Get more independent! Input and Output-Market-Based Dynamic Incentives to Adopt Input-Saving Technology.

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Abstract

We study the long-term incentives for a firm to invest in advanced production technologies, when some new technology is available but an even better technology will be available at some unknown future date. We consider an input and an output market since the incentives in our model are given by the repercussions which the adoption of advanced technology by a certain number of firms create on the input and the output market. Depending on the relative size of adoption fixed costs almost all possible combinations of investment patters may occur in the social optimum. In case of a decentralized decision we can show a analogous result for the market equilibrium. Moreover we show that for input goods like oil or ore, where the input markets are characterized by market power of the provider, there is a greater incentive to adopt new production technologies than in case of a competitive input market. If the input supply is fixed, which for example can be the case if we consider a regulation by permits, where the regulator makes a long term commitment, the contrary is the case.

Keywords: Input-/Output Markets, technology adoption, leapfrogging, option value theory, uncertainty, Poisson distribution, tradeable permits

1 Introduction

t.b.w.

2 The Model

We consider a competitive industry consisting of a continuum of ex ante symmetric firms represented by the interval $[0,1]$. All firms produce a homogenous output good q. The inverse demand function is given by $P(Q)$, where Q is the aggregated output. We assume $P'(Q) < 0$. Furthermore we assume a homogenous input e needed in the production of each firm. For example this can be coal, oil, iron but also worker or pollution or any kind of intermediate good. To produce an output of q each firm faces production cost $C(\theta, q, e)$ which depend on the amount of input e and the technology in use θ. Such kind of production cost function can be derived from a usual production function in the following manner. Assume that there are n other input goods x_i, $i = 1, \ldots, n$ next to good we are interested in. The factor prices are w_i, $i = 1, \ldots, n$. The firms own some production technology represented by a production function $f(x_a, \ldots, x_n, e, \theta)$, where θ specifies the technology and is exogenously given. Then by fixing the output level q and the level of our input e we can derive the function C by

$$min_{x_a,\ldots,x_n} \sum_{i=1}^{n} w_i x_i$$

subject to $q = f(x_a, \ldots, x_n, e, \theta)$.

We make the following assumptions about the function C.

Assumption 2.1. i) For each θ and q there exists a unique laisser-faire input level $e^{max}(\theta, q)$, characterized by $\frac{\partial C}{\partial e}(e^{max}(\theta, q), q, \theta) = 0$. For each input level $e < e^{max}(\theta, q)$ we have $c(e, q, \theta) > 0$, $-\frac{\partial C}{\partial e} > 0$ and $\frac{\partial^2 C}{\partial e^2} > 0$, $\frac{\partial C}{\partial q} > 0$, $\frac{\partial^2 C}{\partial q^2} > 0$ and $\frac{\partial^2 C}{\partial q \partial e} < 0$.

ii) A higher technology parameter induces $\frac{\partial C}{\partial \theta} < 0$, $-\frac{\partial^2 C}{\partial e \partial \theta} < 0$ for $e \leq e^{max}(\theta, q)$ and $\frac{\partial^2 C}{\partial q \partial \theta} < 0$ for $e \leq e^{max}(\theta, q)$.

If we consider a Cobb-Douglas-type production function, it can be shown that the corresponding C fulfills these assumptions.

In the following, we will assume that there are three exogenously given technologies 0 ,a, and b, represented by their corresponding technology parameters θ_0, θ_a and θ_b with $\theta_0 < \theta_a < \theta_b$. To simplify notation, we will write $C_i(\cdot, \cdot\cdot)$ instead of $C(\theta_i, \cdot, \cdot\cdot)$ for $i = 0, a, b$.

Assumption 2.2. We assume that

$$\sum_{i=0,a,b} \xi_i \frac{\partial^2 C_i}{\partial e^2} \cdot \sum_{i=0,a,b} \xi_i \frac{\partial^2 C_i}{\partial q^2} - \left(\sum_{i=0,a,b} \xi_i \frac{\partial^2 C_i}{\partial q \partial e} \right)^2 > 0$$

for all $\xi_0, \xi_a, \xi_b > 0$, where $\sum_{i=0,a,b} \xi_i = 1$

In the appendix it is shown that the cost function $C(\theta, q, e)$ that can be derived from the Cobb-Douglas type production function $f(\theta, x_a, e) = (k(\theta)x_a)^\alpha e^\beta$, where $\alpha + \beta < 1$, $k'(\theta) > 0$, fulfills all assumptions above.

Initially all firms start with technology 0, referred to as the *conventional* technology. Advanced technology a is available yet and can in principle be adopted immediately. Buying and installing this technology causes a fixed cost $F_a > 0$. The even better technology b, will be available in the future with a certain probability. But its arrival time is Poisson-distributed with exogenous arrival parameter λ. Buying and installing that technology costs $F_b > 0$. Investment in one of these technologies is irreversible.

Further we denote total amount of input the industry uses by $E = \int_0^1 e(x)\,dx$. The production costs and damage respectively depends on aggregate input only and is evaluated by the function $V(E)$ which is increasing and convex in E, i.e. $V'(E) > 0$ and $V''(E) > 0$. Finally, we will assume that both the social planner and the firms discount the future at a constant discount rate r. Moreover we will refer to the "first stage" as the time interval where only technology 0 and a are available. In particular the date of first decision making $t = 0$ is called the first stage. By contrast, the "second stage" is referred to as the time interval when the advanced technology b is available.

We denote the input level of a firm using technology $i = 0, a, b$ at stage $j = 1, 2$ by $e_{i,j}$ and the output level of the same firm by $q_{i,j}$. To shorten the notation we will sometimes write $C_{i,j}$ instead of $C_i(q_{i,j}, e_{i,j})$ if there is no opportunity for mistakes.

Furthermore we stipulate the following manner of speaking:

- *Partial adoption of technology* a means that a share $0 \leq x_a \leq 1$ of firms adopts technology a and - in case of the social optimum - the social planner is indifferent between letting the marginal firm invest or letting it postpone the investment or - in case of a market equilibrium - each firm is indifferent between adopting technology a or wait for the arrival of technology b respectively

- *Partial adoption of technology* b means that a share $0 \leq x_{0b} \leq 1 - x_a$ of firms, which have not adopted technology a, adopts technology b and - in case of the social optimum - the social planner is indifferent between letting the marginal firm invest or not or - in case of a market equilibrium - each firm which have not adopted technology a at the first stage is indifferent between adopting technology b or not adopting it respectively

- *Partial replacement of technology* a means that a share $0 \leq x_{ab} \leq x_a$ of firms, which have adopted technology a, replaces this technology by adopting also technology b and - in case of the social optimum - the social planner is indifferent between letting the marginal firm replace technology a or not or - in case of a market equilibrium - each firm, which have adopted technology a in the stage before is indifferent between replacing technology a or not replacing it respectively

- *Full/No adoption of technology* a means that all/none of the firms adopt technology a and the social planner is not indifferent between letting the marginal firm invest or letting it postpone the investment or each firm is not indifferent between adopting technology a or wait for the arrival of technology b respectively

- *Full/No adoption of technology* b adoption of technology b means that all/none of the of firms, which have not adopted technology a, do/do not adopt technology b and the social planner is not indifferent between letting the marginal firm invest or not or each firm which have not adopted technology a at the first stage is not indifferent between adopting technology b or not adopting it respectively

- *Full/No replacement of technology* a means that all7none of the firms, which have adopted technology a, replaces this technology by adopting also technology b and the social planner is not indifferent between letting the marginal firm replace technology a or not or each firm, which have adopted technology a in the stage before is not indifferent between replacing technology a or not replacing it respectively

3 Expected Net Present Value of an Investment Decision

In this section we provide a formula for the net present value of total cost incurred by an economic agent who can invest twice, once immediately and a second time at a later, uncertain date when a further technology is available. This formula is very general and does not only refer to the model of this paper.

Lemma 3.1. *Let F_a and F_b denote the fixed cost incurred when investing into technology a or b, respectively. Further let $C_{0,}$ C_a and C_b denote the current values of the cost flow resulting from not investing, investing into technology a, and investing into technology b, respectively. While technology a is available immediately, the arrival date of technology b is Poisson distributed with mean arrival time λ. If the agent invests immediately into technology a and substitutes technology a by technology b as soon as it is available, the present value of total cost is given by:*

$$\begin{aligned} &F_a + \int_0^\infty \left(\int_0^t C_a \cdot e^{-rs}\, ds + \int_t^\infty C_b \cdot e^{-rs}\, ds + F_b \cdot e^{-rt} \right) \lambda e^{-\lambda t}\, dt \\ =&F_a + \frac{1}{r+\lambda} C_a + \frac{\lambda}{r+\lambda} \left(\frac{C_b}{r} + F_b \right) \end{aligned} \tag{1}$$

If the agent does not invest in technology a, but adopts technology b, as soon as that is available, the net present value of total costs is given by

$$\frac{1}{r+\lambda} C_0 + \frac{\lambda}{r+\lambda} (\frac{C_b}{r} + F_b). \tag{2}$$

4 The Social Optimum

Before considering regulation and the regulated firms' behavior it is useful to study the socially optimal investment pattern. The social planner's problem is to maximize the expected social value by balancing the consumers surplus against the industry's total production costs due to the output good and the production cost for the input. To do so he decides on all, each firm's input and output level at each point of time, and the shares of firms which should either adopt technology a, technology b, or none of both. Note that social value will be constant over time in the two stages before and after technology b is available. Note further that the input and output level of a firm using technology $i = 0, a$ in the first stage may differ from its levels in the second stage even if the firm does not change technology.

Thus, using Lemma 3.1 the social planner maximizes

$$\begin{aligned} \max_{\{q_{i,j}, e_{i,j}, x_a, x_{0b}, x_{ab}\}} \{ & \frac{1}{\lambda + r} [\int_0^{Q_1} P(\tilde{Q}) d\tilde{Q} - (1 - x_a) C_0(q_{0,1}, e_{0,1}) - x_a C_a(q_{a,1}, e_{a,1}) - V(E_1)] \\ & - x_a F_a + \frac{\lambda}{\lambda + r} [\frac{1}{r} [\int_0^{Q_2} P(\tilde{Q}) d\tilde{Q} - (1 - x_a - x_{0b}) C_0(q_{0,2}, e_{0,2}) - (x_a - x_{ab}) C_a(q_{a,2}, e_{a,2}) \\ & - (x_{0b} + x_{ab}) C_b(q_{b,2}, e_{b,2}) - V(E_2)] - (x_{0b} + x_{ab}) F_b] \}, \end{aligned} \tag{3}$$

subject to $1 \leq x_a + x_0 b \ x_{ab} \leq x_a$, $Q_1 = (1 - x_a) q_{0,1} + x_a q_{a,1}$, $E_1 = (1 - x_a) e_{0,1} + x_a e_{a,1}$, $Q_2 = (1 - x_a - x_{0b}) q_{0,2} + (x_a - x_{ab}) q_{a,2} + (x_{0b} + x_{ab}) q_{b,2}$ and $E_2 = (1 - x_a - x_{0b}) e_{0,2} + (x_a - x_{ab}) e_{a,2} + (x_{0b} + x_{ab}) e_{b,2}$.

In the following we will show that depending on F_a and F_b almost every possible adoption scenario can indeed be socially optimal i.e. no adoption of one or both of the technologies or partial adoption of one or both technologies. The only scenarios, which cannot be optimal, are the scenarios where both partial adoption of technology b and partial replacement of technology b occurs.

We will characterize the optimal pattern of technology adoption contingent on the size of F_a and F_b.

For this purpose, we start backwards. [1] For a given number of firms x_a which have adopted

[1]Of course we maximize the expected social value including both stage simultaneously but one will see that this approach delivers much more insights into the structure of the model.

the new technology in the first stage, we determine both the optimal number of firms x_{0b} which should adopt the latest technology and the optimal input levels $q_{0,2}, q_{a,2}$,and $q_{b,2}$ for each technology. Thus in the second stage the social planner's problem can be written as:

$$\max_{\{x_0,x_{0b},x_{ab},q_{i,2},e_{i,2}\}} \{\frac{1}{r}[\int_0^{Q_2} P(\tilde{Q})d\tilde{Q} - x_0 C_0(q_{0,2},e_{0,2}) - (x_a - x_{ab})C_a(q_{a,2},e_{a,2}) \\ - (x_{0b}+x_{ab})C_b(q_{b,2},e_{b,2}) - V(E_2)] + (x_{0b}+x_{ab})F_b\}, \tag{4}$$

subject to $1 = x_0 + x_a + x_{0b}$, $Q_2 = x_0 q_{0,2} + (x_a - x_{0b})q_{a,2} + (x_{0b}+x_{ab})q_{b,2}$ and $E_2 = x_0 e_{0,2} + (x_a - x_{0b})e_{a,2} + (x_{0b}+x_{ab})e_{b,2}$.

First of all we characterize how the aggregated input and output levels depend on the shares of firm that use one of the new technologies:

Lemma 4.1. *Suppose that x_a, x_{0b} are given. Let Q_2^* and E_2^* be the corresponding socially optimal aggregated input and output levels. Then we obtain:*

1. *If $0 \leq x_{0b} < 1$ and $0 \leq x_{0b} < 1 - x_a$ we get $\frac{\partial E_2^*}{\partial x_{0b}} < 0$ and $\frac{\partial Q_2^*}{\partial x_{0b}} > 0$ as well as $\frac{\partial E_2^*}{\partial x_a} < 0$ and $\frac{\partial Q_2^*}{\partial x_a} > 0$. Furthermore $\frac{\partial Q_2^*}{\partial x_{0b}} > \frac{\partial Q_2^*}{\partial x_a}$ and $\frac{\partial E_2^*}{\partial x_{0b}} < \frac{\partial E_2^*}{\partial x_a}$.*

2. *If $0 < x_a < 1$ and $x_{0b} = 1 - x_a$ we obtain $\frac{\partial E_2^*}{\partial x_a} > 0$ and $\frac{\partial Q_2^*}{\partial x_a} < 0$.*

The following result characterizes the optimal rate of adoption of technology b given that a share of x_a has already adopted technology a.

Proposition 4.2 (Adoption pattern in the second stage). Let the share of firms x_a which have adopted technology a be given. Then there exist two interval of fixed costs $[\underline{F}_b^*(x_a), \overline{F}_b^*(x_a)]$ and $[\underline{\underline{F}}_b^*, \overline{\overline{F}}_b^*(x_a)]$ of technology b such that

1. $\overline{\overline{F}}_b^* < \underline{F}_b^*$.

2. No firm should adopt technology b for $F_b \geq \overline{F}_b^*(x_a)$. No adoption of technology b is the case for $F_b > \overline{F}_b^*(x_a)$.

3. For $F_b \in [\underline{F}_b^*(x_a), \overline{F}_b^*(x_a)]$ a partial share $0 < x_{0b}^* < 1 - x_a$ of the $(1 - x_a)$ firms which have not adopted technology a should adopt technology b and no firm should

replace technology a. This share of firms as well as the optimal aggregated output Q_2^* is decreasing in F_b while the optimal aggregated input level E_2^* increases in F_b.

4. All $(1 - x_a)$ firms using the initial technology should adopt technology b for $\overline{\overline{F}}_b^*(x_a) \geq F_b \leq \underline{F}_b^*(x_a)$. Full adoption without replacement of technology b is the case $\overline{\overline{F}}_b^*(x_a) > F_b > \underline{F}_b^*(x_a)$.

5. For $F_b \in (\underline{\underline{F}}_b^*, \overline{\overline{F}}_b^*(x_a))$ there is full adoption of technology b and a partial share $0 < x_{ab}^* < x_a$ of firms which have adopted technology a should replace it by technology b. This share of firms as well as the optimal aggregated output Q_2^* is decreasing in F_b while the optimal aggregated input level E_2^* increases in F_b.

6. All firms, irrespectively if they adopted technology a or not, should adopt technology b for $F_b \leq \underline{\underline{F}}_b^*$. For $F_b < \underline{\underline{F}}_b^*$ full replacement is the case.

It is very intuitive that it depends on the level of F_b whether the social planner prefers that none of the firms, some of the conventional firm, all of the conventional firms or also some of the firms using technology a should adopt technology b. Also it is intuitive that for very small values of F_b it is socially optimal that all firms adopt technology b irrespectively whether they use technology 0 or technology a. Also it is intuitive that the social planner will not choose an allocation where firms using technology a adopt technology b and other firms still use the conventional technology 0 since a firm with conventional technology adopting technology b always adds more surplus to the welfare then a firm using technology a.

To derive the first-stage-result, we first study how a change of x_a affects the second-stage-result.

Proposition 4.3 (Comparative static with respect to x_a). Given the assumptions and results of proposition 4.2 we obtain

1. As x_a increases the lower bound $\underline{F}_b^*(x_a)$ increases while the upper bound $\overline{F}_b^*(x_a)$ decreases. Both converge to a cost level $\check{F}_b^*$ as x_a goes to 1. Furthermore $\overline{\overline{F}}_b^*(x_a)$ increases in x_a while $\underline{\underline{F}}_b^*$ is independent from x_a. $\overline{\overline{F}}_b^*(x_a)$ tends to $\overline{\overline{F}}_b^*$ as x_a goes to 0.

2. Consider F_b such that $F_b \in (\underline{F}_b(x_a), \overline{F}_b(x_a))$. Then the optimal number of firms $x^*_{0b}(x_a)$ adopting technology b is decreasing in x_a if $F_b \in (\underline{F}_b(x_a), \overline{F}_b(x_a))$. The effect on both, the optimal aggregated output Q^*_2 and input E^*_2 is ambiguous in that case, but both effects will have the same direction. i.e. E^*_2 increases (decreases) if and only if E^*_2 increases (decreases).

3. Consider F_b such that $F_b \in (\underline{\underline{F}}^*_b, \overline{\overline{F}}^*_b(x_a))$. Then the optimal number of firms $x^*_{ab}(x_a)$ replacing technology a by technology b increases proportional to x_a, i.e. $\frac{x^*_{ab}}{x_a} = 1$. Both the optimal aggregated output Q^*_2 and the optimal aggregated input E^*_2 do not change in that case.

This result is also intuitive. Given F_b, the higher x_a the less is the incentive for the social planner to let firms adopt technology b.

Now we derive the result for the first stage subject to the socially optimal decision at the second stage:

Proposition 4.4 (Adoption pattern in the first stage). Assume the installment cost F_b of technology b and thus the socially optimal market outcome corresponding to any x_a as being given. Then there exist an interval of fixed costs $[\underline{F}^*_a(F_b), \overline{F}^*_a(F_b)]$ such that

1. No firm should adopt technology a for $F_a \geq \overline{F}^*_a(F_b)$.

2. For $F_a \in [\underline{F}^*_a(F_b), \overline{F}^*_a(F_b)]$ a partial share $0 < x^*_a < 1$ of the firms should adopt technology a. This share of firms as well as the optimal aggregated output Q^*_1 is decreasing in F_a while the optimal aggregated input level E^*_1 increases in F_a.

3. All firms should adopt technology a for $F_a \leq \underline{F}_a(F_b)$.

The intuition behind this result is similar to the intuition behind the corresponding result for the second stage.

To visualize the result, we first study, of the results are affected by the a change of λ and how the interval bounds depend on the installment costs:

Proposition 4.5 (Comperative static). Consider F_b as being given

1. For all F_a where $0 < x_a^* < 1$ is the case $\frac{\partial x_a^*}{\partial \lambda} < 0$ follows. In case of partial adoption of technology b $\frac{\partial x_{0b}^*}{\partial \lambda} > 0$, while in case of partial replacement $\frac{\partial x_{ab}^*}{\partial \lambda} = -\frac{\partial x_a^*}{\partial \lambda} < 0$.

2. In case of partial adoption of both technologies we have $\frac{\partial x_a^* + x_{0b}^*}{\partial \lambda} > 0$. The effect of an increase of λ on both the optimal second stage output and second stage input level is ambiguous. But both have the same sign i.e. if the output increase also the input increases. In case of partial adoption of technology a, full adoption of technology b and no replacement the optimal second stage output increases while the input decreases if λ increases. Finally if replacement of technology a is socially optimal an increase of λ has no effect on the second stage output and input levels.

3. For $F_b < \overline{F}_b^*(0)$ it is $\frac{\partial \overline{F_a}}{\partial F_b} > 0$. Otherwise $\overline{F_a}$ is independent from F_b.

4. For $\overline{\overline{F}}_b^*(1) < F_b < \check{F}_b^*$ it is $\frac{\partial \underline{F}_a^*}{\partial F_b} > 0$. Otherwise $\underline{F}_a^*$ is independent from F_b.

The results of this section can best be illustrated by Figure 1. Line $\overline{AA\prime}$ is the locus of all pairs (F_a, F_b) such that $x_a = 1$, i.e. all firms should adopt technology a but the social planner is indifferent about the last firm to adopt or to wait for the arrival of technology b. The part $\overline{IC}$ of $\overline{AA\prime}$ is increasing since a higher F_a requires a higher F_b to keep x_{0b} equal to zero. In the area bounded by $\overline{ICHJI}$, x_a is strictly smaller than 1, but all the remaining firms $1 - x_a$ adopt technology b, as soon that is available. By contrast in the area bounded by $\overline{A'CGB'}$, we have also $n_a < 1$ but the remaining firms do not adopt any of the new technologies (becauseF_b is too high). Therefore the curve $\overline{CA'}$ is vertical. The curve $\overline{AI}$ is vertical since all firms adopting technology a will replace it by technology b.

Similarly, the line $\overline{BB\prime}$ represents the locus of all pairs (F_a, F_b) where no firm should adopt technology a, but the social planner is just indifferent about having the marginal firm to adopt technology a or not. Below the dotted line to the right of $\underline{F}_b$, the adoption cost of technology b F_b is so low that left of the branch $\overline{BH}$ all the remaining firms $1 - x_a$ should adopt technology b, while to the right of $\overline{BH}$ all firms should wait for technology b. Above the dotted line to the right of $\overline{F}_b$, no firm should ever wait for technology b, no matter how large F_a, because F_b is too large. In that area to the left of $\overline{GB'}$ some firms should adopt technology a, while to the right of $\overline{GB'}$ none of the two technologies should ever been adopted, because costs of both are too high. Along the branch $\overline{GH}$ always some firms should wait for technology b. On that branch an to the right of it n_a is zero, while on the left n_a is positive. Again $\overline{GH}$ is increasing since a higher F_b has to be compensated by a higher F_a to leave it in-attractive for the social planner to let some firms adopt technology a. The branch $\overline{CH}$ is the boundary where some firms adopt technology a and the remaining firms wait for technology b. Note that a higher F_a makes technology a less attractive. Instead of adopting technology b there are two alternatives: wait for technology b or not invest at all. In order to wait for technology two a higher F_a requires a lower F_b. The opposite holds for $\overline{CG}$. Along that branch some but not all firms adopt technology a, while no firm is waiting for technology b on and above $\overline{CG}$. Here a higher F_a requires a higher F_b to keep it in-attractive for the social planner to ever employ technology b.

The branch $\overline{IJ}$ is the boundary where some firms adopt technology a and none of these firms will rpelace it by technology b on and above $\overline{IJ}$. Here an higher F_a requires an lower F_b to keep it in-attractive for the social planner to replace technology a since more firms ($x_{0b} = 1 - x_a$!) already should adopt technology b. Finally the branch $\overline{KJ}$ is the boundary where some firms adopt technology a and all of these firms replace it by technology b on and below $\overline{KJ}$. Since all firms adopt technology b the curve must be horizontal.

5 The Market Equilibrium

We now assume that the input good is provided by some firms. Thus for each demand E of the input good the producers faced a market price $w(E)$ charged by the supplier. We assume asymmetric information in the following sense that these firms do not anticipate the new technologies. They only observe the demand.

To simplify the analysis we will neglect the input-market subgame first. Since the equilibrium on the Input-Market depends on the number of firms using technology a and b respectively, for each stage we consider an equilibrium input price function, in the following denoted by w_i, $i = 1, 2$, depending on x_a at the first and x_a, x_{0b}, x_{ab} at the second stage. We assume that more new technology always lowers the price. i.e. $\frac{\partial w_1}{\partial x_a} < 0$, $\frac{\partial w_2}{\partial x_{0b}} < 0$, $\frac{\partial w_2}{\partial x_{ab}} < 0$ and $\frac{\partial w_2}{\partial x_a} < 0$ as long $x_{0b} < 1 - x_a$. If $x_{0b} = 1 - x_a$ we assume $\frac{\partial w_2}{\partial x_a} > 0$. Later on we will show that the equilibrium input-price, which we derive from a concrete market structure, fulfills these assumptions in many situations.

5.1 The Second Stage

Consider x_a as being given. First of all note that a market equilibrium where $0 \leq x_{0b} < 1 - x_a$ and $x_{ab} > 0$ is the case cannot occur since given an output price P and input price w the cost savings for a firm using technology 0 and adopting technology b - given an output price P - equals

$$P \cdot (q_{b,2} - q_{0,2}) + C_0(q_{0,2}, e_{0,2}) - C_b(q_{b,2}, e_{b,2}) + w_2(e_{0,2} - e_{b,2}) - F_b$$

while the cost saving of a firm which replaces technology a by technology b - given an output price P - is equal to

$$P \cdot (q_{b,2} - q_{a,2}) + C_a(q_{a,2}, e_{a,2}) - C_b(q_{b,2}, e_{b,2}) + w_2(e_{a,2} - e_{b,2}) - F_b.$$

Obviously the first term is always greater. Thus a replacement of technology a can only by a market outcome if all firms with conventional technology adopt technology b since these firms will adopt technology b for larger levels of the installment cost. Thus we can distinguish both types of equilibria.

Then for a given input price w_2 a market equilibrium where partial adoption occurs, i.e. $0 < x_{0,b} < 1 - x_a$ is characterized by the following set of equations:

1.
$$P(Q_2) = \frac{\partial C_i^2}{\partial q}, \ i = 0, a, b. \ \text{(Firms' output rule)} \tag{5}$$

2.
$$w_2 = \frac{\partial C_i^2}{\partial e}, \; i = 0, a, b. \quad \text{(Firms' input rule)} \tag{6}$$

3.
$$Q_2 = (1 - x_a - x_{0b})q_{0,2} + x_a q_{a,2} + x_{0b} q_{b,2} \tag{7}$$

4.
$$E_2 = (1 - x_a - x_{0b})e_{0,2} + x_a e_{a,2} + x_{0b} e_{b,2}. \tag{8}$$

5.
$$\frac{1}{r}[P(Q_2)(q_{b,2} - q_{0,2}) - C_b^2 - C_0^2 - w_2(e_{0,2} - e_{b,2})] = F_b \quad \text{(Allocation rule for } x_{0b}) \tag{9}$$

A market equilibrium where partial replacement occurs is characterized by the same set of equation where only equation (9) will be replaced by

$$\frac{1}{r}[P(Q_2)(q_{b,2} - q_{a,2}) - C_b^2 - C_a^2 - w_2(e_{a,2} - e_{b,2})] = F_b \quad \text{(Allocation rule for } x_{ab}) \tag{10}$$

First of all we derive the ceteris paribus impact of a change of the technology allocation and the market price on the input and the output level:

Lemma 5.1. *1.* $\frac{\partial Q_2}{\partial w_2} < 0$ *and* $\frac{\partial E_2}{\partial w_2} < 0$ *for given* x_a *and* x_{0b}

2. $\frac{\partial Q_2}{\partial x_{0b}} > 0$ *and* $\frac{\partial E_2}{\partial x_{0b}} < 0$ *for given* x_a *and* w_2

3. $\frac{\partial Q_2}{\partial x_a} > 0$ *and* $\frac{\partial E_2}{\partial x_a} < 0$ *if* $x_{0b} < 1 - x_a$ *for given* x_{0b} *and* w_2.

4. $\frac{\partial Q_2}{\partial x_a} < 0$ *and* $\frac{\partial E_2}{\partial x_a} > 0$ *if* $x_{0b} = 1 - x_a$ *for given* x_{0b} *and* w_2.

5. For given x_a *and* x_{0b} *the term* $P(Q_2)(q_{b,2} - q_{i,2}) - C_b^2 - C_i^2 - w_2(e_{i,2} - e_{b,2})$ *increases in* w_2, $i = 0, a$.

We can derive an analogous result to the social optimum in the second stage:

Proposition 5.2. Given a share $0 \leq x_a < 1$ of firms which have adopted technology a in the first stage, we find two intervals $[\underline{F}_b(x_a), \overline{F}_b(x_a)]$ and $[\underline{\underline{F}}_b, \overline{\overline{F}}_b(x_a)]$ such that the market equilibrium contains the following technology allocation:

1. $\overline{\overline{F}}_b(x_a) < \underline{F}_b(x_a)$.

2. None of the remaining $(1 - x_a)$ firms using the conventional technology adopts technology b for $F_b \geq \overline{F}_b(x_a)$.

3. A partial share $0 < x_{0b} < 1 - x_a$ of the remaining $(1 - x_a)$ firms using the conventional technology adopts technology b for $F_b \in (\underline{F}_b(x_a), \overline{F}_b(x_a))$. This share is decreasing in F_b.

4. All $1 - x_a$ firms using the conventional technology adopts technology b for $F_b \leq \underline{F}_b(x_a))$.

5. None of the x_a firms using technology a replaces it by technology b for $F_b \geq \overline{\overline{F}}_b(x_a)$.

6. A partial share of the x_a firms using technology a replace it by technology a for $F_b \in (\underline{\underline{F}}_b, \overline{\overline{F}}_b(x_a))$. This share is decreasing in F_b.

7. All x_a firms using technology a replace it by technology b for $F_b \leq \underline{\underline{F}}_b$.

8. $\overline{F}_b(x_a)$ is decreasing in x_a while $\underline{F}_b(x_a)$ is increasing in x_a. Both tend to the same value $\check{F}_b$ as x_a goes to 1.

9. $\overline{\overline{F}}_b(x_a)$ is increasing in x_a while $\underline{\underline{F}}_2$ is independent from x_a. $\overline{\overline{F}}_b(x_a)$ tends to $\underline{\underline{F}}_b$ as x_a goes to 0.

Given x_a and F_b the corresponding equilibrium is unique.

5.2 The First Stage

Consider F_b as being given. Thus for each x_a we can derive by the result of the last section the market equilibrium after the arrival of technology b.

Thus for a given input price w_1 a market equilibrium where partial adoption of technology a occurs, i.e. $0 < x_a < 1$ is characterized by the following equations:

1.
$$P(Q_1) = \frac{\partial C_i^1}{\partial q},\ i = 0, a. \tag{11}$$

2.
$$e_a = \frac{\partial C_i^1}{\partial e},\ i = 0, a. \tag{12}$$

3.
$$Q_1 = (1 - x_a)q_{0,1} + x_a q_{a,1} \tag{13}$$

4.
$$E_1 = (1 - x_a)e_{0,1} + x_a e_{a,1}. \tag{14}$$

5. If the market outcome in the second stage corresponding to x_a is

   (a) No adoption, no replacement:
$$\frac{1}{r}[P(Q_1)(q_{a,1} - q_{0,1}) - C_a^1 - C_0^1 - w_1(e_{0,1} - e_{a,1})] = F_a \tag{15}$$

   (b) Partial adoption, no replacement:
$$\begin{aligned}&\frac{1}{\lambda + r}[P(Q_1)(q_{a,1} - q_{0,1}) - C_a^1 - C_0^1 - w_1(e_{0,1} - e_{a,1})]\\ &+ \frac{\lambda}{r(\lambda + r)}[P(Q_2)(q_{a,2} - q_{0,2}) - C_a^2 - C_0^2 - w_2(e_{0,2} - e_{a,2})] = F_a\end{aligned} \tag{16}$$

   (c) Full adoption, no or partial replacement:
$$\begin{aligned}&\frac{1}{\lambda + r}[P(Q_1)(q_{a,1} - q_{0,1}) - C_a^1 - C_0^1 - w_1(e_{0,1} - e_{a,1})]\\ &+ \frac{\lambda}{\lambda + r}[\frac{1}{r}(P(Q_2)(q_{a,2} - q_{b,2}) - C_a^2 - C_b^2 - w_2(e_{b,2} - e_{a,2})) + F_b] = F_a\end{aligned} \tag{17}$$

   (d) Full replacement:
$$\frac{1}{\lambda + r}[P(Q_1)(q_{b,1} - q_{a,1}) - C_a^1 - C_0^1 - w_1(e_{0,1} - e_{a,1})] = F_a \tag{18}$$

Proposition 5.3. Given the installment cost F_b of technology b and the results of proposition 5.2 we find an interval $[\underline{F}_a(F_b), \overline{F}_a(F_b)]$ and such that the market equilibrium contains the following technology allocation:

1. None of the firms adopt technology a for $F_a \geq \overline{F}_a(F_b)$.

2. A partial share $0 < x_a < 1$ of the firms adopt technology a for $F_a \in (\underline{F}_a(F_b), \overline{F}_a(F_b))$. This share is decreasing in F_a.

3. All firms adopt technology a for $F_a \leq \underline{F}_b(F_a)$.

6 Competitive input market

Now let us consider that the input market is competitive i.e. for each E we have $w(E) = V'(E)$. It is easy to show that given x_a, x_{b0} and x_{ab} the input-market equilibrium prices $w_1(x_a)$ and $w_2(x_a, x_{0b}, x_{ab})$, which are given by the equation systems (11) - (14) and (5) - (8) respectively, fulfill our assumptions.

Since the input price equals the marginal costs of providing the corresponding quantity, it is intuitive that the technology allocation is the efficient one. Indeed this is the case:

Proposition 6.1. Let the installment cost F_a and F_b of technology a and technology b be given respectively. Moreover assume that $w_i = V'(E_i^*)$ for both stages $i = 1, 2$ and all values of x_a, x_{0b}, x_{ab}. Then market equilibrium $(x_a, x_{0b}, x_{ab}, Q_1, E_1, Q_2, E_2)$ corresponding to F_a and F_b is identical to the socially optimal solution $(x_a^*, x_{0b}^*, x_{ab}^*, Q_1^*, E_1^*, Q_2^*, E_2^*)$ which corresponds to this cost pair.

7 Oil and gold - Market power

On many input markets, like the market for oil, gas or different kinds of ores like gold the supplier form an monopoly or an oligopoly. Thus the input price exceeds the marginal costs of providing the corresponding demand. Thus we will assume that for each given input level E the corresponding input price $w(E)$ exceeds $V'(E)$. This may be the case if we assume an monopolistic supplier of the input. While further assumptions are necessary to show that the corresponding equilibrium input price fulfills our assumptions, for certain functional forms this is rather easy to show. For example if $C(\theta, q, e) = \frac{\alpha\theta}{2}(\beta q - e)^2 + \frac{\gamma}{2}q^2$, where $\alpha(\theta) > 0, \beta > 0, \gamma(\theta) > 0, \alpha' < 0, \gamma' < 0, \alpha'' > 0, \gamma'' > 0$, and if $P(Q)$ is affine linear, this is the case.

We can derive the following result:

Proposition 7.1 (Market Power Induces Over-Investment). Assume that $w(E) > V'(E)$ for all E. Then:

1. Let $1 \leq x_a \leq 0$ and $0 \leq x_{0b} \leq 1 - x_a$. Consider (F_a^*, F_b^*) and (F_a, F_b) be the unique cost pair where partial adoption of both technologies is the social optimal outcome and the market equilibrium respectively. Then $F_a > F_a^*$ and $F_b > F_b^*$.

2. Let $1 \leq x_a \leq 0$ and $0 \leq x_{ab} \leq x_a$. Consider (F_a^*, F_b^*) and (F_a, F_b) the unique cost pair where partial adoption and partial replacement of technology a is the socially optimal outcome and the market equilibrium respectively. Then again $F_a > F_a^*$ and $F_b > F_b^*$.

3. Given a share $0 \leq x_a \leq 1$ for all F_b where F_a^* and F_a exist such that for (F_a^*, F_b) and (F_a, F_b) partial adoption of technology a and no adoption of technology b is the socially optimal outcome and market outcome respectively. Then $F_a^* < F_a$.

4. Given a share $0 \leq x_{0b} \leq 1$ for all F_a where F_b^* and F_b exist such that for (F_a, F_b^*) and (F_a, F_b) partial adoption of technology b and no adoption of technology a is the socially optimal outcome and market outcome respectively. Then $F_b^* < F_b$.

5. Given a share $0 \leq x_{ab} \leq 1$ for all F_a where F_b^* and F_b exist exist such that for (F_a, F_b^*) and (F_a, F_b) partial replacement of technology a and full adoption of technology a is the socially optimal outcome and market outcome respectively. Then $F_b^* < F_b$.

6. Given a share $0 \leq x_a \leq 1$ for all F_b where F_a^* and F_a exist such that for (F_a^*, F_b) and (F_a, F_b) partial adoption of technology a and full adoption of technology b but no replacement of technology a is the socially optimal outcome and market outcome respectively. Then in general it is ambiguous whether $F_a > F_a^*$, $F_a = F_a^*$ or $F_a < F_a^*$.

The intuition behind the result is straight-forward. Since the input price is always higher than the marginal costs, there is a greater incentive for all firms to adopt new technology compared to the competitive situation. If partial adoption or partial replacement is the case, this is reflected by the fact that a higher number of firms adopt both technologies. But if the fixed costs are such that in equilibrium the marginal firm has to decide between adopting technology a and technology b then it depends on which technology is relatively more efficient. For example if technology b is much better than technology a but the the expected arrival time is also very high, it may better for the firms to choose technology a instead of technology b if the input price increases. In contrast to this more firms will decide to adopt technology b instead of a if the expected arrival time is rather small. Thus in this special situation the effect of higher input prices is ambiguous.

8 Fixed Supply

Now assume that for some reason the input level is E is fixed over time. To give an example for this consider emissions. In that case E may be the amount of permits issued by a regulator who commits in the beginning that, for example for some political reasons, this number will not be changed over time. (see e.g. Requate and Unold [2003]). $V(E)$ may be interpreted as the damage corresponding to the emission level E. Furthermore assume that the number of permit $E = E^0$ is equal to the socially optimal emission level if all firms use the conventional technology. It is easy to verify that the corresponding equilibrium permit price - given the number of firms adopting technology a and b respectively - fulfills our assumptions.

The following result generalize the result of Requate and Unold [2003] and Requate and von Döllen [2007] on an input/output market model:

Proposition 8.1 (Fixed Supply Induces Under-Investment).

1. Let $1 \leq x_a \leq 0$ and $0 \leq x_{0b} \leq 1 - x_a$. Consider (F_a^*, F_b^*) and (F_a, F_b) be the unique cost pair where partial adoption of both technologies is the social optimal outcome and the market equilibrium respectively. Then $F_a < F_a^*$ and $F_b < F_b^*$ if and only if either $x_a > 0$ or $x_{0b} > 0$.

2. Let $1 \leq x_a \leq 0$ and $0 \leq x_{ab} \leq x_a$. Consider (F_a^*, F_b^*) and (F_a, F_b) the unique cost pair where partial adoption and partial replacement of technology a is the socially optimal outcome and the market equilibrium respectively. Then $F_b < F_b^*$. Furthermore $F_a = F_a^*$ if $x_a = 0$ and $F_a < F_a^*$ otherwise.

3. Given a share $0 \leq x_a \leq 1$ for all F_b where F_a^* and F_a exist such that for (F_a^*, F_b) and (F_a, F_b) partial adoption of technology a and no adoption of technology b is the socially

optimal outcome and market outcome respectively $F_a^* > F_a$ if and only if $x_a > 0$.

4. Given a share $0 \leq x_{0b} \leq 1$ for all F_a where F_b^* and F_b exist such that for (F_a, F_b^*) and (F_a, F_b) partial adoption of technology b and no adoption of technology a is the socially optimal outcome and market outcome respectively $F_b^* > F_b$ if and only if $x_a > 0$.

5. Given a share $0 \leq x_{ab} \leq 1$ for all F_a where F_b^* and F_b exist exist such that for (F_a, F_b^*) and (F_a, F_b) partial replacement of technology a and full adoption of technology a is the socially optimal outcome and market outcome respectively $F_b^* > F_b$.

6. Let a share $0 \leq x_a \leq 1$ by given. Furthermore consider F_b such that F_a^* and F_a exist such that for (F_a^*, F_b) and (F_a, F_b) partial adoption of technology a and full adoption of technology b but no replacement of technology a is the socially optimal outcome and market outcome respectively. Then in general it is ambiguous whether $F_a > F_a^*$, $F_a = F_a^*$ or $F_a < F_a^*$. But if $x_a = 0$ $F_a > F_a^*$. Furthermore if $\frac{\partial \Delta_{ab}}{\partial w} > 0$ then $F_a < F_a^*$ and if $\frac{\partial \Delta_{ab}}{\partial w} > 0$ then $F_a > F_a^*$.

The result is also very intuitive. Since the quantity of permits is not changed over time the adoption of new technology induces that the input price falls below the virtual[2] marginal costs $V'(E^0)$ which would be equal to the input price in the 'competitive' situation, which could be an ex post anticipation policy (see Requate and Unold [2003], Requate and von Döllen [2007]) in our case. Thus in general the incentive to adopt new technology is lower. For the same reasons as in the latter section the effect is ambiguous if the firms decide between adopting technology a and technology b in equilibrium.

9 Conclusion

We study both the efficient technology allocation and the technology allocation in a decentralized market equilibrium. Depending on the installment cost pair (F_a, F_b) every adoption

[2] Virtual since it is not justified to speak about marginal costs if the supply is fixed.

pattern besides the pattern where both a partial number of firms which did not adopt technology a and a partial number of firms which have adopted technology a adopt technology b, can be efficient as well as a market equilibrium. Thus if the number of firms adopting new technology affects both the input and the output price a dynamic incentive to adopt new technology exists. This incentive explains how an industry structure where ex ante symmetric firms use different kind of technologies can arise. Since the incentives to adopt new technology decrease with the number of firms already using the new technology it also explains why at the same time firms leapfrog a technology while other firms adopt it. One the one hand scenarios exist where the installment costs force the firms to decide whether they adopt technology a or technology b, on the other hand scenarios exist where technology a is only adopted to lower the production cost until technology b becomes available. Studying different kinds of input market structures we learn that a competitive market induces the efficient adoption pattern while in general market power induces over-investment. Furthermore we extend the results about abatement technology adoption induced by a regulation by permits (Requate and Unold [2003], Requate and von Döllen [2007]) to an input/output market model.

A Proofs

Proof of Lemma 4.1: Given x_a and x_{0b} and setting $x_0 := 1 - x_a - x_{0b}$ the firms output and input levels q_i and e_i are given by the equation system 24 and 25. The aggregated levels are given by $E = \sum_{i=0,a,b} x_i e_i$ and $Q = \sum_{i=0,a,b} x_i q_i$. Therefore by differentiating these equations with respect to x_{0b} we get

$$\frac{\partial^2 C_i}{\partial e^2}\frac{\partial e_i}{\partial x_{0b}} + \frac{\partial^2 C_i}{\partial q \partial w}\frac{\partial q_i}{\partial x_{0b}} + V''(E)\frac{\partial E}{\partial x_{0b}} = 0,\ i = 0, a, b \tag{19}$$

$$P'(Q)\frac{\partial Q}{\partial x_{0b}} - \frac{\partial^2 C_i}{\partial q^2}\frac{\partial q_i}{\partial x_{0b}} - \frac{\partial^2 C_i}{\partial e \partial q}\frac{\partial e_i}{\partial x_{0b}} = 0,\ i = 0, 1, 2 \tag{20}$$

$$\frac{\partial Q}{\partial x_{0b}} = q_b - q_0 + \sum_{i=0}^{2} x_i \frac{\partial q_i}{\partial x_{0b}} \tag{21}$$

$$\frac{\partial Q}{\partial x_{0b}} = e_b - e_0 + \sum_{i=0}^{2} x_i \frac{\partial e_i}{\partial x_{0b}} \tag{22}$$

From the equation system 19 and 20 we get for each $i = 0, a, b$

$$\begin{pmatrix} P'(Q)\frac{\partial Q}{\partial x_{0b}} \\ -V''(E)\frac{\partial E}{\partial x_{0b}} \end{pmatrix} = \begin{pmatrix} \frac{\partial^2 C_i}{\partial q^2} & \frac{\partial^2 C_i}{\partial e \partial q} \\ \frac{\partial^2 C_i}{\partial q \partial e} & \frac{\partial^2 C_i}{\partial e^2} \end{pmatrix} \begin{pmatrix} \frac{\partial q_i}{\partial x_{0b}} \\ \frac{\partial e_i}{\partial x_{0b}} \end{pmatrix}$$

By inversion of the matrix we get

$$\begin{pmatrix} P'(Q)\frac{\partial Q}{\partial x_{0b}} \\ -V''(E)\frac{\partial E}{\partial x_{0b}} \end{pmatrix} \begin{pmatrix} \frac{\partial^2 C_i}{\partial e^2}/k_i & -\frac{\partial^2 C_i}{\partial e \partial q}/k_i \\ -\frac{\partial^2 C_i}{\partial q \partial e}/k_i & \frac{\partial^2 C_i}{\partial q^2}/k_i \end{pmatrix} = \begin{pmatrix} \frac{\partial q_i}{\partial x_{0b}} \\ \frac{\partial e_i}{\partial x_{0b}} \end{pmatrix}$$

where $k_i := \frac{\frac{\partial^2 C_i}{\partial e^2}}{\frac{\partial^2 C_i}{\partial e^2}\frac{\partial^2 C_i}{\partial q^2} - \left(\frac{\partial^2 C_i}{\partial q \partial e}\right)^2}$. Clearly $k_i > 0$. Thus we get

$$\frac{\partial q_i}{\partial x_{0b}} = \frac{1}{k_i}[P'(Q)\frac{\partial Q}{\partial x_{0b}}\frac{\partial^2 C_i}{\partial e^2} + V''(E)\frac{\partial E}{\partial x_{0b}}\frac{\partial^2 C_i}{\partial q \partial e}]$$

$$\frac{\partial e_i}{\partial x_{0b}} = \frac{-1}{k_i}[V''(E)\frac{\partial E}{\partial x_{0b}}\frac{\partial^2 C_i}{\partial q^2}P'(Q)\frac{\partial Q}{\partial x_{0b}}\frac{\partial^2 C_i}{\partial q \partial e}]$$

Substituting these equations into equations 21 and 22 delivers

$$\frac{\partial Q}{\partial x_{0b}} = \underbrace{\frac{q_b - q_0}{1 - P'(Q)\sum \frac{x_i}{k_i}\frac{\partial^2 C_i}{\partial e^2}}}_{:=K_1} + \frac{\partial E}{\partial x_{0b}} \underbrace{\left[V''(e)\frac{\sum \frac{x_i}{k_i}\frac{\partial^2 C_i}{\partial e \partial q}}{1 - P'(Q)\sum \frac{x_i}{k_i}\frac{\partial^2 C_i}{\partial e^2}}\right]}_{:=K_2}$$

and

$$\frac{\partial E}{\partial x_{0b}} = \underbrace{\frac{e_b - e_0}{1 + V''(E)\sum \frac{x_i}{k_i}\frac{\partial^2 C_i}{\partial q^2}}}_{:=K_3} + \frac{\partial Q}{\partial x_{0b}} \underbrace{\left[-P'(Q)\frac{\sum \frac{x_i}{k_i}\frac{\partial^2 C_i}{\partial e \partial q}}{1 + V''(E)\sum \frac{x_i}{k_i}\frac{\partial^2 C_i}{\partial q^2}}\right]}_{:=K_4}$$

Or shortly written $\frac{\partial Q}{\partial x_{0b}} = K_1 + \frac{\partial E}{\partial x_{0b}}K_2$ and $\frac{\partial E}{\partial x_{0b}} = K_3 + \frac{\partial Q}{\partial x_{0b}}K_4$. Thus

$$\frac{\partial Q}{\partial x_{0b}} = \frac{K_1 + K_2 K_3}{1 - K_2 K_4}$$

and

$$\frac{\partial E}{\partial x_{0b}} = \frac{K_3 + K_4 K_1}{1 - K_2 K_4}$$

Now $K_1 > 0$, $K_2 < 0$, $K_3 < 0$ and $K_4 < 0$. By assumption 2.2 $1 - K_2 K_4 > 0$ follows.

Analogously we obtain

$$\frac{\partial Q}{\partial x_a} = \frac{\tilde{K}_1 + K_2 \tilde{K}_3}{1 - K_2 K_4}$$

and

$$\frac{\partial E}{\partial x_a} = \frac{\tilde{K}_3 + K_4 \tilde{K}_1}{1 - K_2 K_4}$$

where $\tilde{K}_1 = \frac{q_a - q_0}{1 - P'(Q) \sum \frac{x_i}{k_i} \frac{\partial^2 C_i}{\partial e^2}} < K_1$ and $\tilde{K}_3 = \frac{e_a - e_0}{1 + V''(E) \sum \frac{x_i}{k_i} \frac{\partial^2 C_i}{\partial q^2}} > K_3$. Thus the result follows.

ad 2.) : Again we can approach analogously only noticing that equations (21) and (22) change to

$$\frac{\partial Q}{\partial x_a} = q_a - q_b + x_a \frac{\partial q_a}{\partial x_a} + (1 - x_a) \frac{\partial q_b}{\partial x_a}$$
$$\frac{\partial Q}{\partial x_a} = e_a - e_b + x_a \frac{\partial e_a}{\partial x_a} + (1 - x_a) \frac{\partial e_b}{\partial x_a}$$

This we lead us to

$$\frac{\partial Q}{\partial x_a} = \underbrace{\frac{q_a - q_b}{1 - P'(Q)(\frac{x_a}{k_a} \frac{\partial^2 C_a}{\partial e^2} + \frac{1 - x_a}{k_b} \frac{\partial^2 C_b}{\partial e^2})}}_{:= \hat{K}_1} + \frac{\partial E}{\partial x_{0b}} \underbrace{\left[V''(E) \frac{\frac{x_a}{k_a} \frac{\partial^2 C_i}{\partial e \partial q} + \frac{1 - x_a}{k_b} \frac{\partial^2 C_i}{\partial e \partial q}}{1 - P'(Q)(\frac{x_a}{k_a} \frac{\partial^2 C_a}{\partial e^2} + \frac{1 - x_a}{k_b} \frac{\partial^2 C_b}{\partial e^2})} \right]}_{:= \hat{K}_2}$$

and

$$\frac{\partial E}{\partial x_a} = \underbrace{\frac{e_a - e_b}{1 + V''(E)(\frac{x_a}{k_a} \frac{\partial^2 C_a}{\partial q^2} + \frac{1 - x_a}{k_b} \frac{\partial^2 C_b}{\partial q^2})}}_{:= \hat{K}_3} + \frac{\partial Q}{\partial x_{0b}} \underbrace{\left[-P'(Q) \frac{\frac{x_a}{k_a} \frac{\partial^2 C_i}{\partial e \partial q} + \frac{1 - x_a}{k_b} \frac{\partial^2 C_i}{\partial e \partial q}}{1 + V''(E)(\frac{x_a}{k_a} \frac{\partial^2 C_a}{\partial q^2} + \frac{1 - x_a}{k_b} \frac{\partial^2 C_b}{\partial q^2})} \right]}_{:= \hat{K}_4}$$

This implies that

$$\frac{\partial Q}{\partial x_a} = \frac{\hat{K}_1 + \hat{K}_2 \hat{K}_3}{1 - \hat{K}_2 \hat{K}_4} < 0$$

and

$$\frac{\partial E}{\partial x_a} = \frac{\hat{K}_3 + \hat{K}_4 \hat{K}_1}{1 - \hat{K}_2 \hat{K}_4} > 0$$

since $\hat{K}_1 < 0$, $\hat{K}_2 < 0$, $\hat{K}_3 > 0$ and $\hat{K}_4 < 0$. q.e.d.

Proof of Proposition 4.2 and comperative static (together) :

Ad 1.-3., adoption : Given x_a at stage 2 the social planner solves

$$\min_{\{x_0, x_{0b}, q_{i,2}, w_{i,2}, i=0,1,2\}} \left\{ \frac{1}{r} \left[\int_0^{Q_2} P(\tilde{Q}) d\tilde{Q} - x_0 C_0(q_{0,2}, e_{0,2}) - x_a C_a(q_{a,2}, e_{a,2}) - x_{0b} C_b(q_{b,2}, e_{b,2}) - V(E_2) \right] - x_{0b} F_b \right\}, \tag{23}$$

subject to the constraints $x_0 \geq 0$, $x_a \geq 0$ and $x_0 + x_a + x_{0b} = 1$ with corresponding Kuhn-Tucker multipliers μ_i for non-negative constraint for x_i for $i = 0, b$ and Lagrange multiplier ν w.r.t. $x_0 = 1 - x_a - x_{0b}$. For simplicity we write q_i and e_i instead of $q_{i,2}$ and $w_{i,2}$ for $i = 0, a, b$. The first order conditions w.r.t. q_i, e_i, x_0 and x_{0b} are given by

$$\frac{\partial C_i}{\partial w}(q_i, e_i) + V'(E) = 0, \ i = 0, a, b \tag{24}$$

$$P(Q) - \frac{\partial C_i}{\partial q}(q_i, e_i) = 0, \ i = 0, a, b \tag{25}$$

$$\frac{1}{r}(q_0 P(Q) - C_0(q_0, e_0) - e_0 V'(E)) - \mu_0 - \nu = 0 \tag{26}$$

and

$$\frac{1}{r}(q_b P(Q) - C_b(q_b, e_b) - e_b V'(E)) - F_b - \mu_b - \nu = 0 \tag{27}$$

Eliminating ν yields

$$\frac{1}{r}((q_b - q_0)P(Q) + C_0(q_0, e_0) - C_b(q_b, e_b) + (e_0 - e_b)V'(E)) - \mu_b + \mu_0 = F_b \tag{28}$$

Considering first the interior solutions (i.e. $\mu_0 = \mu_b = 0$), we differentiate the equation system 24, 25 and 28 with respect to F_b . Employing the Envelope Theorem, we obtain:

$$\frac{\partial^2 C_i}{\partial e^2}\frac{\partial e_i}{\partial x_{0b}}\frac{\partial x_{0b}}{\partial F_b} + \frac{\partial^2 C_i}{\partial q \partial e}\frac{\partial q_i}{\partial x_{0b}}\frac{\partial x_{0b}}{\partial F_b} + V''(E)\frac{\partial E}{\partial x_{0b}}\frac{\partial x_{0b}}{\partial F_b} = 0, \ i = 0, a, b \tag{29}$$

$$P'(Q)\frac{\partial Q}{\partial x_{0b}}\frac{\partial x_{0b}}{\partial F_b} - \frac{\partial^2 C_i}{\partial q^2}\frac{q_i}{x_{0b}}\frac{\partial x_{0b}}{\partial F_b} - \frac{\partial^2 C_i}{\partial e \partial q}\frac{e_i}{x_{0b}}\frac{\partial x_{0b}}{\partial F_2} = 0, \ i = 0, a, b \tag{30}$$

and

$$\frac{1}{r}P'(Q)\frac{\partial Q}{\partial x_{0b}}\frac{\partial x_{0b}}{\partial F_b}(q_b - q_0) + \frac{1}{r}V''(E)\frac{\partial E}{\partial x_{0b}}\frac{\partial x_{0b}}{\partial F_b}(e_0 - e_b) = 1 \tag{31}$$

Solving for $\frac{\partial x_{0b}}{\partial F_2}$ yields:

$$\frac{\partial x_{0b}}{\partial F_2} = \frac{r}{P'(Q)\frac{\partial Q}{\partial x_{0b}}(q_b - q_0) + V''(E)\frac{\partial E}{\partial x_{0b}}(e_0 - e_b)} < 0$$

This implies $\frac{\partial E}{\partial F_b} > 0$ and $\frac{\partial Q}{\partial F_b} > 0$. We also have proven that the LHS of equation 28 decreases in x_{0b}. So let $Q(1 - x_a)$ and $E(1 - x_a)$ be the output and input levels, which correspond with $x_{0b} = 1 - x_a$. Then $\underline{F}_b(x_a)$ is given by the LHS of equation (28). For smaller installment cost

F_b we have $\mu_b > 0$ and thus $x_{0b} = 1 - x_a$.

Analogously $\overline{F}_b(x_a)$ is given by the LHS of equation (28) where $Q = Q(0)$ and $E = E(0)$ are the output and input levels which correspond to $x_{0b} = 0$. If F_b gets larger μ_0 must follow.

4.) : Now differentiate equation (28) with respect to x_a. Then we get:

$$\frac{1}{r}P'(Q)[\frac{\partial Q}{\partial x_{0b}}\frac{\partial x_{0b}}{\partial x_a} + \frac{\partial Q}{\partial x_a}](q_b - q_0) + \frac{1}{r}V''(E)[\frac{\partial E}{\partial x_{0b}}\frac{\partial x_{0b}}{\partial x_a} + \frac{\partial E}{\partial x_a}](e_0 - e_b) = 0 \qquad (32)$$

In the following we will write $\frac{dQ}{dx_a} = \frac{\partial Q}{\partial x_{0b}}\frac{\partial x_{0b}}{\partial x_a} + \frac{\partial Q}{\partial x_a}$ and $\frac{dE}{dx_a} = \frac{\partial E}{\partial x_{0b}}\frac{\partial x_{0b}}{\partial x_a} + \frac{\partial E}{\partial x_a}$. Then from equation (32) we obtain

$$\frac{dQ}{dx_a} = \frac{(e_b - e_0)V''(E)}{(q_b - q_0)P'(Q)}\frac{dE}{dx_a} \qquad (33)$$

As we can see both effect will have the same sign. Equation (32) also delivers

$$\frac{\partial x_{0b}}{\partial x_a} = \frac{P'(Q)\frac{\partial Q}{\partial x_a}(q_b - q_0) + V''(E)\frac{\partial E}{\partial x_a}(e_0 - e_b)}{P'(Q)\frac{\partial Q}{\partial x_{0b}}(q_0 - q_b) + V''(E)\frac{\partial E}{\partial x_{0b}}(e_b - e_0)} < 0$$

since by Lemma 4.1 also $\frac{\partial Q}{\partial x_a} > 0$ and $\frac{\partial E}{\partial x_a} < 0$. Now we can directly evaluate $\frac{dQ}{dx_a}$. I.e.

$$\begin{aligned}\frac{dQ}{dx_a} &= \frac{\partial Q}{\partial x_{0b}}\frac{\partial x_{0b}}{\partial x_a} + \frac{\partial Q}{\partial x_a} \\ &= \frac{P'(Q)\frac{\partial Q}{\partial x_a}\frac{\partial Q}{\partial x_{0b}}(q_b - q_0) + V''(E)\frac{\partial E}{\partial x_a}\frac{\partial Q}{\partial x_{0b}}(e_0 - e_b) + P'(Q)\frac{\partial Q}{\partial x_{0b}}\frac{\partial Q}{\partial x_a}(q_0 - q_b) + V''(E)\frac{\partial E}{\partial x_{0b}}\frac{\partial Q}{\partial x_a}(e_b - e_0)}{P'(Q)\frac{\partial Q}{\partial x_{0b}}(q_0 - q_b) + V''(E)\frac{\partial E}{\partial x_{0b}}(e_b - e_0)} \\ &= \frac{V''(E)\frac{\partial E}{\partial x_a}\frac{\partial Q}{\partial x_{0b}}(e_0 - e_b) + V''(E)\frac{\partial E}{\partial x_{0b}}\frac{\partial Q}{\partial x_a}(e_b - e_0)}{P'(Q)\frac{\partial Q}{\partial x_{0b}}(q_0 - q_b) + V''(E)\frac{\partial E}{\partial x_{0b}}(e_b - e_0)} \gtreqless 0\end{aligned}$$

$\underline{F}_2^*(x_a)$*:* Let $\underline{Q}$, $\underline{q}_i$, $\underline{e}_i$, $i = 0, a, b$ and $\underline{E}$ be the socially optimal levels corresponding to $x_{0b} = 1 - x_a$. Then $\underline{F}_b(x_a)$ is given by

$$\frac{1}{r}((\underline{q}_2 - \underline{q}_0)P(\underline{Q}) + C_0(\underline{q}_0, \underline{e}_0) - C_b(q_b, \underline{e}_b) + (\underline{e}_0 - \underline{e}_b)V'(\underline{E})) = \underline{F}_b^*(x_a) \qquad (34)$$

If differentiate this equation by x_a by applying the Envelope-Theorem and Lemma 4.1, 2.), we get:

$$\frac{1}{r}\left((\underline{q}_b - \underline{q}_0)P'(\underline{Q})\frac{\partial \underline{Q}}{\partial x_a} + (\underline{e}_0 - \underline{e}_b)V''(\underline{E})\frac{\partial \underline{E}}{\partial x_a}\right) = \frac{\partial \underline{F}_b^*(x_a)}{\partial x_a} > 0.$$

$\overline{F}_b^*(x_a)$*:* Let $\overline{Q}$, $\overline{q}_i$, $\overline{e}_i$, $i = 0,1,2$ and $\overline{E}$ be the socially optimal levels corresponding to $x_{0b} = 0$. Then $\overline{F}_b^*(x_a)$ is given by

$$\frac{1}{r}((\overline{q}_2 - \overline{q}_0)P(\overline{Q}) + C_0(\overline{q}_0, \overline{e}_0) - C_b(q_b, \overline{e}_2) + (\overline{e}_0 - \overline{e}_2)V'(\overline{E})) = \overline{F}_b^*(x_a) \tag{35}$$

If differentiate this equation by x_a by applying the Envelope-Theorem and Lemma 4.1, 1.), we get:

$$\frac{1}{r}\left((\overline{q}_2 - \overline{q}_0)P'(\overline{Q})\frac{\partial \overline{Q}}{\partial x_a} + (\overline{e}_0 - \overline{e}_2)V''(\overline{E})\frac{\partial \overline{E}}{\partial x_a}\right) = \frac{\partial \overline{F}_b(x_a)}{\partial x_a} < 0.$$

As x_a tends to 1 the value $\underline{Q}$ and $\overline{Q}$ as well as $\underline{E}$ and $\overline{E}$ tend to the same value Q_{lim} and E_{lim} respectively. Thus the LHS of the equations (34) and (35) tend to the same values. Thus also the RHS, $\underline{F}_b^*(x_a)$ and $\overline{F}_b^*(x_a)$ must tend to the same level $\check{F}_b^*$.

Ad 1.-3., replacement : Given x_a at stage 2 the social planner solves

$$\min_{\{x_{0ab}, x_{ab}, q_{i,2}, e_{i,2}, i=1,2\}} \{\frac{1}{r}[\int_0^{Q_2} P(\tilde{Q})d\tilde{Q} - x_{0ab}C_a(q_{a,2}, e_{a,2}) - (x_{0b} + x_{ab})C_b(q_{b,2}, e_{b,2}) - V(W:2)] - x_{ab}F_b\}, \tag{36}$$

subject to the constraints $x_{0ab} \geq 0$, $x_{ab} \geq 0$ and $x_{0ab} + x_{ab} = x_a$ with corresponding Kuhn-Tucker multipliers μ_i for non-negative constraint for x_i for $i = 0ab, ab$ and Lagrange multiplier ν w.r.t. $x_{01} = x_{0ab} + x_{ab}$. For simplicity again we write q_i and w_i instead of $q_{i,2}$ and $w_{i,2}$ for $i = 0, a, b$. Now the first order conditions w.r.t. x_{ab} and x_{0ab} are given by

$$\frac{1}{r}(q_a P(Q) - C_a(q_a, e_a) - e_a V'(E)) - \mu_{0ab} - \nu = 0 \tag{37}$$

and

$$\frac{1}{r}(q_b P(Q) - C_b(q_b, e_b) - e_b V'(E)) - F_b - \mu_{ab} - \nu = 0 \tag{38}$$

Eliminating ν yields

$$\frac{1}{r}((q_b - q_a)P(Q) + C_a(q_a, e_a) - C_b(q_b, e_b) + (e_a - e_b)V'(E)) - \mu_{ab} + \mu_{0ab} = F_b \tag{39}$$

Considering first the interior solutions (i.e. $\mu_{0ab} = \mu_{qb} = 0$), we differentiate equation (39) with respect to F_b . Employing the Envelope Theorem, we obtain:

$$\frac{1}{r}P'(Q)\frac{\partial Q}{\partial x_{ab}}\frac{\partial x_{ab}}{\partial F_2}(q_b - q_a) + \frac{1}{r}V''(E)\frac{\partial E}{\partial x_{ab}}\frac{\partial x_{ab}}{\partial F_b}(e_a - e_b) = 1 \tag{40}$$

We can evaluate $\frac{\partial Q}{\partial x_{ab}}$ and $\frac{\partial E}{\partial x_{ab}}$ by applying Lemma 4.1, 2.) with $x_a = x_{0ab}$ since effectively we have a situation where each firm adopts either technology a or b and the share of firms adopting technology 1 decreases. Thus $\frac{\partial Q}{\partial x_{ab}} = -\frac{\partial Q}{\partial x_{0ab}} > 0$ and $\frac{\partial E}{\partial x_{ab}} = -\frac{\partial E}{\partial x_{0ab}} < 0$. This yields $\frac{\partial x_{ab}}{\partial F_b} < 0$, $\frac{\partial Q}{\partial F_b} < 0$, $\frac{\partial E}{\partial F_b} > 0$. Analogously to the partial adoption case the existence of both, the upper and the lower bound follows.

If we differentiate 39 with respect to x_a we obtain:

$$\frac{1}{r}P'(Q)[\frac{\partial Q}{\partial x_{ab}}\frac{\partial x_{ab}}{\partial x_a} + \frac{\partial Q}{\partial x_a}](q_b - q_a) + \frac{1}{r}V''(E)[\frac{\partial E}{\partial x_{ab}}\frac{\partial x_{ab}}{\partial x_a} + \frac{\partial E}{\partial x_a}](e_a - e_b) = 0$$

Note that e.g. $\frac{\partial Q}{\partial x_a} = -\frac{\partial Q}{\partial x_{ab}}$, since effectively an increase of x_{ab} is an increase of the total share of firms using technology a. Thus, if mimic the approach of the partial adoption case we evaluate $\frac{x_{ab}}{x_a} = 1$ and therefrom $\frac{\partial Q}{\partial x_{ab}}\frac{\partial x_{ab}}{\partial x_a} + \frac{\partial Q}{\partial x_a} = 0$ and $\frac{\partial E}{\partial x_{ab}}\frac{\partial x_{ab}}{\partial x_a} + \frac{\partial E}{\partial x_a} = 0$.

$\underline{\underline{F}}_b^*$*:* Since at this level all firms should adopt technology b, the correspond socially optimal levels $\underline{Q}$, $\underline{q}_i$, $\underline{e}_i$, $i = 0, a, b$ and $\underline{E}$ do not depend on x_a. Since $\underline{\underline{F}}_b^*$ is given by

$$\frac{1}{r}((\underline{q}_b - \underline{q}_a)P(\underline{Q}) + C_a(\underline{q}_a, \underline{e}_a) - C_b(q_b, \underline{e}_b) + (\underline{e}_a - \underline{e}_b)V'(\underline{E})) = \underline{\underline{F}}_b^*$$

it also do not depend on x_a.

$\overline{\overline{F}}_b^*(x_a)$*:* Let $\overline{Q}$, $\overline{q}_i$, $\overline{e}_i$, $i = 0, a, b$ and $\overline{E}$ be the socially optimal levels corresponding to $x_{ab} = 0$. Then $\overline{\overline{F}}_b^*(x_a)$ is given by

$$\frac{1}{r}((\overline{q}_b - \overline{q}_a)P(\overline{Q}) + C_a(\overline{q}_1, \overline{e}_1) - C_b(q_b, \overline{e}_b) + (\overline{e}_1 - \overline{e}_2)V'(\overline{E})) = \overline{\overline{F}}_b^*(x_a) \tag{41}$$

If we differentiate this equation by x_a by applying the Envelope-Theorem and Lemma 4.1, 2.), we get:

$$\frac{1}{r}\left((\overline{q}_b - \overline{q}_a)P'(\overline{Q})\frac{\partial \overline{Q}}{\partial x_a} + (\overline{e}_a - \overline{e}_b)V''(\overline{E})\frac{\partial \overline{E}}{\partial x_a}\right) = \frac{\partial \overline{\overline{F}}_b^*(x_a)}{\partial x_a} > 0.$$

Obviously $\overline{\overline{F}}_b^*(x_a)$ must tend to $\underline{\underline{F}}_b^*$ as x_a tends to 0. q.e.d.

proof of proposition 4.4 First note that given F_b there are six general possible scenarios. If $F_b \geq \overline{F}_b^*(0)$ for all x_a no adoption of technology b will be socially optimal in the second stage since $\overline{F}_b^*$ is decreasing in x_a. If $F_b \leq \underline{\underline{F}}_b^*$ for all x_a full adoption and full replacement of technology b by the remaining will be socially optimal in the second stage. If $F_b \in [\check{F}^*, \overline{F}_b^*(0))$ by proposition 4.2 there will exist an unique $\hat{x}_1 \leq 1$ such that partial adoption of technology b is socially optimal for $x_a < \hat{x}_a$ and no adoption is optimal for $x_a \geq \hat{x}_a$. Conversely $F_b \in (\underline{F}_b^*(0), \check{F}^*]$ by proposition 4.2 there will exist an unique $\hat{x}_a \leq 1$ such that partial adoption of technology b is socially optimal for $x_a < \hat{x}_a$ and at least full adoption of technology b by the remaining firms is optimal for $x_a \geq \hat{x}_a$. Of course there may also exist a unique $\hat{x}_a < \hat{\hat{x}}_a \leq 1$ such that partial replacement will socially optimal for $x_a > \hat{\hat{x}}_a$. If $F_b \in (\underline{\underline{F}}_b^*, \underline{F}_b^*(0)]$ there will exist $0 < \hat{\hat{x}}_a < 1$ such that full adoption of technology b by all $(1 - x_a)$ firms using the conventional technology is optimal for $x_a < \hat{\hat{x}}_a$ and full adoption of technology b by all $(1 - x_a)$ firms using the conventional technology as well as partial replacement is optimal for $x_a > \hat{\hat{x}}_a$. But since in all cases obviously $E_b^*(x_a)$ is continuous in x_a we will easily derive that also $x_a^*(F_a)$ is continuous. Figure xy visualizes these scenarios.

So let us first consider that given F_a we have an inner solution with respect to x_a where no adoption of technology b is optimal. Let $\hat{x}_a$ as described in the first paragraph (where $\hat{x}_a = 0$ is allowed and represent the case that no adoption is always optimal in the second stage) Then in principle this is an one technology case where we can apply the proof of proposition 4.2 for the case of $x_a = 0$. From this we can easily derive that E_1 increases in F_a while x_a decreases and that there will exist an interval $\underline{F}, \overline{F}$ such that no firm will adopt technology 1 for $F_a \geq \overline{F}$ and less then $\hat{x}_a$ firms will adopt technology a for $F_a \leq \underline{F}$.

Secondly consider the case that we have an inner solution with respect to x_a where partial adoption is the optimal outcome of the second stage. Again let $\hat{x}_a$ be as described above. Now the FOC with respect to x_a is

$$\begin{aligned} 0 = &\frac{1}{\lambda + r}(P(Q_1)(q_{a,1} - q_{0,1}) + C_0^1 - C_a^1 + (e_{0,1} - e_{a,1})V'(E_1)) \\ &+ \frac{\lambda}{\lambda + r}\frac{1}{r}(P(Q_2)(q_{a,2} - q_{0,2}) + C_0^2 - C_a^2 + (e_{0,2} - e_{a,2})V'(E_2)) - F_a \end{aligned}$$

By equation 28 we can also rewrite the last equation as

$$\begin{aligned} 0 = &\frac{1}{\lambda+r}(P(Q_1)(q_{a,1}-q_{0,1})+C_0^1-C_a^1+(e_{0,1}-e_{a,1})V'(E_1)) \\ &+\frac{\lambda}{\lambda+r}\left[\frac{1}{r}(P(Q_2)(q_{a,2}-q_{b,2})+C_B^2-C_A^2+(e_{b,2}-e_{a,2})V'(E_2))+F_b\right]-F_a \end{aligned} \tag{42}$$

First we proof that if we differentiate both equations with respect to F_a we will get

$$0=\frac{1}{\lambda+r}\left(P'(Q_1)(q_{a,1}-q_{0,1})\frac{\partial Q_1}{\partial x_a}\frac{\partial x_a}{\partial F_a}+V''(E_1)(e_{0,1}-e_{a,1})\frac{\partial E_1}{\partial x_a}\frac{\partial x_a}{\partial F_a}\right)-1$$

Thus there is no effect with respect to the second stage. To show this we differentiate the terms in equations (42) and (42) corresponding to the second stage. Then, multiplying by $r(r+\lambda)/\lambda$ and using the same notation as in the proof of proposition 4.2, we obtain:

$$P'(Q_2)(q_{a,2}-q_{0,2})\frac{dQ_2}{dx_a}+V''(E_2)(e_{0,2}-e_{a,2})\frac{dE_2}{dx_a} \tag{43}$$

and

$$P'(Q_2)(q_{a,2}-q_{b,2})\frac{dQ_2}{dx_a}+V''(W_2)(e_{b,2}-e_{a,2})\frac{dE_2}{dx_a} \tag{44}$$

respectively. Now from equation (33) we can derive

$$P'(Q_2)((q_{a,2}-q_{0,2})-(q_{a,2}-q_{b,2}))\frac{dQ_2}{dx_a}+V''(E_2)(e_{0,2}-e_{a,2})-(e_{b,2}-e_{a,2}))\frac{dE_2}{dx_a}=0.$$

This implies that the term (43) must be equal to the term (44). But since both terms must have different signs this can be only the case if both are equal to zero, which proves our claim.

Thus

$$\frac{\partial x_a}{\partial F_a}=\frac{r}{P'(Q_1)(q_{a,1}-q_{0,1})\frac{\partial Q_1}{\partial x_a}+V''(E_1)(e_{0,1}-e_{a,1})\frac{\partial E_1}{\partial x_a}}$$

We can apply Lemma 4.1 to derive that $\frac{\partial Q_1}{\partial x_a}>0$ and $\frac{\partial E_1}{\partial x_a}<0$ and thus $\frac{\partial x_a}{\partial F_a}<0$. This implies $\frac{\partial Q_1}{\partial F_a}<0$ and $\frac{\partial E_1}{\partial F_a}>0$ and analogously to the proof of proposition 4.2 we can derive the existence of an interval $[\underline{F},\overline{F}$ such that $F_a>\overline{F}$ induces no adoption of technology a and $F_a<\underline{F}$ induces that a share $x_a>\hat{x}_a$ of firms adopt technology 1 and either no $F_b>\check{F}_2$ or all $F_b<\check{F}_b$ remaining firms with conventional technology adopt technology b.

Thirdly consider that we have an inner solution with respect to x_a where full adoption of technology b by the remaining firms is the optimal outcome of the second stage. Let $\hat{x}_a$ and $\hat{\hat{x}}_a$ be as described above. Now the FOC with respect to x_a is

$$\begin{aligned} 0 = &\frac{1}{\lambda + r}(P(Q1)(q_{a,1} - q_{0,1}) + C_0^1 - C_a^1 + (e_{0,1} - e_{a,1})V'(E_1)) \\ &+ \frac{\lambda}{\lambda + r}\left(\frac{1}{r}(P(Q_2)(q_{a,2} - q_{b,2}) + C_b^2 - C_a^2 + (e_{b,2} - e_{a,2})V'(E_2)) + F_b\right) - F_1 \end{aligned}$$

If we differentiate this equation with respect to F_a we get

$$\begin{aligned} 1 = &\frac{1}{\lambda + r}\left(P'(Q_1)(q_{a,1} - q_{0,1})\frac{\partial Q_1}{\partial x_a}\frac{\partial x_a}{\partial F_a} + V''(E_1)(e_{0,1} - e_{a,1})\frac{\partial E_1}{\partial x_a}\frac{\partial x_a}{\partial F_a}\right) \\ &+ \frac{\lambda}{\lambda + r}\frac{1}{r}\left(P'(Q_2)(q_{a,2} - q_{b,2})\frac{\partial Q_2}{\partial x_a}\frac{\partial x_a}{\partial F_a} + V''(E_2)(e_{b,2} - e_{a,2})\frac{\partial E_2}{\partial x_a}\frac{\partial x_a}{\partial F_a}\right) \end{aligned}$$

Thus

$$\frac{\partial x_a}{\partial F_a} = \frac{\lambda + r}{P'(Q_1)(q_{a,1} - q_{0,1})\frac{\partial Q_1}{\partial x_a} + V''(E_1)(e_{0,1} - e_{a,1})\frac{\partial E_1}{\partial x_a} + \frac{\lambda}{r}\left(P'(Q_2)(q_{a,2} - q_{b,2})\frac{\partial Q_2}{\partial x_a} + V''(E_2)(e_{b,2} - e_{a,2})\frac{\partial E_2}{\partial x_a}\right)}$$

Now if we apply Lemma 4.1 to derive that $\frac{\partial Q_1}{\partial x_a} > 0$, $\frac{\partial E_1}{\partial x_a} < 0$, $\frac{\partial Q_2}{\partial x_a} < 0$ and $\frac{\partial E_2}{\partial x_a} > 0$ and thus we get $\frac{\partial x_a}{\partial F_a} < 0$. From this all other claims follow, especially the existence of an interval $[\underline{F}, \overline{F}]$ such that $F_a > \overline{F}$ induces either no adoption of technology a (if $F_b \leq \underline{F}_b$) or partial adoption of technology a by a share $x_a < \hat{x_a}$ of firms (if $F_b > \underline{F}_b$)). For $F_a < \underline{F}$ a share $x_a > \hat{\hat{x}}_a$ (note that $\hat{\hat{x}}_a = 1$ is allowed) of firms adopt technology 1 and all remaining firms with conventional technology adopt technology b while a partial share of firms will replace technology a.

Fourth consider that we have an inner solution with respect to x_a where full adoption of technology 2 by the remaining firms as well as partial replacement is the optimal outcome of the second stage. Let $\hat{\hat{x}}_a$ be as described above. Now the FOC with respect to x_a is

$$\begin{aligned} 0 = &\frac{1}{\lambda + r}(P(Q_1)(q_{a,1} - q_{0,1}) + C_0^1 - C_a^1 + (e_{0,1} - e_{a,1})V'(E_1)) \\ &+ \frac{\lambda}{\lambda + r}\Big(\underbrace{\frac{1}{r}(P(Q_2)(q_{a,2} - q_{b,2}) + C_b^2 - C_a^2 + (e_{b,2} - e_{a,2})V'(R_2)) + F_b}_{=0}\Big) - F_a \end{aligned}$$

If we differentiate this equation with respect to F_a we get

$$1 = \frac{1}{\lambda + r}\left(P'(Q_1)(q_{a,1} - q_{0,1})\frac{\partial Q_1}{\partial x_a}\frac{\partial x_a}{\partial F_a} + V''(E_1)(e_{0,1} - e_{a,1})\frac{\partial E_1}{\partial x_a}\frac{\partial x_a}{\partial F_a}\right)$$

since by proposition 4.2 Q_2 and E_2 do not depend on x_a. Thus straight-forwardly $\frac{\partial x_a}{\partial F_a} < 0$ and therefrom all other claims follow. Analogously to the cases before the existence of an interval $[\underline{F}, \overline{F}]$ follows such that by proposition 4.2 $F_a > \overline{F}$ induces that $x_a < \hat{\hat{x}}_a$ while for $F_a < \underline{F}$ all firms adopt technology a.

The fifth case is the case where we have an inner solution with respect to x_a where full replacement of technology b by the remaining firms is the optimal outcome of the second stage. The corresponding

$$0 = \frac{1}{\lambda + r}(P(Q_1)(q_{a,1} - q_{0,1}) + C_0^1 - C_a^1 + (e_{0,1} - e_{a,1})V'(E_1)) - F_a \tag{45}$$

In principle the calculation is analogous to the calculations in the former cases. q.e.d.

proof of proposition 4.5: *ad 1): Case 1: Partial adoption of both technologies* The pair (x_a^*, x_{0b}^*) is determined by the equations (28) and (42). Since x_{0b}^* only depends indirectly via x_a^* on λ we only differentiate the second equation with respect to λ. This yields:

$$\begin{aligned}
0 = & \frac{-1}{(\lambda + r)^2}\left[\underbrace{P(Q1^*)(q_{a,1}^* - q_{0,1}^*) + C_0^1 - C_a^1 + (e_{0,1}^* - e_{a,1}^*)V'(E_1^*)}_{:=FSTP}\right] \\
& + \frac{1}{\lambda + r}\left[P'(Q_1^*)(q_{a,1}^* - q_{0,1}^*)\frac{\partial Q_1^*}{\partial x_a}\frac{\partial x_a^*}{\partial \lambda} + (e_{0,1}^* - e_{a,1}^*)V''(E_1^*)\frac{\partial E_1^*}{\partial x_a}\frac{\partial x_a^*}{\partial \lambda}\right] \\
& + \frac{1}{(\lambda + r)^2}\left[\underbrace{P(Q_2^*)(q_{a,2}^* - q_{0,1}^*) + C_0^2 - C_a^2 + (e_{0,2}^* - e_{a,2}^*)V'(E_2^*)}_{SSTP}\right] \\
& + \frac{\lambda}{\lambda + r}\frac{1}{r}\left[P'(Q_2^*)(q_{a,2}^* - q_{0,2}^*)\left(\frac{\partial Q2^*}{\partial x_a} + \frac{\partial Q_2^*}{\partial x_{0b}}\frac{\partial x_{0b}^*}{\partial x_a}\right)\frac{\partial x_a^*}{\partial \lambda}\right] \\
& + \frac{\lambda}{\lambda + r}\frac{1}{r}\left[(e_{0,2}^* - e_{a,2}^*)V''(E_2^*)\left(\frac{\partial E_2^*}{\partial x_a} + \frac{\partial E_2^*}{\partial x_{0b}}\frac{\partial x_{0b}^*}{\partial x_a}\right)\frac{\partial x_a^*}{\partial \lambda}\right]
\end{aligned}$$

By the proof of proposition 4.4 the last two parts of this formula sum up to 0. So set $k := P'(Q_1^*)(q_{a,1}^* - q1_0^*)\frac{\partial Q_1^*}{\partial x_a} + (e_{0,1}^* - e_{a,1}^*)V''(E_1^*)\frac{\partial E_1^*}{\partial x_a} < 0$. Then $\frac{\partial x_a^*}{\partial \lambda} = \frac{[FSTP - SSTP]}{(\lambda + r)k}$. Differentiating SSTP with respect to x_{0b} yields $P'(Q_2^*)(q_{a,2}^* - q_{0,1}^*)\frac{\partial Q_2^*}{\partial x_{0b}} + (e_{0,2}^* - e_{a,2}^*)V''(E_2^*)\frac{\partial E_2^*}{\partial x_{0b}} < 0$. Since $FSTP = SSSP$ for $x_{0b}^* = 0$ it is $FSTP - SSTP > 0$ for $x_{0b}^* > 0$. Thus $\frac{\partial x_a^*}{\partial \lambda} < 0$.

Case 2: Full adoption of technology 2 and no replacement If we differentiate equation (45) with respect to λ we get:

$$
\begin{aligned}
0 =& \frac{1}{(\lambda+r)^2}(\underbrace{P(Q_2^*)(q_{a,2}^* - q_{b,2}^*) + C_b^2 - C_a^2 + (e_{b,2}^* - e_{a,2}^*)V'(E_2^*) + rF_b}_{:=SSTP}) \\
&- \frac{1}{(\lambda+r)^2}(\underbrace{P(Q1^*)(q_{a,1}^* - q_{0,1}^*) + C_0^1 - C_a^1 + (e_{0,1}^* - e_{a,1}^*)V'(E_1^*)}_{:=FSTP}) \\
&+ \frac{1}{\lambda+r}\left[P'(Q_1^*)(q_{a,1}^* - q_{0,1}^*)\frac{\partial Q_1^*}{\partial x_a}\frac{\partial x_a^*}{\partial \lambda} + V''(E_1^*)(e_{0,1}^* - e_{a,1}^*)\frac{\partial W_1^*}{\partial x_a}\frac{\partial x_1^*}{\partial \lambda}\right] \\
&+ \frac{\lambda}{r(\lambda+r)}\left[P'(Q_2^*)(q_{a,2}^* - q_{b,2}^*)\frac{\partial Q_2^*}{\partial x_a}\frac{\partial x_a^*}{\partial \lambda} + V''(E_2^*)(w_{b,2}^* - w_{a,2}^*)\frac{\partial W_2^*}{\partial x_a}\frac{\partial x_2^*}{\partial \lambda}\right]
\end{aligned}
$$

Define

$$
\begin{aligned}
k :=& P'(Q_1^*)(q_{a,1}^* - q_{0,1}^*)\frac{\partial Q_1^*}{\partial x_a} + V''(E_1^*)(e_{0,1}^* - e_{a,1}^*)\frac{\partial W :_1^*}{\partial x_a} \\
&+ \frac{\lambda}{r}\left[P'(Q_2^*)(q_{a,2}^* - q_{b,2}^*)\frac{\partial Q_2^*}{\partial x_a} + V''(E_2^*)(e_{b,2}^* - e_{a,2}^*)\frac{\partial E_2^*}{\partial x_a}\right] < 0
\end{aligned}
$$

Now $\frac{\partial x_a^*}{\partial \lambda} = \frac{FSTP - SSTP}{(\lambda+r)k}$ hence we need to show that $FSTP - SSTP > 0$. Then

$$
\begin{aligned}
&FSTP - SSTP \\
&= P(Q_1^*)(q_{a,1}^* - q_{0,1}^*) + C_0^1 - C_a^1 + (e_{0,1}^* - e_{a,1}^*)V'(E_1^*) \\
&- [P(Q_2^*)(q_{a,1}^* - q_{0,1}^*) + C_0^2 - C_a^2 + (e_{0,2}^* - e_{a,2}^*)V'(E_2^*)] \\
&+ P(Q_2^*)(q_{b,1}^* - q_{0,1}^*) + C_0^2 - C_b^2 + (e_{0,2}^* - e_{b,2}^*)V'(E_2^*) - rF_b
\end{aligned}
$$

The difference between the terms in the second and the third line is positive by reason of the same argument as in the former case. The third line is positive since full adoption is the relevant market outcome.

Case 3: Partial Replacement If we differentiate equation (45) with respect to λ we get:

$$
\begin{aligned}
0 =& \frac{1}{(\lambda+r)^2}(SSTP - FSTP) \\
&+ \frac{1}{\lambda+r}\left[P'(Q_1^*)(q_{a,1}^* - q_{0,1}^*)\frac{\partial Q_1^*}{\partial x_a}\frac{\partial x_a^*}{\partial \lambda} + V''(E_1^*)(e_{0,1}^* - e_{a,1}^*)\frac{\partial E_1^*}{\partial x_a}\frac{\partial x_a^*}{\partial \lambda}\right],
\end{aligned}
$$

where $SSTP$ and $FSTP$ are given as in the former case. Now again

$$
\frac{\partial x_a^*}{\partial \lambda} = \frac{FSTP - SSTP}{(\lambda+r)\left[P'(Q_1^*)(q_{a,1}^* - q_{0,1}^*)\frac{\partial Q_1^*}{\partial x_a} + V''(E_1^*)(e_{0,1}^* - e_{a,1}^*)\frac{\partial E_1^*}{\partial x_a}\right]}.
$$

Analogously to the former case we can show that $FSTP - SSTP > 0$ which induces the result.

Case 4: Full Replacement straight-forward!

ad 2.): $\overline{F_a}$: Case 1: Partial adoption:

$$0 = \frac{1}{\lambda + r}(P(\overline{Q1}^*)(\overline{q}^*_{a,1} - \overline{q}^*_{0,1}) + C^1_0 - C^1_a + (\overline{e}^*_{0,1} - \overline{w}^*_{a,1})V'(\overline{E_1}^*)) + \frac{\lambda}{\lambda + r}\frac{1}{r}(P(Q^*_2)(q^*_{a,2} - q^*_{0,2}) + C^2_0 - C^2_a + (e^*_{0,2} - e^*_{a,2})V'(E^*_2)) - \overline{F}^*_a$$

Differentiating this equation with respect to F_b yields

$$\frac{\partial \overline{F}^*_1}{\partial F_b} = \frac{\lambda}{\lambda + r}\frac{1}{r}\left[P'(Q^*_2)(q^*_{a,2} - q^*_{0,2})\frac{\partial Q^*_2}{\partial F_b} + (e^*_{0,2} - e^*_{a,2})V''(E^*_2)\frac{\partial E^*_2}{\partial F_b}\right] > 0$$

Case 2: full adoption/replacement: Differentiating the corresponding FOC with respect to F_b yields

$$\frac{\partial \overline{F}^*_1}{\partial F_b} = \frac{\lambda}{\lambda + r} > 0$$

ad 3.): $\underline{F_a}$: Case 1: Full adoption:

$$0 = \frac{1}{\lambda + r}(P(\overline{Q1}^*)(\overline{q}^*_{a,1} - \overline{q}^*_{0,1}) + C^1_0 - C^1_a + (\overline{e}^*_{0,1} - \overline{w}^*_{a,1})V'(\overline{E_1}^*)) + \frac{\lambda}{\lambda + r}\left[\frac{1}{r}(P(Q^*_2)(q^*_{a,2} - q^*_{b,2}) + C^2_b - C^2_a + (e^*_{b,2} - e^*_{a,2})V'(E^*_2)) + F_b\right] - \overline{F}^*_1$$

Differentiating this equation with respect to F_b yields

$$\frac{\partial \underline{F}^*_1}{\partial F_b} = \frac{\lambda}{\lambda + r} > 0$$

Case 2: partial replacement: Differentiating equation (45) with respect to F_b yields

$$\frac{\partial \overline{F}^*_1}{\partial F_b} = 0$$

Case 3: full replacement: Differentiating the corresponding FOC with respect to F_b yields

$$\frac{\partial \overline{F}^*_1}{\partial F_b} = 0$$

proof of Lemma 5.1 In principle the proof is analogous to the proof of Lemma 4.1. If we differentiate the equation system (5) and (6) we get for a given $i = 0, a, b$ and setting

$x_0 = 1 - x_a$ wherever it makes sense:

$$\begin{pmatrix} P'(Q)\frac{\partial Q}{\partial w} \\ -1 \end{pmatrix} = \begin{pmatrix} \frac{\partial^2 C_i}{\partial q^2} & \frac{\partial^2 C_i}{\partial e \partial q} \\ \frac{\partial^2 C_i}{\partial q \partial e} & \frac{\partial^2 C_i}{\partial e^2} \end{pmatrix} \begin{pmatrix} \frac{\partial q_i}{\partial w} \\ \frac{\partial e_i}{\partial w} \end{pmatrix}$$

From this analogously to the proof of Lemma 4.1 we derive

$$\frac{\partial q_i}{\partial w} = \frac{1}{k_i}[P'(Q)\frac{\partial Q}{\partial w}\frac{\partial^2 C_i}{\partial e^2} + \frac{\partial^2 C_i}{\partial q \partial e}]$$

$$\frac{\partial e_i}{\partial w} = \frac{-1}{k_i}[\frac{\partial^2 C_i}{\partial q^2} + P'(Q)\frac{\partial Q}{\partial w}\frac{\partial^2 C_i}{\partial q \partial e}]$$

where k_i is given as in the proof of Lemma 4.1. Substituting both to equation 7 and 8 respectively delivers for given and fixed x_{0b}:

$$\frac{\partial Q}{\partial w} = \frac{\sum \frac{x_i}{k_i}\frac{\partial^2 C_i}{\partial q \partial e}}{1 - P'(Q)\sum \frac{x_i}{k_i}\frac{\partial^2 C_i}{\partial e^2}} < 0$$

and therefrom

$$\frac{\partial E}{\partial w} = -\sum \frac{x_i}{k_i}\left[P'(Q)\frac{\partial Q}{\partial w}\frac{\partial^2 C_i}{\partial q \partial e} + \frac{\partial^2 C_i}{\partial q^2}\right]$$
$$= \frac{P'(Q)\left[\sum \frac{x_i}{k_i}\frac{\partial^2 C_i}{\partial q^2}\sum \frac{x_i}{k_i}\frac{\partial^2 C_i}{\partial e^2} - \left(\sum \frac{x_i}{k_i}\frac{\partial^2 C_i}{\partial q \partial e}\right)^2\right] - \sum \frac{x_i}{k_i}\frac{\partial^2 C_i}{\partial q^2}}{1 - P'(Q)\sum \frac{x_i}{k_i}\frac{\partial^2 C_i}{\partial e^2}}$$

Thus $\frac{\partial E}{\partial w} < 0$ by assumption 2.2.

Next we derive $\frac{\partial E}{\partial x_{0b}}$ and $\frac{\partial Q}{\partial x_{0b}}$ for given and fixed w. In that case we have

$$\begin{pmatrix} P'(Q)\frac{\partial Q}{\partial x_{0b}} \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{\partial^2 C_i}{\partial q^2} & \frac{\partial^2 C_i}{\partial e \partial q} \\ \frac{\partial^2 C_i}{\partial q \partial e} & \frac{\partial^2 C_i}{\partial e^2} \end{pmatrix} \begin{pmatrix} \frac{\partial q_i}{\partial x_{0b}} \\ \frac{\partial e_i}{\partial x_{0b}} \end{pmatrix}$$

from which we can straight-forwardly derive

$$\frac{\partial Q}{\partial x_{0b}} = \frac{q_b - q_0}{1 - P'(Q)\sum \frac{x_i}{k_i}\frac{\partial^2 C_i}{\partial e^2}} > 0$$

and

$$\frac{\partial E}{\partial x_{0b}} = e_b - e_0 - P'(Q)\frac{\partial Q}{\partial x_{0b}}\sum \frac{x_i}{k_i}\frac{\partial^2 C_i}{\partial q \partial e} < 0.$$

The impact of an change of x_a for $x_{0b} < 1 - x_a$ can be derived analogously. If $x_{0b} = 1 - x_a$ we analogously to the former calculation can derive that

$$\frac{\partial Q}{\partial x_a} = \frac{q_a - q_b}{1 - P'(Q)(\frac{x_a}{k_a}\frac{\partial^2 C_a}{\partial e^2} + \frac{1-x_a}{k_{0b}}\frac{\partial^2 C_b}{\partial e^2}))} < 0$$

Moreover simply

$$\frac{\partial E}{\partial x_a} = e_a - e_b - P'(Q)\frac{\partial Q}{\partial x_a}\left[\frac{x_a}{k_a}\frac{\partial^2 C_a}{\partial q \partial e} + \frac{1-x_a}{k_{0b}}\frac{\partial^2 C_b}{\partial q \partial e}\right] > 0.$$

Now by the Envelope Theorem

$$\frac{\partial \Delta_{20}}{\partial w_2} = P'(Q_2)(q_{b,2} - q_{0,2})\frac{\partial Q_2}{\partial w_2} + (e_{0,2} - e_{b,2}) > 0.$$

q.e.d.

proof of proposition 5.2 *ad 2.) -4.) :* First of all let us consider that all equations are fulfilled with $0 < x_{0b} < 1 - x_a$. Differentiating the corresponding equations and applying Lemma 5.1 we obtain:

$$r = P'(Q2)(q_{b,2} - q_{0,2})[\frac{\partial Q_2}{\partial w_2}\frac{\partial 2_2}{\partial x_{0b}} + \frac{\partial Q_2}{\partial x_{0b}}]\frac{\partial x_{0b}}{\partial F_b} + (e_{0,2} - e_{b,2})\frac{\partial w_2}{\partial x_{0b}}\frac{\partial x_{0b}}{\partial F_b}$$

Therefrom we get

$$\frac{\partial x_{0b}}{\partial F_b} = \frac{r}{P'(Q_2)(q_{b,2} - q_{0,2})[\frac{\partial Q_2}{\partial w_2}\frac{\partial w_2}{\partial x_{0b}} + \frac{\partial Q_2}{\partial x_{0b}}] + (e_{0,2} - e_{b,2})\frac{\partial w_2}{\partial x_{0b}}}$$

By our assumptions and Lemma 5.1 $\frac{\partial x_{0b}}{\partial F_b} < 0$ follows. Furthermore we have shown that the LHS of equation (9) increases in x_{0b}. Thus any greater x_{0b} would induce that the cost benefits by the new technology would be smaller then the costs - and therefore some firms would prefer not to invest - while any smaller x_{0b} would induce that the cost benefits exceeds the costs F_b - and therefore more firms want to invest. Thus the solution of the equation system, if it exists, is the unique equilibrium. Following the same logic only no adoption is the unique equilibrium if the LHS of equation (9) is smaller then F_b for all $0 \leq x_{0b} \leq 1 - x_a$. If the if the LHS of equation (9) is larger then F_b for all x_{0b} we have to check whether full adoption, partial replacement or full replacement is the (unique) solution.

Furthermore as a result of the former arguments all market outcomes where $0 < x_{0b} < 1 - x_a$ correspond to an F_b are contained in the interval $[\underline{F}_b(x_a), \overline{F}_b(x_a)]$, where $\underline{F}_b(x_a)$ and $\overline{F}_b(x_a)$ are determined by the equation system (5), (6), (7), (8) and (9) given $x_{0b} = 0$ and $x_{0b} = 1 - x_a$ respectively.

ad 5.) -7.) : The proof is analogous to the one in the former case.

ad 1.). Since in both cases, $F_b = \underline{F}_b(x_a)$ and $F_b = \overline{\overline{F}}_b(x_a)$ we have $x_{0b} = 1 - x_a$ and $x_{ab} = 0$, we will have the same equilibrium output price P and equilibrium input price w. From the equations (10) and (9) then obviously follows that $\underline{F}_b(x_a) > \overline{\overline{F}}_b(x_a)$.

ad 8.): Denote with $\underline{Q}_2$, $\underline{q}_{b,2}$, $\underline{e}_{b,2}$, $i = 0, a, b$ and $\underline{w}_2$ the output and input levels and the input price which corresponds to $x_{0b} := 1 - x_a$. As mentioned above $\underline{F}_b(x_a)$ is given by equation (9) corresponding to this values. If we differentiate this equation with respect to x_a and apply Lemma we get

$$\frac{\partial \underline{F}_b(x_a)}{\partial x_a} = P'(\underline{Q}_2)(\underline{q}_{b,2} - \underline{q}_{0,2})[\frac{\partial \underline{Q}_2}{\partial w}\frac{\partial \underline{w}_2}{\partial x_a} + \frac{\partial \underline{Q}_2}{\partial x_a}] + (\underline{e}_{0,2} - \underline{e}_{b,2})\frac{\partial \underline{w}_2}{\partial x_a}.$$

Since $\underline{Q}_2$ corresponds to $x_{0b} = 1 - x_a$ by Lemma 5.1 and our assumptions we get $\frac{\partial \underline{F}_b(x_a)}{\partial x_a} > 0$. Analogously we can derive $\frac{\partial \overline{F}_b(x_a)}{\partial x_a} < 0$. For analogous reason as in the social optimal case both levels tend to the same value as x_a tends to 1.

ad 9.): With analogous calculations as above we can also derive

$$\frac{\partial \overline{\overline{F}}_2(x_a)}{\partial x_a} = P'(\underline{Q}_2)(\underline{q}_{b,2} - \underline{q}_{a,2})[\frac{\partial \underline{Q}_2}{\partial w}\frac{\partial \underline{w}_2}{\partial x_a} + \frac{\partial \underline{Q}_2}{\partial x_a}] + (\underline{e}_{a,2} - \underline{e}_{b,2})\frac{\partial \underline{w}_2}{\partial x_a} > 0.$$

Since for $F_b = \underline{\underline{F}}_b$ all firms use technology 2 the ex post market equilibrium does not depend on the share of firms which have adopted technology a. Thus an change in x_a will not affect these values and therefore also not $\underline{\underline{F}}_b$. Obviously $\overline{\overline{F}}_b(0) = \underline{\underline{F}}_b$ since for $F_b = \overline{\overline{F}}_b(0)$ also all firms use technology b.q.e.d.

proof of proposition 5.3 Substituting $V'(E_i^*)$ by w_i and $V''(E_i^*)\frac{\partial E_i^*}{\partial x_j}$ by $\frac{\partial w_i}{\partial x_j}$ $(i = a, b,$ $j = a, b, ab)$ we can apply the arguments and calculations proof of proposition 4.4. If for example the market solution consists $1 > x_a > 0$ and $1 - x_a > x_{0b} > 0$ by applying these arguments we show that if the share of firms exceeds x_a the cost savings which corresponds to the adoption technology a is smaller then F_a. Thus less firms want to adopt technology a. Vice versa if a smaller share of firms adopt technology a there is still an incentive for firms to adopt technology a. Analogously we can argue in all other cases.

proof of proposition 6.1 If we compare the FOC's which determine the socially optimal allocation with the relevant equation system determining the market solution we see that for $w_1 = V'(E_1^*)$, $w_2 = V'(E_2^*)$, $x_a = x_a^*$, $x_{0b} = x_{0b}^*$ and $x_{ab} = x_{ab}^*$ we are in a market equilibrium. Since this is unique given F_a and $F_b$the proof is completed. q.e.d.

Proof of Proposition 7.1: *ad 1) :* let x_a and x_{0b} be given. Consider F_a, F_b and F_a^*, F_b^* to be the corresponding cost pairs. Then F_a and F_b fulfill the equation pair (9) and (16). Both w_1 and w_2 are greater than the marginal costs of the corresponding input levels. Since given x_a and x_{0b} a decrease of the input price increases the input level and thus the marginal costs, the marginal cost at the socially optimal input level are higher than the marginal costs but lower than the input price in the market equilibrium. Since for given x_a and x_{0b} by Lemma 5.1 Δ_{20} decreases in the input price. This implies $F_b^* < F_b$ since $\Delta_{20}|_{\rho_2 = V'(E_2^*)} = F_b^*$. Furthermore in the market equilibrium we have

$$\begin{aligned} &\frac{1}{\lambda + r}[P(Q_1)(q_{a,1} - q_{0,1}) - C_0^1 - C_a^1 - w_1(e_{0,1} - e_{a,1})] \\ &+ \frac{\lambda}{r(\lambda + r)}[P(Q_2)(q_{a,1} - q_{0,1}) - C_0^2 - C_a^2 - w_2(e_{0,2} - e_{a,2})] = F_a \end{aligned}$$

Since both, the part of the RHS which corresponds to the first stage and the one correspon-ding to the second stage gets smaller if we fill in the socially optimal marginal cost instead of the equilibrium input price, $F_a < F_a^*$ has to hold.

An analogous argumentation holds in case of 2) - 5) taking the relevant cost difference in each case. It is easy to see that in case of scenario 6) this argumentation cannot be applied.q.e.d.

proof of proposition 8.1 : In principle the proof is based on the fact that the FOC's of the social planner and the equation which determines the equilibrium share are basically the same equation which where only the marginal cost of the aggregated input appears instead of the input price. Thus we have only to check whether the equilibrium input price is smaller equal or larger than the socially optimal marginal input costs.

Case 1: partial adoption of both technologies

The social optimal allocation is characterized by

$$\frac{1}{r}((q^*_{b,2}-q^*_{0,2})P(Q^*_2)+C^2_0-C^2_b+(e^*_{0,2}-e^*_{b,2})V'(E^*_2))=F^*_b$$

and

$$\frac{1}{\lambda+r}((q^*_{a,1}-q^*_{0,1})P(Q^*_1)+C^1_0-C^1_b+(e^*_{0,1}-e^*_{a,1})V'(E^*_1))$$
$$+\frac{\lambda}{\lambda+r}\frac{1}{r}((q^*_{a,2}-q^*_{0,2})P(Q^*_2)+C^2_0-C^2_b+(e^*_{0,2}-e^*_{a,2})V'(E^*_2))=F^*_a$$

If $x_a>0$ then obviously $w_1<V'(e_0)$ where w_1 is the market equilibrium input price. Furthermore since w_1 is the input market clearing price and $E^*_1<e_0$ also $w_1<V'(E^*_1)$ since otherwise the demand would be smaller than or equal to E^*_1. Analogously $x_a>0$ or $x_{0b}>0$ implies $w_2<V'(E^*_2)$. Thus by Lemma 5.1 implies

$$\frac{1}{r}((q_{b,2}-q_{0,2})P(Q_2)+C^2_0-C^2_b+(e_{0,2}-e_{b,2})w_2<F^*_b$$

if either $x_a>0$ or $x_{0b}>0$. Thus $F_b<F^*_b$. Analogously if either $x_a>0$ or $x_{0b}>0$ then

$$\frac{1}{\lambda+r}((q_{a,1}-q_{0,1})P(Q_1)+C^1_0-C^1_a+(e_{0,1}-e_{a,1})w_1$$
$$+\frac{\lambda}{\lambda+r}\frac{1}{r}((q^*_{a,2}-q^*_{0,2})P(Q^*_2)+C^2_0-C^2_b+(e^*_{0,2}-e^*_{a,2})w_2<F^*_a,$$

and thus $F_a<F^*_a$.

Case 2: partial adoption and replacement technology a

$$\frac{1}{r}((q^*_{b,2}-q^*_{a,2})P(Q^*_2)+C^2_a-C^2_b+(e^*_{a,2}-e^*_{b,2})V'(E^*_2))=F^*_b$$

and

$$\frac{1}{\lambda+r}((q^*_{a,1}-q^*_{0,1})P(Q^*_1)+C^1_0-C^1_a+(e^*_{0,1}-e^*_{a,1})V'(E^*_1))=F^*_a$$

For all $1\geq x_a\geq 0$ and $x_a\geq x_{ab}\geq 0$ we will have $E^*_2>e_0$ since always $1-x_a$ firms will adopt technology 2 at the second stage. For the same reasons as in the case before $w_2<V'(E^*_2)$

follows and thus $F_b < F_b^*$. At the first stage $E_1^* < e_0$ if and only if $x_a > 0$ thus $F_a < F_a^*$ if $x_a > 0$ and $F_a = F_a^*$ if $x_a = 0$.

Case 3: partial adoption of technology a, no adoption of technology 2

The arguments are analogous to the ones in the former cases.

*Case 4: partial adoption of technology b, no adoption of technology a*As above!

Case 5: partial adoption of technology a, full adoption of technology b, no replacement

$x_a = 0$:

In this case $w_1 = V'(e_0)$ and $Q_1^* = Q_1$ since $E_1^* = e_0$ while $w_2 < V'(E_2^*)$. Since

$$\frac{1}{\lambda + r}((q_{a,1}^* - q_{0,1}^*)P(Q1^*) + C_0^1 - C_a^1 + (e_{0,1}^* - w_{a,1}^*)V'(e_0)) + \frac{\lambda}{\lambda + r}\left[\frac{1}{r}((q_{a,2}^* - q_{b,2}^*)P(Q_2^*) + C_b^2 - C_a^2 + (e_{b,2}^* - e_{a,2}^*)V'(E_2^*)) + F_b\right] = F_a^*$$

we get

$$\frac{1}{\lambda + r}((q_{a,1} - q_{0,1})P(Q1) + C_0^1 - C_a^1 + (e_{0,1} - e_{a,1})V'(e_0)) + \frac{\lambda}{\lambda + r}\left[\frac{1}{r}((q_{a,2} - q_{b,2})P(Q2) + C_b^2 - C_a^2 + (e_{b,2} - e_{a,2})w_2 + F_b\right] > F_a^*$$

since the second stage part increases as ρ^2 decreases. Tuus $F_a > F_a^*$. $x_a = 1$:

In this case $w_1 = w_2 > V'(E_1^*) = W'(E_2^*)$ and $E_1^* = E_2^* < e_0$. Since

$$\begin{aligned}\Delta_{12}(x_a, x_{0b}, V'(E_1^*))) = &\frac{1}{\lambda + r}((q_{a,1}^* - q_{0,1}^*)P(Q_1^*) + C_0^1 - C_a^1 + (e_{0,1}^* - e_{a,1}^*)V'(E_1^*)) \\ &+ \frac{\lambda}{\lambda + r}\left[\frac{1}{r}((q_{a,2}^* - q_{b,2}^*)P(Q_2^*) + C_b^2 - C_a^2 + (e_{b,2}^* - e_{q,2}^*)V'(E_1^*)) + F_b\right] = F_a^*\end{aligned}$$

and

$$\begin{aligned}\Delta_{12}(x_a, x_{0b}, e_1) = &\frac{1}{\lambda + r}((q_{a,1} - q_{0,1})P(Q_1) + C_0^1 - C_a^1 + (e_{0,1} - e_{a,1})w_1 \\ &+ \frac{\lambda}{\lambda + r}\left[\frac{1}{r}((q_{a,2} - q_{b,2})P(Q_2) + C_b^2 - C_a^2 + (e_{b,2} - e_{a,2})w_1 + F_b\right] = F_a\end{aligned}$$

we will have $F_a > F_a^*$ if $\frac{\partial \Delta_{12}}{\partial w} < 0$ and $F_a > F_a^*$ if $\frac{\partial \Delta_{12}}{\partial w} > 0$. Otherwise it is ambiguous.q.e.d

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C Figures

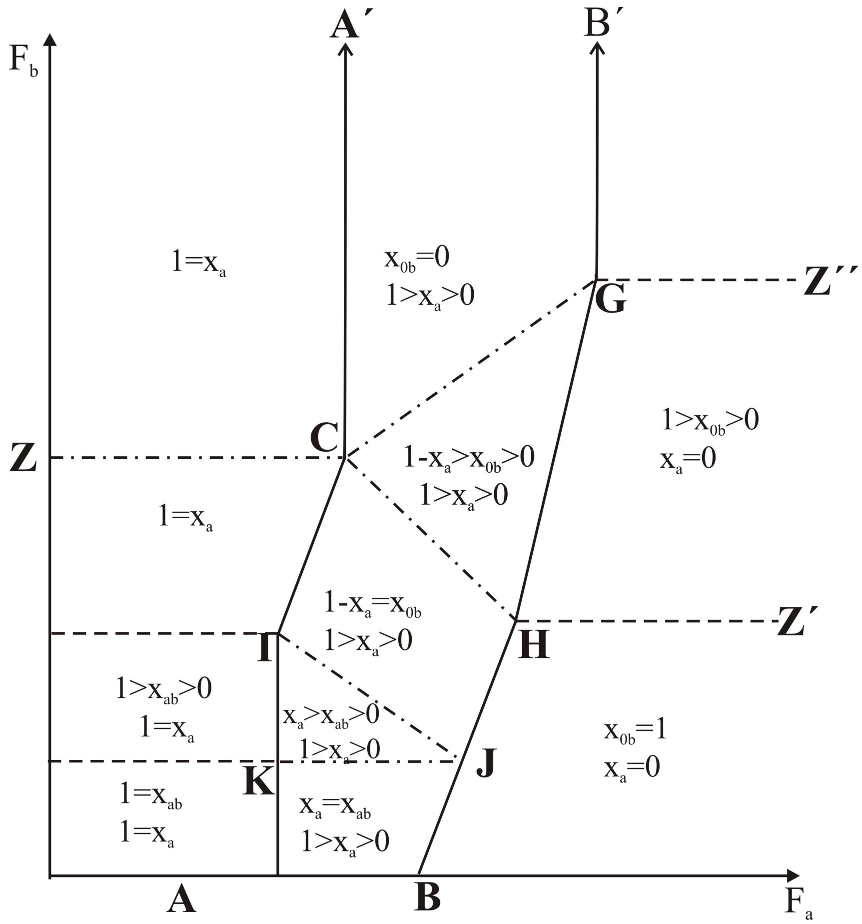

Figure 1. Socially optimal allocation with respect to (F_a, F_b)