Carbon leakage revisited: unilateral climate policy under directed technical change*

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Abstract
This paper analyzes the consequences of unilateral climate policy in the presence of directed technical change. We develop a dynamic two-country model in which two otherwise identical countries differ in their environmental policy: one of the countries enforces a (binding) cap on emissions while the other does not. Focusing on carbon leakage, we show how, compared with a “traditional” endogenous growth model, directed technical change will always lead to lower emissions in the unconstrained country. When clean and dirty goods are good substitutes, it may even be induced to reduce its emissions below the optimum level when both countries are unconstrained, so leakage is negative.

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Keywords: Climate Change, Endogenous Technical Change, International Trade.

1 Introduction
An important threat to climate policy is that action undertaken by a coalition of countries may prove to be ineffective. When emissions of greenhouse gases from different regions are strategic substitutes, the cutback in emissions by the coalition may be partially or even fully offset by countries outside the coalition. The argument behind this claim is simple: if a coalition of technologically advanced (and hence fossil-fuel dependent) economies decides to voluntarily reduce its polluting emissions, this will induce an increase in the price of dirty goods within the coalition. At the same time the world price of fossil fuels may fall due to the reduction in worldwide demand. As a consequence unconstrained countries can

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benefit from both the improved terms of trade and lower fossil fuel prices. They will increase their production of dirty goods and export them to coalition members. If the pollutant was strictly local, the environment of the coalition countries would improve at the expense of the environment elsewhere (the pollution-haven effect). However, since greenhouse gases are global pollutants, the coalition is harmed by any increase in foreign emissions. The pollution-haven effect of unilateral environmental policy thus may be ineffective in case of a global pollutant and coalition countries are penalized twice: while their domestic production (and probably income) decreases, the expected benefits stemming from an improved environment fail to materialize, partly or even fully. This effect has come to be known as carbon leakage.¹

In this paper we study the degree to which carbon leakage is a threat to climate policy when entrepreneurs can target new technologies to either clean or dirty goods (directed technical change in the words of Acemoglu (2002)). When a cap on emission induces a large fraction of countries to specialize in clean production, it might also shift their innovation efforts towards cleaner technologies. When technologies diffuse internationally this affects the degree of carbon leakage. We show that when directed technical change is taken into account, the leakage rate will generally be lower than when it is not. Indeed, carbon leakage may even be negative, as the unconstrained region may find it optimal to reduce its emissions. Furthermore, we show that unilateral climate policy will always be to some extend effective, as the leakage rate will always be less than 100%.

The problem of carbon leakage has been studied extensively using CGE models. However, differences in model assumptions has lead to a wide range of estimates for the leakage rate, both within and between studies. For example, the OECD GREEN model (Burniaux and Oliveira Martins (2000) give a leakage rate of 2.2% or 4.8%, depending on the use of the flexibility mechanisms of the Kyoto Protocol. On the other hand Light et al. (2000) report a leakage rate of 5.6% when internationally differentiated goods are hard to substitute and a rate of 40.6% when trade elasticities approach infinity.

The theoretical literature generally considers emissions of greenhouse gases from different regions as strategic substitutes, so that the cutback in emissions by the coalition may be partially or even fully offset by countries outside the coalition. Contrary to this view, Copeland and Taylor (2005) show that leakage may be negative if one takes into account the effects of a change in emissions on world prices and if the environment is a normal good, provided that a cut in emissions increases the world price of dirty goods. Antweiler et al. (2001) show that there exists a positive or negative income effect from this price increase depending on whether the country is a dirty good exporter or importer. This effect can strongly reduce leakage in case of a local pollutant.

Both the CGE literature and the theoretical literature generally either ignore technological progress or model it as an exogenous process. An exception to this

¹Carbon dioxide (CO₂) is the greenhouse gas with the highest global warming potential since it is both emitted in large amounts and has a low decay rate. Therefore most of the economics literature on climate policy focuses on CO₂. In this paper, we call the global pollutant carbon or carbondioxide as well, although our analysis applies to any other greenhouse pollutant.
is Di Maria and Smulders (2004), who study the pollution haven hypothesis with a North-South model. They allow entrepreneurs to target new technologies to either clean or dirty goods but only the North engages in innovation. In addition they look at optimal environmental policy when the pollutant is strictly local.

The aim of our paper is to study the degree of leakage when international trade affects not only the rate but also the direction of technical change. Contrary to Di Maria and Smulders (2004), we assume economies that are as symmetric as possible. Furthermore we abstract from the effect of environmental quality on utility, such as to focus on the effects of free trade and directed technological change on carbon leakage. Consequently we complement the existing literature in several respects. In the first place we are interested in technical change which is directed. When entrepreneurs are allowed to aim new technologies to the clean or dirty goods sector, countries have an additional instrument to react to a CO$_2$ emission constraint. Second, we study the problem of carbon leakage when a technologically advanced country is outside the coalition, while most of the CGE literature assumes that all developed countries impose the same climate policy. More strongly we assume that our two countries are perfectly symmetric except for their environmental policy. This allows us to focus on the different innovation incentives for coalition and non-coalition countries. Third, the use of an analytical model rather than a CGE model allows us to conveniently disentangle the pure effects arising from trade from the effects of the induced technical change. Finally, and for the same reason, we can successfully highlight the role of the elasticity of substitution between factors of production in determining the direction of technical change and, in the ultimate instance, the pollution outcomes of the model.

The rest of the paper proceeds as follows: After we introduce the model in Section 2, we study the baseline model, without environmental policy in Section 3. Successively, we describe the changes introduced in the model by unilateral environmental policy (Section 4), and we discuss our results in Section 5. We conclude in Section 6.

2 The Model

Our economy consists of two countries that are completely symmetric. With the exception of environmental policy, which is introduced in section 4, the two regions are identical: consumers have the same preferences, and producers the same production possibilities. As long as no environmental policy is implemented, the decisions of the agents in each region will also be identical. In what follows, we only focus on a situation of (potential) free trade, noting that, as long as the two regions are identical, there will be no actual scope for trade.$^2$ In this section we describe the model for one of the two countries.

We start with preferences and assume that both countries admit a represen-
tative consumer who derives her utility from consumption only. We focus on a standard CRRA utility function, so that the infinitely lived consumer has a lifetime utility expressed as

\[ U(C(t)) = \int_0^\infty \frac{(C(t))^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt, \]  

where \( \rho \) is the rate of time preference and \( \theta \) is the coefficient of relative risk aversion or the inverse of the intertemporal elasticity of substitution.  

Each consumer maximizes the present value of her life-time utility subject to the following economy-wide budget constraint:

\[ C + M + R \leq Y \equiv \left[ Y_E^{\frac{1}{\varepsilon}} + Y_L^{\frac{1}{\varepsilon}} \right]^{\frac{\varepsilon}{1-\varepsilon}}, \]  

where \( M \) is total investment (expenditure in machines) and \( R \) is the total amount of research (R&D) expenditure. The production function in (2) shows that final output is obtained as a CES aggregate of two (intermediate) goods, \( Y_E \) and \( Y_L \), with an elasticity of substitution equal to \( \varepsilon \in (0, \infty) \). The two intermediate goods \( Y_E \) and \( Y_L \) are intensive in energy inputs and labour inputs respectively. Expression (2) moreover, requires that consumption, investment and R&D expenditure all come out of the output aggregate. 

The energy intensive good (\( Y_E \)) is produced using energy and a set of differentiated machines. The range of machines that can be used to complement energy is indicated by \( N_E \). The labour-intensive good (\( Y_L \)) is produced using labour and a different set of machines, whose range is \( N_L \). Each type of machine requires a blueprint invented by the R&D sector. We assume that machines developed to complement one factor of production cannot be usefully employed in the other sector so that technology does not diffuse across sectors. The production functions for the intermediate goods are as follows:

\[ Y_E = \frac{1}{1-\beta} \left( \int_0^{N_E} k_E(i)^{(1-\beta)} di \right) E^\beta, \]  

and

\[ Y_L = \frac{1}{1-\beta} \left( \int_0^{N_L} k_L(i)^{(1-\beta)} dt \right) L^\beta. \]  

For a given state of the technology, that is for given \( N_L \) and \( N_E \), both (3) and (4) exhibit constant returns to scale. However the returns will be increasing once \( N_L \) and \( N_E \) grow over time. The expansion of the range of available machines thus determines sustained growth in the long run. 

We assume blueprints to be freely tradable, hence producers in each country can use all machines globally available for their sector, and \( N_L \) and \( N_E \) represent global levels of technology. 

The total amount of labour available each period is equal to \( \bar{L} \). Furthermore energy has to be produced and its production requires some input of labour. Labour

\[ \text{From now on, we will suppress time arguments in order to simplify notation as long as it generates no confusion.} \]
can thus be either directly devoted to the production of the intermediate good \( Y_L \) or employed in energy generation, such that the following market clearing condition holds at each point in time:

\[
\overline{L} = L_L + L_E,
\]

where \( L_E \) is the amount of labour in energy production. Energy generation takes place according to the following production function,

\[
E = L_E^\phi,
\]

where \( \phi < 1 \). We assume that emissions \( Z \) are proportional to the amount of energy that is generated, so that

\[
Z = \zeta E;
\]

this implies that setting a quantitative limit on the amount of emissions allowed, is equivalent to imposing a cap on the production of energy. Indeed, when \( Z^* \) is the maximum amount of emissions permitted in each period, energy production must satisfy: \( E \leq Z^*/\zeta \equiv E^c \).

The last part of the model consists of the R&D sector. We are especially interested in the effect of directed technical change on carbon leakage compared with the effect of “traditional” endogenous growth. That is, what happens to leakage when technology levels in the clean and dirty goods sector (\( N_L \) and \( N_E \) respectively) are allowed to develop differently, compared to the case where they always grow at the same rate? Therefore with directed technical change we allow innovators to invest in the sector that gives the highest profits. Here we follow the literature on directed technical change (see Acemoglu (2002), for example). Moreover we model innovation assuming that only the final good is used in generating innovations, which is often referred to as a lab-equipment specification, following the taxonomy proposed by Rivera-Batiz and Romer (1991). The development of new types of machines takes place according to the following production functions:

\[
\dot{N}_E = \nu R_E, \quad (8)
\]

\[
\dot{N}_L = \nu R_L. \quad (9)
\]

These differential equations imply that each unit of the final good invested in R&D in each sector, generates an amount of new innovations equal to \( \nu \).

In those parts of the paper where we do not allow for directed technical change, we assume that technical change is endogenous, in that it is driven by profit incentives, but that the composition of technology across the two sectors cannot be changed over time. Out of a common pool of R&D expenditures, each period a constant share \( \gamma = N_E/(N_E + NL) \) of innovations turns out to be only applicable.

\[\text{We use a dot on a variable to indicate the time derivative, i.e. } \dot{x} = dx/dt.\]

\[\text{Following the path of perfect symmetry between the two countries, we assume that the productivity of research is the same in both countries. In addition we assume that research is equally productive in both sectors.}\]
to the energy sector, while a share $1 - \gamma$ will be applicable to the labour-intensive sector. This implies that innovators cannot direct their efforts to one specific factor. The cost of R&D will be a weighted average of the costs in each sector, but since we have assumed that R&D is equally productive in both sectors, it will still coincide with $1/\nu$. Our modelling of R&D implies that research is a costly process and that the costs incurred for the development of new machines varieties are sunk. As a consequence, only innovators who expect to wield some monopoly power in the future will actually engage in R&D activities. This requires that each innovator trusts that her right to exploit her innovations will be enforced. That is, she assumes that she can file a patent and that patents will be perfectly enforced, both within and across countries. In terms of the model, this means that each innovation only takes place once, so that there is no overlap in technology development in the two countries.

3 The equilibrium without environmental policy

Our interest lies in analyzing the effects of an asymmetric climate policy measure on the equilibrium level of pollution in our model. In particular, we want to contrast what happens when the direction (and not only the amount) of technical change is endogenous, to a more standard situation, where this additional dimension is not considered. In order to carry out our analysis, we need to analyze the effects of environmental policy starting from a common baseline. We take this baseline to be the long-run equilibrium of the model presented in the previous section. In this section we describe this equilibrium in consecutive steps. We first take the composition of technology, that is $N_E/N_L$, as given and derive a condition that describes the equilibrium on the goods market. Successively, we analyze the process of technical change and derive the condition that guarantees that, along the balanced growth path of our economy, both types of innovation occur at the same rate.

3.1 Equilibrium on the goods market

Here we discuss the equilibrium on the goods market when technology is given. We start by noting that the market for the final good is perfectly competitive, thus, the necessary condition for the optimal demand for the labour- and energy-intensive goods is that the marginal product of intermediate goods equals its price. In relative terms this gives us

$$\frac{Y^d_E}{Y^d_L} = \left( \frac{p_E}{p_L} \right)^{-\epsilon},$$

where we introduced a $d$ superscript to indicate demand and avoid confusion with supply, see (3) and (4). Free trade ensures that all prices will be equalized between the two regions, so throughout the paper prices indicate international ones. We

\footnote{Note that we could not make this assumption if we were studying a North-South model.}
choose the price of the final good as the numeraire, so that the following relation between the prices of the two intermediate goods holds:

\[
\left(p_{1}^{1-\varepsilon} + p_{1}^{1-\varepsilon}\right)^{1/(1-\varepsilon)} = 1. \tag{11}
\]

Producers of the intermediate good \( j \) (with \( j = E, L \)) maximize profits taking prices and technology as given. In particular, they choose the amount of inputs, taking as given the price of their output \( p_{j} \), that of their primary input \( w_{j} \) and those of the machines they use \( p_{k_{j}(i)} \) for a machine of type \( i \) complementing factor \( j \). In addition they take the range of available machines, \( N_{j} \), as given.

From the first-order conditions with respect to each type of machine \( k_{E}(i) \) we can derive the global demand for a machine of type \( i \) in each sector:

\[
k_{E}(i) = \left( \frac{p_{E}}{p_{k_{E}(i)}} \right)^{1/\beta} E \quad \text{and} \quad k_{L}(i) = \left( \frac{p_{L}}{p_{k_{L}(i)}} \right)^{1/\beta} L, \tag{12}
\]

where \( E^{w} \) is the worldwide amount of energy produced and \( L^{y} \) is the global amount of labour in the production of the labour intensive good \( Y_{L} \). By the same token, from the first-order condition with respect to the primary inputs, we can derive the (inverse) global demand for energy and labour,

\[
w_{E} = \frac{\beta}{1-\beta} p_{E} \left( \int_{0}^{N_{E}} k_{E}(i)^{(1-\beta)} di \right)^{E^{\beta-1}}. \tag{13}
\]

\[
w_{L} = \frac{\beta}{1-\beta} p_{L} \left( \int_{0}^{N_{L}} k_{L}(i)^{(1-\beta)} di \right)^{L^{\beta-1}}. \tag{14}
\]

Each machine type \( i \) is invented by the R&D sector. Since intellectual property is perfectly protected, the developer of a new design holds the patent for the unique blueprint. To simplify our analysis, we assume that the holder of a patent can license production to one producer in each region. Consequently, local producers will produce for the local market and act as monopolists there. To derive the price of each type of machines, consider that each technology monopolist will try to maximize her profits subject to the appropriate demand function in (12). We assume that the production of machines in both sectors entails a constant marginal cost equal to \( \omega \). When faced with an iso-elastic demand function, each monopolistic producer will set her price as a constant mark-up over marginal cost, that is \( p_{k_{j}(i)} = \omega(1-\beta) \) for each machine in either sector. Letting \( \omega = 1-\beta \), for convenience, we get that the price of machines in both sectors is equal to 1. Notice that machines are equally productive in production and all command the same cost. Thus, the amount of each machine used in sectorial production will be the same, \( k_{j} \), say. This symmetry greatly simplifies the structure of the sectorial production functions, in fact we may write \( \int_{0}^{N_{j}} k_{j}(i)^{(1-\beta)} di = N_{j} k_{j}^{(1-\beta)} \), for \( j = E, L \).
functions (3) and (4), we conveniently obtain two expressions which are linear in the level of the primary inputs. Taking ratios, we obtain an expression for relative supply of goods that depends on relative prices, relative (primary) factors supplies and relative technology,

$$Y = p^{(1-\beta)/\beta}SN.$$  \hspace{1cm} (15)

Here, and in the rest of the paper, we define variables without a subscript as ratios, with the convention that the variables in the numerator refer to the energy sector $E$. We refer to $N$ as the (global) technology ratio and we let $S \equiv E/L_L$.

Equalling relative supply and relative demand, from (10), yields the market clearing relative price for given technology:

$$p = (NS)^{-\beta/\sigma}.$$  \hspace{1cm} (16)

In this expression, we define $\sigma \equiv 1 + (\varepsilon - 1)\beta$ as the (derived) elasticity of substitution between labour and energy in final goods production.\(^9\)

From (16) we see that a higher level of technology in the dirty goods sector, or a higher relative supply of energy decreases the relative price of the dirty good.

The structure of the model enabled us to reduce the dimensions of the problem. It is clear from (15) and (16) that we can analyze our model in terms of just two (composite or relative) variables, namely the ratio between factors, $S$, and the technology ratio $N$. In the rest of this section we derive the condition for the equilibrium on the goods market, for given technology. In the following section we focus on technical change and derive the equilibrium condition in the R&D sector.

Substituting the machine demands, (12), into (13) and (14) and letting machine prices equal 1, we obtain an expression for the relative factor rewards, as given by:

$$w \equiv \frac{w_E}{w_L} = p^{1/\beta}N.$$  \hspace{1cm} (17)

Using this and (16), we get the following expression for the relative factor rewards for given technology:

$$w = N^{(\sigma-1)/\sigma}S^{-1/\sigma}.$$  \hspace{1cm} (18)

The relative price of energy decreases with energy supply. The effect of the technology ratio $N$ however depends on whether labour and energy are gross substitutes ($\sigma > 1$) or gross complements ($\sigma < 1$).

To fully characterize the equilibrium on the goods market, for given technology, we need to determine the way in which labour is allocated between the two sectors. For this purpose, we use the last expression, together with the first-order condition for labour demand, coming from the maximization of profits in energy generation and the labour market equilibrium condition in (5).

An energy producer can produce her output employing units of labour (at a unit cost of $w_L$) and sell her output at the prevailing market price $w_E$. Her aim is to choose the amount of labour to employ so as to maximize her profits, subject

\(^9\)Indeed, after some manipulation, from (2), (3), (4), (11), and (16), it is possible to write total production in terms of primary factors and technology level as follows: $Y = [(N_L L_L)^{\sigma-1}/\sigma + (N_L E)^{\sigma-1}/\sigma]^{\sigma/(\sigma-1)}$. 

to the production function in (6), and taking prices as given. From this simple maximization problem we get the following first-order condition, which expresses the country’s demand for labour in energy production as a function of relative factor rewards:

$$w = \frac{1}{\phi L_E^{\bar{\phi}}}. \quad (19)$$

Using this expression and (18) yields,

$$N^{(\sigma-1)/\sigma} \left( S \right)^{-1/\sigma} = \phi^{-1} L_E^{1-\phi}, \quad (20)$$

which can be manipulated, using (5) and (6), to obtain the following implicit solution for the optimal amount of labour in energy production:

$$\phi^{-\sigma} N^{1-\sigma} L_E^{\phi(1-\sigma)+\sigma} + L_E = \bar{L}. \quad (21)$$

This expression fully characterize the long-run equilibrium of our model when the ratio of technology is given.

### 3.2 Equilibrium on the market for innovations

We now turn our attention to the process of directed technical change in the two countries. We know from (8) and (9) that in order to produce a new blueprint in any sector, technology monopolists have to invest an amount of money equal to $1/\nu$. Moreover, if we assume that technical change is directed, they also face a further choice in terms of which sector to aim their innovation efforts at. It is natural to conclude that they will invest in the sector which is expected to yield the highest rate of return. Any innovator knows that her profits will be given by the profits on the world market for her machine, in the symmetric case the potential market will be given by twice the size of the home market. Using (12), the instantaneous profits are given by the following expressions:

$$\pi_E = 2 \beta p_E^{1/\beta} E \quad \text{and} \quad \pi_L = 2 \beta p_L^{1/\beta} L_L. \quad (22)$$

Each potential innovator will be interested in the net present value of the stream of future profits that she expects to enjoy over time, rather than in instantaneous profits. Expressing this in standard dynamic programming equations one gets $r(t)V_j(t) - V_j(t) = \pi_j(t)$, where $V_j$ is the value of an innovation in sector $j = E, L$.

This expression relates the present discounted value of developing an innovation, $V_j$, to instantaneous profits and it allows for the flow of profits to change over time through the "capital gain" term $V_j$. Along the balanced growth path of the economy profits will not change over time, so $V_j$ must be zero.\(^{10}\) Moreover, since entry is free in the R&D sector, we know that the value of innovation cannot exceed its cost (see (8)) so that $V_j \leq 1/\nu$ in each sector. Along the balanced growth path both types of innovation must occur at the same time, so that $V_j = 1/\nu$ in

\(^{10}\)We define a balanced growth path as a situation in which prices are constant and $N_E$ and $N_L$ grow at the same constant rate.
both sectors. From this we can derive the following no-arbitrage equation for the research sector:

$$\pi_{E} \nu = \pi_{L} \nu,$$

(23)

which, after substituting for profits from (22), can be rearranged to read

$$p^{1/\beta} S = 1.$$

(24)

This no-arbitrage equation enables us to solve for the equilibrium level of the technology ratio $N$. Indeed, using the expression for relative prices in (16), we may solve (24) for $N$, obtaining the following expression for the balanced growth path equilibrium ratio of technology levels in the two sectors,

$$N = (S)^{\sigma - 1}.$$

(25)

From this expression we see that the effect of a decrease in energy on the direction of technical change, that is on whether $N$ increases or decreases, depends on the size of $\sigma$. When labour and energy are gross complements in final goods production the price effect in (22) outweighs the market size effect and a decrease in energy supply induces an increase in the range of energy complementary machines. However, when $\sigma > 1$ it induces an increase in the range of labour-complementary machines.

### 3.3 General equilibrium under directed technical change

In the previous two sections we derived the two equilibrium conditions for the goods market and for R&D activities, (21) and (25), respectively. Along these lines only one of the two markets is in equilibrium. Quite obviously, the general equilibrium for our model will be obtained when both markets are in equilibrium at the same time. Is it possible to show that at least one interior equilibrium exists for this model, provided that the elasticity of substitution $\sigma$ is not too large.

More specifically, we are able to show that an interior equilibrium always exists for $\sigma < (1 + \phi)/\phi$. Moreover, we can show that the dynamics of the system are such that only one of the possible equilibria is stable, specifically the one that is characterized by the condition that the locus describing the goods market equilibrium is steeper than the one depicting the R&D sector.

Having established that an interior stable equilibrium exists, we can go on to characterize the general equilibrium of our model. Substituting (25) into (20) yields the following implicit solution for labour allocated to the energy sector along a balanced growth path with directed technological change:

$$\left(\frac{S}{\sigma} \right)^{\sigma - 2} = \phi^{-1} L_{E}^{1-\phi}.$$

(26)

The details of this proof are available from the authors upon request. Intuitively, however, when the two intermediate goods are very easily substitutable, production will tend to concentrate in only one of them, specifically in the sector where labour is more productive, that is in the $Y_{E}$ sector. The higher $\phi$, the faster the insurgence of decreasing returns in the energy sector, the smaller the $\sigma$ sufficient to obtain a corner. For the case where technical change is undirected, the no-arbitrage condition (25) is substituted by an expression setting $N$ at some exogenous value. In this case it is possible to show that an interior equilibrium always exists for $N \in (0, +\infty)$. 

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This expression is the directed technical change equivalent of (20) and can be rewritten to look like (21) as:

$$\frac{1}{\sigma - 2} \frac{\phi}{L_E^2 \sigma - 1} \frac{1}{\sigma - 2} + \frac{1}{\sigma - 2} = 1.$$  \hspace{1cm} (27)

The above expression summarizes the long-run equilibrium of our model as it solves implicitly for the optimal level of $L_E$, from which we can immediately obtain $E$, $L_I$ and $N$.

Unfortunately, it is impossible to solve (27) explicitly. Numerical solutions are, on the other hand, straightforward.\footnote{We solve for $L_E$ in the long-run equilibrium of the model as described by equation (27). The baseline parameters value that we use for the simulations are as follows: $T = 5$, $\phi = 0.4$, and $\sigma \in (0, 3.5)$}

Solving the baseline model enables us to underline some characteristics of the solution. In the first place, the elasticity of substitution, $\sigma$, plays a key role in the solution. Intuitively one can imagine that the easier it is to substitute between factors, the less costly it is (in terms of final output) to move labour away from energy generation, and from the decreasing returns that prevails there. Hence the larger $\sigma$, the lower will be the share of labour employed in the energy sector, $L_E$. Indeed, this is what we obtain from our numerical solutions, as can be clearly seen from the lowest line in Figure 1, which represents $L_E$ in the baseline model.\footnote{For the same reason one would expect that the higher $\phi$, i.e. the slower the decrease in labour productivity in the energy sector, the higher $L_E$ for each level of $\sigma$. Indeed, this is confirmed by our robustness checks.}

Apart from the intersectoral labour allocation, the other relevant variable in this framework is the technology ratio $N$. First of all, notice from (25) that, when $\sigma = 1$, that is in the Cobb-Douglas case, $N = 1$. In this case technical change is neutral, i.e. in the long run equilibrium not only the growth rate, but also the level of technology (the number of blueprints) in the two sectors will be the same. Outside of this situation, the development of new technology will benefit the factor that is “scarcer” in economic terms. When $\sigma < 1$ the scarcer factor is energy: it is more profitable to complement energy rather than labour, due to the decreasing returns in the energy sector. As a consequence, the $N$-ratio is larger than one. When $\sigma > 1$, investing in the less abundant and less productive factor is not so appealing. The larger $\sigma$, the larger the incentives to develop innovation complementing labour, hence the smaller $N$. All this is confirmed by Figure 3, where the solid line represents this baseline case.

## 4 Introducing the emission constraint

We now start our analysis of the effects of unilateral climate policy and we proceed introducing an asymmetric emission cap in our analysis. We assume that one of the two countries introduces a cap that is be binding in the steady state and along the transition path. We denote the variables for the constrained country with a $c$-superscript, to distinguish from the case where both countries are unconstrained. Variables for the unconstrained region are denoted with a $u$-superscript.
4.1 Goods market equilibrium

We first remark that in the constrained country energy supply and pollution will be determined by the binding cap. This directly determines the amount of labour in energy production as \( L_E = (E^c)_1^1/\phi \). Since we assume that the cap is binding, the allocation of labour across sectors will be sub-optimal in the constrained country.

In the unconstrained country, however, the split of labour between \( Y_L \) and \( Y_E \) will still be determined optimally, making use of the constrained economy homologue of rule (20). If we abstract from the effects of the cap on technology, the only change induced by the cap will be in raising the international prices of the energy intensive goods. Indeed, in this case the expression for the relative supply of intermediate goods (15) becomes:

\[
Y_s = p(1-\beta)/\beta S^{w}N. \quad (28)
\]

where now \( S^w = E^c + E^u \) is the constrained global factor ratio. As before, solving for the price level and substituting it into the appropriate first-order condition, we are able to derive the goods market equilibrium condition,

\[
N^{(\sigma-1)/\sigma} (S^{w})^{-1/\sigma} = \phi^{-1} (L^u_E)^{1-\phi}; \quad (29)
\]

substituting the definition of \( S^w \) and rearranging, we can rewrite this expression as:

\[
\phi^{-\sigma}N^{1-\sigma} \left[ (L^c_E)^{\phi} (L^u_E)^{\sigma(1-\phi)} + (L^u_E)^{\phi(1-\sigma)+\sigma} \right] + L^c_E + L^u_E = 2\bar{L}. \quad (30)
\]

When the technology ratio \( N \) is given, the model can be considered either a static model, or the steady state of a "traditional" endogenous growth model where \( N_E \) and \( N_L \) grow at the same rate. We now turn our attention to the consequences of the cap on the process of technical progress. We first look at its effects on the level of relative technology, \( N \), and later we present the steady state of this constrained model under directed technical change.

4.2 Equilibrium on the market for innovations

In terms of the direction of technical change, the introduction of the cap changes the potential market for would-be innovators. The instantaneous profit functions need to be modified to reflect this change,

\[
\pi_E = \beta p^{1/\beta}_E (E^c + E^u) \quad \text{and} \quad \pi_L = \beta p^{1/\beta}_L (L^c_L + L^u_L). \quad (31)
\]

Clearly when expected profits change, also the no-arbitrage equation will change. Taking the ratio of profits and setting the result to 1 yields the new no-arbitrage expression:

\[
p^{1/\beta} S^{w} = 1. \quad (32)
\]

Finally, the new long-run technology ratio will be obtained solving this expression for prices, plugging the result into the homologous of (16) and deriving \( N \). This gives us:

\[
N = \left( \frac{E^c + E^u}{L^c_L + L^u_L} \right)^{\sigma-1}. \quad (33)
\]
4.3 General equilibrium under directed technical change

Just as in section 3.3, we start this section by discussing the existence and stability of a general equilibrium. Making use of (30) and (33), we can derive sufficient conditions for the existence of the equilibrium. Yet, it is possible to show that in this case, whenever an equilibrium exists it will be interior. Once again the stability of equilibria requires that the goods market line be steeper than the R&D condition.

We can now complete the presentation of the model under an asymmetric cap, by making use of (33) to substitute for the exogenous \( N \) in (29), which gives

\[
\left( \frac{E^c + E^u}{L^c + L^u} \right)^{\sigma - 2} = \phi^{-1} (L^c)^{1-\phi}.
\] (34)

Remembering the definition of \( E \), and making use of the labour market equilibrium condition, we can finally obtain the condition which provides the implicit solution for \( L^u/E \),

\[
\phi^{1/(\sigma - 2)} \left[ (L^c)^(\phi/(\sigma - 2)) + (L^u)^(\phi(\sigma - 1)/((\sigma - 1)/(\sigma - 2))] + L^c + L^u = 2L. \] (35)

This expression solves for the amount of labour in energy production in the unconstrained region, when the other region faces a binding emission constraint, under directed technical change.

5 Unilateral climate policy and carbon leakage

Now that we have presented implicit solutions for the amount of labour in energy production in both the baseline model and in the constrained one, we are ready to study carbon leakage when entrepreneurs can or cannot aim new inventions at a certain sector. We are particularly interested in comparing the amount of leakage that takes place in the two different regimes. To do so we need to start from a comparable baseline. As already mentioned, we take this benchmark to be the long-run equilibrium of the unconstrained model, which we described in section 3.3. This initial point is (fully) characterized by a certain amount of labour devoted to the energy sector \( L_E \), and by the composition of technology, the ratio \( N \). In this starting equilibrium the two countries are completely symmetric. For the undirected technical change case we set \( N \) equal to the (endogenously determined) bias prevailing in the baseline case.

5.1 Carbon Leakage under undirected technical change

Carbon leakage occurs when the unconstrained region increases its emissions in reaction to a reduction in the other country’s emissions. In terms of our model, there is carbon leakage when

\[
L^u_E > L_E. \] (36)

Intuitively, it would seem clear that there should always be some carbon leakage: when a country exogenously reduces its supply of energy by introducing a limit
to the amount of emissions, the obvious consequence is that the energy intensive
good becomes scarcer on the constrained market, giving rise to an increase in its
relative price. This increase in the relative domestic price creates the necessary
scope for trade: the unconstrained economy now enjoys a comparative advantage
in the production of the dirty good, and is led to expand its production of the
energy intensive good. As a consequence \( L_E^u \), and emissions, increase.

This reasoning makes perfect sense in a world in which the relative technology
is given, but this does not necessarily hold under directed technical change. We
will return to this point later, for the time being we abstract from this and we show
that indeed leakage is always positive in the undirected technical change case.

Consider what happens when a cap is introduced and the bias of technology
is given. First, take the ratio of (20) and (29) and rearrange it to find:

\[
\left( \frac{2L_E^\phi}{(L_E^\phi + (L_E^\phi))} \right)^{-1/\sigma} \left( \frac{2L_E^\phi - L_E^c - L_E^u}{2L_E - 2L_E^c} \right)^{-1/\sigma} = \left( \frac{L_E}{L_E^u} \right)^{1-\phi}. \tag{37}
\]

Now, assume that the leakage rate is non-positive, i.e. \( L_E^u \leq L_E \). The right hand
side of the above expression is larger than or equal to one. However, in this case
both terms on the left hand side are necessarily smaller than one, and we have a
contradiction. Hence, we have shown that leakage is always positive in the case
of undirected technical change.

Numerical simulations provide further evidence on this. Figure 1 plots the
amount of labour allocated to energy production for the different versions on the
model. The lowest dashed line refers to the baseline model, i.e. it plots \( L_E \) in our
notation. The other two lines plot \( L_E^u \) for the undirected technical change case (the
topmost line) and the directed technical change one. Comparing the baseline to
the undirected technical change, we can immediately see that, upon introduction
of the cap, the amount of labour in the energy sector (and hence of pollution)
increases for any given value of \( \sigma \). As discussed above, this result is consistent
with standard trade theory. Our numerical solutions also show that the absolute
level of pollution decreases with the elasticity of substitution between productive
factors.

Now that we have shown that carbon leakage is positive in this case, the ques-
tion arise, whether it can fully off-set the reduction in emissions in the constrained
country. Unfortunately, we are unable to derive any analytical conclusion on this
point from the complete model. To shed some light on this, though, we resort to
a linearization exercise. As shown in the Appendix, when we log-linearize the
model for given \( N \), we are able to show that global pollution will decrease fol-
lowing a marginal tightening of the cap. Thus, the induced increase in pollution
on the part of the unconstrained country will never exceed the decrease brought
about by the cap.\(^{14}\) In other words, leakage will always be less than 100%.

The result obtained with the linearized version of the model is strictly local. To
generalize it, we make use once more of our simulations. To conveniently express
the relative amount of induced leakage, we define the leakage rate as the ratio

\(^{14}\)In the Appendix we present the linearized model, this result is derived in section A.1.
between the induced increase in pollution in the unconstrained country and, the pollution reduction in the constrained region, that is:

$$\frac{E^u - E}{E - E^c}.$$  \hspace{1cm} (38)

Figure 2 presents the leakage rates resulting from our simulations, for increasing values of $\sigma$, for both the undirected and directed technical change scenarios.

The solid line represents the leakage rates under undirected technical change. Just as in the linearized case, the leakage rate is never larger than 1, implying that the induced increase in pollution can never fully off-set the effects of the unilateral policy. Moreover, just like the absolute level of leakage, also the leakage rates are smaller, the larger the elasticity $\sigma$. This result makes perfect economic sense: if the intermediate goods are poor substitutes, the introduction of the cap will induce, \textit{ceteris paribus}, a relatively larger increase in international prices, and this will cause a more pronounced expansion in the production of the “rationed” good.

### 5.2 Carbon leakage under directed technical change

When allowing for directed technical change we give the economy an additional instrument to cope with the consequences of the binding cap in the constrained country. For example, the unconstrained region may now increase the productivity of energy in such a way that, although it faces an increased demand for the energy intensive good, it uses less energy to satisfy the demand. This line of reasoning suggests that the effects of unilateral climate policy are not so easy to predict in a directed technical change setting. Indeed, it might even be the case
that introducing a cap could lead to negative leakage, i.e. to a situation in which the unconstrained region actually uses less energy than before and hence reduces its own emissions. To see whether this theory holds we follow the same steps as in the previous section, using (27) and (35). This gives

\[
\left( \frac{2L_E^\phi}{(L_E^c)^\phi + (L_E^u)^\phi} \right)^{\sigma-2} \left( \frac{2T_c - L_E^c - L_E^u}{2T - 2L_E} \right)^{\sigma-2} = \left( \frac{L_E}{L_E^c} \right)^{1-\phi}. \tag{39}
\]

As expected, things are less straightforward in this situation, since the sign of the exponents can change. We first focus on the case where \(\sigma < 2\). In this case the proof for positive leakage is exactly as for the undirected technical change case. Indeed, we conclude that even with directed technical change, unilateral climate policy will always lead to some positive leakage as long as \(\sigma < 2\).

Consider now the \(\sigma = 2\) case. In this case both terms on the left hand side are equal to one and it is apparent that \(L_E^c = L_E\), and we have exactly zero leakage: the unconstrained region does not change emissions.

What we have shown so far would seem to indicate that as the possibility of substitution between factors increases, the degree of leakage decreases. Unfortunately, we cannot prove it for the complete model. Following our intuition, indeed, we would expect that when \(\sigma > 2\), \(L_E^c < L_E\). Assume that leakage is indeed negative. Then, both sides of (39) are larger than one so this case appears consistent with our model. We can go further and rule out the case in which leakage is zero, i.e. \(L_E^c = L_E\), because in this case the right hand side equals one, while the left hand side is larger than one and we have a contradiction. Hence zero leakage is impossible when \(\sigma > 2\). We are left with the possibility that leakage is positive.
Here we have two choices, we first look at the case where \( L_u^E \leq 2L_E - L_c^E \), and we notice that the first term on the left is larger than one and the second term is larger than or equal to one, hence we have another contradiction and another case we can rule out. To complete our proof we would need to be able to also exclude the case where \( L_u^E > 2L_E - L_c^E \). In this case the second term on the left-hand side is smaller than one, but the first term could be smaller than one as well for large values of emissions from the unconstrained country. So, although the sudden jump in leakage that this kind of result seems to imply makes us suspicious as refers to its optimality, we cannot conclude that leakage will be always negative when \( \sigma > 2 \).

However, we can look at the linearized version of the model again. As detailed in the Appendix, we can positively show that in the linearized model leakage will be negative for \( \sigma > 2 \). The argument behind this result is that a situation in which leakage is positive when \( \sigma > 2 \) can be ruled out since it violates the condition for the stability of the equilibrium.\(^{15}\)

Numerical simulations support the result of the linearized model. In Figure 1, the light dashed line, which plots \( L_u^E \) for the directed technical change model, stays above the dark dashed line (the baseline simulation) for values of \( \sigma < 2 \), thus leakage is positive. At \( \sigma = 2 \) the lines cross, indicating null leakage. Finally, when \( \sigma > 2 \) the unconstrained country reduces rather than expand the units of labour in the energy sector compared to the baseline situation: in this case leakage is negative.

### 5.3 Comparing leakage in the two regimes

As noted before, introducing directed technical change gives the economy an additional instrument to react to a binding emission constraint. We may expect that this additional degree of freedom will lead to a reduction in emissions by the unconstrained country, and hence in the degree of carbon leakage, compared to the case of undirected technical change. Indeed, we have just seen that leakage may be negative for large values of \( \sigma \) under directed technical change.

In this section we compare carbon leakage across technology regimes, and interpret the outcomes form the different versions of the model in light of the theory of directed technical change. Before we turn to comparing directed and undirected technical change, we want to mention one special case of the general model in which directed and undirected technical change are equivalent. When \( \sigma = 1 \), our CES specification in (2) reduces to a Cobb-Douglas production function. As is well known from the growth literature, in this case technical change will always be neutral to the inputs concerned. This can be seen from the fact that in this case (25) and (33) coincide and are equal to 1, showing that the technology levels \( N_E \) and \( N_L \) are the same in the long-run equilibrium in the Cobb-Douglas case. Having said this, it is clear that when \( \sigma = 1 \) (30) and (35) are the same and hence the degree of leakage will be the same across technology regimes.

Comparing the two versions of the model, boils down to an exercise in compar-

\(^{15}\)This result is derived in section A.2 of the Appendix.
ative statics for a maximization problem, when one additional binding constraint is added. We can address this problem in the light of the Le Chatelier principle. Indeed, taking the total differential of (30) and rearranging, we can write:

\[
\left. \frac{\partial L_E}{\partial L^u_E} \right|_{DTC} = \left. \frac{\partial L_E}{\partial L^u_E} \right|_{UTC} + \frac{\partial L^u_E}{\partial N} \frac{dN}{dL^c_E} (40)
\]

We can interpret this expression as saying that the overall effect of the cap can be decomposed in a **trade effect**, represented by the first term at the right-hand side, and a **technology effect**, the remaining term.

To understand the relationship between the outcomes under directed and undirected technical change, we need to sign the components of this equation. We start with the last part. Inspection of (33) informs us that \( \frac{dN}{dL^c_E} < 0 \) when \( \sigma < 1 \), and positive when \( \sigma > 1 \). To analyze \( \frac{\partial L^u_E}{\partial N} \), let us first focus on the case where \( \sigma < 1 \). If we consider expression (35), we see that since in this case when \( N \) increases, \( N^{1-\sigma} \) will also increase, \( L^u_E \) will have to decline to satisfy the equation, *ceteris paribus*. Thus \( \frac{\partial L^u_E}{\partial N} < 0 \), when factors are complements. The opposite will hold when factors are substitutes. We can conclude from this that, irrespective of the value of \( \sigma \), the last term on the right hand side of (40), and hence the technology effect is positive.

From Section 5.1, we know that the trade effect is negative, i.e. \( \frac{\partial L^u_E}{\partial L^c_E}|_{UTC} < 0 \), as carbon leakage will always be positive under undirected technical change. Since the technology effect is instead positive, we can conclude that under directed technical change leakage will always be smaller than under undirected technical change.

Going back to our results from 5.2, we can assess the relative strength of the trade and technology effects for different levels of \( \sigma \). Indeed, we know that, as long as \( \sigma < 2 \), carbon leakage will be positive also under directed technical change, that is, the technology effect will be smaller (in absolute value) than the trade effect and will thus not induce a reversal in the sign of the left-hand side of (40). When \( \sigma > 2 \), instead, the induced technical change will be so strong that it will actually lead the unconstrained country to reduce its polluting emissions and the sign of \( \frac{\partial L^u_E}{\partial L^c_E}|_{DTC} \) become positive. Carbon leakage is positive in this case.

In the rest of this section we will discuss the reasons why allowing for directed technical change leads to such different results. We will proceed in two steps. We will first show how the composition of technology is affected by the introduction of the cap for different values of the elasticity \( \sigma \). Successively we will address the interaction between changes in \( N \), the level of \( \sigma \), and the relative factor productivity, to understand the labour allocation decisions of firms in the unconstrained country.

The composition of technology evolves according to the relative profitability of R&D in the different sectors: more machines will be developed in the sector where the expected profits from innovation are higher. Thus \( N \) will increase whenever relative profits \( \pi \) rise. Recalling the profit equations from (31), and taking the ratio, one gets \( \pi = \frac{p^{1/\beta}}{S^w} \). It is clear that the final effect of introducing a cap (i.e. a change in \( S^w \)) on relative profits will depend both on the change in relative prices, which makes the prospective innovation more valuable, and on the change
in the relative market size, which reduces the potential market. Whether the positive price effect or the negative market size effect dominates, depends on the price elasticity, which is related to $\sigma$.\footnote{Recall that relative prices are given by $p = (NS^u)^{-\beta/\sigma}$ in the constrained model, and that the leakage rate is always less than 100%.
} Since in the long-run equilibrium the technology ratio is given by (33), we can conclude that whenever $\sigma < 1$, the price effect dominates and the introduction of a cap will induce an increase in $N$, when $\sigma > 1$, on the other hand, the market size effect dominates and $N$ decreases. This yields a situation such as the one plotted in Figure 3, where the dashed line represents the ratio of technology under directed technical change, while the solid depicts the baseline (and hence the undirected technical change case).

These differences in the composition of technology, $N$, across versions of the model, determine the differences in the relative productivity of energy and labour, which ultimately drive the results of this section. The relative factor productivity for the constrained model can be written as:

$$w = N^{(\sigma-1)/\sigma} (S^u)^{-1/\sigma}. \quad (41)$$

From this it is clear that, for given $N$, the effect of the cap is unambiguously to increase the relative productivity of energy, and thus to increase pollution in the unconstrained country: leakage is always positive.

Once we allow $N$ to change in response to economic incentives, things become slightly more complicated. From (41), we can see how the effect of a change in $N$ on $w$ depends on $\sigma$. We have seen that, relative to the baseline case, $N$ is higher for $\sigma < 1$ after the cap is introduced. This means that $N^{(\sigma-1)/\sigma}$ is lower than before,
thus the increase in productivity induced by the cap is counteracted by the change in technology. Indeed, the same result can be obtained for $\sigma > 1$, in which case $N$ is below the baseline effect, and the first term at the right-hand side of (41) is lower as well. This shows again that the induced change in technology ($N^{(\sigma-1)/\sigma}$) will mitigate the trade effect (the second term at the right-hand side).

To determine which of the two effects will be stronger, let us substitute (33) in (41), to get the general equilibrium result:

$$w = (S^w)^{-2}. \quad (42)$$

Clearly, as long as $\sigma < 2$ the decrease in the factor ratio induced by the cap will lead to an increase in the relative productivity of energy. Thus leakage will be positive but lower than under undirected technical change. When $\sigma > 2$ instead, the decrease in $S^w$ will reduce the relative productivity of energy. The change in the technology ratio is so strong that it will more than compensate for the trade effect, explaining the result that $L^u_E$ is actually lower after the introduction of the cap under directed technical change and large $\sigma$.

6 Conclusions

The refusal of the United States to ratify the Kyoto Protocol is seen as a serious threat to the Protocol’s effectiveness. If a coalition of technologically advanced (and hence fossil-fuel dependent) economies decides to voluntarily reduce its emissions of carbon dioxide, this will increase the price of dirty goods within this coalition while the world price of fossil fuels may fall due to the lower demand from the coalition. Hence unconstrained countries, such as the US, can produce dirty goods at a lower cost and export them to coalition members, thereby offsetting the decrease in emissions from the coalition.

This paper has studied the problem of carbon leakage when a technologically advanced country is outside the coalition, by focusing on the effect of directed technical change. That is we investigated how, compared with a model of exogenous or “traditional” endogenous technical change, allowing technology levels in the clean and dirty goods sector to develop differently affected energy production in the unconstrained region. A very encouraging result from an environmental point of view, is that directed technical change may induce the unconstrained region to reduce its emissions in reaction to unilateral climate policy by the other region. More generally, with directed technical change the global amount of emissions is always less than (or, in the case of a Cobb-Douglas specification, equal to) it would be with undirected technical change. In addition unilateral climate policy will always be effective to some degree, as the leakage rate will always be less than 100%.

Of course, there are interesting ways to improve our model. Here we propose just two. In the first place, since we take policy to be exogenous we disregard the effect of income on climate policy which, as shown in the trade literature, may have important consequences. Furthermore one may model energy as a non-renewable resource, instead of being produced from labour.
A Appendix: the log-linearized model

In this appendix we (log-)linearize the model around the steady state and derive several results.

Using the goods market equilibrium condition (29), we get the corresponding linearized version, that is:

\[
(\sigma - 1) \hat{N} = [(1 - \phi) \sigma + \eta \phi + \nu] \hat{L}_E + \left[(1 - \eta) \phi + \nu \frac{L_E}{L_u} \right] \hat{L}_E,
\]

where a hat, \(\hat{\cdot}\), over a variable denotes a growth rate, and where we have used the following definitions:

\[
\eta \equiv \frac{(L_u^\phi)_\phi}{(L_u^\phi + (L_u^\phi)_\phi)} \in (0, 1), \quad \text{and} \quad \nu \equiv \frac{L_u^\phi}{2L_u^\phi - L_c^\phi - L_u^\phi}.
\]

When we linearize the equilibrium condition for the market for innovations, (33), we find:

\[
\hat{N} = (\sigma - 1) \left[(1 - \eta) \phi + \nu \frac{L_c^\phi}{L_c^\phi} \right] \hat{L}_E + (\sigma - 1) (\eta \phi + \nu) \hat{L}_u.
\]

The growth rates denote any marginal change in the respective variable. For example, a decrease in \(L_c^\phi\) (that is a \(\hat{L}_c^\phi < 0\)) from \(L_c^\phi = L_E^\phi\) would represent the introduction of a marginal emissions cap in the country, while a decrease from any \(L_c^\phi < L_E^\phi\) would represent any marginal tightening of an existing cap.

A.1 Undirected technical change

We now derive two results for the case of undirected technical change. In this case we only have to look at the goods market equilibrium expression (A.1).

We first prove that the introduction of a marginal cap in a country (or the marginal tightening of an existing cap) will lead to an increase in emissions by the other country. In other words: carbon leakage will be positive. To see this, rewrite (A.1) and set \(\hat{N}\) equal to zero (as \(N\) does not change with undirected technical change) to find:

\[
\frac{\hat{L}_u}{L_u^\phi} = -\frac{(1 - \eta) \phi + \nu \frac{L_c^\phi}{L_c^\phi}}{(1 - \phi) \sigma + \eta \phi + \nu} < 0.
\]

Carbon leakage will be positive when the unconstrained region increases its emissions \((\hat{L}_u^\phi > 0)\) in reaction to a decrease in emissions by the constrained region \((\hat{L}_c^\phi < 0)\). Hence, with undirected technical change, carbon leakage will always be positive after a marginal tightening of the cap.

Secondly, we show that carbon leakage will always be less than 100% with undirected technical change, that is unilateral climate policy is always effective to some extent. For this to be true, global emissions should not increase due to unilateral climate policy. In growth rates global emissions are given as: \(\hat{E}^w = \eta \phi \hat{L}_u^\phi + (1 - \eta) \phi \hat{L}_c^\phi 0.\) Using the result above, and after some manipulation we find:

\[
\frac{\hat{E}^w}{L_L^\phi} = \frac{(1 - \eta) \phi (1 - \phi) \sigma + \nu \phi \eta \frac{L_u^\phi - L_c^\phi}{L_c^\phi}}{(1 - \phi) \sigma + \eta \phi + \nu} > 0.
\]

This follows since the denominator and the first term in the numerator are positive, so the sign of (A.5) is determined by the last term in the numerator. This term will be larger.
than or equal to zero provided that \( L_u \geq L_c \), that is if emissions by the unconstrained country are larger than or equal to emissions by the constrained region. This condition always hold since, by symmetry, the cap cannot be binding for the unconstrained country if it is not binding for the other. Since the introduction of, or a tightening in climate policy implies \( \hat{L}_c < 0 \), we conclude that leakage will always be less than 100%.

A.2 Directed technical change

To see whether carbon leakage will always be positive in case of directed technical change, we have to substitute (A.3) into (A.1) and rewrite to find

\[
\frac{\hat{L}_u}{\hat{L}_c} = \frac{(\sigma - 2) \left( (1 - \eta) \phi + \nu \frac{\hat{L}_c}{\hat{L}_u} \right)}{(2 - \sigma) (\eta \phi + \nu) + 1 - \phi}.
\]

(A.6)

Carbon leakage will be positive when this expression is negative. From this expression it is easy to see that carbon leakage will be positive (that is, (A.6) will be negative) when \( \sigma < 2 \). When \( \sigma > 2 \) however, things are less apparent. In that case carbon leakage will be negative when the denominator is positive: \((2 - \sigma) (\eta \phi + \nu) + 1 - \phi > 0\). As this expression is also a necessary condition for a stable equilibrium in the model when \( \sigma > 2 \), we conclude that in this case leakage will be negative. Unilateral climate policy stimulates the unconstrained country to voluntarily decrease its emissions when substitution possibilities are large enough.

A.3 Directed technical change vs. undirected technical change

We are now ready to compare the leakage rates with directed and with undirected technical change. For this we take the ratio of (A.4) and (A.6) to find

\[
\left. \frac{\hat{L}_u}{\hat{L}_c} \right|_{UTC} / \left. \frac{\hat{L}_u}{\hat{L}_c} \right|_{DTC} = \frac{\eta \phi + \nu + (1 - \phi) (2 - \sigma)^{-1}}{\eta \phi + \nu + (1 - \phi) \sigma}.
\]

(A.7)

From this expression it is clear that, as argued in the main text, when \( \sigma = 1 \) both technology regimes give the same degree of carbon leakage. Moreover, when \( \sigma = 2 \) the expression will diverge to infinity (recall that in this case leakage is zero under directed technical change and thus the denominator is also zero). For all other values of \( \sigma \), we see that the degree of leakage will be larger in the model with undirected technical change. Indeed, the numerator of (A.7) is always larger than its denominator, since \( 1/(2 - \sigma) > \sigma \) or, more clearly, \((\sigma - 1)^2 > 0\). So, with the exception of the case where technology is Cobb-Douglas, the introduction of a marginal cap or a marginal tightening in an existing cap will always lead to a larger increase in emissions by the unconstrained region in the model without directed technical change, relative to the model with directed technical change.
References


