Overstatements in the grandfathering process - the opposite of tax evasion?*

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Abstract

The grandfathering process of an emissions trading scheme provides an example of informational asymmetries that require strict verification. Since this is costly the question arises in how far audit costs could be saved by introducing penalties for misreporting. The problem resembles those in the tax evasion context and is therefore solved following the approach that Chander and Wilde (1998) have used for the characterization of optimal income tax and enforcement schemes. It turns out that, in addition to policy recommendations concerning the efficient verification of firm data, the analysis provides interesting insights into the structural differences between problems where firms have an incentive to *overstate* their emissions, and problems where there is an incentive to *understate* the private data, as in the case of tax evasion. Although the structure is similar, results prove to be quite different for the two cases.

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1 Introduction

With the start of the EU emissions trading scheme in 2005, and further trading schemes in several states and under the Kyoto Protocol ahead, it comes apparent that this instrument has become one of the central means in complying with greenhouse gas emissions obligations. At the start of each of the firm-level trading schemes, initial allowances will be allocated to the firms, usually based on historic emissions. Since this initial allocation of greenhouse gas allowances to firms is primarily based on the firms' own emissions reports, it is commonly agreed that a thorough verification of these reports is necessary.

The problem of the verification of self-reported emissions data resembles the one of tax evasion, where firms have to pay a tax based on their (voluntarily) reported income. Following the approach that Chander and Wilde (1998) have used for the characterization of optimal income tax and enforcement schemes, I set up a principal-agent model with costly auditing, exogenously constrained penalties, and risk neutral firms and government. Verification schemes are introduced which consist of three elements: Firstly, allowances are allocated on the basis of the reports of the firms via an allocation function. Secondly, audit probabilities are determined for each firm, based as well on the reported data. Thirdly, penalties are imposed for overstatements.

An optimal verification scheme fulfills two requirements: high verification standards, which are in this model given by incentive compatibility, and efficiency, where an efficient scheme has the property, that, for a given penalty structure, incentive compatibility can not be achieved with less auditing. I derive and discuss the interplay between the allocation function, audit probability and penalty structure in efficient verification schemes.

The analysis deviates from the existing literature on emissions trading, where aspects like the comparison of emissions trading with other policy instruments or the international emissions trading scheme under the Kyoto Protocol received much attention, like in Hahn and Hester (1989), or Montero (2005), and others. It also does not belong to the field of compliance and enforcement[1], since firms incur no costs for reporting false data, unless they are audited and punished. The literature on the verification of firm data in emissions trading schemes mainly concentrates on technical or legal issues, not on economic efficiency. In the model that I present, the introduction of penalties for overstatements allows the reduction of audit costs. Too high penalties are, however, likely to be politically not desirable, and therefore the efficiency condition is further specified.

For the comparison with the results in the similar tax evasion model, I analyze a penalty that is proportional to the difference between the true and the reported value. Under such a penalty there are two allocation/audit-combinations that fulfill the required standards and in addition lead to efficiency. These combinations are (i) a regressive allocation of allowances combined with an audit probability that is actually decreasing in the reported parameter and given by a multiple of average allocations, and (ii) a progressive allocation of allowances combined with an audit probability that is increasing in the reported parameter and given by a multiple of marginal allocations. This implies in particular, that for a proportional allocation of emission allowances, which can be observed in reality, the audit probability should be the same for all reports, i.e. there is no need to audit higher reports stronger, if the penalty and audit probability are chosen respectively.

The corresponding result in the tax evasion model differs in two aspects. Firstly there is only one efficient tax/audit-combination, and secondly the

[1]For an overview over the extensive literature on this see Heyes (2000), and Kaplow and Shavell (1994) for a model with self-reporting of the compliance status, where the verification is conditioned on these reports.

audit probability in efficient schemes is determined by the marginal tax rate and never by average taxes.

The remainder of this paper is organized as follows: In chapter 2 the basic model is set up and the notion of efficiency is introduced. In chapter 3 I characterize efficient verification schemes in the grandfathering process of an emissions trading scheme. In chapter 4 I compare the results to the respective results of the corresponding tax evasion model and in chapter 5 I conclude.

2 Analytical Framework

In the model the true emission value $\theta \in [0, \bar{\theta}]$ of a firm is given exogenously and considered to be private information of the firm. Emission allowances are then grandfathered to the firm, solely based on its own report of its emissions. The model sets in at the point where the firm reports an emission value $\hat{\theta} \in [0, \bar{\theta}]$ to the government. Since the firm may have incentives to report a value $\hat{\theta} \neq \theta$, the government may prefer to perform costly audits and to impose penalties for misreporting. In this principal-agent framework it is assumed that the principal and agent are risk-neutral and that there is no commitment problem.

The government introduces a *verification scheme* $(e(\cdot), a(\cdot), f(\cdot))$, which consists of the *allocation function*,

$$e : [0, \bar{\theta}] \to \mathbb{R}_+, \tag{1}$$

the *audit probability*, which also depends on the firm's report $\hat{\theta}$,

$$a : [0, \bar{\theta}] \to [0, 1], \tag{2}$$

and the *penalty function* or *fine*, which depends on the firm's report $\hat{\theta}$ as well as the true emission value θ,

$$f : [0, \bar{\theta}] \times [0, \bar{\theta}] \to \mathbb{R}_+. \tag{3}$$

Initial allocations of emission allowances, as well as fines, are non-negative. All functions are assumed to be continuous, but differentiability is not required. Thus, a firm which reports $\hat{\theta}$ receives an initial amount of emission allowances $e(\hat{\theta})$. In addition, it will be audited with probability $a(\hat{\theta})$ and will then have to pay the fine $f(\hat{\theta}, \theta)$. It is assumed, that the true parameter is discovered, if a firm is audited[2]. The government, which determines the audit probability, also pays the audit costs[3].

In this model the penalty also consists of GHG allowances and represents the amount of allowances that is transferred from the firm to the government. *Expected allowances* for the firm are then given by initial allocations minus expected penalties:

$$\pi(\hat{\theta}, \theta) = e(\hat{\theta}) - a(\hat{\theta}) f(\hat{\theta}, \theta). \tag{4}$$

Since costs are lump sum and since allowances are grandfathered free of charge, it is the objective of the firm to choose $\hat{\theta}$ such that expected allowances are maximized.

The objective of the government is to construct the verification mechanism in an optimal way, where optimality is characterized by incentive compatibility and efficiency, and abstracts from redistributive considerations. The single components of the mechanism are required to fulfill certain social norms, which

[2] In addition to the assumption that it is technically possible to observe the true emission value, this includes the assumption that bribing does not take place.

[3] The placing of the burden of the actual audit costs onto the side of the government has the advantage that the question of the appropriate audit frequency is separated from the firms' problem of minimizing audit costs. This modeling does not conflict with the perception that audit costs should be paid by firms. In such a case the underlying assumption of lump sum payments from firms to the government holds.

will lead to a set of feasible verification schemes which is displayed below.

The choice of the penalty function plays a crucial role in the analysis. To start off, I consider the penalty function to be a choice variable of the planner, subject only to slight restrictions resulting from the discussions in the political context. It should provide neither a reward nor a fine for understating or truth-telling, i.e. $f(\hat{\theta}, \theta) = 0$ should hold for $\hat{\theta} \leq \theta$. This implies that honest reports are not treated differently depending on whether they are audited or not. In addition, the penalty should "fit the crime", i.e. the penalty function should be bounded, and it should be nondecreasing in the degree of misreporting.

Truthful reporting will always lead to nonnegative expected allowances, however, (4) may become negative if $\hat{\theta} > \theta$ and the allocation function is small, or the expected penalty high enough. Since allocations and penalties are measured in allowances and since firms cannot give more allowances than they receive, it is assumed that "negative allowances" can be transferred into monetary equivalents in order to assure that the penalty is credible.

The *feasibility conditions* are given by:

$$e(0) = 0, \tag{5}$$

$$e(\cdot) \not\equiv 0, \tag{6}$$

$$f(\hat{\theta}, \theta) = 0 \quad \forall \, \hat{\theta} \leq \theta, \tag{7}$$

$$f(\cdot) \text{ nondecreasing in the degree of misreporting and bounded} \tag{8}$$

A firm which reports zero emissions will not receive any allowances, which is given by equation (5). Equation (6) simply technically rules out the possibility of not allocating any allowances. (7) and (8) follow from above.

In addition, two technical conditions are introduced, which are necessary to make use of the revelation principle:

$$\pi(\hat{\theta}, 0) \leq 0 \qquad \forall\, \hat{\theta} \in [0, \bar{\theta}], \tag{9}$$

$$f(x, z) \leq f(x, y) + f(y, z) \quad \forall\, x, y, z \in [0, \bar{\theta}] \quad \text{with } x \geq y > z. \tag{10}$$

Inequality (9) ensures, that a firm with zero emissions has no incentive to overstate, since it does not expect to receive any allowances. Together with $a(\cdot) \in [0, 1]$ this implies

$$e(\hat{\theta}) \leq a(\hat{\theta}) f(\hat{\theta}, 0) \leq f(\hat{\theta}, 0) \quad \forall\, \hat{\theta} \in [0, \bar{\theta}]. \tag{11}$$

For the specific penalty function used later on in the analysis, condition (10) implies subadditivity of the penalty function. Thus, the penalty function has the property that the fine for a single "large" deviation is small relative to the fine of other combinations of "smaller" deviations that add up to the large deviation. Such a penalty might restrain firms from splitting their plants and making overstatements in the single reports.

The set F of *feasible verification mechanisms* is given by:

$$F = \{(e, a, f) : \quad (5) \text{ to } (10) \text{ hold}\}. \tag{12}$$

One may argue, that the problem of the allocation of the initial allowances to firms is not a principle-agent problem, since there are many firms and since firms know that there is an upper limit of allowances and thus act strategically and take their rivals' actions into consideration. Two explanations, however, justify the choice of the principle-agent framework. The intuitive explanation is that each firm receives an amount of greenhouse gas allowances that is small relative to the number of total allowances. The other is a technical explanation. Suppose firms considered that there is a cap on allowances and that the other firms' decisions have an influence on how much they get and vice versa. This would result in a rent-seeking contest where players compete for emission allowances. It can then be shown, however, that it is optimal for firms not to take their rivals' actions into consideration[4]. The result is driven by the fact that the penalty function f does not depend on the other firms' actions.

In some parts of the analysis it will be necessary to consider the entire firms in the economy. The set-up for this is as follows: In the model there are n firms, indexed by i, which are distinguished only by their emission values. The emission values θ are distributed over the interval $[0, \bar{\theta}]$ according to a common probability density function $g : [0, \bar{\theta}] \to \mathbb{R}_+$. The government knows the probability distribution of the emission parameters, but not the realizations of specific firms. If the sum over all firms' initial allowances exceeds the cap on overall allowances, it is assumed that they can be adjusted proportionally via a compliance factor. Such a procedure was proposed in the context of the EU emissions trading scheme by Harrison and Radov (2002).

All mechanisms in F have the characteristic that firms are asked to report their emissions parameter, so for each firm the message space coincides with the set of possible types $[0, \bar{\theta}]$. Such mechanisms are called *direct revelation mechanisms*. A direct revelation mechanism is said to be *incentive compatible* if it is optimal for each firm to report its emission parameter truthfully. In the context of this analysis firms will report truthfully if this maximizes their

[4] Eckermann and Guse (2006) provide a proof of this result

expected allowances, i.e. if

$$\pi(\theta,\theta) \geq \pi(\hat{\theta},\theta) \quad \forall\,\hat{\theta},\ \theta. \tag{13}$$

The government performs costly audits in order to induce truth-telling by the firms. I make the assumption that it is considered politically desirable that *all* firms report their emissions truthfully. However, from an economic perspective the restriction of the analysis to incentive compatible mechanisms can only be approved if it is assured that no, possibly optimal, outcome is lost. The tool that is usually applied to ensure this, is the *revelation principle.* It states that without loss of generality the government can restrict its attention to incentive compatible direct revelation mechanisms.

In the context I consider, the standard arguments do, however, not apply due to requirements that originate from political considerations. I do not foresee rewards for truthful reporting[5]. Feasibility conditions (5) and (7) fix the image of the allocation- and the penalty function respectively at the point zero and make additional conditions necessary, see appendix. From the proof of Lemma 1 it becomes apparent, that (9) and (10) are sufficient to ensure that without loss the analysis can be restricted to incentive compatible mechanisms in F.

Lemma 1 *Let $(e,a,f) \in F$. Then there exists an incentive compatible revelation mechanism $(e',a',f') \in F$ such that e' is nondecreasing and (e',a',f') replicates the equilibrium outcome arising from (e,a,f).*

The proof is given in the appendix. Note that e nondecreasing is not an

[5]While in the tax evasion literature several authors establish the result that the optimal solution requires that there should be rewards for honest reporting (see Mookherjee and Png (1989) or Cremer and Gahvari (1996)), Boadway and Sato (2000) also point out, that in the absence of rewards for truthful reporting there is no guarantee that the revelation principle will apply.

assumption, but an implication of the model.

It is assumed that firms report their emission parameter truthfully if reporting truthfully is optimal. Due to Lemma 1 the analysis can then be restricted to mechanisms $(e, a, f) \in F$ which satisfy:

1. e is nondecreasing:
$$\hat{\theta}^i > \hat{\theta}^j \;\Rightarrow\; e(\hat{\theta}^i) \geq e(\hat{\theta}^j), \tag{14}$$

2. incentive compatibility holds:
$$e(\theta) \geq e(\hat{\theta}) - a(\hat{\theta}) f(\hat{\theta}, \theta) \quad \forall\, \hat{\theta},\, \forall \theta. \tag{15}$$

Denote this subset of mechanisms with F_I:

$$F_I = \{(e, a, f) : (e, a, f) \in F \text{ and (14) and (15) hold}\}. \tag{16}$$

Conditions (14) and (15) imply that $a(\cdot) \not\equiv 0$, unless $e(\cdot) \equiv \mathit{const}$. So, whenever the allocation of allowances is differentiated with respect to the emissions of the firms (which of course is the intention), some auditing is necessary to achieve incentive compatibility.

High verification standards, which are in this model represented by incentive compatibility, are one of the characteristics that an optimal verification scheme should possess. The other feature is efficiency. *Efficiency* in this model is defined for exogenously given penalty functions. An efficient verification scheme has the characteristic, that incentive compatibility can not be achieved with less auditing. The formal definition is given by:

For any given penalty function f, a mechanism $(e, a, f) \in F_I$ is *efficient* in

F_I if there is no other mechanism $(e', a', f) \in F_I$ with

$$e'(\cdot) \geq e(\cdot),\ a'(\cdot) \leq a(\cdot) \text{ and } a'(\cdot) \neq a(\cdot). \tag{17}$$

Other things being equal, smaller audit probabilities are always preferred, as they produce lower audit costs. The idea is, that total audit costs will be as low as possible, if they are as low as possible for every single firm.

3 Grandfathering

Incentive compatibility (15) is equivalent to

$$a(\hat{\theta})f(\hat{\theta}, \theta) \geq e(\hat{\theta}) - e(\theta) \quad \forall\, \hat{\theta}, \forall\, \theta. \tag{18}$$

In order to induce truth-telling by the firms the government has to choose the allocation function e, the penalty function f, and the audit probability a such that for each firm the expected fine is not smaller than the "gross gain from misreporting", i.e. the gain the firm made if there were no penalties or if it were not audited. Since $a \leq 1$ this implies in particular that for all mechanisms element F_I it holds $f(\hat{\theta}, \theta) \geq e(\hat{\theta}) - e(\theta)$ for all $\hat{\theta} \in [0, \bar{\theta}]$. Note that firms will not underreport, since e is nondecreasing and $f(\hat{\theta}, \theta) = 0$ for $\hat{\theta} < \theta$. In the following the analysis therefore focuses on ensuring (15) for $\hat{\theta} > \theta$.

A firm which reports a value $\hat{\theta}$ may thus truly have any emissions value $\theta < \hat{\theta}$. With respect to the audit probability that guarantees incentive compatibility, (15) implies

$$a(\hat{\theta}) \geq \sup_{\theta < \hat{\theta}} \frac{e(\hat{\theta}) - e(\theta)}{f(\hat{\theta}, \theta)} \quad \forall\, \hat{\theta}. \tag{19}$$

For any given pair of allocation and penalty functions, the audit probability for a reported type $\hat{\theta}$ has to be large enough to outweigh the potential expected gain for any true type $\theta < \hat{\theta}$. (19) states that this is achieved by setting the audit probability for any report at least equal to the largest possible ratio of gross gain to penalty for an overstatement. It then holds:

Proposition 1 *Let $(e, a, f) \in F_I$ be efficient in F_I. The audit probability is given by*

$$a(\hat{\theta}) = \sup_{\theta < \hat{\theta}} \frac{e(\hat{\theta}) - e(\theta)}{f(\hat{\theta}, \theta)} \quad \forall\, \hat{\theta}. \tag{20}$$

The proof is given in the appendix. The result is intuitive: as stated above, the audit probability has to be large enough to induce incentive compatibility for any possible type $\theta < \hat{\theta}$. However, an audit probability that is higher than this minimal necessary one is inefficiently high, in the sense that incentive compatibility can already be achieved with less auditing and therefore less audit costs. Efficiency can therefore only be achieved if equality holds in (19).

Proposition 1 makes the strong interrelation apparent that the three functions have in an efficient verification scheme. Not surprisingly is the audit probability lower for a higher penalty. This reflects the well known result that enforcement costs are kept at the minimum, if penalties for misreporting are increased as far as possible and thus the probability of costly auditing is as small as possible (Becker (1968)). The schemes I consider are therefore necessarily second-best. They result from the assumption that too high penalties are likely to be politically not desirable. In addition there might be small unintentional errors by the firm. The current political regulations foresee no explicit punishment.

For the further analysis a *proportional penalty* is used, which is given by

$$f_{prop}(\hat{\theta}, \theta) = \max\{0, \gamma(\hat{\theta} - \theta)\}. \tag{21}$$

The idea behind such a penalty structure is that higher absolute deviations from the true emissions value should be penalized stronger, independent of whether the true emissions value is high or low. Proportional penalties lead to expected allowances of

$$\pi_{prop}(\hat{\theta},\theta) = \begin{cases} e(\hat{\theta}) - a(\hat{\theta})\gamma[\hat{\theta}-\theta] & \text{if } \hat{\theta} > \theta \\ e(\hat{\theta}) & \text{if } \hat{\theta} \leq \theta, \end{cases} \tag{22}$$

From Proposition 1 it follows that the audit probability of an efficient verification scheme $(e, a, f_{prop}) \in F_I$ is given by

$$a(\hat{\theta}) = \sup_{\theta<\hat{\theta}} \frac{e(\hat{\theta}) - e(\theta)}{\hat{\theta}-\theta}\frac{1}{\gamma} \quad \forall\, \hat{\theta}. \tag{23}$$

If more is known about the structure of the allocation function, the audit probability can be further specified:

Proposition 2 *Let $(e, a, f_{prop}) \in F_I$ be an efficient mechanism in F_I.*

1. *For concave allocation functions e, the audit probability a is nonincreasing in the reported parameter and given by*

$$a(\hat{\theta}) = \frac{e(\hat{\theta})}{\hat{\theta}\gamma}. \tag{24}$$

2. *For convex functions e, it is nondecreasing and given by*

$$a(\hat{\theta}) = \frac{1}{\gamma}D^{-}e(\hat{\theta}), \tag{25}$$

where D^{-} denotes the left-hand derivative.

3. *In particular for linear allocation functions e, the audit probability a is*

constant. Let $e(\hat{\theta}) = \lambda\hat{\theta}$, $\lambda \in \mathbb{R}_+$, *then*

$$a(\hat{\theta}) = \frac{\lambda}{\gamma}. \tag{26}$$

The proof is given in the appendix. As expected, a stronger penalty (i.e. a higher γ) leads to a lower audit probability. The interplay between initial allocations and audit probabilities depends on the form of the allocation function.

A *concave allocation function* describes a regressive allocation of allowances. The at first sight most striking result is surely that in an efficient verification scheme the audit probability is nonincreasing (strictly decreasing in the case of a strictly concave allocation function) in the reported parameter. This means that companies which present higher emission reports are less likely to be audited, which seems to contradict the aim of detecting large deviations. One has to consider, however, that the penalty and allocation function were chosen in a way that firms have incentives to announce their true values. This implies that a higher report does not stand for a bigger false report, but for a higher true value. Reporting a bigger false report than the true value is too costly for the firms, since additional gains from overstating are outweighed by larger penalties. Auditing higher reports with a higher probability would then mean that those reports were audited with a probability that is higher than necessary for achieving incentive compatibility, which, of course, can not be efficient.

Equation (24) shows that the audit probability, thought of as a decimal number, should then be given by a multiple of average allocations, where the multiplier is determined by the strength of the penalty function. Consider exemplary the case that $\gamma = 1$, i.e. the penalty function is given by

$$f_{prop}(\hat{\theta}, \theta) = \max\{0, (\hat{\theta} - \theta)\}. \tag{27}$$

In this case the audit probability should equal average allocations. If, for example a firm reports the emission value $\hat{\theta} = 1000$ and allowances are allocated such that $e(1000) = 900$, the respective firm should be audited with a probability of 90%.

Figure 1 below displays a concave and nondecreasing allocation function e, corresponding to a regressive allocation of allowances. γ is assumed to be 1, therefore the audit probability is equal to the average allocation of allowances, as shown in (24). In the figure it is given by the slope of the straight line through (0,0) and $(\hat{\theta}^i, e(\hat{\theta}^i))$. It can easily be seen that a higher reported value (e.g. $\hat{\theta}^2$ instead of $\hat{\theta}^1$) leads to a lower audit probability.

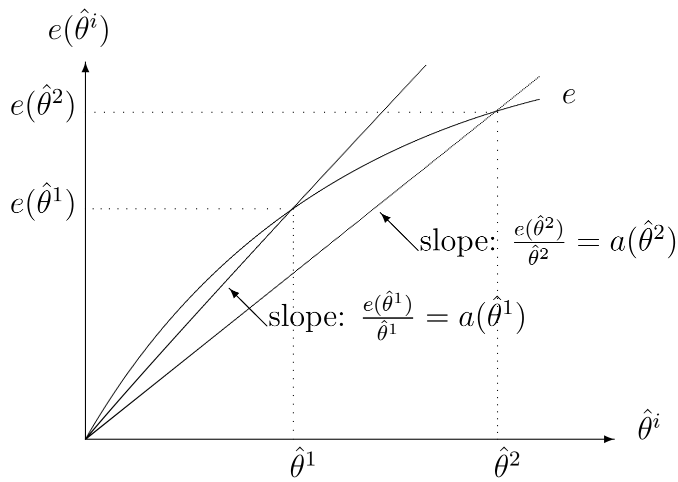

Figure 1: Regressive allocation of allowances

If the penalty is stronger than the difference between the reported and the true value, i.e. if $\gamma > 1$, the audit probability will be accordingly lower. Given the numerical example from above, a penalty factor of $\gamma = 10$ would reduce the audit probability that is necessary to induce incentive compatibility to 9%.

A simple insertion of a penalty factor $\gamma < 1$ seems to lead to contradictions in the above numerical example at first sight: a factor of $\gamma = 1/2$, for example, would lead to an audit probability of 180%, which is of course not possible. This example points up the significance of feasibility condition (9), which requires incentive compatibility for firms with no emissions ($\theta = 0$) and implies (11). For proportional penalties the latter is given by

$$e(\hat{\theta}) \leq a(\hat{\theta})\gamma\hat{\theta} \leq \gamma\hat{\theta} \quad \forall\, \hat{\theta} \in [0, \bar{\theta}]. \tag{28}$$

It can immediately be seen that in the numerical example above $e(1000) = 900 \not\leq \frac{1}{2}1000 = 500$, thus such a verification scheme is not feasible.

It is interesting to note that a higher initial allocation of allowances should be combined with higher penalties.

A *convex allocation function*, on the other hand, describes a progressive allocation of emission allowances. Efficiency then implies the reverse effect on audit intensity: a plant which reports higher values will be audited with a higher probability. Equation (25) shows that the audit probability should be given by a multiple of the left-hand derivative, and of the derivative where it exists. The multiplier is again determined by the strength of the penalty function. The relationship between a convex allocation function and the audit probability in an efficient verification scheme is illustrated in Figure 2.

The displayed allocation function is differentiable, therefore the audit probability for report $\hat{\theta}$ is given by the first derivative of e in $\hat{\theta}$, multiplied with $\frac{1}{\gamma}$. It is easily seen, that the audit probability is increasing in the reported parameter. Exemplary it is shown to be higher for the report $\hat{\theta}^2$ than for the lower report $\hat{\theta}^1$.

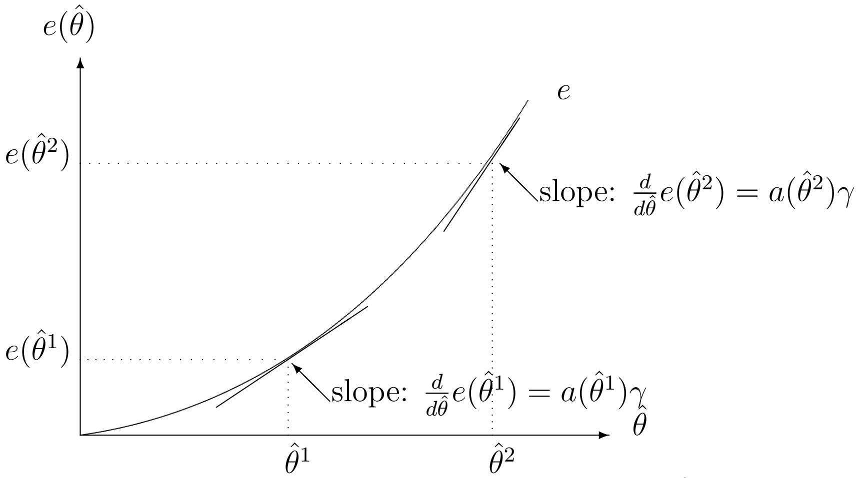

Figure 2: Progressive allocation of allowances

Reformulating (25) and using $a \in [0, 1]$ yields

$$D^{-}e(\hat{\theta}) = \gamma a(\hat{\theta}) \leq \gamma. \tag{29}$$

This condition makes clear that only those mechanisms can be efficient, which have an allocation function with a first (left-hand) derivative smaller than or equal to γ. The smaller the penalty the smaller the slope of the allocation function, and vice versa. However, the restriction is even stronger. The incentive compatibility condition (15) can for convex allocation functions be reformulated to

$$a(\hat{\theta}) \geq \sup_{\theta<\hat{\theta}} \frac{e(\hat{\theta}) - e(\theta)}{\gamma(\hat{\theta} - \theta)} = \frac{1}{\gamma} D^{-}e(\hat{\theta}), \tag{30}$$

which is also equivalent to $\gamma \geq D^{-}e(\hat{\theta})$ for all $\hat{\theta}$. Thus, mechanisms with an allocation function that has a slope larger than γ for any value $\hat{\theta} \in [0, \bar{\theta}]$ can not be incentive compatible. Looking at this from the other side, the formula simply states that for any given allocation function the penalty has to be chosen accordingly, in order to guarantee incentive compatibility.

The allocation schedule that is most popular in reality, is a proportional allocation of allowances. Such an allocation is described by a *linear allocation function*. Result (26) makes clear that the audit probability of an efficient verification scheme with proportional allocations and penalties is the same for all reports. Furthermore, the audit probability is not equal to one, as in the schemes that can be observed in reality, but dependent on the allocation and penalty factor. If, for example, $e(\hat{\theta}) = 0.9\hat{\theta}$ for all $\hat{\theta}$ and $\gamma = 1$, the audit probability is 90% for all firms. Not surprisingly, it is increasing in the allocation factor and decreasing in the penalty factor. Figure 3 displays an exemplary allocation function $e(\hat{\theta}) = \lambda\hat{\theta}$ with $\lambda < 1$.

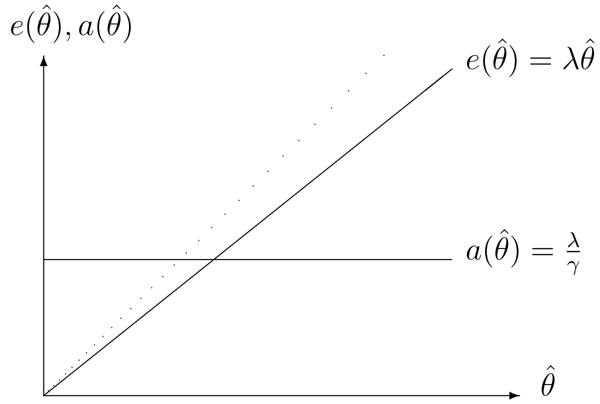

Figure 3: Proportional allocation of allowances

The results depend, of course, on the underlying penalty function.

4 Comparison with Tax Evasion

The reporting and verification of firm data in the grandfathering process of an emissions trading scheme displays strong parallels to the tax evasion problem. Not surprisingly many of the results in the two fields have similar structures. However, they also reveal important differences.

Chander and Wilde (1998), for example, analyze the interplay between optimal tax rates, audit probabilities and penalties for misreporting in a principal-agent model with a structure similar to the one of this analysis. In their notation the "penalty" is really a "post-audit payment", i.e. the payment after the audit has occurred and after the true income has been discovered. Let y denote income, x reported income, $t(\cdot)$ the tax function and $f_t(\cdot)$ the penalty function. Under the penalty

$$f_t(x,y) = t(x) + \max\{0, y-x\} \text{ for all } x \text{ and } y \tag{31}$$

the taxpayer will have to pay the tax $t(x)$ and all of the underreported income. This penalty structure is comparable to the proportional penalty $f_{prop}(\hat{\theta},\theta) = \max\{0, \gamma(\hat{\theta}-\theta)\}$ in my model, since this leads to a post-audit allocation with the same structure: $e(\hat{\theta}) + \max\{0, \gamma(\theta-\hat{\theta})\}$, due to

$$\begin{aligned} \pi_{prop}(\hat{\theta},\theta) &= e(\hat{\theta}) - a(\hat{\theta})\max\{0,\gamma(\hat{\theta}-\theta)\} && (32)\\ &= (1-a(\hat{\theta}))e(\hat{\theta}) + a(\hat{\theta})(e(\hat{\theta}) - \max\{0,\gamma(\hat{\theta}-\theta)\}). && (33) \end{aligned}$$

The difference is in the interpretation: (31) determines a value that firms have to pay, whereas the parallel in the emissions trading model determines a value that firms receive. In the following I use the notation $x, y, t(\cdot)$ to indicate the tax evasion model, and $\hat{\theta}, \theta, e(\cdot)$ to refer to the emissions trading model. Chander and Wilde (1998) find, that efficiency is given only if the tax function t is concave and nondecreasing and if the audit probability is nonincreasing. In particular the audit probability is determined by the marginal tax rate. These results differ from the results in the emissions trading model in two aspects.

Remark 1 *The audit probability* $p(x)$ *of an efficient tax scheme with a concave tax function is given by*

$$p(x) = D^+ t(x), \tag{34}$$

the right-hand derivative of the tax function, whereas the audit probability of an efficient verification scheme with a concave allocation function is given by

$$a(\hat{\theta}) = \frac{e(\hat{\theta})}{\gamma \hat{\theta}}, \tag{35}$$

a multiple of average and not marginal allocations.

This difference is due to the fact that in the tax evasion context firms have an incentive to *understate* their income, whereas in the grandfathering context they have an incentive to *overstate* their emissions. Technically this leads to

$$p(x) = \sup_{y>x} \frac{t(y) - t(x)}{y - x} \tag{36}$$

versus

$$a(\hat{\theta}) = \sup_{\theta<\hat{\theta}} \frac{e(\hat{\theta}) - e(\theta)}{\gamma(\hat{\theta} - \theta)}. \tag{37}$$

Remark 2 *In the tax evasion context a mechanism is efficient only if* t *is concave, whereas in the grandfathering context there is no such restriction for the allocation function.*

This results also from the different direction of misreporting. Figure 4 presents an intuition for the effect of the direction of misreporting on audit efficiency.

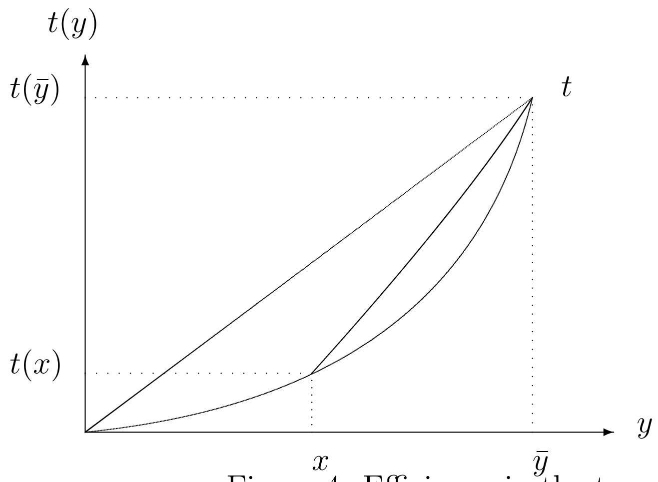

Figure 4: Efficiency in the tax evasion model

For any efficient (t, p, f_t), equation (36) holds. For t convex the supremum is attained at $y = \bar{y}$ and thus

$$p(x) = \frac{t(\bar{y}) - t(x)}{\bar{y} - x}. \tag{38}$$

Now a linear function t' through the points $(0, t(0))$ and $(\bar{y}, t(\bar{y}))$ can be constructed. Being a linear combination of two points of a convex function, t' is bigger than t, with $t' > t$ for t strictly convex. With regard to the audit probability it holds

$$p'(x) = \frac{t(\bar{y})}{\bar{y}} < \frac{t(\bar{y}) - t(x)}{\bar{y} - x} = p(x), \tag{39}$$

since $\frac{t(\bar{y})}{\bar{y}} > \frac{t(x)}{x}$ for t strictly convex. Thus there is no efficient (t, p, f_t) with t convex.

For any efficient (e, a, f_{prop}) on the other hand, equation (37) holds, the supremum is attained at $\theta = \hat{\theta}$ with $a(\hat{\theta}) = D^{-}e(\hat{\theta})$ and no function e' can be found with $e' \geq e$, $a' \leq a$ and $e' \neq e$ or $a' \neq a$.

Chander and Wilde (1998) analyze a second more general penalty structure with

$$f(x, y) = y \text{ for } x \neq y, \tag{40}$$

i.e. if the firm is found to have misreported it will have to pay all of its income. In the emissions trading context such a penalty would be comparable to a penalty that takes away all of the allowances that the firm would have received otherwise and leaves it with zero allowances:

$$f_{zero}(\hat{\theta}, \theta) = \begin{cases} e(\hat{\theta}) & \text{if } \hat{\theta} > \theta \\ 0 & \text{if } \hat{\theta} \leq \theta \end{cases}, \tag{41}$$

leading to post-audit allowances of zero in case of an overstatement, and of $e(\hat{\theta})$ in case of $\hat{\theta} \leq \theta$. Incentive compatibility requires $e(\theta) \geq e(\hat{\theta}) - a(\hat{\theta})e(\hat{\theta})$ for all $\hat{\theta} > \theta$ and audit efficiency implies (cf. Proposition 1) that

$$a(\hat{\theta}) = \sup_{\theta < \hat{\theta}} (1 - \frac{e(\theta)}{e(\hat{\theta})}) = 1 \quad \forall \, \hat{\theta}, \tag{42}$$

because the supremum is attained at $\theta = 0$ for all $\hat{\theta}$. Although the penalty seems to be very strict, it has to be combined with full verification in order to achieve incentive compatibility. It can easily be seen that this is due to firms with a small θ-value. For these firms the penalty is after all not very strong and if the audit probability were low it would be worthwhile to overstate to a great extend. This result is different from the result in the tax evasion model, where Chander and Wilde (1998) find that the marginal payment rates determine the audit probabilities and thus efficient schemes may involve random or deterministic auditing. The difference in the results is again due to the different directions of misreporting.

5 Conclusion

The allocation of the initial allowances is one of the important elements of an emissions trading scheme. Grandfathering, which is the commonly applied procedure, makes strict verification necessary. The question is how strict the verification procedure should be, since it usually imposes costs on the one who verifies the information. For the general setup of this analysis the Becker-solution holds. Under more specific penalties, which are proportional to and increasing in the degree of misreporting and therefore hit large deviations stronger, it is the form of the allocation function that drives the results. For a regressive allocation of initial allowances, for example, the efficient audit probability is shown to be nonincreasing in the reported emissions value and it is determined by average allocations, whereas the audit probability is the same for all reports, if allowances are allocated proportionally.

Although the structure of the model used for this analysis parallels the structure of principal-agent models in the tax evasion context, the results differ with respect to the number of efficient allocation (respectively tax) and audit - combinations, as well as the form of each variety. These differences are driven by the fact that in one case firms have an incentive to overstate, and in the other case to understate their private data.

Appendix

Proof of Lemma 1:

Without conditions (5) and (7) it is easily seen that the revelation principle holds: if the mechanism $(e(\cdot), a(\cdot), f(\cdot))$ results in an optimal report $\alpha(\theta)$ for an emission value θ, the mechanism given by $(e'(\hat{\theta}), a'(\hat{\theta}), f'(\hat{\theta}, \theta)) := (e(\alpha(\hat{\theta})), a(\alpha(\hat{\theta})), f(\alpha(\hat{\theta}), \theta))$ has an optimal report which is equal to the true value θ. The payoffs to both the government and the firm are unaffected by this change, since $a'(\theta) = a(\alpha(\theta))$ and $\pi'(\theta,\theta) = e'(\theta) - a'(\theta)f'(\theta,\theta) = e(\alpha(\theta)) - a(\alpha(\theta))f(\alpha(\theta),\theta) = \pi(\alpha(\theta),\theta)$. However, the penalty function of the new mechanism would possibly be positive even for truth-telling firms, $f'(\theta,\theta) := f(\alpha(\theta),\theta) \geq 0$, since $(e(\cdot), a(\cdot), f(\cdot))$ is not incentive compatible and $\alpha(\theta) \geq \theta$. Since this does not seem to be politically desirable in the current context it is ruled out by feasibility condition (7). Similar arguments apply to feasibility condition (5): $e'(0) = e(\alpha(0))$ need not necessarily be zero. This shows that the mechanism $(e'(\hat{\theta}), a'(\hat{\theta}), f'(\hat{\theta},\theta)) := (e(\alpha(\hat{\theta})), a(\alpha(\hat{\theta})), f(\alpha(\hat{\theta}),\theta))$ can not be used to prove the revelation principle in this setting. A different mechanism is therefore put up:

Let $(e,a,f) \in F$. Define (e',a',f') from (e,a,f) as follows:

$$\begin{aligned}
e'(\hat{\theta}) &:= e\left(\alpha(\hat{\theta})\right) - a\left(\alpha(\hat{\theta})\right) f\left(\alpha(\hat{\theta}),\hat{\theta}\right) = \pi(\alpha(\hat{\theta}),\hat{\theta}), \\
a'(\hat{\theta}) &:= a\left(\alpha(\hat{\theta})\right), \\
f'(\hat{\theta},\theta) &:= f(\hat{\theta},\theta),
\end{aligned}$$

where $\alpha(\theta), (\alpha(\theta) \geq \theta)$, is an optimal signal for a firm of type θ under the mechanism (e,a,f). Note that the penalty function is exactly the same in both mechanisms.

(a) Show that $(e',a',f') \in F$: $e : [0,\bar{\theta}] \to \mathbb{R}_+$ holds, since it equals the expected allocation for a firm of type θ under the mechanism (e,a,f) if it pronounces $\alpha(\theta)$. This is not smaller than 0, since $e(\hat{\theta}) \geq 0$ for all $\hat{\theta}$ and the firm can secure itself a punishment of 0 if it pronounces the true type θ. As $\alpha(\theta)$ is an optimal signal it holds $e'(\theta) \geq 0$. $e'(0) = 0$ since $\pi(\alpha(0),0) = 0$ due to condition (9), $\pi(\hat{\theta},0) \leq 0$, and the optimality of $\alpha(\cdot)$. $e(\cdot) \not\equiv 0$ induces $\pi(\alpha(\hat{\theta}),\hat{\theta}) \not\equiv 0$ and therefore $e'(\cdot) \not\equiv 0$. The argument is again that $\alpha(\cdot)$ is an optimal signal and therefore $\pi(\alpha(\hat{\theta}), \hat{\theta}) \geq e(\hat{\theta})$ has to hold. $\pi'(\hat{\theta}, 0) = e'(\hat{\theta}) - a'(\hat{\theta})f'(\hat{\theta}, 0) = \pi(\alpha(\hat{\theta}), 0) - a(\alpha(\hat{\theta}))f(\hat{\theta}, 0) \leq 0$, since $\pi(\alpha(\hat{\theta}), 0) \leq 0$ and $a(\alpha(\hat{\theta}))f(\hat{\theta}, 0) \geq 0$. In addition $0 \leq a'(\hat{\theta}) \leq 1$ and the conditions regarding $f(\cdot)$ carry over to $f'(\cdot)$ and thus $(e', a', f') \in F$.

(b) Show that $e'(\cdot)$ is nondecreasing: Let $\theta^1 > \theta^2$, show that $e'(\theta^1) \geq e'(\theta^2)$:

$$\begin{aligned} e'(\theta^2) &= e(\alpha(\theta^2)) - a(\alpha(\theta^2))f(\alpha(\theta^2), \theta^2) \leq e(\alpha(\theta^2)) - a(\alpha(\theta^2))f(\alpha(\theta^2), \theta^1) \\ &\leq e(\alpha(\theta^1)) - a(\alpha(\theta^1))f(\alpha(\theta^1), \theta^1) = e'(\theta^1). \end{aligned}$$

To see why this holds note that due to $\theta^1 > \theta^2$ and f nondecreasing in the degree of misreporting it holds that $f(\alpha(\theta^2), \theta^2) \geq f(\alpha(\theta^2), \theta^1)$ for both $\alpha(\theta^2) > \theta^1$ and $\alpha(\theta^2) \leq \theta^1$. In addition $\alpha(\theta^1)$ is optimal for θ^1 and therefore the second inequality holds.

(c) Show that (e', a', f') is incentive compatible: The mechanism is incentive compatible if expected allowances are maximized for reporting the true emissions value, i.e. if $\pi'(\theta, \theta) \geq \pi'(\hat{\theta}, \theta) \quad \forall\, \hat{\theta}, \theta \in [0, \bar{\theta}]$. This is equivalent to $e'(\theta) \geq e'(\hat{\theta}) - a'(\hat{\theta})f'(\hat{\theta}, \theta) \quad \forall\, \hat{\theta}, \theta \in [0, \bar{\theta}]$. However, since $f'(\hat{\theta}, \theta) = f(\hat{\theta}, \theta) = 0$ for all $\hat{\theta} \leq \theta$ and since e is nondecreasing as shown above, no firm will have an incentive to report a parameter $\hat{\theta} < \theta$. It is therefore now sufficient to show that $e'(\theta) \geq e'(\hat{\theta}) - a'(\hat{\theta})f'(\hat{\theta}, \theta)$ for all $\hat{\theta} > \theta$. Let therefore $\hat{\theta} > \theta$, then $e'(\theta) \geq e'(\hat{\theta}) - a'(\hat{\theta})f'(\hat{\theta}, \theta)$ is equivalent to

$$\begin{aligned} e(\alpha(\theta)) - a(\alpha(\theta))f(\alpha(\theta), \theta) &\geq e(\alpha(\hat{\theta})) - a(\alpha(\hat{\theta}))f(\alpha(\hat{\theta}), \hat{\theta}) - a(\alpha(\hat{\theta}))f(\hat{\theta}, \theta) \\ &= e(\alpha(\hat{\theta})) - a(\alpha(\hat{\theta}))[f(\alpha(\hat{\theta}), \hat{\theta}) + f(\hat{\theta}, \theta)]. \end{aligned}$$

In order to show that this holds, the fact that $\alpha(\theta)$ is an optimal signal for a firm with emission parameter θ, and condition (10), $f(\alpha(\hat{\theta}), \theta) \leq f(\alpha(\hat{\theta}), \hat{\theta}) + f(\hat{\theta}, \theta)$, are used:

$$\begin{aligned} e(\alpha(\theta)) - a(\alpha(\theta))f(\alpha(\theta),\theta) &\geq e(\alpha(\hat{\theta})) - a(\alpha(\hat{\theta}))f(\alpha(\hat{\theta}),\theta) \\ &\geq e(\alpha(\hat{\theta})) - a(\alpha(\hat{\theta}))[f(\alpha(\hat{\theta}),\hat{\theta}) + f(\hat{\theta},\theta)]. \end{aligned}$$

(d) Show that (e', a', f') replicates the equilibrium outcome arising from (e, a, f): The outcome for the government is audit it has to perform and therefore given by the audit probability. This is the same under the two mechanisms from the definition of $a'(\hat{\theta})$. The outcome for the firm is the amount of expected allowances. This is the same under the two mechanisms since $\pi(\alpha(\theta),\theta) = e(\alpha(\theta)) - a(\alpha(\theta))f(\alpha(\theta),\theta) = e'(\theta) = e'(\theta) - a'(\theta)f'(\theta,\theta) = \pi'(\theta,\theta)$, due to $f'(\theta,\theta) = 0$. The amount of allowances that the firm expects to receive under the new mechanism (e', a', f') when it tells the truth equals just the amount of allowances that it expected to receive under the old mechanism (e, a, f) and the announcement of the optimal (i.e. expected allowances maximizing) report $\alpha(\theta)$. ■

Proof of Proposition 1: Let $(e, a, f) \in F_I$ be efficient in F_I. Since the mechanism is incentive compatible, (19) holds:

$$a(\hat{\theta}) \geq \sup_{\theta<\hat{\theta}} \frac{e(\hat{\theta}) - e(\theta)}{f(\hat{\theta},\theta)} \quad \forall\, \hat{\theta}.$$

If the incentive constraint is not binding, i.e. if there exist some $\hat{\theta}$ with

$$a(\hat{\theta}) > \sup_{\theta<\hat{\theta}} \frac{e(\hat{\theta}) - e(\theta)}{f(\hat{\theta},\theta)},$$

the inequality still holds, and therefore incentive compatibility is still provided, if the audit probability $a(\hat{\theta})$ is reduced, while $e(\cdot)$ and $f(\cdot)$ are not changed.

However, this contradicts the assumption that (e, a, f) is efficient in F_I and therefore $a(\hat{\theta}) = \sup_{\theta<\hat{\theta}} \frac{e(\hat{\theta})-e(\theta)}{f(\hat{\theta},\theta)}$ for all $\hat{\theta}$ has to hold. ■

Proof of Proposition 2: $a(\hat{\theta}) = \sup_{\theta<\hat{\theta}} \frac{e(\hat{\theta})-e(\theta)}{\hat{\theta}-\theta}$ simply gives the slope of the secant through the points $(\theta, e(\theta))$ and $(\hat{\theta}, e(\hat{\theta}))$.
For e *concave* and $\hat{\theta} \in [0, \bar{\theta}]$ the slope through $(\theta, e(\theta))$ and $(\hat{\theta}, e(\hat{\theta}))$ with $\theta < \hat{\theta}$, $\theta \in [0, \bar{\theta}]$, is steepest if $\theta = 0$. The supremum is therefore attained at $\theta = 0$ and (24) holds due to $e(0) = 0$. $a(\hat{\theta}) = \frac{e(\hat{\theta})}{\hat{\theta}\gamma}$ is a multiple of average allocations. Since e is concave the average allocations, and therefore a, are nonincreasing in $\hat{\theta}$.
If e is *convex* the supremum is attained at $\theta = \hat{\theta}$. $a(\hat{\theta})$ is given by a multiple of the left-hand derivative, and of the derivative where it exists. e is convex, therefore $D^{-}e$ is nondecreasing in the reported parameter and it holds that a is nondecreasing.
The linear case follows immediately. ■

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