Optimal use of a polluting capital stock

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!!! very preliminary and incomplete !!!

Abstract. Sustainable economic growth necessitates sufficient provision of energy. In view of the finiteness of fossil fuels, ensuring sustainable growth requires the transition towards the usage of renewable resources. Past and recent work in environmental growth theory has dealt intensively with this problem. Most of the existing models however, tend to neglect one essential aspect of this problem. Namely, that renewables and non-renewable resources are associated with their own, hardly shiftable, capital stock. As a consequence, policy implications extracted from these models are based on a distorted picture about the easiness of the transitional dynamics of resource use. The reason for this is that a homogenous capital stock offers an additional degree of freedom about the substitution away from the non-renewable resources. Resource depletion and environmental damages can be counteracted at a large scale by simply shifting capital between sectors. This paper takes a different approach. We develop a growth model of an economy with two capital stocks associated with different technologies, one of them being clean, the other one causing environmental damages. The technologies are completely embedded in the corresponding stock of physical capital. The usage of one technology can be intensified in two ways. First, via investment in the associated capital stock. Since we implement convex adjustment costs of investment a quick built up of the alternative capital stock becomes more expensive in terms of forgone consumption. Second, we add a capacity utilization rate as an exogenous variable. It is defined as the intensity with which the capital stock is utilized. However, a higher rate of of capacity utilization causes the capital stock to depreciate faster (endogeneous rate of depreciation). These two mechanisms are the major drivers of our model which we use to examine the the socially optimal use of the capital stocks and hence, the socially optimal mix of the technologies involved.

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1 Introduction

Sustainable economic growth necessitates a sufficient provision of energy. In view of the finiteness of fossil fuels, ensuring sustainable growth requires the transition towards the usage of renewable resources. Past and recent work in environmental growth theory has dealt intensively with this problem. The works by Dasgupta and Heal [4], Pezzey and Withagen [6] and Tahvonen and Salo [7] in particular analyze the transitional dynamics away from the usage of conventional technologies and towards the adoption of backstop technologies. Most of these models however, tend to neglect one essential aspect of this problem. Namely, that renewable and non-renewable resources are associated with their own, hardly shiftable, capital stock. As a consequence, policy implications extracted from these models are based on a distorted picture about the easiness of the transitional dynamics of resource use. The reason for this is that a homogenous capital stock offers an additional degree of freedom about the substitution away from the non-renewable resources. Resource depletion and environmental damages can be counteracted at a large scale by simply "shifting capital" between sectors.

However, if we assume that installed capital is fully embodied in an existing technology, the average productivity characteristics and environmental impact of the total capital stock will change only slowly, as new capital goods fill the gaps left by the physical decay and scrapping of old capital goods.

If we link a capital stock in a putty-clay[2] manner to a technology which relies on a depleting resource the consequence is that the stock has to be used up eventually, or at least underutilized and left idle.

In case the long run level of the capital stock associated with conventional resources is less than the current one, the capital stock inevitably has to be reduced in the transition process. In a closed economy[3] such a reduction

[2]The advantage of using a putty-clay structure is that it implements the idea of irreversible investment decisions.

[3]We assume that it is not possible to sell part of the idle capital stock abroad.

can occur by investing less than needed to compensate physical decay. By analogy, if the economic environment requires a sudden change of the energy mix, an economy characterized by a heterogeneous capital structure can not react without severe time lags. This is due to the ex post clay nature of investment. Installment of the desired capital stock simply takes time if one does not want to abstain from smooth consumption patterns.

There are only a few models so far which have analyzed optimal investment with several stocks of physical capital. (Ryder [8], Pitchford[5]). Pitchford [5] deals with the optimal investment into two regions of an economy. In a dynamic growth model with two heterogeneous capital stocks, the author investigates the optimal investment decisions and their change towards the steady state. These and the vast majority of other standard aggregate models assume that capital depreciates at an exogenous rate; thus neglecting the increasing user cost of capital with respect to the intensity of usage. This causality however, is crucial for the case at hand, where we want to analyze the dynamic interplay between using up an old and polluting capital stock and building up a less polluting and sustainable capital stock. Especially in a setting where we are interested in how a polluting capital stock is maintained and accumulated and/or used up optimally. In order to analyze the inter- and intrasectoral tradeoffs between capacity building and capacity using which guide the economy's transition process towards a balanced growth equilibrium we develop a model with two production sectors that generate a homogenous consumption good. The production processes in these two sectors differ with respect to the technology which is used. While in one sector the process is clean, generating output in the other sector also creates environmental damage. The technologies are completely embedded in the corresponding stock of physical capital. Hence, the usage of one technology can only be intensified by investing more in the associated capital stock or utilizing it more intensively.

Similar to Chatterjee [4] the combination of capital utilization and an endogenous depreciation rate severely affects the related sector of an economy. Since output in a given period does not only depend on the capital stock in place but also on the flow of capital services extracted from it, the control over capital utilization enables firms to adjust faster to exogenous shocks or to other changes in the economic environment. In addition, firms influence the rate at which the installed capital stock depreciates by utilizing it only to a certain degree. Hence, depreciation is of the wear and tear form and firms are faced with the tradeoff between extracting extra output from a given capital stock and the costs of overutilizing it.

The paper is organized as follows. The next section outlines the model and stresses some important features of the transitional dynamics. Section 3 then, presents the results of some simulations which have been carried out. Section 4 concludes.

2 The Model

Similar to Pitchford [5] there are two output sectors in the economy producing one homogenous consumption good. Capital in its broad notion is the only factor of production in both output sectors. The output sectors differ with respect to the production process of the consumption good. While sector 1 is assumed to apply a clean technology, the production process in sector 2 generates pollution. In order to keep things as simple as possible, we assume that these technologies enter the production structure of a firm via the firm's capital stock. We do not model the technologies explicitly, as we assume that physical capital is the only factor of production in the economy.

2.1 Capacity installment and utilization

Effective capital in each sector is given by the product of the installed capital stock and the intensity with which the capital stock is utilized.

$$K_1^e = \kappa_1 \cdot K_1 \quad (1)$$
$$K_2^e = \kappa_2 \cdot K_2 \quad (2)$$

As in Chatterjee[4] we will define the rate of capital utilization κ_i as the intensity with which the installed stock of capital K_i is utilized in sector i[4]. Let $\delta_i(\kappa_i)$ be the rate of depreciation depending on κ_i the intensity with which the capital stock is utilized. Therefore, in any period the instantaneous change in the capital stocks in both sectors is given by[5]

$$\dot{K}_1 = I_1 - \delta_1(\kappa_1) \cdot K_1 \quad (3)$$
$$\dot{K}_2 = I_2 - \delta_2(\kappa_2) \cdot K_2 \quad (4)$$

Output in each sector is a twice differentiable function of effective capital.

$$Y_1 = Y_1(K_1^e) \quad (5)$$
$$Y_2 = Y_2(K_2^e) \quad (6)$$

with its derivatives satisfying $f_i' > 0$ and $f_i'' < 0$. Furthermore, we

[4]Alternatively we could model capacity utilization as a fraction/part of the existing capital stock in use as in Fisher *et al.*[7]. Our simple approach has the advantage of getting rid of two additional Kuhn-Tucker conditions (one for each sector). This comes in very handy in the analysis of the optimal growth path since we can evade performing a path connection procedure with 36 paths (In our model with four control variables we would have 6 Kuhn-Tucker conditions). In addition, the functional form for the depreciation rate ensures that capacity utilization will globally never be zero. (see Proposition 1)

[5]where we denote $\dot{x} = \frac{\partial x}{\partial t}$

assume that the Inada-Uzawa conditions hold, i.e.:

$$\lim_{K_i^e \to \infty} Y_i' = 0,\ \lim_{K_i^e \to 0} Y_i' = \infty,\ Y_i(0) = 0,\ Y_i(\infty) = \infty \tag{7}$$

The total output of the economy (and hence, total production of the homogenous good) is simply the sum of the sector-specific output levels.

$$f = Y_1 + Y_2 \tag{8}$$

Total output is used for consumption and investment where C denotes the contemporary consumption level of the economy and I is the total investment volume in the same period.

$$C = f - I \tag{9}$$

Total investment is distributed over the two sector-based capital stocks, where

$$I = I_1 + I_2 \tag{10}$$

2.2 Adjustment costs

Similar to Fisher *et al.*[7] the installment of additional capital is costly. However, in our model the adjustment costs of capital are not measured in labor units but rather in consumption forgone. In this model we will allow for disinvestment. Hence, capital can be converted into consumption. This is a trivial assumption, especially since we have a capacity utilization rate. Thus, it is always inefficient to disinvest capital. If one needs to use less of it one simply *uses* less.[6] Nevertheless, we are using this assumption since again,

[6]Even without a capacity utilization rate, disinvestment would be a rare occurrence in this model. In the neighborhood of the steady state it is obvious that investment would

we do not need to implement additional Kuhn-Tucker conditions. Denote by $A_i(I_i)$ the instantaneous adjustment costs occurring from installing additional capacity I_i in sector i. $A_i(I_i)$ is chosen such as to satisfy

$$\begin{aligned} A_i(0) &= 0,\ A_i(I_i) \geq 0,\ A_i' \begin{cases} > 0, & I_i > 0 \\ = 0, & I_i = 0 \\ < 0, & I_i < 0 \end{cases} \\ A_i''(I_i) &> 0,\ \lim_{I_i \to \infty} A_i' = \infty,\ \lim_{I_i \to -\infty} A_i' = -\infty,\ \lim_{I_i \to \pm 0} A_i' = 0. \end{aligned} \tag{11}$$

2.3 Social optimum

Consumption of the produced commodity generates utility $U(C)$ which is a standard CRRA function[7] satisfying

$$U_C \geq 0,\ U(0) = 0,\ U' > 0,\ \ U'' < 0. \tag{12}$$

However, in generating the consumption good, the usage of the effective capital stock in sector 2, $K_2^e = \kappa_2 \cdot K_2$ causes environmental damages which are perceived by the consumers. Let $D(\kappa_2 \cdot K_2)$ denote the disutility generated by using the environmental bad[8]. We assume that $D(\kappa_2 \cdot K_2)$ has the following properties:

$$D(K_2^e) \geq 0,\ D(0) = 0,\ D' > 0,\ D'' < 0 \tag{13}$$

Total utility, as perceived by the representative individual at time t, is simply the difference between the utility derived from consumption of the

have to be positive per definition of the steady state. Disinvestment would only occur for the case of an initially too high polluting capital stock that has to be used. In this example one would have to adjust the set of allowable parameter values.

[7]e.g.: $U(C) = \frac{C^{1-\theta}}{1-\theta}$, $0 < \theta < 1$

[8]e.g.: $D(K_2^e) = \frac{K_2^{e(1+\omega)}}{1+\omega}$, $\omega > 0$

homogeneous commodity and the disutility caused by the flow of pollution[9] accrued due to the usage of the polluting capital stock in the production process.

Setting up the dynamic maximization problem the objective function is given by

$$J = \int_{t=0}^{\infty} e^{-\rho \cdot t} \cdot [U(C) - D(\kappa_2, K_2)] \, dt \tag{14}$$

subject to the following constraints

$$\begin{aligned}
\dot{K}_1 &= I_1 - \delta_1(\kappa_1) \cdot K_1 && (15)\\
\dot{K}_2 &= I_2 - \delta_2(\kappa_2) \cdot K_2 && (16)\\
C &= f(\kappa_1, \kappa_2, K_1, K_2) - A_1(I_1) - A_2(I_2) && (17)\\
\kappa_1 &\geq 0 && (18)\\
\kappa_2 &\geq 0 && (19)
\end{aligned}$$

In addition, the transversality condition requires

$$\lim_{t \to \infty} e^{-\rho \cdot t} \cdot \lambda_i \cdot K_i = 0 \qquad \forall\, x \in \{1, 2\} \tag{20}$$

The current value Hamiltonian is then given by

$$\begin{aligned}
H &= U\big(f(\kappa_1, \kappa_2, K_1, K_2) - A_1(I_1) - A_2(I_2)\big) && (21)\\
&- D(\kappa_2, K_2) + \lambda_1 \cdot [I_1 - \delta_1(\kappa_1) \cdot K_1] + \lambda_2 \cdot [I_2 - \delta_2(\kappa_2) \cdot K_2]
\end{aligned}$$

[9]It would have been more appropriate to model pollution as a stock variable to investigate the history effect of investment on the polluting capital stock. This is however beyond the scope of this paper, since we are rather interested in the inter- and intrasectoral tradeoffs between capacity building and capacity using which guide the transition process towards the steady state. Therefore, we choose the less appropriate alternative.

and the corresponding Lagrangian is

$$L = H + \mu_1 \cdot \kappa_1 + \mu_2 \cdot \kappa_2 \tag{22}$$

Applying the maximum principle yields the following F.O.C.

$$\begin{aligned}
\frac{\partial L}{\partial I_1} &= 0 \Rightarrow U' \cdot A_1' = \lambda_1 && (23)\\
\frac{\partial L}{\partial I_2} &= 0 \Rightarrow U' \cdot A_2' = \lambda_2 && (24)\\
\frac{\partial L}{\partial \kappa_1} &= 0 \Rightarrow U' \cdot f_{\kappa_1}' - \lambda_1 \cdot \delta_1' \cdot K_1 + \mu_1 = 0 && (25)\\
\frac{\partial L}{\partial \kappa_2} &= 0 \Rightarrow U' \cdot f_{\kappa_2}' - D_{\kappa_2}' - \lambda_2 \cdot \delta_2' \cdot K_2 + \mu_2 = 0 && (26)\\
\frac{\partial L}{\partial K_1} &= \lambda_1 \cdot \rho - \dot{\lambda}_1 \Rightarrow U' \cdot f_{K_1}' - \lambda_1 \cdot \delta_1 = \lambda_1 \cdot \rho - \dot{\lambda}_1 && (27)\\
\frac{\partial L}{\partial K_2} &= \lambda_2 \cdot \rho - \dot{\lambda}_2 \Rightarrow U' \cdot f_{K_2}' - D_{K2}' - \lambda_2 \cdot \delta_2 = \lambda_2 \cdot \rho - \dot{\lambda}_2 && (28)
\end{aligned}$$

The necessary Kuhn Tucker conditions are

$$\kappa_1 \geq 0, \quad \mu_1 \geq 0, \quad \kappa_1 \cdot \mu = 0 \tag{29}$$

$$\kappa_2 \geq 0, \quad \mu_2 \geq 0, \quad \kappa_2 \cdot \mu = 0 \tag{30}$$

In the next step we want to show that the complementary slackness conditions in conditions (25) and (26) imply that μ_1 and μ_2 are both zero.

Proposition 1. *The system described by (23)-(30) has a singular solution.*

Proof. We show that $\forall t \in \{0, \infty\}$ $\mu_i = 0$ and hence, $\kappa_i > 0$. First, consider κ_1. We have to show that if starting from $\kappa_1 = 0$, welfare at time t can be increased by setting $\kappa_i > 0$.

From (25) we have $\frac{\partial L}{\partial \kappa_1} = U' \cdot f'_{\kappa_1} - \lambda_1 \cdot \delta'_1 \cdot K_1 + \mu_1$. Since by equations (7) and (11) $\lim_{\kappa_1 \to 0} f'_i = \infty$, $\lim_{\kappa_1 \to 0} \delta'_i = 0$, and $\mu_1 \geq 0$ it follows that $\lim_{\kappa_1 \to 0} \frac{\partial L}{\partial \kappa_1} = \infty$. We can apply the same approach to κ_2:
From (26) we have $\frac{\partial L}{\partial \kappa_2} = U' \cdot f'_{\kappa_2} - \lambda_2 \cdot \delta'_2 \cdot K_2 - D'_{\kappa_2} + \mu_2$. Since by equations (7), (11) and (13) $\lim_{\kappa_2 \to 0} f'_2 = \infty$, $\lim_{\kappa_2 \to 0} \delta'_2 = 0$, $\lim_{\kappa_2 \to 0} D'_{\kappa_2} = 0$ and $\mu_2 \geq 0$ it follows that $\lim_{\kappa_2 \to 0} \frac{\partial L}{\partial \kappa_2} = \infty$

$\square$

Proposition 1 states that it is always welfare improving to utilize capital because the marginal welfare gains for the first unit of capital intensity are infinite. This line of reasoning holds even for the case with a polluting capital stock because the marginal polution of the first unit of capital is nil.

2.4 Intersectoral tradeoffs

This sections purpose is to derive an equation which captures the link between optimal capital installment in the two sectors. Dividing equation (24) by (23), differentiating w.r.t. time and using (25) and (26) we can eliminate the co-state variables. Reformulating, we obtain the following optimality condition (see Appendix X for a derivation):

$$\frac{A''_2 \cdot \dot{I}_2}{A'_2} - \frac{A''_1 \cdot \dot{I}_1}{A'_1} = (\delta_1 - \delta_2) + \frac{f'_{K_1} \cdot U'_f}{A'_1 \cdot U'_A} - \frac{f'_{K_2} \cdot U'_f}{A'_2 \cdot U_A} + \frac{D'_{K_2}}{A'_2 \cdot U'_A}$$

We can w.l.o.g. assume that $U'_f = U'_A$. The condition above reduces to:

$$\frac{A''_2 \cdot \dot{I}_2}{A'_2} - \frac{A''_1 \cdot \dot{I}_1}{A'_1} = (\delta_1 - \delta_2) + \frac{f'_{K_1}}{A'_1} - \left(\frac{f'_{K_2}}{A'_2} - \frac{D'_{K_2}}{A'_2 \cdot U'_A} \right) \tag{31}$$

The LHS of equation (31) is the difference in the growth rates of the marginal adjustment costs. The RHS consists of two parts. The first term is the difference in the depreciation rates. The second term is the difference between the adjusted marginal social products in the two sectors.

...to be completed...

2.5 Intrasectoral tradeoffs

Within a sector of our model economy there exists the tradeoff between capital usage and capital build up, which translates into the optimal choice of κ_i and I_i. Proposition 1 has established that capital will always be utilized to some positive degree. Regarding the investment decision we can make the following claim:

Proposition 2. *There will be no disinvestment in any sector*

Proof. We show that $\forall t \in \{0, \infty\}, I_i \geq 0$. Let us first consider sector 1 and assume that $I_1 \leq 0$. Because of equation (11) $A_1' \leq 0$ and equation (23) would imply that $\lambda_1 < 0$. From equation (25) we would obtain that $\kappa_1 < 0$ which violates equation (29) and proposition 1. The same line of reasoning can be applied to sector 2. We assume that $I_2 \leq 0$. Because of equation (11) $A_2' \leq 0$ and equation (24) would imply that $\lambda_2 < 0$. From equation (26) we would obtain that $U' \cdot f_{\kappa_2}' - D_{\kappa_2}' < 0$. Since, the economy will never utilize capital such that the marginal social product (in utility terms) of capital utilization is negative, it must hold that $\kappa_2 \leq 0$ which in turn violates equation (30) and proposition 1. □

The intuition behind proposition 2 seems trivial. In terms of efficiency, disinvestment makes only sense if the capital stock needs to be decreased faster than by economic decay with zero gross investment. However, since we have included into our model κ_i, the capacity utilization rate, we have an additional channel by which we can steer how much capital is being used in the production process.

2.5.1 Sector 1

From the optimality conditions (23) and (25) we can derive an optimal rule for the tradeoff between the two control variables in sector 1. It is given by:

$$f'_{\kappa_1} = A'_1 \cdot \delta'_1 \cdot K_1 \tag{32}$$

The LHS of equation (32) is the marginal product (marginal social product) of capacity intensity. The RHS of (32) describes the marginal costs of expanding capacity by one extra unit. This is due to the fact that the new capital stock needs to be maintained. If e.g. due to technological progress the marginal product of capacity intensity increases, then it will be accompanied by either a higher investment volume, larger capital stock, higher capacity or any combination of these effects. Because of the properties of f, δ and A the LHS is strictly convex, and the RHS is strictly concave. Hence, for each level of capacity intensity there is exactly one optimal volume of investment.

2.5.2 Sector 2

We can conduct a similar analysis for sector 2. From the optimality conditions (24) and (26) we can derive an optimal rule for the tradeoff between the two control variables in sector 2. It is given by:

$$f'_{\kappa_2} - \frac{D'_{\kappa_2}}{U'} = A'_2 \cdot \delta'_2 \cdot K_2 \tag{33}$$

The only difference to sector 1 is the LHS. Because of the disutility from pollution due to the capital usage, the marginal product of capacity intensity is larger than its marginal social product which is the LHS of (33). It is equated to the marginal costs of expanding capacity by one extra unit. If e.g. the marginal disutility from pollution rises the RHS term becomes smaller.

To keep equation (33) in balance κ_2 can be decreased or we could lower the investment volume I_2 which in turn will slowly lower the capital stock K_2.

2.6 Transition dynamics and steady state analysis

The next step is to eliminate the co-state variables from equations (34)-(39) to obtain the equations of motion for our four controls and two states[10] (See Appendix X for a derivation)

$$\begin{aligned}
\dot{I}_1 &= \frac{U'_A \cdot A'_1}{U''_A \cdot A'_1 + U'_A \cdot A''_1} \cdot \left(\delta_1 + \rho - \frac{U'_f \cdot f'_{K1}}{U'_A \cdot A'_1} \right) && (34) \\
\dot{I}_2 &= \frac{U'_A \cdot A'_2}{U''_A \cdot A'_2 + U'_A \cdot A''_2} \cdot \left(\delta_2 + \rho - \frac{U'_f \cdot f'_{K2}}{U'_A \cdot A'_2} + \frac{D'_{K2}}{U'_A \cdot A'_2} \right) && (35) \\
\dot{\kappa}_1 &= \frac{(\delta_1 + \rho) \cdot (U'_f \cdot f'_{\kappa 1}) - U'_f \cdot f'_{\kappa 1} \cdot \delta'_1 \cdot K_1 + \frac{U'_f \cdot f'_{\kappa 1} \cdot \dot{K}_1}{K_1}}{U''_f \cdot f'_{\kappa 1} + U'_f \cdot f''_{\kappa 1} - \frac{U'_f \cdot f'_{\kappa 1} \cdot \delta''_1}{\delta'_1}} && (36) \\
\dot{\kappa}_2 &= \frac{(U'_f \cdot f'_{\kappa 2} - D'_{\kappa 2}) \cdot (\delta_2 + \rho) - (\delta'_2 \cdot K_2)(U'_f \cdot f'_{\kappa 2} + D'_{K2})}{\frac{U''_f \cdot f'_{\kappa 2} + U'_f \cdot f''_{\kappa 2} - D''_{\kappa 2} - \delta''_2 K_2 (U'_f \cdot f'_{\kappa 2} - D'_{\kappa 2})}{\delta'_2 \cdot K_2}} \\
&+ \frac{\dot{K}_2 \cdot \delta'_2 \cdot (U'_f \cdot f'_{\kappa 2} - D'_{\kappa 2})}{U''_f \cdot f'_{\kappa 2} + U'_f \cdot f''_{\kappa 2} - D''_{\kappa 2} - \delta''_2 K_2 (U'_f \cdot f'_{\kappa 2} - D'_{\kappa 2})} && (37) \\
\dot{K}_1 &= I_1 - \delta_1(\kappa_1) \cdot K_1 && (38) \\
\dot{K}_2 &= I_2 - \delta_2(\kappa_2) \cdot K_2 && (39)
\end{aligned}$$

We define the balanced growth equilibrium as a path along which all control and stock variables grow at zero growth rate. From equations (27)-(32)

[10] We can also solve the system in a different way. Combining equation (23) and (27) we obtain $\dot{C} = -\frac{U'}{U''} \cdot [\frac{f'_{K_1}}{A'_1} - \rho - \delta_1 - \frac{A''_1 \cdot \dot{I}_1}{A'_1}]$ Assuming that $A'_1 = 1$, $A''_1 = 0$ and $\delta_1 = \delta$ we can reproduce the standard Ramsey type equation for the change in consumption. Similarly we can combine equation (24) and (28) to obtain $\dot{C} = -\frac{U'}{U''} \cdot [\frac{f'_{K_2}}{A'_2} - \rho - \delta_2 - \frac{A''_2 \cdot \dot{I}_2}{A'_2} - \frac{D'_{K_2}}{U' \cdot A'_2}]$ which for $A'_2 = 1$, $A''_2 = 0$ and $\delta_2 = \delta$ reproduces the standard result of a neoclassical model with pollution. Since we are interested in the paths of investments we rather solve the system for the *direct* control variables.

we can establish conditions which hold in the balanced growth equilibrium. These are given by:

$$\dot{I}_1 = 0 \Rightarrow \delta_1 + \rho = \frac{U_f' \cdot f_{K1}'}{U_A' \cdot A_1'} \tag{40}$$

$$\dot{I}_2 = 0 \Rightarrow \delta_2 + \rho = \frac{U_f' \cdot f_{K2}'}{U_A' \cdot A_1'} + \frac{D_{K2}'}{U_A' \cdot A_1'} \tag{41}$$

$$\dot{\kappa}_1 = 0 \Rightarrow (\delta_1 \cdot \rho) \cdot f_{\kappa 1}' = f_{\kappa 1}' \cdot \delta_1 \cdot K_1 - \frac{f_{\kappa 1}' \cdot \dot{K}_1}{K_1} \tag{42}$$

$$\dot{\kappa}_2 = 0 \Rightarrow (\delta_2 \cdot \rho) \cdot \left(f_{\kappa 2}' - \frac{D_{\kappa 2}'}{U_f'} \right) = (\delta_2 \cdot K_2) \left(f_{\kappa 2}' + \frac{D_{K2}'}{U_f'} \right) - \frac{\dot{K}_2 \left(f_{\kappa 2}' - \frac{D_{\kappa 2}'}{U_f'} \right)}{K_2} \tag{43}$$

$$\dot{K}_1 = 0 \Rightarrow I_1 = \delta_1 \cdot K_1 \tag{44}$$

$$\dot{K}_2 = 0 \Rightarrow I_2 = \delta_2 \cdot K_2 \tag{45}$$

In order to analyze the balanced growth equilibrium and its stability properties we have to implement functional forms for the endogenous variables of the model.

These functional forms are:

$$
\begin{aligned}
U(C) &= \frac{C^{1-\theta}}{1-\theta} && (46)\\
D(K_2^e) &= \frac{K_2^{e\,1+\omega}}{1+\omega} && (47)\\
f(\kappa_1,\kappa_2,K_1,K_2) &= (\kappa_1 \cdot K_1)^{\beta_1} + (\kappa_2 \cdot K_2)^{\beta_2} && (48)\\
\delta_1(\kappa_1) &= \bar{\delta}_1 + \kappa_1^{\gamma_1} && (49)\\
\delta_2(\kappa_2) &= \bar{\delta}_2 + \kappa_2^{\gamma_2} && (50)\\
A_1(I_1) &= I_1^{\alpha_1} && (51)\\
A_2(I_2) &= I_2^{\alpha_2} && (52)
\end{aligned}
$$

Using equations (46)-(52) we can apply conditions (40)-(45) to solve for the steady state values of I_1, I_2, κ_1, κ_2, K_1, K_2 which are denoted by $\tilde{I}_1$, $\tilde{I}_2$, $\tilde{\kappa}_1$, $\tilde{\kappa}_2$, $\tilde{K}_1$, $\tilde{K}_2$ respectively.(see Appendix A2 for a derivation)

...to be completed...

3 Simulations

In order to make the analysis less complicated, the model has not been calibrated. Instead, it has been simulated by using arbitrary values for the parameters as well as arbitrary data for the exogenous variables and lagged endogenous variables. We intend to conduct more extensive research to find out about the working of the model in other regions of the parameter space. Our aim now, however, is to illustrate the qualitative behavior of the model and to find out by means of an experiment how the different channels within the structure of the model affect the choice of the control variables.

3.1 The base run

Table 1 depicts the parameter values that have been chosen for the simulation of the base run.

Parameter	Value	Parameter	Value
α_1	2	α_2	2
β_1	.5	β_2	.8
γ_1	2	γ_2	2
$\bar{\delta}_1$	.01	$\bar{\delta}_2$	.01
θ	3	ω	.02
K_1^0	1	K_2^0	5

Table 1: Parameter Values Base run

Using the parameter values outlined above, we have obtained the development over time of a number of important variables relative to their initial values. These are displayed in Figure 1.

The first thing one should observe is the monotonic behavior of the two capital stocks over time. While K_1 is increasing steadily over the simulation period we observe the opposite for K_2, the capital stock associated with the polluting technology. This behavior over time displays that the while K_1 is initially below its steady state value, K_2 is far above its steady state value.

K_1 is initially only of about one fifth of its steady state value. This huge difference has to be overcome which explains the high initial volumes of I_1. Also, since for low levels of the capital stock, high utilization is relatively inexpensive we observe a quite high utilization rate as compared to the steady state. Along the adjustment path towards the steady state K_1 increases rapidly up to around t=40 and slows down thereafter. The utilization of the capital stock decreases sharply due to the high maintenance cost of rising levels of the installed and operating capital stock. The direction of the essential variables in sector 2 is different. The capital stock is initially

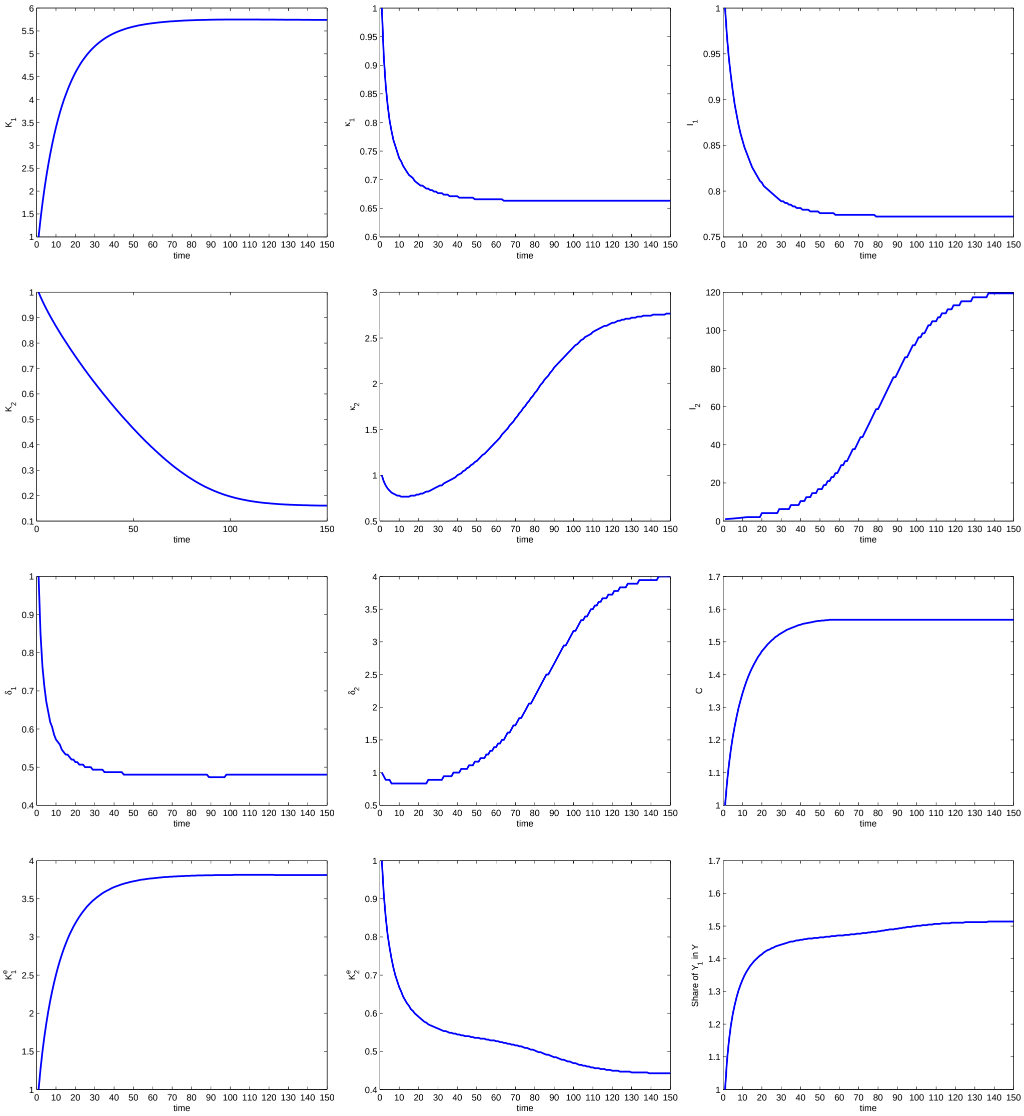

Figure 1: $K_1^0 = 1$ and $K_2^0 = 5$

far above its steady state level. As already laid out in chapter 2, investment levels are close to zero when the related capital stock is far above its steady state value[11]. In this phase the economy is effectively eating up its capital

[11]The magnitude of increase in I_2 over the simulation period might be misleading since

stock.

Note the slight and short change in the curvature for K_2^e at about t=70.

...to be completed...

3.2 Different initial values

This sections objective is to analyze the dynamics of K_1 and K_2 towards the steady state for different initial values. Figure 2 displays the adjustment of the capital stocks in the K_1-K_2 space.

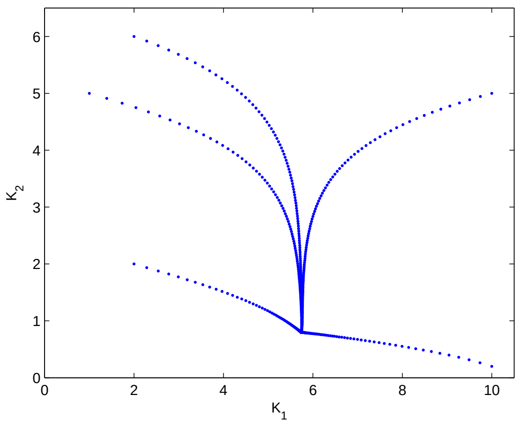

Figure 2: Different initial values for K_1 and K_2

the figures depict the behavior of variables relative to their initial levels. Investment in sector 2 starts at a zero level and eventually it is still significantly low in the steady state

As can be seen from figure 2 the common steady state is given by (5.742, 0.805). We have deliberately chosen initial values from different directions around the steady state. These are besides the base run (1, 5) also (2, 2); (2, 6); (10, 0.2) (10, 5). We observe a monotonic movement towards the steady state in all cases. Consider the initial point (10, 5). The capital allocation is to the north-east of the steady state implying that both capital stocks need to be

...to be completed...

It is obvious that we cannot put all control and state variables into one plot. We can however combine the time path of K_1, K_2 and consumption into one figure to show how consumption (and thus total investment) adjusts to the steady state if we choose different initial values for the two capital stocks. Figure 3 displays this adjustment in the K_1-K_2-C space.

...to be completed...

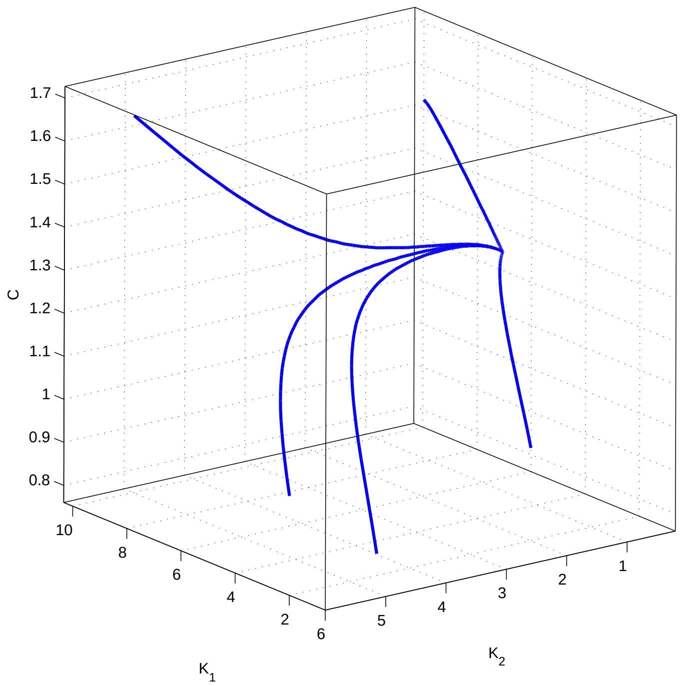

Figure 3: capital stocks and consumption

3.3 Exogenous shocks

In this section we describe the behavior of some model variables due to some exogenous shocks. The procedure is as follows. We run the base run simulation up to period t=50 to omit any possible initial value problems and implement the shock which persists until the end of the simulation period (t=150). Figure 4 displays the results of the scenario in which the usage of the polluting capital stock has an increasing adverse impact on welfare. More specifically: we double the perceived disutility from the effective usage of the installed capital stock in sector 2.

...to be completed...

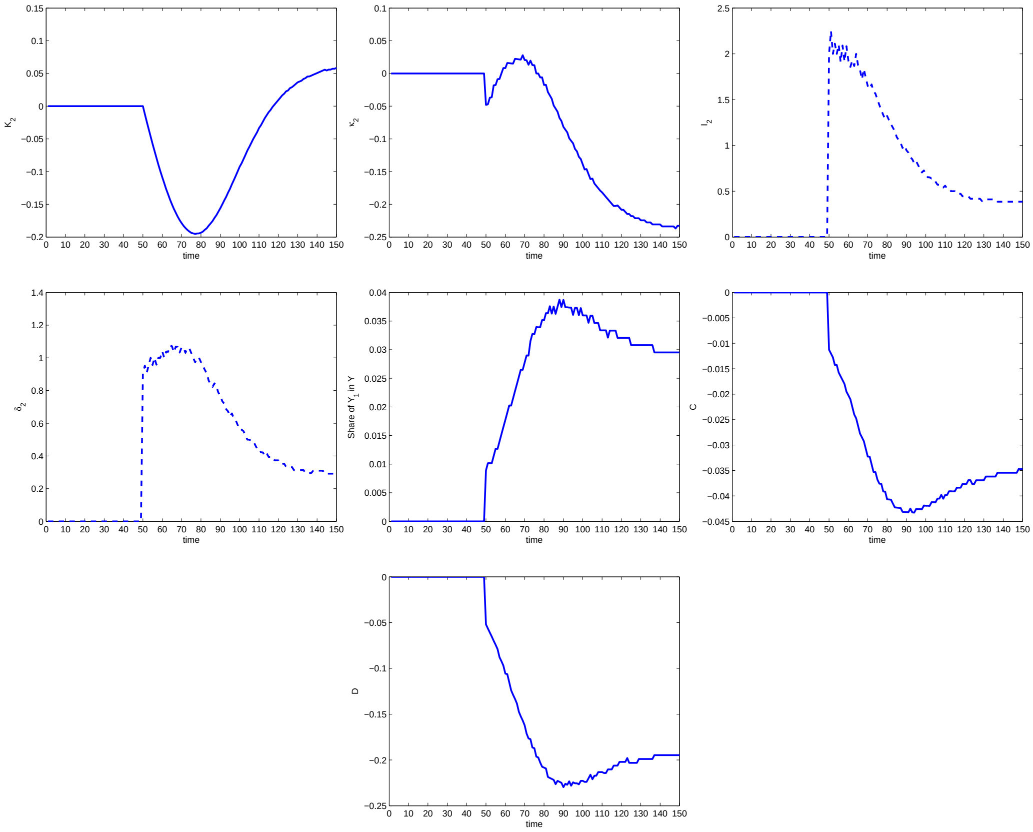

Figure 4: pollution more severe - % deviations from base run

4 Conclusion

Within a growth setting we have analyzed the dynamics of an economy in its transition process towards the steady state. The economy operates two sector-specific capital stocks which embody different technologies. While the usage of capital in one sector has no externality, the production process in the other sector causes environmental damage. In addition we have included a capital utilization rate. Thus, the economy must choose the intensity with which the installed stocks of capital are utilized. In this manner it is possible to extract more volume of capital than what is available. This comes in very handy when capital shortages have to be overcome. However, it does not come at no cost. The depreciation rate of capital has been endogenized such that capital depreciates at an increasing rate with the intensity of its usage. The characteristics of this model as laid out analytically and numerically provide some useful insights that should be considered in the debate about the transition towards more use of environmentally friendly energy technologies. The combination of heterogeneous capital, endogenous depreciation and capital intensity is in our view essential for extracting qualitative and quantitative implications for policy makers about the easiness of a technology switch. If the economic environment requires a sudden change of the energy mix, an economy driven by our model structure can not react without severe time lags, due to the ex post clay nature of investment. Installment of the desired capital stock simply takes time if one does not want to abstain from smooth consumption patterns.

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5 Appendix

A Derivation of the equation of motion

The first order conditions (1.1)-(2.2) are rewritten as follows: Differentiating (1.2) w.r.t time and rearranging (See appendix A) we

$$
\begin{aligned}
\lambda_1 &= U' \cdot A_1' && (53)\\
\lambda_2 &= U' \cdot A_2' && (54)\\
U' \cdot f_{\kappa_1}' &= \lambda_1 \cdot \delta_1' \cdot K_1 + \mu_1 && (55)\\
U' \cdot f_{\kappa_1}' &= \lambda_2 \cdot \delta_2' \cdot K_2 - D_{\kappa_2}' + \mu_2 && (56)\\
\dot{\lambda}_1 &= \lambda_1 \cdot (\delta_1 + \rho) - U' \cdot f_{K_1}' && (57)\\
\dot{\lambda}_2 &= \lambda_2 \cdot (\delta_2 + \rho) - U' \cdot f_{K_2}' + D_{K2}' && (58)\\
& && (59)
\end{aligned}
$$

...to be completed