Zero-carbon electricity markets with grid-scale electricity storage

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January 31, 2020

WORK-IN-PROGRESS.

Abstract

Decarbonizing the electricity sector, through incorporating renewable generation, is of central importance to emission reduction policies around the world. However, many renewables are intermittent; incorporating largely intermittent generation into the grid poses significant challenges since power can only be dispatched up the random level of supply determined by nature. This feature of intermittent supply has raised concerns about the potential volatility and viability of electricity spot markets with intermittent generation. This paper builds a dynamic general equilibrium framework of speculative storage to understand the potential role of commercially provided electricity storage in attenuating the volatility of a spot market with intermittent supply. Our computed results show that grid-scale storage can serve to dispatch power when needed, stabilising a market; however, for there to be sufficient incentives for investment in storage capacity, the supply volatility must be high enough to generate incentives for storage speculators to invest in storage capacity.

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§We are grateful for funding from the Svenska Handelsbanken foundation and the ANU Zero Carbon Energy for the Asia-Pacific Grand Challenge project.

1 Introduction

Motivated by the large contribution of the electricity sector to carbon emissions, policy makers have placed central importance on de-carbonizing electricity generation to reduce aggregate emissions. Many governments have adopted policies – varying from premium tariffs to quantity mandates with tradable certificates – to promote renewable electricity generation. These mandates have already led to a significant share of electricity generation from intermittent renewables.

However, integrating significant shares of intermittent generation sources ("intermittent sources", henceforth), such as wind turbines ("wind") and solar photovoltaics ("solar"), into the electricity system poses unique challenges due to the characteristics of electricity markets and intermittent generation. Among markets for goods, electricity markets are unique because the market must clear, demand must equal supply, at every instant for the grid to be stable. Generation from hitherto common sources such as coal and natural gas has been "dispatchable" i.e. its load can be varied continuously and predictably up to the maximum generation capacity even for short periods. Consequently, so long as spare capacity is available, meeting varying power demands, and thus balancing the system, is a relatively easy task. In a market with only intermittent renewable supply, power can only be dispatched up to the random level determined by nature, regardless of consumers' willingness to pay. This fact has led policy makers to raise concerns about the potential volality of markets with intermittent supply. [1]

This paper develops a dynamic general equilibrium framework to understand the role a key anticipated mechanism to address the challenges posed by renewable, grid scale storage, can play in addressing the challenges to market stability posed by intermittent supply. Owing to significant predicted cost reductions in electricity storage technologies, the literature and policy makers have anticipated that grid-scale electricity storage ("storage") can play a key role in addressing the challenges posed by integrating renewables into electricity markets (Heal (2016); Newbery (2016); Sinn (2017); Zerrahn et al. (2018)). Storage can address the challenges we raised above by becoming a predictable dispatchable source of power, smoothing out supply shocks. Storage may also be able to help stabilize the market price, allowing for investment in generation capacity. The main mechanism through which storage providers can "stabilize" the market is by taking advantage of arbitrage opportunities; commercial storage providers can buy electricity when prices are low and dispatch electricity when demand is high and/or supply low.

Grid-scale storage, in fact, is anticipated to play different roles, at different time scales, in electric systems with significant share of intermittent generation, including (Heal (2016); Newbery (2016); Antweiler (2018)): shifting generation between more to less expensive period (energy arbitrage); minimising curtailment by allowing generators to store excess electricity (the supply-side storage option); alternatively, intermittent generators may them-

[1] Intermittent sources face two sources of variability. The first is predictable, for example, the diurnal cycle. The second is unpredictable variability in generation induced by unpredictable variation in e.g. wind . We focus on the latter aspect in our analysis.

selves deal with the possibility of curtailment by installing significant storage capacity allowing them to capture these arbitrage opportunities.

Despite the significant role storage can play in electric systems with intermittent generation, the literature on the economics of electricity storage is sparse (Heal, 2016; Newbery, 2016; Antweiler, 2018). In addition, the current literature in economics evaluating the trade-offs between storage and intermittent generation (Pommeret et al., 2019; Helm and Mier, 2018; Abrell et al., 2019) is focused largely on predictable variability (e.g. diurnal) in intermittent generation and on policies assisting the transition to intermittent-dominant electricity generation (see section 2), largely in a non-market setting (see e.g. Newbery et al. (2018); Riesz and Milligan (2015) and the references therein). The significance of unpredictable variability in intermittent generation has been discussed before (Heal (2016) and references therein) and the possibility that the variance of aggregate generation increases with aggregate intermittent capacity has been demonstrated for Germany in Sinn (2017). However, we are unaware of any dynamic general equilibrium analysis of a market-based determination of price and investment in generation and storage capacity when intermittent generation is stochastic. In particular, a detailed understanding of the effects of grid-level storage upon price and supply volatility and the effects of this on the ability of the market to facilitate sufficient generation and storage investment is missing.

Our intention is to fill this gap: we build a theoretical model to understand how electricity markets of the envisaged future, which are largely renewable, will work.[2] We focus on the *post-transition* setting, where intermittent sources of generation are already dominant, where electricity is traded in a wholesale (spot) market (as today in many countries) and where the day ahead prices determine both capacity investment and storage dynamics. In this context, we model storage both as a source of demand and supply, storage providers can buy when the price is low in anticipation of supplying power when the price is high. The decision to invest in storage and generation capacity is also made in the market taking into account the future path of returns from owning storage or generation capital.

The key questions we address include: the nature of the relationship between storage and generation capacity, such as whether storage complements, or substitutes for, generation; the dependence of this relationship upon volatility of supply; the effect of storage upon price volatility; and key questions involved in how price, and demand, volatility relate to welfare. Questions such as these are at the forefront of the policy discussion on integrating significant share of renewable generation in the electric grid and the literature in economics examining them in a rigorous framework is sparse.

The model here draws on an established tradition of speculative models of commodity storage dynamics (Wright and Williams, 1982; Scheinkman and Schechtman, 1983; Deaton and Laroque, 1992; Bobenreith H. et al., 2002). (See Ernesto Guerra et al. (2018) for a recent discussion of technical issues.) We build on the standard commodity storage model in key ways by including a demand shock, endogenous generation capacity and storage capacity decisions. This project also seeks to make a technical contribution by establishing new

[2]Renewable-dominance is in fact a key goal of many policy makers including e.g. the EU, which aims at achieving a target of at least 70% of total generation being renewable by 2045.

results on the existence and convergence properties of the model with endogenous storage capacity decisions on unbounded state-space.

In our computational experiments, we seek to understand the role storage can play in attenuating the effect of supply volatility on equilibrium demand volatility. We seek to understand when and how storage can play the role of dispatchable power in a fully intermittent system. Our main finding is that storage can play the role of dispatchable power, effectively "breaking the link" between supply shocks and equilibrium demand. However, for there to be sufficient incentives for investment in storage capacity, surprisingly, intermittent supply must be sufficiently volatile. Without sufficient volatility in intermittent generation there may not exist the arbitrage opportunities to drive investment in storage capacity.

The rest of the paper proceeds as follows: Section 2 places our work in the context of the relevant literature, while the model details are developed in Section 3. The results of preliminary numerical simulations will be presented in Section 4.

2 Related Literature

Our work relates to an increasing number of studies dealing with the energy transition with storage, where intermittent generation is beginning to arise in a system comprised largely of conventional generation sources, with storage being a potential source of demand or supply. While many studies evaluate storage along with intermittent generation as an explicit strategy in the energy transition in a static set up, treating in essence storage capacity as fixed (e.g. Sinn (2017); Zerrahn et al. (2018)), only a few papers explicitly consider storage in a dynamic set up (or at least allow for capacity addition), including Pommeret et al. (2019); Helm and Mier (2018); Abrell et al. (2019). Some of this literature, in turn, builds on models focused on the design of electric system where intermittent generation is accounted for (e.g. Ambec and Crampes (2012)). The motivation is climate related, and policies considered in this setting reflect it, a tax (e.g. upon conventional fuels) or subsidy (for intermittent generation or storage) leading to the appropriate emissions level.

Of the three most relevant studies, Helm and Mier (2018) only consider the predictable variability involved in the provision of electricity from intermittent sources, not the randomness in generation from intermittent sources arising from the inherent uncertainty in actual wind or solar-related conditions. The set up here is an extended version of a classical discrete-time peak-load pricing set up with storage, with conventional and intermittent generation technologies and a storage technology, and comprises of three-stages; in the first stage, government chooses policy (capacity subsidies/taxes); in a second stage, capacity choice occurs; finally, day-to-day operations occur. Variability is accounted for by letting generation depend upon a deterministic function of time, $\alpha(t)$ (over a cycle, defined arbitrarily). Storage, as always, involves conversion losses. In view of the cyclic nature of variability, the model leads to periods of storage followed by periods of discharge.

Capacities (hence generation) are provided by competitive firms in respective sector. Demand and supply both being subject to no uncertainty, market clearing is simple, and the market maximises consumer surplus. The optimal policy here is more complex than the simple pigouvian tax and depends upon relative fossil and intermittent capacity choice, and whether conventional generation is allowed to access storage; for low intermittent share, renewable subsidies and taxation on storage maximise welfare; at higher levels of penetration, this scheme is second-best.

Pommeret et al. (2019), on the other hand, consider the case of a planner in continuous time, and evaluate the effects of both variability and (a very simplified form of) intermittency. A day is is defined by two periods, 'day' and 'night', with intermittent generation available only during the 'day'. Consumers evaluate consumption during these periods differently, have a low elasticity of demand, and are unwilling to substitute demand between these two time periods (demand is separable across the periods). Storage plays two roles here; in the first role, it provides the needed "time shifting" (detailed above) while in the second role, it can "smooth out" some of the uncertainties involved in intermittent generation. Uncertainty is modelled very simply, as leading to one of two outcomes ("sunny" or "cloudy") with known probabilities. The planner maximizes utility from consumption in these two periods, net of the cost of electricity provision. Storage capacity is available at no investment cost, subject to the usual conversion losses. Intermittent generation has a capacity factor, there is a carbon budget ceiling, and capacity is costly, comes online instantaneously. In the case with no intermittency, a key finding is that investment in solar panels here may be monotonic or not, depending upon many aspects of the problem. With intermittency, much depends upon how "significant" the uncertainty involved is: if low enough, it can be effectively ignored; if high enough, storage optimally commences significantly early, since "smoothing out" is now rather important. While no specific policies were considered (e.g. taxes/subsidies), the presence of a carbon budget drives much of the system.

The study of Antweiler (Antweiler, 2018) can be considered complementary to ours in a sense. That study considers stylized models of both the energy arbitrage and the supply side option channel of grid-scale storage, and provides an exposition of the effect of different parameters in a counter-factual setting. The basic set up for the energy arbitrage framework is similar to the others detailed above, but capacity choices are exogenous, and no explicit price determination occurs. In any case, we are unaware of any rigorous analysis of the scope of the energy arbitrage channel in the context of stochastic weather conditions and endogenous price and capacity choice. Our analysis, similar to other aggregate analyses detailed above, considers aggregate grid-level storage that combines both channels identified above.[3]

[3]Very little is known regarding the economic aspects of the supply side option, investigated only in Antweiler (2018) in a stylized setting, which may be an optimal response when curtailment is uneconomical (it occurs frequently or is of significant duration)

3 Equilibrium storage and investment with stochastic demand and supply

3.1 Model Outline

We develop a speculative storage model enriched with stochastic consumer demand and supply and an endogenous choice of storage capacity and generation capital. Speculators make hour-to-hour decisions about how much power to charge or discharge subject to the wattage of their storage and "round-trip" line losses. Speculators also make an initial decision about the total storage capacity (total gigawatt hours of of "battery capacity"). Generators make an initial investment decisions on the level of intermittent generation capacity (total gigawatts of name-plate generation capacity). Consumers provide the ultimate source of demand, and an equilibrium corresponds to a price that clears storage demand/supply of power, consumer demand and supply from generators for each hour and "state of the world".

We note that the storage framework developed here model bears a strong resemblance to the speculative storage model in e.g. Deaton and Laroque (1992), with the addition of endogenous decisions about storage capacity and generation capital, stochastic demand non-linear constraints on flow and friction-related elements in storage losses.

A key way in which the market with storage differs from that without, encapsulated in the conventional L-shaped (or step) supply curve, is in the inherently dynamic nature of determination of price. Storage is by definition an arbitrage decision over time, and hence, the equilibrium price for any day will be connected to expected price at all future periods and states.[4]

While we enrich the standard speculative storage model with many features of electricity markets, our initial model does make a number of simplifying assumptions; in particular, we assume i.i.d shocks, no curtailment, no negative prices and a continuum of homogeneous price taking generators.

A few aspects of the model are described next, before mathematical details of the model are presented.

Demand Unlike in the existing storage literature, demand is both somewhat elastic and random, meaning the overall degree of mismatch between demand and supply is larger than when demand is treated as a fixed amount per hour. Consequently, the significance of storage is potentially enhanced. For now, the demand curve does not allow negative prices.

Storage Storage acts both as a source of supply and of demand, depending upon the price. Storage capacity is determined as a fixed one-off capital investment by representative

[4] While this dynamic is equally true in hydro-dominant systems, there is a key difference between storage and hydro-power, which is that equilibrium demand satisfied need not equal total generation (it can be more or less).

speculators. The investment decision takes into account the discounted sum of future profits from owning the storage capacity.

Generation We only consider intermittent generators, who, with a known capacity at every time period and with a zero marginal cost of generation, face a random weather event. The resolution of the weather shock at each time period determines the total generation. In the absence of curtailment (and of storage capacity restrictions), all electricity generated must be supplied to the market. In other words, the supply curve of *generation* is vertical at the generation level given current weather conditions.

Investment in Generation Capacity is made as a fixed one-off decision that takes into account the

Market Structure The market structure we consider is that of a spot or day ahead market, which is becoming common the world over.[5] The interaction between random demand and supply in our model can lead to many scenarios, including significant over or under-supply (respectively on days with too much demand and too little wind and vice-versa). In contrast to many other settings with either no uncertainty at all (Abrell et al. (2019); Helm and Mier (2018)) or only a high-low type of uncertainty in supply (Pommeret et al. (2019)), our model not only accommodates the full range of scenarios but also ensures that the resulting market price takes into account these dynamics. The market outcomes here represent a benchmark for a decentralised setting, and its structure is consistent with both the energy and the speculative storage literature.

Curtailment For the initial version of the model, we do not allow for curtailment of intermittent generation. With the introduction of unlimited storage, clearly curtailment will never be needed, since prices can never go negative (even with round-trip losses, it is clearly never sensible for the speculators to not store all available energy at non-positive prices). Two cases may however lead to curtailment being necessary: (i) without storage, it is profitable to curtail when prices can go negative i.e. in cases where demand curve can accommodate negative prices; and (ii) with storage, when there is a fixed storage capacity (since negative prices are possible when capacity limits are binding). In any case, curtailment is also determined in equilibrium and will be considered in a later analysis.[6]

[5]Many EU countries operate and share markets of this form e.g. the Nordic and Baltic countries together participate in the NORDPOOL market, France and Germany in the EPEX-SPOT and Belgium and Netherlands in the APX-ENDEX; many regional spot markets in the U.S. share many of these characteristics as well; and the U.K. also operates a day-ahead market of the form considered here, the N2EX.

[6]It is important to note that in reality, curtailment is largely driven by two considerations: (i) grid capacity and system frequency considerations, with this reason being by far the most important one for curtailment (citations); (ii) contractual obligations–e.g. minimum offtake contracts– with existing conventional fossil generation plants (citations). In any case, at present, curtailment often occurs due either to a mix of intermittent and non-intermittent generation, and limited transmission capacity. In most models of generation, the latter reason is ignored and only the former is incorporated, an approach we will follow.

3.2 Model

3.2.1 Model environment in renewable only model

We now proceed to formally describe the model. Let $t = 0, 1, 2, \ldots$ index time in the model. Each time period represents an hour.

We begin with the structure of probability in the model. Let all uncertainty be defined on a probability space $(\Omega, \Sigma, \mathbb{P})$. Let $(z_t)_{t=0}^{\infty}$ and $(e_t)_{t=0}^{\infty}$ be two random variables defined on $(\Omega, \Sigma, \mathbb{P})$ with support Z and E respectively. The shocks (z_t) are generator supply shocks and the shocks (e_t) are consumer demand shocks.

Assumption 1. *Uncertainty satisfies:*

1. *the random variables (z_t) are independently and identically distributed (i.i.d) with support $[0, 1]$*
2. *the random variables (e_t) are i.i.d demand shocks with finite variance.*

We will use the term $(\mathscr{F}_t)_{t=0}^{\infty}$ to denote the filtration generated by the sequence $(z_t, e_t)_{t=0}^{\infty}$. Throughout the paper, whenever we use equalities or inequalities involving random variables defined on $(\Omega, \Sigma, \mathbb{P})$, the inequalities will hold $\mathbb{P}$ almost everywhere.

There is a continuum of two agents: generators and speculators. Both generators and speculators face a two-stage problem. At the beginning of time 0, before shocks have been realised, the generators decide upon a name plate generation capacity K that can be purchased at price P_K that we assume does not depreciate through time. Similarly, storage providers decide on a storage capacity (number of batteries) $\bar{S}$ that does not depreciate through time and costs $P_{\bar{S}}$. After the first stage decisions have been made, generators inelastically supply $z_t K$ of electricity to the spot market. Storage providers can also storage electricity up to the level $\bar{S}$. We assume the marginal cost of generating renewables is zero in each period, thus generators would supply up to their capacity if prices are non-negative.

At each t, storage providers make a second stage decision about how much total power to take into the next period. The storage providers' choices are constrained by, first, an upper bound $\bar{S}$ on total storage. Storage also depreciates period to period and the stored electricity taken by speculators in from the previous period into period t is $(1-\gamma)S_t$. Storage providers also face a constraint on how much power can be charged/ discharged in each period. That is, stored power must satisfy:

$$-\zeta\bar{S} \leqslant (1-\gamma)S_t - S_{t+1} \leqslant \zeta\bar{S}, \qquad \zeta \in (0,1), \tag{1}$$

where $\Delta S_t := S_{t+1} - (1-\gamma)S_t$ represents the net change in storage between periods $t, t+1$. When $S_{t+1} - (1-\gamma)S_t > (<) 0$, the battery is charging (discharging) in period t.[7]

[7] We note that our model is directly written in terms of the next period stock, S_{t+1}, rather than the amount charged or discharged. This is due to the fact (clarified in §) that (as in many dynamic stochastic frameworks) we can directly control S_{t+1}.

Finally, storage providers face round-trip losses. Let $\phi(x) = \frac{\eta_1}{1+e^{-\eta_2 x}} + \eta_3$ be a logistic round-trip loss function, where x is the change in storage, which can be positive or negative. If at time t, storage providers decide to increase their stock of stored power, they need to purchase $\phi(S_{t+1} - (1-\delta)S_t)$(a value greater than 1) times the amount of energy "pumped" into storage to make up for losses incurred by taking power from the grid and placing it in storage. Conversely, if at time t, storage providers decide to use their stored power and sell it on the market, then $\phi(S_{t+1} - (1-\delta)S_t)$(a value less than 1) times the amount of power pumped out of storage will become available on the spot market. Each period, prices must clear the market with no free disposal. The round trip looses will imply the following market clearing condition:

$$D(P_t, e_t) - z_t K = \phi(\Delta S_t)(\Delta S_t) \tag{2}$$

The expression says that total new electricity storage (or discharged into the market), subject to round-trip losses, must equal the balance of generation and consumer demand. To ease the length of notation, we will define:

$$\Xi(\Delta S_t) :\; = \phi(\Delta S_t)(\Delta S_t) \tag{3}$$

as the total power bought/sold on the spot market by storage providers at any time t.

Turning now to consumer demand, we assume a demand function $D\colon \mathbb{R}^2 \to \mathbb{R}$, whose first argument is price and second argument is an exogenous shock.

Assumption 2. *The demand function takes the form $D(P_t, e_t) = \bar{D}e^{e_t}P_t^{-\frac{1}{\eta}}$, where $\bar{D}$ is an exogenous demand shifter and $-\eta$ is the elasticity of demand.*

Note the function D is invertible in the first argument and $\lim_{x\to\infty} D(x,e) = 0$ for all e. Let H denote the inverse of D.

3.2.2 Second stage maximisation problem

We now turn to the speculators' second stage maximization problem. After the decision for the level of storage capacity has been made, the speculators choose a sequence of history contingent storage decisions to maximize the net present value of their profits. Thus, we have defined the second stage problem as a sequential problem. We will later show that the sequential equilibrium is equivalent to a recursive equilibrium, where firms follow policy rules that define next period storage based only on the current period state. A sequential equilibrium depends on the states in all periods to date.

The speculators' maximization problem can be written as:

$$\max_{(S_t)_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t P_t \Xi(\Delta S_t) \tag{4}$$

such that $(S_t)_{t=0}^{\infty}$ is a sequence of random variables adapted to the filtration $(\mathscr{F}_t)_{t=0}^{\infty}$, $0 \leqslant S_t \leqslant \bar{S}$ and (1) holds almost everywhere.

We characterize the equilibrium using first order conditions of the storage speculators. Begin by noting that for each t, speculators maximize (4) if and only if speculators maximize the "one period deviation problem":[8]

$$\max_{\bar{S} \geqslant S_{t+1} \geqslant 0} P_t \Xi\left(\Delta S_t\right) + \beta \mathbb{E}_t \left\{P_{t+1} \Xi\left(\Delta S_{t+1}\right)\right\} \tag{5}$$

such that S_{t+1} is $\mathscr{F}_t$ measurable and (1) holds. The first order condition yield:

$$\begin{aligned}
&(BD) \quad P_t \Xi'\left(\Delta S_t\right) \geqslant \beta(1-\gamma)\mathbb{E}_t P_{t+1} \Xi'\left(\Delta S_{t+1}\right) \quad \text{if} \quad S_{t+1} = \max\{(1-\gamma)S_t - \zeta\bar{S}, 0\} && \text{(6a)}\\
&(I) \quad P_t \Xi'\left(\Delta S_t\right) = \beta(1-\gamma)\mathbb{E}_t P_{t+1} \Xi'\left(\Delta S_{t+1}\right) \quad \text{if} \quad S_{t+1} \geqslant \max\{(1-\gamma)S_t - \zeta\bar{S}, 0\} && \text{(6b)}\\
&(BC) \quad P_t \Xi'\left(\Delta S_t\right) \leqslant \beta(1-\gamma)\mathbb{E}_t P_{t+1} \Xi'\left(\Delta S_{t+1}\right) \quad \text{if} \quad S_{t+1} = \min\{\zeta\bar{S} + (1-\gamma)S_t, \bar{S}\} && \text{(6c)}
\end{aligned}$$

The first order conditions are divided into three regions: the speculators' constraint binding while discharging (BD), an interior region where constraints do not bind (I) and the constraint binding while charging (BC). The three regions are represented in the figure below.

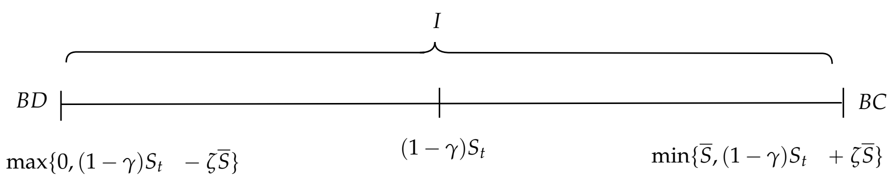


Figure 1: Equilibrium regions for values of S_{t+1} within each period.

Discharge constraint binds When the storage constraint binds at BD and speculators cannot discharge any more, the price of selling power ($P_t\Xi'(\Delta S_t)$) cannot be less than the benefit of holding onto the power till the next period ($\beta(1-\gamma)\mathbb{E}_t P_{t+1}\Xi'\left(\Delta S_{t+1}\right)$). This is because speculators have no incentive to deviate to the right of the point BD and discharge less power. On the other hand, speculators may have an incentive to deviate to the left of the point BD and discharge more power, but are not able to do so because of the binding constraint. This implies (6a) may hold with a strict inequality and speculators may make profits here.

Interior Turning to the case where a point in region I is an equilibrium, speculators have no incentive to move to the left or right of the equilibrium point. There is no marginal

[8]The proof of this statement is straight-forward and will be provided in an appendix

profit from deviation, hence the price of buying/selling power today must equal the benefit of holding onto the power till the next period and (6b) holds with equality

Charge constraint binds Finally, when the equilibrium point is where the storage constraint binds at BC and speculators cannot charge anymore, the price of buying power today ($P_t\Xi'(\Delta S_t)$) cannot be more than the benefit of holding power till the next period. This is because speculators have no incentive to deviate to the left of point BC and charge less power. On the other hand, speculators may have an incentive to deviate to the right of the point BC and charge more power, but are not able to because of the binding constraint. This implies (6c) may hold with a strict inequality and speculators may also make profits here.

Note speculators only make second stage profits when the storage capital choice limits how much they can charge/ discharge. When we turn to the first stage problem, the cost of storage capital in the first period will equal the expected discounted value of these second stage-profits.

Before turning to the first stage problem, we can can write the first-stage problem in a more tractable form as follows as:

$$\begin{aligned}
&P_t\Xi'\left(\Delta S_t\right) = \min\{\max\{ \\
&\quad \beta(1-\gamma)\mathbb{E}_t P_{t+1}\Xi'\left(\Delta S_{t+1}\right), H(z_tK + (1-\gamma)S_t - \underline{B}_t, e_t)\}, H(z_tK + (1-\gamma)S_t - \bar{B}_t, e_t)\} \quad (7)
\end{aligned}$$

where $\underline{B}_t = \max\{(1-\gamma)S_t - \zeta\bar{S}, 0\}$ and $\bar{B}_t = \min\{\zeta\bar{S} + (1-\gamma)S_t, \bar{S}\}$.

3.2.3 First stage problem

We first present a heuristic derivation of the first stage problem for the stock of storage. A formal proof will be presented the Appendix.

To evaluate the value of a marginal increase in storage capital, we need to evaluate the marginal returns to equilibrium storage and how a marginal increase in storage capital effects equilibrium storage. Let $\bar{\Pi}_t$ denote be the expected marginal return of equilibrium storage providers per period:

$$\bar{\Pi}_t = -P_t\Xi'(\Delta S_t) + \beta(1-\gamma)\mathbb{E}_t P_{t+1}\Xi'(\Delta S_{t+1})$$

Following the discussion of eq.6(a)-(c), the marginal value storage capacity is strictly positive only when either the battery capacity or charge/discharge rate constraints are binding. Moreover, when the charge/discharge rates constraints are binding and the parameter ζ is not one, then the marginal effect of storage capital on equilibrium storage will depend on ζ; ζ will be the marginal change in equilibrium storage from an increase in storage capital. Consequently, the marginal value to storage providers of relaxing the constraint $\bar{S}$ in any period will be:

$$
\begin{aligned}
-\zeta\bar{\Pi}_t \qquad & \text{if} \qquad (1-\gamma)S_t - S_{t+1} = \zeta\bar{S} && \text{(8a)}\\
\zeta\bar{\Pi}_t \qquad & \text{if} \qquad S_{t+1} - (1-\gamma)S_t = \zeta\bar{S} && \text{(8b)}\\
\bar{\Pi}_t \qquad & \text{if} \qquad S_{t+1} = \bar{S} && \text{(8c)}
\end{aligned}
$$

The three cases above correspond to the three cases when speculators can gain positive profits. Equation (8a)) corresponds to the case when no more power can be discharged (charged) due to the pipe constraint; here the marginal value of expanding the stock of storage capital will be the marginal effect of expanding the stock of storage capital on how much extra can be discharged (charged), the charge/discharge rate ζ, multiplied by the marginal profit made from the discharge (charge), $-(+)\bar{\Pi}_t$. And eq. (8c) corresponds to the case when no more power can be charged due to the overall battery capacity.

The storage capital decision will be characterized by the point where the discounted sum of marginal returns from increasing the stock of storage capital equals the price of storage capital, $P_{\bar{S}}$. To ease notation, denote by $\mathcal{D}, \mathcal{R}, \mathcal{C}$ regions where profits are made and discharge occurs (Equation (8a)), charge occurs (Equation (8b)), and capacity is reached (Equation (8c)), respectively. The storage capital decision will then be characterised by:

$$\sum_{t=0}^{\infty} \beta^t \Big[-\mathbb{E}\left(\mathbb{1}_{\mathcal{D}}\right)\zeta\bar{\Pi}_t + \mathbb{E}\left(\mathbb{1}_{\mathcal{R}}\right)\zeta\bar{\Pi}_t + \mathbb{E}\left(\mathbb{1}_{\mathcal{C}}\right)\bar{\Pi}_t \Big] = P_{\bar{S}} \tag{9}$$

where $\mathbb{1}\{.\}$ is the indicator function for the event (set) in question.

The first stage problem for the renewable energy generators is more straight-forward. The maximization problem is:

$$\max_K \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \left(P_t z_t K\right)\right] - P_K K \tag{10}$$

where P_K is the cost of generation capital. This yields the following condition characterizing the equilibrium choice (assuming stationarity):

$$\frac{1}{1-\beta}\mathbb{E}\left(P_t z_t\right) = P_K \tag{11}$$

The following defines the equilibrium in the economy:

Definition 3.1. A sequential equilibrium is a tuple $\mathscr{E} = \left((S_t)_{t=0}^{\infty}, (P_t)_{t=0}^{\infty}, \bar{S}, K\right)$ such that:

1. the sequences $(P_t)_{t=0}^{\infty}$ and $(S_t)_{t=0}^{\infty}$ are sequences of random variables adapted to $(\mathscr{F}_t)_{t=0}^{\infty}$
2. given $(P_t)_{t=0}^{\infty}$, $\bar{S}$ and K, $(S_t)_{t=0}^{\infty}$ solves (4) (agents are optimizing in the second stage)
3. given (S_t) and and (P_t), $\bar{S}$ and K solves the storage capital and generation capital first stage problems respectively

4. the market clearing condition, (2), holds almost everywhere for each $t \in \mathbb{N}$.

A sequential equilibrium gives us stochastic sequences (S_t) and (P_t) defined on the probability space $(\Omega, \Sigma, \mathbb{P})$. However, for computation and analysis, we seek a recursive structure to the competitive equilibrium. That is, we seek a measurable pricing function function ρ and a measurable policy function σ such that:

$$P_t = \rho(z_t, e_t, S_t) \tag{12a}$$

$$S_{t+1} = \sigma(z_t, e_t, S_t) \tag{12b}$$

Before we move to characterizing such a recursive equilibrium, we formalize its definition.

Definition 3.2. A sequential equilibrium is a recursive equilibrium if there exists a Borel measurable function σ such that $S_{t+1} = \sigma(z_t, e_t, S_t)$.[9]

Note that if an equilibrium is a recursive equilibrium, then, we must have $P_t = H(Kz_t + (1-\gamma)S_t - S_{t+1}, e_t))$. And, a such, P_t can be written in the form (12a).

3.2.4 Functional equations characterizing the pricing function

By defining a recursive equilibrium in the previous section, we collapsed the study of an infinitely long, infinite dimensional sequence of the sequential equilibrium (s_t) to the study of one function, either the policy function σ, or the pricing function ρ. In this section we turn to how we can show that the pricing function is a solution to a functional equation that can be computed. The approach of this section mirrors that by Deaton and Laroque (1992) and is closely related to the literature on time iteration or Coleman-Reffett operators (see Ernesto Guerra et al. (2018)).

Assuming a recursive equilibrium exists, we seek a measurable pricing function such that $P_t = \rho(z_t, e_t, S_t)$ for all t. The FOC, equation (7) will imply (where primes indicate values in the next period and double primes in two periods' time):

$$\begin{aligned}\rho(z,e,s)\Xi'((1-\gamma)s - s') = \min \big\{ &\max\{\beta(1-\gamma)\mathbb{E}\rho(z',e',s')\Xi'((1-\gamma)s' - s''), \\ &H(zK + (1-\gamma)s - \underline{B}, e)\}, H(zK + (1-\gamma)s - \bar{B}, e)\big\}\end{aligned} \tag{13}$$

where $(1-\gamma)s - s' = \Xi^{-1}(D(\rho(z,e,s),e) - Kz)$, $(1-\gamma)s' - s'' = \Xi^{-1}(D(\rho(z',e',s'),e') - Kz')$ and $\underline{B} = \max\{(1-\gamma)s - \zeta\bar{S}, 0\}$ and $\bar{B} = \min\{\zeta\bar{S} + (1-\gamma)s, \bar{S}\}$.

The arguments of the pricing function are the lower case alphabets. The expectation on the right hand side of (14) can be taken with respect to e' and z'.

[9] The definition implies that a sequential equilibrium is a recursive equilibrium if the sequence of storage is a stochastic recursive sequence (see (Stachurski, 2009), Definition 9.2.3.

We can now define an operator $\mathbb{T}$ that takes a measurable pricing function and returns a pricing function $\mathbb{T}\rho$. Given ρ, $\mathbb{T}\rho(z,e,s)$ satisfies:

$$\begin{aligned}\mathbb{T}\rho(z,e,s)\Xi'((1-\gamma)s-s') = \min\Big\{ & \max\{\beta(1-\gamma)\mathbb{E}\rho(e',z',s')\Xi'((1-\gamma)s'-s'') \\ & ,H(zK+(1-\gamma)s-\underline{B},e)\} \\ & \qquad\qquad ,H(zK+(1-\gamma)s-\bar{B},e)\Big\}\end{aligned} \tag{14}$$

where $(1-\gamma)s-s' = \Xi^{-1}(D(\mathbb{T}\rho(z,e,s),e)-Kz)$ and $(1-\gamma)s'-s'' = \Xi^{-1}(D(\rho(z',e',s'),e')-Kz')$. An equilibrium pricing function will solve $\rho = \mathbb{T}\rho$.

3.2.5 The social planner's problem and existence

So far, we have assumed a recursive solution does indeed exist and showed how it can be characterised. The previous section did not, however, show that a solution to the planning problem exists. In this section, we turn to the question of existence. The standard approach to showing existence, used by, for instance, Deaton and Laroque (1992), is to show that a fixed point to the operator $\mathbb{T}$ exists. However, in our study, the study of existence must also include a solution to the first stage problem. Thus, we depart from the standard approach and proceed to show the existence of a sequential competitive equilibrium first. We then recover the recursive competitive equilibrium through a projection argument discussed in Shanker (2017), which, to the best of our knowledge is novel.

To show the existence of a sequential competitive equilibrium, we first show existence to a planning problem and then prove that the planners solution is in-fact the sequential competitive equilibrium. Let us start by considering the planning problem as the sum of producer plus consumer surplus. Such a formulation of the planning problem is a natural starting point in analysis of welfare in models with complete and incomplete markets (for example, see Gowrisankaran et al. (2016), equations (5) and (6)). The discounted sum of consumer and producer surplus can be defined as:

$$
\begin{aligned}
W((S_t, K, \bar{S}) :&= \underbrace{\mathbb{E}\sum_{t=0}^{\infty}\beta^t \int_{P_t}^{\infty} D(p, e_t)\, dp}_{\text{Consumer surplus}} + \overbrace{\mathbb{E}\sum_{t=0}^{\infty}\beta^t P_t K z_t - P_K K}^{\text{Generator surplus}} \\
&\quad + \underbrace{\mathbb{E}\sum_{t=0}^{\infty}\beta^t P_t((1-\gamma)S_{t+1} - (1-\gamma)S_t) - P_{\bar{S}}\bar{S}}_{\text{Speculator surplus}} \\
&= \mathbb{E}\sum_{t=0}^{\infty}\beta^t \left\{ \int_{P_t}^{\infty} D(p, e_t)\, dp + P_t D(P_t, e_t) \right\} - P_K K - P_{\bar{S}}\bar{S} \qquad (15) \\
&= \mathbb{E}\sum_{t=0}^{\infty}\beta^t \int_{0}^{D(P_t, e_t)} H(y, e_t)\, dy - P_K K - P_{\bar{S}}\bar{S} \\
&= \sum_{t=0}^{\infty}\beta^t \int_{0}^{K z_t + (1-\gamma)S_t - S_{t+1}} H(y, e_t)\, dy - P_K K - P_{\bar{S}}\bar{S}
\end{aligned}
$$

The first equality uses the standard definition of consumer and producer surplus. The second equality follows from collecting terms in the infinite sums and the per period market clearing condition from eq. (2) (with demand expressed as the sum of supply and change in storage). The third equality follows from properties of integrals of inverse functions and the final equality also follows from the market clearing condition.[10]

It is also possible to express the welfare function in terms of a direct utility function, U, which will be more amenable to an intuitive understanding of welfare and connect the welfare solution explicitly to the decentralised. To do so, define U as follows

$$
U(x, e_t) := \bar{D}^{-\theta} e^{\frac{e_t}{\theta}} \frac{x^{1-\theta}}{1-\theta}, \qquad (16)
$$

where $\theta = \frac{1}{\eta}$. Now, computing the integral on the RHS on the final line of (15), we can

[10]The law of integral of inverse function states that:

$$
\int_c^d f^{-1}(x)\, dx = -\int_a^b f(y) dy + bd - ac
$$

where $f(a) = c$ and $f(b) = c$. Applying this rule to our case gives:

$$
\begin{aligned}
\int_{P_t}^{\infty} D(p, e_t)\, dp + P_t D(P_t, e_t) &= \int_{H(D(P_t, e_t), e_t)}^{D(0, p_t)} D(p, e_t)\, dp + P D(P_t, e_t) \\
&= -\int_{D(P_t, e_t)}^{0} H(y, e_t)\, dy
\end{aligned}
$$

write welfare for a sequence (S_t) and choice of capital stocks as:[11]

$$W((S_t), K, \bar{S}) = \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t U\Big(Kz_t + (1-\gamma)S_t - S_{t+1}\Big)\right] - P_K K - P_{\bar{S}}\bar{S}. \tag{18}$$

Definition 3.3. The social planner's problem (SPP) is to maximise (18) with respect to $\{(S_t), \bar{S}, K\}$ such that (1) holds, $S_t \leqslant \bar{S}$ and $S_t \in m\mathscr{F}_t$ for each t.

We are now ready to connect the planner's problem to the decentralised market allocation. In the absence of externalities or other market imperfections, the planner's choices will be reproduced by the market, and these two problems are linked by the spot price. To see this connection more explicitly, let $\{(S_t), \bar{S}, K\}$ be a solution to the social planner's problem. The solution generates a sequence of prices as follows:

$$P_t = H\Big(Kz_t + (1-\gamma)S_t - S_{t+1}, e_t\Big), \tag{19}$$

yielding the sequential market equilibrium consisting of the 4-tuple $\{(S_t), \bar{S}, K, (P_t)\}$.

Proposition 3.1. *The tuple $\{(S_t), \bar{S}, K\}$ is a solution to the SPP if and only if the tuple $\{(S_t), (P_t), \bar{S}, K\}$, with (P_t) satisfying,* (19) *is a sequential competitive equilibrium*

Theorem 3.1. *If Assumptions 1 - 2 hold, then there exists a solution (S_t) to the social planner's problem*

Since there exists a solution to the social planner's problem, by the above theorem, there exists a sequential competitive equilibrium. Our next step is to connect the sequential competitive equilibrium to a recursive competitive equilibrium. Consider a solution to the sequential SPP as a sequence $\{\bar{S}, K, (Y_t)\}$ and construct the sequence:

$$\begin{aligned} S_0 &= Y_0 \\ S_t &= \mathbb{E}(Y_t | S_{t-1}, e_{t-1}, z_{t-1}) \qquad \forall t > 0 \end{aligned} \tag{20}$$

Lemma 3.1. *If $\{(Y_t), \bar{S}, K\}$ is a solution to the SPP and (S_t) satisfies* (20)*, then $\{(S_t), \bar{S}, K\}$ is a solution to the SPP*

Note that (S_t) has recursive in the sense that $S_{t+1} = g(S_t, e_t)$ for a measurable function g by the proprieties of conditional expectation at (20). Moreover, let (P_t) satisfy (19), then $P_t = \rho(S_t, e_t, z_t)$ for a measurable function ρ

[11] The definition of welfare, as the sum of consumer and producer surplus is connected to the notion of Value of Lost Load as follows:

$$VOLL = \frac{\int_{P_t}^{\infty} D(p, e_t)\, dp + P_t D(P_t, e_t)}{D(P_t, e_t)} \tag{17}$$

Thus gross consumer and producer surplus is equal to Demand × VOLL (see Gowrisankaran et al. (2016), equation (6)).

Corollary 3.1. *If* $\{(Y_t), \bar{S}, K\}$ *is a solution the SPP and* (P_t) *and* (S_t) *satisfy* (20) *and* (19)*, then* $\{(S_t), \bar{S}, K\}$ *is a recursive competitive equilibrium*

Finally, we present the main existence proof for a recursive competitive equilibrium.

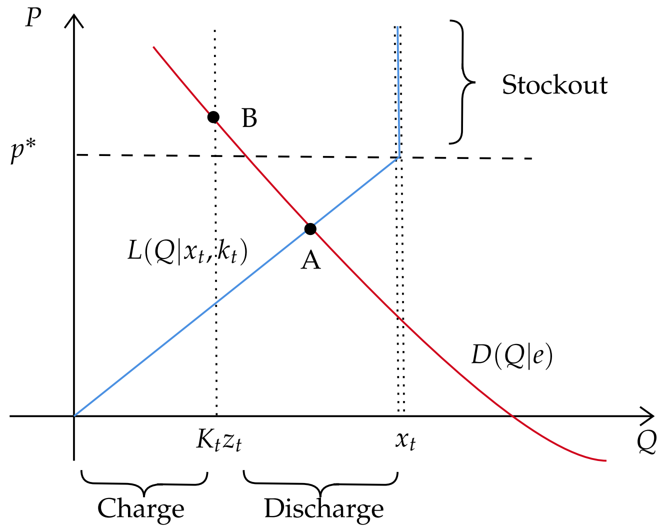
Theorem 3.2. *If Assumptions 1- 2 hold, then there exists a recursive competitive equilibrium,* $\{(S_t), (P_t), \bar{S}, K\}$.

Proof. By Theorem 3.1, there exists a solution to the social planner's problem, which we denote by $\{(Y_t), \bar{S}, K\}$. Now construct a sequence (S_t) satisfying (20). By Lemma 3.1, $\{(S_t), \bar{S}, K\}$ will also be a solution to the social planner's problem. Moreover, letting (P_t) satisfy (19), by Proposition 3.1, $\{(S_t), (P_t), \bar{S}, K\}$ will be a sequential competitive equilibrium. Note the sequence (S_t) satisfies (20), thus there exists a measurable function ρ such that $S_{t+1} = \rho(S_t, e_t, z_t)$ for each t. The tuple $\{(S_t), (P_t), \bar{S}, K\}$ thus satisfies the definition of a recursive competitive equilibrium. $\square$

Corollary 3.2. *There exists a fixed point to the operator* $\mathbb{T}$

3.3 Graphical analysis of supply-demand curves

Before we begin a formal computational and mathematical analysis, we develop graphical intuition of the modelled market based on our existing understanding of storage dynamics. We know from Deaton and Laroque (1992) that there exists $p^\star$ such that for any price greater than $p^\star$, it is not profitable for storage providers to carry any stocks to the next period. For now, we assume $p^\star$ does not depend on the demand shock. Thus, for equilibrium prices below $p^\star$, some stock of power will be carried over to the next day. If the equilibrium quantity demanded, D, is above $K_t z_t$, then batteries will discharge. On the other hand, if the equilibrium demand is below $K_t z_t$, batteries will charge. For demand shocks that generate prices greater than $p^\star$, there will be a "stock out", in that all available power will be discharged to meet demand.



The hypothesized market equilibrium is characterized by Figure 3.3. The figure characterizes the supply/ demand curves for a given realization of the state of the system — the shocks and stock of capital. The function D is the demand curve in kWh (red line), the vertical line at $K_t z_t$ represents the inelastic supply (in kWh) with no storage and the vertical line x_t is the stock at hand (i kWh). The function L (blue line) gives the supply curve of power supplied to meet consumer demand, again conditioned on the state. In terms of the sequential equilibrium, the function L gives the quantity $x_t - S_{t+1}$ for different prices (not necessarily the equilibrium price).

The intersection point of L with the demand curve, point A (at the price where (2) holds), gives the equilibrium with storage while the point B is the equilibrium without storage. The equilibrium at A is an equilibrium with "high" demand, where prices are depressed compared to the case without storage. An equilibrium with low demand, in the charge region, on the other hand, would have higher prices and lower quantity supplied than without storage. Understanding the dynamics of the equilibrium between the charge and discharge regions will be a key focus of our analysis: presumably, a feasible path must spend an "equal" amount of its time in the charge and discharge region for storage not to explode or converge to zero but that is among the aspects to be established. Of course, to truly understand how the market quantity is affected by storage, one would also need to relate the level of K_t to the availability of storage.

Another point of focus of our analysis will be the shape and volatility of the curve L. Under the assumption that $p^\star$ is fixed, observing the possible share of L, a high level of expected x_t would reduce the volatility of L. Note, however, the variance of x_t is bounded below by $z_t K_t$ at-least in the case of i.i.d shocks. We can derive an implicit expression for L given a recursive equilibrium. Recall that if S_{t+1} is a recursive solution, then there exists a function ϕ mapping the current state to the stock of power tomorrow. The function $q \mapsto L(q|x, k)$ is then given by the root of the function:

$$p \mapsto x - \phi(x, E(q, p), k) - q \tag{21}$$

where $E(q, p)$ is the solution e to the equation $q = D(e, p)$. The root of the function above is the price that would clear supply given a quantity q, implicitly defined by the demand shock that would generate a demand for q at the clearing price. A key aspect of the subsequent formal analysis will be to understand the shape and variance of the function L defined above.

4 Quantitative results

In this section, we present preliminary simulation results for our model. For these preliminary simulations, we assume mean intermittent generation capital is exogenous, equal to the current total non-intermittent capacity in Swedish grid region 4. That is, 6.7 giggawatts (Gw). While Swedish region 4has 6.7 Gw of capacity, equilibrium demand is on average, 1.51 Gw. The excess capacity is dispatched in response to a positive demand shock. In our simulation, we compute the equilibrium market level of storage capacity investment. Using the simulations, we ask whether storage can play the role of dispatchable power and allow the demand profile to look like what it is today — with equilibrium demand responding mainly to demand, rather supply shocks.

The simulations below are carried out assuming $\beta = .96$, the supply shocks have a beta distribution with mean .5 and variance ν. We de-trend the Swedish demand data and discretize the distribution of the demand residual, drawing our demand shocks from this discrete distribution. The investment adjustment cost function is assumed to be the identity function. Demand given a shock e and price p is assumed to be $D(p, e) = \exp^{\frac{e}{\zeta}} p^{\frac{-1}{\zeta}}$, ζ is 2 and the parameter γ is held at .05.[12] Further details on the computational procedure can be found in Appendix A

[12]To compute a solution, we follow a modified time-iteration algorithm. In particular, guessing ρ_0 and σ_0, we iterate on the sequence $\rho_{t+1} = \mathbb{T}(\rho_t, \sigma_t)$ and $\sigma_{t+1} = \mathbb{C}(\rho_{t+1}, \sigma_t)$ till (ρ_t) and (σ_t) both converge in the supremum norm. At each iteration, we use piece-wise linear interpolation to approximate the functions.Using package interpolation. See `https://github.com/EconForge/interpolation.py`. The grid-size for capacity and stock at hand is 1000. Simulations are run on a Google Cloud 96 core Intel Skylake Compute Engine. Code can be found at `https://github.com/akshayshanker`.

	Kz_t(Gw)	$\bar{S}$ (GwH)	P(MUSD/GwH)	D (Gw)	S (GwH)	$P_S \mid P_K$
Today	1.51 (.15)	0.00	.05(.11)	1.51 (.15)	0.00 (0.00)	465.26 ∣ 1400
No storage	6.70 (5.09)	0.00	7×10^7 (1.4×10^8)	6.70 (5.09)	0.00 (0.00)	$9.3\times10^{12} \mid 6\times10^6$
Baseline	6.70 (5.06)	20.68	0.01 (0.01)	6.00 (3.60)	18.82 (2.53)	465 ∣ 162
High variance	6.69 (6.28)	123.68	0.07 (0.01)	1.08 (0.23)	122.39 (1.94)	468 ∣ 5363
Low s cost	6.71 (5.05)	28.36	0.00 (0.00)	5.93 (3.02)	25.13 (3.79)	200.57 ∣ 163.24

Table 1: Supply shocks have a beta distribution. The no storage, baseline and low s cost scenario have a supply shock variance of .1. The High variance scenario has a variance of .2. The columns are denote generation, storage capacity, mean (std. dev.) spot market price, mean (std. dev.) equilibrium demand, mean (std. dev.) storage level and the shadow value of storage capacity and generation capacity in the first stage problem.

Table 1 gives the result our simulations for five scenarios. The first line is the profile of the market today, average demand and generation is 1.51 Gw (while total capacity is 6.7 Gw), the mean price is .05 Million USD per GwH, the level of storage is zero and the price of intermittent generation capital today is 1400 USD per Gw.

No storage scenario. The second line gives the result for the case where there is no storage, we simply set the price of storage, P_S to $9.3 \times 10^{12} USD/Gw$, which is sufficiently high for close to zero investment in storage capacity. Without storage, average demand is also 6.7 Gw, since all intermittent power is cleared in the market and supplied to consumers. However, as expected, demand and price variance is high, with a standard deviation of 5.09. We also note the case without storage has a very high shadow value of capital. This reflects a high willingness to pay for power when demand shocks are high and intermittent generation low.

The baseline. The third lines gives a baseline scenario with endogenous storage capacity and a "low" standard of supply of .1. In this scenario, storage capacity rises to just below three hours of power (each time step is one hour in our model). Storage serves to dispatch power somewhat, and the variance of equilibrium demand falls from 5.90 in the no storage scenario to 3.60 in the scenario with storage. However, in this case, the shadow value of generation capital falls to 162 USD per Gw, much below its current price, implying a fall in generation capacity if that were to also be determined in the market.

The high variance scenario. The fourth lines gives a scenario where the variance of the supply shocks is now doubled to .2. In this case, the variance provides increased arbitrage opportunities to storage providers and increases the returns to storage investment. Storage capacity increases to around one hundred days of equilibrium demand (capacity of 122.39 GwH); here storage functions to effectively smooth the effect of the supply shocks on equilibrium demand. The level and variance of demand falls to levels similar to today. The shadow value of capital also rises with higher supply variance.

Low storage cost scenario. In the final line, we compare the effect of doubling the variance of the supply shocks to reducing the price of the storage capital to 200 USD per GwH. While storage capacity does increase, we find the effect modest and not close the the magnitudes required to reduce equilibrium volatility to the levels seen without intermittent supply.

Figures 2 and 3 show us the distribution of price and equilibrium demand under the four scenarios. The case with no storage generates a high power price along with prices of a very high magnitude. The baseline level of storage serves to reduce the price and price volatility, however, only the levels of storage under the high shock variance scenario reduce the volatility of the market to close to levels in the maretk today without intermittent supply.

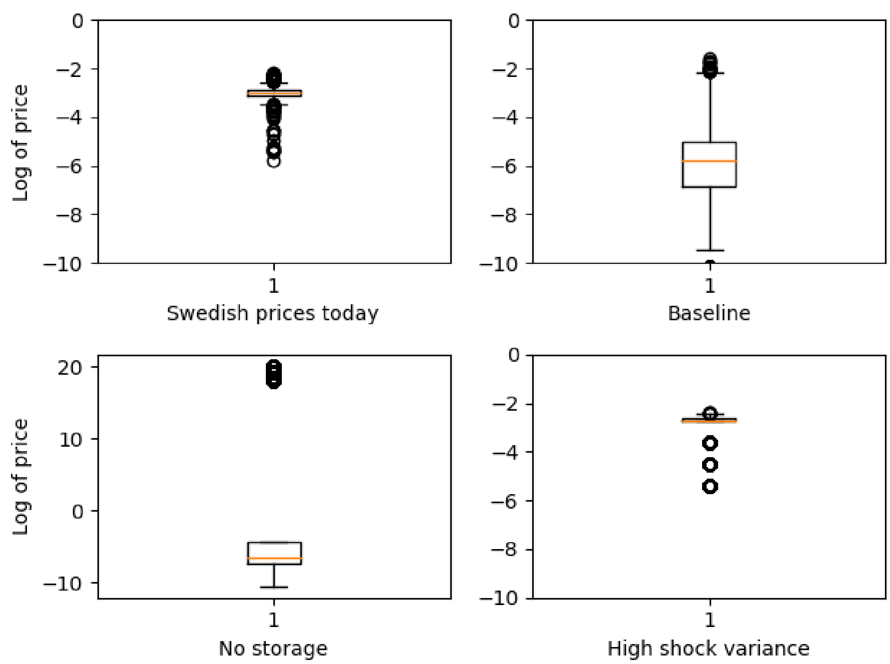


Figure 2: Box and whisker plots for equilibrium price.

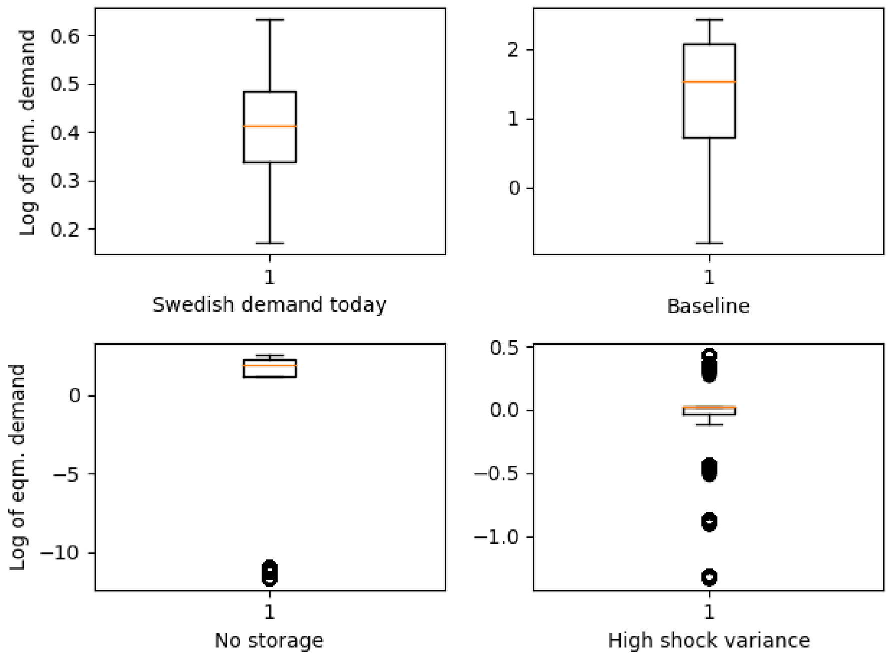


Figure 3: Box and whisker plots for equilibrium demand.

Finally, another way to understand how storage affects the relationship between equilibrium power and supply intermittent is to examine the co-variance between equilibrium demand and the supply shocks in table 2. In a market with no intermittent supply, the co-variance between equilibrium supply and demand is zero. In a market with no storage and fully intermittent supply, the co-variance is one since supply is inelastic. The role of storage is to make equilibrium demand less dependent on supply, and we see this as the co-variance between equilibrium demand and supply falls to .2 in the case with high supply variability.

Scenario	Covariance
No storage	*1.00*
Baseline	0.97
High supply var	0.19
Halved storage cost	0.94

Table 2: Covariance of supply and equilibrium demand.

5 Conclusion

This paper presented a dynamic equilibrium model of electricity generation, storage and generation capacity investment. We demonstrated that the equilibrium model is well-defined and can be computed to help us understand the potential role of storage in addressing the challenges of integrating renewables into electricity markets. Through our computational experiments, we explored how demand and supply uncertainty affects the market equilibrium in the presence of storage. We found that storage may rise to levels needed to act as a source of dispatchable power. With sufficient storage capacity, the link between equilibrium demand and supply shocks can be broken, resulting in price and demand distributions similar to a market with no intermittent supply. However, investment in storage capacity requires there to be sufficient supply uncertainty to generate the rents for storage providers to invest in capacity in the first place.

A Appendix: Computational method

Under the assumption that (S_t) has a stationary distribution (to be established subsequently) and S_0 follows the equilibrium stationary distribution, the dependence on time of the summand at equation (9) disappears. Let $\psi^\star$ be the joint stationary distribution of $\{S_t, z_t, e_t\}$. To solve for the optimal storage decision, we can compute the root of $\frac{1}{1-\beta}\hat{\Pi} - P_{\bar{S}}$, where $\hat{\Pi}$ is defined as:

$$\hat{\Pi} = -\mathbb{E}_{\psi^\star}(\mathbb{1}_{\mathcal{D}})\,\zeta\bar{\Pi} + \mathbb{E}_{\psi^\star}(\mathbb{1}_{\mathcal{R}})\,\zeta\bar{\Pi} + \mathbb{E}_{\psi^\star}(\mathbb{1}_{\mathcal{C}})\,\bar{\Pi} \tag{22}$$

where $\mathbb{E}_{\psi^\star}$ is the expectation taken with respect to the the joint stationary distribution of storage and the shocks.

For given K and $\bar{S}$, let us use G to define the condition implied by (22), that is $G(K, \bar{S}) = \hat{\Pi}(\bar{S}, K) - P_{\bar{S}}$. Let us also use F to denote the condition implied by (11), that is, $F(K, \bar{S}) = \mathbb{E}\rho(\bar{S}, K)(z, s, e)z - P_K$. The functions G and F depend on $\bar{S}$ and K both through the equilibrium distribution of S_t and the pricing function ρ.

Our solution will be a value of K and $\bar{S}$ that is a root of G and F. For a given K, let $G^\star(K)$ be the root of $\bar{S} \mapsto G(K, \bar{S})$. Similarly, define $F^\star$ as the root of the function $K \mapsto F(K, \bar{S})$ for given $\bar{S}$. To find a solution, we can iterate $K_{i+1} = F^\star(G^\star(K_i))$ till the sequence (K_i) converges.

A more detailed pseudo-code for computation is as follows:

1. Guess K_0 and $\bar{S}_0$, set $i = 0$
2. Compute $G^\star(K_i)$ given $\bar{S}_i$ as the initial value of $\bar{S}$ using Brent's method. To compute $G(K_i, x)$ in each evaluation of Brent's method:
    - compute the fixed point ρ of $\mathbb{T}$ given K_i, x

- draw a sequence of size (S_t) of size $10,0000$
- evaluate $\hat{\Pi}$ using the sample (S_t) and function ρ

3. set $S_{i+1} = G^\star(K_i)$

4. Compute $F^\star(\bar{S}_{i+1})$ given K_i as the initial value of K using Brent's method. To compute $F(x, \bar{S}_{i+1})$ for each value of x iterated by Brent's method:

   - compute the fixed point ρ of $\mathbb{T}$ given $x, \bar{S}_{i+1}$
   - draw a sequence of size (S_t) of size $10,0000$
   - evaluate $\mathbb{E}Pz$ using the sample (S_t) and function ρ

5. set $K_{i+1} = F^\star(\bar{S}_{i+1})$

6. if $|K_{i+1} - K_{i+1}| >$ tol, then set $i = i + 1$ and return to 2

References

Abrell, J., Rausch, S., Streitberger, C., 2019. Buffering volatility: Storage investments and technology-specific renewable energy support. CER-ETH–Center of Economic Research at ETH Zurich, Working Paper 19, 310.

Ambec, S., Crampes, C., 2012. Electricity provision with intermittent sources of energy. Resource and Energy Economics 34 (3), 319–336.

Antweiler, W., 2018. The economics of grid-scale electricity storage: Location heterogeneity and business models.

Bobenreith H., E. S., Bobenreith H., J. R., Wright, B. D., 2002. A commodity price process with a unique continuous invariant distribution having infinite mean. Econometrica 70 (3).

Deaton, A., Laroque, G., 1992. On the Bheaviour of Commodity Prices. The Review of Economic Studies.

Ernesto Guerra, V., Bobenreith H., E. S., Bobenreith H., J. R., 2018. Comments on : On the behavior of commodity prices when speculative storage is bounded , 1–36.

Gowrisankaran, G., Reynolds, S. S., Samano, M., 2016. Intermittency and the Value of Renewable Energy Gautam Gowrisankaran. Journal of Political Economy 124 (4).

Heal, G., 2016. Notes on the economics of energy storage. Tech. rep., National Bureau of Economic Research.

Helm, C., Mier, M., 2018. Subsidising renewables but taxing storage? second-best policies with imperfect carbon pricing. Tech. rep., Oldenburg Discussion Papers in Economics.

Newbery, D., 2016. A simple introduction to the economics of storage: shifting demand and supply over time and space.

Newbery, D., Pollitt, M. G., Ritz, R. A., Strielkowski, W., 2018. Market design for a high-renewables european electricity system. Renewable and Sustainable Energy Reviews 91, 695–707.

Pommeret, A., Schubert, K., et al., 2019. Energy transition with variable and intermittent renewable electricity generation. In: CESifo Working Paper Series 7442. CESifo Group Munich.

Riesz, J., Milligan, M., 2015. Designing electricity markets for a high penetration of variable renewables. Wiley Interdisciplinary Reviews: Energy and Environment 4 (3), 279–289.

Scheinkman, J. A., Schechtman, J., 1983. A Simple Competitive Model with Production and Storage. The Review of Economic Studies 50 (3), 427–441.

Shanker, A., 2017. Existence of Recursive Constrained Optima in the Heterogenous Agent Neoclassical Growth Model. Working Paper.
URL `https://papers.ssrn.com/sol3/papers.cfm?abstract{_}id=3011662`

Sinn, H.-W., 2017. Buffering volatility: A study on the limits of germany's energy revolution. European Economic Review 99, 130–150.

Stachurski, J., 2009. Economic Dynamics: Theory and Computation. MIT Press Books, Cambridge, MA.

Wright, B. D., Williams, J. C., 1982. The Economic Role of Commodity Storage. The Economic Journal 92 (367), 596–614.

Zerrahn, A., Schill, W.-P., Kemfert, C., 2018. On the economics of electrical storage for variable renewable energy sources. European Economic Review 108, 259–279.