$), while innovation and growth jump up ($L_A, g_c \uparrow$). Along the transition, effective resource stock, production labor and labor cost share rise ($E, L_N, L_X, \phi_{LN} \nearrow$), while innovation and growth decline over time ($L_A, g_c \searrow$). The economy converges to a new steady state with zero resource prices, higher effective resource stock, and lower growth ($\phi_{LN}^{0'} = 1, E^{0'} > E_0, g_c^{0'} < g_c$). Higher impatience leads to lower consumption both in the short and long run.

2. In a steady state with positive resource prices ($\beta > \bar{\beta}$), the shock leads to an immediate surge in production ($L_X, L_N \uparrow$) and drop in innovation and growth ($L_A, g_c \downarrow$). Along the transition, effective resource stock, production labor and labor cost share decline ($E, L_N, L_X, \phi_{LN} \searrow$), while relative resource price, innovation and growth rise over time ($\frac{p_R}{w}, L_A, g_c \nearrow$). The economy converges to a new steady state with positive resource prices, lower effective resource stock, and higher growth ($\phi_{LN}^{1'} < \phi_{LN}^{1} < 1, E^{1'} < E^{1}, g_c^{1'} > g_c$). Higher impatience leads to higher consumption both in the short and long run.

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13 Such preference shock could be due to new scientific findings that point to higher probability of catastrophic consequences of climate change, for example.
Proposition 3 offers a few counterintuitive results. Starting from a steady state with positive resource prices, the immediate effect of a higher impatience is to increase production and consumption, as more impatient households value present consumption more. Increased production on one hand absorbs more labor and crowds out innovation, and on the other hand increases resource use. Both effects lead to a faster increase in resource scarcity and as a result, the relative resource price $\frac{p_r}{w}$ rises, which rewards resource saving innovations and raises research input along the transition. Towards the new steady state, the price signal is so strong that the economy accumulates technology at a faster pace that exactly counterbalances the higher resource extraction rate in steady state due to more impatience (recall equation ??). In the very long run, the faster pace of innovation means that both growth rate and consumption level will be higher than in the absence of the preference shock. Thus, the effect of the preference shock is to raise consumption both in the short and very long run, while decreases it in between. Put differently, more impatient agents “let the intermediate generations pay”.

Paradoxically, the impact of higher impatience almost reverses itself if the economy were to start from a steady state with zero resource price. For a “zrp” steady state, the incentives for saving and investing is lowered for a more impatient economy and it is to be expected that the steady state innovation and growth rate will be negatively affected. This is different from a “prp” steady state, where changing resource scarcity is reflected in a strictly positive resource price that regulates innovation incentives. In a steady state where resource price is near zero, this price channel fails and higher impatience simply leads to lower innovation in steady state. To counterbalance the lower pace of technology accumulation, the more impatient economy can only sustain a steady state by maintaining a higher effective resource stock at $(E^0' > E^0)$ at steady state, which requires the economy to extract resource at a lower pace than its technology accumulation along the transition. Thus, lower willingness to invest coupled with the lack of price mechanism from the market means that the only possibility for the more impatient economy to sustain long run consumption is, paradoxically, to sacrifice its consumption and increase innovation in the short run. Despite the short run sacrifice and the higher effective resource stock in the long run, lower innovation still means that the more impatient economy has a lower consumption in the very long run.

4.2 Shock of Resource Stock $S$

Another phenomenon that has affected the course of sustainable development has been the unexpected oil discoveries. Such discoveries pose positive shocks to the resource stock, upon which dynamic adjustments of price paths and factor allo-
cations are necessary. The following proposition summarizes the impact of such 
fortuitous shocks.

**Proposition 4.** Consider an economy at steady state, while at $t = 0$ there is a 
sudden resource discovery that raises $S_0$ to $S'_0$.

1. At a steady state with zero or negligible resource prices, the economy is not 
affected by the discovery and the newly discovered resource remains slack 
and unused.

2. With positive resource prices, the economy experience a temporary surge of 
production and consumption, while lowering innovation and growth, until 
the effective resource stock converges back to the steady state value $E^3$; the 
relative resource price $p_R$ immediately jumps down at the time of discovery, 
but gradually rises up again along the transition; back to the steady state, both 
technology and consumption level will be lower than without the discovery.

With negligible resource prices, additional resources do not affect economic 
decisions as resource is not scarce and not the limiting factor of production. The 
newly discovered resource is simply a slack resource. When resource is scarce and 
there is a positive resource price at steady state, the discovery loosens the immediate 
resource scarcity and gives production an immediate boost. Production labor 
demand and consumption go up accordingly. However, since the lowered resource 
scarcity is reflected in a drop in the relative resource price, it also lowers innovation incentives, which negatively affects growth. The economy thus substitutes 
technology away by the now more abundant resource. In the long run, the lowered technology negatively affects the consumption level, even though innovation and growth converge back to the same pre-discovery level. Resource discovery and resource abundance, in this case, thus lower welfare and lead to a sort of “resource curse”.

5 Conclusions

Circular economy has been a buzzword in recent years, and continues to arouse 
increasing enthusiasm among policymakers, environmentalists and private businesses. It is often taken for granted that reorganizing our global economy into 
a circular design will not only saves resources and alleviate environmental problems, but also provides benefits in terms of welfare and living standards. It is often overlooked in related discussions that in a dynamic environment such as our global economy, relative scarcity and economic tradeoffs are themselves continuously evolving and affected by technology progress. It is thus crucial to consider
how implementing a circular economy would affect innovation and technological change, and what it means for growth and welfare. Using an endogenous growth model with creative destruction in which monopolists of newly created patents face competition from a secondary sector that supplies reusable old modules, we show that a circular economy does not promote higher long run growth. Increasing refurbishing can nevertheless increase consumption, both in the short and very long run, if complemented with existing resource markets that price resource scarcity. Without such market, refurbishing may deteriorate sustainability of the economy.

By pointing to the dynamic tradeoff between immediate resource saving and innovation, the present paper hopes to serve as a starting point for further discussions and future investigations on the economic impact of a paradigm shift towards a circular economy. A few points of interest for future work follow immediately from the present setting. First, while in the present setting innovations augment labor and resource productivity at the same time, it will be interesting to see how the direction of technical change can arise endogenously in a circular economy as a response to changing resource scarcity. Second, by focusing on the competitive tension between new module monopolists and refurbishers, we have treated the technically feasible refurbishing rate as an exogenous parameter. Engendering this parameter through vertical integration can complement the present work and offer further insights on what market structure better assists the goals of circular economy. And finally, the present work considers a closed economy while currently the frontier nations with ambitions on circular economy are mostly small open economies. Adapting the model to a small open economy setting and investigating the impact of a unilateral implementation of a circular economy design should provide valuable insights for policymaking.
References


Sormunen, Kirsi and Kimmo Tiilikainen, “The circular economy enters the world stage, with Finland leading the way,” 2017.


Appendices

A Proofs

A.1 Proof of Lemma 2

Combining (28) with (26) and (24) to eliminate $r$ and $\hat{w}$ and using (23) gives us

$$\hat{\phi}_{LN} = (1 - \phi_{LN}) \left[ \Delta + \lambda \theta^* - (\Delta + \lambda \theta^*) \zeta L_N - (\xi - 1) \lambda \theta^* \phi_{LN}^{-1} L_N \right].$$

Combining (24) and (25) to eliminate $r$ and $\hat{w}$, we have

$$\hat{L}_N = \lambda \theta^* \left[ (\xi - 1) \phi_{LN}^{-1} L_N + \zeta L_N - \left( 1 + \frac{\rho}{\lambda \theta^*} \right) \right] + \hat{\phi}_{LN}.$$

Simplifying, we arrive at (29) and (30), respectively.

By definition of $E$, we have

$$\hat{E} = (\mu_R - \mu_L) \ln \gamma \lambda L_A - \frac{R_N}{S} = \Delta L_A - \frac{R_N}{S}. \quad (A.1)$$

Since $R_N = L_N^* (A^*)^{\mu_L - \mu_R}$, we have

$$\frac{R_N}{S} = \frac{L_N}{E}. \quad (A.2)$$

Therefore

$$\hat{E} = \Delta - [\Delta \zeta + E^{-1}] L_N. \quad (A.3)$$

A.2 Proof of Lemma 3

Suppose a steady state exists. At the steady state, $\hat{L}_N = \hat{\phi}_{LN} = 0$. (25) then implies that the required return on investment must satisfy $r - \hat{w} = \rho$. From (24), the steady state research effort is then given by

$$L_A = \max \left\{ \frac{\pi^*}{V^*} - \frac{\rho}{\lambda \theta^*}, 0 \right\}. \quad (A.4)$$

For positive innovation ($L_A > 0$) to occur at a steady state, the expected flow profit per share of innovation (or the innovation dividend), $\frac{\pi^*}{V^*}$, must exceed the utility cost of postponing consumption $\rho$. Using (9) and the definition of $\pi^*$ and $\chi$, $\frac{\pi^*}{V^*}$ can be written as

$$\frac{\pi^*}{V^*} = \lambda \theta^* \chi \frac{1 - L_A}{\phi_{LN}}. \quad (A.5)$$
From (29), a steady state labor cost share is either $\phi_{LN} = 1$ or

$$\phi_{LN} = \frac{\lambda \theta^* \chi}{\Delta + \lambda \theta^*} \frac{1 - L_A}{L_A} \leq 1. \quad (A.6)$$

Plugging the two expressions above into (A.4), we have

$$L_A = \begin{cases} \max \left\{ \frac{\lambda \theta^* \chi - \rho}{\lambda \theta^* \chi + \lambda \theta^*}, 0 \right\}, & \text{if } \phi_{LN} = 1; \\ \frac{\rho}{\Delta}, & \text{if } \phi_{LN} < 1. \end{cases} \quad (A.7)$$

A positive $L_A$ at steady state is therefore only possible if $\chi > \bar{\chi} \equiv \frac{\rho}{\lambda \theta^*}$ and $\phi_{LN} = 1$ (Case 0), or if $\chi \leq \bar{\bar{\chi}} \equiv \frac{\rho}{\lambda \theta^*} \frac{\Delta + \lambda \theta^*}{\Delta - \rho}$ and $\phi_{LN} \leq 1$ (Case 1). It is easily verified that $\chi$ decreases monotonically with $\beta$, and further $\lim_{\beta \to 0} \chi = \frac{1}{\epsilon - 1}$ and $\lim_{\beta \to 1} \chi = 0$.

The existence of the two thresholds $\bar{\beta}$ and $\bar{\bar{\beta}}$ follows directly from $\bar{\chi}$ and $\bar{\bar{\chi}}$. If $\beta > \bar{\beta}$, positive innovation is only feasible in Case 1, where a positive resource price prevails; if $\beta < \bar{\bar{\beta}}$, positive innovation is only feasible within Case 0, where resource price is zero; if $\bar{\beta} < \beta < \bar{\bar{\beta}}$, both cases are feasible.

### A.3 Proof of Proposition 1

**Part 1**

Any long term equilibrium that satisfies consumption smoothing and optimal resource extraction must follow

$$\hat{R}_N = -\rho. \quad (A.8)$$

However, the Leontief production function implies (22), which gives rise to

$$\hat{R}_N = \hat{L}_N - \Delta L_A. \quad (A.9)$$

Since $L_N$ is bounded between zero and one, the persistent growth rate of $R_N$ is bounded below, where the lower bound could only be reached asymptotically if $L_N \to 0$ and $L_A \to 1$. That is:

$$\min \hat{R}_N = -\Delta, \quad (A.10)$$

In other words, $R_N$ can at most decline at the rate of $\Delta$ for a long period of time. If $\Delta \leq \rho$, it is technically infeasible to reduce extraction fast enough to sustain a long term equilibrium. The cumulative resource demand surpasses the available resource stock

$$S^D = \int_0^\infty R_{N,t} dt > \frac{R_{N,0}}{\Delta} \geq \frac{R_{N,0}}{\rho} = S_0, \quad (A.11)$$
where $R_{N,0}$ is the initial resource extraction required by a long term equilibrium. Consequently, the effective resource stock will declines towards zero, as

$$
\tilde{E} = \Delta L_A + \tilde{S} < \Delta - \rho \leq 0.
$$

(A.12)

Part 2 to 4

We start by deriving the loci of the differential equation system given by Lemma 2. The $\dot{\phi}_N = 0$ locus is given by

$$
\begin{cases}
\phi_N = 0, \\
\phi_N = 1, \\
L^\phi_N = \zeta^{-1} \left[ 1 + \frac{\lambda \phi^{\ast \chi - \rho}}{\Delta + \lambda \phi^\ast} \phi_{LN} \right]^{-1}, \quad (0 \leq \phi_N \leq 1),
\end{cases}
$$

where $L^\phi_N$ is increasing in $\phi_N$ and bounded between $L_N = 0$ and $L_N = \zeta^{-1} \left[ 1 + \chi \frac{\phi^\ast}{\Delta + \phi^\ast} \right]^{-1} < \zeta^{-1}$ since $\phi_N$ is bounded between 0 and 1.

The $L_N = 0$ locus, on the other hand, is given by

$$
\begin{cases}
L_N = 0 \\
L^L_N = \zeta^{-1} \left[ 1 + \frac{\lambda \phi^{\ast \chi - \rho}}{\Delta - \lambda \phi^\ast (\Delta + \phi^\ast)} \phi_{LN} \right], \\
\phi_N = \frac{\Delta - \rho}{\Delta + \phi^\ast} < 1, \\
\text{if } \chi = \frac{\rho}{\chi^\ast} \text{ (or } \beta = \bar{\beta}).
\end{cases}
$$

(A.14)

which suggests that for any steady state $L_N$ there is a unique $E$ that can stay stationary.

Besides the degenerate case with $E$, $L_N$ and $\phi_{LN}$ all being zero, the $\dot{\phi}_N = 0$ and the $\dot{L}_N = 0$ loci have two other possible intersections, leading to the following three possible steady states:

$$
\begin{cases}
\text{Case "zrp": } \phi_{LN} = 1, \quad L_N = \frac{\lambda \phi^{\ast \chi + \rho}}{\zeta (\lambda \phi^\ast + \lambda \phi^\ast) + \lambda \phi^\ast}, \quad E = \frac{\lambda \phi^{\ast \chi + \rho}}{\Delta \zeta (\lambda \phi^\ast + \lambda \phi^\ast) + \lambda \phi^\ast}, \quad \text{if } \phi_{LN} = 1; \\
\text{Case "prp": } \phi_{LN} = \frac{\lambda \phi^\ast (\Delta - \rho)}{(\Delta + \lambda \phi^\ast) \rho}, \quad L_N = \frac{\Delta - \rho}{\zeta \Delta}, \quad E = \frac{\Delta - \rho}{\Delta \rho}, \quad \text{if } 0 \leq \phi_{LN} \leq 1; \\
\text{Case "deg": } E^{deg} = L^{deg}_N = \phi^{deg}_{LN} = 0, \quad \text{(degenerate case)}.
\end{cases}
$$

(A.16)

That is, if we ignore the degenerate case, there are always in principle two steady states: one with $0 < \phi_{LN} < 1$ (i.e. positive resource price) and another with $\phi_{LN} = 1$ (i.e. zero resource price).
In the following, we analyze the dynamics of the system in different parameter ranges, since the slope of \( L_N^L \) depends on the size of \( \chi \) and thus on \( \beta \). In particular, \( L_N^L \) is upward sloping for \( \beta > \bar{\beta} \), vertical for \( \beta = \bar{\beta} \) and downward sloping for \( \beta < \bar{\beta} \). The existence and uniqueness of the steady states follow from checking the intersections of the \( L_N = 0 \) and \( \phi_{LN} = 0 \) loci, while the stability of the steady states can be seen from the phase diagram and checked by the eigenvalues of the steady states.

For a low refurbishing rate \( (\beta \leq \bar{\beta}) \), the phase diagram of the dynamic system is given by Figure 1. Illustrated is the case of \( \beta < \bar{\beta} \), where the boundary case \( \beta = \bar{\beta} \) is embedded with the three lines \( L_N^L, L_N^\phi \) and \( \phi_{LN} = 1 \) intersect at the same point. For this range of \( \beta \), the “zrp” steady state is the unique steady state. There is a one-sided stable path towards this steady state if the initial effective resource stock is beneath the steady state level, that is, if \( E_0 < E^0 \). In this case, there is a unique stable path, along which \( L_N, \phi_{LN} \) and \( E \) increase monotonically towards their respective value at the “zrp” steady state. If \( E_0 \geq E^0 \), the unique long-term equilibrium is that \( L_N \) and \( \phi_{LN} \) immediately take on the steady state values, while \( E \) either stays at \( E^0 \) or increases towards infinity, leading to slack resource.

There are two interesting points to notice. First, the unique “zrp” steady state is saddle path stable, although the saddle path is truncated at \( \phi_{LN} = 1 \) or \( E = E^0 \). An initially resource rich economy \( E_0 > E^0 \) thus always follow an unstable path

\[ L_N^L \] always has two branches but one of the two branches is always outside of the admissible range of \( L_N \) and can be ignored. In particular, let \( \phi_{LN} = \frac{\Delta - \lambda \phi^* L \chi}{\Delta + \lambda \phi^* L \chi} \). For \( \beta < \bar{\beta} \), both branches are increasing in \( \phi_{LN} \); the right branch \( (\phi_{LN} > \phi_{LN}^*) \) is bounded above by \( L_N = \zeta^{-1} \frac{\lambda \phi^o + \lambda \phi^*}{\lambda \phi^o + \lambda \phi^* L \chi} < \zeta^{-1} \) and thus within the admissible range, while the left branch \( (\phi_{LN} < \phi_{LN}^*) \) is bounded below by \( L_N = \zeta^{-1} \frac{\Delta - \lambda \phi^* L \chi}{\Delta + \lambda \phi^* L \chi} > \zeta^{-1} \) and is thus ruled out. For \( \beta > \bar{\beta} \), both branches are decreasing in \( \phi_{LN} \); the left branch \( (\phi_{LN} < \phi_{LN}^*) \) is bounded above by \( L_N = \zeta^{-1} \frac{\Delta - \lambda \phi^* L \chi}{\Delta + \lambda \phi^* L \chi} < \zeta^{-1} \) and thus within the admissible range, while the right branch \( (\phi_{LN} > \phi_{LN}^*) \) is bounded below by \( L_N = \zeta^{-1} \frac{\lambda \phi^o + \lambda \phi^* L \chi}{\lambda \phi^o + \lambda \phi^* L \chi} > \zeta^{-1} \) and is thus ruled out. For \( \beta = \bar{\beta} \), the two branches collapse into a horizontal line \( L_N = \zeta^{-1} \).

\[ L_N^L \] always has a unique intersection with \( \phi_{LN} = 1 \), while the intersection only yields \( L_N < \zeta^{-1} \) if \( \beta < \bar{\beta} \). Second, it can be shown that for \( \beta \geq \bar{\beta} \), either \( L_N^L \) or \( \phi_{LN} = \phi_{LN}^* \) (if \( \beta = \bar{\beta} \) always has a unique intersection with \( L_N^\phi \), which always results in \( L_N < \zeta^{-1} \) (i.e. \( L_A > 0 \)). To see this, notice that the following relations apply: if \( \beta < \bar{\beta} \), \( \lim_{\phi_{LN} \to \phi_{LN}^*} L_N^L < \lim_{\phi_{LN} \to \phi_{LN}^*} L_N^\phi \) and \( \lim_{\phi_{LN} \to 0} L_N^L > \lim_{\phi_{LN} \to 0} L_N^\phi \); if \( \beta = \bar{\beta} \), \( L_N^L > L_N^\phi \) for all \( \phi_{LN} \) so there is no intersection between \( L_N^L \) and \( L_N^\phi \), but \( L_N^\phi \) and \( \phi_{LN} = \phi_{LN}^* \) has a unique intersection; if \( \beta > \bar{\beta} \), \( \lim_{\phi_{LN} \to 0} L_N^L > \lim_{\phi_{LN} \to 0} L_N^\phi \) and \( \lim_{\phi_{LN} \to 1} L_N^L < \lim_{\phi_{LN} \to 1} L_N^\phi \).
with ever increasing resource stock. Second, whether an economy starts as resource
rich or resource poor, at no point in time should the production labor exceeds its
steady state value, that is, \( L_N \leq L_N^0 \) always holds in equilibrium. The implication
is that equilibrium in this case always involves positive innovation at any point in
time.

The phase diagram for the case With a moderate refurbishing rate \((\bar{\beta} < \beta < \bar{\beta})\)
is provided in Figure 2. Ignoring the degenerate case, we now have two steady
states. While the interior, “prp” steady state is saddle path stable, the “zrp”
steady state is unstable. Economies with different initial effective resource stocks
could now end up in different steady states. We can now categorize economies
according to their initial resource endowment as resource rich \((E \geq E^0)\), resource
sufficient \((E^1 \leq E < E^0)\) and resource poor \((E < E^1)\). A resource rich economy
will immediately jump to the steady state labor allocation \((L_N = L_N^0)\) and have a
zero resource price \((\phi = 1)\), while its effective resource stock is either constant
(knife-edge case) or increasing over time leading to slack resource. A resource
sufficient economy will converge to the “prp” steady state along a unique stable
path, along which the relative resource price increases over time, resulting in in-
creasing innovation effort until reaching the steady state. And finally, a resource
poor economy needs to build up its effective resource stock by having high level
of innovation initially, while gradually allocating more labor to production along
the unique stable path towards the “prp” steady state. Accordingly, the relative
resource price falls over time, reflecting the increasing effective resource stock.

Finally, the case of high refurbishing rate \((\beta \geq \bar{\beta})\) is illustrated by Figure 5 and
3. Although the boundary case \(\beta = \bar{\beta}\) looks slightly different due to an additional
line for the \(L_N = 0\) locus, the qualitative results are the same as with \(\beta > \bar{\beta}\). With
\(\beta \geq \bar{\beta}\), there is again only a unique steady state with a positive resource price,
although we might interpret the “zrp” as an asymptotic steady state, which is never
reached. With a high refurbishing rate, all economies fall within the categories of
resource sufficient or resource poor. No economy can be called resource rich as
this would require an infinitely large initial resource endowment \((E^0 = \infty)\). The
statements concerning the resource sufficient and resource poor economies thus
carry through.

The stability property of the “zrp” and “prp” steady states are already illus-
trated in the phase diagrams. Analytically, the stability can also be shown by
log-linearizing the dynamic system around the respective steady states.
A.4 Proof of Lemma 4

Cumulative resource use in steady state is given by

$$S_0 = \frac{E^s}{(A_0^s)^{\mu_R - \mu_L}},$$

where the superscript $s$ again denotes the type of steady state ($s = 0$ or $s = 1$). With the same $A_0^s$, higher (lower) $E^s$ translates to higher (lower) cumulative resource use.

The instantaneous resource use is given by

$$R_N^s = \Delta L_s^A S_0 = (A_0^s)^{-1} (\mu_R - \mu_L) L_N^s.$$  

If $s = 1$, higher $\beta$ leads to lower $L_1^N$ and lower instantaneous resource use. If $s = 0$, instantaneous resource use could be higher or lower. For the assumption $\delta \leq 1$, higher $\beta$ leads to higher $L_0^N$ and higher instantaneous resource use.

Consumption level in steady state is given by

$$c_s = \frac{L_s X}{\zeta (1 - \beta)} (A_t^s)^{\mu_L}. \quad \text{(A.17)}$$

In the “zrp” steady state, the growth rate is lower for higher $\beta$. Thus consumption in the very long run is always lower for higher $\beta$. In the short run, consumption is given by

$$c_0^s = \frac{\lambda \theta^s + \rho}{\lambda \theta^s} (A_t^s)^{\mu_L} [(\chi + 1) \zeta (1 - \beta)]^{-1} = \frac{\lambda \theta^s + \rho}{\lambda \theta^s} (A_t^s)^{\mu_L} \left[ \frac{\epsilon}{1 - \beta - 1} + \beta \delta \right]^{-1}, \quad \text{(A.18)}$$

and with the same $A_0^s$, we have

$$\frac{\partial c_0^s}{\partial \beta} = -\frac{c_0^s}{(\chi + 1) \zeta (1 - \beta)} (\delta - \xi^2). \quad \text{(A.19)}$$
Therefore $\frac{\partial \xi}{\partial \beta} < 0$ if $\delta > \xi^2$. Since $\delta > \xi^2$ if $\beta > \frac{(\epsilon \delta^{1/2} - \epsilon + 1)}{1}$, for $\frac{\partial \xi}{\partial \beta} > 0$ to hold, it is sufficient if $\delta < \overline{\xi}$, where $\overline{\xi} \equiv \frac{\epsilon}{\epsilon-1}$.

In the “prp” steady state, the growth rate is not affected by $\beta$. With the same initial $A_0^*$, $A_t^*$ is the same for different $\beta$ at any point in time in steady state. Steady state consumption is thus

$$c_1 = \frac{L_X^*}{\zeta (1-\beta)}(A_t^*)^{\mu_L} = \frac{L_X^*}{1-\beta(1-\delta)}(A_t^*)^{\mu_L} \quad (A.20)$$

and we have

$$\frac{\partial c_1}{\partial \beta} = c_1 \frac{\zeta (1-\beta)}{\zeta (1-\beta) (1-\delta)}. \quad (A.21)$$

Thus steady state consumption at any point in time is higher (lower) if $\delta < 1$ ($\delta > 1$).

### A.5 Proof of Proposition 2

**Log-linearization around the steady state**

Use tilde to denote the log-deviation of a variable from its steady state value, e.g. $\tilde{E}^j \equiv \ln E - \ln E^j$, where the superscript $j$ indicates which steady state is concerned (0 for the “zrp” and 1 for the “prp” steady state). We then have\(^\text{16}\)

$$\begin{bmatrix} \tilde{E}^0 \\ \tilde{L}_N^0 \\ \tilde{\phi}_{LN}^0 \end{bmatrix} = \begin{bmatrix} \Delta L_A^0 & -\Delta & 0 \\ 0 & \lambda \theta^* + \rho & - (\Delta + \lambda \theta^*) L_A^0 \\ 0 & 0 & \rho - \Delta L_A^0 \end{bmatrix} \times \begin{bmatrix} \tilde{E}^0 \\ \tilde{L}_N^0 \\ \tilde{\phi}_{LN}^0 \end{bmatrix} \equiv K^0 \quad (A.22)$$

and

$$\begin{bmatrix} \tilde{E}^1 \\ \tilde{L}_N^1 \\ \tilde{\phi}_{LN}^1 \end{bmatrix} = \begin{bmatrix} \rho & -\Delta & 0 \\ 0 & (\Delta - \rho) + (\Delta + \lambda \theta^*) \phi_{LN}^1 & - (\Delta + \lambda \theta^*) \phi_{LN}^1 L_A^1 \\ 0 & - (\Delta + \lambda \theta^*) (1 - \phi_{LN}^1) L_A^1 & (\Delta + \lambda \theta^*) (1 - \phi_{LN}^1) L_A^1 \end{bmatrix} \times \begin{bmatrix} \tilde{E}^1 \\ \tilde{L}_N^1 \\ \tilde{\phi}_{LN}^1 \end{bmatrix} \equiv K^1 \quad (A.23)$$

The three eigenvalues of $K^0$ are given by $v_1^0 = \Delta L_A^0 > 0, v_2^0 = \lambda \theta^* + \rho > 0$ and $v_3^0 = \rho - \Delta L_A^0$. The sign of the last eigenvalue depends on $\beta$. In particular, $v_3^0 \leq 0$ \(^{16}\phi_{LN}^- \) only makes sense if $\phi_{LN}^- < 0$, otherwise $\phi_{LN}^- = 0$.  

\(^{16}\phi_{LN}^- \) only makes sense if $\phi_{LN}^- < 0$, otherwise $\phi_{LN}^- = 0$.  

35
if $\beta \leq \bar{\beta}$, and $v_0^3 > 0$ if $\beta > \bar{\beta}$. It can further be shown that $v_0^3 > \Delta L_A^0 - \Delta$. For $K^1$, there is obviously a positive eigenvalue $v_1^3 = \rho$, and it is easily verified that the determinant of $K^1$ is given by:

$$detK^1 = -\rho(\Delta - \rho)(\Delta + \lambda^*\rho - \rho\lambda^*\chi(\Delta - \rho)).$$

Thus $K^1$ has altogether two positive and one negative values, again showing that the “zrp” steady state is saddle-point stable. Let the negative eigenvalue of $K^1$ be $v_3^1$, and denote

$$T^1 \equiv K^1_{22} + K^1_{33} = \lambda^*\rho + \rho - \Delta - \rho\frac{\rho}{\Delta}[((\Delta + \lambda^*\rho) - \rho\lambda^*\chi(\Delta - \rho))]$$

$$D^1 \equiv K^1_{22} - K^1_{23} - K^1_{32} = -\Delta - \rho\frac{\rho}{\Delta}[((\Delta + \lambda^*\rho) - \rho\lambda^*\chi(\Delta - \rho))]$$

$v_3^1$ is the negative root of $G(v_3^1) \equiv (v_3^1)^2 - T^1v_3 + D^1 = 0$. Without solving for $v_3^1$ explicitly, it can be easily verified that the following holds for $v_3^1$:

1. $K^1_{11} + K^1_{12} < v_3^1 < K^1_{23} + K^1_{33}$, since $G(K^1_{22} + K^1_{23}) < 0$ and $G(K^1_{11} + K^1_{12}) > 0$;

2. $\frac{\partial v_3^1}{\partial \beta} < 0$, since $\frac{\partial G(v_3^1)}{\partial v_3^1} < 0$, $v_3^1$ is on the downward sloping part of the the curve) and $\frac{\partial G(v_3^1)}{\partial \beta} = \frac{\partial G(v_3^1)}{\partial v_3^1} \frac{\partial v_3^1}{\partial \beta} < 0$ (due to $\frac{\partial T^1}{\partial \chi} > 0$, $\frac{\partial D^1}{\partial \chi} > 0$ and $\frac{\partial \chi}{\partial \beta} < 0$). Since $|v_3^1|$ is the adjustment speed, we know also that the higher for a deviation from a “prp” steady state, the higher $\beta$, the faster the transition back to the steady state.

Let $v_3^s$ be the negative eigenvalue of the eigenmatrix $K^s$, where $s$ denotes the type of stable steady state and is 0 for the “zrp” or 1 for “prp” steady state. Along the saddlepath, we have $\tilde{E}_t^s = \tilde{E}_0^se^{v_3^st} + \tilde{E}_\infty^s(1 - e^{v_3^st})$ from which we know $\tilde{E}_t^s = v_3^s(\tilde{E}_t - \tilde{E}_\infty^s)$, where $\tilde{E}_\infty^s = 0$. Close to the steady state, the log derivation from the steady state is given by $\tilde{E}_t^s = K^s_1\tilde{E}_t^s + K^s_2\tilde{E}_N^s$. Equating the two expressions and dropping the time subscript, we derive the following expression for the saddlepath close to the steady state:

$$\tilde{L}_N^s = \frac{v_3^s}{K^s_1} \tilde{E}_s.$$
each of the two steady states as

\[
\begin{align*}
\tilde{L}_X = \tilde{L}_N &= \frac{v_3^s - K_{11}^s}{K_{12}^s} \tilde{E}^s = \alpha_1^s \tilde{E}^s \quad \text{(A.24)} \\
\phi_{LN} &= \frac{v_3^s - K_{22}^s}{K_{23}^s} \alpha_1^s \tilde{E}^s = \alpha_2^s \alpha_1^s \tilde{E}^s = \alpha_2^s \frac{\alpha_1^s}{\theta} \tilde{L}_N^s. 
\end{align*}
\]

Using the fact that \((v_3^s)^2 - (K_{22}^s + K_{33}^s)v_3^s + (K_{22}^s K_{33}^s - K_{23}^s K_{32}^s) = 0\), we can write \(\alpha_1^s \alpha_2^s\) as follows:

\[
\begin{align*}
\alpha_1^s \alpha_2^s &= \frac{(v_3^s)^2 - (K_{11}^s + K_{22}^s)v_3^s + K_{11}^s K_{22}^s}{K_{12}^s K_{23}^s} \\
&= \frac{(K_{33}^s - K_{11}^s)v_3^s - (K_{33}^s - K_{11}^s)K_{22}^s + K_{23}^s K_{32}^s}{K_{12}^s K_{23}^s} \\
&= 1 + \frac{(K_{33}^s - K_{11}^s)v_3^s - (K_{33}^s - K_{11}^s)K_{22}^s + K_{23}^s K_{32}^s - K_{12}^s K_{23}^s}{K_{12}^s K_{23}^s}. 
\end{align*}
\]

1. For \(s = 1\), notice that \(K_{22}^1 = K_{11}^1 + K_{12}^1 + K_{12}^1 K_{11}^1 \) and \(K_{33}^1 = \frac{K_{12}^1 K_{11}^1}{K_{12}^1}\), we can simplify (A.26) to

\[
\alpha_1^1 \alpha_2^1 = 1 + \frac{(K_{33}^s - K_{11}^s)(v_3^s - K_{11}^s)}{K_{12}^s K_{23}^s} \\
= 1 + \frac{v_3^s - K_{11}^1 - K_{12}^1 \lambda \theta^* (\Delta - \rho)}{K_{12}^1 K_{23}^1} \left(\frac{\rho}{\Delta - \rho} - \chi\right). 
\]

Thus whether \(\alpha_1^1 \alpha_2^1 \geq 1\) depends on the size of \(\chi\) (and therefore \(\beta\)). Since \(\chi(\tilde{\beta}) = \frac{\rho}{\Delta - \rho} \lambda \theta^* > \frac{\rho}{\Delta - \rho} > 0\) and \(\chi(1) = 0 < \frac{\rho}{\Delta - \rho}\), by monotonicity of \(\chi(\beta)\) there exists a unique \(\tilde{\beta} = \chi^{-1} \left(\frac{\rho}{\Delta - \rho}\right) \in (\tilde{\beta}, 1)\), and

\[
\alpha_1^1 \alpha_2^1 \begin{cases} < 1, & \text{if } \tilde{\beta} < \beta < \tilde{\beta}, \\ = 1, & \text{if } \beta = \tilde{\beta}, \\ > 1, & \text{if } \beta > \tilde{\beta}. \end{cases} 
\]

\textsuperscript{17}For \(\alpha_1^0 < 1\) we need the assumption \(\lambda \theta^* > \Delta - 2 \rho\).
2. For \( s = 0 \), we have

\[
\alpha_1^0 \alpha_2^0 = 1 + \frac{1}{\Delta + \lambda \theta^*} L_A^0 \left[ \frac{2\Delta^2 L_A^0}{2 + \Delta(\lambda \theta^* - \Delta - \rho)L_A^0 - \rho \lambda \theta^*} \right] \quad \text{(A.29)}
\]

Since \( \frac{\partial \alpha(L_A^0)}{\partial L_A^0} = 4\Delta^2 L_A^0 - \Delta(\lambda \theta^* - \Delta - \rho) \geq \Delta(\lambda \theta^* - 3\rho + \Delta) \) (as \( L_A^0 \geq \rho / \Delta \)), \( \alpha(L_A^0) \) is non-decreasing in \( L_A^0 \) if \( \lambda \theta^* > \Delta - 3\rho \). Further, since, as \( L_A^0 \to 1 \), \( \alpha(L_A^0) \to (\Delta + \lambda \theta^*)(\Delta - \rho) > 0 \), and as \( L_A^0 \to \rho / \Delta \), \( \alpha(L_A^0) \to \rho(\rho - \Delta) < 0 \), there exits a \( 0 \leq \beta < \tilde{\beta} \) such that \( \alpha(L_A^0(\beta)) = 0 \), and

\[
\alpha_1^0 \alpha_2^0 \begin{cases} 
> 1, & \text{if } 0 \leq \beta < \tilde{\beta}, \\
= 1, & \text{if } \beta = \tilde{\beta}, \\
< 1, & \text{if } \beta < \beta \leq \tilde{\beta}
\end{cases} \quad \text{(A.30)}
\]

Finally, to find the size of the initial jump of a variable immediately after a shock, we find the log-deviation with respect to the old steady state by making use of the relation \( \ln x_0 / x_s = \ln x_0 / x_s' + \ln x_s'/x_s \), where \( s \) denotes the old and \( s' \) the new steady state:

\[
\begin{align*}
\widetilde{L}_{N,0}^s &= \ln \frac{L_{N,0}}{L_N} = \tilde{L}_{N,0}^s \quad \ln \frac{L_N'}{L_N} = \alpha_1^s \tilde{E}^s + \ln \frac{L_N'}{L_N}, \\
\widetilde{\phi}_{L_{N,0}}^s &= \ln \frac{\phi_{L_{N,0}}}{\phi_L} = \phi_{L_{N,0}}^s \quad \ln \frac{\phi_L}{\phi_L} = \alpha_2^s \tilde{E}^s + \ln \frac{\phi_L}{\phi_L},
\end{align*} \quad \text{(A.31)}
\]

\[
\begin{align*}
\tilde{E}_{N,0}^s &= \frac{\bar{E}^s}{\alpha_1^s}, \\
\tilde{E}_{N,0}^s &= \frac{\bar{E}^s}{\alpha_2^s}.
\end{align*} \quad \text{(A.32)}
\]

**Initial Steady State with Positive Resource Prices**

If initial \( \beta > \tilde{\beta} \), the economy is initially at a “prp” steady state. The new steady state will be a “prp” steady state with a lower effective resource stock \( E^1 > E_0 = E^1 \). Permanent increase of \( \beta \) thus create an expected resource abundance. Economy-wide production thus jumps up \( (L_X,0 > L_X^1) \), while innovation and growth drop \( (L_A,0 < L_A^1, g_c,0 < g_c^1) \).

For the initial jump after the shock, using (A.31), \( E^1 > E^1 \) and the fact that at a “prp” steady state \( E^1 = L_X^1 / \rho \) always holds, we have

\[
\tilde{L}_{N,0}^1 = \alpha_1^1 \tilde{E}_0^1 + \ln \frac{L_N^1}{L_N} = (1 - \alpha_1^1) \ln \frac{L_N^1}{L_N} < 0. \quad \text{(A.33)}
\]
That is, relative to the old steady state, \( L_{N,0} \) is lower. This means that immediately following the shock, production labor in the primary sector jumps down.

Similarly, for \( \phi_{LN} \), using (A.32) and noting that \( \phi_1^{LN} \) can be rewritten as:

\[
\phi_1^{LN} = \frac{\lambda_{n}^{\ast}(\xi-1)}{\Delta + \lambda_{n}^{\ast}} \frac{L_{N}^{0}}{1-L_{X}^{0}},
\]

we have:

\[
\tilde{\phi}_{LN,0}^{-1} = \alpha_1^{1} \alpha_2^{1} \tilde{E}_{0}^{1'} - \ln \frac{L_{N}^{0}}{L_{N}^{1}} + \ln \frac{\xi^{1'}-1}{\xi-1} = (1 - \alpha_1^{1} \alpha_2^{1}) \ln \frac{L_{N}^{0}}{L_{N}^{1}} + \ln \frac{\xi^{1'}-1}{\xi-1}. \quad (A.34)
\]

For \( \bar{\beta} < \beta < \tilde{\beta} \), \( \phi_{LN,0}^{-1} < 0 \) and \( \phi_{LN} \) immediately jumps down following the shock, corresponding to an increase in relative resource price; while for large \( \beta \), \( \phi_{LN} \) jumps up corresponding to a decrease in relative resource price.

Consumption is again given by (A.20). In the short run, technology level is unchanged. Following (A.21), short run consumption is therefore higher (lower) than without the shock if \( \delta < 1 \) (\( \delta > 1 \)). Along the transition, since \( L_{A}^{1'} = L_{A}^{1} \), consumption growth is slower than the counterfactual of no shock.

As the steady state growth rate is unaffected by the shock, whether long run consumption is higher or lower depends on both \( \delta \) and the adjustment speed \(|v^{1'}_{3}|\).

As the adjustment speed is an increasing function of \( \beta \), assuming \( \delta < 1 \) it follows that for very high refurbishing rate \( \beta \), both the short and long run consumption will be higher following the shock.

### A.6 Proof of Proposition 3

For the “prp” steady state, a higher \( \rho \) corresponds to a lower effective resource stock, lower production labor, more innovation and a higher growth rate (\( E_{1}^{0'} < E_{1}^{1} \), \( L_{N}^{1'} < L_{N}^{0} \), \( L_{X}^{1'} < L_{X}^{0} \), \( L_{A}^{1'} > L_{A}^{0} \)) and \( g_{c}^{1'} > g_{c}^{1} \). For the “zrp” steady state, a higher \( \rho \) corresponds to a higher effective resource stock, higher production labor, less innovation and a lower growth rate (\( E_{0}^{0'} > E_{0}^{1} \), \( L_{N}^{0'} > L_{N}^{0} \), \( L_{X}^{0'} > L_{X}^{0} \), \( L_{A}^{0'} < L_{A}^{0} \)) and \( g_{c}^{0'} < g_{c}^{0} \).

To find the initial jump of the variables after the shock, we check the log-deviation of the variables with respect to the old steady state immediately after the shock. Since prior to the shock the economy is at a steady state, \( E_{0}^{s} = E_{s}^{s} \), where \( s \) denotes the type of steady state (\( s = 0 \) or \( s = 1 \)), we have:

\[
\tilde{E}_{0}^{s} = \ln \frac{E_{s}^{s}}{E_{0}^{s}}.
\]

Notice that \( E_{s} = \frac{L_{N}^{s}}{\Delta L_{A}^{s}} = \frac{1-L_{X}^{s}}{\Delta L_{A}^{s}} \) and make use of the relations \( 1-x \approx= -\frac{x}{1-x} \bar{x} \).
and \( \left( \frac{x}{1-x} \right) \approx -\frac{1}{x^2} \approx \frac{1}{1-x^2} \) for any variable \( x \), we have

\[
\ln \frac{E^s}{E^{s'}} = \tilde{E}_0^{s'} = -\frac{1}{1-L_A'} \ln \frac{L_A^s}{L_A^{s'}}
\]

\[
\ln \frac{L_N^s}{L_N^{s'}} = \ln \frac{L_X^s}{L_X^{s'}} = -\frac{L_A'}{1-L_A'} \ln \frac{L_A^s}{L_A^{s'}}.
\]

Using (A.24) and (A.31), the log-deviation of \( L_{N,0} \) with respect to the old steady state is finally given by

\[
\tilde{L}_{N,0}^s = \alpha_1 E_0 \tilde{E}_0^{s'} + \ln \frac{L_N^s}{L_N^{s'}} = \frac{L_A^s - \alpha_1 s'}{1-L_A'} \ln \frac{L_A^s}{L_A^{s'}} \begin{cases} < 0, & \text{if } s = 0 \\ > 0, & \text{if } s = 1 \end{cases}.
\]

Therefore, starting from a “prp” steady state, immediately after the shock, \( L_N \) and \( L_X \) jump up, while \( L_A \) jump down. Starting from a “zrp” steady state, immediately after the shock, \( L_N \) and \( L_X \) jump down, while \( L_A \) jump up.

As for \( \phi_{LN} \), starting from a “zrp” steady state \( \phi_{LN} \) can only jump down initially before increasing back to 1. For a “prp” steady state, notice that \( \phi_{LN}^1 = \frac{\lambda_0'}{\Delta + \lambda_0'} \frac{1-L_A'}{L_A'} \). Therefore

\[
\phi_{LN,0}^{-1} = \alpha_2 \alpha_1 E_0 \tilde{E}_0^{s'} + \ln \frac{\phi_{LN}^{s'}}{\phi_{LN}} = \frac{1 - \alpha_2 \alpha_1'}{1-L_A'} \ln \frac{L_A^s}{L_A^{s'}}.
\]

### A.7 Proof of Proposition 4

Increasing in \( S_0 \) does not affect the steady state, but raises \( E_0 \). At a “zrp” steady state, additional resource does not affect economic decisions and the additional resource is simply a slack resource. Starting from a “prp” steady state, innovation effort needs to jump down first, before converging back to the same steady state level as the economy builds up the effective resource stock. The transition of lower innovation and growth means that technology \( A^* \) is lower than the counterfactual when the economy converges back to the steady state. Since \( \frac{c}{(A^*)^2L} \) does not change in the steady state, lower \( A^* \) translates to lower consumption level.