# Closing the Loop in a Circular Economy: Saving Resources or Suffocating Innovations?

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**Abstract**

Policymakers around the world are increasingly embracing the idea of a "circular economy" (CE), an economy built on the principle of re-use of materials and produced goods through recycling, refurbishing, and extended product life. By using less new materials per unit of value added, a CE is considered both a solution to our environmental issues and good for the economy. Yet closing the material loop also changes the structure of the economy and the incentives for labor- and resource-productivity enhancing innovations. The overall economic impact is thus not so clear. This paper develops a two-sector endogenous growth model with Schumpeterian innovation, where the primary sector continuously develops new products and uses primary resources in production, while the secondary sector refurbishes retired products for re-use. We show that increased refurbishing increases short-run consumption, but reduces the incentives for developing new, possibly less resource-intensive products. If innovations are strongly resource-saving, raising the refurbishing rate leads to a net economic loss.

**JEL codes:** Q55, O30, O41, Q30

**Keywords:** Circular economy, refurbishing, innovation, creative destruction, resource market

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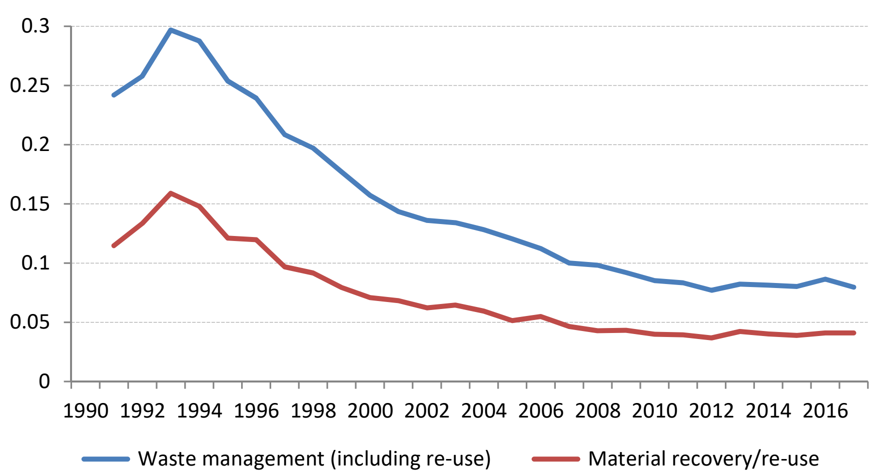
# 1 Introduction

The call for a "circular economy", an economy in which materials and produced goods are re-used and generate less waste, through recycling, refurbishing, and extended product life, has gained momentum in recent years. In March 2020, a new Circular Economy Action Plan was adopted by the European Commission as one of the main building blocks of the European Green Deal. At this year's annual meeting, the World Economic Forum intensified their call for joint actions among its members to scale up the circular economy on a global level. Apart from the environmental benefits,[1] the popularity of this sustainability movement reflects the high level of optimism concerning the economic benefits of a circular economy. The European Commission suggests that circular economy measures could "increase the EU's GDP by an additional 0.5% by 2030" (European Commission, 2020). The World Economic Forum calls it "a trillion-dollar opportunity, with huge potential for innovation, job creation and economic growth" (World Economic Forum, 2014).

While this "win-win" belief is mobilizing policymakers around the globe, little research has been done to assess the robustness of the purported economic benefits. For one, while circular economy measures are believed to stimulate innovation and growth, patent data from the OECD technology development dataset (OECD, 2019) suggest that among all environment-related patents, the shares of patents in the categories of waste management in general or material re-use in particular have been declining since the 90s, see Figure 1.

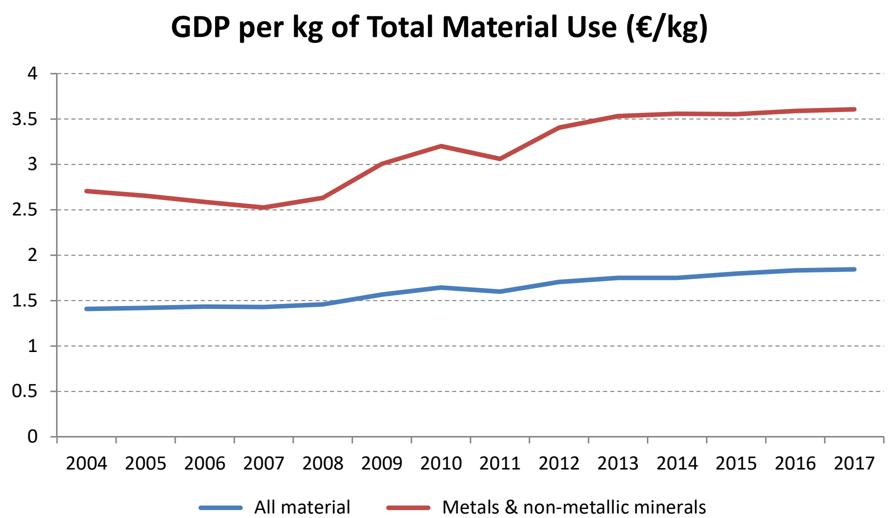



Note: Included here are patent families by inventors from the EU-28. Similar patterns emerge when considering patent families by inventors from the OECD countries or the entire world.

Figure 1: Share of CE-related patents in all environment-related patents (EU-28)

[1] Globally, resource extraction and processing is estimated to be responsible for half of total greenhouse gas emissions and over 90% of biodiversity loss and water use (see International Resource Panel, 2019), while secondary production such as recycling are considered much less energy-, carbon-, and water-intensive (see e.g. EPA, 2016).

Relatedly, material productivity has been rising in the EU even after controlling for the rise of the circular material use rate, suggesting that the production technology is becoming more resource-saving over time. This is illustrated in Figure 2 for real GDP per kg of total material use, which consists of both primary material (i.e. newly extracted) and secondary material (i.e. regained from waste streams).




Data source: circular material use rate (Eurostat, 2020a), GDP (Eurostat, 2020b), material use (Eurostat, 2020d)

Figure 2: Rising material productivity in the EU-28

Taking these two trends together, it seems that other, circular-economy-unrelated innovative processes have been playing a bigger role in increasing resource efficiency and driving green growth in the past two decades. This raises the question of how closing the material loop could affect these other innovative processes, and whether the purported economic benefits will remain if such effects are taken into account. This is the question that we address in this paper.

We develop a two-sector endogenous growth model with Schumpeterian innovation. The primary sector is innovative: primary producers continuously develop new products and use raw materials in production. The secondary sector refurbishes retired products for re-use. Refurbishing saves on raw materials for production, but also reduces the incentives for developing new, possibly less resource-intensive, products. This negative effect on innovation incentives works through two channels. On the one hand, refurbished products compete with new products for market share, leading to a "business-stealing" effect. On the other hand, being more labor-intensive than the primary sector, refurbishing drives up labor cost relative to resource price, leading to what we call a "cost-of-innovation" effect.

In the baseline where the refurbishing rate is exogenous, we find that increased refurbishing increases short-run consumption, but lowers growth either in the short run, or in both the short and long run. The overall economic impact depends on how resource-saving innovations are. If innovations are not strongly resource-saving, for low levels of re-use, an increase leads to increased lifetime consumption utility. If innovations are strongly resource-saving, more refurbishing lowers lifetime consumption utility and leads to a net economic loss.

The baseline result is robust to several extensions. When considering vertical integration of the primary sector that conducts its own refurbishing, we find that the dynamic trade-off between higher short-run consumption and lower growth is still present due to the cost-of-innovation effect. By endogenizing the refurbishing rate, we find that the net economic loss of higher refurbishing could be even larger, as higher refurbishing is associated with higher production cost.

**Related Literature** Our paper is most closely related to two strands of the literature. Firstly, by focusing on the economic impact of a circular economy, our paper is similar to early interest on the effect of competitive recycling in the industrial organization literature. Inspired by the judicial complexities of the Alcoa case in 1945, this literature has focused on the erosion of monopoly power through competitive recycling. Gaskins (1974) finds that a secondhand market undercuts the monopolist's revenues, and can lead to higher prices and lower output in the short run. Martin (1982) emphasizes the role of vertical integration, and shows that in the long run consumers always (weakly) benefit from recycling, with strict benefits if scrap recovery is independent. Swan (1980) studies the role of endogenous scrap rates and the scrap market, and finds ambiguous welfare effects with potentially inefficiently high recycling rates. Grant (1999) connects prior studies through a more detailed description of material flows. His empirical estimates suggest that competitive recycling decreases welfare. Complementing these early studies, our paper adds a general equilibrium view of competitive refurbishing that interacts with the primary sector through multiple markets, and provides a dynamic assessment of the overall economic effect.

Secondly, our paper contributes to the literature that connects recycling to long-run resource markets and scarcity. Hoel (1978) studies recycling as a substitute for resource extraction with less negative environmental impacts. Di Vita (2001) explores transitional dynamics and differences between developing and developed countries. Considering waste as a valuable production input, Pittel et al. (2010) characterize market inefficiencies when recycling markets are incomplete, and identify policy measures for correcting the market failures to achieve optimal material recycling. Hoogmartens et al. (2018) study the interaction between recycling and resource prices in a Hotelling model, and numerically find the optimal resource extraction path. Sørensen (2018) studies optimal recycling in a Ramsey model, and suggests that a Pigouvian tax on nonrecycled materials can ensure the transition towards a circular economy. Akimoto and Futagami (2018) set up a model where capital accumulation affects the incentives to recycle, and identify the optimal tax-subsidy policy for a transition from the linear economy (back) to the circular economy. Different from the above studies, our paper puts innovation at the center of our analysis. By assessing the dynamic resource efficiency and lifetime consumption, our paper is thus able to directly address the overall economic benefit of a circular economy.

The rest of the paper is organized as follows. Section 2 introduces the model, and Section 3 solves the model for its steady state and transitional dynamics. Section 4 investigates the overall economic impact of refurbishing, while Section 5 assesses the robustness of the results in a few key extensions. Section 6 briefly addresses the environmental and overall welfare impact of refurbishing. And finally, Section 7 concludes with a few final remarks.

# 2 The model

## 2.1 Final good

There is one final good, which is produced using a continuum of components indexed by $i$. Production of the final good is essentially an assembly process of the various components, and is subject to perfect competition:

$$y = \left[\int_0^1 x_i^{\frac{\epsilon-1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}} \tag{1}$$

$$p_{x_i} = p_y \left(\frac{x_i}{y}\right)^{-\frac{1}{\epsilon}}, \tag{2}$$

where $\epsilon > 1$ is the elasticity of substitution between the different components, $p_{x_i}$ is the price of component $i$, and $p_y = \left[\int_0^1 p_{x_i}^{1-\epsilon} di\right]^{1/(1-\epsilon)}$ is the ideal price index of the final good. Each component $i$ can be either newly produced ($x_{N_i}$) or refurbished from old components ($x_{R_i}$). The new and refurbished components are perfect substitutes so that $x_i = x_{N_i} + x_{R_i}$.

In the rest of the paper, we refer to a component type $i$ as an industry. Within each industry, we refer to the new and refurbished components as the primary and secondary sectors, and denote them by subscripts $N$ and $R$, respectively. We omit the time subscript whenever it does not cause confusion.

## 2.2 New component producer (primary sector)

New components of industry $i$ are produced by a primary producer, who possesses the most advanced technology of that industry. The production follows a Leontief technology[2] making use of labor and raw material:

$$x_{N_i} = A_i \min\left\{(A^*)^{\psi_L} L_{N_i}, (A^*)^{\psi_M} M_i\right\}, \tag{3}$$

[2] The Lenotief technology is chosen mainly for traceability, and is not crucial for the results. Our results carry through if instead a CES production function is used, as long as the elasticity of substitution between labor and raw material is less than 1 (that is, if the substitutability is weaker than that in the Cobb-Douglas case).

where $L_{N_i}$ and $M_i$ represent labor and raw material employed in the primary sector of industry $i$, $A_i$ is industry $i$'s technology stock, and $A^*$ is the so-called frontier technology, that is, the highest technology among all industries.[3]

The terms $A_i(A^*)^{\psi_L}$ and $A_i(A^*)^{\psi_M}$ represent the productivity (factor augmentation levels) of labor and material inputs, respectively. While the industry-specific knowledge $A_i$ determines the overall productivity of an individual primary producer, the frontier technology $A^*$ drives the relative productivity of raw material versus labor in production. This set-up therefore captures the idea that while an industry's overall productivity is constrained by their own knowledge, they benefit from the advance of the technology frontier and are able to adopt the "best practice" in terms of input combinations. A bias of technical change arises if the magnitude of spillovers to labor productivity differs from that of material productivity: if $\psi_M = \psi_L$, technical change is neutral as it does not alter the productivity of raw material relative to labor; if $\psi_M > \psi_L$, technical change is resource-saving since the advance of the technology benefits resource productivity relatively more; if $\psi_M < \psi_L$, technical change is labor-saving.

Given the production technology, the unit cost of new component $i$ is given by

$$c_{N_i} = A_i^{-1}\left((A^*)^{-\psi_L} w + (A^*)^{-\psi_M} p_M\right) = a_i^{-1} c_N^*, \tag{4}$$

where $w$ is the wage of workers, $p_M$ is the resource price, $a_i \equiv A_i/A^*$ represents an inverse measure of the technology distance of industry $i$ to the technology frontier $A^*$, and $c_N^* \equiv (A^*)^{-1}\left((A^*)^{-\psi_L} w + (A^*)^{-\psi_M} p_M\right)$ is the unit cost of the primary producer of the frontier industry.

### 2.3 Refurbishers (secondary sector)

The refurbishing sector of each industry takes retired components and refurbish them to be re-used. Refurbishing uses only labor as an input, according to

$$x_{R_i} = \min\left\{A_i(A^*)^{\psi_L} L_{R_i}, Z_i\right\}, \tag{5}$$

where $L_{R_i}$ is labor input and $Z_i$ is retired components. Naturally, one unit of retired component is required to make available one refurbished component. This is also a matter of accounting: non-refurnishable components are accounted as waste as we see below. Refurbishers have the same per unit labor requirement as primary producers within their industry, $A_i(A^*)^{\psi_L}$. This simplifying assumption is not crucial for the results and will be relaxed in Section 5.2.

The collection, sorting and supply of retired components are conducted by a central governmental agency, who sets the price for retired components to target a refurbishing

[3] Throughout the paper, the asterisk symbol denotes the frontier industry.

rate $\beta$, while the refurbishing sector is subject to perfect competition and free entry.[4] Regarding the supply of retired components, we assume the following:

**Assumption 1.** *Collecting retired goods and sorting reusable components are costless. There is an exogenous feasibility constraint such that the maximum fraction of reusable components in all old components is given by* $0 < \beta^u < 1$.

In equilibrium, for any refurbishing rate target $\beta \leq \beta^u$, the governmental agency charges a price for retired components equal to the rent, that is, the difference between the price of new components and refurbishing costs $(p_{x_i} - A_i^{-1}(A^*)^{-\psi_L} w)$, and transfers this revenue to the households. Refurbishers make zero profits and enter until a fraction $\beta$ of all components is refurbished. The amount of retired components supplied to refurbishers in industry $i$ is thus given by $Z_i = \beta x_i$.[5] Essentially, the equilibrium refurbishing rate in this setting is determined by the target rate, while a policy that increases the circularity is interpreted as an increase of the target rate $\beta$.

## 2.4 Equilibrium price setting

Primary producers anticipate that for any quantity they produce, a fraction re-enters the market as refurbished components.[6] Since the equilibrium goods market share for refurbishers is the same across all industries ($x_{R_i}/x_i = \beta$), primary producers thus maximize their flow profit $\pi_i = (p_{x_i} - c_{N_i})(1-\beta)x_i$, subject to the sectoral demand (2), which leads to the standard pricing rule:[7]

$$p_{x_i} = \frac{\epsilon}{\epsilon - 1} c_{N_i}. \tag{6}$$

In equilibrium, therefore, there is a uniform markup $\frac{\epsilon}{\epsilon-1}$ for all industries. This markup is unaffected by the competitive pressure from refurbishing, since a forward-looking primary producer realizes that their market share is unaffected by their pricing strategy and will thus rationally set the same markup as in the absence of competitive refurbishing.

[4] This assumption of the central governmental collection agency together with a competitive refurbishing sector reflects the characteristics of the waste management industry in the EU. Waste management is historically a matter of the municipalities, and is still characterized by the presence of many public companies. In addition, the industry consists of mainly micro companies (less than 10 employees, 77%) and SMEs (less than 250 employees, 99%), see Eurostat (2020c)

[5] This continuous time equation is the limit case of the discrete time version with the discrete period length $dt$ goes to zero, that is, $Z_{i,t} = \beta X_{i,t-dt}$.

[6] In a discrete time setting, pricing decision of the primary producers in one period affects their next period profit as a fraction of the products sold of the current period will come back next period as refurbished products. Profit maximizing in such a setting thus involves a dynamic pricing decision. As the period length approaches zero, the refurbished products will return to the market immediately. The analysis in a continuous time setting thus simplifies into a static profit maximization problem.

[7] The setting here considers the primary producer to be the first-mover in the Stackelberg game in prices. Alternatively, the primary producer and the refurbishers may engage in a Nash game, resulting in $p_{x_i} = \frac{\epsilon}{\epsilon - 1 + x_{R_i}/x_i} c_{N_i}$, where in equilibrium $x_{R_i}/x_i = \beta$. All results carry through under this pricing rule.

### 2.5 Material balance

Resource exists either in the form of low entropy raw material or as high entropy waste. The raw material needed for producing the new components is extracted from a non-renewable resource stock $S$, while the waste is generated by consumption and is deposited in a waste stock $W$. As mentioned earlier, in our continuous-time setting refurbishing is immediate. Thus the total quantity of material is accounted for in the two stocks and satisfies Lavoisier's law of mass conservation. The equations of motion for the two stocks are given by:

$$\text{Raw material stock:} \qquad \dot{S} = -M, \tag{7}$$

$$\text{Waste stock:} \qquad \dot{W} = M, \tag{8}$$

where $M$ is the net material flow through the economy, measured in mass per unit of time (e.g. kg/year), and the dot notation denotes time derivative.

Resource demand comes from the primary producers only so the aggregate resource demand is given by $M = \int_0^1 M_i \, \mathrm{d}i = \int_0^1 (A^*)^{-\psi_M} A_i^{-1} x_{N_i} \, \mathrm{d}i$. The secondary sector does not demand resource directly, but has an embedded material flow of $\int_0^1 \frac{\beta}{1-\beta} M_i \, \mathrm{d}i = \frac{\beta}{1-\beta} M$. Thus refurbishing effectively scales up the productive material flow by a multiplier of $\frac{1}{1-\beta}$:[8]

$$M_{\text{Eff}} = \frac{1}{1-\beta} M.$$

### 2.6 Extraction

Competitive price-taking resource extractors manage the resource stock and supply raw material. Resource extraction costs are zero. Maximization of net present value of extraction profits thus leads to an equilibrium material price that equals the scarcity rent, which grows at the rate of interest. That is, the simplest form of the classic Hotelling rule applies:[9]

$$\hat{p}_M = r. \tag{9}$$

### 2.7 Research and development

Our modeling of research and development (R&D) closely follows Aghion and Howitt (1998). Research occurs in all industries. Successful R&D in industry $i$ increases the

[8] The fact that there is repeated refurbishing and that not only new goods, but also refurbished goods are re-used means that in the extreme case of maximum refurbishing ($\beta = 1$), closing the cycle can bring resource extraction to zero. The more realistic case of $\beta < 1$ means there are limited number of times a component can be re-used.

[9] Throughout the paper, the hat notation denotes growth rates.

productivity of that industry, and brings it from the incumbent's level $A_i$ to the frontier $A^*$. The frontier itself increases over time proportionally to aggregate R&D.

The Poisson arrival rate of innovation per unit of time for each industry $i$ is given by $\lambda L_{A_i}$, where $\lambda > 0$ is a research productivity parameter and $L_{A_i}$ is research labor input devoted to that industry. By the law of large numbers, $\lambda L_{A_i}$ is the likelihood that the incumbent primary producer with productivity level $A_i$ will be replaced by a new entrant that operates at frontier productivity level.

There is free entry in R&D, so that in equilibrium the cost of R&D, $wL_{Ai}$, equal the expected benefits of R&D, $\lambda L_{Ai}V^*$, where $V^*$ denotes the value of a patent for the frontier technology. The free-entry condition for R&D can thus be written as:

$$\lambda V^* \leq w \quad \perp \quad L_{Ai} \geq 0. \tag{10}$$

This condition shows that the costs and benefits of research are the same across all industries, which implies that innovators are indifferent with respect to which industry to target. We focus on the symmetric equilibrium in the analysis, in which all industries have the same equilibrium flow of research labor, that is, $L_{A_i} = L_A$, where $L_A$ is also the aggregate research effort since we normalized the mass of sectors to unity.

The technology frontier expands at a rate proportional to aggregate research effort:

$$\hat{A}^* = \lambda L_A \ln \gamma, \tag{11}$$

where $\gamma > 1$ represents the size of technology improvement of each innovation, and $\lambda \ln \gamma > \rho$ is assumed to hold throughout the paper so that technology can grow at a faster pace than the time preference rate $\rho$, if all labor is devoted to research (that is, if $L_A = 1$).

Although the distribution of the technology stocks across industries, $A_i$, changes over time, the distribution of the relative technology distance, $a_i$, is independent of the absolute levels of the technology and is stationary in the long run with the following cumulative distribution function (see appendix in Aghion and Howitt (1998)):

$$H(a) \equiv a^{\frac{1}{\ln \gamma}},\ 0 \leq a \leq 1. \tag{12}$$

### 2.8 Labor market clearing and households

The representative household supplies inelastically one unit of labor. Labor market clearing thus requires:

$$L_X + L_A = 1, \tag{13}$$

where $L_X \equiv L_N + L_R$ is the aggregate amount of labor allocated to production.

The instantaneous utility of the households is given by $u_t = \ln y_t - \mu n_t$, where $\ln y_t$ represents the utility from consumption, $n_t$ captures the disutility from environmental damage, and $\mu$ is the relative weight given to the environmental concerns. The environmental damage constitutes an externality, which is not taken into account by agents' behavior. Since our main purpose in this paper is to assess the overall economic impact of a circular economy, we mostly focus on the consumption utility $\ln y_t$. We shall return to the environmental consideration in Section 6.

The households hold the equity of material extraction firms and intermediate goods firms, and maximize lifetime utility $U_0 = \int_0^\infty u_t e^{-\rho t}\,\mathrm{d}t$ subject to an intertemporal budget constraint $\dot{\mathcal{A}} = w + r\mathcal{A} + p_M M + \mathcal{T} - p_y y$, where $\mathcal{A}$ is total wealth and $\mathcal{T}$ is lump sum taxes or transfers from the government. The maximization results in the Ramsey rule for optimal saving:

$$r = \rho + \hat{p}_y + \hat{y}. \tag{14}$$

# 3 Equilibrium

## 3.1 Equilibrium industry shares and factor shares

We first solve for the share of labor in total cost of primary production, to be denoted by $\phi_N$, and the value share of any industry $i$ in total production, $\theta_i$.

We first note from (4) that the labor share in primary production cost is the same across all industries:

$$\frac{wL_{N_i}}{c_{N_i}} = \frac{w(A^*)^{-\psi_L}}{w(A^*)^{-\psi_L} + p_M(A^*)^{-\psi_M}} \equiv \phi_N. \tag{15}$$

From now on we will call $\phi_N$ the labor (cost) share for short. It reflects both the state of technology and relative factor prices, $w/p_M$. We will use it as the key price variable in the model when analyzing the dynamics.

Similarly, from (3) we note that relative factor use is the same across $i$, so that material-labor ratio at the industry level as well as in aggregate can be expressed in terms of frontier technology:

$$\frac{M_i}{L_{N_i}} = \frac{M}{L_N} = (A^*)^{\psi_L - \psi_M}. \tag{16}$$

Relative demand for material versus labor either stays constant, falls, or rises over time as the frontier technology increases due to innovation. It depends on whether technology is unbiased ($\psi_M = \psi_L$), relatively resource saving ($\psi_M > \psi_L$), or labor saving ($\psi_M < \psi_L$).

Second, we define $\theta_i \equiv p_{x_i} x_i / p_y y$ as the market share of industry $i$ (relative to the average industry).[10] Uniform markup together with demand (2) and marginal costs (4) indicate a market share of each industry determined by its distance to the frontier: $\theta_i =$

[10] Recall that we normalized the number of industries to unity so that the average industry has a market share of 1; if $\theta_i = 4$, spending on industry $i$ is four times as large as on the average industry.

$a_i^{\epsilon-1}\theta^*$. Since market shares of all industries add up to 1, that is, $\int_0^1 \theta_i \,\mathrm{d}i = 1$, the frontier industry market share is given by

$$\theta^* = \left[\int_0^1 a_i^{\epsilon-1} di\right]^{-1}.$$

Substituting the stationary distribution for $a_i$ (12), we find a constant market share for the frontier industry:

$$\theta^* = \left[\int_0^1 h(a)a^{\epsilon-1} da\right]^{-1} = 1 + (\epsilon - 1)\ln\gamma. \tag{17}$$

Though the market share of the frontier industry does not change over time, the identity of the frontier industry changes through creative destruction.

Because factor shares and markups are the same across $i$, employment and materials shares equal market shares: $L_{N_i} = \theta_i L_N$ and $M_i = \theta_i M$. Considering total labor use in production, we also need to account for secondary production. Since in each industry a share $\beta$ of the components is supplied as refurbished components and both primary and secondary production have the same labor requirement, the share of primary production labor $L_N$ in total production labor $L_X$ is then simply

$$L_N/L_X = 1 - \beta. \tag{18}$$

Given (17) and (18), the total output of the final goods is given by

$$y = (\theta^*)^{-\frac{\epsilon}{\epsilon-1}} x^* = (\theta^*)^{-\frac{\epsilon}{\epsilon-1}} \frac{(A^*)^{1+\psi_L} L_N^*}{1-\beta} = (\theta^*)^{-\frac{1}{\epsilon-1}} (A^*)^{1+\psi_L} L_X. \tag{19}$$

That is, the total output is directly proportional to the total labor employed in production and the labor augmenting capacity of the frontier technology. The more production labor ($L_X$), the higher the frontier technology ($A^*$), and the more labor-augmenting ($\psi_L$) technology is, the higher is the total output.

### 3.2 Rates of return to investment and saving

The value of a primary producer firm with productivity $A_i$, denoted $V_i$, is the expected net present value of profits, accounting for the risk $\lambda L_A$ of being replaced. Written as an arbitrage equation, this implies

$$rV_i = \pi_i + \dot{V}_i - (\lambda L_A)V_i, \tag{20}$$

meaning that the return equals the profit flow, capital gains while still in business, minus the expected capital loss due to creative destruction.

Because all primary producers charge the same markup, their relative profits equal relative market value shares, $\pi_i/\pi_j = p_i x_i / p_j x_j = \theta_i/\theta_j = (A_i/A_j)^{\epsilon-1}$. Because all primary producers face the same risk-corrected discount rate, $r + \lambda L_A$, their relative expected net present value of profits, i.e. relative firm value, equals relative profits,[11] $V_i/V_j = \pi_i/\pi_j = (A_i/A_j)^{\epsilon-1}$. Hence, firm value can be expressed relative to the value of the frontier firm:

$$V_i = (A_i/A^*)^{\epsilon-1} V^*. \tag{21}$$

Recalling that $V^* = w/\lambda$ by free entry in research and that $\hat{A}^* = \lambda \ln \gamma L_A$, we can write the growth rate of firm value as:

$$\hat{V}_i = \hat{w} - (\epsilon - 1)\lambda \ln \gamma L_A \quad \text{ if } L_A > 0. \tag{22}$$

The profits of a primary producer are proportional to total cost because of the constant markup. Using the definitions of labor cost share $\phi_N$ and industry share $\theta_i$, we may write total cost as $c_{Ni} x_{Ni} = w L_{Ni}/\phi_N = w \theta_i L_N/\phi_N$. Using (18) and the free entry condition (10), the frontier primary producer's profit to firm value ratio (that is, the dividends paid to households who hold the assets) is found to be proportional to labor use in production, and inversely proportional to the labor cost share:

$$\frac{\pi^*}{V^*} = B(\beta) \frac{L_X}{\phi_N}, \tag{23}$$

where

$$B(\beta) \equiv \frac{1-\beta}{\epsilon - 1} \lambda \theta^*. \tag{24}$$

Substituting (23), (22), and (13) into (20), we find (for $L_A > 0$)

$$r - \hat{w} = B(\beta) \frac{L_X}{\phi_N} - \lambda \theta^* (1 - L_X). \tag{25}$$

This equation characterizes the return to R&D in terms of our key variables: production employment $L_X$ and wage cost indicator $\phi_N$. With higher production employment, primary producers make higher profits and innovators realize a higher rate of return to innovation. How much innovators profit from a large economy and low labor cost is summarized by the term $B$, which is a composite parameter representing the product of research productivity $\lambda$, frontier industry market share $\theta^*$, primary sector employment share $L_N/L_X = 1 - \beta$, and profit-to-cost ratio $\pi/(c_N x) = 1/(\epsilon - 1)$. We interpret $B$ as an indicator of the relevant *business size* for innovators. Business size indicator $B$ falls with the refurbishing rate $\beta$. More refurbishing lowers the share of the total market for the frontier primary producer as

[11] Let $R_s \equiv e^{-\int_t^s (r_u + \lambda L_{A,u}) du}$ denote the risk-corrected discount factor, so that expected net present value of profits equals $V_{i,t} = \int_t^\infty \pi_{i,s} R_s ds = \int_t^\infty (\pi_{i,s}/\pi_{j,s}) \pi_{j,s} R_s ds$. Substituting $\pi_{i,s}/\pi_{j,s} = (A_{i,s}/A_{j,s})^{\epsilon-1}$, which is constant over time, we find the proportionality result.

refurbishers steal a bigger part of their market; as a result the return to innovation falls, other things equal.

Households invest their wealth not only in production firms but also in extraction firms. The latter manage the resource stock and the rate of return to investing in this stock is the rate of material price increase according to the Hotelling rule (9). Noting from our definition of the labor share (15) that $p_M = w(A^*)^{\psi_M - \psi_L}(1-\phi_N)/\phi_N$, we write the Hotelling rule in terms of our key variables as:

$$r - \hat{w} = \Psi(1 - L_X) - (1-\phi_N)^{-1}\hat{\phi}_N. \tag{26}$$

where the composite parameter

$$\Psi \equiv (\psi_M - \psi_L)\lambda \ln \gamma \tag{27}$$

is the productivity of research labor $L_A = 1 - L_X$ in generating resource-saving technical change, which we refer to as the *technology bias* indicator. Equation (26) states that the return on resource holdings increases with resource-saving technical change when wage rate and factor share remain the same. This reflects the fact that higher future technology levels increase the productivity and hence price of resources. However, an alternative – more precise – interpretation of (26) is the rate of change of relative factor prices: since $\hat{p}_m = r$ by the Hotelling rule, the left hand side represents $\hat{p}_M - \hat{w}$. If material prices grow faster than wages, the wage share must fall (second term on the right hand side of (26)) unless offset by sufficiently high bias of technical change (first term on the right hand side).

Households save part of their income optimally such that the Ramsey rule (14) holds. We write also this equation in terms of our key variables, $L_X$ and $\phi_N$. Given a constant markup rate and refurbishing rate, the value of final output is proportional to the aggregate profits of primary producers and their costs. We can thus substitute the change in expenditures of production. Since $wL_X$ are labor costs, and $\phi_N$ is the labor cost share, we have $wL_X/\phi_N$ as production costs that is proportional to the value of consumption. The Ramsey rule describes the return that households demand for their investments:

$$r - \hat{w} = \rho + \hat{L}_X - \hat{\phi}_N, \tag{28}$$

which gives the capital supply by households.

Equations (25), (26), and (28) provide three expressions for the rate of return. The capital market is in equilibrium if they are equalized while innovators are active. An equilibrium might also arise without innovation, in which case (25) no longer holds with equality (but with its left hand side larger than its right hand side). An equilibrium with zero scarcity rent (and thus zero material price) implies $\phi_N = 0$, in which case (26) still holds.

## 3.3 Effective resource stock

The productive capacity of the economy at any point in time is represented by two predetermined variables: the stock of remaining raw material $S$ and the stock of accumulated technology $A^*$. While the former captures the scarcity of production inputs,[12] the latter determines how efficient the economy is at using production inputs. Summarizing both aspect, we can define the effective resource stock as

$$E \equiv (A^*)^{\psi_M - \psi_L} S, \tag{29}$$

which provides a productivity-augmented measure of resource scarcity.

## 3.4 Equilibrium dynamics

Equalizing the rates of returns ((25), (26), and (28)) and deriving the equation of motion for the effective resource stock, the dynamic equilibrium of the model is fully captured by three reduced-form differential equations, provided in Lemma 1. The proof for this lemma, as all other proofs, is provided in Appendix A.

**Lemma 1.** *The equilibrium in its reduced form can be expressed by the differential equation system of the variables $\phi_N$, $L_X$, and $E$:*

$$\hat{\phi}_N = (1 - \phi_N)(\Psi + \lambda\theta^*) \left[1 - \left(1 + \frac{B(\beta)}{\Psi + \lambda\theta^*}\phi_N^{-1}\right) L_X\right] \tag{30}$$

$$\hat{L}_X = [\Psi - \rho - (\Psi + \lambda\theta^*)\phi_N] - [\Psi - B(\beta) - (\Psi + \lambda\theta^*)\phi_N] L_X \tag{31}$$

$$\hat{E} = \Psi - \left(\Psi + (1 - \beta)E^{-1}\right) L_X \tag{32}$$

*where $B(\beta)$ is the business size indicator defined in (24), $\Psi$ the technology bias indicator defined in (27), and $E$ is the effective resource stock defined in (29).*

Intuitively, the three differential equations summarize how the effective resource scarcity ($E$) and its expression through relative wage ($\phi_N$) affect labor allocation between production and research (or equivalently, between consumption and saving), which in turn affects future effective resource scarcity. In this process, the technology bias ($\Psi$) and business size ($B$) indicators play a crucial role. While the former controls how much current research can affect future resource scarcity, the latter directly affects the allocative decisions.

To derive the dynamic equilibrium, notice that the dynamics of $\phi_N$ and $L_X$ are independent of the level of $E$. We can thus first build a two-dimensional phase diagram in the $(\phi_N, L_X)$ plane. Subsequently, the dynamics of $E$ are independent of $\phi_N$, and we build a second phase diagram in the $(E, L_X)$ plane. Since both $\phi_N$ and $L_X$ are bounded between

[12] With a finite stock, raw material is the more scarce production input compared to labor, which is available at a constant flow.

0 and 1, we use $\Psi$ and $B(\beta)$ to partition the parameter space such that the steady states in each parameter region are within these bounds. For $\Psi$, we distinguish between the cases of large ($\Psi > \rho$) versus small ($\Psi \leq \rho$) technology bias. For $B(\beta)$, we introduce two refurbishing rate thresholds, implicitly defined as follows:

$$B(\bar{\beta}) = \rho \tag{33}$$

$$B(\bar{\bar{\beta}}) = \rho\frac{\Psi + \lambda\theta^*}{\Psi - \rho}, \tag{34}$$

where $\beta \in (0,1)$, while $\bar{\bar{\beta}} \in (-\infty, \bar{\beta})$ if $\Psi > \rho$ and $\bar{\bar{\beta}} > 1$ if $\Psi < \rho$.[13] The phase diagrams for small $\Psi$ are provided in Figure 3, and for large $\Psi$ in Figure 4.

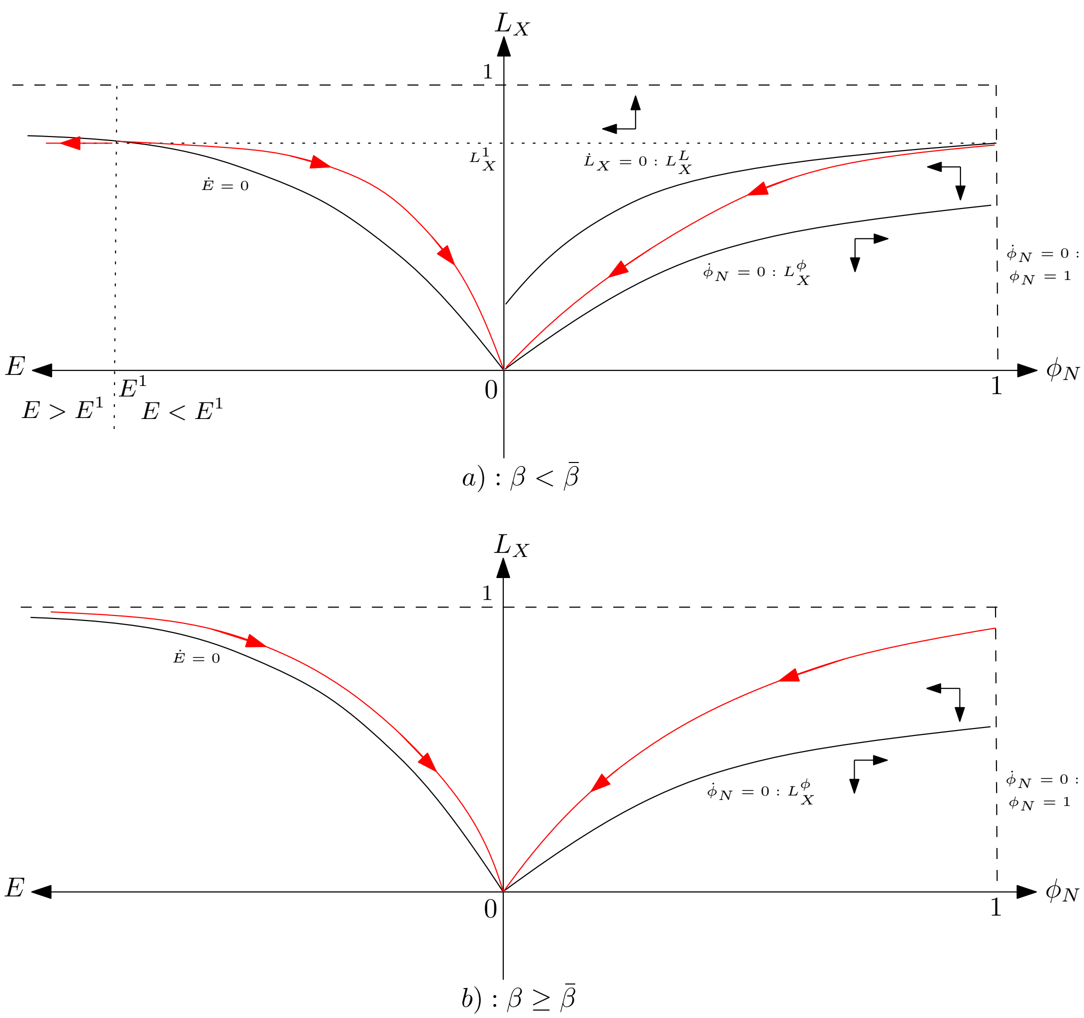
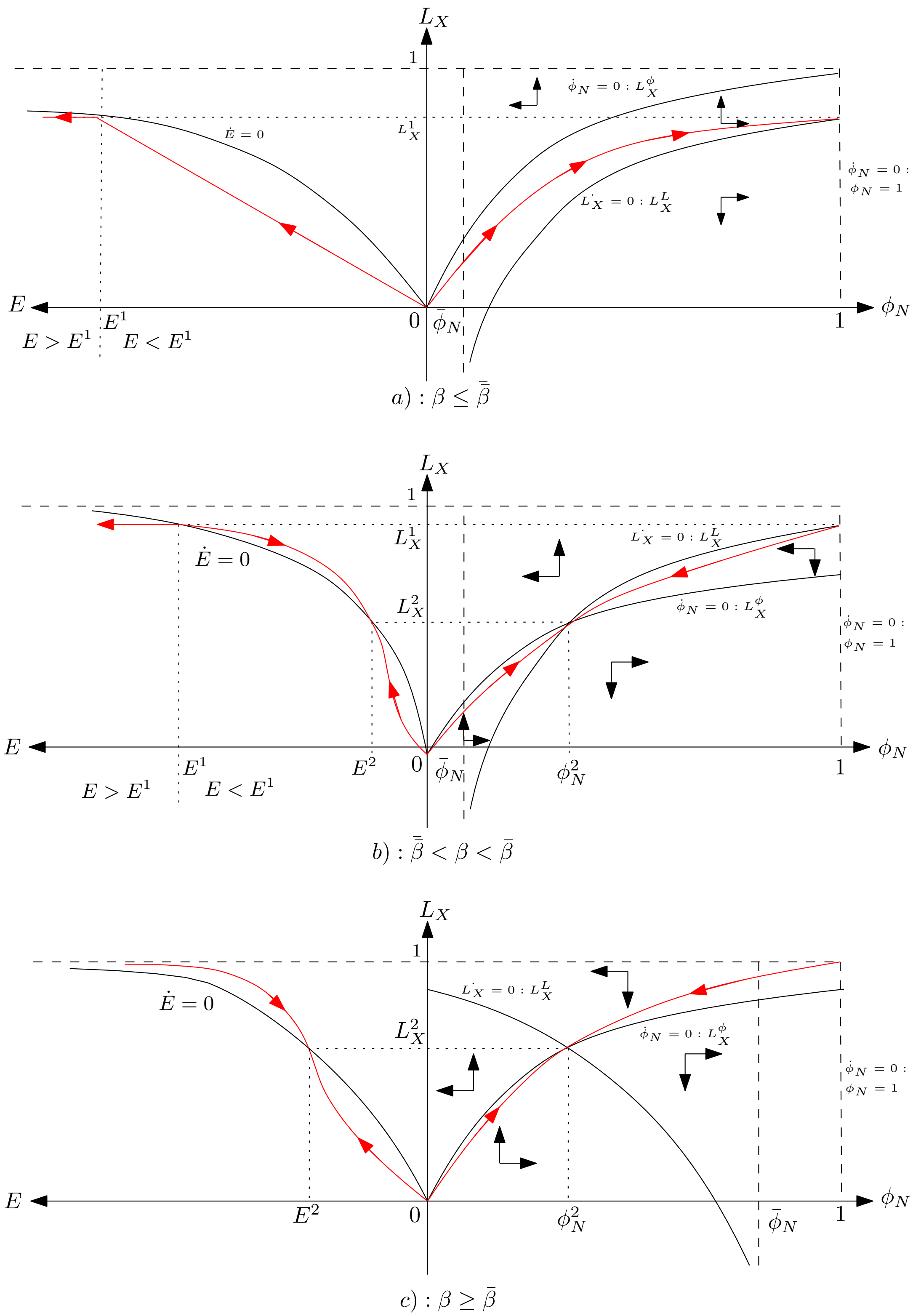



Figure 3: Phase diagrams when $\Psi < \rho$

[13] Since $\epsilon >$ and $\lambda \ln \gamma > \rho$, $B(0) = \frac{\lambda}{\epsilon - 1} + \lambda \ln \gamma > \rho > 0 = B(1)$ holds. By $\frac{\partial B(\beta)}{\partial \beta} < 0$, $\beta \in (0,1)$. The statement on $\bar{\bar{\beta}}$ follows from $\frac{\partial B(\beta)}{\partial \beta} < 0$. Further, since $\frac{\partial B(0)}{\partial \epsilon} < 0$ and $\frac{\partial \theta^*}{\partial \epsilon} > 0$, $\bar{\bar{\beta}} > 0$ if $\epsilon$ is not too large.




Note: Panel a) illustrates the case when $\bar{\bar{\beta}} > 0$, which occurs if $\epsilon$ is not too large. In Panel c), as $\beta \to \bar{\beta}$, the $\dot{L_N} = 0$ becomes a vertical line $\phi_N = \bar{\phi}_{L_N}$ but the dynamics are the same as in the case of $\beta > \bar{\beta}$.

Figure 4: Phase diagrams when $\Psi > \rho$

### 3.5 Equilibrium resource scarcity

Based on the phase diagrams in Figures 3 and 4, the equilibria can be characterized into four different regimes, summarized in Proposition 1 and illustrated in Figure 5.

**Proposition 1** (Characterization of Equilibrium)**.** *For each combination of the initial effective resource stock $E(0) > 0$ and circularity parameter $\beta \in [0, 1)$, there is a unique saddle-point stable equilibrium.*

*I. ("convergence to balanced growth") If innovation is sufficiently resource-saving compared to impatience ($\Psi > \rho$), the refurbishing rate is high ($\beta > \bar{\bar{\beta}}$), and the resource stock falls short of a critical level ($E(0) < E^1$), then the labor share $\phi_N$, the effective resource stock $E$, and research labor $L_A$ converge to the steady state levels given by:*

$$SS^I: \quad \phi_N^I = \frac{B(\Psi - \rho)}{\rho(\Psi + \lambda\theta^*)}, \qquad L_X^I = 1 - \frac{\rho}{\Psi}, \qquad L_A^I = \frac{\rho}{\Psi}, \qquad E^I = \frac{(1-\beta)L_X^I}{\Psi L_A^I}. \tag{35}$$

*Convergence is monotonic, with $E$ and $\phi_N$ rising if $L_A$ falls and vice versa.*

*II. ("vanishing scarcity") If innovation is sufficiently resource-saving compared to impatience ($\Psi > \rho$), the refurbishing rate is low ($\beta \leq \bar{\bar{\beta}}$), and the effective resource stock falls short of a critical level ($E(0) < E^1$), then equilibrium wages grow faster than resource prices; the labor share $\phi_N$ and the effective resource stock $E$ grow, while research labor falls, with the economy converging to the steady state given by:*

$$SS^1: \quad \phi_N^1 = 1, \qquad L_X^1 = \frac{\rho + \lambda\theta^*}{B + \lambda\theta^*}, \qquad L_A^1 = \frac{B - \rho}{B + \lambda\theta^*}, \qquad E^1 = \frac{(1-\beta)L_X^1}{\Psi L_A^1}. \tag{36}$$

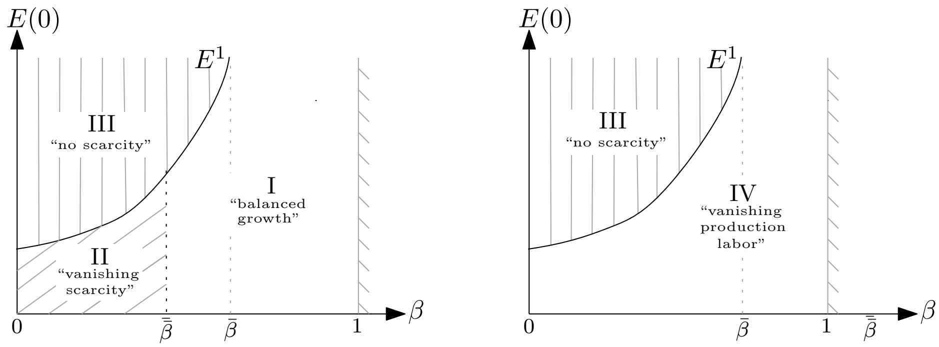
*III. ("no scarcity") If the refurbishing rate is low ($\beta < \bar{\beta}$), and the initial effective resource exceeds a critical level ($E(0) \geq E^1$), the equilibrium resource price is zero ($\phi_N = 1$) and innovation labor is constant at level $L_A^1$ as given by (36) for all $t > 0$.*

*IV. ("vanishing production labor") If innovation is insufficiently resource-saving compared to impatience ($\Psi \leq \rho$), and either the refurbishing rate is high ($\beta \geq \bar{\beta}$), or the resource stock is small ($E(0) < E^1$), then the equilibrium converges monotonically to a steady state with vanishing wage share and vanishing labor in production:*

$$SS^0: \quad \phi_N^0 \to 0, \qquad L_X^0 \to 0, \qquad L_A^0 \to 1, \qquad E^0 \to 0. \tag{37}$$

*In the long run, production and consumption vanish (stay constant, grow unboundedly) if $\psi_M < (=, >) \frac{\rho}{\lambda \ln \gamma} - 1$.*

With an exhaustible resource as essential input, long run consumption and welfare crucially depend on whether technical change can compensate for increasing resource scarcity.




$(a)\ \Psi > \rho\ \left(\psi_M - \psi_L > \frac{\rho}{\lambda \ln \gamma}\right)$ $(b)\ \Psi \leq \rho\ \left(\psi_M - \psi_L \leq \frac{\rho}{\lambda \ln \gamma}\right)$

Figure 5: Characterization of steady states

Essentially, the equilibrium characterization in Proposition 1 answers this questions for different combinations of $E(0)$, $\Psi$ and $\beta$. While $E(0)$ describes the initial scarcity, together with the technology bias indicator $\Psi$ it determines how much innovation will be needed. The refurbishing rate $\beta$, on the other hand, influences the profits of the primary producers and thus determines how much innovation the economy is willing to generate.

Several insights can be gleamed from the above proposition. Firstly, given insufficient initial endowment ($E(0) < E^1$), equilibrium depends crucially on whether or not the resource-saving potential of innovation ($\Psi$) surpasses the depletion incentives ($\rho$). If $\Psi \leq \rho$, innovation is ineffective in creating or keeping high resource abundance. As a result, the effective resource supply quickly falls. In the resulting "vanishing production" regime, scarcity of resources ultimately drives all labor out of production, while innovation is at maximum speed but still incapable of offsetting depletion. In contrast, if $\Psi > \rho$, the economy converges to a steady state with constant effective resource stock and increasing consumption.

Secondly, given insufficient initial endowment ($E(0) < E^1$) and large resource-saving potential of innovation ($\Psi > \rho$), the long run resource scarcity depends on the refurbishing rate. If the refurbishing rate is low ($\beta \leq \bar{\bar{\beta}}$) resulting in high innovation incentives, resource price converges to zero relative to wage, as scarcity vanishes in the long run. If the refurbishing rate is high, innovators see a sizable market share taken away by refurbishers. To maintain sufficient incentives for innovation, resource prices must remain high, as scarcity increases in balanced growth.

And finally, only if the refurbishing rate is not too high, an effectively high endowment of resources results in zero resource prices. In the resulting "no scarcity" regime, cumulative demand for resources is lower than total supply because innovation offsets depletion, where innovation can be sufficiently high because innovators see little of their market being stolen away by the secondary sector. As the refurbishing rate becomes higher, however, innovation incentives are reduced and the required initial resource endowment for covering the cumulative demand quickly increases. Once the refurbishing rate becomes too high ($\beta \geq \bar{\beta}$), it is practically no longer possible to have a high enough initial endowment ($E^1 = \infty$). In this sense, refurbishing increases resource scarcity by reducing resource-saving innovations.

For the remainder of the paper, we restrict our attention to the empirically more relevant case, where $E(0) < E^1$ (that is, resource is scarce) and $\Psi > \rho$ (that is, technical change is sufficiently resource saving, see for example Figure 2). We thus concern ourselves only with the balance growth and vanishing scarcity equilibria. We further note that the vanishing scarcity equilibrium is only relevant if $\bar{\bar{\beta}} > 0$, which requires that $\epsilon$ is not too large. We nevertheless continue with both equilibria for completeness.

**Assumption 2.** *Technology bias towards resource-saving is sufficiently strong that $\psi_M - \psi_L > \frac{\rho}{\lambda \ln \gamma}$, that is, $\Psi > \rho$, where $\lambda \ln \gamma > \rho$ is assumed to hold.*

# 4 The economic impact of refurbishing

We now turn to the main question of the paper, the economic impact of raising the refurbishing rate. We start with the comparative statics of the steady state. Table 1 presents how the steady-state effective resource stock ($E$), extraction rate ($\frac{M}{S}$), labor cost share ($\phi_N$), R&D labor share ($L_A$), and consumption growth rate ($g \equiv \hat{y}$) are affected by the refurbishing rate ($\beta$). In this table, +, −, and 0 represent a positive, negative, and no effect, respectively.

| | $E$ | $\frac{M}{S}$ | $\phi_N$ | $L_A$ | $g$ |
|---|---|---|---|---|---|
| $SS^I$ ($\bar{\bar{\beta}} < \beta \leq 1$) | − | 0 | − | 0 | 0 |
| $SS^1$ ($0 \leq \beta < \bar{\bar{\beta}}$) | + | − | 0 | − | − |

Table 1: Steady state comparative statics

From Table 1, we see that the refurbishing rate affects the steady state quite differently depending on which regime the economy is in. Particularly, while a higher refurbishing rate does not affect the long-run growth rate in the balance growth regime, it lowers the long-run growth rate in the vanishing scarcity regime. To understand why this occurs, let us revisit the return on R&D equation (25). On the right hand side of (25), we see that refurbishing affects the dividend ratio $B(\beta)\frac{L_X}{\phi_N}$ (and thus the innovation incentive) through both the business-stealing ($B(\beta)$) and the cost-of-innovation ($\phi_N$) channels. Since $B(\beta)$ decreases with $\beta$, higher refurbishing always lowers the primary producers' market share and makes innovation less attractive. While more refurbishing pulls labor into production and – together with the increase in effective resource flow – tends to increase wage relative to resource price, lower innovation raises resource scarcity and has the opposite effect on relative wage. In a balance growth regime, the latter effect dominates and the steady state labor cost share $\phi_N$ decreases with $\beta$, leading to a positive cost-of-innovation effect that exactly offsets the business-stealing effect. The net effect on innovation is zero. This self-correcting mechanism, however, is absent in the vanishing scarcity regime: as $\phi_N$ approaches 1 the cost-of-innovation effect is negligible. Thus the business-stealing effect dominates and innovation decreases with the refurbishing rate.

Apart from the growth rate, Table 1 shows that the steady state effective resource stock also changes with the refurbishing rate. One implication of this is that when raising the refurbishing rate, the transition from one steady state to another cannot be immediate, as it takes time for $E$ to decrease (balance growth) or increase (vanishing scarcity regime) towards the new steady state level. The next lemma summarizes the transitional dynamics.

**Lemma 2.** *Consider an economy in steady state hit by a shock that permanently raises the refurbishing rate $\beta$.*

1. *If the economy is in a balanced growth regime, immediately following the shock, economy-wide production and consumption increase ($L_X, y \uparrow$), while research effort and growth drop ($L_A, g \downarrow$). Along the transition, production labor and labor cost share decline ($L_X, \phi_N \searrow$), while relative resource price, research effort and growth rise ($p_M/w, L_A, g \nearrow$). The economy converges to a new steady state with the same growth rate but a lower technology level compared to business-as-usual (BAU).*
2. *If the economy is in a vanishing scarcity regime, immediately following the shock, economy-wide consumption can increase or decrease, corresponding to lower or higher research effort and growth. Along the transition, production labor and labor cost share increase ($L_X, \phi_N \nearrow$), while research effort and growth rate fall ($L_A, g \searrow$). The economy converges to a new steady state with a lower growth rate compared to BAU.*

In the balance growth regime, raising the refurbishing rate effectively increases the immediate resource availability, leading to more production and consumption. However, more refurbishing also leads to an unambiguous fall in innovation in the short run, as the business-stealing and cost-of-innovation effects reinforce each other, contrary to the long run outcome. To see why the cost-of-innovation effect behaves differently in the short run, note from (15) that the labor cost share depends both on the relative price and on the technology level. In the short run, both the immediate resource abundance and the higher production raise relative wage. While in the long run the adjustment in the technology level eventually raises resource scarcity and lowers the labor cost share, before the technology level can adjust sufficiently, the labor cost share rises in the short run and makes innovation more costly.

In the vanishing scarcity regime, the short run response is not unambiguous. The economy either experiences higher short-run consumption accompanied by permanently lower innovation and growth, or lower short-run consumption with higher short-run but lower long-run growth. Whichever situation, while raising the refurbishing rate may or may not lead to higher short-run consumption, it always lowers innovation and growth in the long run.

Lemma 2 makes clear that a rise in the refurbishing rate always brings a trade-off between the short- and long-run efficiency. While more refurbishing could mean less resource extraction and higher consumption in the short run, the increased activities in production crowd out innovation and lead to lower long-run resource efficiency and consumption. By showing that the potential static benefits is always accompanied by a dynamic cost, this lemma thus casts doubt on whether an overall economic benefit will be present.

To compare the trade-off between the short and long run, we now turn to the lifetime consumption utility of the households, given by $U_{y,0} = \int_0^\infty \ln y_t e^{-\rho t}\ \mathrm{d}t$. Since $y_t = y_0 e^{\int_{s=0}^t g_s\ \mathrm{d}s}$, where $g_s$ is consumption growth rate at time $s$, a marginal increase in the refurbishing rate changes the consumption utility relative to BAU according to

$$\mathrm{d}U_{y,0} = \int_{t=0}^\infty \left[ \mathrm{dln} y_0 + \int_{s=0}^t \mathrm{d}g_s\ \mathrm{d}s \right] e^{-\rho t}\ \mathrm{d}t.$$

That is, the total effect on the consumption utility consists of both the immediate response ($\ln y_0$) and the growth effect ($\mathrm{d}g_s$). The next proposition summarizes the net impact on lifetime consumption utility.

**Proposition 2.** *The effect of higher refurbishing on lifetime consumption utility $U_y$ depends on the resource bias of technology ($\psi_M - \psi_L$) and the initial refurbishing rate ($\beta$). In particular,*

1. *if $\frac{\rho}{\lambda \ln \gamma} < \psi_M - \psi_L < \frac{(1+\psi_L)\rho}{(1+\psi_L)\lambda \ln \gamma - \rho}$, $U_y$ is always increasing in $\beta$ for all $\beta \in [0, 1)$;*
2. *if $\psi_M - \psi_L \geq \frac{(1+\psi_L)\rho}{(1+\psi_L)\lambda \ln \gamma - \rho}$, $U_y$ is generally hump-shaped in $\beta$ and there exists a unique $\beta^* \in [0, \bar{\bar{\beta}}]$ such that higher refurbishing increases the consumption utility for $0 \leq \beta < \beta^*$ while decreasing it for $\beta^* < \beta < 1$. The consumption-utility-maximizing refurbishing rate $\beta^*$ is decreasing in the elasticity of substitution $\epsilon$.*

Given that raising the refurbishing rate crowds out innovation either in the short run or in the long run, the results in Proposition 2 is not surprising. Whether or not a potential short-run consumption gain outweighs the loss of lower innovation depends on how costly it is to crowd out innovation, which in turn depends on how resource-saving innovation is. The stronger the resource-saving bias, the more costly it is to crowd out innovation and the less likely an overall economic benefit exists. Furthermore, the easier it is for the economy to substitute away from the less efficient industries (the higher $\epsilon$), the easier the efficiency loss rises and the less likely refurbishing will lead to an overall economic benefit.

By showing that an overall economic benefit does not necessarily exist, the result of Proportion 2 thus challenges the "win-win" claim that a circular economy is good for both the environmental and the economy. Perhaps somewhat paradoxically, it is exactly due to the ability of the economy to adapt to more environmentally friendly production technology that a lack of economic benefit occurs. More generally, the result of Proposition 2 thus

| Parameter | Value | Source |
|---|---|---|
| | *Structural Parameters* | |
| $\beta$ | 11.7% | Eurostat |
| $\epsilon$ | 7 | Literature |
| $\rho$ | 0.05 | Literature |
| $\psi_L$ | 0 | Normalization |
| | *Steady State Values (Target)* | |
| $g_c$ | 0.01 | Eurostat |
| $L_A$ | 0.088 | Eurostat |
| $\frac{p_M M_N}{p_y y}$ | 0.027 | World Bank |
| | *Calibration* | |
| $\psi_M$ | 5 | Calibrated |
| $\lambda$ | 0.29 | Calibrated |
| $\ln \gamma$ | 0.39 | Calibrated |

Table 2: Parameter Values

points to the importance of taking existing mechanisms into account when evaluating new policy measures or environmental paradigms.

**Numerical example** We now provide a simple numerical example to illustrate the economic response to an increase of the refurbishing rate.

The model consists of seven structural parameters $(\beta, \epsilon, \rho, \psi_L, \psi_M, \lambda, \gamma)$. We set the refurbishing rate $\beta$ to the 2017 circularity rate of the EU-28 (Eurostat, 2020a), and set the time preference rate $\rho$ to a commonly used value in the literature. For the elasticity of substitution $\epsilon$, we follow the estimates of the product-level elasticity from the literature (see Broda and Weinstein, 2006; Hottman et al., 2016).[14] We further normalize $\psi_L$ to 0.

The remaining three parameters are calibrated to match the steady state GDP growth rate $g$, R&D labor share $L_A$ and share of resource rent in GDP $\frac{p_M M}{p_y y}$. For $g$, we use the average GDP growth rate of the EU-28 between 2000 and 2019 from the Eurostat national accounts database (Eurostat, 2020b). For $L_A$, we use the employment category

[14] Existing empirical estimates tend to vary by a large range. In general, as shown by Broda and Weinstein (2006), the more we disaggregate, the more substitutable the goods becomes. They report an average elasticity of substitution of 7 at the three-digit level (in the Tariff System of the USA) during 1972-1988 and 17 at the seven-digit level for the same period; for the period 1990 to 2001, the average elasticities estimated are 4 at the three-digit level (in the Harmonized Tariff System) and 12 at the ten-digit level. Hottman et al. (2016) argue for the use of scanner data for the elasticity estimate as this data measure corresponds more closely to the level at which firms and consumers make their product choice decisions. Using either 12-digit Universal Product Codes or 13-digit European Article Numbers, their estimates of product substitution elasticity range from 4.7 to 17.6 with a median elasticity of 6.9.

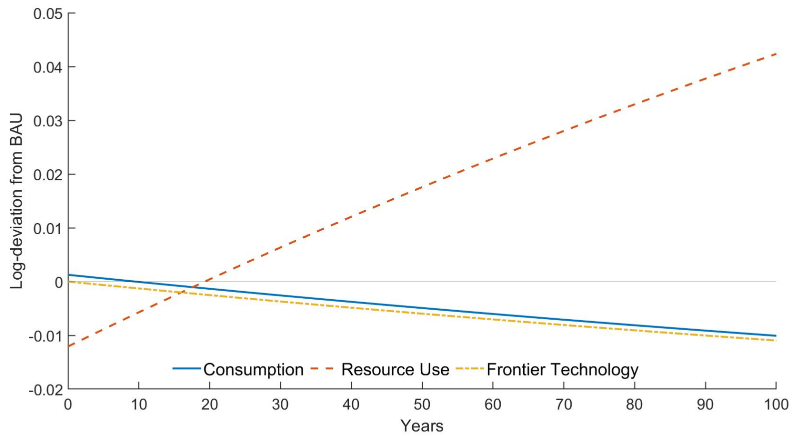



Figure 6: Response to a 10% increase in $\beta$ (initial $\beta = 11.7\%$)

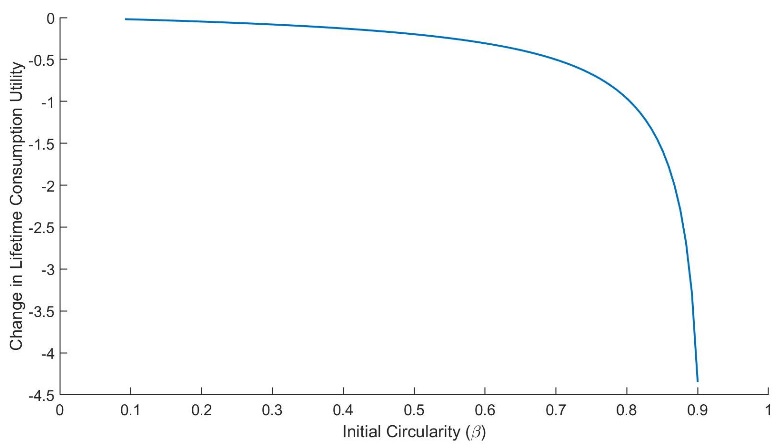



Figure 7: Effect of a 10% increase in $\beta$ on lifetime consumption utility

"professional, scientific and technical activities" as a proxy for R&D workers and derive the 2017 R&D labor share for the EU-28 from the Eurostat national accounts employment data (Eurostat, 2020e). And finally, from the World Bank data on the share of total resource rent (that is, the sum of oil rents, natural gas rents, coal rents (hard and soft), mineral rents, and forest rents) in GDP (World Bank, 2020), we set $\frac{p_M M}{p_y y}$ to the 1970-2015 average share of the world. Table 2 provides an overview of the parameter values and their sources.

Given the targeted share of resource rent, the model is calibrated to the balance growth regime. Using the calibrated parameters, we simulate the economic response to a 10% increase in the refurbishing rate. Figure 6 illustrates the timepaths of the key variables following the increase, starting from an initial $\beta$ of 11.7%. For all variables, the timepaths illustrated are the log-deviation with respect to the BAU scenario. The dynamic trade-off is apparent, as consumption gradually falls below the BAU level, whereas resource use increases beyond the BAU level. In Figure 7, the effect on lifetime consumption utility is illustrated for different initial values of $\beta$. For all $\beta > \bar{\bar{\beta}}$ (i.e. the range of $\beta$ for which a balance growth steady state is possible, here $\bar{\bar{\beta}} = 0.08$), a 10% increase in the refurbishing rate lowers lifetime consumption utility.

# 5 Alternative model specifications and robustness

The lack of economic benefit when increasing the refurbishing rate is a consequence of the crowding out of resource-saving innovations. To check the robustness of this crowding out mechanism, we now consider two alternative specifications that are most likely to challenge it.[15] In the first, we consider vertically-integrated primary producers who conduct refurbishing themselves, which can directly shut down or reduce the business-stealing effect of refurbishing. In the second alternative specification, we endogenize the size of the refurbishing sector by allowing it to respond to price signals.

## 5.1 Vertically-integrated primary producer

We now assume that primary producers are vertically integrated in the sense that they also collect and refurbish used components. Primary producers do not manage to take back all but only a fraction of used components from their customers, while the rest continues to be collected by the governmental agency and supplied to the competitive refurbishing sector. The governmental agency takes the internal refurbishing of primary producers into account, and targets an overall refurbishing rate of $\beta$.

Due to imperfect collection and refurbishing, the maximum share of refurbished components in each primary producer's output is less than one. We denote this maximum share by $\beta^I \in [0, \beta)$. We further use $\beta^C \in [0, 1)$ to denote the maximum share of all used components that can be refurbished by the competitive refurbishing sector. The total refurbishing rate is thus given by

$$\beta = \beta^C + (1 - \beta^C)\beta^I = 1 - (1 - \beta^C)(1 - \beta^I). \tag{38}$$

Primary producers maximize their flow profit by setting their product price (the markup decision) and deciding on how much to refurbish (the refurbishing decision). Since the overall refurbishing target $\beta$ is set by the governmental agency, primary producers foresee that a higher internal refurbishing means a smaller competitive refurbishing sector. At the

[15] Another potential challenge concerns the factor market interaction, through which refurbishing affects the cost of innovation. While we have assumed a single labor market and a uniform skill level, these assumptions can be relaxed. Allowing research to be conducted only by high-skilled workers or allowing different wage rates for high- and low-skilled workers will not affect the result, as long as low-skilled labor is scarce and high-skilled labor is mobile between both types of labor markets.

same time, since with an internal refurbishing rate $b_i^I \in [0, \beta^I]$ the unit production cost becomes

$$c_i = (1 - b_i^I)c_{N_i} + b_i^I c_{R_i}^I,$$

where $c_{N_i} = A_i^{-1}((A^*)^{-\psi_L} w + (A^*)^{-\psi_M} p_M)$ and $c_{R_i}^I = A_i^{-1}(A^*)^{-\psi_L} w$, it is clear that $c_{N_i} > c_{R_i}$ and internal refurbishing also lowers unit production cost. Primary producers thus rationally conduct the maximum amount of refurbishing that is possible to them, i.e. $b_i^I = \beta^I$, while continuing to set a markup of $\frac{\epsilon}{\epsilon-1}$ over the marginal cost.

Before we proceed, we note that the baseline model is embedded in this more general framework. If $\beta^I = 0$, meaning the primary producer has no market power in the refurbishing sector, we return to the baseline model. This is for example the case if the primary producer cannot collect used components directly from their customers, but must purchase old components from the governmental agency the same way as competitive refurbishers. At the other extreme, if we set $\beta^I = \beta$, the primary producer has full power over the refurbishing process. This would be the case if there is no separate collection sector, and the primary producers are fully vertically integrated.

The rest of the analysis follows the baseline model almost exactly. In particular, we continue to use the primary producer' labor cost share $\phi_N$, total production labor $L_X$ and the effective resource stock $E$ as the key variables for the reduced form relations, while defining the composite parameter

$$B^C(\beta^C) \equiv \frac{1 - \beta^C}{\epsilon - 1} \lambda \theta^*.$$

With internal refurbishing, the primary producers' material and labor demand now satisfies

$$\frac{M}{L_N} = (A^*)^{\psi_L - \psi_M}(1 - \beta^I),$$

while the labor share in primary production cost is now:

$$\phi_N = \frac{w(A^*)^{-\psi_L}}{w(A^*)^{-\psi_L} + (1 - \beta^I) p_M (A^*)^{-\psi_M}}.$$

Accordingly, with internal refurbishing, the reduced form differential equation system in Lemma 1 now changes to:

$$\hat{\phi}_N = (1 - \phi_N)(\Psi + \lambda\theta^*) \left[1 - \left(1 + \frac{B^C(\beta^C)}{\Psi + \lambda\theta^*} \phi_N^{-1}\right) L_X\right] \tag{39}$$

$$\hat{L}_X = [\Psi - \rho - (\Psi + \lambda\theta^*)\phi_N] - \left[\Psi - B^C(\beta^C) - (\Psi + \lambda\theta^*)\phi_N\right] L_X \tag{40}$$

$$\hat{E} = \Psi - \left(\Psi + (1 - \beta)E^{-1}\right) L_X. \tag{41}$$

Compared to Lemma 1, while the dynamics of the effective resource stock $E$ are still affected by the aggregate refurbishing rate $\beta$, $\phi_N$ and $L_X$ are now only affected by the rate of competitive refurbishing $\beta^C$, as $B^C(\beta^C)$ replaces $B(\beta)$ in (30) and (31).

The similarity of the reduced form equations indicates that the dynamic behavior of this generalized framework will be qualitatively similar to that of the baseline. Particularly, as internal refurbishing already at its upper bound, raising the refurbishing rate $\beta$ translates directly into raising $\beta^C$ and all results in the baseline will carry through directly. However, what motivates us to this particular model extension is whether or not the crowding out of innovation will still occur if refurbishing is done by the primary producers themselves. The answer to this question, for the balance growth regime, is provided by the next proposition.[16]

**Proposition 3.** *Consider an economy in a balance growth steady state hit by a shock that permanently raises the internal refurbishing rate $\beta^I$, while keeping the competitive refurbishing rate $\beta^C \in [0, 1)$ unchanged.*

1. *Immediately following the shock, economy-wide production and consumption increase ($L_X, y \uparrow$), while research effort and growth drop ($L_A, g \downarrow$). Along the transition, production labor and labor cost share fall ($L_X, \phi_N \searrow$), while relative resource price, research effort and growth rise ($p_M/w, L_A, g \nearrow$). The economy converges to a new steady state with the same growth rate but a lower technology level compared to BAU.*

2. *The effect of higher internal refurbishing on lifetime consumption utility depends on the resource bias of technology ($\psi_M - \psi_L$): lifetime consumption utility is always increasing in $\beta^I$ if $\frac{\rho}{\lambda \ln \gamma} < \psi_M - \psi_L < \frac{(1+\psi_L)\rho}{(1+\psi_L)\lambda \ln \gamma - \rho}$, and always decreasing in $\beta^I$ if $\psi_M - \psi_L \geq \frac{(1+\psi_L)\rho}{(1+\psi_L)\lambda \ln \gamma - \rho}$.*

Proposition 3 shows that the result of the dynamic trade-off induced by increased refurbishing is robust to internal refurbishing. While this might seem surprising at a first glance, it is in fact logical since refurbishing crowds out innovation not just by its business-stealing effect but also by competing with research for labor. Although internal refurbishing does not affect the market share of the primary producers ($B(\beta^C)$ unchanged), by raising labor demand and lowering material scarcity it drives up the labor cost share $\phi_N$ immediately following the shock, which raises the cost of innovation. Before the labor cost share converges back to the steady state level, the innovation incentives are reduced due to the increased cost of innovation. Whether the loss of innovation outweighs the short-run gains of consumption again depends on the resource bias of technical change. Since internal refurbishing does not affect the steady state labor allocation, the impact on lifetime consumption utility no longer depends on the initial refurbishing rate. The short-run benefit

[16] If the economy is in the vanishing scarcity regime, raising $\beta^I$ while keeping $\beta^C$ unchanged will only reduce the steady state effective resource stock $E^1$. This means that $E(0) > E^1$ and there is no longer scarcity.

always dominates the long-run cost of lower technology level if technical change is insufficiently resource-saving, while the long-run cost dominates if technology is sufficiently resource-saving.

### 5.2 Endogenous competitive refurbishing

We now endogenize the size of the competitive refurbishing sector, by assuming free entry in this sector and a refurbishing cost that increases with the refurbishing rate. To focus on competitive refurbishing, we again assume away the possibility of internal refurbishing by the primary producers.

Instead of facing fixed labor requirement and paying a maximal fee to the government for collection, we now assume that refurbishers constitute a perfectly competitive fringe that collects used components and brushes them up to make them as new. This integrated process of refurbishing requires labor effort. The bigger the fraction of used components that is refurbished, the higher the average cost of refurbishing. In particular, refurbishers need a factor $\delta$ more labor than primary producers, where this factor increases with the rate of refurbishing, $\beta_i$, which continues to face a technical upper bound. That is, (5) changes to

$$x_{R_i} = \min\left\{(\delta(\beta_i))^{-1} A_i (A^*)^{\psi_L} L_{R_i}, Z_i\right\},$$

where $\delta(\beta_i)$ and $\beta_i$ satisfy the following assumption:

**Assumption 3.** *The refurbishing rate is bounded above by $\beta^u \in (0,1)$. The unit labor requirement in refurbishing increases with the refurbishing rate, and approaches infinity as $\beta_i$ approaches its upper bound: $\delta'(\beta_i) > 0$ and $\lim_{\beta_i \to \beta^u} \delta(\beta_i) = \infty$.*

Free entry of the refurbishing sector requires that the unit refurbishing cost equals the market price of components:

$$p_i = \tau^{-1}\delta(\beta_i) A_i^{-1} (A^*)^{-\psi_L} w, \tag{42}$$

where $\tau \geq 1$ is a governmental subsidy on refurbishing.

The primary producer of industry $i$ sets the price to maximize profits, anticipating that it will sell only share $(1-\beta_i)$ of the total market, that total market demand has a price elasticity $\epsilon$ and that refurbishers will respond to price changes according to the above free-entry condition. That is, the primary producer maximizes $(1-\beta_i)(p_i - c_{N_i})x_i$ subject to (2) and the endogenous response of $\beta_i$, which results into markup pricing, $p_i = m_i \cdot c_{N_i}$, where $m_i$ denotes the markup. It is easily verified that there is again a uniform refurbishing rate $(\beta_i = \beta)$ across all sectors, and consequently also a uniform markup given by

$$m_i = m = \frac{\epsilon + \eta}{\epsilon + \eta - 1}, \tag{43}$$

where $\eta$ captures the equilibrium response of refurbishers to price changes derived from the free entry condition

$$\eta \equiv -\frac{\partial \ln(1-\beta)}{\partial \ln p} = \frac{\delta(\beta)}{(1-\beta)\delta'(\beta)}. \tag{44}$$

Examining (43), we see that the adjusted markup essentially follows the standard markup rule but uses the effective elasticity of substitution, $\epsilon + \eta$. This is intuitive as primary producers not only face *inter-industrial* competition from producers of other industries ($\epsilon$), but also *intra-industrial* competition from the refurbishers ($\eta$) of the same industry. In the baseline model, collection is done by a governmental agency and the unit refurbishing cost does not respond to the refurbishing rate, so $\delta'(\beta) = 0$ and $\eta = 0$. Without the endogenous response of the refurbishing market share, there is no need for the primary producers to adjust their markup. With the endogenous response of the refurbishing sector, profit maximizing primary producers need to consider the impact of their pricing decision on the intensity of intra-industrial competition.

We do not change the cost structure for primary producers, so we still have $p_i = m \cdot c_{N_i} = mA_i^{-1}(A^*)^{-\psi_L} w\phi_N^{-1}$. The free-entry condition of refurbishers is thus also:

$$m = \tau^{-1}\delta(\beta)\phi_N. \tag{45}$$

Since $m > 1$ and $\phi_N \leq 1$, (45) means that $\delta(\beta) > 1$ in equilibrium even if $\tau = 1$ (that is, without a refurbishing subsidy). This suggests that refurbishing is less labor efficient when compared to primary production.

Equations (43) and (45) define the best-response of the primary producer and the competitive fringe of refurbishers in the Stackelberg game in prices. Together, (43), (44), and (45) solve for $\beta, \eta, m$ and $\delta$ as functions of $\phi_N$, which depends on the exact specification of the cost function $\delta(\beta)$.

The rest of the model is as in the baseline. When finding the reduced form of the model, we now need to account for the fact that the refurbishing rate and labor requirement in refurbishing depend on $\phi_N$. As a result, the share of primary production in total production ($L_N/L_X$), the business size indicator of the primary producers ($B$), and total spending relative to wage cost ($\frac{yp_y}{wL_X}$) now become functions of $\phi_N$:

$$n(\phi_N) \equiv \frac{L_N}{L_X} = \frac{1}{1+\delta\beta/(1-\beta)}, \tag{46}$$

$$B(\phi_N) \equiv \lambda\theta^*(m-1)n(\phi_N), \tag{47}$$

$$\frac{1}{s(\phi_N)} \equiv \frac{yp_y}{wL_X} = \frac{m}{1-\beta}\frac{n(\phi_N)}{\phi_N}. \tag{48}$$

The variable $s$ can be interpreted as an inverse measure of spending or a measure of the savings rate.[17] We further denote the elasticity of spending with respect to the labor share by $\epsilon_s$, which has the following properties:

$$\epsilon_s \equiv \frac{\partial \ln s}{\partial \ln \phi_N}; \qquad \epsilon_s < 1; \qquad \lim_{\phi_N \to 0} s(\phi_N) = (\theta^*)^{\frac{1}{\epsilon-1}} \beta^u; \qquad \lim_{\phi_N \to 0} \epsilon_s = 0.$$

We again have three expressions of capital returns: returns to innovation, to resource conservation, and to savings, as given below:

$$r - \hat{w} \begin{cases} = \frac{B(\phi_N)}{\phi_N} L_X - \lambda\theta^*(1 - L_X), & \text{if } L_X < 1 \\ < \frac{B(\phi_N)}{\phi_N}, & \text{if } L_X = 1 \end{cases} \tag{49}$$

$$r - \hat{w} = \Psi(1 - L_X) - (1 - \phi_N)^{-1}\hat{\phi}_N, \tag{50}$$

$$r - \hat{w} = \rho + \hat{L}_X - \epsilon_s(\phi_N)\hat{\phi}_N. \tag{51}$$

Combining the three expressions to eliminate $r - \hat{w}$ and adding the equation for the extraction of the effective stock, the dynamics of the model can again be summarized in three differential equations in $L_X, \phi_N, E$, as a counterpart of Lemma 1:

$$\hat{\phi}_N = (1 - \phi_N)(\Psi + \lambda\theta^*) \left[ 1 - \left( 1 + \frac{B(\phi_N)}{\Psi + \lambda\theta^*} \phi_N^{-1} \right) L_X \right] \tag{52}$$

$$\begin{aligned} \hat{L}_X &= \epsilon_s(1 - \phi_N)\Psi - \rho - [1 - \epsilon_s(1 - \phi_N)]\lambda\theta^* \\ &\quad + \left[ [1 - \epsilon_s(1 - \phi_N)] \left( \frac{B(\phi_N)}{\phi_N} + \lambda\theta^* \right) - \epsilon_s(1 - \phi_N)\Psi \right] L_X \end{aligned} \tag{53}$$

$$\hat{E} = \Psi - \left( \Psi + n(\phi_N) E^{-1} \right) L_X. \tag{54}$$

As a first step towards characterizing the steady state, we consider a candidate steady state with both $\phi_N$ and $L_X$ constant. This requires:

$$\bar{\bar{B}} \equiv \rho \frac{\Psi + \lambda\theta^*}{\Psi - \rho} = \frac{B(\phi_N)}{\phi_N}. \tag{55}$$

In the above equation, $B(\phi_N)$ in the numerator captures the business-stealing effect, where higher wages, as reflected in higher $\phi_N$, affects refurbishing and thus the primary producer's market size, while $\phi_N$ in the denominator captures the cost-of-innovation effect, where higher wages make R&D more costly. While in the baseline the business-stealing effect is exogenous (exogenous $\beta$), here the refurbishing rate and thus also the business-stealing effect endogenously adjust according to the labor cost share.

[17] Conventionally, the savings rate in a closed economy is defined as investment over the sum of investment and consumption, which amounts to $wL_A/(yp_y + wL_A)$ here. Since $L_A = 1 - L_X$, our variable $s$ is directly related to the conventional savings rate.

Whether (55) can hold at equality determines the existence of an interior or corner steady state. The stability of such a steady state then depends on the sign of $\frac{\partial(B(\phi_N)/\phi_N)}{\partial\phi_N}$ near the steady state. Accordingly, we find three steady states, as summarized below.

**Lemma 3.** *Define $B^1 \equiv B(1)$ and $B^0 \equiv \lim_{\phi_N\to 0}\frac{B(\phi_N)}{\phi_N}$. If $\Psi > \rho$, then a steady state with bounded $E$ has the following properties:*

*I. ("balance growth") If $B^1 < \bar{\bar{B}} < B^0$, for any $\phi_N^I \in (0,1)$ with $B(\phi_N^I)/\phi_N^I = \bar{\bar{B}}$ and $lim_{\phi_N\to\phi_N^I} d(B(\phi_N)/\phi_N)/d\phi_N < 0$ there is a saddle-point stable steady state characterized by:*

$$SS^I: \quad \phi_N = \phi_N^I, \quad L_X = 1 - \frac{\rho}{\Psi}, \quad E = \frac{n(\phi_N^I)L_X}{\Psi(1-L_X)}. \tag{56}$$

*II. ("no resource scarcity") If $\bar{\bar{B}} \le B^1$, there is a saddle-point stable steady state characterized by:*

$$SS^1: \quad \phi_N = 1, \quad L_X = \frac{\rho+\lambda\theta^*}{B^1+\lambda\theta^*}, \quad E = \frac{n(1)L_X}{\Psi(1-L_X)}. \tag{57}$$

*III. ("no labor scarcity") If $B^0 \le \bar{\bar{B}}$, there is a saddle-point stable steady state characterized by:*

$$SS^0: \quad \phi_N \to 0, \quad L_X = \min\left\{\frac{\rho+\lambda\theta^*}{B^0+\lambda\theta^*}, 1\right\}, \quad E \to 0. \tag{58}$$

*Consumption vanishes (stays constant, grows unboundedly) if $(1+\psi_M)\lambda\ln\gamma L_A <$ $(=,>)$ $\rho$. Employment grows in the refurbishing sector, while falling in primary production: $\hat{L}_N < 0$ and $L_R \to L_X$.*

Lemma 3 presents all locally saddle-point stable steady states that are consistent with our current setting. It is apparent that the current setting is much more flexible than the one considered in the baseline, and uniqueness of the steady state is not generally warranted. If $B(\phi_N)/\phi_N$ is monotonically increasing or hump-shaped in $\phi_N$, for example, multiplicity and self-fulfilling expectations can be easily generated. This suggests that the comparability with the baseline, and consequently also the robustness of the baseline reults, will depend on the shape of $B(\phi_N)/\phi_N$ as a function of $\phi_N$, which in turn depends on the properties of the refurbishing cost $\delta(\beta)$. To proceed, we focus on a case that is more directly comparable to the baseline.

**Assumption 4.** *Suppose $\delta(\beta)$ is such that $B(\phi_N)/\phi_N$ is monotonically decreasing in $\phi_N$ and $B^0 > \bar{\bar{B}} > B^1$.*

Given Assumption 4, the model features a unique saddle-point stable steady state characterized by (56), which is directly comparable to the balance growth steady state in the baseline. To evaluate the economic impact of raising the refurbishing rate in this extended framework, we now consider the impact of introducing a refurbishing subsidy. The results are summarized in the next proposition.

**Proposition 4.** *Consider an economy in a balance growth steady state hit by a policy shock that permanently raises the refurbishing subsidy $\tau$.*

1. *Immediately following the shock, employment in production increases ($L_X \uparrow$), but research effort and growth fall ($L_A, g \downarrow$). Consumption can rise or fall depending on the immediate response of the refurbishing rate. If the refurbishing rate falls or increases moderately, consumption rises immediately following the shock; if the refurbishing rate increases sufficiently, consumption falls.*

2. *Along the transition, production labor and labor cost share fall ($L_X, \phi_N \searrow$), while the refurbishing rate, research effort, and growth rise ($\beta, L_A, g \nearrow$). The economy converges to a new steady state with the same growth rate but a lower technology level compared to BAU.*

Not surprisingly, the response of the labor allocation between production and research is very similar to that of the baseline. This similarity, however, does not directly translates to conclusions on the consumption response. While higher $L_X$ means more production, since the refurbishing sector is less labor efficient, a larger refurbishing sector means a lower average labor productivity, which tends to decrease output. The net effect on output and consumption thus depends on the endogenous response of the refurbishing rate, which again depends on the specific cost function $\delta(\beta)$. Independent of the short-term consumption response, it is clear from Proposition 4 that the dynamic cost of lower innovation and growth remains. The ambiguous short-term benefit coupled with unambiguous long-term cost thus hints at a more pessimistic prediction concerning the overall economic impact.

**Numerical example** We now illustrate the economic response to the introduction of a refurbishing subsidy using a numerical example. In line with Assumption 3, we assume the following refurbishing cost function

$$\delta(\beta) = \bar{\delta}(\beta^u - \beta)^{-1/\bar{\eta}},$$

which introduces three additional parameters $(\beta^u, \bar{\eta}, \bar{\delta})$. For $(\epsilon, \rho, \psi_L, \psi_M, \lambda, \gamma)$, we take the same values as in the baseline. We choose $\beta^u$ and $\bar{\eta}$ such that Assumption 4 is satisfied and there is a unique, balance-growth steady state. Finally, we calibrate $\bar{\delta}$ to match the 11.7% circularity as in the baseline. The parameter values are summarized in Table 3.

| Parameter | Value | Source |
|---|---|---|
| $\epsilon$ | 7 | Baseline |
| $\rho$ | 0.05 | Baseline |
| $\psi_L$ | 0 | Baseline |
| $\psi_M$ | 5 | Baseline |
| $\lambda$ | 0.29 | Baseline |
| $\ln \gamma$ | 0.39 | Baseline |
| $\beta^u$ | 0.435 | Assumption |
| $\bar{\eta}$ | 1.7 | Assumption |
| $\bar{\delta}$ | 0.6088 | Calibrate to target $\beta = 11.7\%$ |

Table 3: Parameter Values

The economy at the above parameter values is characterized by the following:

$$\frac{\partial \eta}{\partial \beta} < 0, \quad \frac{\partial m}{\partial \beta} > 0; \tag{59}$$

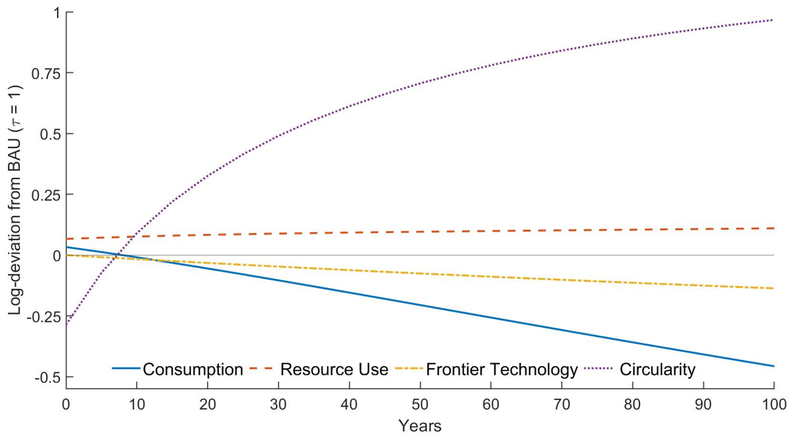
$$\frac{\partial \beta}{\partial \phi_N} < 0, \quad \frac{\partial \delta}{\partial \phi_N} < 0; \tag{60}$$

$$\frac{\partial n}{\partial \phi_N} > 0, \quad \frac{\partial B}{\partial \phi_N} > 0, \quad \frac{\partial (B/\phi_N)}{\partial \phi_N} < 0. \tag{61}$$

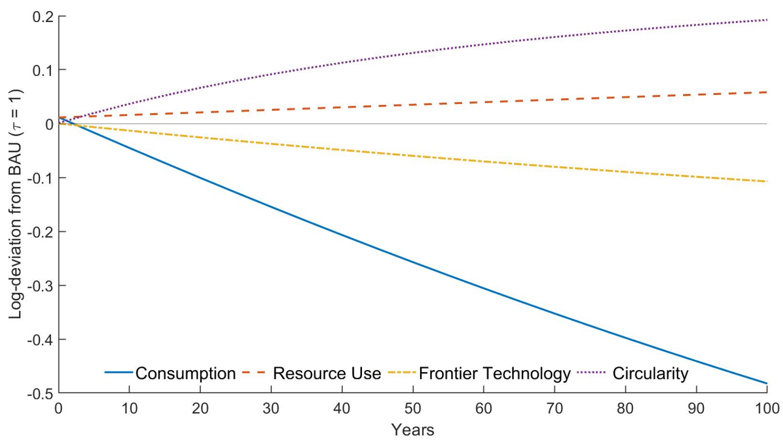
Intuitively, (59) means that the higher the refurbishing rate, the less the market size of the refurbishing sector responds to a price change. From the primary producer's perspective, the intra-industrial competition decreases with $\beta$, allowing them to set a higher markup. (60) reflects the fact that refurbishing is more labor intensive than primary production. Consequently, higher wage relative to resource price (as reflected by a higher $\phi_N$) increases the cost of refurbishing relative to its return, and lowers the market size of the refurbishing sector. Relatedly, (61) suggests the share of the primary sector in production $n$ and its profitability $B$ increase as labor becomes relatively more expensive (as reflected by a higher $\phi_N$). Consequently, the business-stealing and the cost-of-innovation effects work in opposite directions: higher $\phi_N$ increases innovation incentives through a larger market size for the primary producers, but lowers innovation incentives through more expensive labor. By fulfilling Assumption 4, however, the latter dominates the former.

Starting from an economy without refurbishing subsidy ($\tau = 1$), we now introduce a 10% permanent refurbishing subsidy ($\tau = 1.1$) and simulate the economic response of the key variables. The results are illustrated in Figure 8. In addition to the calibrated value of $\bar{\delta}$ (Panel (c)), we show the results for two more $\bar{\delta}$ values (Panel (a) and Panel (b)) to illustrate all possible cases mentioned in Proposition 4.

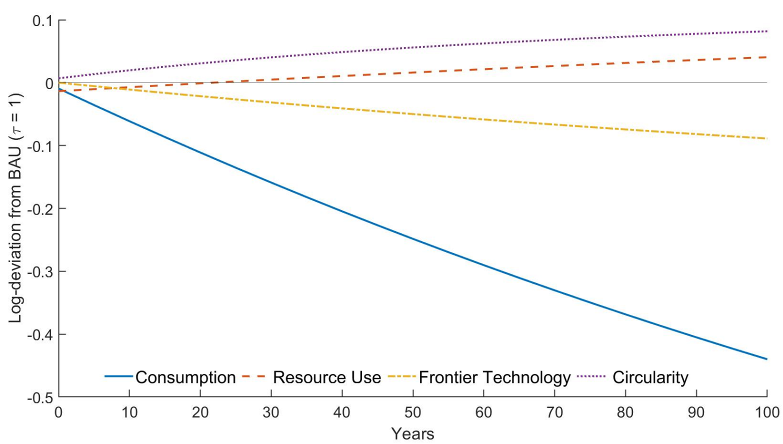
While in Panel (a) with a relatively large $\bar{\delta}$ value $\beta$ falls initially, in Panels (b) and (c) $\beta$ increases immediately. This difference is due to the different immediate response of $\phi_N$. While in all three cases, $\phi_N$ increases immediately after the policy shock, the magnitude




(a) $\bar{\delta} = 0.7$




(b) $\bar{\delta} = 0.655$




(c) $\bar{\delta} = 0.6088$

Figure 8: Response to a 10% refurbishing subsidy ($\tau = 1.1$)

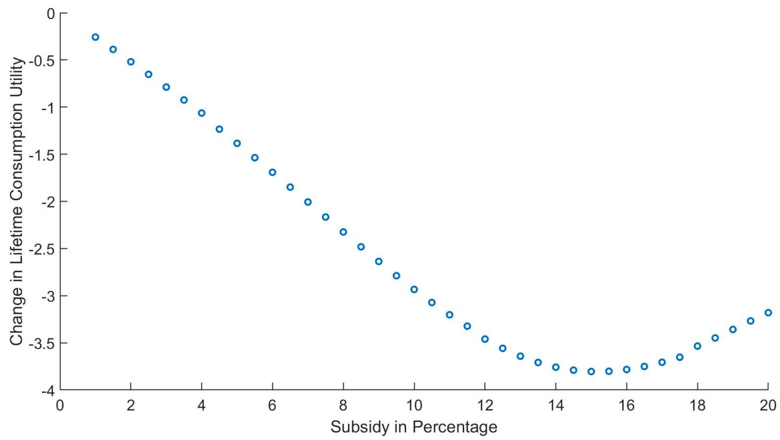



Figure 9: Effect of refurbishing subsidy on lifetime consumption utility ($\bar{\delta} = 0.6088$)

of the change decreases with $\bar{\delta}$. Intuitively, the larger the cost coefficient of refurbishing, the larger is the potential effect of a proportional refurbishing subsidy on the relative labor cost. Since $\eta$ and thus the markup $m$ are unaffected by $\bar{\delta}$, from (45) it follows that if the immediate increase of $\phi_N$ is larger than the subsidy rate (so that $\phi_N$ increases), $\beta$ will fall immediately after the policy shock.

The different immediate responses of $\beta$ affect how consumption and resource use change relative to BAU. In Panel (a), the fall in $\beta$ together with a rise in $L_X$ means that the economy is more labor efficient in the short run and thus output and consumption rise. As the refurbishing rates rises over time and the technology grows more slowly than in BAU, however, consumption eventually falls behind the BAU level. As for the resource use, immediate fall in the refurbishing rate means that in the short run the resource use increases. Over time, lower technology level means that resource use continues to be above the BAU level.

In Panel (b), $\beta$ also falls immediately after the policy shock, which explains the similar pattern of resource use as in Panel (a). Since the immediate rise of the refurbishing rate is rather small, consumption rises moderately in the short run due to higher $L_X$.

In Panel (c), $\beta$ jumps up immediately after the policy shock. While resource use accordingly falls in the short run relative to BAU, the lowered average labor productivity means that the short run consumption also falls. Over time, the lower pace of innovation together with rising refurbishing rate means that consumption continues to be lower than that of BAU, while resource use rises beyond the BAU level.

In terms of the overall effect on the lifetime consumption utility, the picture is similar for all three $\bar{\delta}$ values and is illustrated in Figure 9 for the case of $\bar{\delta} = 0.6088$. Starting from $\tau = 1$ (no tax or subsidy), the graph illustrates the effect of a permanent refurbishing subsidy ranging from 1 to 20 percent. The overall effect on the lifetime consumption utility is negative for the entire range of $\tau$ considered. Thus with the endogenous response of the refurbishing sector, the overall economic impact of raising the refurbishing rate continues to be negative.

## 6 The environmental impact

Our main interesting in this paper has been the economic impact of a circular economy in the presence of resource-saving technical change. The analysis so far shows that, particularly when technical change has a strong resource-saving bias, promoting re-use and refurbishing does not necessarily bring an economic benefit. This result, however, does not preclude the overall benefit of a circular economy. Indeed, a major incentive for promoting a circular economy is the reduction of the negative environmental impact associated with resource extraction and processing. Even if raising circularity lowers households' lifetime consumption utility, a circular economy can still be welfare improving.

To illustrate this point, we turn to the overall welfare impact of raising the refurbishing rate. We return to the baseline model and focus on the balance growth regime for this purpose. Since the lifetime utility of the households is given by $U_0 = \int_0^\infty u_t e^{-\rho t}\,\mathrm{d}t$, where $u_t = \ln y_t - \mu n_t$, the overall welfare can be written as the sum of two terms:

$$U_0 = \underbrace{\int_0^\infty \ln y_t e^{-\rho t}\,\mathrm{d}t}_{\equiv U_{y,0}} - \mu \underbrace{\int_0^\infty n_t e^{-\rho t}\,\mathrm{d}t}_{\equiv U_{n,0}} = U_{y,0} - \mu U_{n,0}. \tag{62}$$

While $U_{y,0}$ captures the discounted sum of consumption utility, $U_{n,0}$ is the discounted sum of environmental damages.

The fact that resource extraction and processing have a higher environmental impact than refurbishing can be captured by $\frac{\partial n_t}{\partial M_t} > 0$. A permanent increase of the refurbishing rate leads to the following change in environmental damage relative to BAU:

$$\mathrm{d}U_{n,0} = \int_0^\infty \mathrm{d}n_t\, e^{-\rho t}\,\mathrm{d}t = \int_0^\infty \left(\frac{\partial n_t}{\partial M_t} M_t\,\mathrm{dln}M_t\, e^{-\rho t}\right)\mathrm{d}t,$$

where $\mathrm{dln}M_t$ captures how resource extraction and thus the waste flow at any point in time compare to the BAU level. According to Lemma 2, in the balance growth regime resource extraction falls immediately after the shock to the refurbishing rate but gradually increases and eventually surpasses the BAU level. Thus when raising the refurbishing rate, $\mathrm{dln}M_t$ is first negative in the short run, but will turn positive after some time. The next lemma states the condition in order for an overall environmental benefit to occur.

**Lemma 4.** *Increasing the refurbishing rate leads to an environmental benefit (i.e. lowering $U_d$) only if the marginal damage ($\frac{\partial n_t}{\partial M_t}$) grows at a slower rate than the reduction rate of emission $M_t$.*

Intuitively, as the waste flow and the environmental damage caused by it falls in the short run but rises over time relative to BAU, an environmental benefit only occurs if the short-run reduction of the waste flow is sufficiently valuable. Since the change of waste flow relative to the BAU level is evaluated by the level of damage (as measured by $\frac{\partial n_t}{\partial M_t} M_t$) at each point in time, an overall environmental benefit requires that current damage is already relative high compared to future damages. Given that the absolute level of the waste flow $M_t$ falls over time, an environmental benefit occurs as long as the marginal damage does not rise too fast, which in turn poses restrictions on the curvature of the damage function $n(\cdot)$.

Since the magnitude of the environmental benefit, if present, depends on the relative utility weight $\mu$ and the curvature of the damage function, among other things, we now turn to specific examples and numerical exercises for the overall welfare impact.

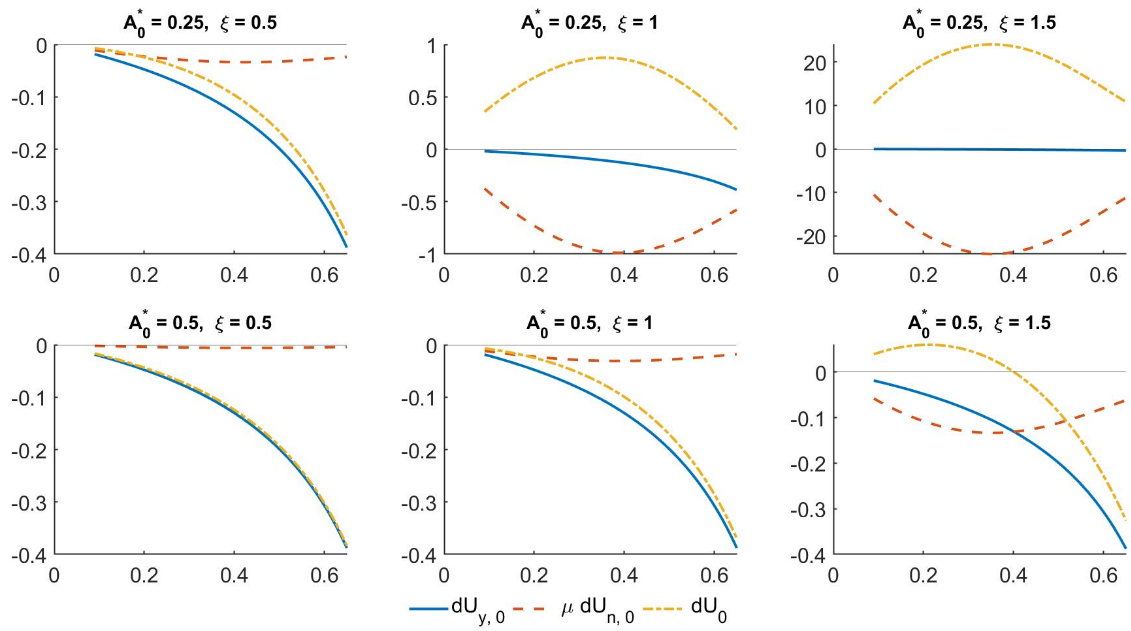
**Numerical example** Let us consider a simple flow damage function given by

$$n(M_t) = \frac{M_t^{\xi}}{\xi},$$

where $\xi$ is the elasticity of marginal damage. With this damage function, households essentially suffer a disutility from the amount of newly generated waste, for example, due to the reduction of amenity services of the natural environment caused by the waste flows. To assess the magnitude of the environmental impact, three parameters need to be determined: $(\mu, \xi, A_0^*)$. $A_0^*$ affects the size of the initial waste flow, $\xi$ captures how quickly environmental damage rises when the waste flow increases, and finally, $\mu$ is the relative utility weight of environmental damage to consumption utility. We set the utility weight of environmental damage to 0.01 following Gradus and Smulders (1993), while allowing the $A^*$ and $\xi$ to vary.

For an initial refurbishing rate ranging from 0.08 (that is, $\bar{\bar{\beta}}$) to 0.65, Figure 10 illustrate the overall welfare impact of raising the refurbishing rate by 10% for different parameter combinations. While the blue solid curves illustrate the difference of lifetime consumption utility compared to the BAU level, the red dashed curves show the changes to the environmental damage and the yellow dash-dotted curves show the overall welfare impact.

A first thing to notice is that for all combinations of the parameters considered, an environmental benefit is present, reflected by the reduction in environmental damage as compared to BAU. Further, the reduction in environmental damage seems U-shaped in the initial refurbishing rate. This is because while a higher initial refurbishing rate corresponds to a lower resource use (that is, lower $M_t$), it also means a larger reduction of the resource flow compared to the BAU level (that is, a large magnitude of $\mathrm{dln}M_t$). And finally, the reduction in environmental damage increases in magnitude with higher $\xi$ and lower $A_0^*$. A higher $\xi$ means that the marginal damage falls more quickly, as $M_t$ decreases over time. Thus the initial damage reduction compared to BAU is weighted more heavily, leading




(Horizontal axes: initial circularity)

Figure 10: Welfare effect of a 10% increase of $\beta$

to larger reduction of overall environmental damage over time. A lower $A_0^*$, on the other hand, simply scales down the environmental damage and its reduction for all time.

While an environmental benefit is present for all parameter combinations considered, the economic impact in Figure 10 is always negative. Adding up the two, the overall welfare impact is ambiguous: whether an overall welfare gain exists depends on the specific parameter combination and the initial refurbishing rate.

To summarize, this numerical exercise thus shows on the one hand that a circular economy could be welfare improving despite of bringing an economic loss, but on the other hand, also highlights the danger of failing to account for and address the potential negative economic impact.

# 7 Final remarks

The increasing interest for a circular economy in recent years is a welcome development for sustainability. Casting a circular economic transition as a no-cost, win-win solution to our environmental problems, however, can be misleading. As closing the material loop changes the structure of the economy, it can very well have unintended effect on existing green growth mechanisms. This paper demonstrates that promoting re-use and refurbishing can crowd out innovation for creating newer, more efficient products. If technical change is strongly resource-saving, crowding out innovation could be very costly and reduce or even eliminate the overall economic benefit.

Inasmuch as our result sounds the alarm, it points out potential opportunities. Firstly, questions can be raised in terms of how policies can mitigate the potential negative effect on innovation and growth. To dampen the innovation crowd-out, it should for example be investigated if certain R&D promoting policies should be introduced simultaneously with circular economic measures, or if and how existing R&D policies can be modified to accommodate the change introduced by a circular economy. Secondly, crowding out innovation in the primary sector turns out to be costly in our paper also because the secondary sector is much less innovative. While existing patent data seems to corroborate this assumption, it can nevertheless be asked if the secondary sector can be made more innovative. This is of course also a question of market structure and of the role of the public sector. To which degree the municipalities can and should be more of a market facilitator instead of a service provider in the waste management sector, for example, merits more research and discussions. This will affect how innovative the secondary sector can be and have important implications for the overall economic benefit of promoting a circular economy.

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# Appendices

## A Proofs

### A.1 Proof of Lemma 1

Combining (25) and (26) to eliminate $r$ and $\hat{w}$ gives us

$$\begin{aligned}\hat{\phi}_N &= (1-\phi_N)\left[\Psi(1-L_X)+\lambda\theta^*(1-L_X)-B(\beta)\frac{L_X}{\phi_N}\right]\\ &= (1-\phi_N)(\Psi+\lambda\theta^*)\left[1-(1+\frac{B(\beta)}{\Psi+\lambda\theta^*}\phi_N^{-1})L_X\right].\end{aligned}$$

Combining (25) and (28) to eliminate $r$ and $\hat{w}$ and plugging in (30), we have

$$\begin{aligned}\hat{L}_X &= B(\beta)\frac{L_X}{\phi_N}-\lambda\theta^*(1-L_X)+\hat{\phi}_N-\rho\\ &= [\Psi-\rho-(\Psi+\lambda\theta^*)\phi_N]-[\Psi-B(\beta)-(\Psi+\lambda\theta^*)\phi_N]\,L_X\end{aligned}$$

By definition of $E$, we have

$$\hat{E} = (\psi_M-\psi_L)\ln\gamma\lambda L_A-\frac{M}{S} = \Psi L_A-\frac{M}{S}. \tag{A.1}$$

By (16) and (18),

$$\frac{M}{S} = \frac{L_N}{E} = \frac{(1-\beta)L_X}{E}. \tag{A.2}$$

Therefore

$$\hat{E} = \Psi-\left[\Psi+(1-\beta)E^{-1}\right]L_X. \tag{A.3}$$

### A.2 Proof of Proposition 1

We first state and proof a lemma concerning the steady states.

**Lemma 5.** *There are three possible steady states.*

1. *For all $\beta\in[0,\bar{\beta})$, there exists a steady state characterized by (36).*
2. *There exist $\bar{\epsilon}>1$ such that if $\Psi>\rho$ and $\epsilon<\bar{\epsilon}$, $\bar{\bar{\beta}}>0$ and for all $\beta\in(\bar{\bar{\beta}},1)$, there exists a steady state characterized by (35).*
3. *For $\Psi\le\rho$, there exists an asymptotic steady state characterized by (37).*

*Proof of Lemma 5.*

**The Case of $\phi_N = 1$**

If $\phi_N = 1$, we have

$$\dot{\phi}_N = 0$$
$$\dot{L}_X = L_X\left((B + \lambda\theta^*)L_X - (\rho + \lambda\theta^*)\right)$$

$\dot{L}_X = 0$ if $L_X = \frac{\rho+\lambda\theta^*}{B+\lambda\theta*}$ or if $L_X = 0$. $L_X = 0$, however, violates the Ramsey rule. To see this, note that

$$p_y \propto c_N^* = (A^*)^{-1-\psi_L} w \left(1 + \frac{p_M (A^*)^{\psi_L - \psi_M}}{w}\right) = (A^*)^{-1-\psi_L} w \left(1 + \frac{1-\phi_N}{\phi_N}\right)$$

and with $\phi_N = 1$, we have $p_y \propto (A^*)^{-1-\psi_L} w$. Thus, using (19), we have

$$\hat{y} + \hat{p}_y = (1 + \psi_L)\hat{A}^* + \hat{L}_X - (1 + \psi_L)\hat{A}^* + \hat{w} = \hat{L}_X + \hat{w}.$$

Plugging in (25) and $L_X = 0$, we have

$$\hat{y} + \hat{p}_y = -\rho - \lambda\theta^* + r - \frac{\pi^*}{V^*} + \lambda\theta^* = r - \rho - \frac{\pi^*}{V^*} < r - \rho.$$

So if $\phi_N = 1$, $\dot{L}_X = 0$ is only possible if $L_X = \frac{\rho+\lambda\theta^*}{B+\lambda\theta*}$, which is only feasible if $B > \rho$, or equivalently, $\beta < \bar{\beta}$.

**The Case of $\phi_N \in (0, 1)$**

If $\phi_N \in (0, 1)$, $\dot{\phi}_N = 0$ is given by $L_X = \left[1 + \frac{B}{\Psi+\lambda\theta^*}\phi_N^{-1}\right]^{-1}$, and $\dot{L}_X = 0$ is given by $L_X = 0$ or $L_X = 1 + \frac{B-\rho}{\Psi - B - (\Psi+\lambda\theta^*)\phi_N}$. For $\dot{\phi}_N = \dot{L}_X = 0$, the only possibility is when $\phi_N = \frac{B(\Psi-\rho)}{\rho(\Psi+\lambda\theta^*)}$, which is only feasible (that is, $\phi_N \in (0, 1)$ ) if $\Psi > \rho$ and $B < \rho\frac{\Psi+\lambda\theta^*}{\Psi-\rho}$ (or equivalently, $\beta > \bar{\bar{\beta}}$).

**The Case of $\phi_N \to 0$**

If $\phi_N = 0$, we have $\dot{L}_X = L_X (\Psi - \rho - (\Psi - B)L_X)$ so that $\dot{L}_X = 0$ if $L_X = 0$ or if $L_X = \frac{\Psi-\rho}{\Psi-B}$. The case of $L_X = \frac{\Psi-\rho}{\Psi-B}$ violates the Ramsey rule. To see this, note that

$$p_y \propto c_N^* = (A^*)^{-1-\psi_M} p_M \left(1 + \frac{w}{p_M (A^*)^{\psi_L - \psi_M}}\right) = (A^*)^{-1-\psi_M} p_M \left(1 + \frac{\phi_N}{1-\phi_N}\right).$$

If $\phi_N = 0$, $p_y \propto (A^*)^{-1-\psi_M} p_M$. Therefore

$$\hat{y} + \hat{p}_y = (1 + \psi_L)\hat{A}^* + \hat{L}_X - (1 - \psi_M)\hat{A}^* + \hat{p}_M = r + \hat{L}_X - \Psi L_A.$$

If $L_X = \frac{\Psi-\rho}{\Psi-B}$,

$$\hat{y} + \hat{p}_y = r - \Psi\frac{\rho - B}{\Psi - B} > r - \rho.$$

($L_X = \frac{\Psi-\rho}{\Psi-B}$ is feasible if $\Psi > \rho > B$ or if $B > \rho > \Psi$. For either case $r - \Psi\frac{\rho-B}{\Psi-B} > r - \rho$ holds. )

Thus if $\phi_N = 0$, $\dot{L}_X = 0$ is only possible with $L_X = 0$. In this case, $\hat{L}_X = \Psi - \rho$. For this to be consistent with $\phi_N \to 0$ and $L_X \to 0$, it must be that $\Psi < \rho$.

Given the steady state $L_X$, the steady state $E$ is given by $E = \frac{(1-\beta)L_X}{\Psi L_A}$.

□

The four different regimes and their stability properties follow directly from Lemma 5 and the phase diagrams in Figures 3 and 4. The derivation of the loci for the phase diagrams can be provided upon request. We now show the additional equilibrium properties for the "vanishing production labor" regime mentioned in Proposition 1.

If $\phi_N \to 0$ and $L_X \to 0$, $\hat{L}_X \to \Psi - \rho$, which is only consistent with a declining $L_X$ if $\Psi < \rho$. In addition, for $\phi_N \to 0$ to be consistent with $L_X \to 0$, effective resource scarcity must increase, that is, $E$ decreases, even if $L_A = 1$. However, if $E \geq E^1$, then E always increases for $L_A = 1$ and we cannot have vanishing production labor. Thus the "vanishing production labor" regime can only occur when $E < E^1$ and $\Psi < \rho$.

Since the production function is Leontief, and all output is consumed, we have $y \propto (A^*)^{1+\psi_L} L_X$ and

$$\hat{y} = (1 + \psi_L)\hat{A}^* + \hat{L}_X. \tag{A.4}$$

In the vanishing production labor regime, we have $L_X \to 0$ and $\phi_N \to 0$, which, plugging into (31), gives $\hat{L}_X \to \Psi - \rho$. Thus in the long run, consumption growth rate is given by

$$\hat{y} \to (1 + \psi_L)\hat{A}^* + \Psi - \rho = (1 + \psi_M)\lambda \ln \gamma - \rho.$$

Thus consumption increases if $\psi_M > \frac{\rho}{\lambda \ln \gamma} - 1$, stays constant if $\psi_M = \frac{\rho}{\lambda \ln \gamma} - 1$, and decreases if $\psi_M < \frac{\rho}{\lambda \ln \gamma} - 1$

### A.3 Proof of Lemma 2

The effect of higher $\beta$ on the steady state innovation and growth is given by Table 1. The direction of change along the transition towards the new steady state follows from the phase diagrams. Here in this proof we focus on the immediate response to the shock.

To see the immediate response to a $\beta$ shock, we log-linearize around the steady states. Using tilde to denote the log-deviation of a variable from its steady state value, e.g. $\widetilde{E}^j \equiv \ln E - \ln E^j$, where the superscript $j$ indicates which steady state is concerned (1 for the "vanishing scarcity" and 2 for the "balanced growth" steady state), the log-linearized systems for the two steady states are given by

$$\begin{bmatrix} \dot{\widetilde{E}}^1 \\ \dot{\widetilde{L}}_X^1 \\ \dot{\widetilde{\phi}}_N^1 \end{bmatrix} = \underbrace{\begin{bmatrix} \Psi L_A^1 & -\Psi & 0 \\ 0 & \lambda\theta^* + \rho & -(\Psi + \lambda\theta^*)L_A^1 \\ 0 & 0 & \rho - \Psi L_A^1 \end{bmatrix}}_{\equiv K^1} \times \begin{bmatrix} \widetilde{E}^1 \\ \widetilde{L}_X^1 \\ \widetilde{\phi}_N^1 \end{bmatrix}. \tag{A.5}$$

and

$$\begin{bmatrix} \dot{\widetilde{E}}^2 \\ \dot{\widetilde{L}}_X^2 \\ \dot{\widetilde{\phi}}_N^2 \end{bmatrix} = \underbrace{\begin{bmatrix} \rho & -\Psi & 0 \\ 0 & \frac{L_X^2}{L_A^2}(B(\beta) - \rho) & -B(\beta)L_X^2 \\ 0 & -\frac{1-\phi_N^2}{\phi_N^2}\frac{L_X^2}{L_A^2}B(\beta) & \frac{1-\phi_N^2}{\phi_N^2}L_X^2 B(\beta) \end{bmatrix}}_{\equiv K^2} \times \begin{bmatrix} \widetilde{E}^2 \\ \widetilde{L}_X^2 \\ \widetilde{\phi}_N^2 \end{bmatrix}. \tag{A.6}$$

Based on the log-linearized systems given above, the log derivation from the new steady state at any time $t$ is given by

$$\dot{\widetilde{E}}_t^j = K_{11}^j \widetilde{E}_t^j + K_{12}^j \widetilde{L}_{X,t}^j$$

where $K_{lm}^j$ represents the element of $K^j$ in row $l$ and column $m$.

Let $v^1$ and $v^2$ be the absolute value of the negative eigenvalues (i.e. the adjustment speed) of $K^1$ and $K^2$, respectively. Along the saddlepath towards the new steady state (after the shock), we have $\widetilde{E}_t^j = \widetilde{E}_0^j e^{-v^j t} + \widetilde{E}_\infty^j (1 - e^{-v^j t})$ from which we know

$$\dot{\widetilde{E}}_t^j = -v^j \left(\widetilde{E}_0^j - \widetilde{E}_\infty^j\right) e^{-v^j t} = -v^j (\widetilde{E}_t^j - \widetilde{E}_\infty^j) = -v^j \widetilde{E}_t^j$$

where the third equality sign follows from $\widetilde{E}_\infty^j = 0$ as both steady states are saddlepath stable. Equating the two $\dot{\widetilde{E}}_t^j$ expressions, we derive the following expression for the saddlepath close to the new steady state:

$$\widetilde{L}_{X,t}^j = -\frac{v^j + K_{11}^j}{K_{12}^j} \widetilde{E}_t^j. \tag{A.7}$$

Immediately after the shock, $\widetilde{E}_0^j$ with respect to the new steady state is given by $\widetilde{E}_0^j = \ln \frac{E_0}{E^j} = \ln \frac{E^{o,j}}{E^{n,j}}$, with the superscripts "o" and "n" denoting the old and the new steady state, respectively. Similarly, $\tilde{L}_{X,0}^j = \ln \frac{L_{X,0}^j}{L_X^{n,j}} = \ln \frac{L_{X,0}^j}{L_X^{o,j}} + \ln \frac{L_X^{o,j}}{L_X^{n,j}} = \tilde{L}_{X,0}^o + \ln \frac{L_X^{o,j}}{L_X^{n,j}}$. We thus have

$$\tilde{L}_{X,0}^{o,j} = \tilde{L}_{X,0}^j - \ln \frac{L_X^{o,j}}{L_X^{n,j}} = -\frac{v^j + K_{11}^j}{K_{12}^j} \ln \frac{E^{o,j}}{E^{n,j}} - \ln \frac{L_X^{o,j}}{L_X^{n,j}}. \tag{A.8}$$

**“Balance growth” steady state:**
For the “balance growth” steady state, we have

$$\ln \frac{L_{X,0}^{2}}{L_{X}^{o,2}} = \underbrace{-\frac{v^2 + K_{11}^2}{K_{12}^2}}_{>0} \underbrace{\ln \frac{E^{o,2}}{E^{n,2}}}_{>0} - \underbrace{\ln \frac{L_X^{o,2}}{L_X^{n,2}}}_{=0} > 0. \tag{A.9}$$

Therefore, the immediate response of the economy is to allocate more labor towards production, which lowers innovation and growth. Since consumption is given by

$$y_t = (\theta^*)^{-\frac{1}{\epsilon-1}} L_{X,t} (A_t^*)^{1+\psi_L}, \tag{A.10}$$

immediately after the shock, production and consumption level goes up.

**“Diminishing scarcity” steady state:**
For the “diminishing scarcity” steady state, notice first that $v^1 = \Psi L_A^1 - \rho$.

$$\ln \frac{L_{X,0}^{1}}{L_{X}^{o,1}} = \frac{2\Psi L_A^{n,1} - \rho}{\Psi} \ln \left[\frac{(1-\beta^o)(B^n - \rho)}{(1-\beta^n)(B^o - \rho)}\right] - \ln \frac{B^n + \lambda\theta^*}{B^o + \lambda\theta^*}. \tag{A.11}$$

Let $\tilde{\beta}^o \equiv \ln \frac{\beta^n}{\beta^o}$. By Taylor expansion, we have[18]

$$\begin{aligned}
\ln \frac{B^o - \rho}{B^n - \rho} &= \frac{B^o - B^n}{B^n - \rho} = \frac{B^n}{B^n - \rho} \ln \frac{1-\beta^o}{1-\beta^n} \\
\ln \frac{B^o + \lambda\theta^*}{B^n + \lambda\theta^*} &= \frac{B^n}{B^n + \lambda\theta^*} \ln \frac{1-\beta^o}{1-\beta^n} = \frac{B^n}{B^n - \rho} L_A^{n,1} \ln \frac{1-\beta^o}{1-\beta^n} \\
\ln \frac{1-\beta^o}{1-\beta^n} &= \frac{\beta^n - \beta^o}{1-\beta^n} = \frac{\beta^o}{1-\beta^n} \tilde{\beta}^o
\end{aligned}$$

so that

$$\begin{aligned}
\tilde{L}_{X,0}^{o,1} &= \left[-\frac{2\Psi L_A^{n,1} - \rho}{\Psi} \frac{\rho}{B^n - \rho} + \frac{B^n}{B^n - \rho} L_A^{n,1}\right] \ln \frac{1-\beta^o}{1-\beta^n} \\
&= \underbrace{L_A^{n,1} \ln \frac{1-\beta^o}{1-\beta^n}}_{>0} \underbrace{\left[1 - \frac{\rho}{B^n - \rho}\left(1 - \frac{\rho}{\Psi} \frac{1}{L_A^{n,1}}\right)\right]}_{\equiv \Omega(\beta,\epsilon)},
\end{aligned}$$

which can be positive, zero, or negative depending on the size of $\beta$ and thus $B^n$ and $L_A^{n,1}$.

[18] In general, for any variable $y$, the following holds in the if $y$ is in the neighborhood of $\bar{y}$: $\ln y = \ln \bar{y} + \frac{y - \bar{y}}{\bar{y}}$.

### A.4 Proof of Proposition 2

The consumption utility at the time of shock ($t = 0$) is given by

$$U_{y,0} = \int_{t=0}^{\infty} \ln y_t e^{-\rho t} \, \mathrm{d}t = \int_{t=0}^{\infty} \left[ \ln y_0 + \int_{s=0}^{t} g_s ds \right] e^{-\rho t} \, \mathrm{d}t,$$

where $g_s$ is the growth rate of consumption at time $s$. The change in consumption utility w.r.t. the old steady state (i.e. BAU) due to a marginal increase of $\beta$ is given by[19]

$$\mathrm{d}U_{y,0} = \int_{t=0}^{\infty} \left[ \mathrm{dln} y_0 + \int_{s=0}^{t} \mathrm{d}g_s \, \mathrm{d}s \right] e^{-\rho t} \, \mathrm{d}t = \int_{t=0}^{\infty} \left[ \tilde{y}_0^o + \int_{s=0}^{t} \mathrm{d}g_s \, \mathrm{d}s \right] e^{-\rho t} \, \mathrm{d}t,$$

where $\tilde{y}_0^o \approx \frac{dy_0}{y_0}$ denotes the log-deviation w.r.t. the old steady state (BAU). Immediately after the shock, the economy jumps to the saddle-path towards the new steady state. Log-linearizing around the new steady state gives

$$g_s - g^n \approx g^n \tilde{g}_s = g^n \left[ \tilde{g}_0 e^{-\nu s} + \tilde{g}_\infty (1 - e^{-\nu s}) \right] = g^n \tilde{g}_0 e^{-\nu s},$$

where $\nu$ the magnitude of the adjustment speed towards the new steady state and the last equality follows from the fact that $\tilde{g}_\infty = 0$ holds along the saddle path. Since $dg_s = (g_s - g^n) + (g^n - g^o)$, we thus have

$$\mathrm{dln} y_t = \tilde{y}_0^o + \int_{s=0}^{t} \left[ g^n \tilde{g}_0 e^{-\nu s} + (g^n - g^o) \right] ds = \tilde{y}_0^o + g^n \tilde{g}_0 \frac{1 - e^{-\nu t}}{\nu} + (g^n - g^o) t, \tag{A.12}$$

and consequently

$$\mathrm{d}U_{y,0} = \int_{t=0}^{\infty} \left[ \tilde{y}_0^o + g^n \tilde{g}_0 \frac{1 - e^{-\nu t}}{\nu} + (g^n - g^o) t \right] e^{-\rho t} \, \mathrm{d}t = \frac{\tilde{y}_0^o}{\rho} + \frac{g^n \tilde{g}_0}{\rho(\rho + \nu)} + \frac{g^n - g^o}{\rho^2}. \tag{A.13}$$

From (19), we have

$$\tilde{y}_0^o = \widetilde{L}_{X,0}^o = \widetilde{L}_{X,0} - \ln \frac{L_X^o}{L_X^n}. \tag{A.14}$$

and

$$g_0 = \hat{L}_{X,0} + (1 + \psi_L) \hat{A}_0^* = \dot{\hat{L}}_{X,t} + (1 + \psi_L) \lambda \ln \gamma L_{A,0}.$$

[19] Alternatively, we can derive the same result by using:

$$\mathrm{d}U_{y,0} = \int_{0}^{\infty} \left[ \mathrm{dln} L_{X,t} + (1 + \psi_L) \mathrm{dln} A_t \right] \, e^{-\rho t} \, \mathrm{d}t = \int_{0}^{\infty} \left[ \widetilde{L}_{X,t}^o + (1 + \psi_L) \int_{s=0}^{t} \mathrm{d}g_{A,s} \mathrm{d}s \right] \, e^{-\rho t} \, \mathrm{d}t.$$

Since along the saddle path to the new steady state, $\tilde{L}_{X,t} = \tilde{L}_{X,0}e^{-\nu t}$ and thus $\dot{\tilde{L}}_{X,t} = -\nu\tilde{L}_{X,t}$ holds, we have

$$\tilde{g}_0 \approx \frac{g_0 - g^n}{g^n} = \frac{-\nu\tilde{L}_{X,t} + (1+\psi_L)\lambda\ln\gamma L_A^n \tilde{L}_{A,0}}{(1+\psi_L)\lambda\ln\gamma L_A^n} = -\left(\frac{\nu}{g^n} + \frac{L_X^n}{1-L_X^n}\right)\tilde{L}_{X,0}. \tag{A.15}$$

Finally,

$$g^n - g^o = (1+\psi_L)\lambda\ln\gamma(L_A^n - L_A^o) = (1+\psi_L)\lambda\ln\gamma L_X^n \ln\frac{L_X^o}{L_X^n}. \tag{A.16}$$

Using (A.14), (A.15), (A.16) together with (A.7) and (A.8), $\mathrm{d}U_{y,0}$ can be written as

$$\begin{aligned} dU_{y0} &= \frac{1}{\rho}\left(1 - \frac{(1+\psi_L)\lambda\ln\gamma}{\rho}L_X^n\right)\left(\frac{\rho}{\rho+\nu}\tilde{L}_{X,0} - \ln\frac{L_X^o}{L_X^n}\right) \\ &= \begin{cases} \frac{1}{\rho}\ln\frac{1-\beta^o}{1-\beta^n}F_1(\beta), & \text{if } 0 \leq \beta \leq \bar{\bar{\beta}} \\ \frac{1}{\rho}\ln\frac{1-\beta^o}{1-\beta^n}F_2(\beta), & \text{if } \bar{\bar{\beta}} < \beta < 1 \end{cases} \end{aligned} \tag{A.17}$$

where

$$F_1(\beta) \equiv \frac{\rho}{\Psi}\left(1 - \frac{(1+\psi_L)\lambda\ln\gamma}{\rho}L_X^n\right)\left[-2 + \frac{\rho}{\Psi L_A^n} + \frac{B^n}{\rho}\frac{\Psi L_A^n}{\rho}\right]\frac{\rho}{B^n - \rho} \tag{A.18}$$

$$F_2(\beta) \equiv \frac{\rho}{\Psi}\left(1 - \frac{(1+\psi_L)\lambda\ln\gamma}{\rho}L_X^n\right) \tag{A.19}$$

Notice that $\frac{1}{\rho}\ln\frac{1-\beta^o}{1-\beta^n} > 0$. Further, for $F_1(\beta)$ we have $\left[-2 + \frac{\rho}{\Psi L_A^n} + \frac{B^n}{\rho}\frac{\Psi L_A^n}{\rho}\right]\frac{\rho}{B^n-\rho} \geq 1$, with the equality sign holds when $\beta = \bar{\bar{\beta}}$, since $\frac{\partial\left[-2+\frac{\rho}{\Psi L_A^n}+\frac{B^n}{\rho}\frac{\Psi L_A^n}{\rho}-\frac{B^n-\rho}{\rho}\right]}{\partial\beta} \leq 0$ if $\beta \leq \bar{\bar{\beta}}$. Thus $\lim_{\beta\to\bar{\bar{\beta}}} F_1(\beta) = F_2(\beta)$.

To determine the sign of $dW_0$ we can thus distinguish between the following two cases.

1. $\psi_M - \psi_L < \frac{(1+\psi_L)\rho}{(1+\psi_L)\lambda\ln\gamma - \rho}$
   In this case, $1 - \frac{(1+\psi_L)\lambda\ln\gamma}{\rho}L_X^{n,2} > 0$. And since $L_X^{n,1} \leq L_X^{n,2}$, $1 - \frac{(1+\psi_L)\lambda\ln\gamma}{\rho}L_X^n > 0$ for all $\beta$. Consequently, $dW_0 > 0$ for all $\beta$.

2. $\psi_M - \psi_L \geq \frac{(1+\psi_L)\rho}{(1+\psi_L)\lambda\ln\gamma - \rho}$
   $F_2(\beta) \leq 0$ holds always, but $F_1(\beta) > 0$ possible if $1 - \frac{(1+\psi_L)\lambda\ln\gamma}{\rho}L_X^{n,1} > 0$. Since $\frac{\partial L_X^{n,1}}{\partial\beta} > 0$ and $\frac{\partial L_X^{n,1}}{\partial\epsilon} > 0$, there generally exists a unique $\beta^* \in [0, \bar{\bar{\beta}}]$, where $\frac{\partial\beta^*}{\partial\epsilon} < 0$, such that $F_1(\beta) > 0$ if $\beta \in [0, \beta^*)$ and $F_1(\beta) \leq 0$ if $\beta \in [\beta^*, 1)$.

## A.5 Proof of Proposition 3

The phase diagrams are the same as in Figure 4 with $B(\beta^C)$ replacing $B$ of the baseline. From (39)-(41), it is clear that in the balance growth regime an increase in $\beta^I$ does not affect the steady state $\phi_N$ or $L_X$, but lowers steady state $E$ (see also (36) and (35)). Thus along the transition towards the new steady state, both $L_X$ and $E$ fall. The immediate response of labor allocation after the shock is given by (A.8). Since steady state labor allocation is not affected by the shock but steady state $E$ falls, it follows from (A.8) that immediately after the shock, $L_X$ jumps up.

The overall welfare impact again follows from (A.19). Thus $dW0 > 0$ if $\psi_M - \psi_L < \frac{(1+\psi_L)\rho}{(1+\psi_L)\lambda \ln \gamma - \rho}$, and $dW0 \leq 0$ if $\psi_M - \psi_L \geq \frac{(1+\psi_L)\rho}{(1+\psi_L)\lambda \ln \gamma - \rho}$.

## A.6 Proof of Lemma 3

(I.) A steady state with $L_X \in (0,1), \phi_N \in (0,1)$ and hence $\hat{L}_X = \hat{\phi}_N = 0$ requires $L_X = 1 - \rho/\Psi$ and $B(\phi_N)/\phi_N = \bar{\bar{B}}$ for some $\phi_N^I \in (0,1)$. The solution exists and is saddle-point stable if $B/\phi_N$ cuts $\bar{\bar{B}}$ from above for $\phi_N = \phi_N^I$, which requires $d(B(\phi_N)/\phi_N)/d\phi_N < 0$ evaluated at $\phi_N^I$. The solution is consistent with $L_X > 0$ only if $\Psi > \rho$.

(II.) A steady state with $L_X \in (0,1), \phi_N = 1$ and hence $\hat{L}_X = \hat{\phi}_N = 0$ requires $L_X = (\rho + \lambda\theta^*)/(B^1 + \lambda\theta^*)$. It is saddle-point stable if the $\dot{\phi}_N = 0$ locus is above the $\dot{L}_X = 0$ locus at $\phi_N = 1$, which requires $B^1 \geq \bar{\bar{B}}$.

(IIIa.) A steady state with $L_X \in (0,1), \phi_N \to 0$ is consistent with (51) and (49) if $r - \hat{w} = \rho = B^0 L_X - \lambda\theta^*(1 - L_X)$ and hence $L_X = (\rho + \lambda\theta^*)/(B^0 + \lambda\theta^*)$. To be consistent with $L_X < 1$, we need $B^0 > \rho$. Next, from (51) and (50) we have $\hat{\phi}_N = \Psi(1 - L_X) - \rho$ which needs to be negative to be consistent with $\phi_N \to 0$. Substituting the solution for $L_X$ we find that $\hat{\phi}_N < 0$ requires $B^0 < \bar{\bar{B}}$. Combining the two conditions we find that this steady state exists if $\rho < B^0 < \bar{\bar{B}}$.

From (48), $\epsilon_s = 0$, $\hat{L}_X = 0$, and $\hat{p}_y = \hat{p}_M - (1 + \psi_M)\hat{A}^*$, consumption growth is $\hat{y} = \hat{w} - \hat{p}_M + (1 + \psi_M)\hat{A}^*$. Substituting $r - \hat{w} = \rho$ and $\hat{A}^* = \lambda \ln \gamma L_A$ we get the condition for positive consumption growth.

Employment in primary production and its growth rate are $L_N = nL_X$ and $\hat{L}_N = \epsilon_n[\Psi L_A - \rho]$, respectively, where $\epsilon_n \equiv n'\phi_N/n > 0$ (with $\lim_{\phi_N \to 0} \epsilon_n = 1$) and the term in brackets equals $\hat{\phi}_N < 0$. Hence $\hat{L}_N = \Psi L_A - \rho < 0$. Employment in refurbishing equals $L_R = L_X - L_N$ where $L_X$ is constant and $L_N$ falls to zero. Hence $L_R \to L_X$.

From (46) and (45) we find $n(0) = 0$, so that (54) and $\dot{E} = 0$ require $E = n(0)L_X/\Psi(1 - L_X) = 0$.

(IIIb.) A steady state with $L_X = 1, \phi_N \to 0$ is consistent with (51) and (50) if $r - \hat{w} = \rho = -\hat{\phi}_N$ and with (49) and (51) if $r - \hat{w} = \rho \geq B^0$. Hence this steady state exists if $B^0 \leq \rho$. From (54) and $L_X = 1$, we find $\hat{E} < 0$ so that $E$ converges to zero.

(IV.) A steady state with $L_X = 0, \phi_N = 0$ implies from (49) and (50) that $r - \hat{w} = -\lambda\theta^* = \Psi - \hat{\phi}_N \Leftrightarrow \hat{\phi}_N = \Psi + \lambda\theta^* > 0$ which contradicts $\phi_N \to 0$. Hence, this steady state cannot arise as equilibrium.

## A.7 Proof of Proposition 4

Combining (55) with (47) and (45), we see that $\frac{\partial \phi_N^{ss}}{\partial \tau} < 0$. Consequently, a higher refurbishing subsidy leads to a higher $\beta^{ss}$, a lower $n^{ss}$, and a lower $E^{ss}$.

To derive the transition path towards the new steady state, we now log-linearize around the steady state, which gives us

$$\begin{bmatrix} \dot{\tilde{E}} \\ \dot{\tilde{L}}_X \\ \dot{\tilde{\phi}}_N \end{bmatrix} = \underbrace{\begin{bmatrix} \rho & -\Psi & -\rho\epsilon_n^{ss} \\ 0 & \left[(1-(1-\phi_N^{ss})\epsilon_s^{ss})\,\omega^{ss} - \rho\right]\frac{L_X^{ss}}{L_A^{ss}} & (1-(1-\phi_N^{ss})\epsilon_s^{ss})\,\omega^{ss} L_X^{ss}\epsilon_\omega^{ss} \\ 0 & -\frac{1-\phi_N^{ss}}{\phi_N^{ss}}\frac{L_X^{ss}}{L_A^{ss}}B(\phi_N^{ss}) & -\frac{1-\phi_N^{ss}}{\phi_N^{ss}}L_X^{ss}B(\phi_N^{ss})\epsilon_\omega^{ss} \end{bmatrix}}_{\equiv K} \times \begin{bmatrix} \tilde{E} \\ \tilde{L}_X \\ \tilde{\phi}_N \end{bmatrix}. \tag{A.20}$$

where $\omega \equiv \frac{B(\phi_N)}{\phi_N}$, $\epsilon_\omega \equiv \frac{\phi_N}{\omega}\frac{\partial \omega}{\partial \phi_N} < 0$, $\epsilon_n \equiv \frac{\phi_N}{n}\frac{\partial n}{\partial \phi_N} > 0$, and $\epsilon_s \equiv \frac{\phi_N}{s}\frac{\partial s}{\partial \phi_N}$.

Since $\rho$ is obviously an eigenvalue of $K$ and since

$$\det K = \rho\,(K_{22}K_{33} - K_{23}K_{32}) = \rho^2 \frac{1-\phi_N^{ss}}{\phi_N^{ss}} \left(\frac{L_X^{ss}}{L_A^{ss}}\right)^2 B(\phi_N^{ss})\epsilon_\omega^{ss},$$

as long as $\frac{\partial \omega}{\partial \phi_N} < 0$ near the steady state, $\det K < 0$ and the steady state is saddle-path stable. Again, use $\nu$ to denote the magnitude of the negative eigenvalue, we have

$$\begin{aligned} \tilde{E}_t &= \tilde{E}_0 e^{-\nu t} \\ \tilde{L}_{X,t} &= \frac{\nu + K_{11}}{-K_{12} + \frac{K_{13}K_{32}}{K_{33}+\nu}}\tilde{E}_t \\ \tilde{\phi}_{N,t} &= -\frac{K_{32}}{K_{33}+\nu}\tilde{L}_{X,t}. \end{aligned}$$

Since the new steady state has a lower $E^{ss}$, $\tilde{E}_0 > 0$ holds and thus also $\tilde{E}_t > 0$, $\tilde{L}_{X,t} > 0$ and $\tilde{\phi}_{N,t} > 0$. Immediately after the shock, $L_X$ jumps up, while the $\beta$ response is ambiguous.

## A.8 Proof of Lemma 4

As in (A.13), we have

$$\mathrm{dln}M_t = \tilde{M}_0^o + g_M^n \tilde{g}_{M,0} \frac{1-e^{-\nu t}}{\nu} + (g_M^n - g_M^o)t.$$

From (16), $M_t = (1-\beta)L_{X,t}(A_t^*)^{-(\psi_M - \psi_L)}$ holds and thus for the balanced growth regime,

$$\begin{aligned}
\tilde{M}_0^o &= \tilde{L}_{X,0}^o + \mathrm{dln}(1-\beta) = \tilde{L}_{X,0} + \mathrm{dln}(1-\beta) \\
g_{M,t} &= \hat{L}_{X,t} - (\psi_M - \psi_L)\hat{A}^* = -\nu \tilde{L}_{X,0} - \Psi L_{A,t} \\
\tilde{g}_{M,0} &= \frac{g_{M,0} - g_M^n}{g_M^n} = \frac{-\nu \tilde{L}_{X,0} - \Psi L_A^n \tilde{L}_{A,0}}{-\Psi L_A^n} = \left(\frac{\nu}{\Psi L_A^n} - \frac{L_X^n}{L_A^n}\right)\tilde{L}_{X,0}.
\end{aligned}$$

Thus for the balanced growth regime, $\mathrm{dln}M_t$ is given by

$$\mathrm{dln}M_t = \underbrace{\ln\frac{1-\beta^o}{1-\beta^n} L_A^n \left(\frac{\Psi L_X^n}{\nu} - 1\right)}_{>0} \left[1 - \frac{\nu+\rho}{\rho} e^{-\nu t}\right]. \tag{A.21}$$

Clearly, $\mathrm{dln}M_0$ is increasing in $t$, and $\mathrm{dln}M_0 < 0$ and $\lim_{t\to\infty} \mathrm{dln}M_t > 0$ hold. There thus also exists a unique $t_0 > 0$ such that $\mathrm{dln}M_0 < 0$ for all $t < t_0$ and $\mathrm{dln}M_0 \geq 0$ for all $t \geq t_0$. It is further easily verified that $\int_{t=0}^{\infty} \mathrm{dln}M_t e^{-\rho t}\,\mathrm{d}t = \int_{t=0}^{\infty}\left(1 - \frac{\nu+\rho}{\rho}e^{-\nu t}\right)e^{-\rho t}\,\mathrm{d}t = 0$.

Since $\mathrm{d}U_{n,0} = \int_0^\infty \left(\frac{\partial n_t}{\partial M_t} M_t \,\mathrm{dln}M_t\, e^{-\rho t}\right)\,\mathrm{d}t$, whether or not an environmental benefit is present depends on how $\mathrm{dln}M_t$ is weighted over time. Suppose $\frac{\partial n_t}{\partial M_t}M_t$ is increasing or constant over time. Then

$$\begin{cases} \frac{\partial n_t}{\partial M_t}M_t \leq \frac{\partial n_{t0}}{\partial M_{t0}}M_{t0}, & \text{if } t \leq t_0 \\ \frac{\partial n_t}{\partial M_t}M_t \geq \frac{\partial n_{t0}}{\partial M_{t0}}M_{t0}, & \text{if } t \geq t_0 \end{cases}$$

Consequently,

$$\mathrm{d}U_{n,0} \propto \int_{t=0}^{\infty}\left(\frac{\partial n_t}{\partial M_t}M_t \mathrm{dln}M_t\right)e^{-\rho t}\,\mathrm{d}t \geq \frac{\partial n_{t0}}{\partial M_{t0}}M_{t0}\int_{t=0}^{\infty}\mathrm{dln}M_t e^{-\rho t}\,\mathrm{d}t = 0.$$

Thus for an environmental benefit to be present ($\mathrm{d}U_{n,0}$), the marginal damage must rise at a slower pace than $\hat{M}_t$.