

# Measuring Residential Energy Efficiency Improvements with DEA

Peter Grösche

RWI Essen\*

January 30, 2007

## **Abstract**

This paper measures energy efficiency improvements of US single-family homes between 1997 and 2001 using Data Envelopment Analysis (DEA). Energy efficiency is captured by an indicator that is related to the energy conservation potential. Nonparametric tests confirm that the conservation potential of the households sampled in 2001 is significantly smaller compared with households sampled in 1997, suggesting a significant efficiency improvement.

**JEL:** C6, D13, Q41,

**Key words:** Household Production, Data Envelopment Analysis

---

\*Rheinisch-Westfälisches Institut für Wirtschaftsforschung, Hohenzollernstr. 1-3, 45128 Essen, Germany. Email: groesche@rwi-essen.de

# 1 Introduction

Improvements of energy efficiency is often cited as a possible option to alleviate both the greenhouse effect and import dependencies on fossil fuels. It is expected that the residential sector, in particular, can contribute substantially to efficiency improvements. Not only do residents account for a large share of final energy consumption, but their homes are often equipped with out-of-date and energy-inefficient appliances.

A number of approaches and concepts to measure energy efficiency have evolved in the literature (see Ang (2006) for a recent review). The spectrum of indicators ranges from the simple ratio of energy usage per capita to sophisticated composite index approaches, measuring efficiency on a very disaggregated level. However, such sophisticated indices come at a cost of extensive data requirements for the efficiency component. The option for the analyst to fulfill these requirements are rare, especially in the household sector. He can either use exact but expensive metering, with the undesirable feature that the metered values would frequently base on very few observations. The alternatives are to combine survey data with regression techniques (EIA 1999) or comparing energy consumption between groups of households (Schipper et al. 1985). But still, the meager empirical data situation raises the desire to have a meaningful efficiency index that avoids extensive data requirements.

This paper measures residential energy efficiency for US single-family homes using *Data Envelopment Analysis* (DEA). DEA is a nonparametric frontier estimation technique that is firmly anchored in production theory (see e.g. Seiford and Thrall, 1990). One of the key advantages of the approach are its light data requirements. We use household survey data, publicly available from the US energy department. After deriving the efficiency indicator from two samples drawn in 1997 and 2001, we test whether the estimated efficiency improvements are statistically significant.

Applications of DEA in the context of energy efficiency are sparse. Ferrier and Hirschberg (1992) employed DEA to measure energy efficiency of US commercial buildings, Mukherjee (forthcoming) uses DEA to estimate energy efficiency trends in the US manufacturing sector. Phylipsen et al. (1997, 1998) applied a comparable benchmarking procedure for the European cement industry. To our knowledge this is the first paper that explicitly measures and tests for residential efficiency improvements with DEA.

The outline of the paper is as follows. In Section 2, we introduce the DEA methodology and show how it fits into the usual definition of energy efficiency. Section 3 provides an overview of our data set. In Section 4, we discuss our results, while section 5 concludes.

## 2 Measuring energy efficiency

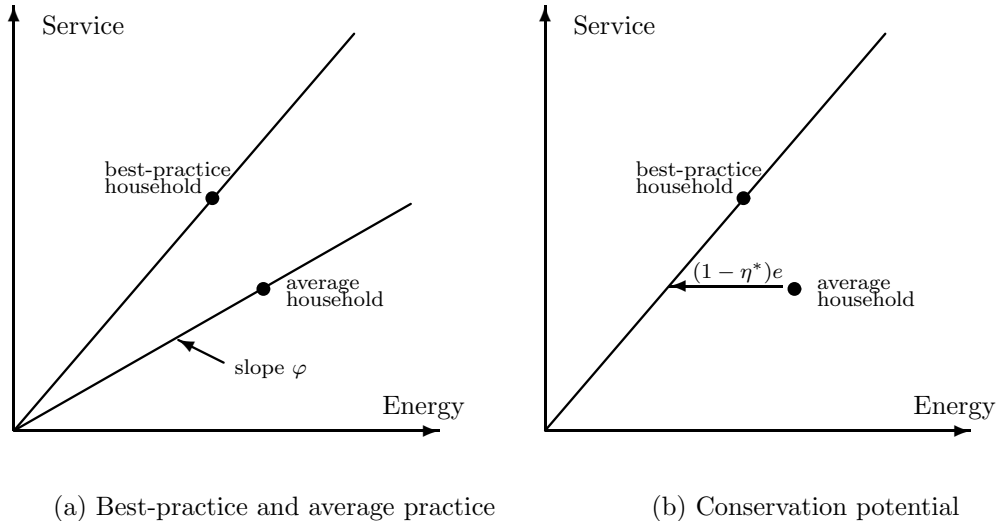
Residential energy consumption derives from the demand for energy services, such as the demand for thermal comfort. Households ‘produce’ those services with their energy commodities (e.g. heating equipment) by using a set of fuel inputs. The standard approach to measure residential energy efficiency draws on the framework of Becker’s (1965) home-production function. Along these lines, Wirl (1997:17) defines residential energy efficiency as the ratio between the output of a certain service  $s$  and the energy  $e$  consumed to produce it:

$$(1) \quad \varphi := \frac{s}{e}.$$

An improvement of energy efficiency would result in an increase of  $\varphi$ .

Frequently, the literature uses the inverse measure  $1/\varphi$  (often called energy intensity) to express efficiency tendencies for a certain service. For instance, the so called ASI approach combines  $1/\varphi$  with non-efficiency related indices to decompose changes in residential energy consumption (Haas 1997, Schipper et al. 2001, Schipper et al 1985). The major drawback of ASI is its extensive data re-

Figure 1: Benchmarking against the best-practice frontier



quirements, as it requires separate figures of energy intensities for each produced service. This means, the researcher needs information on how much energy was consumed e.g. for preparing one liter of hot water. Such data are rarely available in the residential sector and must be obtained either from costly monitoring, due to regression techniques (EIA 1999), or must base on comparisons between groups of households (Schipper et al. 1985). A more desirable approach would encompass the convenience of the ASI methodology with less onerous data requirements.

Basically, DEA can be considered as a generalization of the energy efficiency definition (1). Figure 1(a) illustrates the similarities between DEA and  $\varphi$ . Typically,  $\varphi$  is computed for an average household and is the slope of the ray through the origin and the average household. In contrast, DEA computes a best-practice frontier, which is in the one-input one-output case the steepest ray through the origin that has support from at least one data point.<sup>1</sup> The production plans of

---

<sup>1</sup>To be more precise: such a DEA model would assume a technology with constant returns to scale.

all households are bounded by the best-practice frontier, and are benchmarked against this frontier.

To formalize things, let  $\mathbf{s}_l = (s_{1l}, \dots, s_{Jl})'$  be a vector of produced services  $s_{jl}$  ( $j = 1, \dots, J$ ) from household  $l$  ( $l = 1, \dots, L$ ), and let  $e_l$  be  $l$ 's total energy input.<sup>2</sup> Each household uses a positive amount of energy to produce at least one service. Assuming that household  $o$  wants to appear as efficient as possible, the goal is to:

$$(2a) \quad \max_{\mathbf{u}_o, v_o} \eta_o = \frac{\mathbf{u}'_o \mathbf{s}_o}{v_o e_o} = \frac{\sum_{j=1}^J u_{jo} s_{jo}}{v_o e_o} \quad \text{subject to}$$

$$(2b) \quad \frac{\sum_j u_{jo} s_{jl}}{v_o e_l} \leq 1 \quad l = 1, \dots, L,$$

with weights,  $\mathbf{u}_o = (u_{1o}, \dots, u_{Jo})'$ ,  $u_{jo} \geq 0$ , and  $v_o \geq 0$ , assigned to the outputs and the input, respectively.

Problem (2) must be solved for each household. It is a ratio of weighted service output to weighted fuel input, subject to the condition that the similar ratio for each of the  $L$  households is less than or equal to unity. An important implicit assumption is that the underlying technology exhibits constant returns to scale (Charnes, Cooper and Rhodes, 1978). Note that  $\eta_o$  resembles definition (1) of energy efficiency. Due to its weighting scheme, problem (2) can handle several services simultaneously such that a decomposition of total energy demand is not necessary.

Let  $(\eta_o^*, \mathbf{u}_o^*, v_o^*)$  describe the optimal solution of model (2) for household  $o$ . The product  $\eta_o^* e_o$  is a measure of how much energy consumption is justified for the service production of household  $o$  such that  $o$  will become efficient. Thus,  $\eta_o^* = 1$  indicates a position on the (technically) efficient frontier, as depicted for the best-practice household in Figure 1(b). If  $0 < \eta_o^* < 1$ , household  $o$  can reduce its energy consumption by  $(1 - \eta_o^*)$  percent, or by  $(1 - \eta_o^*)e_o$  units, without

---

<sup>2</sup>We restrict our analysis on the case of only one input (energy) and multiple outputs (services), although DEA can easily deal with multiple inputs and outputs. See e.g. Seiford and Thrall (1990).

being forced to diminish the service level. The same amount of services may be maintained by improved efficiency. In Figure 1(b) the corresponding conservation potential is illustrated for the average household as distance to the frontier.

By setting  $\tilde{u}_j = u_{jo}/v_o > 0$ ,  $\tilde{\mathbf{u}} = (\tilde{u}_1, \dots, \tilde{u}_J)'$ , and  $h_o = \eta_o e_o > 0$ , we obtain a simpler model:

$$(3a) \quad \max_{\tilde{\mathbf{u}}} h_o = \tilde{\mathbf{u}}' \mathbf{s}_o = \sum_j \tilde{u}_j s_{jo} \quad \text{subject to}$$

$$(3b) \quad \sum_j \tilde{u}_j s_{jl} \leq e_l \quad l = 1, \dots, L,$$

which goes back to Dyson and Thanassoulis (1988). Similar to model (2), the optimal value  $h_o^* = \eta_o^* e_o$  is a measure of how much energy consumption is justified for the service production of household  $o$ .

The weights  $\tilde{u}_j$  can be interpreted as the amount of energy consumed by household  $o$  in the production of one unit of  $s_j$ , as it can be seen in (3b). For example, if service 1 stands for space heating (measured in squared-meters,  $\text{m}^2$ ) and energy consumption  $e$  is expressed in kilowatthours (kWh), then  $\tilde{u}_1$  is measured in  $\text{kWh}/\text{m}^2$ . Loosely speaking, the vector  $\tilde{\mathbf{u}}$  can be thought of as the energy intensities from household  $o$ .

The benchmarking process works as follows. By maximizing (3a), household  $o$  proposes a certain pattern of intensities  $\tilde{\mathbf{u}}$ , and a respective amount of energy  $e_o$ . In (3b), these intensities are plugged into the production plans  $(\mathbf{s}_l, e_l)$  of all  $L$  households. If the pattern  $\tilde{\mathbf{u}}$  is not feasible for at least one household  $l$ , meaning that  $\sum_j \tilde{u}_j s_{jl} > e_l$ , household  $l$  is able to produce at least one service with a lower energy intensity. The process will then be reiterated with a different pattern of intensities, until the optimal pattern  $\tilde{\mathbf{u}}^*$  for household  $o$  is feasible for any household. This reiteration leads to the decrease of at least one  $\tilde{u}_j$ , such that the justified amount of energy  $h_o^* = \sum_j \tilde{u}_j^* s_{jo}$  will fall below actual energy consumption ( $h_o^* < e_o$ ). Because  $\eta_o^* = h_o^*/e_o$ , household  $o$  is inefficient with

$0 < \eta_o^* < 1$ . Since  $\tilde{\mathbf{u}}^*$  is also feasible for the best-practice households,  $h_o^*$  reflects the amount of energy that a best-practice household would consume *at most*, given that it would produce the same amount of services as household  $o$ .

### 3 The Data

We use data from the US *Residential Energy Consumption Survey* (RECS), conducted regularly by the US Energy Information Administration (EIA).<sup>3</sup> For our purpose, we use the surveys of 1997 and 2001 to check for energy efficiency improvements. Each survey contains household micro data of energy consumption, dwelling characteristics and the number of electric appliances. We concentrate our attention only on households living in single-family homes.

We had to drop a couple of observations from both years because of missing or implausible data. Because of lacking or inappropriate consumption figures we exclude households using coal, wood, district heating, renewable energies and households that state that their energy consumption covers demand from non-residential purposes or were disconnected from energy supply (e.g. due to an unpaid utility bill). Further we remove all households that had reported an annual energy consumption of less than 40 kWh per square-meter living space (including electricity and space heating/cooling).<sup>4</sup> The remaining sample comprises 4,284 households in total, from which 2,432 come from the 1997 survey, and 1,852 households from the 2001 survey.

Table 1 summarizes the employed input and output data. The only input is the households' total energy consumption, measured in kWh. We consider the following end uses: space heating and cooling, water heating, cooking, and electric appliances. The number of household members are used as proxy for hot water and cooking demands. To account for energy consumption due to the use

---

<sup>3</sup>The data are available online at <http://www.eia.doe.gov/emeu/recs/contents.html>.

<sup>4</sup>Such a low consumption appears implausible, since it would even outperform a standard of low-energy buildings, which requires an energy consumption of annually at most 40 kWh/m<sup>2</sup>.

Table 1: Data Summary

		1997		2001		Total	
		Mean	Median	Mean	Median	Mean	Median
Total energy	kWh	35,310	32,765	32,400	30,406	34,052	31,775
Heatnorm	m <sup>2</sup>	152	138	179	159	164	145
Coolnorm	m <sup>2</sup>	121	104	195	150	153	120
Persons	number	2.6	2	2.7	2	2.7	2
Electr. Appliances	number	3.8	3	4.7	4	4.2	4
Fridges, Freezers	number	1.6	1	1.6	2	1.6	1

of electric appliances, we incorporate the joint number of TV-sets, videos, DVDs, and computers. The overall number of refrigerators and freezers in the household are likewise included in our estimation.

To account for climate conditions, we calculate the following normalized values for thermal conditioned living space:

$$(4) \quad \text{heatnorm}_l = \text{space}_l \times NHDD_l$$

$$(5) \quad \text{coolnorm}_l = \begin{cases} \text{space}_l \times NCDD_l, & \text{if household has air-conditioning,} \\ 0, & \text{else,} \end{cases}$$

where  $\text{space}_l$  describes the living space of household  $l$  in m<sup>2</sup>. The factors  $NHDD_l$  and  $NCDD_l$  control for the deviation of (household specific) actual heating ( $HDD_l$ ) and cooling degree days ( $CDD_l$ ) from their 30 year average (1961 to 1990, NCDC 2002a, 2002b):<sup>5</sup>

$$(6) \quad NHDD_l = \frac{HDD_l}{\text{average } HDD},$$

$$(7) \quad NCDD_l = \frac{CDD_l}{\text{average } CDD}.$$

$HDD$  are calculated as the difference between 65° F indoor temperature and the daily average outdoor temperature below 65° F, summed over all days of a year.

<sup>5</sup>Since the most precise location we have for the households is their affiliation to a US census division, we used 30 year averages on a division basis.



*CDD* are calculated in a like manner. The effect of the adjustment is twofold. A household that has to bridge a large gap between indoor and outdoor temperature gets in this way comparable to households living in moderate climatic regions of the USA. Further, it is controlled for intertemporal changing weather conditions.

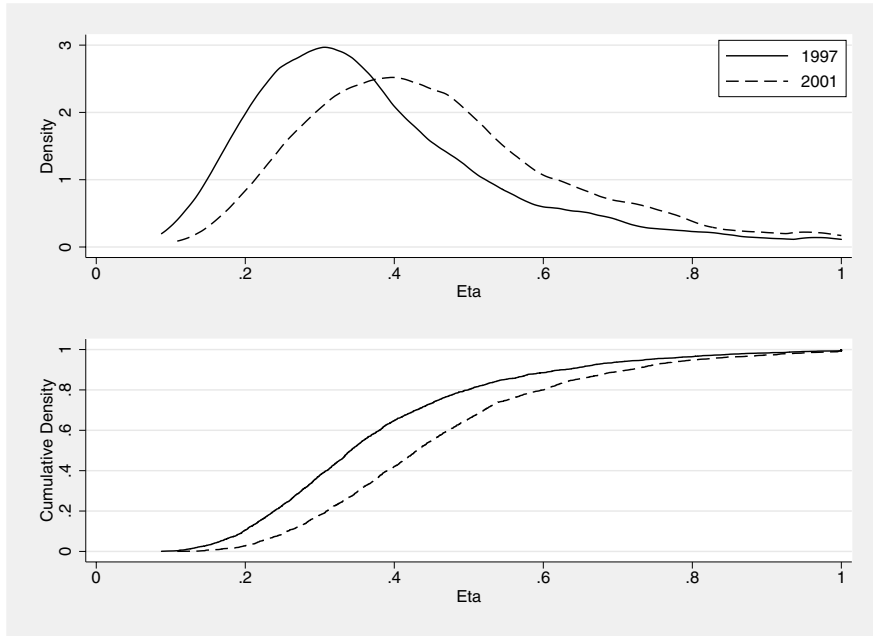
By pooling all households from both years and solving model (3) for each household, we benchmark each household against an intertemporal best-practice frontier (Tulkens and Vanden Eeckhaut 1995), and obtain for every household an efficiency estimates  $\eta_i^*$ . We continue with two empirical efficiency distributions, one for each of the respective years. If energy efficiency has improved between 1997 and 2001, we generally expect a comparably larger  $\eta_i^*$  (meaning a smaller conservation potential) for households belonging to the 2001 survey. Thus, we can focus on deviations between the efficiency distributions from 1997 and 2001.

To test whether such a difference is significant, we employ simple nonparametric tests, which are in line with the nonparametric nature of the DEA methodology. The *Kolmogorov-Smirnov (KS) Two-Sample Test* focuses on the maximal vertical deviation between the two cumulative densities and tests if one distribution is stochastically larger than the other. Further, we use the *Robust-Rank-Order Test* which draws inference from a rank vector. Both tests are discussed in more detail in the appendix.

## 4 Results

Figure 2 shows the empirical and the cumulative densities of the two distributions. The bulk of observations lie within a range of  $0.1 < \eta_i^* < 0.6$ , with a tail to the right. There are 33 households with an  $\eta_i^* = 1$ , thus, serving as best-practice benchmark. The average  $\eta^*$  of 1997 amounts to 0.38, whereas the average  $\eta^*$  of 2001 is 0.46. This increase gives a first hint that some efficiency improvements

Figure 2: Efficiency estimates  $\eta^*$  in single-family homes

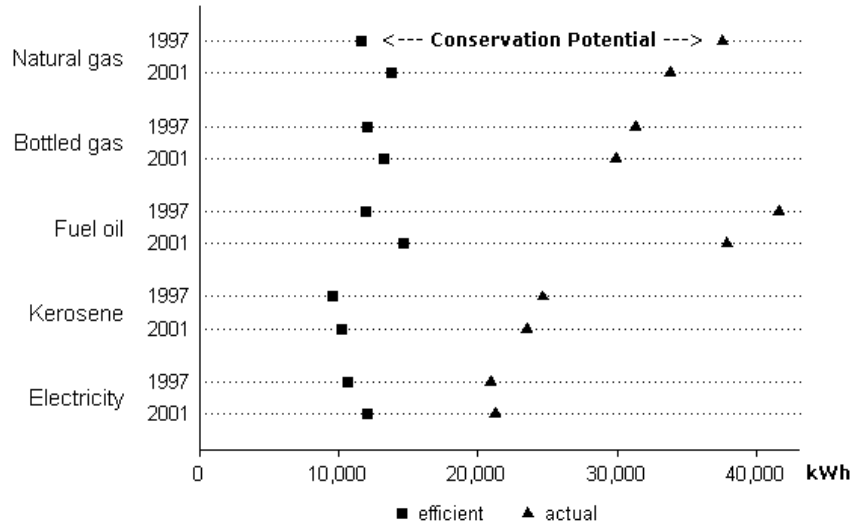


between 1997 and 2001 have occurred.

As can be seen in Figure 2, the 2001 density has more mass in higher efficiency regions, compared to its counterpart. Accordingly, the cumulative density of 2001 lies well below the cumulative density of 1997: The (one-side) Kolmogorov-Smirnov test computes a maximal distance of  $D = 0.2355$  between the two cumulative densities, which is significantly different from zero ( $p$ -value: 0). Likewise, the Robust-Rank-Order test computes a test statistic of  $\tilde{U} = 16.48$ , being far higher than any popular critical value. In short, both tests indicate a statistical significant improvement in energy efficiency for US single-homes between 1997 and 2001.

The long tails of the two distributions in Figure 2 imply, that an average household is quite inefficient, compared to the best-practice benchmarks, and exhibits a large conservation potential. A look at the mean distance between actual energy consumption and efficient consumption gives some hint on the

Figure 3: Mean Efficient and Mean Actual Energy Consumption



currently remaining conservation potential. Figure 3 shows actual (black triangle) and computed efficient consumption (black square) for the average household in both years and for each main heating fuel. The average fuel oil consumer demands the most energy per household: in 1997 the mean fuel oil consumption adds up to 40,157 kWh. The average household who heats mainly with electricity demands just about 50% of this amount. In the wake of this difference, households heating with oil exhibit on average a much larger conservation potential.<sup>6</sup>

It is plausible to assume that an inverse relationship exists between the conservation potential and the cost of conserving energy. Thus, one can expect that the biggest efficiency improvements come from heating systems with a large conservation potential. As Figure 3 reveals, the most notable efficiency improvements came in fact from households heating either with natural gas or fuel oil.

<sup>6</sup>Note that the reported household consumption refers to site or delivered energy. The proportions between oil and electricity heaters would not appear such drastically, if we would consider primary energy, and account thereby for conversion leakage in the power plants. The EIA (1997) reports that households heating mainly with electricity consume on average 34,172 kWh of primary energy.

Their conservation potential decreased most striking between the years, whereas the mean actual consumption for electricity remained quite stable. But it is apparent, that especially oil users exhibit still the largest conservation potential.

A substantial share of the observed declines in conservation potential stem from an increase of the respective efficient consumption value. This rise of the benchmark might appear counterintuitive, especially because we are faced with efficiency improvements. The benchmark varies between fuels and years, since it depends on the certain characteristics included in model (3) of the particular average household. For example, in 2001 the normalized heated living space ('heatnorm') of the mean fuel oil consumer adds up to 211 m<sup>2</sup>, whereas only 130 m<sup>2</sup> are heated on average by electric heaters. Accordingly, a larger energy consumption is justified in 2001 for the average fuel oil consumer. The increase in the produced amount of services between the years (Table 1) likewise explains the rise of the justified amount of energy consumption. Unfortunately, this growth in service production narrows the currently remaining conservation potential and limits the capability of efficiency improvements to counteract the greenhouse effect and fuel import dependencies.

## 5 Summary

This paper has measured energy efficiency improvements in US single-family homes between 1997 and 2001. Treating a household as a producing entity (Becker, 1965), we use an intertemporal version of Dyson and Thanassoulis (1988) DEA model. With simple nonparametric tests we checked for efficiency improvements.

DEA has a strong theoretical background in production theory (Seiford and Thrall, 1990) and is as easy to interpret as the traditional efficiency measurement approaches. In favor of DEA we argue, that it has light data requirements and can avoid the burdensome demands that accompany the traditional approaches.

Our analysis reveals that, in fact, US households living in single-family homes improved their energy efficiency by narrowing their distance to the efficient best-practice frontier. But this distance shows also the currently remaining capability of improving efficiency to alleviate the greenhouse effect and fuel import dependencies. This capability is increasingly limited, since the observed decline in conservation potential is partially caused by a growth of households' service production. Hence, if all households had already operated efficiently, residential energy consumption would even have increased.

## Acknowledgments

The author is indebted to Manuel Frondel and Colin Vance who contributed very helpful comments on this paper.

## References

ANG, B.W. (2006), Monitoring Changes in Economy-Wide Energy Efficiency: From Energy-GDP Ratio to Composite Efficiency Index, *Energy Policy*, Vol.34: 574-582.

BECKER, G.S. (1965), A Theory of the Allocation of Time, *The Economic Journal*, Vol.75: 493-517.

DYSON, R.G. AND E. THANASSOULIS (1988), Reducing Weight Flexibility in Data Envelopment Analysis, *Journal of the Operational Research Society*, Vol.39: 563-576.

EIA (2006), U.S. Residential Energy Intensity Using Weather-Adjusted Site Energy by Census Region and Type of Housing Unit, 1980-2001, Energy Information Administration, Washington DC, Internet: <http://www.eia.doe.gov/emeu/efficiency>.

EIA (2004), Square Footage Measurements and Comparisons: Caveat Emptor, 2001 Residential Energy Consumption Survey, Energy Information Administration, Washington DC, Internet: <http://www.eia.doe.gov/emeu/recs/sqft-measure.html>.

EIA (1999), End-Use Estimation Methodology, in: *A Look at Residential Energy Consumption in 1997* (Appendix C), Energy Information Administration, Washington DC, Internet: <http://www.eia.doe.gov/emeu/recs/pubs.html>.

EIA (1997), Residential Energy Consumption Surveys: 1997 Consumption and Expenditure Tables - Total Consumption Tables, Energy Information Administration, Washington DC, Internet: <http://www.eia.doe.gov/emeu/recs/contents.html>.

FERRIER, G.D., AND J.G. HIRSCHBERG (1992), Climate Control Efficiency, *The Energy Journal*, Vol.13: 37-54.

HAAS, R. (1997), Energy Efficiency Indicators in the Residential Sector: What Do We Know and What Has to Be Ensured, *Energy Policy*, Vol.25: 789-802.

MUKHERJEE, K. (forthcoming), Energy Use Efficiency in U.S. Manufacturing: A Nonparametric Analysis, forthcoming in *Energy Economics*.

NCDC (2002a), State, Regional, and National Monthly Cooling Degree Days Weighted by Population (2000 Census), *Historical Climatology Series 5-2*, National Climatic Data Center, Asheville.

NCDC (2002b), State, Regional, and National Monthly Heating Degree Days Weighted by Population (2000 Census), *Historical Climatology Series 5-1*, National Climatic Data Center, Asheville.

PHYLIPSEN, G.J.M., K. BLOK AND E. WORRELL (1998), *Handbook on International Comparisons of Energy Efficiency in the Manufacturing Industry*, Department of Science, Technology and Society, Utrecht University, The Netherlands.

PHYLIPSEN, G.J.M., K. BLOK AND E. WORRELL (1997), International Comparisons of Energy Efficiency-Methodologies for the Manufacturing Industry, *Energy Policy*, Vol.25: 715-725.

SCHIPPER, L., A. KETOFF, AND A. KAHANE (1985), Explaining Residential Energy Use by International Bottom-Up Comparisons, *Annual Review of Energy*, Vol.10: 341-405.

SCHIPPER, L., F. UNANDER, S. MURTISHAW AND M. TING (2001), Indicators of Energy Use and Carbon Emissions: Explaining the Energy Economy Link, *Annual Review of Energy and the Environment*, Vol.26: 49-81.

SEIFORD, L.M. AND R.M. THRALL (1990), Recent Developments in DEA: The Mathematical Programming Approach to Frontier Analysis, *Journal of Econometrics*, Vol.46: 7-38.

SIEGEL, S. AND N.J. CASTELLAN (1988), *Nonparametric Statistics for the Behavioral Sciences*, McGraw-Hill, New York.

TULKENS, H. AND P. VANDEN EECKAUT (1995), Non-Parametric Efficiency, Progress and Regress Measures for Panel Data: Methodological Aspects, *European Journal of Operational Research*, Vol.80: 474-499.

WIRL, F. (1997), *The Economics of Conservation Programs*, Kluwer Academic Publishers, Dordrecht.

## Appendix: Nonparametric tests

The *Kolmogorov-Smirnov (KS) Two-Sample Test* examines whether two independent samples come from the same population.<sup>7</sup> It focuses on the maximal vertical deviation between the two empirical cumulative density distributions. As a one-tail test, it tests  $H_0$ : “No difference in the distributions” against  $H_1$ : “One distribution is stochastically larger”, where ‘stochastically larger’ means in our context ‘more efficient’, since efficiency increases with  $\eta^*$ . Basically, the KS test computes along the range of  $\eta^*$  the difference between the two cumulative density functions:

$$(8) \quad D = \max\{F_{1997}(\eta^*) - F_{2001}(\eta^*)\}$$

and tests if this difference is “large enough” to reject  $H_0$ . The KS test is standard in most statistical software packages and can therefore be easily adopted. If no such option is available, Siegel and Castellan (1988) report tables for critical values.

Another method of comparing the difference between two distributions is to rank all  $L$  households by their  $\eta_i^*$  in ascending order, such that the largest  $\eta_i^*$  leads

---

<sup>7</sup>For more than two samples, one can use the Jonckheere test for ordered alternatives, described in e.g. Siegel and Castellan (1988).

to the highest rank. Now, if the households surveyed in 2001 are more efficient than the households surveyed in 1997, the bulk of the in 2001 inquired households should be assigned with comparably higher ranks. On the other hand, if no difference can be found, the observations of both years should be distributed randomly across the rank vector. The *Robust-Rank-Order Test* (Siegel and Castellan, 1988) inspects whether the assigned ranks for one year are significantly larger. For each household, this test counts how many observations *of the other survey year* have a smaller rank, viz. are less efficient. For example, if a household  $l$  is surveyed in 1997 and has rank 15 we count for the ranks 1 to 14 how many households were surveyed in 2001. If this count is 6, then  $U_l^{1997} = 6$ .

After doing this count for all members in both sets, the respective means and variabilities can be calculated:

$$(9a) \quad U^{2001} = \sum_{l \in 2001} U_l^{2001} / L^{2001}$$

$$(9b) \quad U^{1997} = \sum_{l \in 1997} U_l^{1997} / L^{1997}$$

$$(9c) \quad V^{2001} = \sum_{l \in 2001} [U_l^{2001} - U^{2001}]^2$$

$$(9d) \quad V^{1997} = \sum_{l \in 1997} [U_l^{1997} - U^{1997}]^2,$$

where  $L^{2001}$  and  $L^{1997}$  describes the number of households surveyed in 2001 and 1997, respectively. The test statistics:

$$(10) \quad \tilde{U} = \frac{L^{1997} U^{1997} - L^{2001} U^{2001}}{2\sqrt{V^{1997} + V^{2001} + U^{1997}U^{2001}}}$$

approaches the unit normal distribution for large samples. For small samples (e.g.  $L^{1997} \leq L^{2001} \leq 12$ ) tabulated values are available (Siegel and Castellan, 1988).

A natural occurrence with DEA are ties across groups in the rank assignment. At least if we find several efficient households with  $\eta_l^* = 1$  while belonging to different survey years, we have ties for the highest ranks. In such a case, households



of the other survey year having the same rank are judged only with  $1/2$  (Siegel and Castellan, 1988). This means for a certain household  $l$  from 1997: Count all households surveyed in 2001 with a lower rank +  $1/2$  times the households surveyed in 2001 having the same rank like household  $l$ .