Policies for electrification of the car fleet in the short and long run – subsidizing purchases or subsidizing long-lived investments in infrastructure?

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Abstract

Our starting point is a country with emission targets for both the short and long run. Abatement can be performed by measures that have an impact on present emissions, but no lasting effect, and by long-lived infrastructure investments with minor effect on present emissions, but with an effect over a long time. We study the optimal combination of short and long-lived options for reducing emissions, by specifying abatement cost functions depending on abatement from these two options. Electrification of the transport sector is used as an example. A transition from internal combustion engines vehicles (ICEVs) to electric vehicles (EVs) can be incentivized by both subsidies on purchases of EVs and increased density of fast chargers. Investments in the grid and fast charging stations have longer lifetime than an EV. Hence, subsidizing the purchase of EVs only leads to emissions reductions in the “short” run (static option), whereas investment in infrastructure also will reduce mitigation costs in the years to come (dynamic option). We find that the present marginal mitigation cost of the dynamic alternative exceeds the costs of static mitigation in optimum. The emergence of an emission permit market and flexibility in emission reductions across time periods affects the optimal combinations of domestic static and dynamic mitigation options. A higher expected abatement cost in later periods makes it more profitable to use the dynamic policy instrument. This framework is used for a numerical study on electrification of the transport sector in Norway.

Keywords: Emission permit market; infrastructure investments; electric vehicles.

JEL classifications: C63, H21, Q54, R42.

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1 Introduction

The Paris agreement that was signed in 2015, has as its aim to reduce emissions of greenhouse gases so that the increase in the average world temperature does not exceed two degrees Celsius. This is implemented in many countries and regions as short term and long-term goals for emissions and/or energy efficiency. For instance, the EU aims to reduce greenhouse gas emissions by at least 40% below 1990 levels by 2030, while for 2050, the aim is to become carbon neutral.4

The transition to a low carbon society requires investments in new clean production capital. Some of the abatement investments may have long-lasting effect on emissions and abatement costs, others may have significant effect on present emissions, but last for a shorter time period. Examples of long-lasting investments are electrification of offshore oil platforms, building or extending a public transport system, and building infrastructure for charging stations for electric vehicles. Infrastructure investments may have low impact on immediate emissions so that emissions reductions are initially costly. However, once the investments are made, future mitigation may become cheaper. For instance, present investment in charging stations for electric vehicles may have low impact on present emissions, but may be vital for the cost of a decarbonized transport sector in the future (Sierzchula, 2014). Postponing investment in infrastructure may lead to high demand for infrastructure over a short time period in the future. With positive adjustment costs (convex per period investment costs), postponing investment may become a costly policy. As argued by Vogt-Schilb et.al (2018), starting with expensive options may make sense when reducing emissions requires investments in long-lived goods, which takes time to deploy. Policy options that work in a similar way is investing in solar panels or battery production that may reduce costs in the future due to learning by doing (Wigley et al., 1996; Kverndokk and Rosendahl, 2007).

In this paper, we study the optimal combination of short and long-lived investment options for reducing emissions through an electrification of the transport sector. In economics, abatement costs are usually specified as functions of the level of abatement, for instance to find the marginal abatement cost curves, see, e.g., McKitrik (1999). One contribution of this paper is to specify abatement costs as a function of abatement from static and dynamic

abatement separately. In this way, we can study the marginal abatement costs from the two options.

In our example, replacing internal combustion engines vehicles (ICEVs) with electric vehicles (EVs) is the static option, whereas long-lived investment in grids and fast charging stations is the dynamic option, as the charging infrastructure has a longer lifetime than cars. A third option for meeting abatement obligations is international trade in emissions reduction (permit trade). Permit trading has been implemented in several regions in the world. EU has for instance its Emissions Trading System – ETS, where companies in the countries that are part of EU ETS, decide whether they want to buy permits allowing them to emit a certain amount of greenhouse gases or to reduce their emissions domestically. EU ETS does not cover emissions from the transport sector. However, EU’s Effort Sharing Regulations opens for emission trading also for non-ETS sectors.\(^5\) An emissions trading regime can also be designed to allow for various degrees of intertemporal flexibility in abatement.\(^6\)

In general, for most countries, it will not be cost effective to meet the emission targets by permit purchases only, even in the presence of a well-functioning permit market. It is always cost effective to implement domestic abatement options with lower cost than the present permit price. Furthermore, a nation can also set restrictions on how many permits their companies are allowed to buy, so that a certain share of emission reductions has to be taken at home.\(^7\)

Based on the discussion above, a nation or a large company then faces the following options to meet its climate goals; mitigate emissions by short-lived options, buy emissions permits in the international market, or make infrastructure investments that also makes it cheaper to meet future emissions targets. These choices are studied in this paper, where we focus on how the choices depend on different assumptions about the flexibility in emission targets over time and flexibility in the emissions across countries through permit markets.

We first build an analytical model to analyze the implications of the emission targets and permit marked on the profitability of investments in long-lived infrastructure compared to


\(^6\) For instance, in EU-ETS, banking permits to be used in a future time period is allowed, whereas borrowing from future commitments is not allowed

\(^7\) One example of this is Norway’s political agreement on climate policy from 2008 that specifies that two-thirds of emissions reductions up to 2020 should be taken nationally when reforestation is included.
other short-term options. Infrastructure investments can be seen as a dynamic abatement option as they will affect the abatement costs in future periods, while the short-term options are static, as they will have no impact on future abatement. An abatement cost function is specified as function of the levels following from the two abatement options respectively. To simplify the presentation, we stick to a two-period analytical model. We find that restrictions on permit trading affects the profitability for infrastructure investments. Without any restrictions on banking and borrowing, the permit price in the two periods are the same (in present value), and a higher price gives an incentive for more abatement of both types. However, with restrictions, the prices are no longer equal. A higher price in the first period will give a larger incentive for static abatement, while dynamic abatement will be more stimulated by a higher future permit price.

We next present a numerical model for the cost of Norwegian abatement policies, where we use electrification of the car fleet in Norway as an example. Norway is the world leader in supporting the transition from ICEVs to EVs. In 2019, more than 40% of the new private cars in Norway were EVs\(^8\), and this share is expected to increase in the coming years. The electrification of the car fleet is encouraged by the government through several types of subsidies and privileges (exemption from VAT, free parking, free charging, exemption from tolls, right to use the bus lanes). Furthermore, investments in fast charging stations outside the big cities are subsidized. It has been argued that electrification of the transport sector is an expensive way of reducing current greenhouse gas (GHG) emissions (van Vliet et al. 2011; Bjertnæs, 2013; Holtmark and Skontoft, 2014). On the other hand, it takes time to build a charging infrastructure and replace the car fleet (in Norway, the average lifetime of an ICE car is 18 years\(^9\)). Thus, large investments in both EVs and EV-infrastructure today may pay off in the future if emission targets become tighter (Thiel et al., 2010). However, we distinguish between the lifetime of the two types of EV support, where investments in infrastructure (distribution grid) for fast charges have a significantly longer life time than an EV, NVE (2019). There is a large literature pointing to the significance of charging infrastructure for EV adoption (Sierzchula et al., 2014; Zang et al., 2016; Mersky et al., 2016; Yu et al., 2016; Li et al., 2017; Figenbaum, 2018).


Norway is a part of EU-ETS and does cooperate with the EU on emissions reductions in the non-ETS sectors that also includes transport. At present, there are emissions targets for these sectors for 2030 (annual emission allocations-AEAs), and according to the Effort Sharing Regulations, Member States can buy and sell allocations from and to other Member States.\footnote{https://ec.europa.eu/clima/policies/effort/regulation_en (accessed 27.1.2020).} Hence, a permit market for AEAs may emerge in the future. Such market will have implications for policies to reduce domestic emissions. Based on the literature on future permit prices for the non-ETS sectors, we discuss under what conditions the present EV policy in Norway can be considered as a cost-effective climate policy, and discuss the optimal combination of EV purchase subsidies, public support of investments in fast charging stations and trade in emission permits.

The numerical simulations show that, in the absence of permit trading, optimal abatement policy is a combination of both static and dynamic abatement. The direct subsidies to car owners are responsible for the largest emissions reductions. Flexibility in the domestic target over time affects the combination of static and dynamic abatement due to discounting. The more flexibility, the lower is the dynamic abatement relative to the static abatement in the first period. Further, domestic abatement is more expensive than buying emissions permits, meaning that no domestic abatement is performed under international permit trading. This also indicates that the Norwegian EV policy is a very costly way of reducing CO$_2$ emissions with a price of more than 400 Euro per ton.

2 A two-period mitigation model

Assume that a country has signed an agreement that puts restrictions on its greenhouse gas emissions in period 1 (e.g., 2030) and period 2 (e.g., 2050). Abatement follows from replacing fossil-based consumption by consumption derived from a non-emitting energy sources (e.g., by replacing fossil fueled cars with EVs). The replacement of fossil-based consumption is induced through two types of measures; direct support to purchase and use of the non-emitting consumption capital (e.g., by subsidizing use and purchase of EVs), $a_f$, and long-lived investments in infrastructure which reduces the user cost of the non-emitting consumption capital (e.g., by investing in charging infrastructure, pipelines for storing...
captured carbon, wind mills etc.), \( a_e \). Investments in long lasting infrastructure in period 1 affect the abatement costs also in the second period. Thus, total abatement level at a certain time \( t \), \( a_t \), is the sum of abatement due to the static policy instrument, \( a_{sf} \), and the dynamic policy instrument, \( a_{se} \).

As is standard in economics, we express the cost of abatement as a function of the total level of abatement. However, based on the discussion above, we can specify the abatement function as a function of static and dynamic abatement separately. Thus, the cost of abatement in period 1 and 2 are given by

\[
C_1 = C_1(a_{sf}, a_{se})
\]

\[
C_2 = C_2(a_{sf}, a_{se}, a_{ie})
\]

where the per period cost of abatement is increasing in both types of emission reductions within each period, but abatement following from investment in infrastructure in period 1 decreases the abatement cost in period 2:

\[
\frac{\partial C_1(a_{sf}, a_{se})}{\partial a_{sf}} \equiv C'_{1sf} > 0, \quad \frac{\partial C_1(a_{sf}, a_{se})}{\partial a_{se}} \equiv C'_{1se} > 0,
\]

\[
\frac{\partial C_2(a_{sf}, a_{se}, a_{ie})}{\partial a_{sf}} \equiv C'_{2sf} > 0, \quad \frac{\partial C_2(a_{sf}, a_{se}, a_{ie})}{\partial a_{se}} \equiv C'_{2se} > 0, \quad \frac{\partial C_2(a_{sf}, a_{se}, a_{ie})}{\partial a_{ie}} \equiv C'_{2ie} < 0.
\]

Abatement costs are convex in \( a_f \) and \( a_e \) respectively:

\[
\frac{\partial^2 C_1(a_{sf}, a_{se})}{\partial a_{sf}^2} \equiv C''_{1sf} > 0, \quad \frac{\partial^2 C_1(a_{sf}, a_{se})}{\partial a_{se}^2} \equiv C''_{1se} > 0,
\]

\[
\frac{\partial^2 C_2(a_{sf}, a_{se}, a_{ie})}{\partial a_{sf}^2} \equiv C''_{2sf} > 0, \quad \frac{\partial^2 C_2(a_{sf}, a_{se}, a_{ie})}{\partial a_{se}^2} \equiv C''_{2se} > 0, \quad \frac{\partial^2 C_2(a_{sf}, a_{se}, a_{ie})}{\partial a_{ie}^2} \equiv C''_{2ie} > 0.
\]

In addition, we take into account the limited ability for an economy to switch from low to high level of infrastructure over a short time period, by assuming convex investments cost, see Vogt-Schilb et.al (2018). In our model setup, this means that the cost of reducing
emissions through increased stock of infrastructure in period 2 is lower the higher abatement following from infrastructure in period 1, i.e., $C^*_2 < 0$.

Furthermore, within each periods, increasing abatement of one type decreases the marginal cost of the other (complements):

$$\frac{\partial^2 C_2(a_{1f}, a_{2e}, a_{ie})}{\partial a_{2e} \partial a_{ie}} \equiv C^*_2 < 0,$$

$$\frac{\partial^2 C_1(a_{1f}, a_{ie})}{\partial a_{ie} \partial a_{ie}} \equiv C^*_{1ie} < 0, \quad \frac{\partial^2 C_2(a_{1f}, a_{2e}, a_{ie})}{\partial a_{1f} \partial a_{2e}} \equiv C^*_{21e} < 0.$$

### 2.1 No Permit trade

In the absence of permit trading, and with no intertemporal flexibility the country must ensure that the sum of abatement in each period equals the sum of abatement obligations in each period, $A_t$:

$$a_{1f} + a_{ie} = A_t, \quad t = 1, 2$$

This leads to the following optimizing problem where we have omitted discounting for simplicity:

$$\text{Min} \quad TC(a_{1f}, a_{ie}, a_{1f}, a_{2e}) = C_1(a_{1f}, a_{ie}) + C_2(a_{1f}, a_{2e}, a_{ie})$$

$$\lambda_t(A_t - a_{1f} - a_{ie}) + \lambda_2(A_t - a_{2f} - a_{2e})$$

where $\lambda_t$ is the shadow cost of the emission constraint in period $t$. This gives the following first order conditions:

$$C'_{1f} = C'_{1e} + C'_{2e} = \lambda_1$$

$$C'_{2f} = C'_{2e} = \lambda_2$$

In the case of full intertemporal flexibility, abatement over both periods equals the sum of targets over both periods, where $A_1 + A_2 = \bar{A}$

The abatement constraint is thus given by:

$$a_{1f} + a_{ie} + a_{2f} + a_{2e} = \bar{A}$$

The first order conditions are:

$$C'_{1f} = C'_{1e} + C'_{2e} = C'_{2f} = C'_{2e} = \lambda_3$$
where $\lambda$ is the shadow cost of the emission constraint given by (9).

2.2 Permit trade

As part of the agreement, there is an international market for emissions permit trade, where $p_t$ denote the exogenous permit price in period $t$. For compliance, the country must ensure that abatement in each period plus permit purchase ($d_t$) equals the initial abatement obligation for each period $A_t$, where the abatement obligations are based on a business-as-usual (BAU) scenario.\(^{11}\) Note that this put restrictions on both banking and borrowing:

\[(11) \quad d_t = A_t - a_{te} - a_{ui} \quad t = 1, 2\]

In addition, the country may set restrictions on maximum permit purchase within each period, denoted $Q_t$:

\[(12) \quad A_t - a_{te} - a_{ui} \leq Q_t \quad t = 1, 2\]

If $Q_t \leq 0$, all abatement must be taken at home,\(^{12}\) while for $Q_t > 0$, a certain amount of abatement has to be taken at home.

Full flexibility means that both borrowing ($a_{ef} + a_{te} + d_1 < A_1$) and banking ($a_{ef} + a_{te} + d_1 > A_1$) of permits from the first period to the second are allowed. This is specified as:

\[(13) \quad A_1 + A_2 = \bar{A}\]

This gives:

\[(14) \quad a_{1f} + a_{1e} + a_{2f} + a_{2e} + d_1 + d_2 = \bar{A}\]

We first study the case with restrictions in the permit market as given by equation (11). This leads to the following optimizing problem:

\[\ldots\]

\[^{11}\text{E.g., emissions reductions in BAU to reach the targets.}\]

\[^{12}\text{If } Q_t < 0 \text{ means that the country has set its own targets that are stricter than the international agreement.}\]
where $\lambda^p_t$ is the shadow cost of the permit purchase constraint in period $t$. This gives the following first order conditions:

$$C'_{1f} = C'_{2s} = p_1 + \lambda^p_t$$
$$C'_{2f} = p_2 + \lambda^p_2$$

(16)

where

$$\lambda^p_1 \geq 0 \quad (= 0 \text{ for } A_1 - a_{1f} - a_{1e} < Q_1)$$
$$\lambda^p_2 \geq 0 \quad (= 0 \text{ for } A_2 - a_{2f} - a_{2e} < Q_2)$$

In contrast, when there is full flexibility in the permit market (see Appendix A), the permit prices must be equal in the two periods, i.e., $p_1 = p_2 = p$, so that the first order conditions become:

$$C'_{1f} = C'_{1s} + C'_{2s} = p_1 + \lambda^p_1$$
$$C'_{2f} = p_2 + \lambda^p_2$$

(17)

Independent of flexibility, we see that the total marginal cost of emissions reductions through $a_{ie}$ consists of two terms; marginal abatement cost in period 1 and the impact on the marginal abatement cost in period 2 (which is zero or negative). Hence, cost-effective climate policy implies that the marginal abatement cost of e.g., electrification policies may exceed the marginal cost of short-term fossil-based mitigation in period 1 ($C'_{1f} < C'_{1s}$). This confirms the conclusion in Vogt-Schilb et al. (2018) that it may be optimal to start with an expensive abatement option even if there are cheaper options available.

From (17), we can also derive the following conclusion:

**Conclusion 1:** Given, full flexibility in the permit market ( $p_1 = p_2 = p$), we find it profitable to increase both types of abatement when the market price of permits increases.
Now, consider the case with restrictions in the permit market. The marginal abatement cost should be equal to the marginal cost of emissions, which consists of the permit price and the shadow cost on the constraint. Let \( P_j \) be the marginal cost of emissions in period \( j \), i.e.,

\[ P_j = p_j + \lambda_j. \]

Total differentiating the first order condition and solving for \( \frac{da_j}{dP_j} \) yields (see Appendix B):

\[
\begin{align*}
\frac{da_{1f}}{dP_1} > 0, \quad \frac{da_{1f}}{dP_2} = 0, \quad \frac{da_{ie}}{dP_1} > 0, \quad \frac{da_{ie}}{dP_2} > 0, \\
\frac{da_{2f}}{dP_1} = 0, \quad \frac{da_{2f}}{dP_2} > 0, \quad \frac{da_{2e}}{dP_1} > 0, \quad \frac{da_{2e}}{dP_2} > 0.
\end{align*}
\]

From (18) we derive the second conclusions:

**Conclusion 2:** Given constraints on banking and borrowing on the international permit market, or domestic restrictions on permit purchase, such that \( P_1 \neq P_2 \), an increase in the price of emissions in period 1 makes it profitable to increase the abatement of both types in period 1, whereas an increase in the period 2 price only makes it profitable to increase \( a_{ie} \), but not \( a_{1f} \).

Thus, this shows that restrictions on permit trade have an impact on the profitability of infrastructure investments. The larger increase in the permit price over time, the more profitable to invest in long lasting infrastructure today, relative to short-lived policies.

### 3 An application of the model - electrification of the car fleet

We now derive how policies to induce abatement by an electrification of the car fleet can be represented by the model in section 2 (eqs. (1)-(5)). See also Appendix C for a comprehensive presentation of the model below.

The social cost of electrifying the car fleet consist of three main elements. First, there is a price difference (user cost) between cars running on fossil fuels (internal combustion engine vehicles - ICEVs) and EVs. Second, there is a disutility associated with electric cars compared to ICEVs due to the need for charging the batteries. This brings about both “range

\[ 13 \text{ Note that if (12) is binding, an exogenous increase in } p_j \text{ will not affect } P_j, \text{ as this will lead to a corresponding decrease in } \lambda_j. \]
anxiety” and “charging anxiety” (Hidrue et al., 2011). The first is due to the limited range for the batteries, and the concern for not reaching the charging station (on long trips) before the battery is empty. The other type of anxiety, charging anxiety, is related to the concern for long queues in front of the fast chargers. The range anxiety is higher the less charging stations in total (and the geographical distributions), whereas the charging anxiety is higher the more EVs per charging station. Both these types of social costs can be reduced by increased density of fast charger stations (Zhang et al., 2016 and Sierzchula, 2014). The third main cost is the investment in new infrastructure (not covered by the driving cost/price on fast charging).

As a starting point, we assume a given number of driving distance per car and a given number of cars, $E$, in each period. Abatement is achieved by an increase in the share of electric cars, $\mu$, where $0 \leq \mu \leq 1$. The disutility costs as mentioned above, can be reduced by public investment, $I$, in fast charging infrastructure $K$. The amount of capital (fast chargers) in period 1 and 2 respectively, is given by

$$
K_1 = I_1 + (1-\delta)K_0,
K_2 = I_2 + (1-\delta)K_1,
$$

where $\delta$ is the depreciation of capital in each period.\(^{14}\)

Both range anxiety and charging anxiety can be specified as a disutility cost function in year $i$ in the following way:

$$
D_i = \frac{(\mu E_i)^{\beta}}{K_i + G_i}
$$

Here, $G_i$ is the infrastructure capital that would be available in the absence of public investment, i.e., commercially profitable investments in fast chargers, and $\beta > 1$ is a parameter which captures that the marginal disutility is increasing in the share of EVs.\(^{15}\) As seen from (20), the disutility is increasing in the number of EVs on the road (charging anxiety) and the fewer the number of fast chargers is (range anxiety).

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\(^{14}\) Note that in the simulations below, the time periods are not equal as we study the years 2020, 2030 and 2050. Thus, the depreciation rates in equation (19) will differ.

\(^{15}\) With heterogeneous consumers, it is reasonable to assume that the disutility of a transition from ICEVs to EVs is unevenly distributed across consumers. A convex disutility function of a representative consumer can be seen as a representation of heterogeneous consumers’ disutility, in merit order.
The representative consumer maximises its utility of driving \( (V_i) \) at a given time period, \( i \), with respect to the fraction of EVs, \( (\mu_i) \), given the costs of driving, which depends the disutility of driving an EV, the subsidies for electric cars \( (s_i) \), and the difference in driving cost of the two types of cars \( (q_e \text{ and } q_f) \):

\[
V_i(K_i,s_i) = \max_{\mu_i} \left\{ \Pi(E_i) - \left( \frac{\mu_i E_i}{K_i + G_i} \right)^{\beta} - (q_e - s_i) \mu_i E_i - q_f (1 - \mu_i) E_i \right\}
\]

The government can influence the decision of the representative consumer by transport policies. In the model, it has to policy instruments; subsidizing driving of EVs and investing in fast chargers. Thus, the value of \( \mu_i \) depends on \( s_i \) and \( K_i \).

From the first order conditions, we find the optimal share of EVs:

\[
\mu_i(s_i,K_i) = E_i^{-1} \left( \frac{G_i + K_i}{\beta} \right)^{\frac{1}{\beta - 1}} \left( q_f + s_i - q_e \right)
\]

As seen, the share of EVs is increasing in both policy instruments.

Note from this expression that if \( q_f < q_e \), then \( s_i > 0 \) in order for \( \mu_i > 0 \). In the absence of transport policies, we get that:

\[
\mu(0,0) = E_i^{-1} \left( \frac{G_i}{\beta} \right)^{\frac{1}{\beta - 1}} (q_f - q_e)
\]

We can decompose the emission reductions that follow from \( s_i \) and \( K_i \) in the following manner. Let \( \sigma \) be an emission coefficient, i.e., CO\(_2\) emissions from driving an ICEV in the given time period, while there are no CO\(_2\) emissions from driving an EV. Total abatement in period \( i \) is then given by:

\[
a_i = \sigma \left[ \mu_i(s_i,K_i) E_i - \mu(0,0) E_i \right]
\]

\[
= \sigma \left[ \mu_i(0,K_i) E_i - \mu(0,0) E_i \right] + \sigma \left[ \mu_i(s_i,K_i) E_i - \mu_i(0,K_i) E_i \right]
\]

\[\text{subscript } a_i = \text{Abatement induced by } K_i\]

\[\text{subscript } a_s = \text{Abatement induced by } s_i\]

\[16\text{ Note from this equation that if } q_e > q_f \text{ in the absence of transport policies, } \mu(0,0) \text{ is negative.}\]
The last line in this equation separates the effect of subsidies and infrastructure investment. We can write out this line to give us three equations in five variables: $a_{ie}, a_{if}, a, s, K_i$:

\[(25) \qquad \sigma\left[\mu_i(0, K_i) E_i - \mu(0, 0) E_i\right] = a_{ie}\]

\[(26) \qquad \sigma\left[\mu_i(s, K_i) E_i - \mu(0, K_i) E_i\right] = a_{if}\]

\[(27) \qquad a_{ie} + a_{if} = a\]

Here $a_{if}$ is the abatement created by the subsidy and $a_{ie}$ is the additional abatement from infrastructure abatement, see section 2 above. We can solve (25) and (26) with respect to $s_i$ and $K_i$. This yields $s_i$ as $s_i(a_{ie}, a_{if})$ and $K_i$ as $K_i(a_{ie})$. We can thus write the fraction of cars, optimally chosen by the representative consumer and the investment in infrastructure as functions of the two types of abatements; $\mu_i(a_{ie}, a_{if})$ and $I_i(a_{ie})$ (see eq.(19) and see appendix C).

3.1 The Government’s decision problem

The governmental decision problem is to decide the values of $a_{ie}$ and $a_{if}$ which minimize social cost, taken into account the abatement targets and permit trade restrictions. We define the consumer’s cost as the loss in welfare (eq.(21)) of the transport policy. As derived above, the fraction of EVs can be expressed as a function of the two types of abatement, and therefore, the welfare of the representative consumer is a function of $a_{ie}$ and $a_{if}$. In addition, the government must also consider the cost of investing in $K_i$. If we denote the cost of $I_i$ by $\kappa(I_i)$, we show in Appendix C that we can write the social cost of abatement in each period by:

\[\ldots\]

17 Note that the subsidy is not included in the social cost as this is only an internal domestic transfer. We have not included marginal cost of public fund in the model.
The first order conditions for an optimal transport policy in the presence of permit trading are thus given by (16) and (17).

### 3.2 Calibration of the model

The model is calibrated to data for Norway with 2020 as the base year. We further consider the years 2030 and 2050.

The total number of private cars in 2018 is according to Statistics Norway (2019) about 2,750,000, with a share of electric vehicles equal to 7.1%. Predictions for the number of private cars and private transport for the next decades vary (see, e.g., Madslien et al., 2014; KPMG, 2018), but based on these and the current trends, such as car sharing and self-driving cars, it may be reasonable to assume that the total number of private cars will not increase significantly. We have therefore, set the total number to 2,900,000 in both 2030 and 2050.

The capital stock in the model is defined as the number of fast charges or supercharges for electric vehicles. 94% of the owners of electric vehicles in Norway charge their cars at home (Figenbaum, 2018). However, fast chargers are important for long distance driving, and the investments in chargers the last few years have mainly been fast chargers in the traffic corridors between cities. Figenbaum (2018) points to investments in fast chargers as a main instrument to reduce distance anxiety. According to Figenbaum (2018, 2019, also personal communication) and Elbilforeningen (2019), there were 1,852 fast chargers in Norway per 30 June 2019. The Government subsidizes fast chargers outside the main cities through the governmental body ENOVA. The subsidy is 40% of the cost. We assume that there are 2000 fast charges in Norway in the year 2020, where 50%, i.e., 1,000, is the number of commercial fast chargers ($G$) not receiving subsidies, while the rest ($K$) have received governmental subsidies.

Fast chargers consist of several components with different lifetimes. The lifetime of the charger itself is about 10-15 years (Schroeder and Trabe, 2012). However, the cost of the
charger is only about 25% of the total costs.\textsuperscript{18} Physical facilitation is required such as casting and burial of cables, in addition to power connection. These components have longer lifetimes. For road projects as well as power connections, the lifetime is set to 40 years (Statens vegvesen, 2018; NVE, 2019). We therefore choose a lifetime of fast chargers of 40 years, using linear depreciation. The lifetime of EVs is set to 10 years, so that no EVs are left over from 2020 to 2030.

The cost of fast chargers vary. Schroeder and Trabe (2012) report numbers for Germany. For all chargers, the numbers vary between €1000 and €125,000, where the highest number is for public superchargers. Figenbaum (2018, also personal communication) reports prices for fast chargers from NOK 400,000 (€40,000) and higher, and that the fast chargers that have received governmental support cost between NOK 500,000 and 700,000 (€50,000 and €70,000). Based on this, we set the price of fast chargers to €60,000 and the subsidy to €24,000 (40%). In the present simulation, the investment cost is assumed linear.

The costs of driving a car are taken from Smarte Penger (2019). Based on a set of assumptions,\textsuperscript{19} the annual cost of an EV and an ICEV vehicle is set to €8,600 and €10,900 respectively. These numbers include taxes and subsidies. The subsidy rate is taken from Kverndokk et al. (2020), who calculate the subsidy to electric vehicles to be 20%. Thus, the cost of driving an electric vehicle without subsidies is set to €10,300. Further, the tax rate on ICEV is set to 40% based on Kverndokk et al. (2020).

Based on the numbers above, the parameter in the distance anxiety function ($\beta$) is calibrated to be 2.195.

CO$_2$ emissions per year from a car driving on fossil fuels, is set to 2.2 ton, and is held constant in the analysis. This calculation is based on a driving distance of 16,000 kilometers per year, and emissions of 140g CO$_2$ per kilometer. This is a little bit higher than the numbers calculated for 2016 from Statistics Norway (2018), which is 153g per kilometer. However, the car park has been less energy intensive over the last few years. For ease of reference, parameters are summarized in Appendix D.

\textsuperscript{18} Erik Figenbaum, personal communication.

\textsuperscript{19} E.g., the retail price is NOK 400,000 (€40,000), annual driving distance is 16,000 kilometers, and the car is sold after three years.
3.3 Permit market assumptions

We will explore how the possibilities for emission trade across countries affect the optimal distribution of the two types of abatement policies. For this task we will need estimates of the future prices of tradable emissions. EU wants to cut emissions by 40% by 2030 and 80% by 2050 (compared to 1990), and as part of the GHG policies, EU has specified that sectors not covered by the EU-ETS (non-ETS) must reduce emissions by 30% by 2030 compared to 2005. In addition to EU Member States, Iceland and Norway has agreed to implement the Effort Sharing Regulations which translates this commitment into binding annual GHG emission for each Member States (including Norway and Iceland). The Effort Sharing Regulations allows for some flexibility in terms of banking and borrowing across time periods, and limited flexibility in terms of access to credits from the land use sectors and EU-ETS allowances. Member States can also buy and sell allocations from and to other Member States. In the following we refer to this potential trade in allowances across countries as the non-ETS permit market. Although permit trade is an option, there is yet no permit market, and in remains to be seen whether a permit market will emerge.

The literature investigating the potential price path for non-ETS permits is scarce, and obviously, price estimates will be highly uncertain. The permit market price, if a permit market emerge, will not only depend on the abatement costs in the non-ETS sectors, but also on the use of the flexible mechanisms of the Effort Sharing Regulations, and the use of other policy instruments and legislations for achieving other energy and environmental targets (e.g., renewable shares and energy efficiency), see Aune and Golombek (2018) and EC (2018).

Bye et al. (2019) construct marginal abatement costs under EU’s Effort Sharing Regulation based on a multi sector CGE model. For estimating the permit market price in 2030, they have as starting point two reference scenarios for projected 2030 emission of CO2 in EU (with current policies), EC (2016) and EU (2017), and they consider different options for flexibility mechanism and abatement also for non-CO2 GHG. They pick out two key scenarios, one which lead to a 2030 permit price of 64 Euro per ton CO2, and another (the highest cost scenario) which gives a permit price of 158 Euro per ton CO2. We will use these price scenarios for 2030 in our numerical model.

---

20 2011-prices.
Bye et al. (2019) do not present any figures for 2050, but they report that the marginal abatement cost curve is convex. For 2050 with an 80 percent reduction target in total GHGs in Europe, we choose to operate with marginal abatement costs (permit prices) 2.5 times higher than the 2030 prices in our numerical illustration. Thus, the cost in the low permit price scenario we have that the permit price at in 2030 is €64 in 2030 and €160 in 2050. In the high permit price scenario the permit price is €158 in 2030 and €395 in 2050.

4 An Numerical results – the case for Norway

The starting point is the emissions reductions in the effort sharing program in the EU, i.e., in the non-ETS sectors. Norway has a massive program for electrification of cars, but transportation is only a part of non-ETS. It also includes agriculture, buildings and waste. However, we restrict the analyses to the transport sector in this numerical illustration, and assume that the other sectors reduce their emissions according to the program.

To capture the uncertainty for flexibility across periods and across countries, we consider five scenarios for the flexibility in non-ETS:

1. No emission caps

2. Emission targets in each period without non-ETS permit trade (40% and 80% reductions in emissions from non-ETS sectors)

3. Total emissions as in scenario 1, but full intertemporal flexibility. No permit trade.

4. Emission targets in each period and permit trade in Non-ETS. Permit price set to €64 in 2030 and €160 in 2050.

5. Emission targets in each period and permit trade in Non-ETS. Permit price set to €158 in 2030 and €395 in 2050.

The results are summarized in Table 1. In the scenario with no emission caps, no new abatement policies are implemented, but there is still a positive share of electric cars due to an existing charging infrastructure in 2020. Note that the lifetime of electric cars is set to 10 years in the simulations so that all EVs existing in 2030 are due to the present charging network in this year.
Imposing constraints on emissions over the two periods decreases the social welfare ($\Delta V$ in Table 1) These constraints are more costly (almost 1 billion €) when there is no flexibility across periods compared to full flexibility, as is to be expected.

<table>
<thead>
<tr>
<th></th>
<th>No emission cap</th>
<th>No flexibility across periods</th>
<th>Full flexibility across periods</th>
<th>No flexibility Quota trading €64/160</th>
<th>No flexibility Quota trading at €158/395</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_e$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$a_f$</td>
<td>0</td>
<td>27%</td>
<td>15%</td>
<td>5%</td>
<td>24%</td>
</tr>
<tr>
<td>$\mu(\zeta</td>
<td>\kappa)$</td>
<td>0.02</td>
<td>40%</td>
<td>80%</td>
<td>20%</td>
</tr>
<tr>
<td>$I$</td>
<td>0</td>
<td>15986</td>
<td>24050</td>
<td>6538</td>
<td>38046</td>
</tr>
<tr>
<td>$K$</td>
<td>750</td>
<td>16736</td>
<td>32418</td>
<td>7289</td>
<td>41690</td>
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<tr>
<td>$\text{in €}$</td>
<td>0</td>
<td>1585</td>
<td>2055</td>
<td>1442</td>
<td>2114</td>
</tr>
<tr>
<td>$\Delta V$ in €</td>
<td>0</td>
<td>-6.43 billion</td>
<td>-5.58 billion</td>
<td>-0.21 billion</td>
<td>-0.82 billion</td>
</tr>
<tr>
<td>Quota purchase</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>100%</td>
<td>100%</td>
</tr>
</tbody>
</table>

Table 1. Outcomes of different scenarios

As seen from the table, in all scenarios with domestic abatement, the direct subsidies to car owners are responsible for the largest emissions reductions.

Considering no permit trade, we see that both $a_e$ and $a_f$ is higher in period 2 than in period 1 when intertemporal flexibility is allowed. This follows from positive discounting of future costs. The present value of the total cost decreases when more abatement effort is postponed to the future. In the model section 2, we ignored discounting for the sake of simplicity, but if we had included a discount factor, the first order condition (10) would read:

$$C^I_{h_1} = C^I_{h_0} + \delta C^I_{2_{h_0}} = \delta C^I_{2_{h_0}}$$

where $\delta < 1$.

With a positive discount factor, the cost reduction in period 2 following from $a_e$ in period 1 declines. Hence, it is relatively less beneficial to invest in $a_e$ than $a_f$ when there is flexibility.
The share $\frac{a_{i'}, a_i}{a_{i'}}$ in period 1 falls from 0.44 to 0.38 when changing from a system without flexibility to a system with flexibility.

Allowing a permit trading scheme is however the most effective way to meet quota requirements. Indeed, in our simulations permit trading voids all requirements to reduce emissions and the reduction in costs is by an order of magnitude bringing compliance costs down from being measured in billions of euros to hundreds of millions of euros. Preliminary numerical experiments indicate that the permit price in 2030 may have to increase to levels above €400 per ton in order for domestic policy to become important in this sector.

This also implies that the present EV-policy is very costly, with a marginal abatement costs above €400 per ton CO₂.

5 Conclusions

Policy makers and owners of large companies face many options to reduce greenhouse gas emissions. Some of these options are static in the way that they do not affect the costs of future emission reductions. Examples of this is buying permits in the international permit market or reducing consumption of fossil fuel based goods. Other options are dynamic in the sense that they will affect future costs of meeting emissions targets. Such options may be infrastructure investments that for instance makes it possible to use other fuel types or reduce our carbon footprint of consumption activities.

This paper studies the choice of policy instruments to reduce greenhouse gas emissions when there are restrictions in flexible instruments such as permit trading. Independent on flexibility, we find that the marginal mitigation cost of the dynamic alternative may exceed the costs of static mitigation, such as the permit price. Thus, expensive mitigation investments today are justified if the benefits lasts over several time periods. However, introducing restrictions on flexibility, so that permit prices across periods are not equal, have impacts on the choice of policy instrument. A higher expected price in later periods makes it more profitable to use the dynamic policy instrument, while the opposite is result will hold for a higher permit price today.

The framework is used for a numerical study on electrification of the transport sector in Norway, where infrastructure investments such as building charging stations across the country, makes it expensive to reduce emissions in the transport sector from electrification
today, but will reduce costs in later period. The numerical simulations show how the permit market may influence the transition from petrol-based fuel to electricity in the transport sector. We find that most of the abatement comes from using the static abatement option, subsidies to EVs. Flexibility in emissions abatement postpones both abatement options due to discounting of abatement costs. Further, domestic abatement is more expensive than buying emissions permits, meaning that no domestic abatement is performed under international permit trading.
References


Kverndokk, S., E. Figenbaum and J. Hovi (2020): Would my driving pattern change if my neighbor were to buy an emission-free car?, forthcoming in Resource and Energy Economics.


Appendix A: The optimization problem with full flexibility in the permit markets

With full flexibility, the optimization problem becomes:

\[
\begin{align*}
\text{Min} & \quad TC(a_{1f}, a_{1e}, a_{2f}, a_{2e}, d_1, d_2) = C_1(a_{1f}, a_{1e}) + C_2(a_{2f}, a_{2e}) \\
& \quad + p_1 d_1 + p_2 d_2 + \lambda (\bar{A} - a_{1f} - a_{1e} - a_{2f} - a_{2e} - d_1 - d_2)
\end{align*}
\]

This gives the following first order conditions:

\[
\begin{align*}
C_{1f}' & = \lambda \\
C_{1e}' + C_{2f}' & = \lambda \\
C_{2f}' & = \lambda \\
C_{2e}' & = \lambda \\
p_1 & = \lambda \\
p_2 & = \lambda
\end{align*}
\]

Thus, \( p_1 = p_2 = \lambda \).
Appendix B: Finding the abatement as a function of prices under restrictions in the permit market

By total differentiating (16), we find:

\[
\begin{aligned}
C^\sigma_{i_{1,1}} \, da_{i_f} & \quad C^\sigma_{i_{1,1}} \, da_{i_e} = dP^1 \\
C^\sigma_{i_{1,1}} \, da_{i_f} & \quad (C^\sigma_{i_{1,1}} + C^\sigma_{i_{2,1,2}}) da_{i_e} \quad C^\sigma_{i_{2,1,2}} \, da_{i_e} = dP^1 \\
C^\sigma_{2_{1,1,2}} \, da_{i_f} & \quad C^\sigma_{2_{1,1,2}} \, da_{i_e} \quad C^\sigma_{2_{1,1,2}} \, da_{i_e} = dP^2 \\
C^\sigma_{2_{1,1,2}} \, da_{i_e} & \quad C^\sigma_{2_{1,1,2}} \, da_{i_f} \quad C^\sigma_{2_{1,1,2}} \, da_{i_e} = dP^2
\end{aligned}
\]

Solving the system of equations gives:

\[
\begin{aligned}
da_{i_f} &= \frac{C^\sigma_{h_{1,1}_{1,1}} \left[ C^\sigma_{h_{1,1}_{1,1}} + C^\sigma_{h_{1,2,2}} \right] \cdot C^\sigma_{2_{1,1}_{1,2}} \cdot C^\sigma_{2_{1,1}_{2,2}} - \left[ C^\sigma_{h_{1,1}_{2,2}} + C^\sigma_{h_{1,2,2}} \right] \cdot C^\sigma_{2_{1,1}_{1,2}} \cdot C^\sigma_{2_{1,1}_{2,2}}}{D} \cdot dP^1 \\
da_{i_f} &= \frac{-C^\sigma_{h_{1,1}_{2,2}} \cdot C^\sigma_{h_{1,2,2}} \cdot C^\sigma_{2_{1,1}_{2,2}} - C^\sigma_{h_{1,1}_{1,1}} \cdot C^\sigma_{h_{1,2,2}} \cdot C^\sigma_{2_{1,1}_{1,2}} + C^\sigma_{h_{1,1}_{2,2}} \cdot C^\sigma_{h_{1,2,2}} \cdot C^\sigma_{2_{1,1}_{2,2}}}{-D} \cdot dP^2 \\
da_{i_e} &= \frac{C^\sigma_{h_{1,2,2}} \cdot C^\sigma_{2_{1,1}_{2,2}} + C^\sigma_{h_{1,2,2}} \cdot C^\sigma_{2_{1,1}_{2,2}} - C^\sigma_{h_{1,2,2}} \cdot C^\sigma_{2_{1,1}_{1,2}}}{D} \cdot dP^1 - \frac{C^\sigma_{h_{1,1}_{1,1}} \cdot C^\sigma_{2_{1,1}_{2,2}}}{D} \cdot dP^2 \\
da_{i_e} &= \frac{-C^\sigma_{h_{1,2,2}} \cdot C^\sigma_{2_{1,1}_{2,2}} - C^\sigma_{h_{1,2,2}} \cdot C^\sigma_{2_{1,1}_{1,2}}}{-D} \cdot dP^2 - \frac{C^\sigma_{h_{1,1}_{1,1}} \cdot C^\sigma_{2_{1,1}_{2,2}}}{D} \cdot dP^1
\end{aligned}
\]

The determinant D is positive by assumption, and consist of five terms, which we denote by capital letters from E to I:
\[
D = C^n_{1,1,1} \cdot (C^n_{1,1,2} + C^n_{2,1,2}) \cdot C^n_{2,2,2} \\
- C^n_{1,1,1} \cdot (C^n_{1,1,2} + C^n_{2,1,2}) \cdot C^n_{2,2,2} \\
- C^n_{1,1,1} \cdot C^n_{1,1,2} \cdot C^n_{2,2,2} \\
+ C^n_{1,1,1} \cdot C^n_{1,1,2} \cdot C^n_{2,2,2} \\

\equiv E - F - G - H + I > 0 \text{ by assumption}
\]

\[-H + I = \left[ C^n_{1,1,1} \cdot C^n_{1,1,2} \cdot (C^n_{2,2,2} \cdot C^n_{2,2,2} - C^n_{2,2,2} \cdot C^n_{2,2,2}) \right] \]

\[-F + I = \left[ C^n_{2,2,2} \cdot C^n_{2,2,2} \cdot (C^n_{1,1,1} + C^n_{2,2,2}) - C^n_{1,1,1} \cdot C^n_{1,1,1} \right] \]

We can thus rewrite $XX$ and find

\[
\text{Nominator} \quad \frac{d{a_{1,f}}}{d{P_1}} = \frac{1}{C^n_{1,1,1}} \cdot [E - F - G] + \frac{1}{C^n_{1,1,1}} \cdot [-H + I] \\
\text{Nominator} \quad \frac{d{a_{2,f}}}{d{P_2}} = \frac{1}{C^n_{2,2,2}} \cdot [E - G - H] + \frac{1}{C^n_{2,2,2}} \cdot [-F + I] \\
\text{Nominator} \quad \frac{d{a_{1,e}}}{d{P_1}} = \frac{1}{(C^n_{1,1,1} + C^n_{2,2,2})} \cdot [E - F - 0 \cdot G] + \frac{1}{C^n_{1,1,1}} \cdot [-H + I] \\
\text{Nominator} \quad \frac{d{a_{2,e}}}{d{P_2}} = \frac{1}{C^n_{2,2,2}} \cdot [E - H - 0 \cdot G] + \frac{1}{C^n_{2,2,2}} \cdot [-F + I] \\
\text{Nominator} \quad \frac{d{a_{1,e}}}{d{P_2}} = -\frac{1}{C^n_{2,2,2}} \cdot G \\
\text{Nominator} \quad \frac{d{a_{2,e}}}{d{P_1}} = -\frac{1}{C^n_{2,2,2}} \cdot G
\]

By assuming that $|C^n_{j,k,l}| > |C^n_{j,k,l}|$, $j=1,2$, $k=e,f$, $l=e,f$, and $k \neq l$ we find that

\[-H + I < 0, \quad -F + I < 0. \text{ Furthermore } G > 0. \text{ Since } D > 0 \text{ and } C^n_{j,k,l} < 0, \text{ we can sign all the derivatives:}
\]

\[
\frac{d{a_{1,f}}}{d{P_1}} > 0, \quad \frac{d{a_{1,f}}}{d{P_2}} = 0, \quad \frac{d{a_{1,e}}}{d{P_2}} > 0, \quad \frac{d{a_{1,e}}}{d{P_1}} > 0 \\
\frac{d{a_{2,f}}}{d{P_2}} > 0, \quad \frac{d{a_{2,f}}}{d{P_1}} = 0, \quad \frac{d{a_{2,e}}}{d{P_2}} > 0, \quad \frac{d{a_{2,e}}}{d{P_1}} > 0.
\]
Appendix C: The model for electrification of the car fleet  (To be completed)

The model assumes a given number of driving distance per car and a given number of cars, $\bar{E}$. Let $\mu$ be the fraction of cars that are electric. A representative consumer chooses the fraction of electric cars to be driven. There is a disutility cost associated with driving electric cars due to “distance anxiety.” This cost may be reduced by public investment in electric loading station infrastructure $K_i$. The amount of capital in period 1 is $K_1 = I_1 + (1 - \delta)K_0$ and the amount in period 2 is $K_2 = I_2 + (1 - \delta)K_1$. The distance anxiety cost in year $i$ is given by

$$D_i = \left( \frac{\mu \bar{E}_i}{K_i + G_i} \right)^\beta$$

(1)

Here $G_i$ is the amount infrastructure that would be available in the absence of public investment. $\beta > 1$ is a parameter.

The maximal welfare of a representative consumer van be expressed as:

$$V_i(K_i, s_i) = \max_{\mu} \left\{ \Pi \left( \bar{E}_i \right) - \frac{(\mu \bar{E}_i)^\beta}{K_i + G_i} - (q_e - s) \mu \bar{E}_i - q_f (1 - \mu) \bar{E}_i \right\}$$

(2)

From the optimization problem, the value of $\mu_i$ is given by

$$\mu_i(s_i, K_i) = \bar{E}_i^{-1} \left( \frac{G_i + K_i (q_f + s_i - q_e)}{\beta} \right)^{\frac{1}{\beta - 1}}$$

(3)

Note from this expression that if $q_f < q_e$, then $s_i > 0$ in order for $\mu_i > 0$. In the absence of policy we get that:

$$\mu(0, 0) = \bar{E}_i^{-1} \left( \frac{G_i (q_f - q_e)}{\beta} \right)^{\frac{1}{\beta - 1}}$$

(4)

We can decompose the emission reductions that follow from $s_i$ and $K_i$ in the following manner. Total abatement in period $i$, where $\sigma$ is the emission coefficient, is given by:
\[ a_i = \sigma \left[ \mu_i (s_i, K_i) \bar{E}_i - \mu (0,0) \bar{E} \right] \]
\[ = \sigma \left[ \mu_i (0, K_i) \bar{E}_i - \mu (0,0) \bar{E} \right] + \sigma \left[ \mu_i (s_i, K_i) \bar{E}_i - \mu_i (0, K_i) \bar{E}_i \right] \]

The last line in this equation separates the effect of subsidies and infrastructure investment.

We can write out the last line to give us three equations in five variables:

\[ \sigma \left[ \mu_i (0, K_i) \bar{E}_i - \mu (0,0) \bar{E} \right] = a_{ie} \] (5)

\[ \sigma \left[ \mu_i (s_i, K_i) \bar{E}_i - \mu_i (0, K_i) \bar{E}_i \right] = a_{if} \] (6)

\[ a_{ie} + a_{if} = a_i \] (7)

Here \( a_{if} \) is the abatement created by the subsidy and \( a_{ie} \) is the additional abatement from infrastructure abatement. We can solve (5) and (6) with respect to \( s_i \) and \( K_i \). This yields

\[ s_i = (q_f - q_e) \left( a_{ie} + a_{if} + \left( \frac{G_i (q_f - q_e)}{\beta} \right)^{\frac{1}{\beta - 1}} \right) \left( \frac{G_i (q_f - q_e)}{\beta} \right)^{\frac{1}{\beta - 1}} \left( a_{ie} + \mu (0,0) \bar{E} \right)^{\frac{1}{\beta - 1}} - (q_f - q_e) \]

\[ = (q_f - q_e) \left( a_{ie} + a_{if} + \mu (0,0) \bar{E} \right)^{\frac{1}{\beta - 1}} \left( a_{ie} + \mu (0,0) \bar{E} \right)^{\frac{1}{\beta - 1}} - 1 \] (8)

\[ K_i = \left. \frac{\beta \left( \frac{G_i (q_f - q_e)}{\beta} \right)^{\frac{1}{\beta - 1}} + a_{ie}}{q_f - q_e} \right| - G_i = \frac{\beta (\mu (0,0) \bar{E} + a_{ie})^{\frac{1}{\beta - 1}}}{q_f - q_e} - G_i \] (9)

To indicate that \( s_i \) and \( K_i \) are functions of abatement requirements and the exogenous path of \( G \) we denote \( s_i \) as \( s(a_{ie}, a_{if}) \) and \( K_i \) as \( K(a_{ie}, G_i) \). Inserting from (8) and (9) into (3) gives:

\[ \mu (a_{ie}, a_{if}) = \frac{a_{ie} + a_{if} + \mu (0,0)}{\bar{E}} \] (10)
\[ \mu(a_{ie}, a_{if}) E = a_{ie} + a_{if} + \mu(0,0) \]  

(11)

We can combine the expression in (9) and the difference equation for \( K_i \) to derive a difference equation for \( a_{ie} \).

\[ a_{i+1,e} = \left( \frac{G_z(q_f - q_e)}{\beta} \right)^{\frac{1}{\beta-1}} + \left( 1 - \delta \right) \left( \frac{G_1(q_f - q_e)}{\beta} \right)^{\frac{1}{\beta-1}} + a_i + \frac{(I_{i+1} - (1 - \delta)G_i + G_z)(q_f - q_e)}{\beta} \]  

(12)

Alternatively we can write \( I_{i+1} \) as a function of \( a_{ie} \) and \( a_{i(i+1)e} \). This yields

\[ I_{i+1}(a_{i+1}, a_i) = (1 - \delta)G_i - G_{i+1} + \frac{\beta \left( \left( \mu_i^0 + a_{i+1} \right)^{\beta-1} - (1 - \delta) \left( \mu_i^0 + a_i \right)^{\beta-1} \right)}{q_f - q_e} \]  

(13)

If we denote \( \mu(0,0) \) given \( G_i \) as \( \mu_i^0 \) we can simplify this expression to

\[ I_{i+1} = (1 - \delta)G_i - G_2 + \frac{\beta \left( \left( \mu_i^0 + a_{i+1} \right)^{\beta-1} - (1 - \delta) \left( \mu_i^0 + a_i \right)^{\beta-1} \right)}{q_f - q_e} \]  

(14)

The Governments decision problem

The regulating authority takes the expressions derived above as constraints. When fixing \( a_{ie} \) and \( a_{if} \) the authority takes into account that \( \mu \) is determined by these variables and determine the welfare of the consumers. However, the authority must also consider the cost of subsidies and the cost of investing in \( K_i \). If we denote the cost of \( I_i \) by \( \kappa(I_i) \), then after inserting from (8), (9), (14) and recalling that \( a_{i0e} = 0 \) by assumptions we can write the government cost (not welfare) in each period may be written.
\[ C_2(a_{j_1}, a_{j_2}, a_{j_3}) = \left( \frac{\mu_j \beta_2}{K_j(a_{j_2}) + C_j} + q_j \mu_j \beta_1 + q_j (1 - \mu_j) \beta_1 + \kappa (I_j(a_{j_2}, a_{j_3})) \right) \]

\[ C_1(a_{f_1}, a_{f_2}) = \left( \frac{\mu_1 \beta_1}{K_1(a_{f_1}) + C_1} + q_j \mu_1 \beta_1 + q_j (1 - \mu_j) \beta_1 + \kappa (I_j(a_{f_1}, 0)) \right) \]

Completely writing out \( C_2 \) and \( C_2 \) gives

\[ C_2 = \frac{(q_j - q_e)(\mu^0_2 \beta_2 + a_{j_2} + a_{j_2})^{\beta_1}}{\beta (\mu^0_2 \beta_2 + a_{j_2})^{\beta_1} + (q_j - q_f)(a_{j_2} + a_{j_2} + \mu^0_2 \beta_2)} - q_j \beta_2 \]

\[ \frac{\delta C_2}{\delta a_{j_2}} = -\kappa \frac{(1 - \delta)}{q_f - q_e} \beta (\beta - 1) \left( \frac{1}{\mu^0_2 \beta_2 + a_{j_2}} \right)^{\beta_2 - 2} < 0 \]

for \( \beta = 2 \):

\[ \frac{\delta C_2}{\delta a_{j_2}} = -\kappa \frac{(1 - \delta)}{q_f - q_e} \]

\[ \frac{\delta C_2}{\delta a_{j_2} \delta a_{j_3}} = 0 \]

\[ \frac{\delta C_2}{\delta a_{j_3}} = \beta (q_j - q_e)(\mu^0_2 \beta_2 + a_{j_3})^{\beta_2 - 2} \beta (\mu^0_2 \beta_2 + a_{j_3})^{\beta_2 - 1} - (\beta - 1) \beta (\mu^0_2 \beta_2 + a_{j_3})^{\beta_2 - 2} (q_j - q_e)(\mu^0_2 \beta_2 + a_{j_3} + a_{j_3})^{\beta_2 - 1} \]

\[ + \kappa \frac{\beta}{q_f - q_e} (\beta - 1) (\mu^0_2 \beta_2 + a_{j_3})^{\beta_2 - 2} \]

(19)
\[
\frac{\delta C_2}{\delta a_{zf}} = \frac{(q_f - q_e)\beta(\mu^0_2 E + a_{\nu} + a_{\mu})^{\beta-1}}{\beta(\mu^0_2 E + a_{\nu})^{\beta-1}} + (q_e - q_f) = \frac{(q_f - q_e)(\mu^0_2 E + a_{\nu} + a_{\mu})^{\beta-1}}{\mu^0_2 E + a_{\nu})^{\beta-1}} + (q_e - q_f)
\]

\[
(q_f - q_e)\left[\frac{(\mu^0_2 E + a_{\nu} + a_{\mu})^{\beta-1}}{\mu^0_2 E + a_{\nu})^{\beta-1}} - 1\right] > 0
\]

\[
\frac{\delta^2 C_2}{\delta a_{zf} \delta a_{zf}} = \frac{(q_f - q_e)(\mu^0_2 E + a_{\nu} + a_{\mu})^{\beta-2}}{(\mu^0_2 E + a_{\nu})^{\beta-1}} > 0
\]

\[
\frac{\delta^2 C_2}{\delta a_{zf} \delta a_{zf}} = \frac{(q_f - q_e)}{(\mu^0_2 E + a_{\nu})^{\beta-1}}\left[1 - \frac{(\mu^0_2 E + a_{\nu} + a_{\mu})}{(\mu^0_2 E + a_{\nu})}\right] < 0
\]

(20)

\[
C_1 = \frac{(q_f - q_e)(\mu^0_2 E + a_{\nu} + a_{\mu})^{\beta}}{\beta(\mu^0_2 E + a_{\nu})^{\beta-1}} + (q_e - q_f)(\mu^0_2 E + a_{\nu} + a_{\mu}) + q_j E_2
\]

\[
+ \kappa \left(\beta \left(\frac{\mu^0_2 E + a_{\nu}}{\beta-1} - (1 - \delta)(\mu^0_2 E + a_{\mu})^{\beta-1}\right) - G_1\right)
\]

(21)
Appendix D. Parameter values.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_1$</td>
<td>Number of cars in 2030</td>
<td>2,900,000</td>
</tr>
<tr>
<td>$E_2$</td>
<td>Number of cars in 2050</td>
<td>2,900,000</td>
</tr>
<tr>
<td>$G_0$</td>
<td>Number of commercial fast chargers in 2020</td>
<td>1,000</td>
</tr>
<tr>
<td>$K_0$</td>
<td>Number of fast chargers with governmental support in 2020</td>
<td>1,000</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>Depreciation rate of infrastructure, 2020 to 2030</td>
<td>0.25</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>Depreciation rate of infrastructure, 2030 to 2050</td>
<td>0.5</td>
</tr>
<tr>
<td>$q_e$</td>
<td>Annual user cost for an EV before subsidies</td>
<td>€ 10,300</td>
</tr>
<tr>
<td>$q_f$</td>
<td>Annual user cost for an ICEV including taxes</td>
<td>€ 10,900</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Investment cost for a fast charger before subsidies</td>
<td>€ 60,000</td>
</tr>
<tr>
<td></td>
<td>Subsidies to fast chargers</td>
<td>€ 24,000</td>
</tr>
<tr>
<td>$\mu_0$</td>
<td>Share of EVs in 2020</td>
<td>0.071</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Parameter in disutility function</td>
<td>2.195</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Emission coefficient; tons of CO$_2$ per year for an ICEV</td>
<td>2.2</td>
</tr>
</tbody>
</table>

Table D1: Parameters in the numerical model